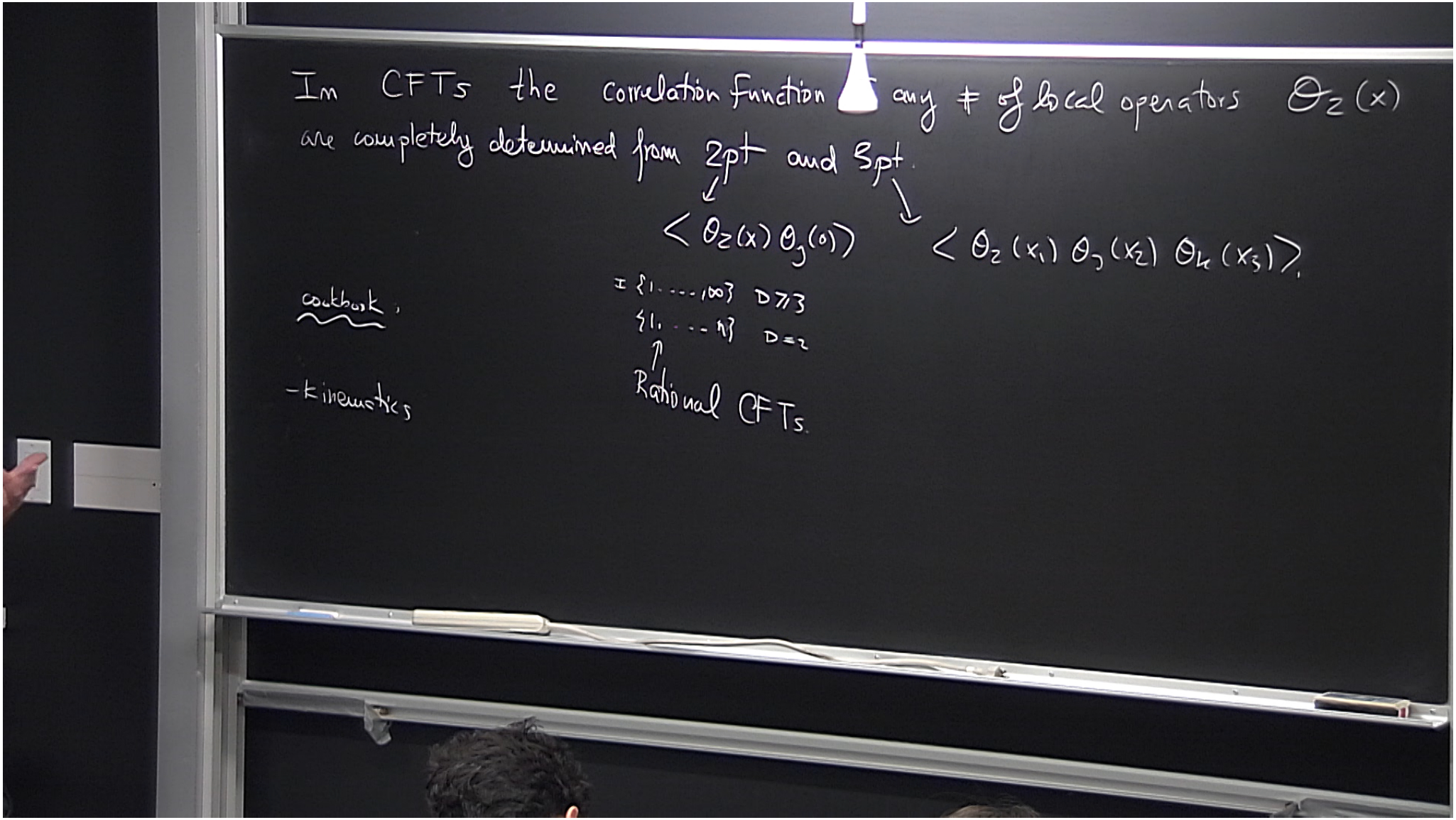
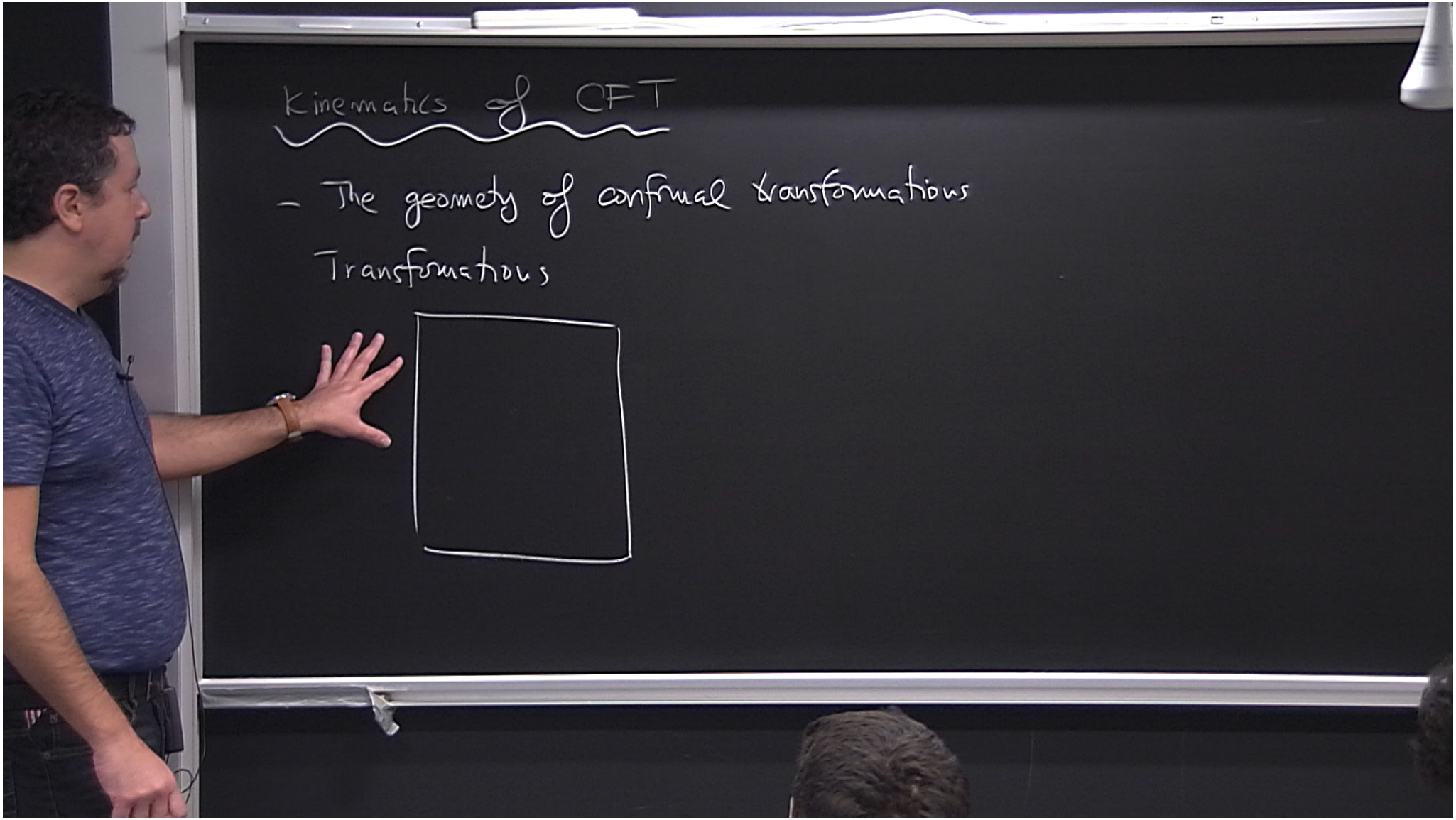
Title: PSI 17/18 - Quantum Field Theory III - Lecture 3

Date: Jan 31, 2018 09:00 AM

URL: http://pirsa.org/18010056

Abstract:

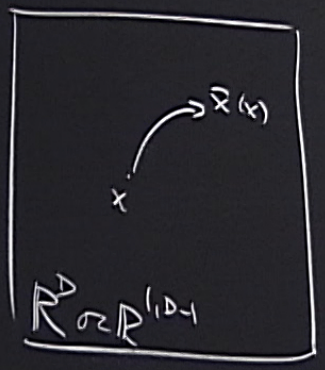

In CFTs the correlation function of any # of local operators $\mathcal{O}_i(x)$
are completely determined from 2pt and 3pt.
$\langle \mathcal{O}_i(x)\mathcal{O}_j(0)\rangle$
$\langle \mathcal{O}_i(x_1)\mathcal{O}_j(x_2)\mathcal{O}_k(x_3)\rangle$.
cookbook.
$i \in \{1,\dots,\infty\}$ $D \neq 3$
$\{1,\dots,N\}$ $D=2$
Rational CFTs.
-kinematics


Kinematics of CFT
- The geometry of conformal transformations
Transformations

Kinematics of CFT

- The geometry of conformal transformations

Transformations $x^\mu \longrightarrow \tilde{x}^\mu(x)$ that



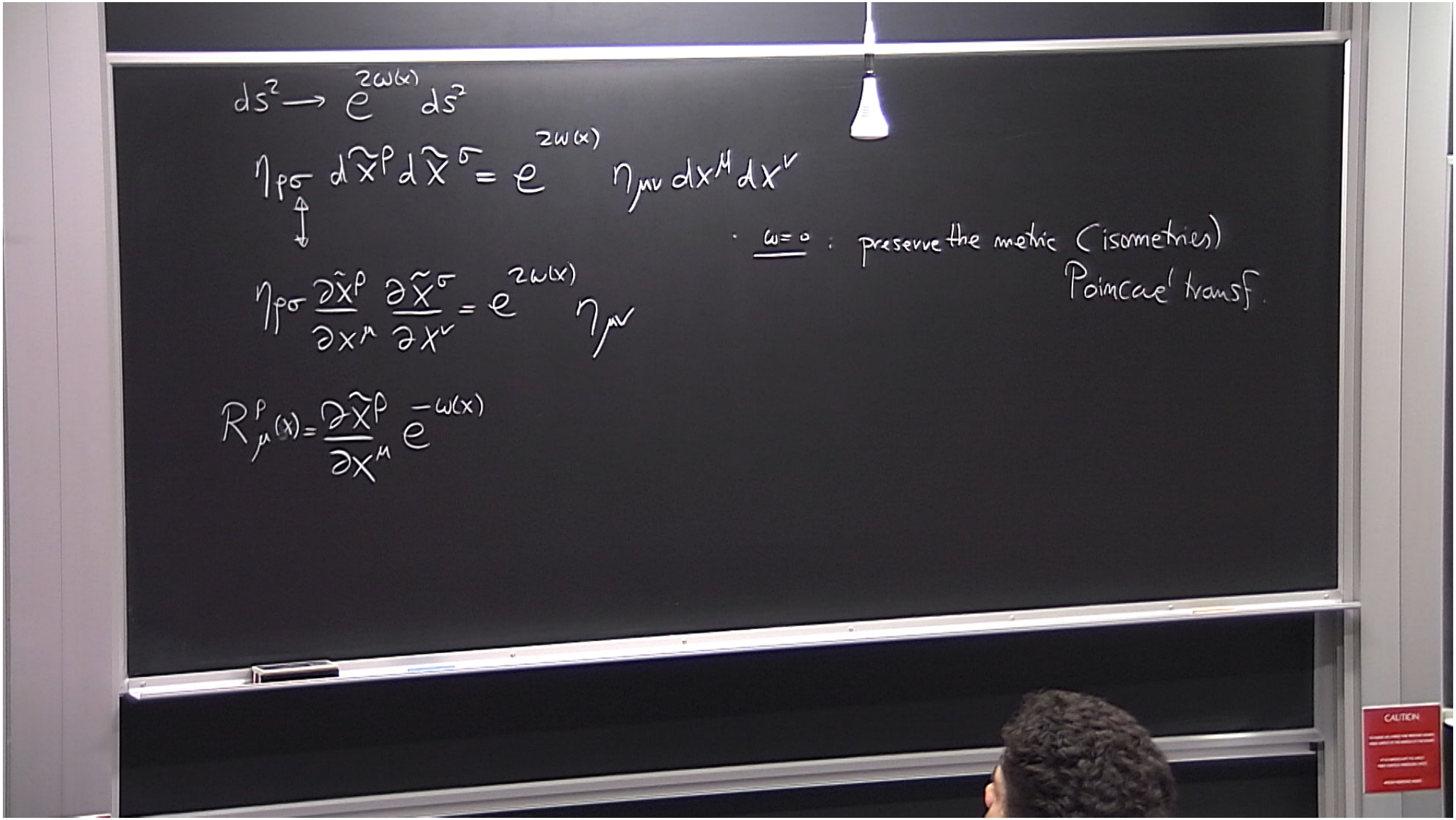
1. preserves angles between vectors
2. preserves the light-cones.

$$ds^2 \longrightarrow e^{2\omega(x)} ds^2$$

$$\eta_{\rho\sigma} d\tilde{x}^\rho d\tilde{x}^\sigma = e^{2\omega(x)} \eta_{\mu\nu} dx^\mu dx^\nu$$

$$\eta_{\rho\sigma} \frac{\partial \tilde{x}^\rho}{\partial x^\mu} \frac{\partial \tilde{x}^\sigma}{\partial x^\nu} = e^{2\omega(x)} \eta_{\mu\nu}$$

- $\omega = 0$: preserve the metric (isometries)
  Poincaré transf.


$ds^2 \longrightarrow e^{2\omega(x)} ds^2$
$\eta_{\rho\sigma} d\tilde{x}^\rho d\tilde{x}^\sigma = e^{2\omega(x)} \eta_{\mu\nu} dx^\mu dx^\nu$
$\eta_{\rho\sigma} \frac{\partial \tilde{x}^\rho}{\partial x^\mu} \frac{\partial \tilde{x}^\sigma}{\partial x^\nu} = e^{2\omega(x)} \eta_{\mu\nu}$
$R^\rho_\mu(x) = \frac{\partial \tilde{x}^\rho}{\partial x^\mu} e^{-\omega(x)}$
· $\underline{\omega = 0}$: preserve the metric (isometries)
Poincaré transf.
CAUTION

$$ds^2 \longrightarrow e^{2\omega(x)} ds^2$$

$$\eta_{\rho\sigma} d\tilde{x}^\rho d\tilde{x}^\sigma = e^{2\omega(x)} \eta_{\mu\nu} dx^\mu dx^\nu$$

$$\eta_{\rho\sigma} \frac{\partial \tilde{x}^\rho}{\partial x^\mu} \frac{\partial \tilde{x}^\sigma}{\partial x^\nu} = e^{2\omega(x)} \eta_{\mu\nu}$$

$$R^\rho_\mu(x) = \frac{\partial \tilde{x}^\rho}{\partial x^\mu} e^{-\omega(x)} \qquad R^T \eta R = \eta.$$

· $\underline{\omega = 0}$: preserve the metric (isometries)
Poincaré transf.

$ds^2 \longrightarrow e^{2\omega(x)} ds^2$

$\eta_{\rho\sigma} d\tilde{x}^\rho d\tilde{x}^\sigma = e^{2\omega(x)} \eta_{\mu\nu} dx^\mu dx^\nu$

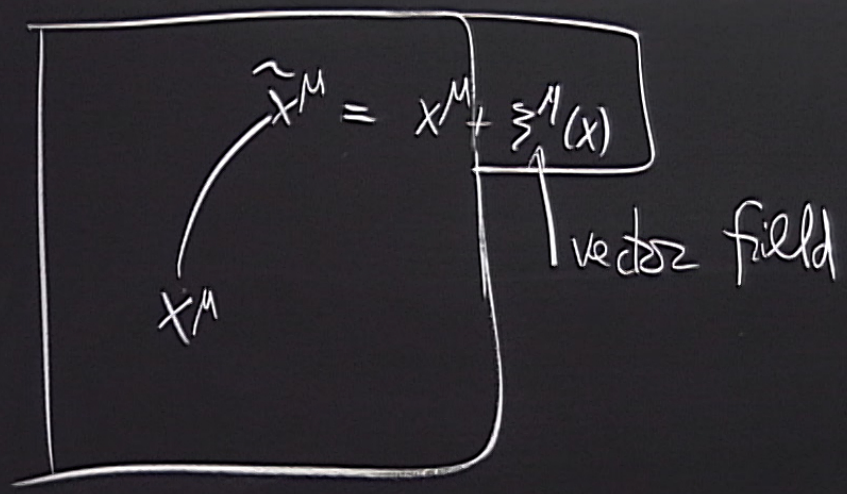
$\eta_{\rho\sigma} \frac{\partial \tilde{x}^\rho}{\partial x^\mu} \frac{\partial \tilde{x}^\sigma}{\partial x^\nu} = e^{2\omega(x)} \eta_{\mu\nu}$

$R^\rho_\mu(x) = \boxed{\frac{\partial \tilde{x}^\rho}{\partial x^\mu}} e^{-\omega(x)}$ $\quad R^T \eta R = \eta.$

- $\underline{\omega = 0}$: preserve the metric (isometries)
  Poincaré transf.

- Local Lorentz transformation $\times$ dilatation

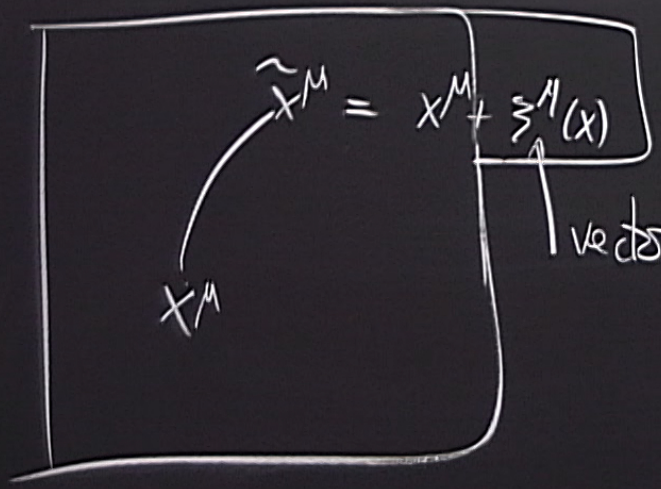
Transf. that are connected to identity



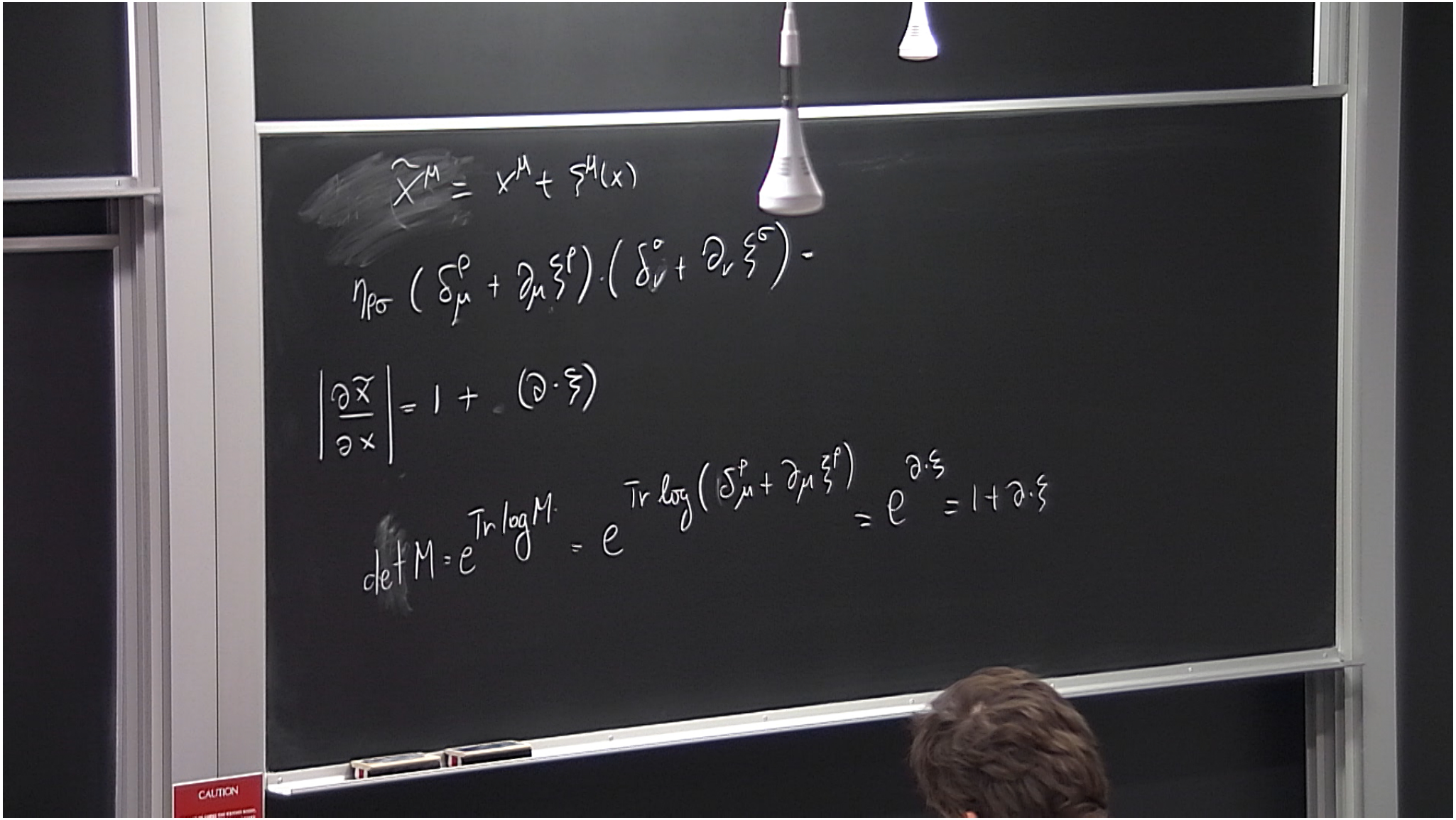
- Local Lorentz transformation $\times$ dilatation

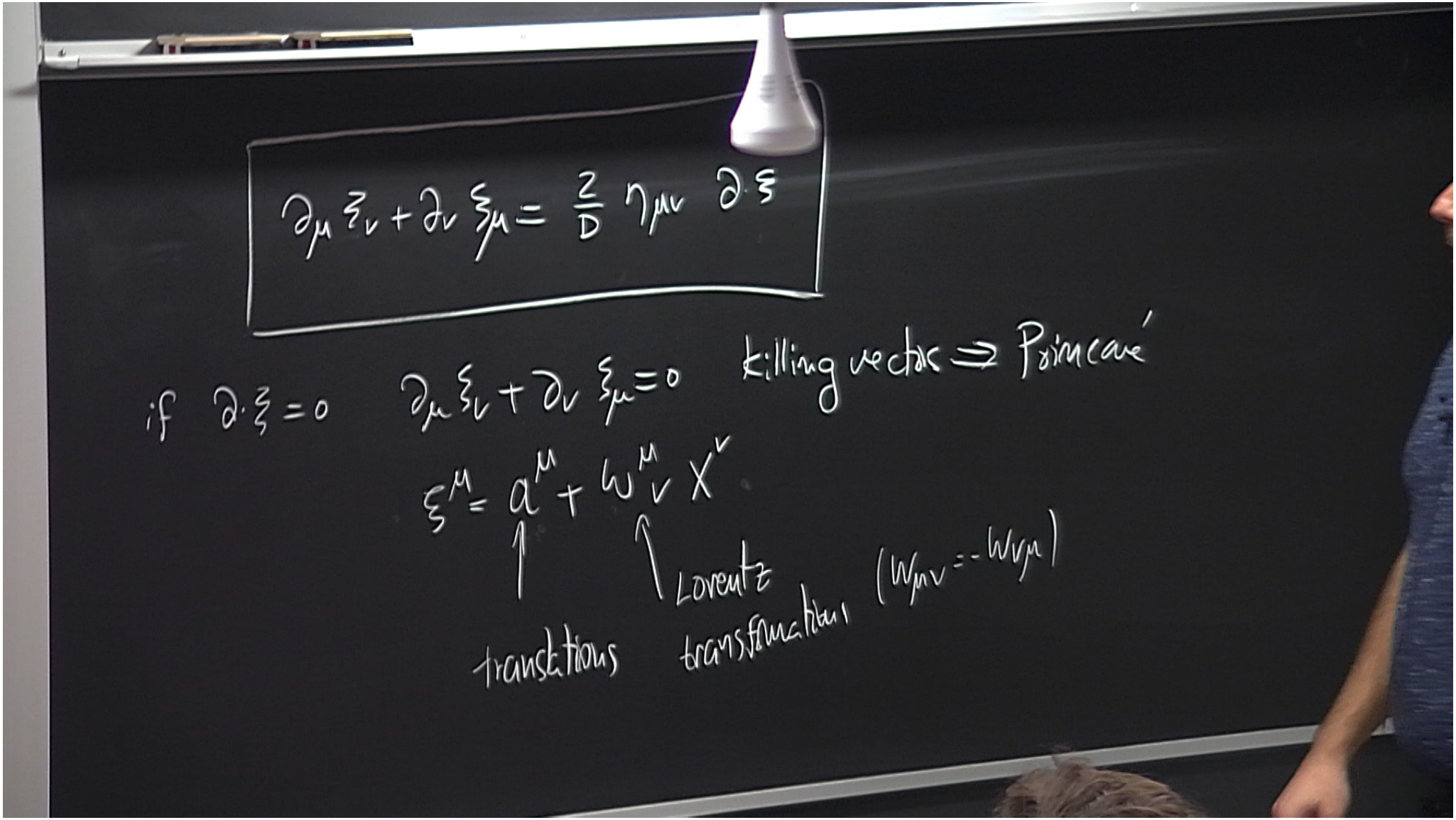
$\left|\frac{\partial x}{\partial x^\mu}\right| = e$

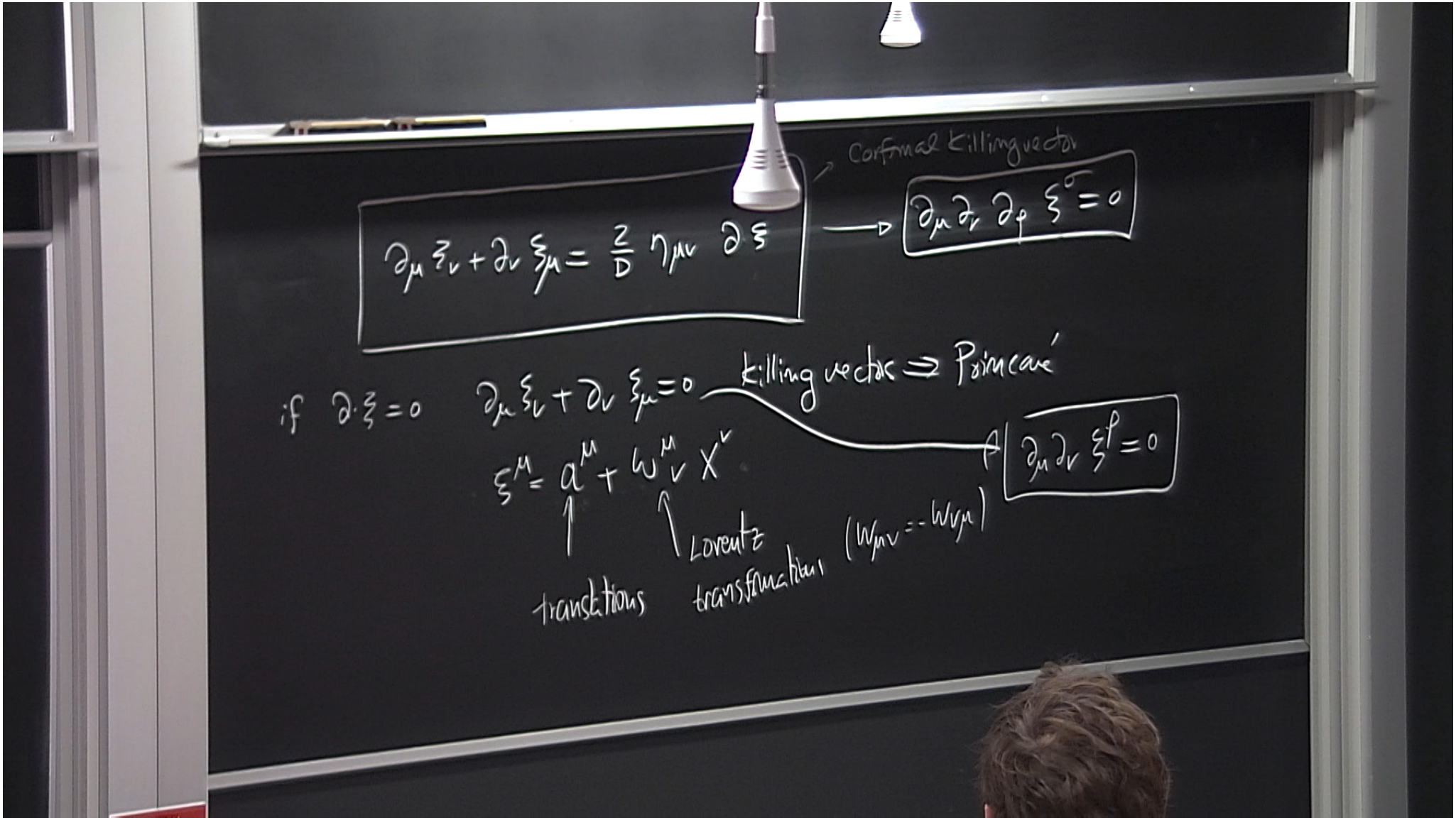
Transf. that are connected to identity

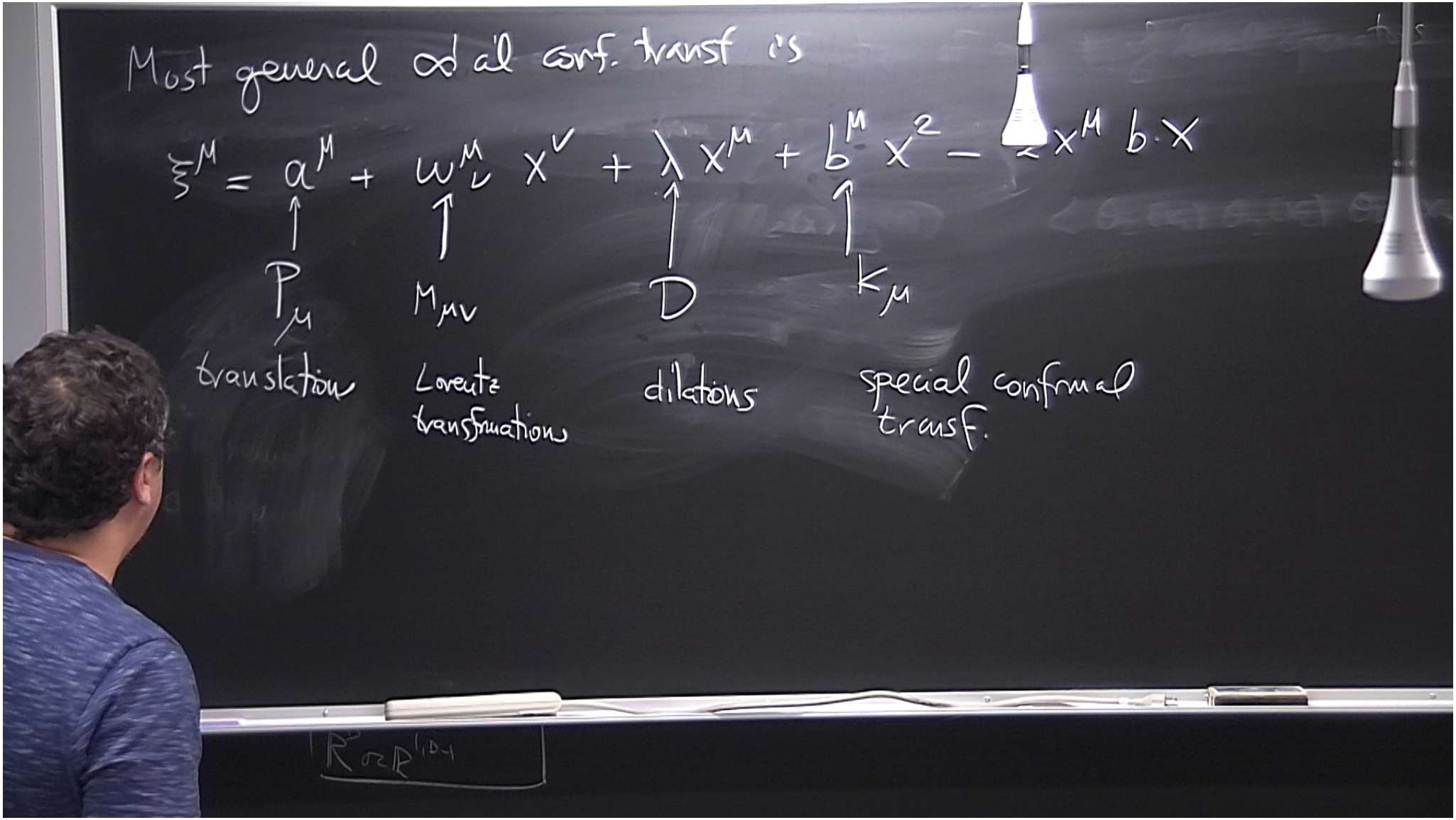


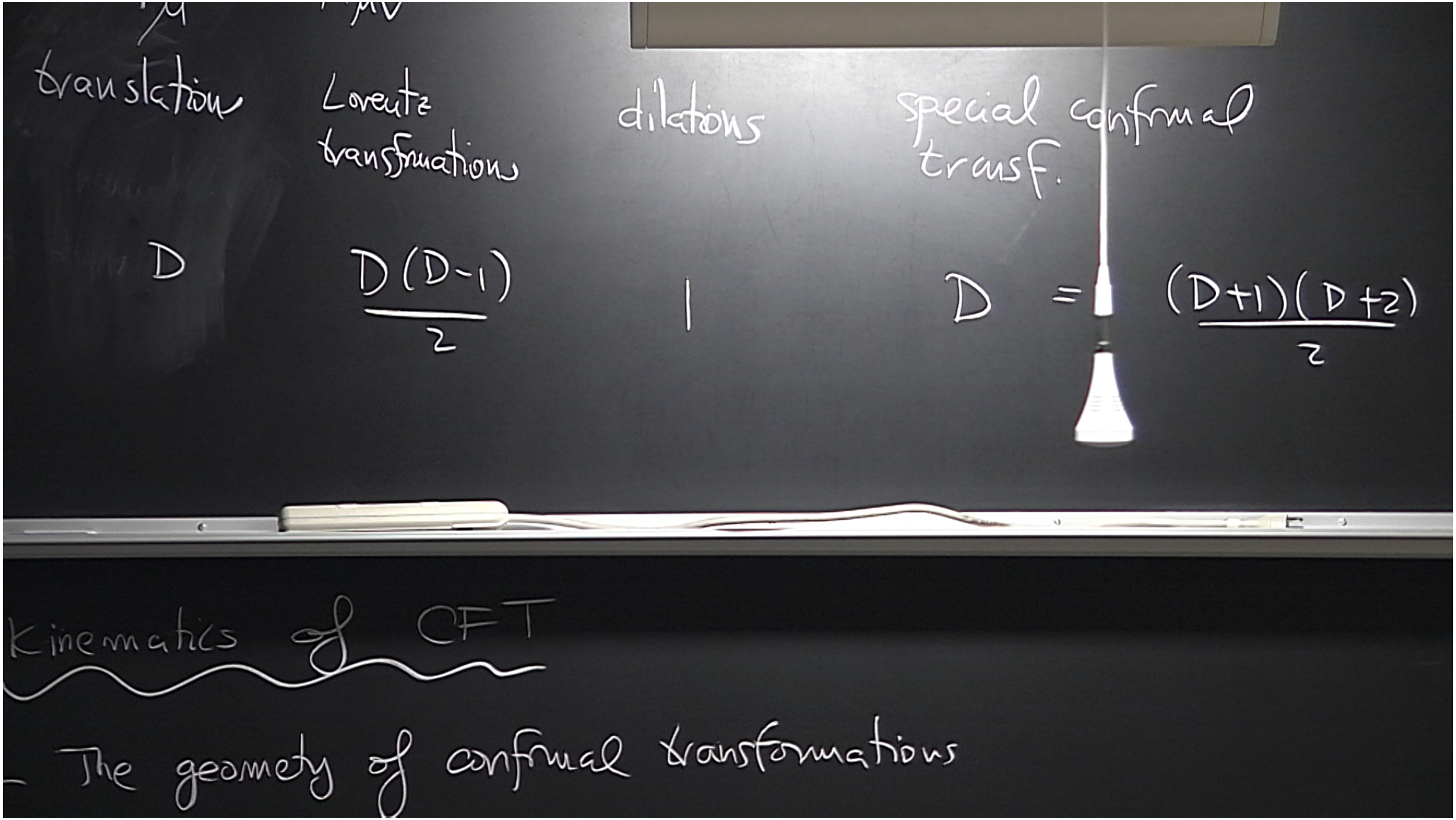
vector field $\xi = \xi^\mu \partial_\mu$

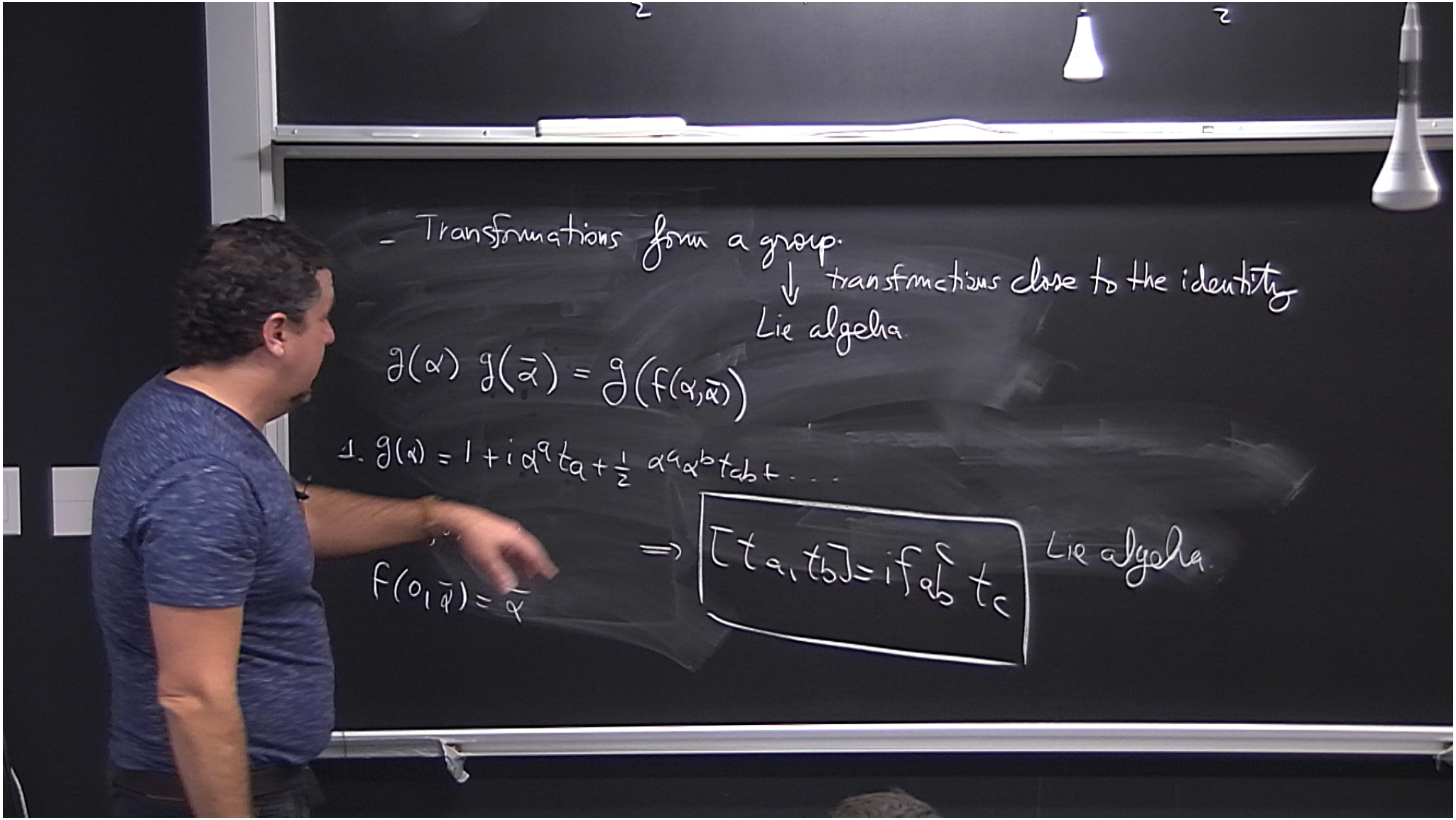

$\tilde{x}^\mu = x^\mu + \xi^\mu(x)$
$\eta_{\rho\sigma}\,(\delta^\rho_\mu + \partial_\mu \xi^\rho)\cdot(\delta^\sigma_\nu + \partial_\nu \xi^\sigma) =$
$\left|\frac{\partial \tilde{x}}{\partial x}\right| = 1 + (\partial\cdot\xi)$
$\det M = e^{\mathrm{Tr}\log M} = e^{\mathrm{Tr}\log(\delta^\rho_\mu + \partial_\mu \xi^\rho)} = e^{\partial\cdot\xi} = 1 + \partial\cdot\xi$
CAUTION

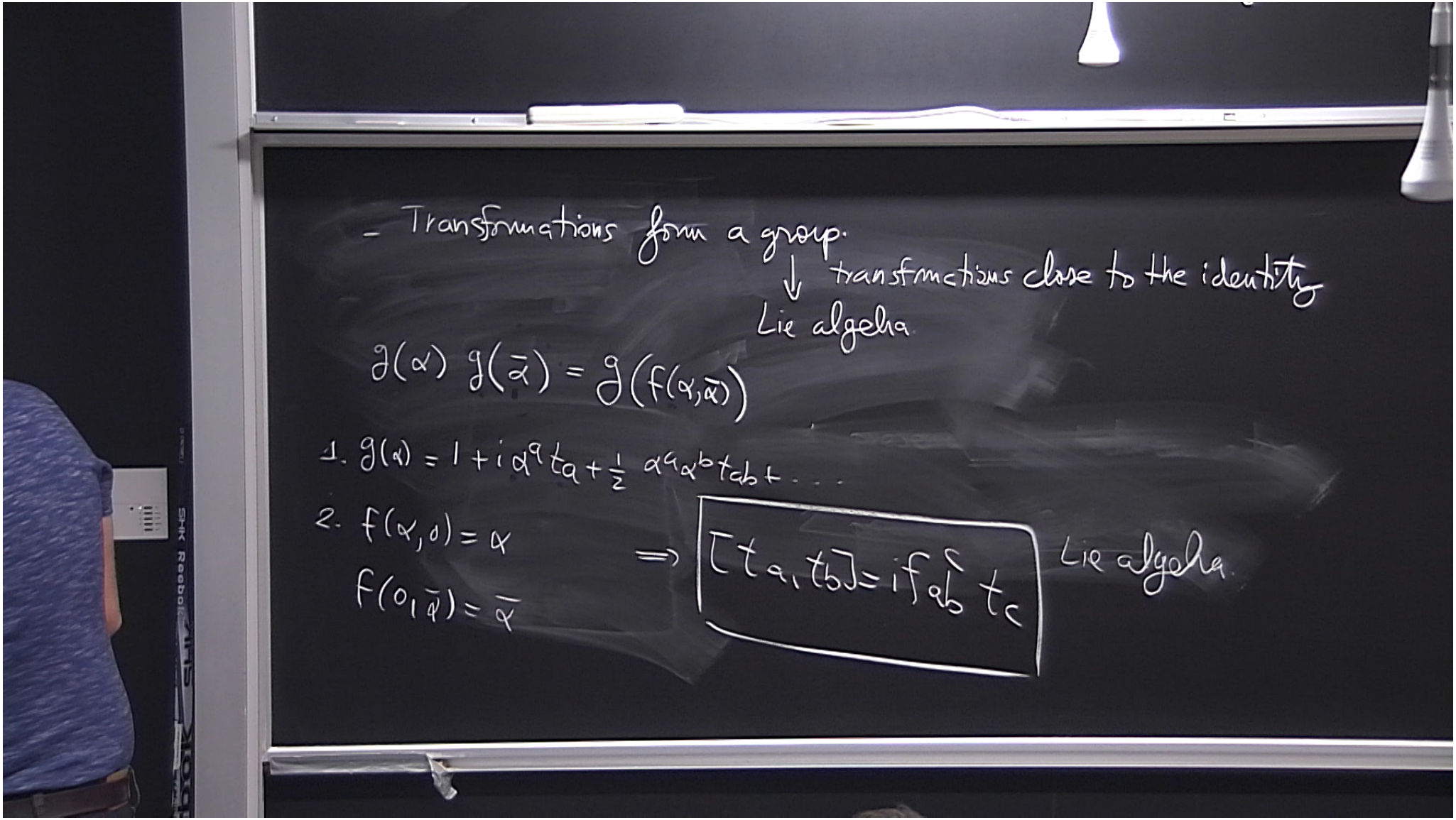

$\partial_\mu \xi_\nu + \partial_\nu \xi_\mu = \frac{2}{D} \eta_{\mu\nu} \partial \cdot \xi$
if $\partial \cdot \xi = 0$ $\partial_\mu \xi_\nu + \partial_\nu \xi_\mu = 0$ killing vector $\Rightarrow$ Poincaré
$\xi^\mu = a^\mu + \omega^\mu{}_\nu x^\nu$
translations
Lorentz transformations $(\omega_{\mu\nu} = -\omega_{\nu\mu})$

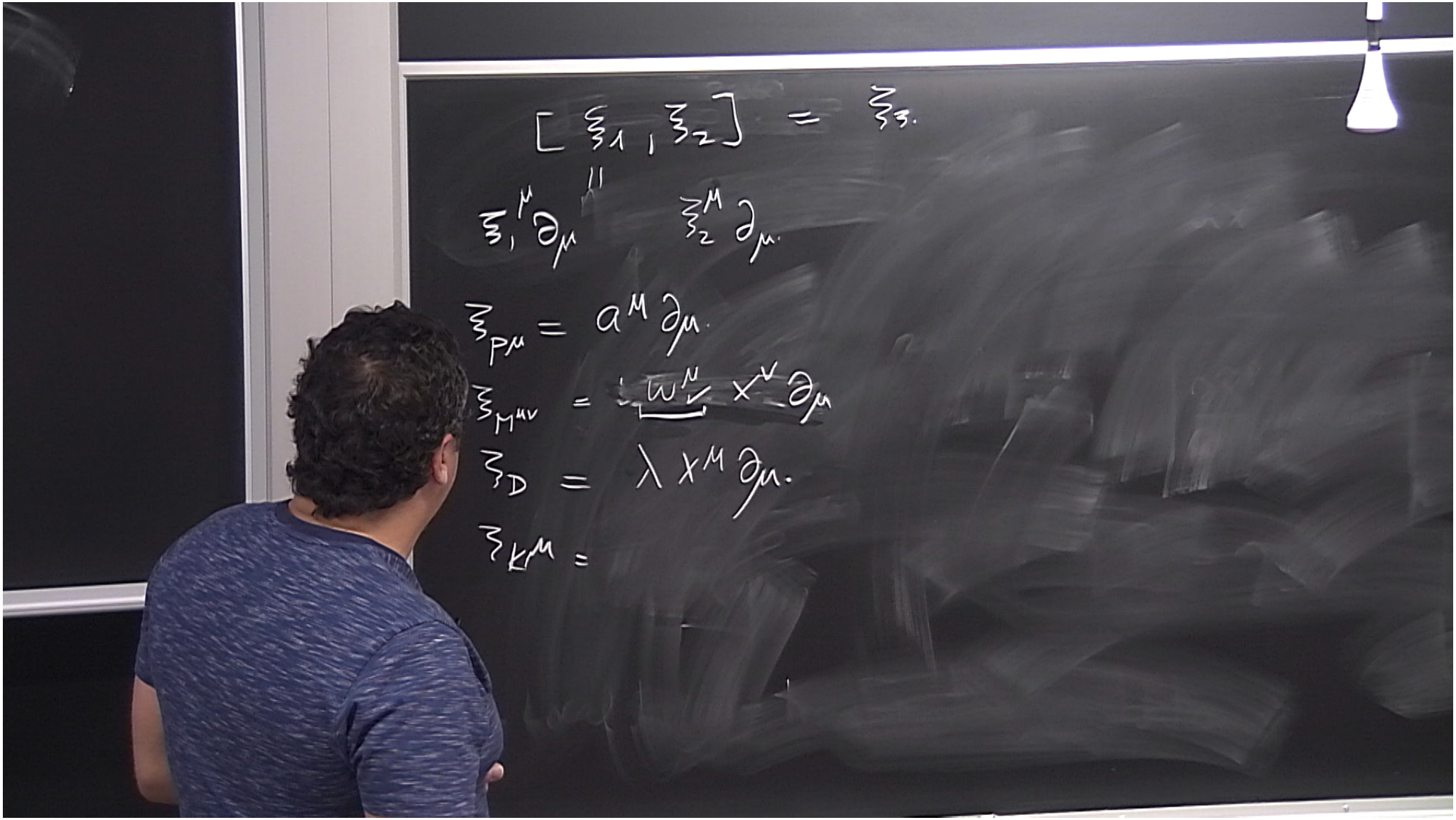

Conformal Killing vector
$\partial_\mu \xi_\nu + \partial_\nu \xi_\mu = \frac{2}{D} \eta_{\mu\nu} \partial \cdot \xi$
$\partial_\mu \partial_\nu \partial_\rho \xi^\sigma = 0$
if $\partial \cdot \xi = 0$ $\partial_\mu \xi_\nu + \partial_\nu \xi_\mu = 0$
Killing vector ⇒ Poincaré
$\partial_\mu \partial_\nu \xi^\rho = 0$
$\xi^\mu = a^\mu + \omega^\mu{}_\nu x^\nu$
translations
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$(\omega_{\mu\nu} = -\omega_{\nu\mu})$

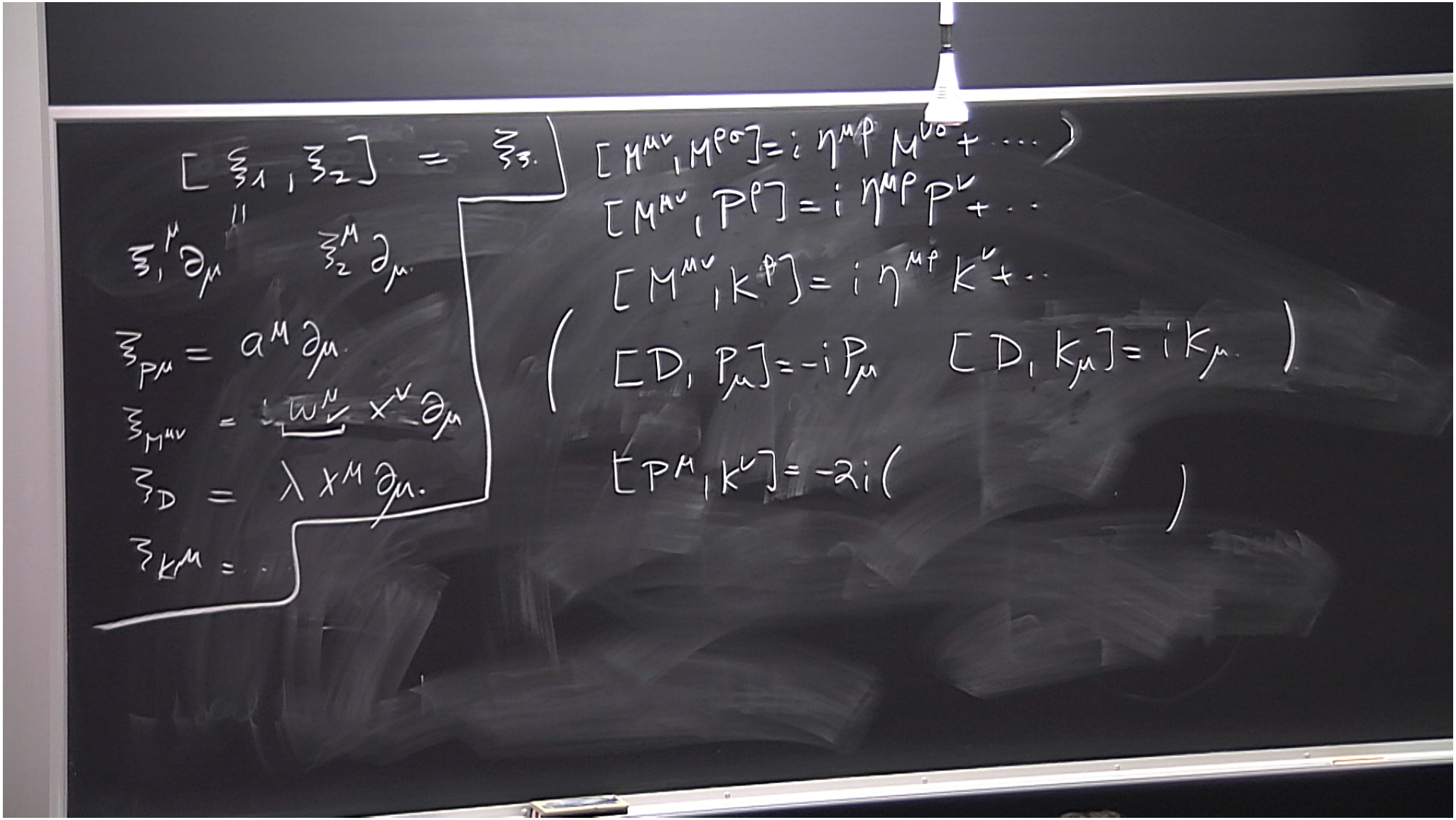

Most general conf. transf is
$\xi^\mu = a^\mu + \omega^\mu_{\ \nu} x^\nu + \lambda x^\mu + b^\mu x^2 - 2 x^\mu\, b\cdot x$
P_μ
$M_{\mu\nu}$
D
K_μ
translation
Lorentz transformations
dilations
special conformal transf.
$\mathbb{R}^2 \cong \mathbb{R}^{1,1}$

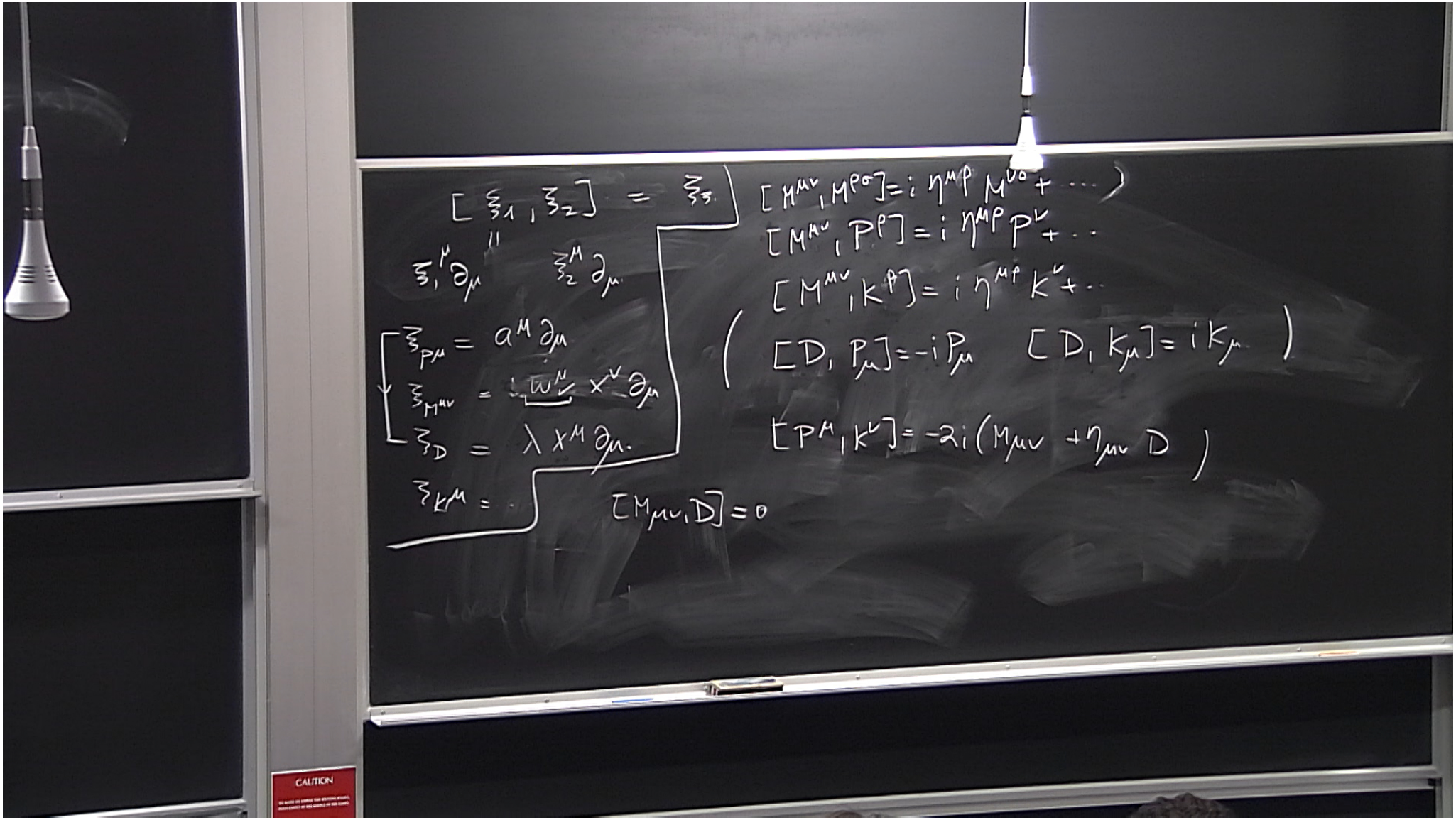

translation
Lorentz transformations
dilations
special conformal transf.
D
D(D-1)/2
1
D = (D+1)(D+2)/2
Kinematics of CFT
The geometry of conformal transformations

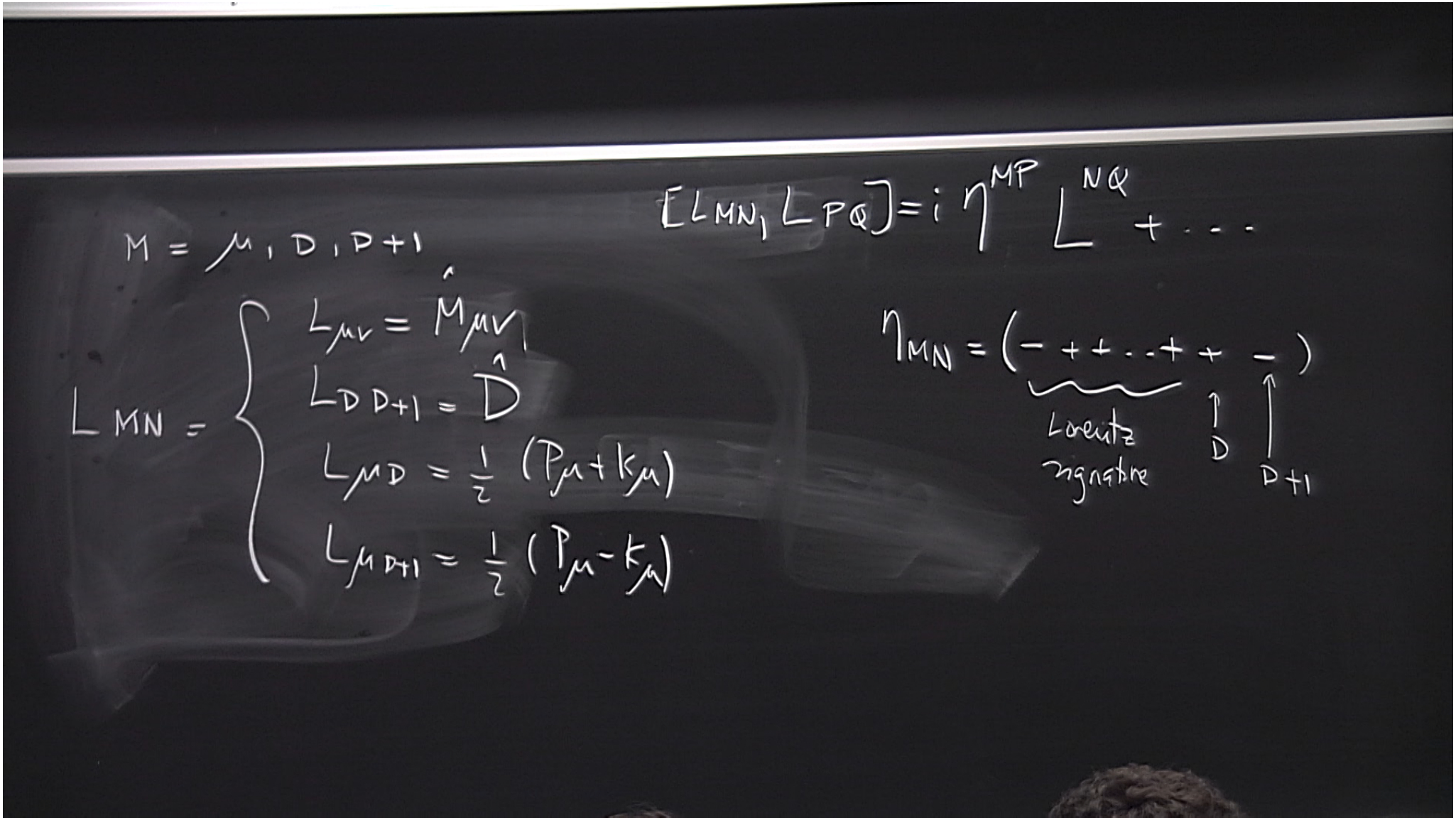

- Transformations form a group.
↓ transformations close to the identity
Lie algebra
$g(\alpha)\, g(\bar{\alpha}) = g(f(\alpha,\bar{\alpha}))$
1. $g(\alpha) = 1 + i\alpha^a t_a + \frac{1}{2}\alpha^a\alpha^b t_{ab} + \ldots$
$f(0,\bar{\alpha}) = \bar{\alpha}$
$\Rightarrow$ $[t_a, t_b] = i f_{ab}^{\ c}\, t_c$
Lie algebra

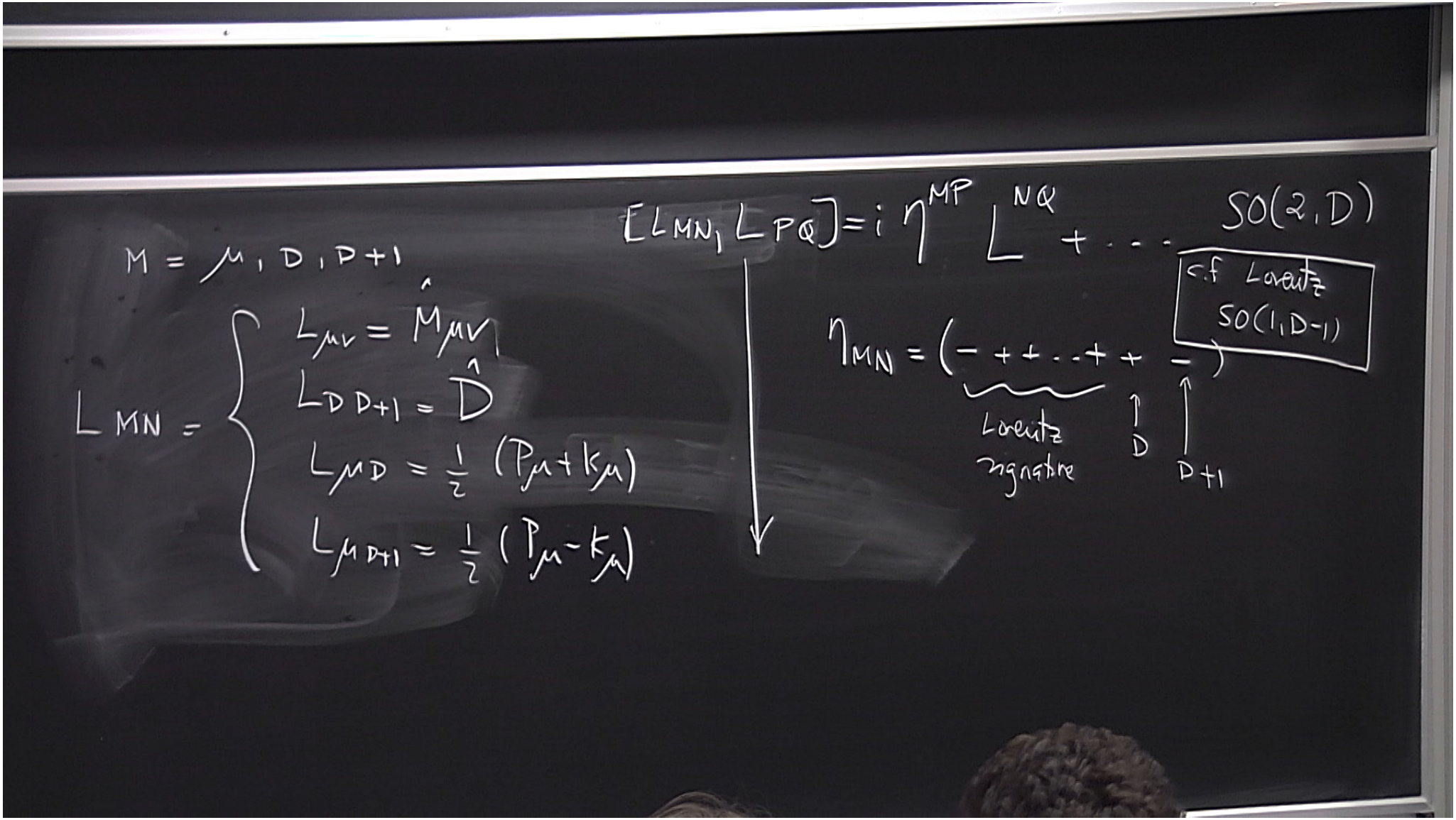

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1. $g(\alpha) = 1 + i\alpha^a t_a + \frac{1}{2}\alpha^a\alpha^b t_{ab} + \ldots$
2. $f(\alpha,0) = \alpha$
$f(0,\bar{\alpha}) = \bar{\alpha}$
$\Rightarrow \boxed{[t_a, t_b] = i f_{ab}^{\ c}\, t_c}$
Lie algebra


$[\xi_1, \xi_2] = \xi_3$
$\xi_1^\mu \partial_\mu$ $\xi_2^\mu \partial_\mu$
$\xi_{P\mu} = a^\mu \partial_\mu$
$\xi_{M\mu\nu} = \omega^\nu_{\ \nu} x^\nu \partial_\mu$
$\xi_D = \lambda x^\mu \partial_\mu$
$\xi_{K\mu} =$


$[\xi_1, \xi_2] = \xi_3$.
$\xi_1^\mu \partial_\mu$ $\xi_2^\mu \partial_\mu$
$\xi_{P^\mu} = a^\mu \partial_\mu$.
$\xi_{M^{\mu\nu}} = \omega^\mu{}_\nu x^\nu \partial_\mu$
$\xi_D = \lambda x^\mu \partial_\mu$.
$\xi_{K^\mu} =$
$[M^{\mu\nu}, M^{\rho\sigma}] = i\eta^{\mu\rho} M^{\nu\sigma} + \dots$
$[M^{\mu\nu}, P^\rho] = i\eta^{\mu\rho} P^\nu + \dots$
$[M^{\mu\nu}, K^\rho] = i\eta^{\mu\rho} K^\nu + \dots$
$[D, P_\mu] = -iP_\mu$ $[D, K_\mu] = iK_\mu$
$[P^\mu, K^\nu] = -2i($ $)$


$[\xi_1, \xi_2] = \xi_3$
$\xi_1^\mu \partial_\mu$ $\xi_2^\mu \partial_\mu$
$\xi_{P\mu} = a^\mu \partial_\mu$
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$[M^{\mu\nu}, M^{\rho\sigma}] = i\eta^{\mu\rho} M^{\nu\sigma} + \cdots$
$[M^{\mu\nu}, P^\rho] = i\eta^{\mu\rho} P^\nu + \cdots$
$[M^{\mu\nu}, K^\rho] = i\eta^{\mu\rho} K^\nu + \cdots$
$[D, P_\mu] = -iP_\mu$ $[D, K_\mu] = iK_\mu$
$[P^\mu, K^\nu] = -2i(M_{\mu\nu} + \eta_{\mu\nu} D)$
$[M_{\mu\nu}, D] = 0$
CAUTION


$M = \mu, D, D+1$
$L_{MN} = \begin{cases} L_{\mu\nu} = \hat{M}_{\mu\nu} \\ L_{D\,D+1} = \hat{D} \\ L_{\mu D} = \frac{1}{2}(P_\mu + k_\mu) \\ L_{\mu\,D+1} = \frac{1}{2}(P_\mu - k_\mu) \end{cases}$
$[L_{MN}, L_{PQ}] = i\,\eta^{MP} L^{NQ} + \dots$
$\eta_{MN} = (- + + \dots + + -)$
Lorentz signature
D
D+1


$M = \mu, D, D+1$
$L_{MN} = \begin{cases} L_{\mu\nu} = \hat{M}_{\mu\nu} \\ L_{D\,D+1} = \hat{D} \\ L_{\mu D} = \frac{1}{2}(P_\mu + K_\mu) \\ L_{\mu\,D+1} = \frac{1}{2}(P_\mu - K_\mu) \end{cases}$
$[L_{MN}, L_{PQ}] = i\,\eta^{MP} L^{NQ} + \cdots$
$SO(2,D)$
c.f Lorentz $SO(1,D-1)$
$\eta_{MN} = (- + + \cdots + + \; - \;)$
Lorentz signature
D
$D+1$

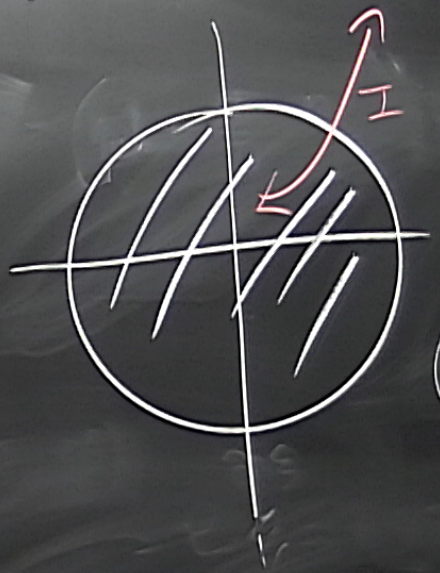
Discrete conformal transformations:

$$I: \quad x^\mu \longrightarrow \frac{x^\mu}{x^2}$$

1) $\mathbb{Z}_2$ transf.

2) changes orientation of spacetime

Conformal transf can be obtained by combining Poincaré + I



$I P_\mu I = K_\mu$

$I M_{\mu\nu} I = M_{\mu\nu}$

$I K_\mu I = P_\mu$

$I D I = -D$