Title: PSI 17/18 - Quantum Field Theory III - Lecture 1

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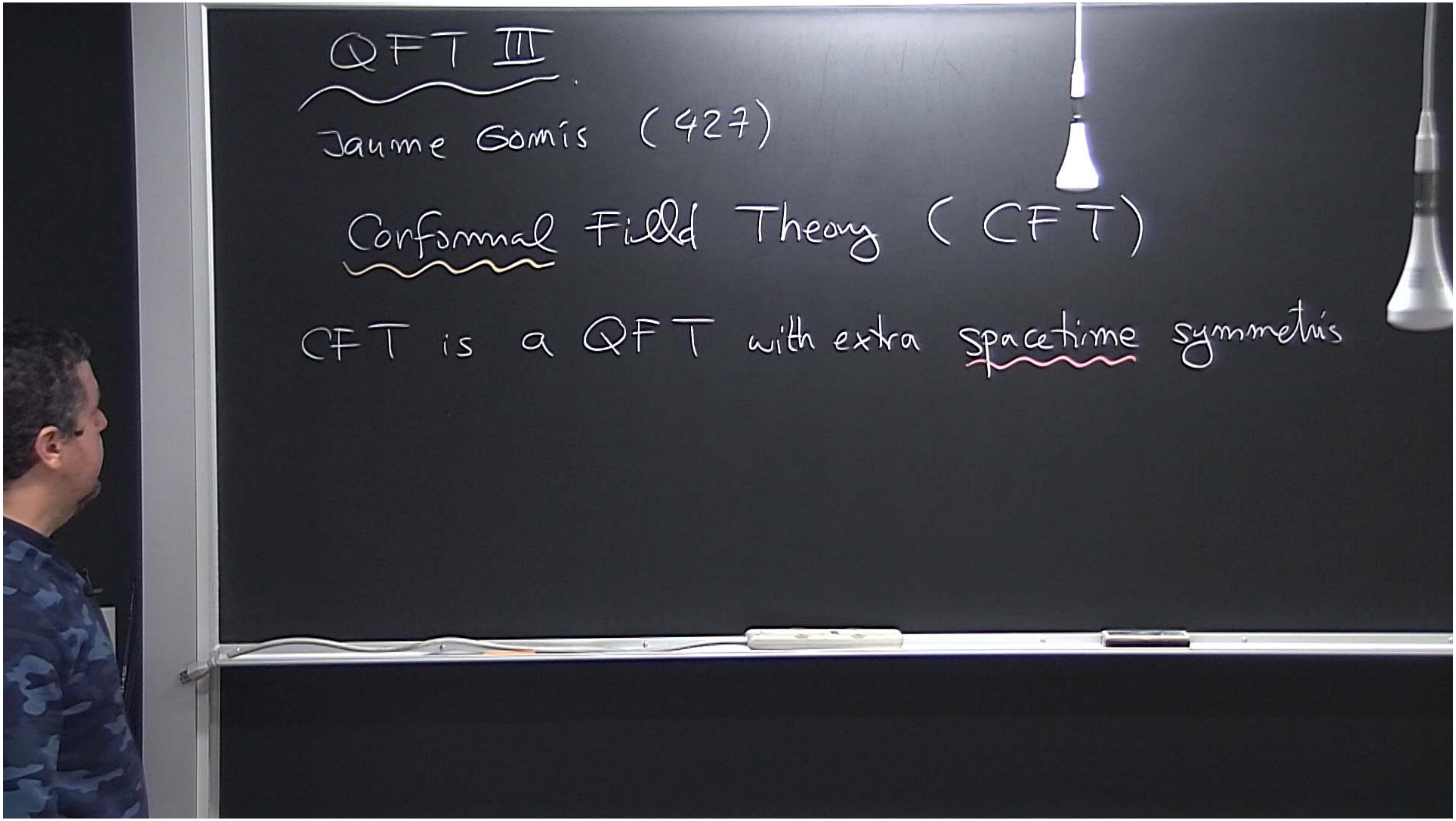
URL: http://pirsa.org/18010054

Abstract:


QFT III
Jaume Gomis (427)
Conformal Field Theory (CFT)
CFT is a QFT with extra spacetime symmetries

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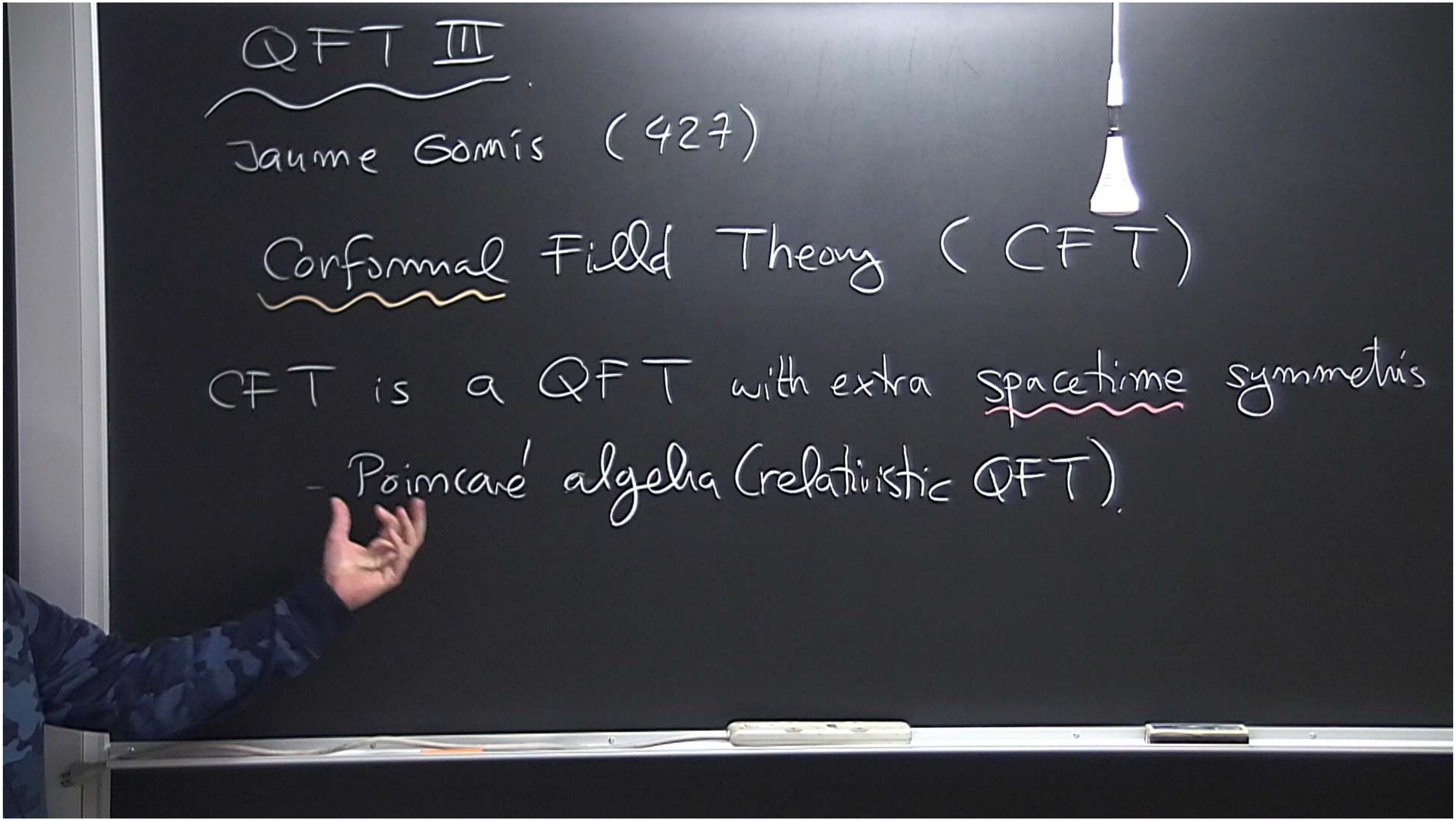
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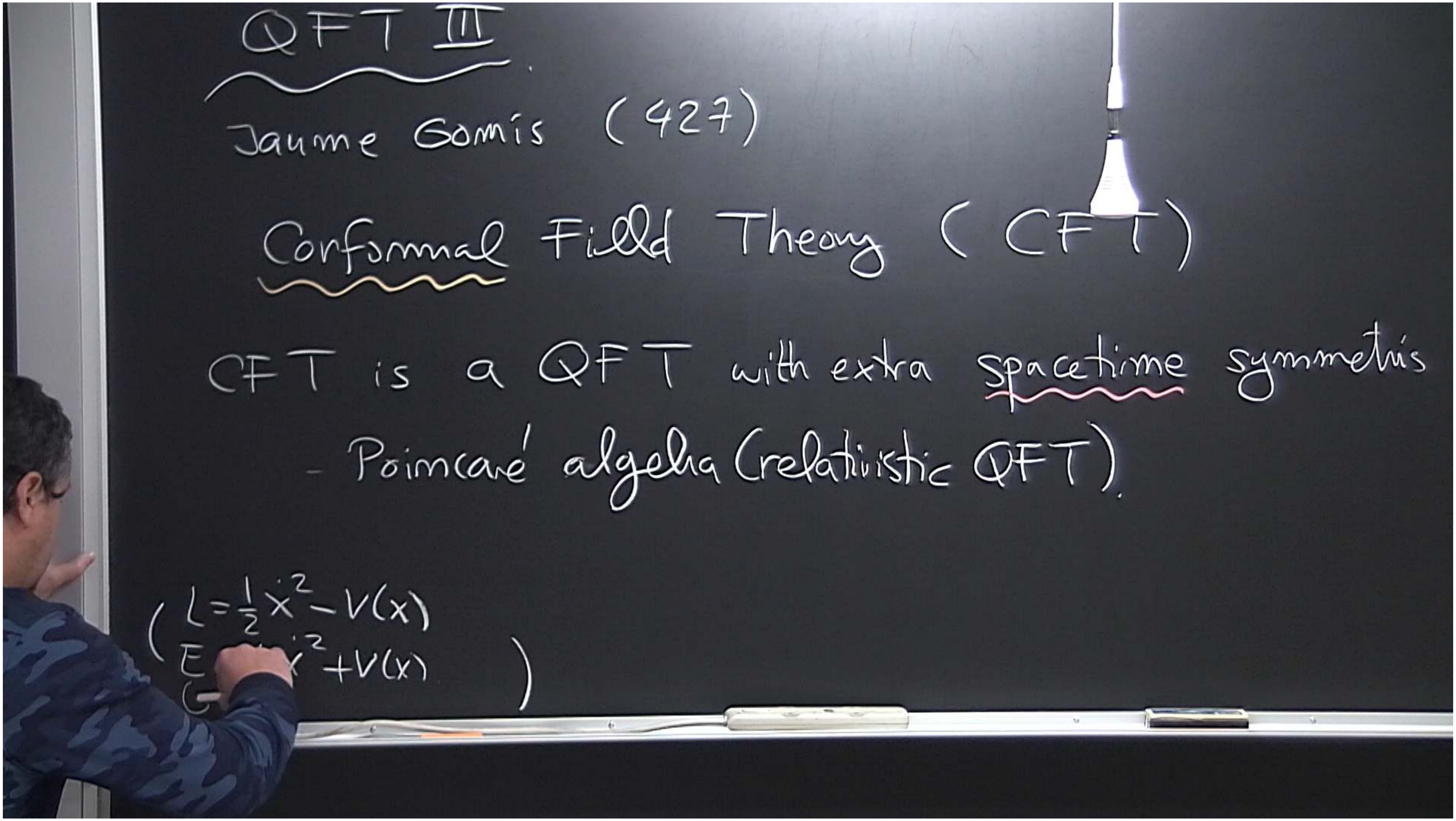
Conformal Field Theory (CFT)
CFT is a QFT with extra spacetime symmetries
- Poincaré algebra (relativistic QFT)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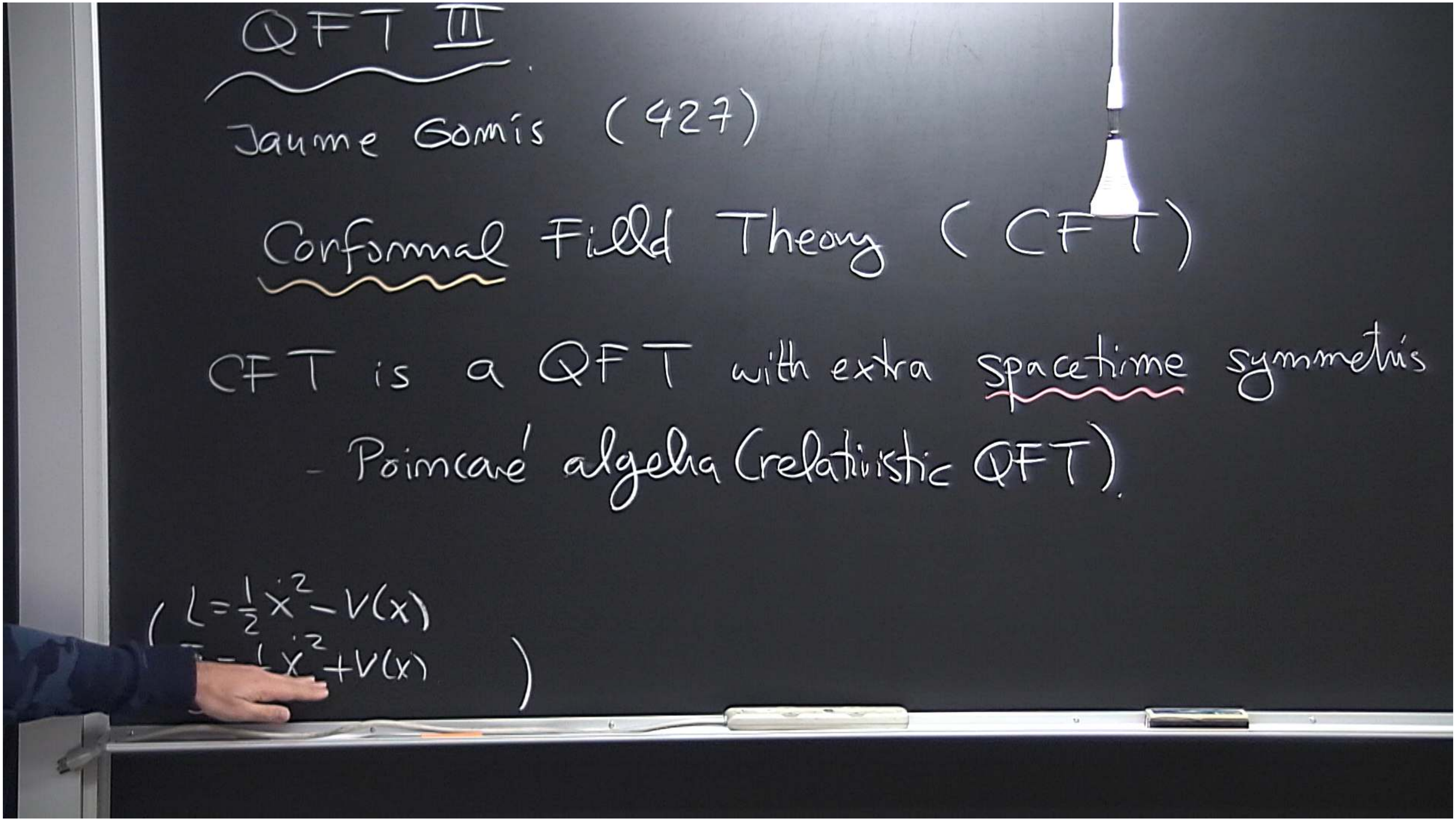
$L=\frac{1}{2}\dot{x}^2-V(x)$
$+V(x)$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- Poincaré algebra (relativistic QFT).

$$\left(\begin{array}{l} L = \frac{1}{2}\dot{x}^2 - V(x) \\ E = \frac{1}{2}\dot{x}^2 + V(x) \\ Q \end{array} \right)$$

QFT III

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Conformal Field Theory (CFT)

CFT is a QFT with extra spacetime symmetries

- Poincaré algebra (relativistic QFT) ⟶ conformal symmetry.
- Galilean algebra (non-relativistic QFTs).

- Poincaré algebra (relativistic QFT)
- Galilean algebra (non-relativistic QFTs)

$\rightsquigarrow$ conformal symmetry.

$(D>2)$

- conformal symmetry is the largest spacetime symmetry of interacting QFT.

Scale transformations:

- conformal symmetry is the largest spacetime symmetry of interacting QFT !

Scale transformations (dilatations)

$$\vec{x} \longrightarrow \lambda \vec{x} \qquad \lambda \in \mathbb{R}.$$

$$t \longrightarrow \lambda^z t \qquad z: \text{dynamical critical exponent}$$

- Relativistic ($z=1$) $x^\mu \longrightarrow \lambda x^\mu$
- Nonrelativistic $\quad -\frac{1}{2}\nabla^2\psi = -i\frac{\partial}{\partial t}\psi$

- conformal symmetry is the largest spacetime symmetry of interact[illegible] QFT !.

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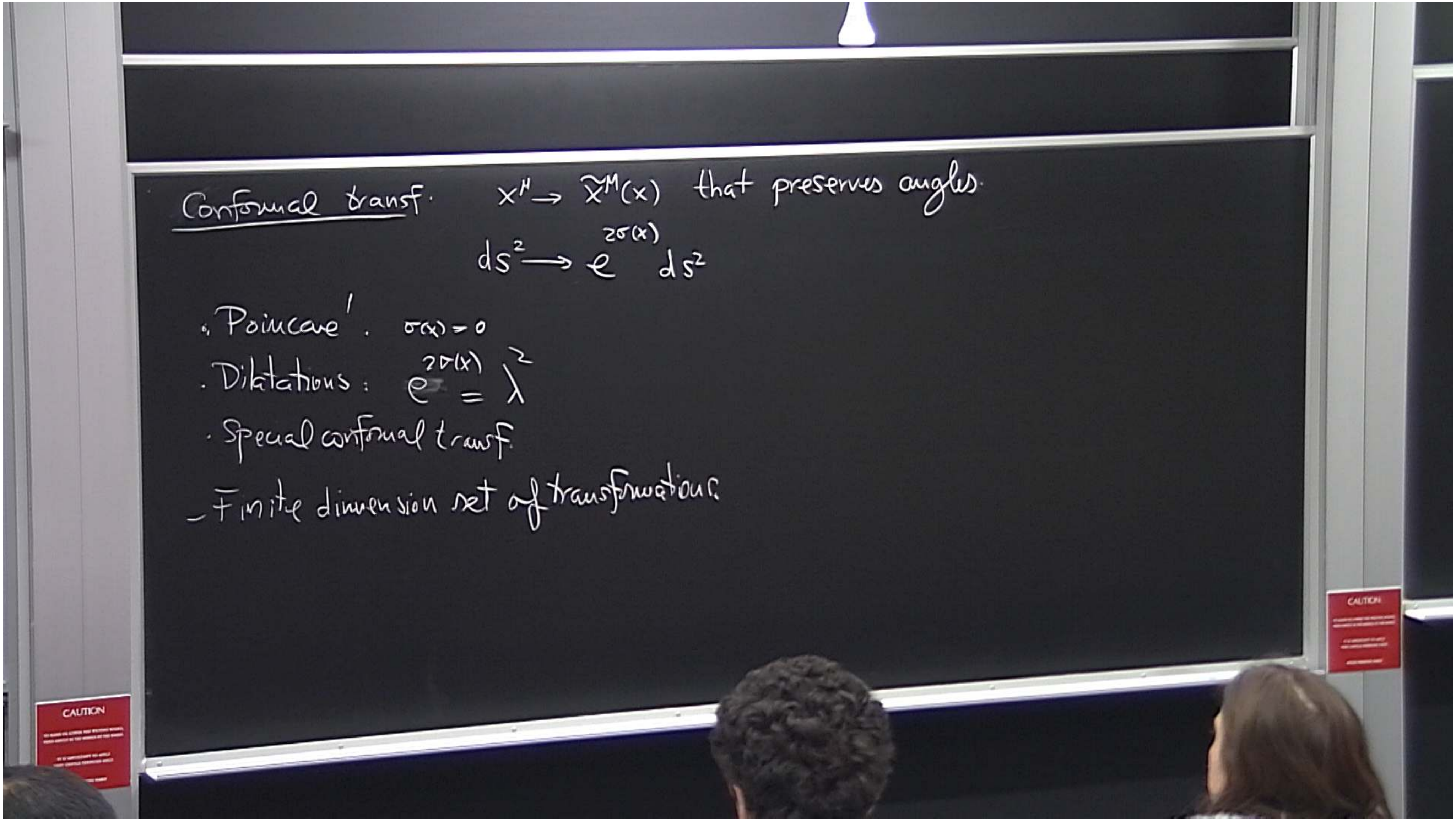
- Relativistic ($z=1$) $x^{\mu} \longrightarrow \lambda x^{\mu}$

- Nonrelativistic $\quad -\frac{1}{2}\nabla^2 \psi = -i \frac{\partial}{\partial t} \psi \quad (z=2) \qquad \mathcal{L} = \frac{1}{2}(\partial_t \phi)^2 + (\nabla^3 \phi)^2$

$(z=3)$


Conformal transf. $x^\mu \to \tilde{x}^\mu(x)$ that preserves angles.
$ds^2 \to e^{2\sigma(x)} ds^2$
. Poincaré: $\sigma(x) = 0$
. Dilatations: $e^{2\sigma(x)} = \lambda^2$
. Special conformal transf.
- Finite dimension set of transformations

Why CFTs?

- Asymptotic large distance behaviour of QFT

1. Trivial/gapped/(massive).

$$\langle O(x)\, O(0)\rangle \sim e^{-\frac{|x|}{\xi}}.$$

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1. Trivial/gapped/massive). $\xrightarrow{\text{extreme IR}}$ no local operators

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Non-trivial topological quantum field theory (TQFT)

1. Trivial/gapped/(massive). $\xRightarrow{\text{extreme IR}}$ no local operators

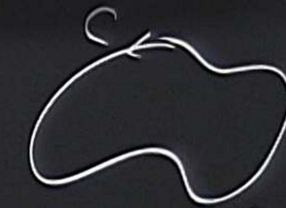
$$\langle O(x)\, O(0)\rangle \sim e^{-\frac{|x|}{\xi}}.$$

- Non-trivial topological quantum field theory (TQFT)
  - No local operators
  - $\exists$ non-local operators (Wilson loops, ...)

$\Rightarrow$ Describe IR dynamics
- QCD in 2+1 d
- condensed matter.

Why CFT's?

- Asymptotic large distance behaviour of QFT



1. Trivial/gapped/massive) $\xrightarrow{\text{extreme IR}}$ no local operators

$$\langle O(x)\, O(0)\rangle \sim e^{-\frac{|x|}{\xi}}.$$

$$U(1):\ e^{i\oint_C A_\mu \frac{dx^\mu}{d\tau}}$$

- Non-trivial topological quantum field theory (TQFT)
  - No local operators
  - ∃ non-local operators (Wilson loops, ...)

$\Rightarrow$ { Describe IR dynamics
- QCD in 2+1 d
- condensed matter

2. Scale invariant theory.

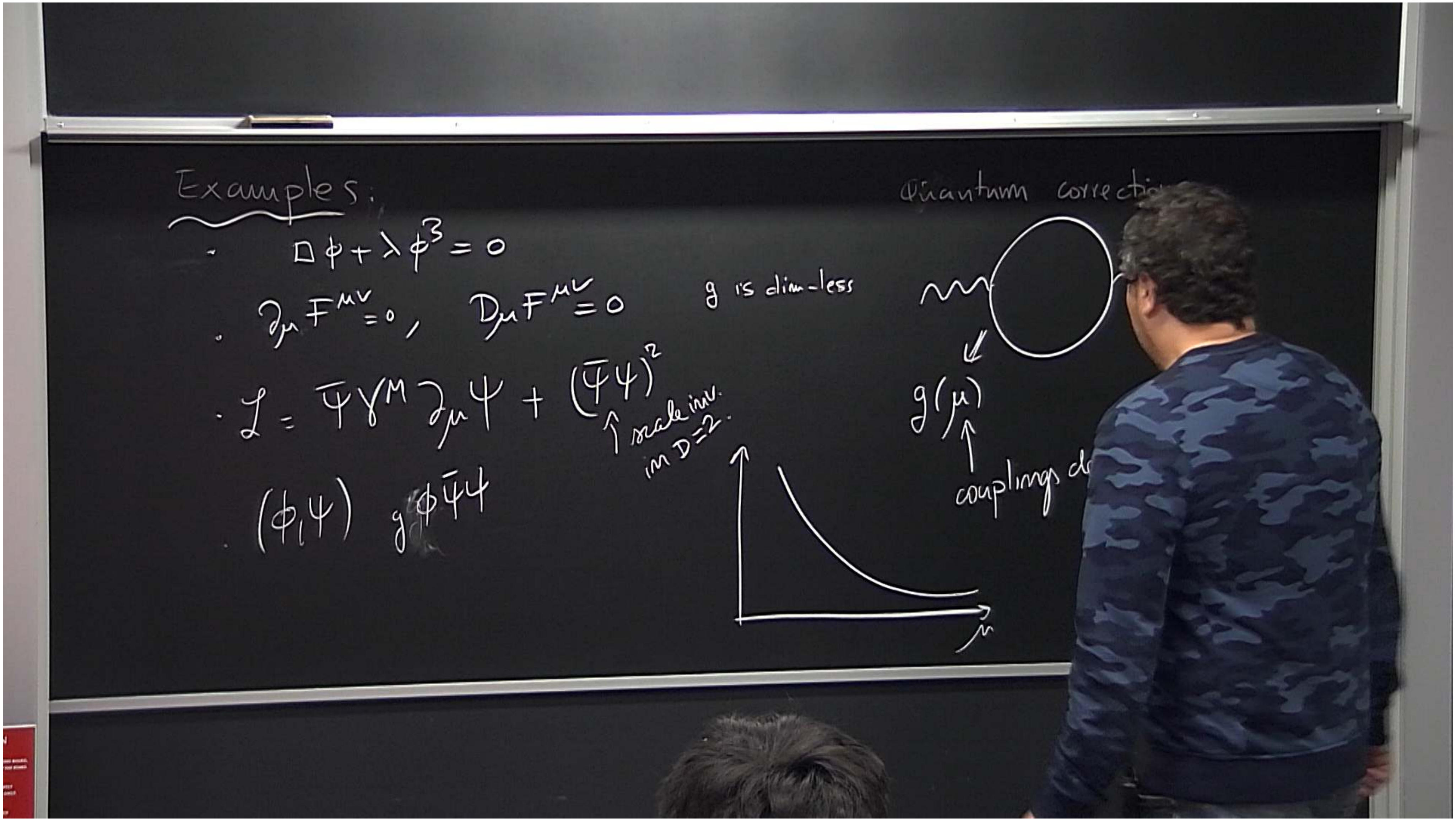
$$\text{Poincare} \longrightarrow \text{Poincare}' \rtimes \mathbb{R} \longrightarrow \text{Conformal}$$

$\Downarrow$ $\uparrow$ dilatations.

$$\langle \mathcal{O}(x)\, \mathcal{O}(0) \rangle = \frac{1}{x^{\Delta}}$$

Δ scaling dimensions of operator

CFTs provide an ordering in the space of QFTs.

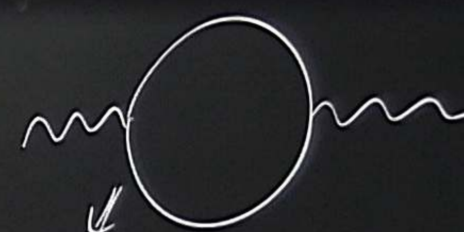

Examples:
$\Box\phi + \lambda\phi^3 = 0$
$\partial_\mu F^{\mu\nu} = 0$, $D_\mu F^{\mu\nu} = 0$
g is dim-less
$\mathcal{L} = \bar\Psi \gamma^\mu \partial_\mu \Psi + (\bar\Psi\Psi)^2$
scale inv. in D=2.
(ϕ, ψ) $g\phi\bar\Psi\Psi$
Quantum correcti
$g(\mu)$
couplings d
μ

Examples:

- $\Box\phi + \lambda\phi^3 = 0$
- $\partial_\mu F^{\mu\nu} = 0$, $D_\mu F^{\mu\nu} = 0$ $\quad$ g is dim-less
- $\mathcal{L} = \bar\Psi\gamma^\mu\partial_\mu\Psi + (\bar\Psi\Psi)^2$

  ↑ scale inv. in $D=2$.
- (ϕ,ψ) $\quad g\phi\bar\psi\psi$

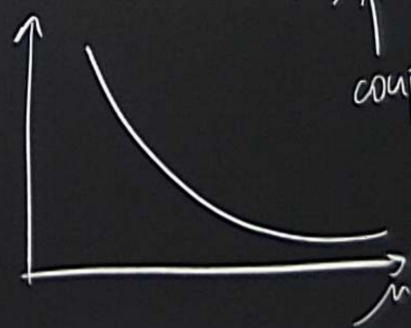
Quantum corrections

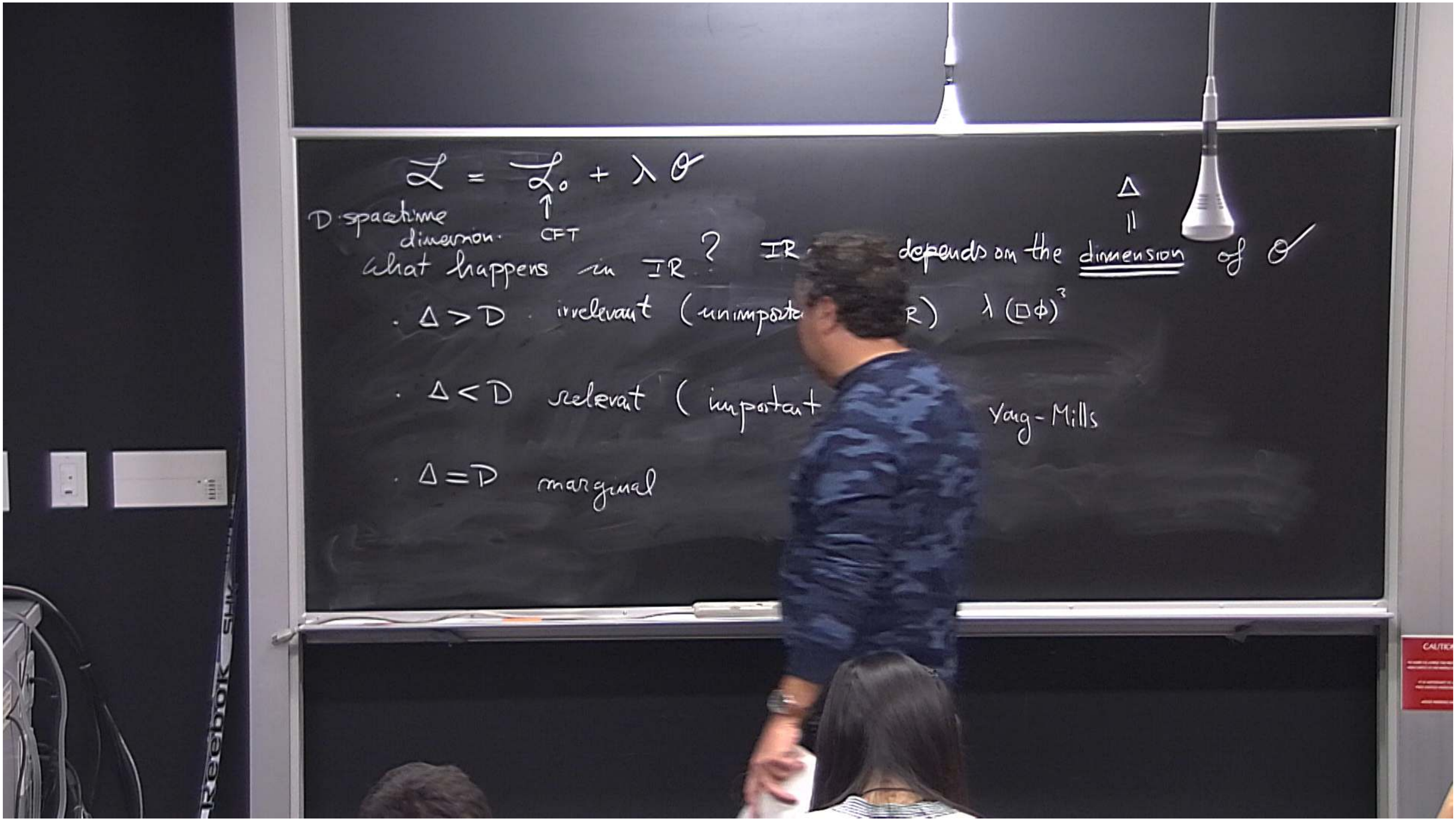


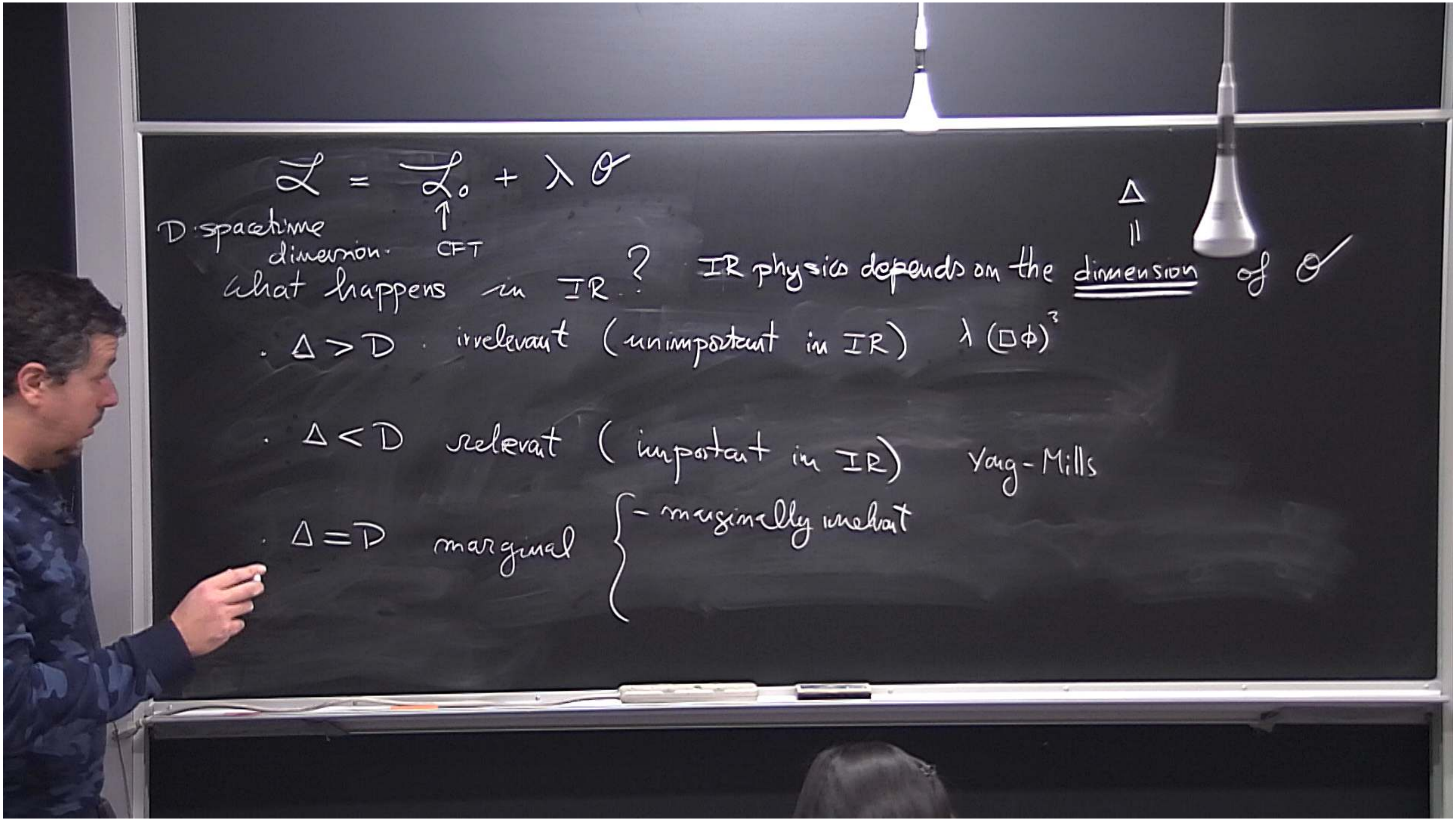
$g(\mu)$

↑ couplings depend on E scale

$$\beta(\lambda) = \mu\frac{d}{d\mu}\lambda(\mu) \neq 0$$




$\mathcal{L} = \mathcal{L}_0 + \lambda \mathcal{O}$
D: spacetime dimension.
CFT
What happens in IR ?
IR
depends on the dimension of $\mathcal{O}$
Δ
$\|$
· $\Delta > D$ · irrelevant (unimporta R) $\lambda (\Box\phi)^3$
· $\Delta < D$ relevant (important
Yang-Mills
· $\Delta = D$ marginal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what happens in IR?
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· $\Delta > D$ · irrelevant (unimportant in IR) $\lambda (\Box \phi)^3$
· $\Delta < D$ relevant (important in IR) Yang-Mills
· $\Delta = D$ marginal { - marginally irrelevant

$$\mathcal{L} = \mathcal{L}_0 + \lambda \mathcal{O}$$

$\uparrow$ CFT

D: spacetime dimension.

What happens in IR? IR physics depends on the $\underline{\text{dimension}}$ of $\mathcal{O}$ (Δ = dimension)

- $\Delta > D$: irrelevant (unimportant in IR) $\lambda (\Box\phi)^3$
- $\Delta < D$ relevant (important in IR) Φ^4
- $\Delta = D$ marginal
  - marginally irrelevant e.g. $\lambda\phi^4$ $\beta > 0$
  - marginally relevant e.g. Yang-Mills, $\beta < 0$

$$\mathcal{L} = \underset{\substack{\uparrow \\ \text{CFT}}}{\mathcal{L}_0} + \lambda \mathcal{O}$$

D: spacetime dimension.

What happens in IR? IR physics depends on the dimension (Δ) of $\mathcal{O}$

- $\Delta > D$: irrelevant (unimportant in IR) $\lambda (\Box\phi)^3$

$$\boxed{\begin{array}{ll} \cdot\ \Delta < D & \text{relevant (important in IR)} \quad \bar{\psi}\psi \\ \cdot\ \Delta = D & \text{marginal} \begin{cases} \text{- marginally irrelevant e.g. } \lambda\phi^4 \quad \beta > 0 \\ \text{- marginally relevant e.g. Yang-Mills } \beta < 0 \end{cases} \end{array}}$$

$(z=3)$