Title: PSI 17/18 - Cosmology - Lecture 3

Date: Jan 31, 2018 11:30 AM

URL: http://pirsa.org/18010053

Abstract:

LAST TIME: HOM & ISOTROPY CONSTRAIN GEOMETRY OF OUR UNIVERSE TO FRW FORM:

$$\boxed{ds^2 = -dt^2 + a^2(t)\, g^{(3,0)K}_{ij}\, dx^i dx^j}$$

(FRAME ADAPTED TO COMOVING OBSERVERS)

OTHER COORDS: $ds^2 = -dt^2 + a^2\left(\frac{dr^2}{1-Kr^2} + r^2 d\Omega^2\right)$

$= -dt^2 + a^2\left(d\chi^2 + S_K^2(\chi)\, d\Omega^2\right)$

OR INTRODUCING CONFORMAL TIME η: $\boxed{dt = a\, d\eta}$

$$ds^2 = a^2(\eta)\left(-d\eta^2 + d\chi^2 + S_K^2(\chi)\, d\Omega^2\right)$$

GEOMETRY OR INTRODUCING CONFORMAL TIME η: $\boxed{dt = a\,d\eta}$

$$\boxed{ds^2 = a^2(\eta)\left(-d\eta^2 + d\chi^2 + Sk^2(\chi)\,d\Omega^2\right)}$$

- **HORIZONS**

CONSIDER RADIAL PHOTONS $\quad ds^2 = 0,\ d\Omega^2 = 0$

$$d\eta = \pm d\chi = \frac{dt}{a}$$

OBSERVATIONAL FACT: $\dot{a} > 0$

- $a = a_0 \left(\frac{t}{t_0}\right)^p \sim t^p \quad p = \text{CONST} > 0$

(TRUE IN A GIVE EPOCH OF DOMINATION)

- $t \in (0, \infty)$

BB

- $\ddot{a} \sim p(p-1) t^{p-2}$

$$\eta = \int \frac{dt}{a} \sim \int t^{-p} dt \sim \frac{1}{1-p} t^{1-p}$$

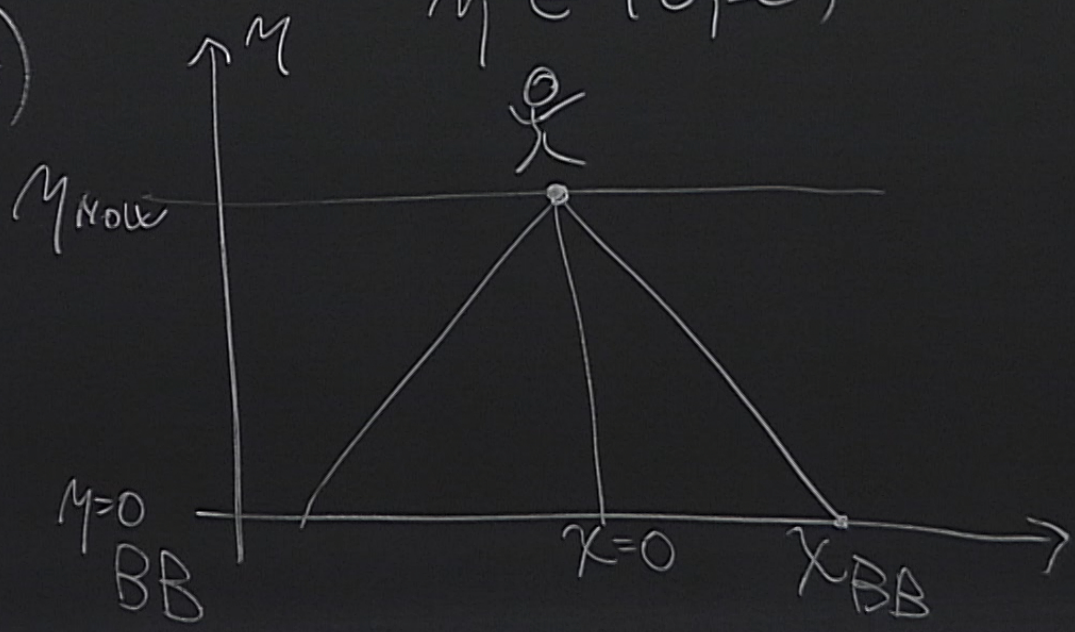
DECELERATING $\ddot{a} < 0$

$p \in (0,1)$

$\eta \in (0,\infty)$

ONST > 0

OF DOMINATION)



$\frac{1}{1-p} t^{1-p}$

DECELERATING $\ddot{a} < 0$

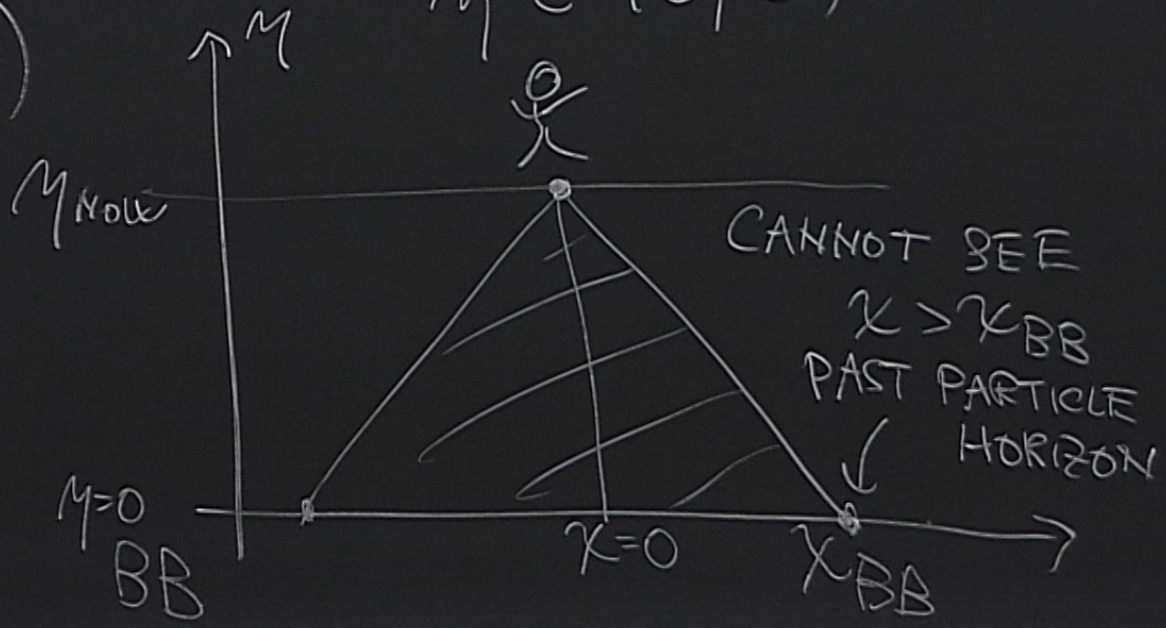
$p \in (0, 1)$

$\eta \in (0, \infty)$

CONST > 0

H OF DOMINATION)

$\frac{1}{1-p} t^{1-p}$



ATING $\ddot{a}<0$

$(0,1)$

$(0,\infty)$

CANNOT SEE

$\chi > \chi_{BB}$

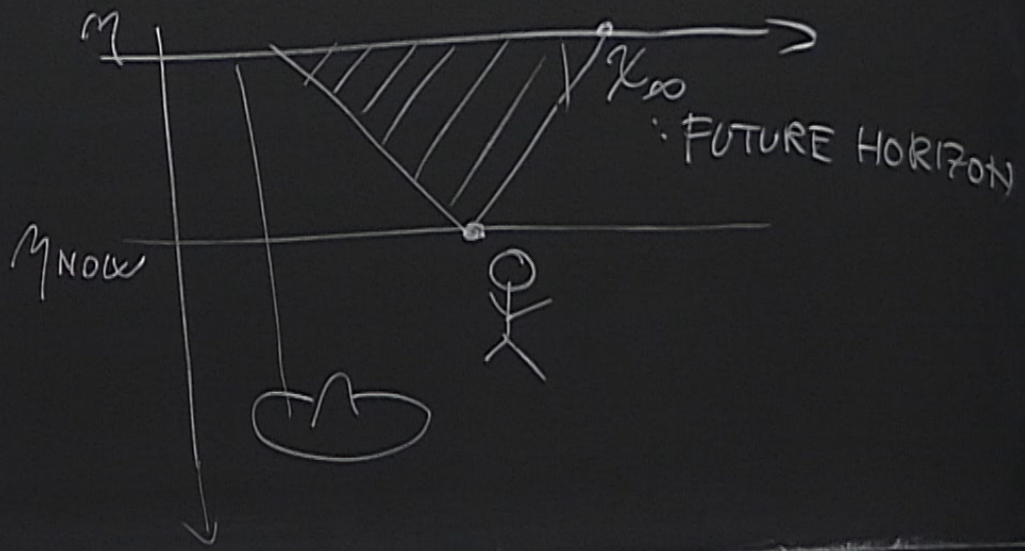
PAST PARTICLE HORIZON

χ_{BB}

ACCELERATING $\ddot{a}>0$

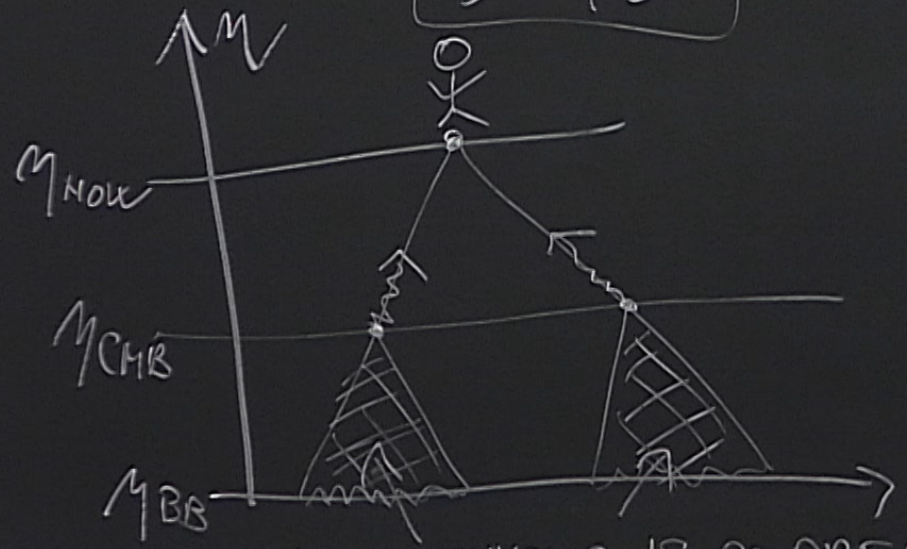
$p>1$

$\eta \in (-\infty, 0)$



HORIZON PUZZLE: BACK IN 70'S

ρ_m, ρ_Λ $\ddot{a} < 0$ ALL TIMES

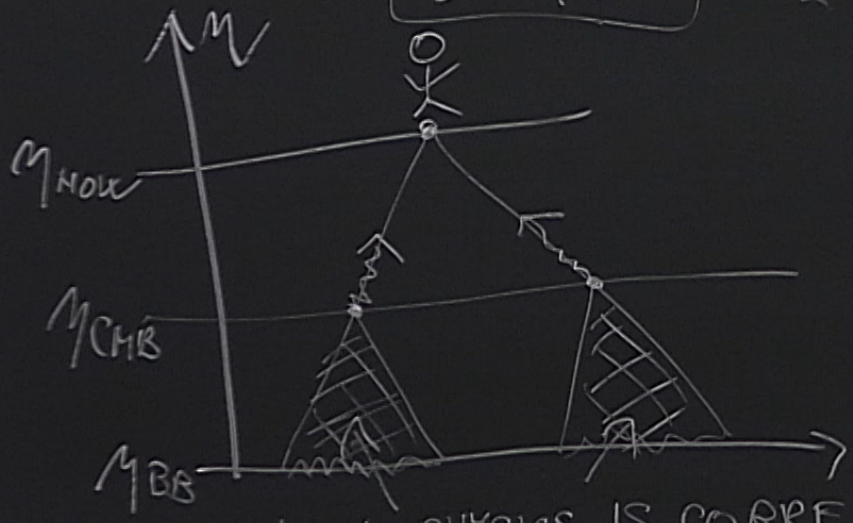


WHY PHYSICS IS CORRELATED (?)

HORIZON PUZZLE: BACK IN 70's

ρ_m, ρ_Λ

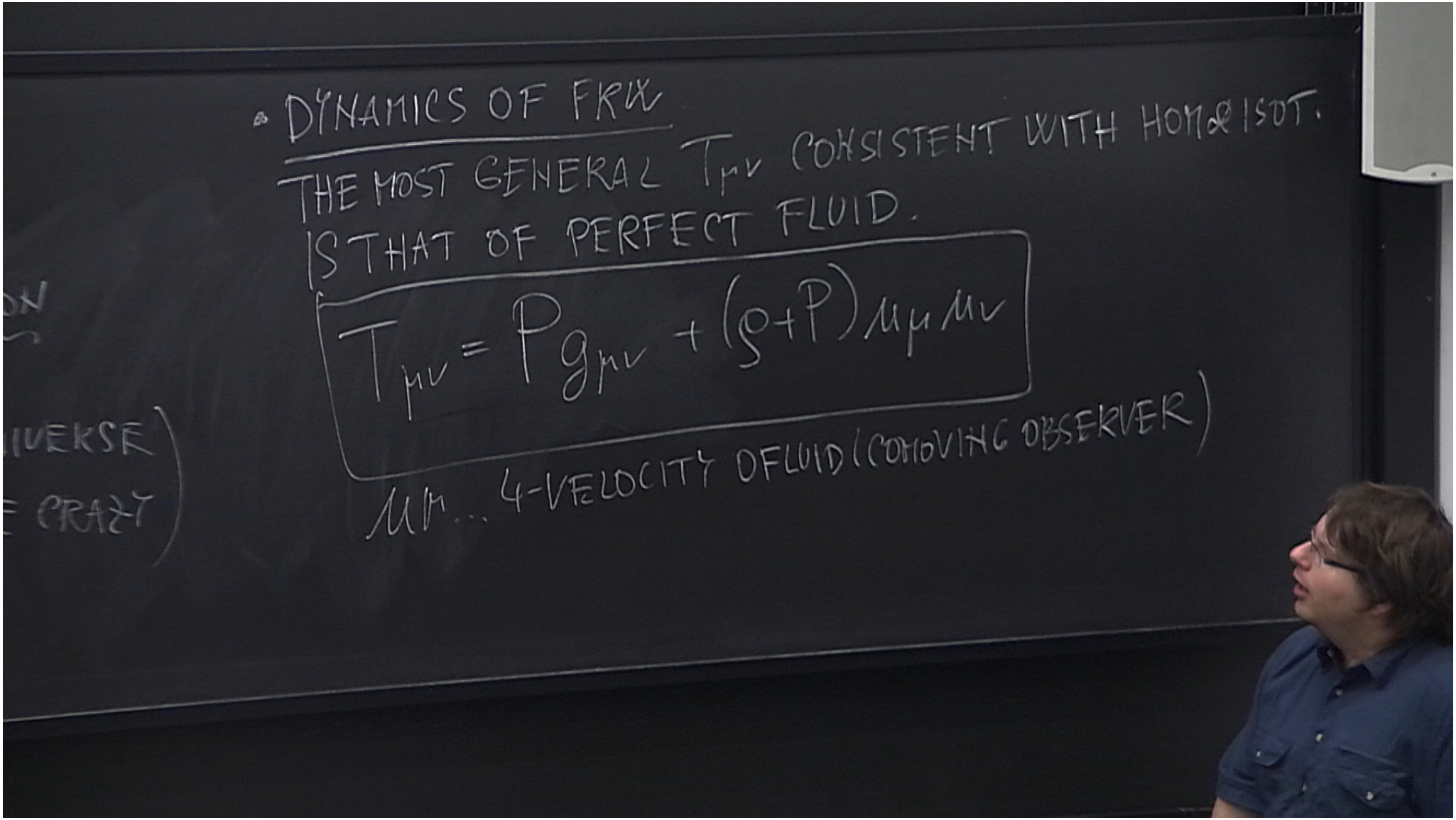
$\ddot{a} < 0$ ALL TIMES

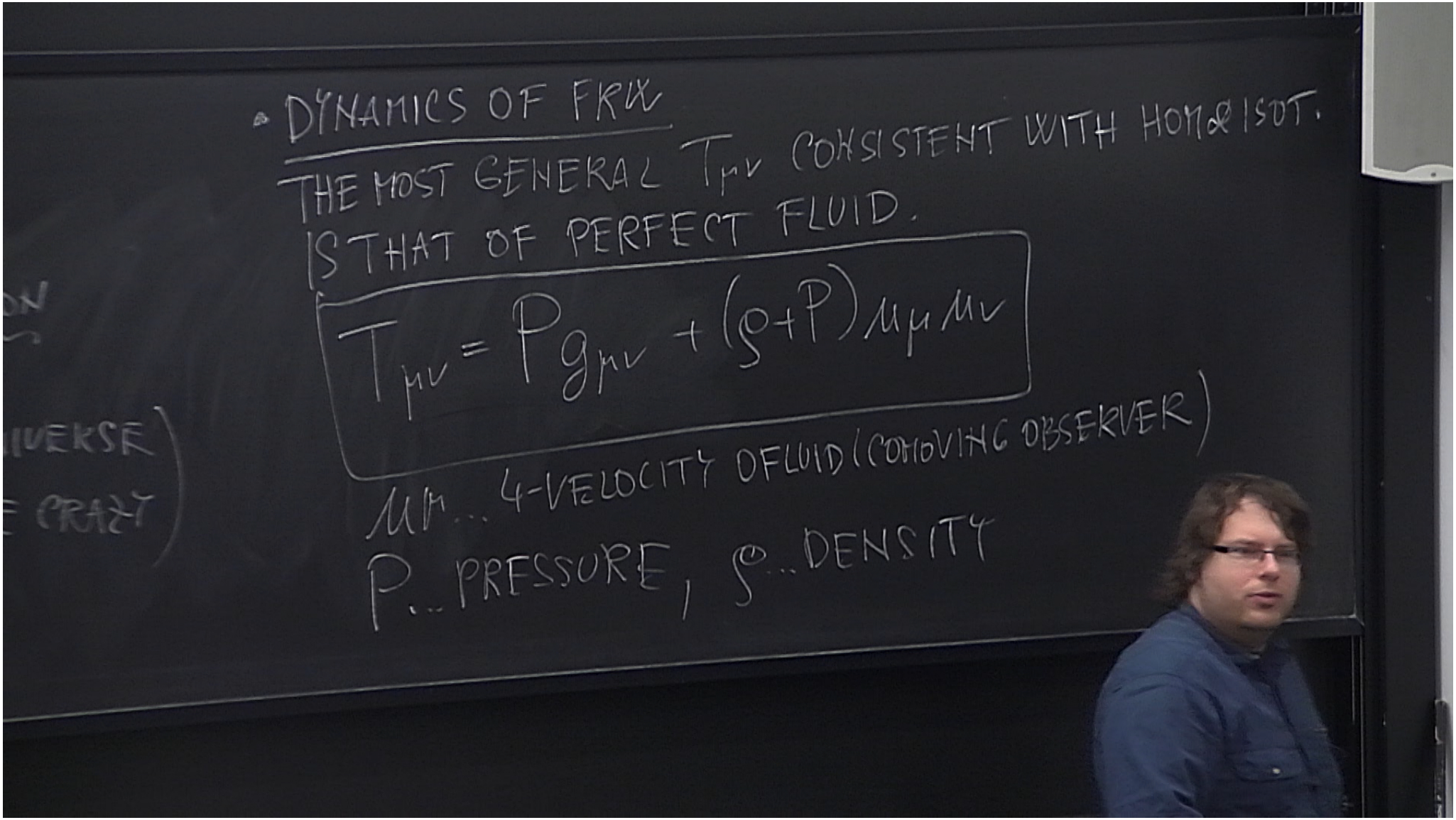


⇒ IDEA OF INFLATION

(AFTER BIG BANG
≃ PERIOD WHEN UNIVERSE
EXPANDING LIKE CRAZY)

WHY PHYSICS IS CORRELATED (?)


• DYNAMICS OF FRW
THE MOST GENERAL $T_{\mu\nu}$ CONSISTENT WITH HOM & ISOT.
IS THAT OF PERFECT FLUID.
$T_{\mu\nu} = P g_{\mu\nu} + (\rho + P) u_\mu u_\nu$
u_μ ... 4-VELOCITY OF FLUID (COMOVING OBSERVER)


DYNAMICS OF FRW
THE MOST GENERAL $T_{\mu\nu}$ CONSISTENT WITH HOM & ISOT.
IS THAT OF PERFECT FLUID.
$T_{\mu\nu} = P g_{\mu\nu} + (\rho + P) u_\mu u_\nu$
u_μ... 4-VELOCITY OF FLUID (COMOVING OBSERVER)
P... PRESSURE, ρ... DENSITY

2 INDEPENDENT EINSTEIN EQS:

$$H^2 = \frac{8\pi G_N}{3}\rho - \frac{k}{a^2}$$

$$\dot{\rho} = -3H(\rho + P)$$

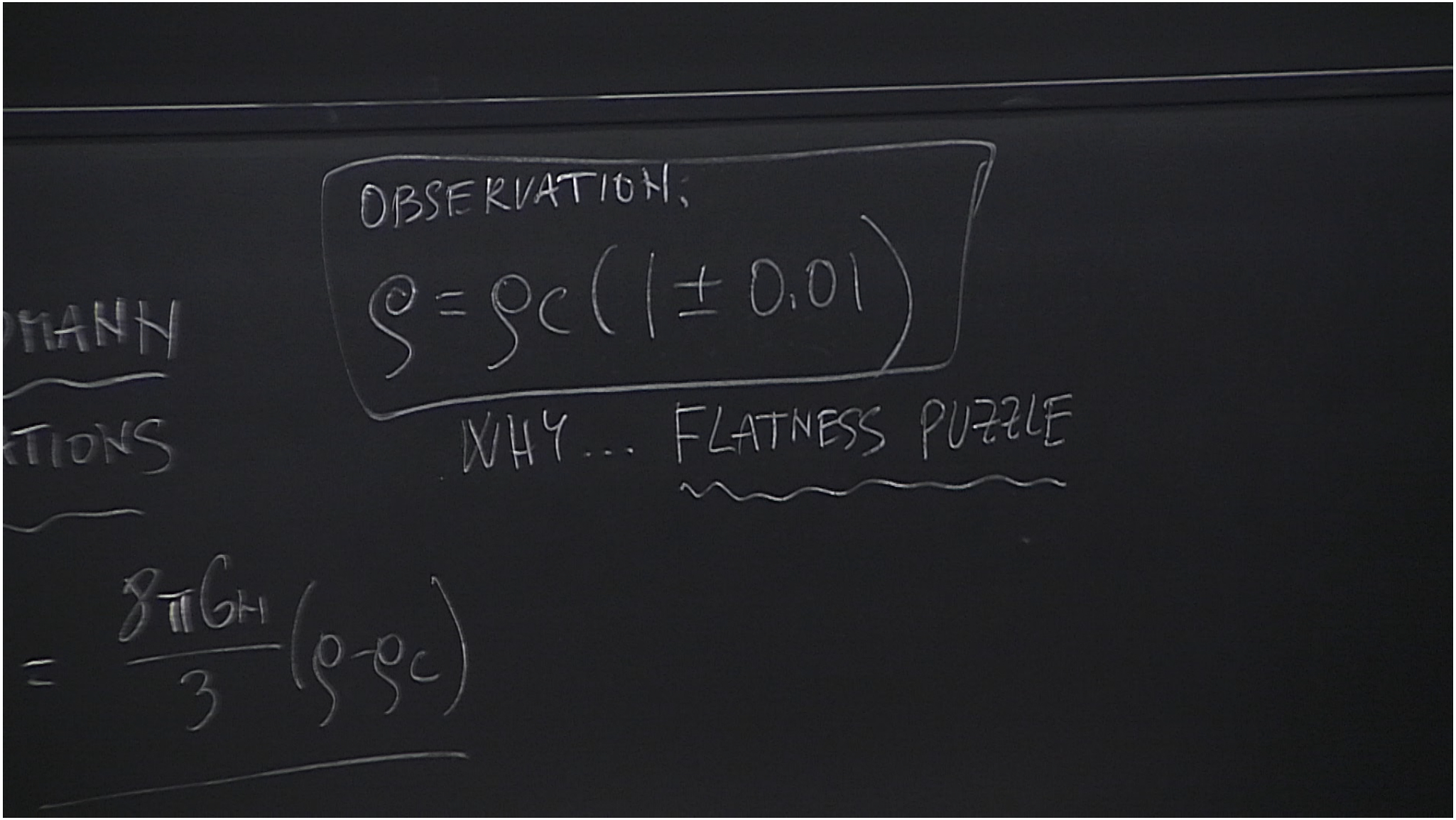
FRIEDMANN EQUATIONS

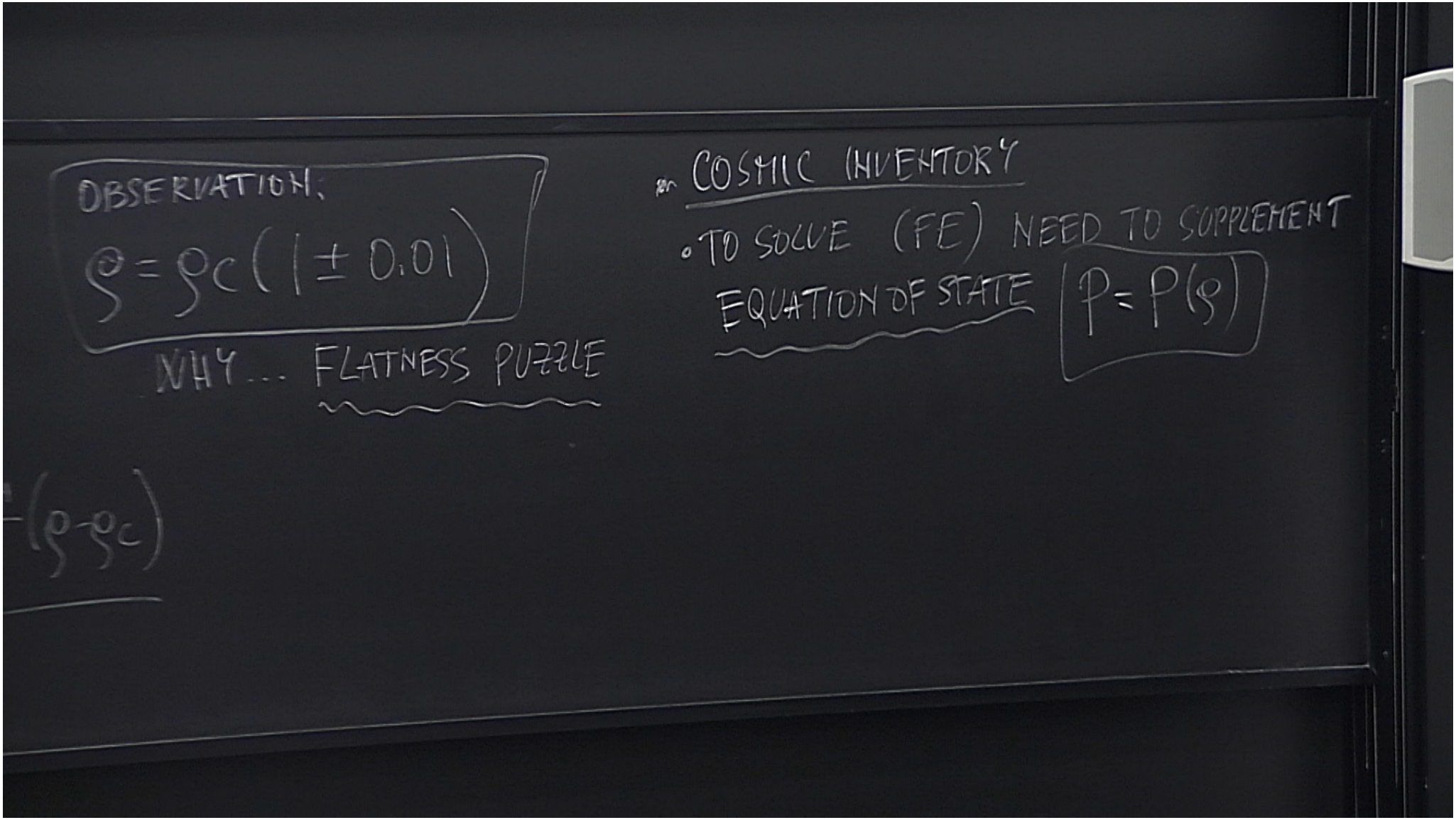
- CRITICAL DENSITY

$$\rho_c(t) = \frac{3H^2(t)}{8\pi G_N}$$

$$\frac{k}{a^2} = \frac{8\pi G_N}{3}(\rho - \rho_c)$$

OBSERVA


OBSERVATION:
$\rho = \rho_c(1 \pm 0.01)$
WHY... FLATNESS PUZZLE
$= \frac{8\pi G_N}{3}(\rho - \rho_c)$


OBSERVATION:
$\rho = \rho_c (1 \pm 0.01)$
WHY... FLATNESS PUZZLE
$-(\rho - \rho_c)$
COSMIC INVENTORY
• TO SOLVE (FE) NEED TO SUPPLEMENT
EQUATION OF STATE $P = P(\rho)$

OBSERVATION:

$\rho = \rho_c (1 \pm 0.01)$

WHY... FLATNESS PUZZLE

$\rho - \rho_c$)

COSMIC INVENTORY

- TO SOLVE (FE) NEED TO SUPPLEMENT EQUATION OF STATE $P = P(\rho)$

TYPICALLY $P = w\rho$ $w = \text{CONST}$

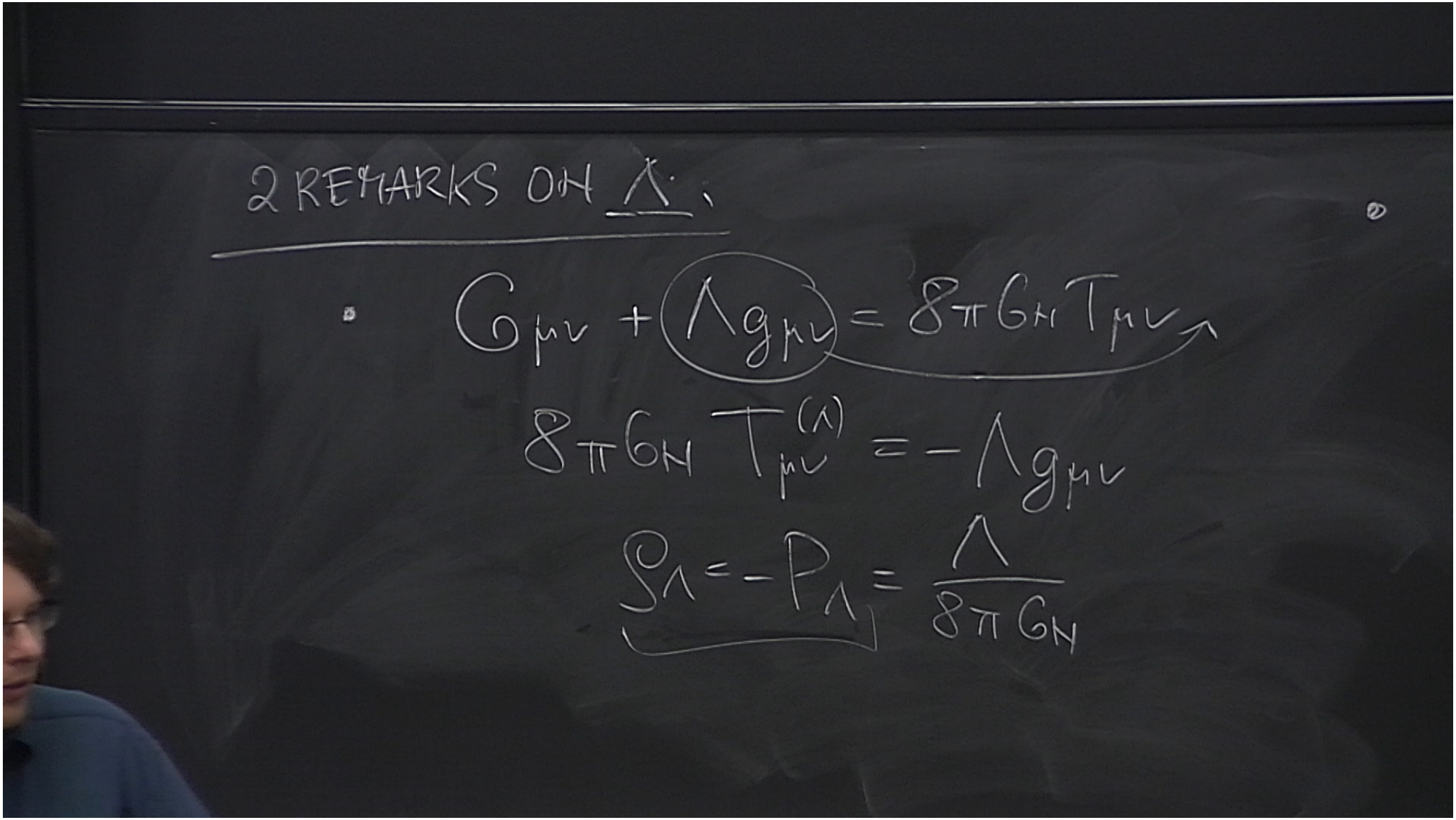
AND (FE)

$$\dot{\rho} = -3H(\rho + w\rho) = -3H\rho(1+w) = -3\rho(1+w)\frac{\dot{a}}{a}$$

$$\frac{\dot{\rho}}{\rho} = -3(1+w)\frac{\dot{a}}{a} \Rightarrow \boxed{\rho \propto a^{-3(1+w)}}$$

TYPICAL MATTER CONTENT

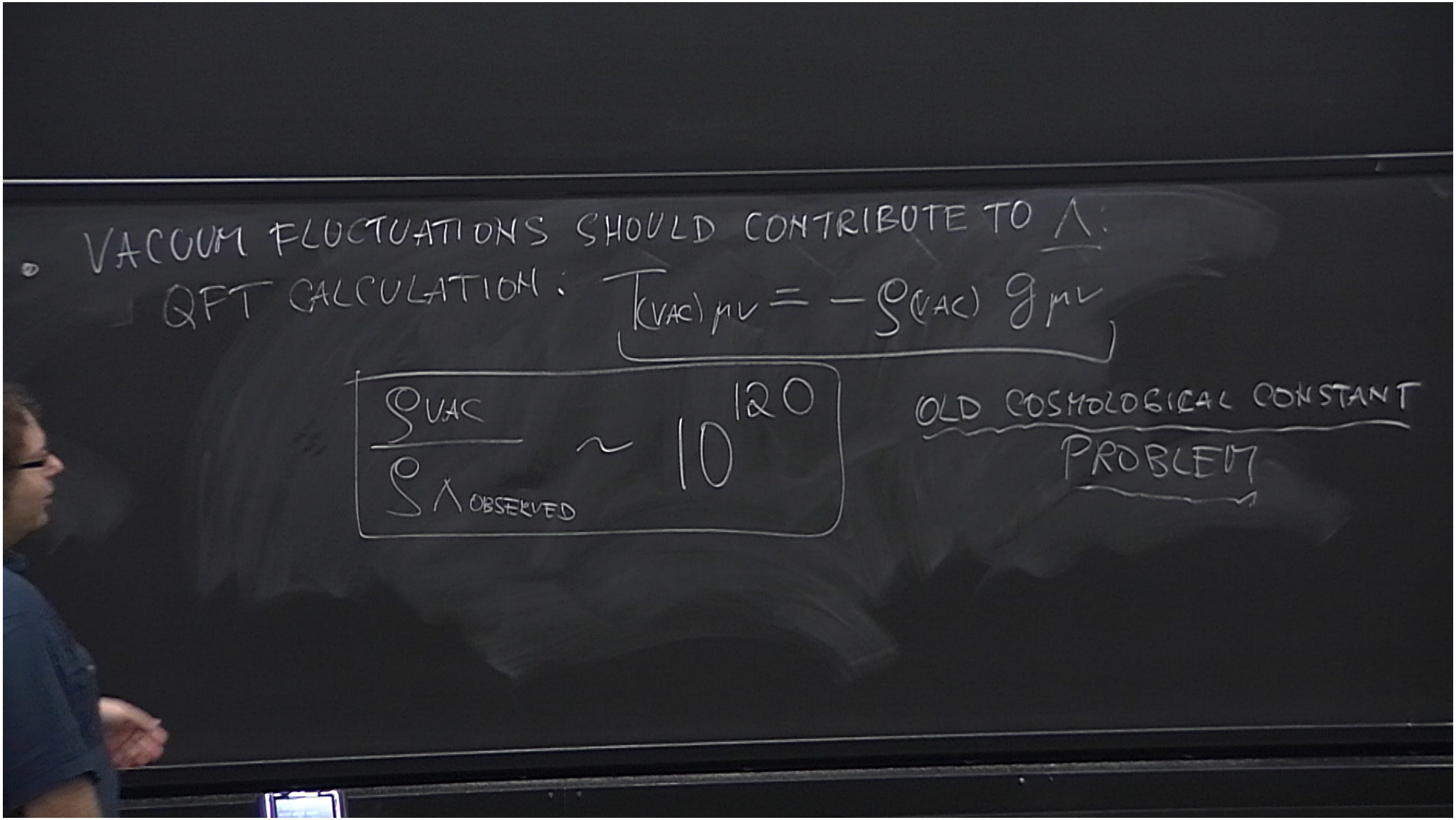
COMPONENT	EXAMPLE	EQ OF STATE	DENSITY
MATTER	GALAXY, DUST, DM	$P \approx 0$, $w = 0$	$\rho \sim a^{-3}$
RADIATION	PHOTONS, GRAVITONS, NEUTRINOS	$P = \frac{1}{3}\rho$, $w = \frac{1}{3}$	$\rho \sim a^{-4}$
COSMOLOGICAL CONSTANT	Λ	$P = -\rho$, $w = -1$	$\rho = \text{CONST}$
CURVATURE	K	$P = -\frac{1}{3}\rho$, $w = -\frac{1}{3}$	$\rho \sim \frac{1}{a^2}$

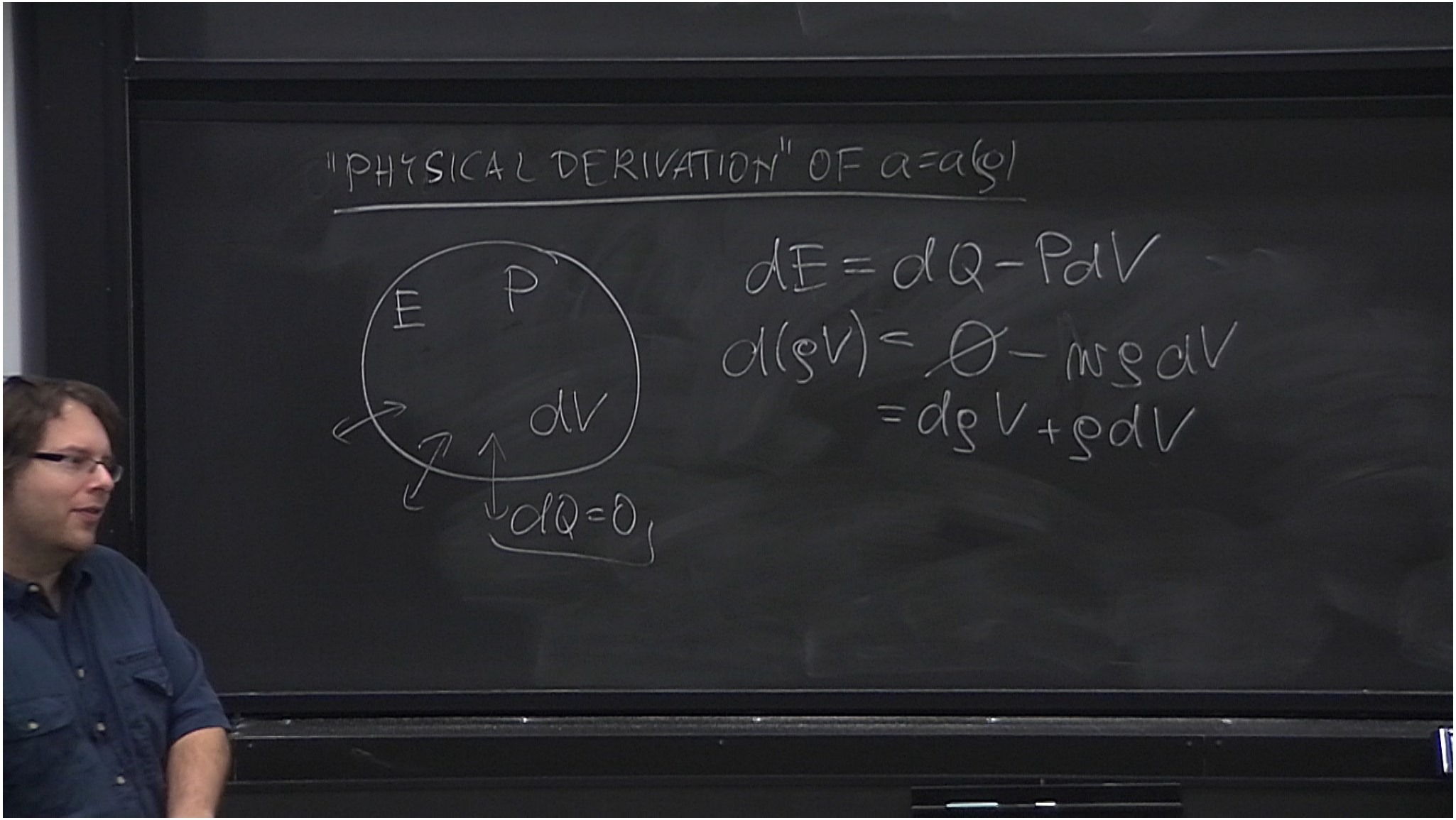

2 REMARKS ON Λ:
$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G_N T_{\mu\nu}$
$8\pi G_N T^{(\Lambda)}_{\mu\nu} = -\Lambda g_{\mu\nu}$
$\rho_\Lambda = -P_\Lambda = \frac{\Lambda}{8\pi G_N}$

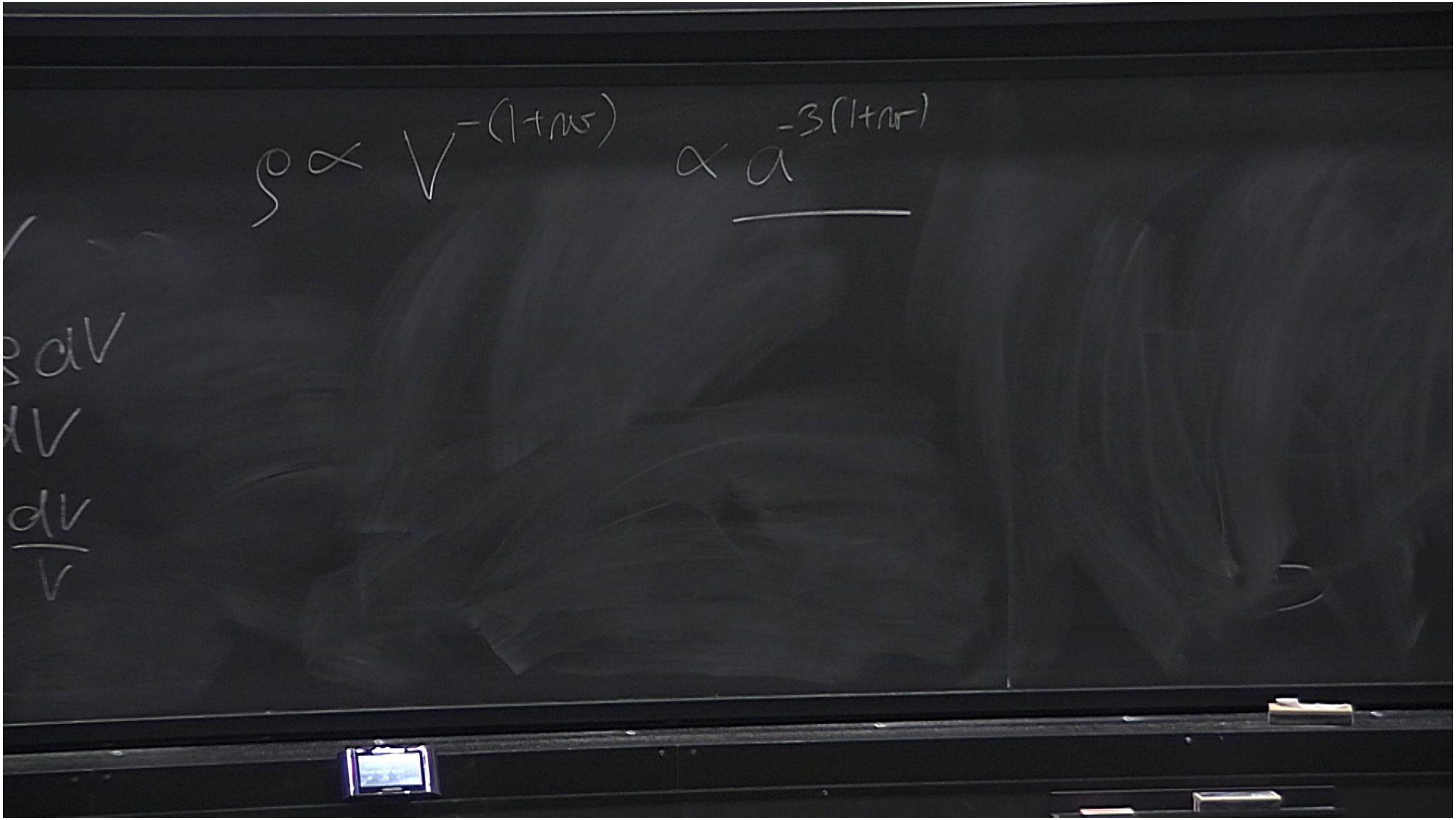
- VACUUM FLUCTUATIONS SHOULD CONTRIBUTE TO Λ:

  QFT CALCULATION: $T_{(VAC)\mu\nu} = -\rho_{(VAC)}\, g_{\mu\nu}$

$$\frac{\rho_{VAC}}{\rho_{\Lambda\ \text{OBSERVED}}} \sim 10^{120}$$


VACUUM FLUCTUATIONS SHOULD CONTRIBUTE TO Λ:
QFT CALCULATION: $T_{(VAC)\mu\nu} = -\rho_{(VAC)}\, g_{\mu\nu}$
$\frac{\rho_{VAC}}{\rho_{\Lambda\,OBSERVED}} \sim 10^{120}$
OLD COSMOLOGICAL CONSTANT PROBLEM


"PHYSICAL DERIVATION" OF a=a(ρ)
E
P
dV
dQ=0
dE = dQ - PdV
d(ρV) = 0 - nρdV
= dρV + ρdV


$\rho \propto V^{-(1+n)}$
$\propto a^{-3(1+n)}$
$\int dV$
dV
$\frac{dV}{V}$