Title: PSI 17/18 - Cosmology - Lecture 2

Date: Jan 30, 2018 11:30 AM

URL: http://pirsa.org/18010052

Abstract:

YESTERDAY: ALL MAXIMALLY SYMMETRIC SPACES: ARG

$$ds^2 = \eta^{(p,q)\,K}_{AB}\, dx^A dx^B = \eta^{(p,q)}_{ab}\, dx^a dx^b + K (dx^{n+1})^2$$

$$K\rho^2 = \eta^{(p,q)\,K}_{AB}\, X^A X^B$$

$$\Rightarrow \quad g^{(p,q)\,K}_{ab} = \eta^{(p,q)}_{ab} + \frac{K x_a x_b}{\rho^2 - K x^2}$$

ES:

$(dx^{n+1})^2$

ARGUED THAT AT MOST WE CAN HAVE

$$n + \binom{n}{2} = \frac{n(n+1)}{2} \quad \underline{\text{KVs}}$$

YESTERDAY: ALL MAXIMALLY SYMMETRIC SPACES: ARGUED TH

$$ds^2 = \eta^{(p,q)K}_{AB}\, dX^A dX^B = \eta^{(p,q)}_{ab}\, dx^a dx^b + K(dx^{n+1})^2$$

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$$\Rightarrow \quad g^{(p,q)K}_{ab} = \eta^{(p,q)}_{ab} + \frac{K x_a x_b}{\rho^2 - K x^2}$$

EX: S_n, H_n, E_n

M_n, AdS_n, dS_n

BUILDING BLOCK FOR FRW

ETRIC SPACES:

$$\ldots + K\,(dx^{n+1})^2$$

EX: S_n, H_n, E_n — BUILDING BLOCK FOR FRW

M_n, AdS_n, dS_n

ARGUED THAT AT MOST WE CAN HAVE

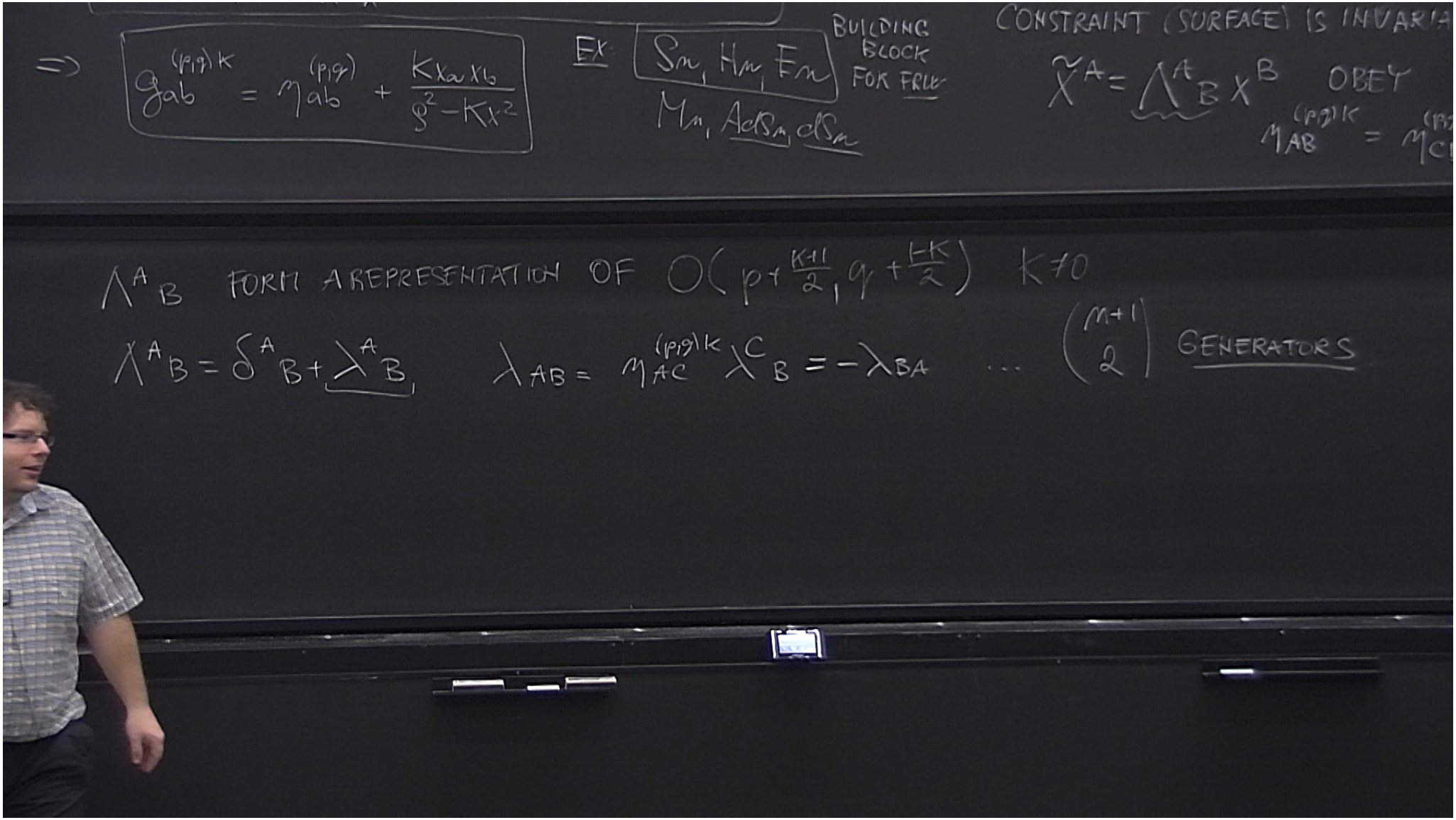
$$n + \binom{n}{2} = \frac{n(n+1)}{2} \quad \text{KVs}$$

ξ^a $\quad L^{ab} = \nabla^a \xi^b$

CONSTRAINT (SURFACE) IS INVARIANT UNDER

$$\tilde{X}^A = \Lambda^A{}_B X^B \quad \text{OBEY}$$

$$M_{AB}^{(p,q)k} = M_{CD}^{(p,q)k} \Lambda^C{}_A \Lambda^D{}_B$$


$\Rightarrow$
$g_{ab}^{(p,q)k} = \eta_{ab}^{(p,q)} + \frac{k x_a x_b}{s^2 - k x^2}$
Ex: S_n, H_n, F_n
M_n, AdS_n, dS_n
BUILDING BLOCK FOR FREE
CONSTRAINT (SURFACE) IS INVARIA
$\tilde{X}^A = \Lambda^A{}_B X^B$ OBEY
$M_{AB}^{(p,q)k} = \eta_{C}^{(p}$
$\Lambda^A{}_B$ FORM A REPRESENTATION OF $O(p+\frac{k+1}{2}, q+\frac{1-k}{2})$ $k \neq 0$
$\Lambda^A{}_B = \delta^A{}_B + \lambda^A{}_B$, $\lambda_{AB} = \eta_{AC}^{(p,q)k} \lambda^C{}_B = -\lambda_{BA}$ ... $\binom{n+1}{2}$ GENERATORS

$\Rightarrow$ $g_{ab}^{(p,q)k} = \eta_{ab}^{(p,q)} + \frac{K x_a x_b}{\beta^2 - K x^2}$

Ex: S_n, H_n, E_n BUILDING BLOCK FOR FRW

M_n, AdS_n, dS_n

CONSTRAINT (SURFACE) IS INVARIANT UNDER $\tilde{X}^A = \Lambda^A{}_B X^B$ OBEY

$\eta_{AB}^{(p,q)k} = \eta_{CD}^{(p,q)k} \Lambda^C$

$\Lambda^A{}_B$ FORM A REPRESENTATION OF $O(p + \frac{K+1}{2}, q + \frac{1-K}{2})$ $K \neq 0$

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b) FRW SPACETIME

- SYMMETRY OF UNIVERSE: ROTATIONAL SYMMETRY (ISOTROPY) + TRANSLATIONAL SYMMETRY (HOMOGENEITY)

3 KVs



$\Rightarrow$ $g_{ab}^{(p,q)k} = \eta_{ab}^{(p,q)} + \frac{K x_a x_b}{\beta^2 - K x^2}$

Ex S_n, H_n, E_n BUILDING BLOCK FOR FRW

M_n, AdS_n, dS_n

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b) FRW SPACETIME

- SYMMETRY OF UNIVERSE : ROTATIONAL SYMMETRY (ISOTROPY) + TRANSLATIONAL SYMMETRY (HOMOGENEITY)

DON'T HAVE "BOOSTS" 3 KVs 3 KVs

$\Rightarrow$ $g_{ab}^{(p,q)k} = \eta_{ab}^{(p,q)} + \frac{K x_a x_b}{\rho^2 - K x^2}$

Ex: S_n, H_n, E_n BUILDING BLOCK FOR FRW

M_n, AdS_n, dS_n

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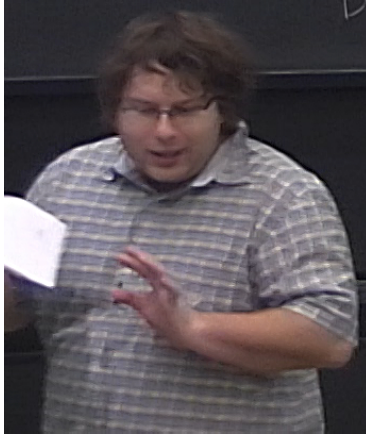
$\Lambda^A{}_B = \delta^A{}_B + \lambda^A{}_B$, $\lambda_{AB} = \eta_{AC}^{(p,q)k} \lambda^C{}_B = -\lambda_{BA}$ ... $\binom{n+1}{2}$ GENERATORS

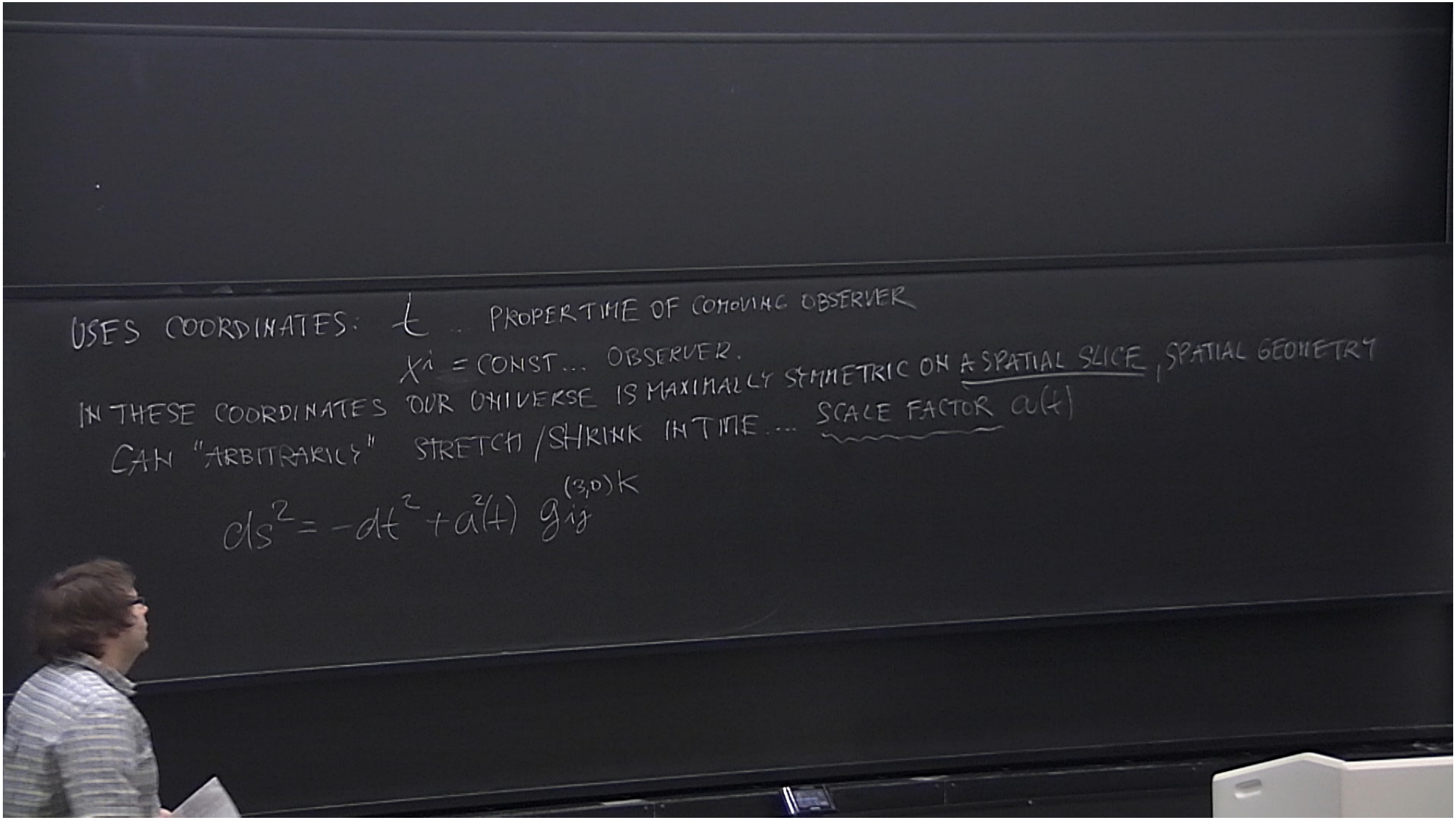
b) FRW SPACETIME

- SYMMETRY OF UNIVERSE: ROTATIONAL SYMMETRY (ISOTROPY) + TRANSLATIONAL SYMMETRY (HOMOGENEITY)

3 KVs 3 KVs

DON'T HAVE "BOOSTS" $\Rightarrow \exists$ PREFERRED FRAME $\simeq$ COMOVING OBSERVER




USES COORDINATES: t .. PROPER TIME OF COMOVING OBSERVER
x^i = CONST... OBSERVER.
IN THESE COORDINATES OUR UNIVERSE IS MAXIMALLY SYMMETRIC ON A SPATIAL SLICE, SPATIAL GEOMETRY
CAN "ARBITRARILY" STRETCH/SHRINK IN TIME... SCALE FACTOR $a(t)$
$ds^2 = -dt^2 + a^2(t)\, g_{ij}^{(3,0)k}$

USES COORDINATES: t .. PROPER TIME OF COMOVING OBSERVER

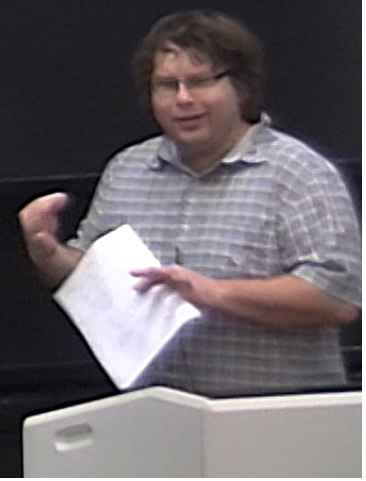
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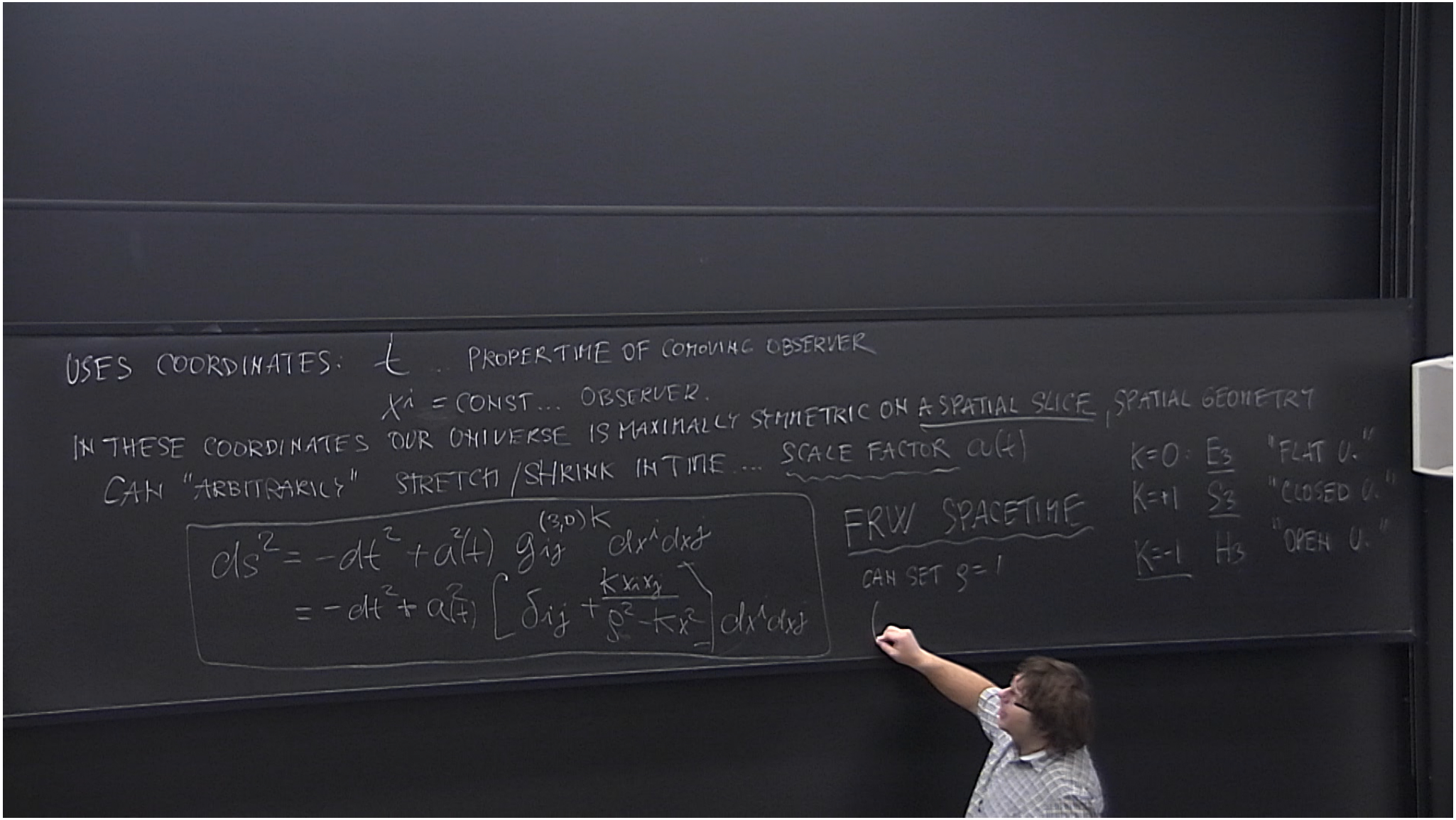
IN THESE COORDINATES OUR UNIVERSE IS MAXIMALLY SYMMETRIC ON A SPATIAL SLICE, SPATIAL GEOMETRY CAN "ARBITRARILY" STRETCH/SHRINK IN TIME... SCALE FACTOR $a(t)$

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$$= -dt^2 + a^2(t)\left[\delta_{ij} + \frac{K x_i x_j}{\rho^2 - K x^2}\right] dx^i dx^j$$

FRW SPACETIME




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FRW SPACETIME
CAN SET $\rho = 1$
$K=0$: E_3 "FLAT U."
$K=+1$ S_3 "CLOSED U."
$K=-1$ H_3 "OPEN U."

USES COORDINATES: t .. PROPER TIME OF COMOVING OBSERVER

x^i = CONST... OBSERVER.

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FRW SPACETIME

CAN SET $g=1$

$(x^i = g\tilde{x}^i,\ a = \tilde{a}/g)$ $\Rightarrow$ $a[a]$ RADIUS OF CURVATURE

$K=0$: E_3 "FLAT U."

$K=+1$: S_3 "CLOSED U."

$K=-1$: H_3 "OPEN U."



USES COORDINATES: t ... PROPER TIME OF COMOVING OBSERVER

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FRW SPACETIME

CAN SET $\rho = 1$

$(\tilde{x}^i = \rho x^i,\ a = \tilde{a}/\rho)$ $\Rightarrow$ $a, |a|$ RADIUS OF CURVATURE

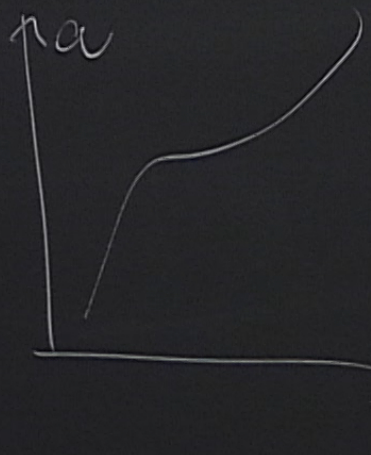
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• SCALE FACTOR . DETERMINED BY MATTER CONTENT

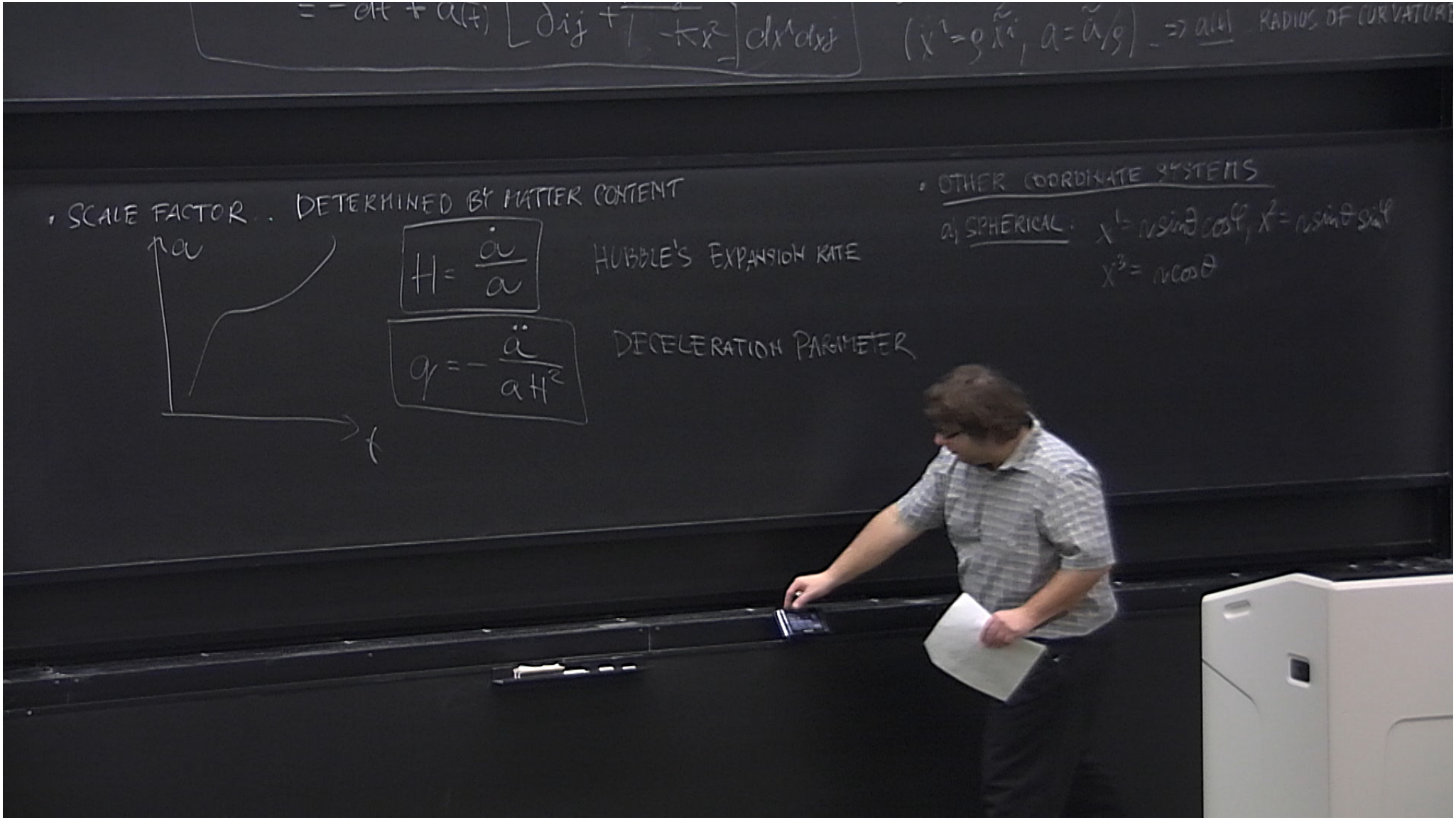


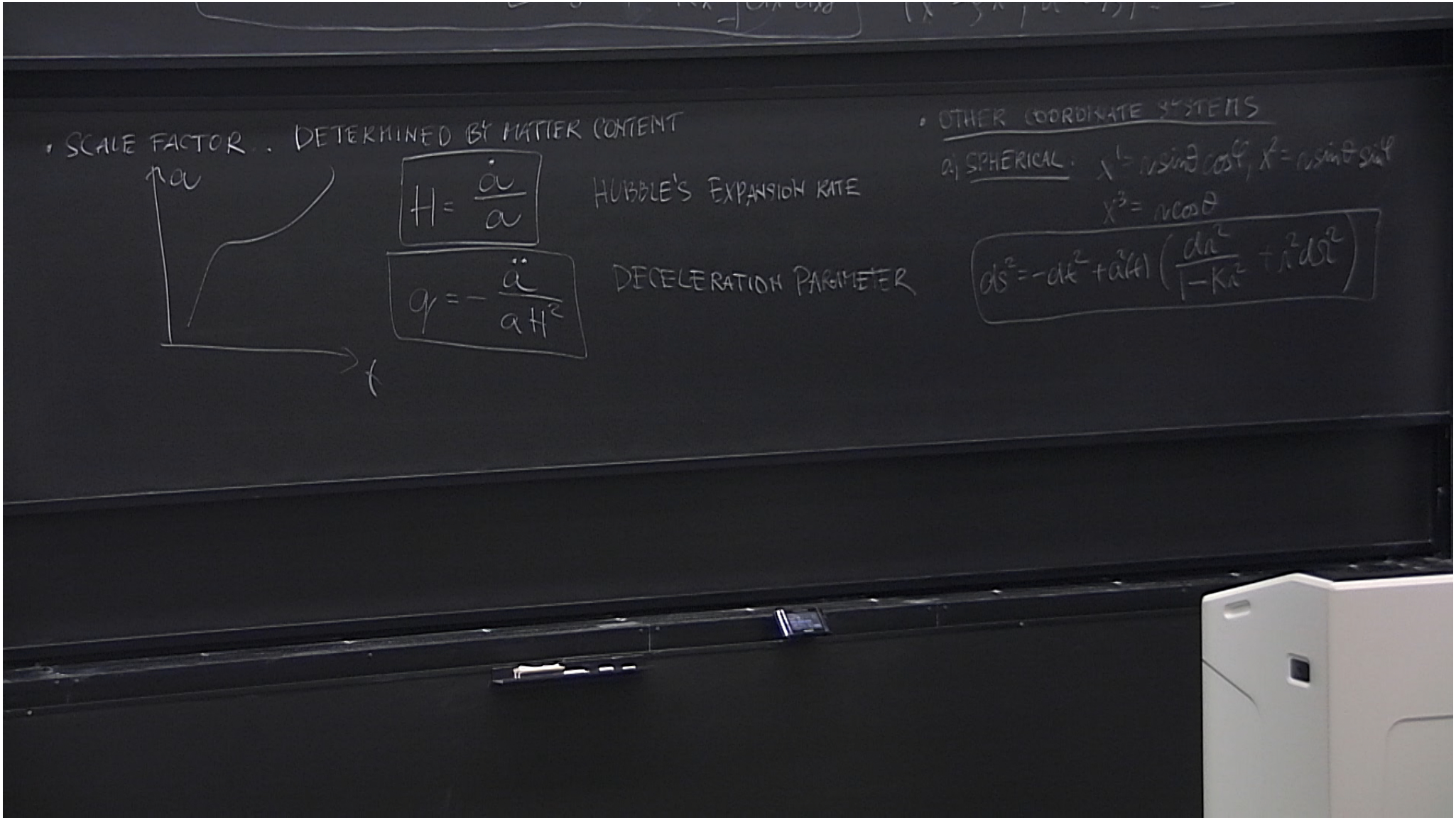
$$H = \frac{\dot{a}}{a}$$

HUBBLE'S EXPANSION RATE

$$q = -\frac{\ddot{a}}{aH^2}$$

DECELERATION PARAMETER


• SCALE FACTOR . DETERMINED BY MATTER CONTENT
a(t)
t
$H = \frac{\dot{a}}{a}$
HUBBLE'S EXPANSION RATE
$q = -\frac{\ddot{a}}{aH^2}$
DECELERATION PARAMETER
• OTHER COORDINATE SYSTEMS
a) SPHERICAL:
$x^1 = r\sin\theta\cos\varphi$, $x^2 = r\sin\theta\sin\varphi$
$x^3 = r\cos\theta$
RADIUS OF CURVATURE


• SCALE FACTOR . DETERMINED BY MATTER CONTENT
a
t
$H = \frac{\dot{a}}{a}$
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$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-Kr^2} + r^2 d\Omega^2\right)$

EXPANSION RATE

RATION PARAMETER

a) SPHERICAL: $x^1 = r\sin\theta\cos\varphi$, $x^2 = r\sin\theta\sin\varphi$

$x^3 = r\cos\theta$

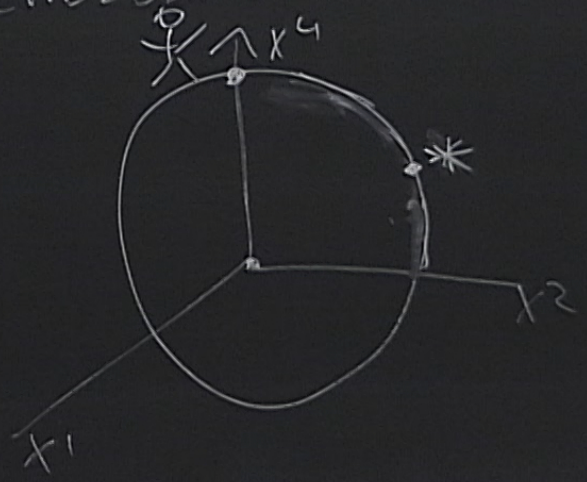
$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-Kr^2} + r^2 d\Omega^2\right)$$

b) NEW RADIAL COORDINATE χ

$$r = S_K(\chi)\begin{cases}\sin\chi & K=+1\\ \chi & K=0\\ \sinh\chi & K=-1\end{cases}$$

$$ds^2 = -dt^2 + a^2\left(dx^2 + S_k^2(x)\,d\Omega^2\right)$$

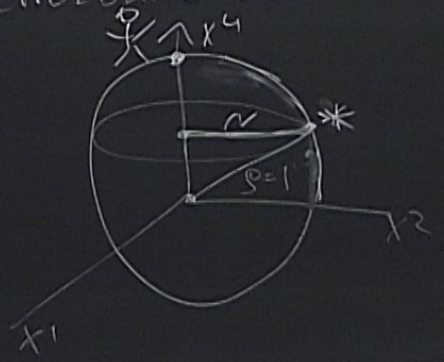
K=+1 EMBEDDING VIEW POINT



$$q = -\frac{}{aH^2}$$

$$ds^2 = -dt^2 + a^2\left(dx^2 + S_K^2(x)\,d\Omega^2\right)$$

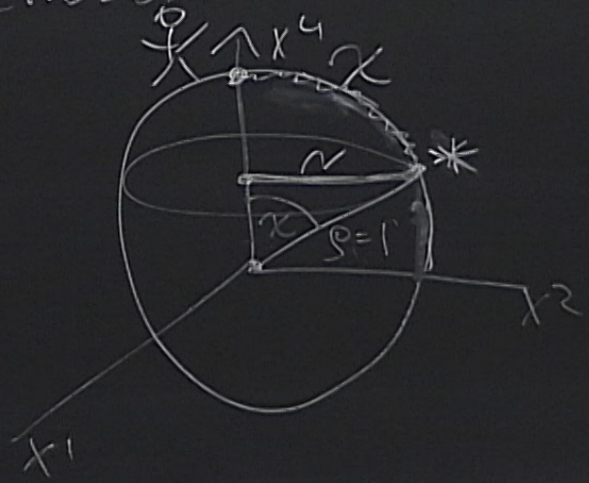
K=+1 EMBEDDING VIEW POINT



$r = \sin\chi$

$$ds^2 = -dt^2 + a^2\left(d\chi^2 + S_K^2(\chi)\, d\Omega^2\right)$$

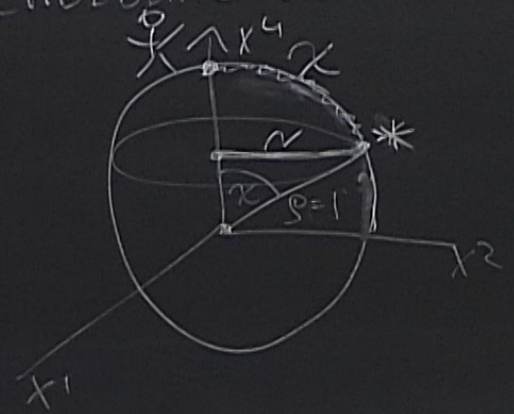
K=+1 EMBEDDING VIEW POINT



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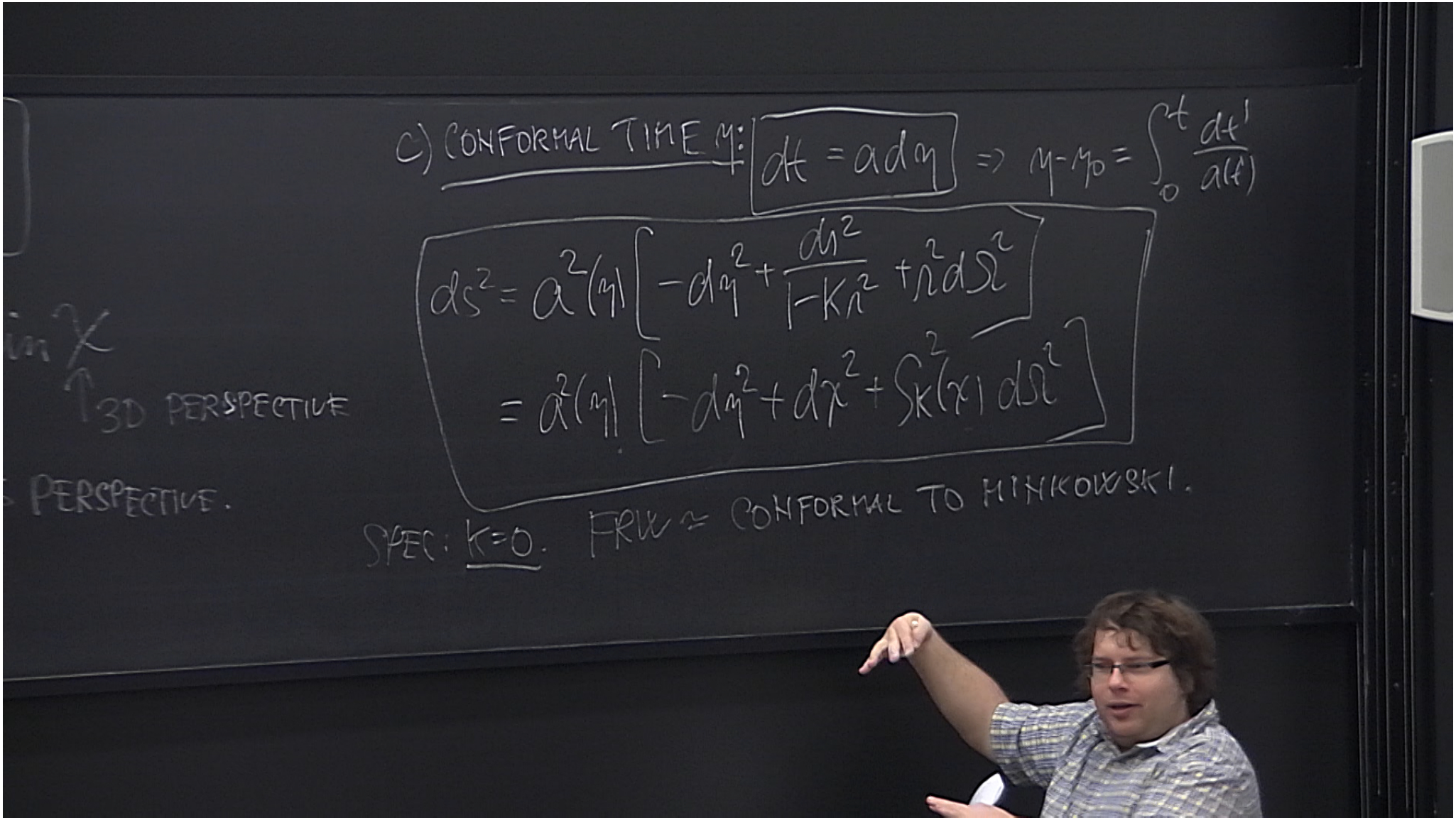
K=+1 EMBEDDING VIEW POINT



$r = \sin\chi$

↑ 3D PERSPECTIVE

↑ EMBEDDING PERSPECTIVE.


c) CONFORMAL TIME η: $dt = a\,d\eta$ $\Rightarrow \eta - \eta_0 = \int_0^t \frac{dt'}{a(t')}$
$ds^2 = a^2(\eta)\left[-d\eta^2 + \frac{dr^2}{1-Kr^2} + r^2 d\Omega^2\right]$
$= a^2(\eta)\left[-d\eta^2 + d\chi^2 + S_K^2(\chi)\,d\Omega^2\right]$
SPEC: K=0. FRW ≃ CONFORMAL TO MINKOWSKI.
3D PERSPECTIVE
PERSPECTIVE.

$\Lambda^A{}_B$ FORM A REPRESENTATION OF $O(p+\frac{K+1}{2}, q+\frac{1-K}{2})$ $K \neq 0$

- HUBBLE FLOW: $(K=0)$

PHYSICAL DISTANCE: $\boxed{x^i_p = a(t)x^i}$

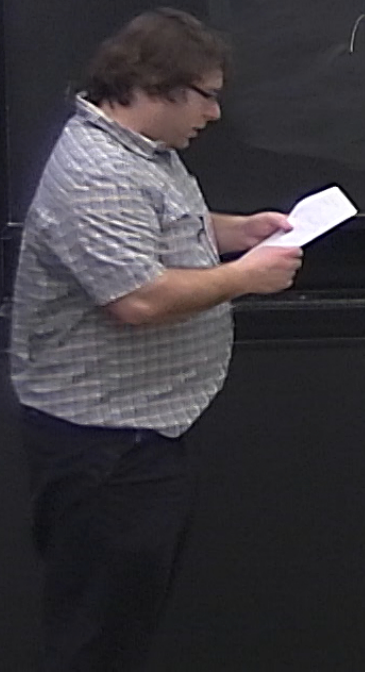
HUBBLE FLOW : VELOCITY DUE TO EXPANSION OF UNIVERSE.

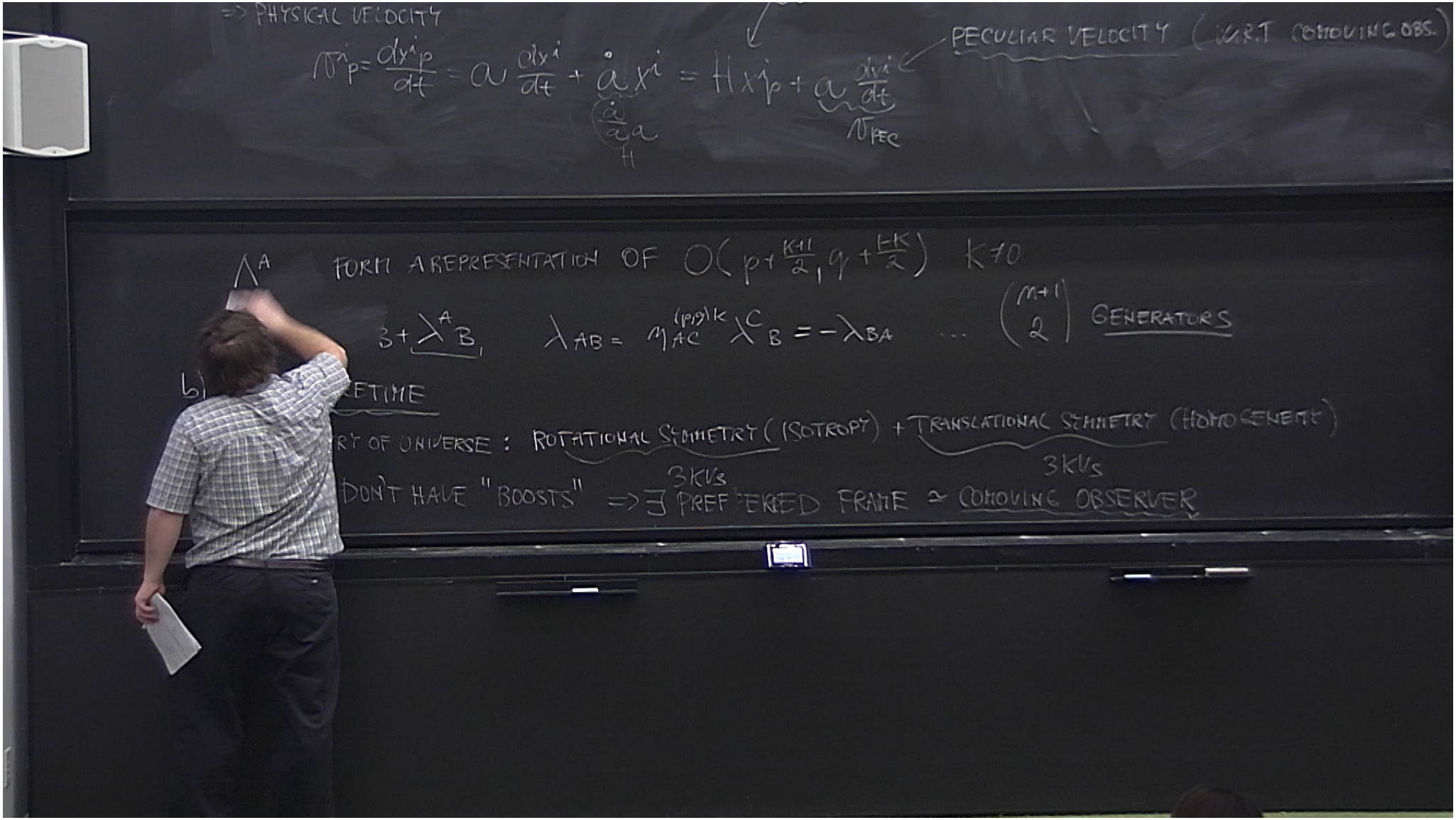
=> PHYSICAL VELOCITY

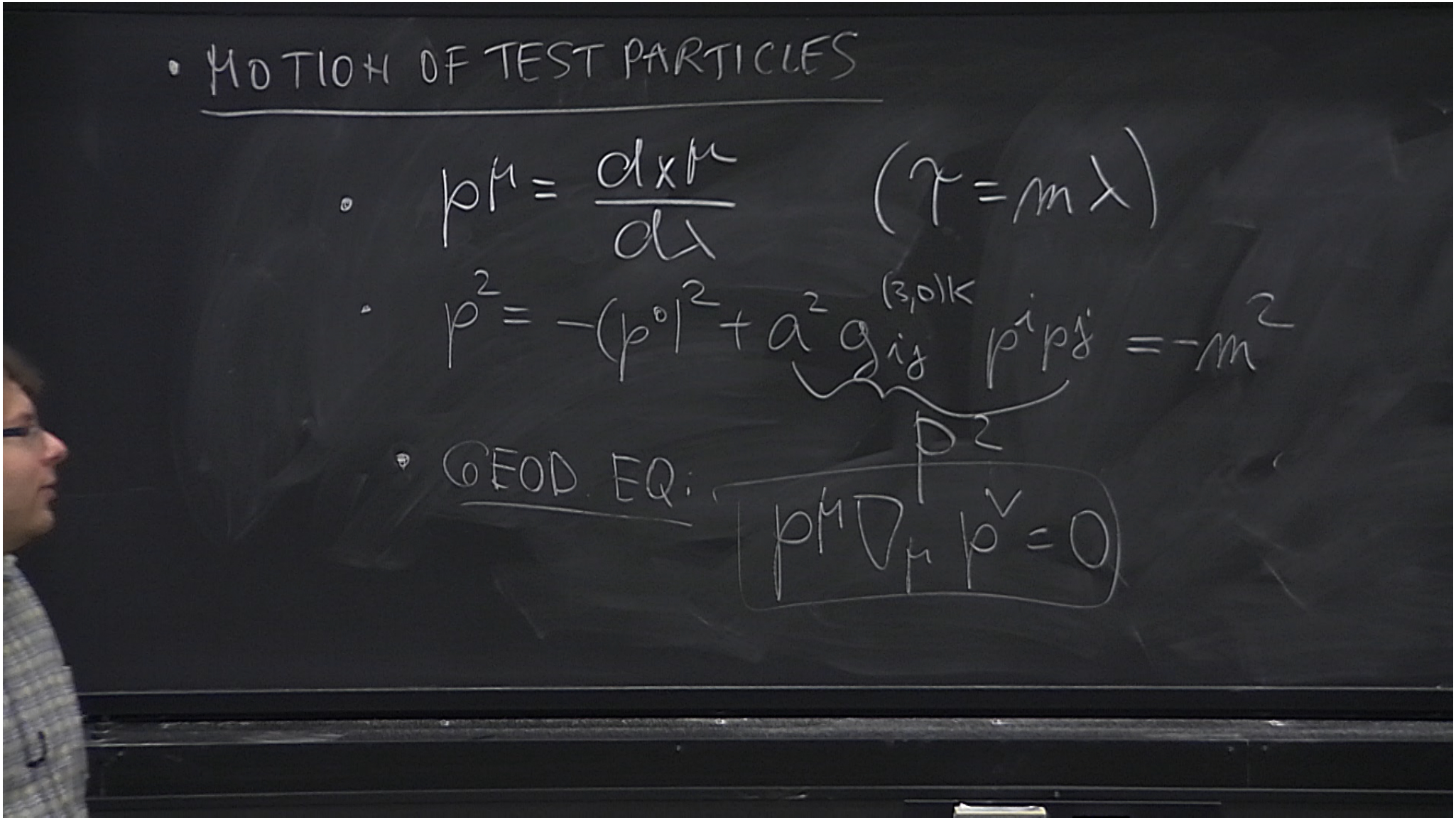
$$v^i_p = \frac{dx^i_p}{dt} = a\frac{dx^i}{dt} + \dot{a}x^i = Hx^i_p + a\frac{dx^i}{dt}$$

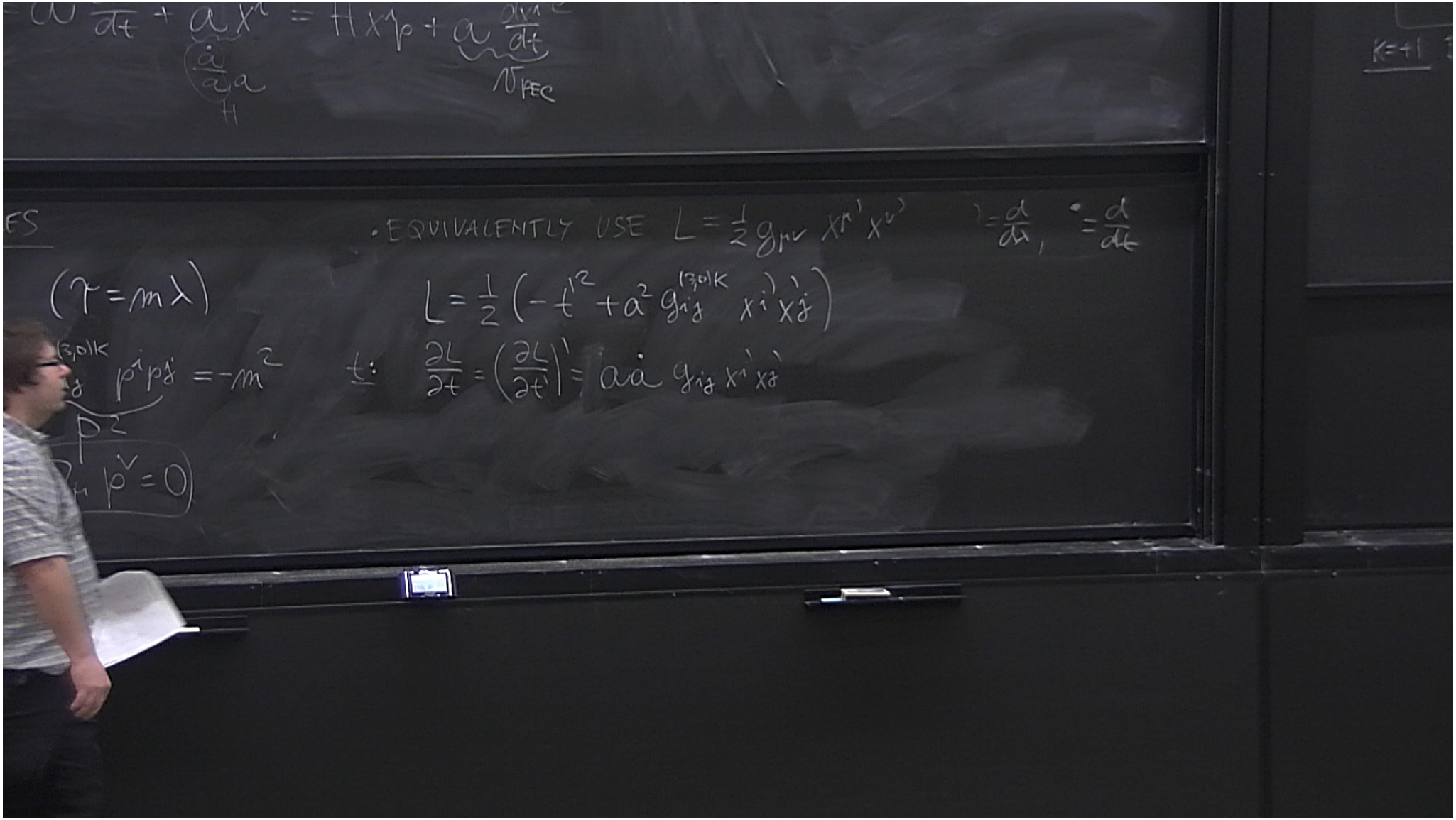
$\frac{\dot{a}}{a} = H$

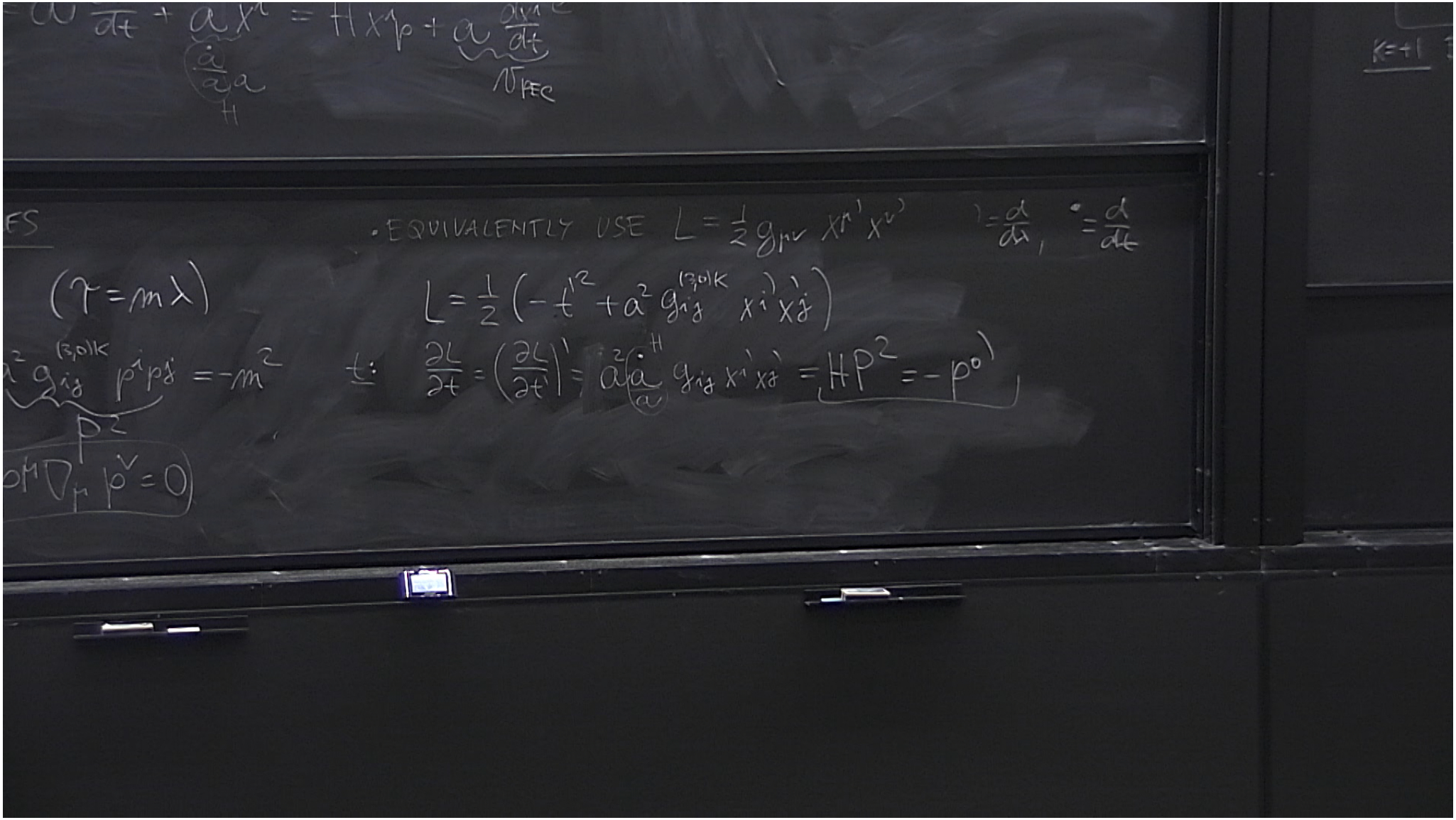
v_{PEC}




=> PHYSICAL VELOCITY
PECULIAR VELOCITY (W.R.T COMOVING OBS.)
FORM A REPRESENTATION OF
GENERATORS
RT OF UNIVERSE : ROTATIONAL SYMMETRY (ISOTROPY) + TRANSLATIONAL SYMMETRY (HOMOGENEITY)
3KVs
DON'T HAVE "BOOSTS"
3KVs
COMOVING OBSERVER


• MOTION OF TEST PARTICLES
• $p^\mu = \frac{dx^\mu}{d\lambda}$ $(\tau = m\lambda)$
• $p^2 = -(p^0)^2 + a^2 \underbrace{g_{ij}}_{(3,0)\,k} p^i p^j = -m^2$
p^2
• GEOD EQ: $p^\mu \nabla_\mu p^\nu = 0$


$\frac{dv}{dt} + \frac{\dot{a}}{a} a v^i = f^i v^i p + a \frac{dv^i}{dt}$
$\frac{\dot{a}}{a} = H$
∇_{FEC}
K=+1
ES
• EQUIVALENTLY USE $L = \frac{1}{2} g_{\mu\nu} x^{\mu'} x^{\nu'}$ $' = \frac{d}{d\lambda}$, $\dot{} = \frac{d}{dt}$
$(p = m\lambda)$
$L = \frac{1}{2}(-t'^2 + a^2 g^{(3,0)K}_{ij} x^{i'} x^{j'})$
$g^{(3,0)K}_{ij} p^i p^j = -m^2$
t: $\frac{\partial L}{\partial t} = \left(\frac{\partial L}{\partial t'}\right)' = a\dot{a} g_{ij} x^{i'} x^{j'}$
p^2
$p^\nu = 0$


$\frac{da}{dt} + \frac{a}{a}a = H x^1 p + a \frac{dv_{rec}}{dt}$
v_{rec}
K=+1
FS
• EQUIVALENTLY USE $L = \frac{1}{2} g_{\mu\nu} x^{\mu\prime} x^{\nu\prime}$ $' = \frac{d}{d\lambda}$, $\dot{} = \frac{d}{dt}$
$(\tau = m\lambda)$
$L = \frac{1}{2}(-t'^2 + a^2 g^{(3,0)K}_{ij} x^{i\prime} x^{j\prime})$
$g^{(3,0)K}_{ij} p^i p^j = -m^2$
p^2
t: $\frac{\partial L}{\partial t} = \left(\frac{\partial L}{\partial t'}\right)' = a\dot{a} g_{ij} x^{i\prime} x^{j\prime} = Hp^2 = -p^0$
$p^\mu \nabla_\mu p^\nu = 0$

$\Pi^i_p = \frac{dx^i p}{dt} = a\frac{dx^i}{dt} + \dot{a}x^i = Hx^i p + a\frac{dx^i}{dt}$ ($\dot{a} = aH$; $\frac{dx^i}{dt} = \Pi_{REC}$)

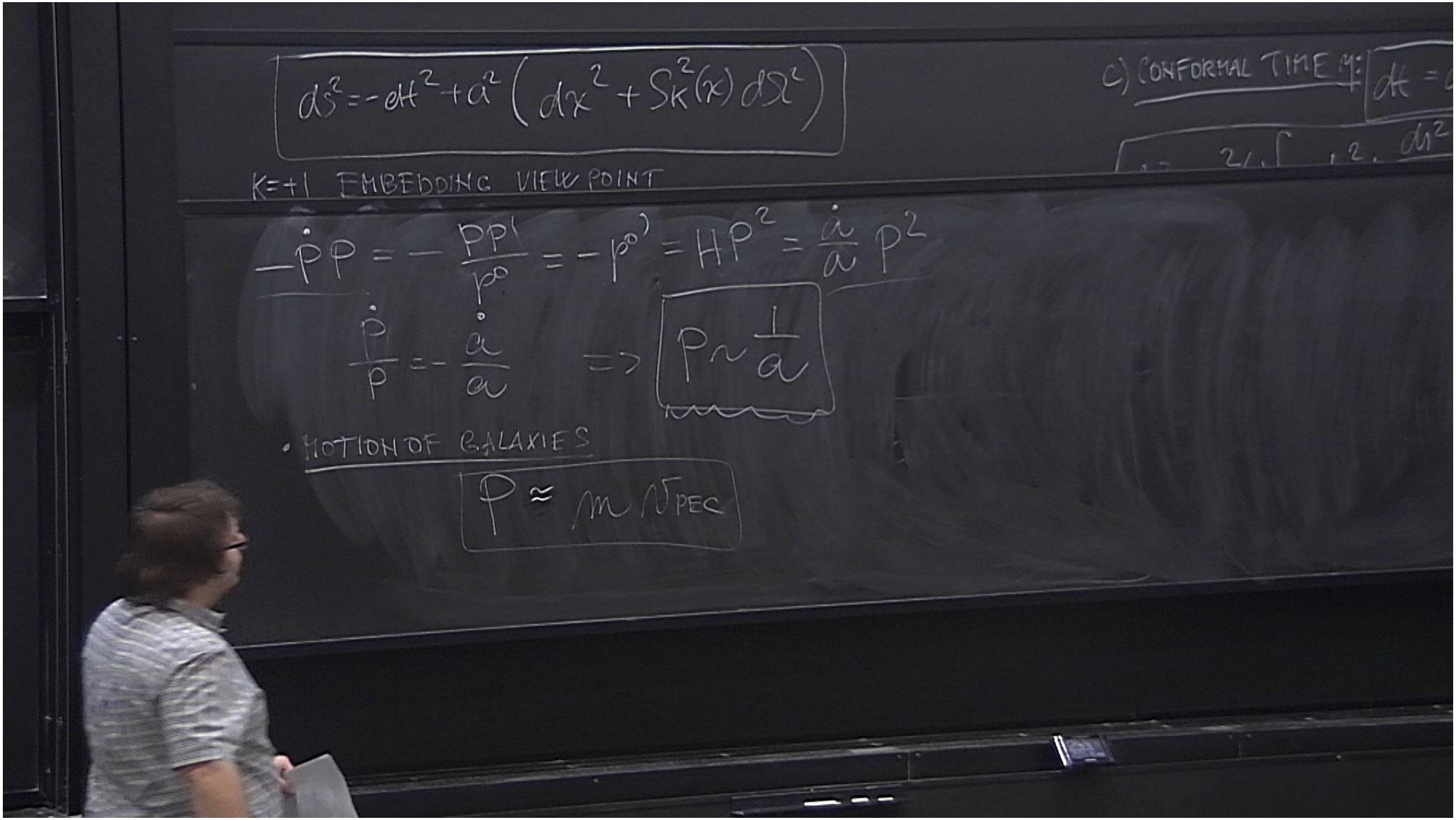
- MOTION OF TEST PARTICLES
  - $p^\mu = \frac{dx^\mu}{d\lambda}$ $\quad (\tau = m\lambda)$
  - $p^2 = -(p^0)^2 + a^2 g^{(3,0)K}_{ij} p^i p^j = -m^2$ ($g^{(3,0)K}_{ij} p^i p^j = p^2$)
  - GEOD EQ. $p^\mu \nabla_\mu p^\nu = 0$

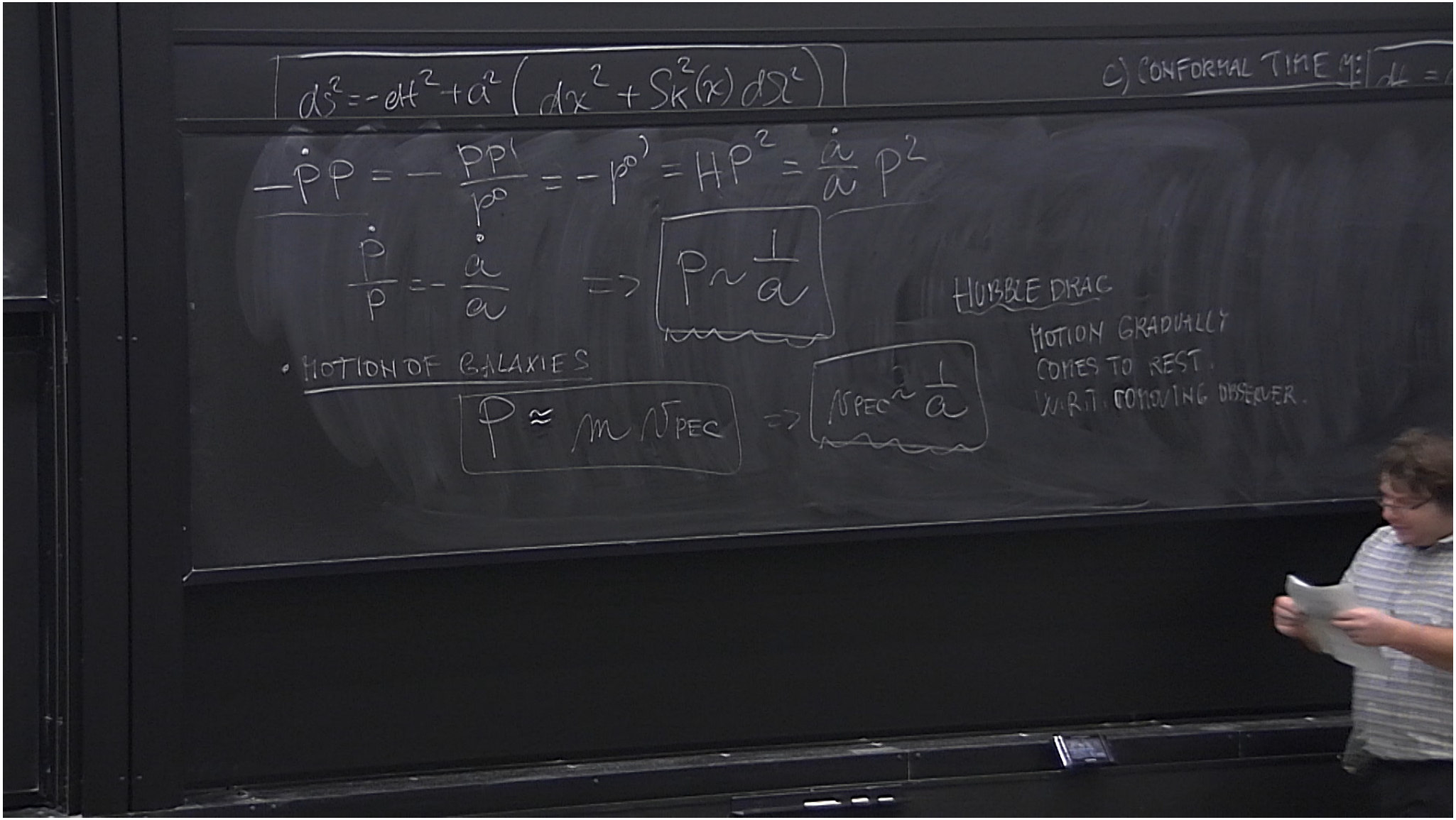
- EQUIVALENTLY USE $L = \frac{1}{2} g_{\mu\nu} x^{\mu\prime} x^{\nu\prime}$ $\quad (' = \frac{d}{d\lambda}, \ \dot{} = \frac{d}{dt})$

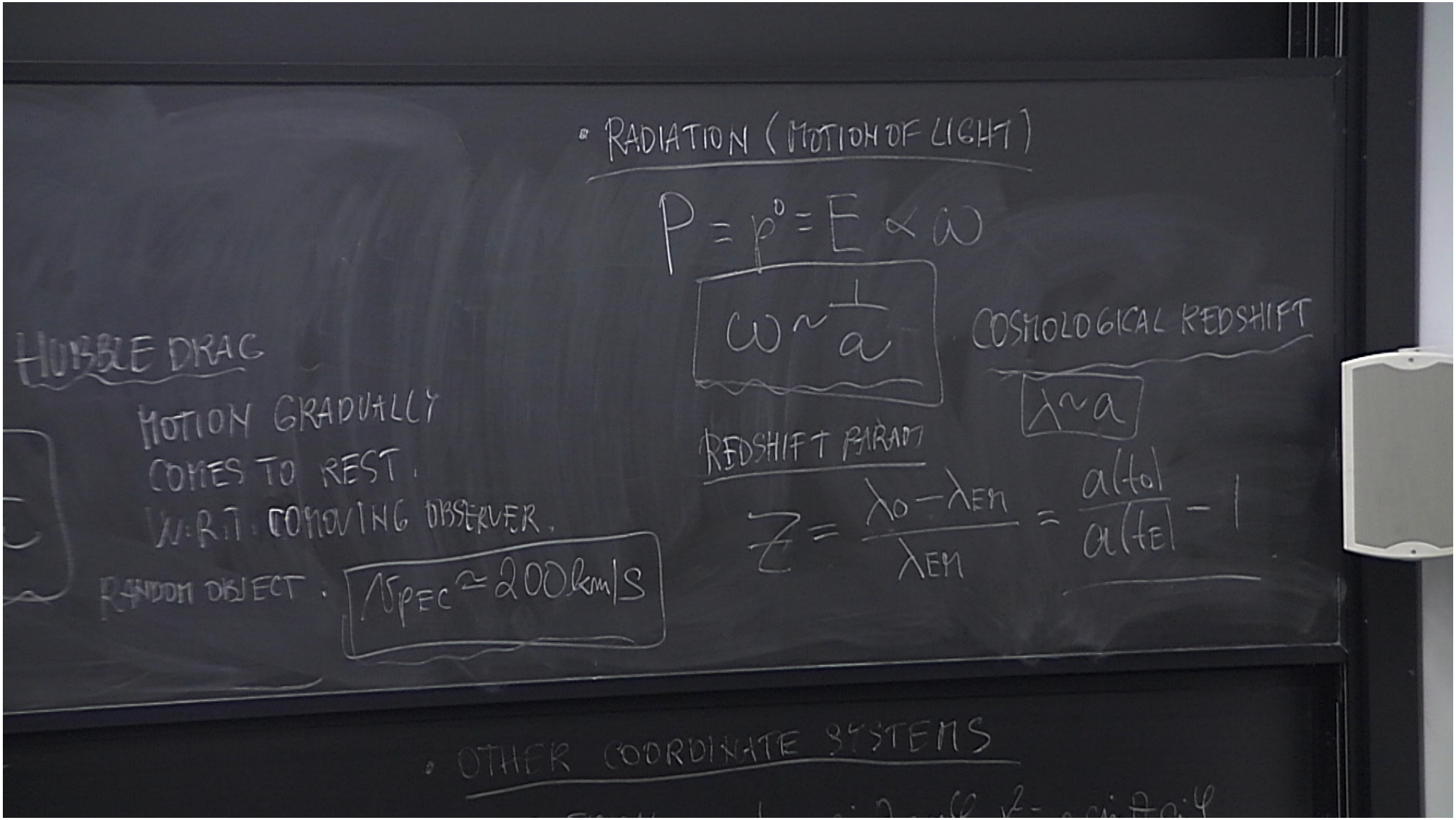
$L = \frac{1}{2}\left(-t'^2 + a^2 g^{(3,0)K}_{ij} x^{i\prime} x^{j\prime}\right)$

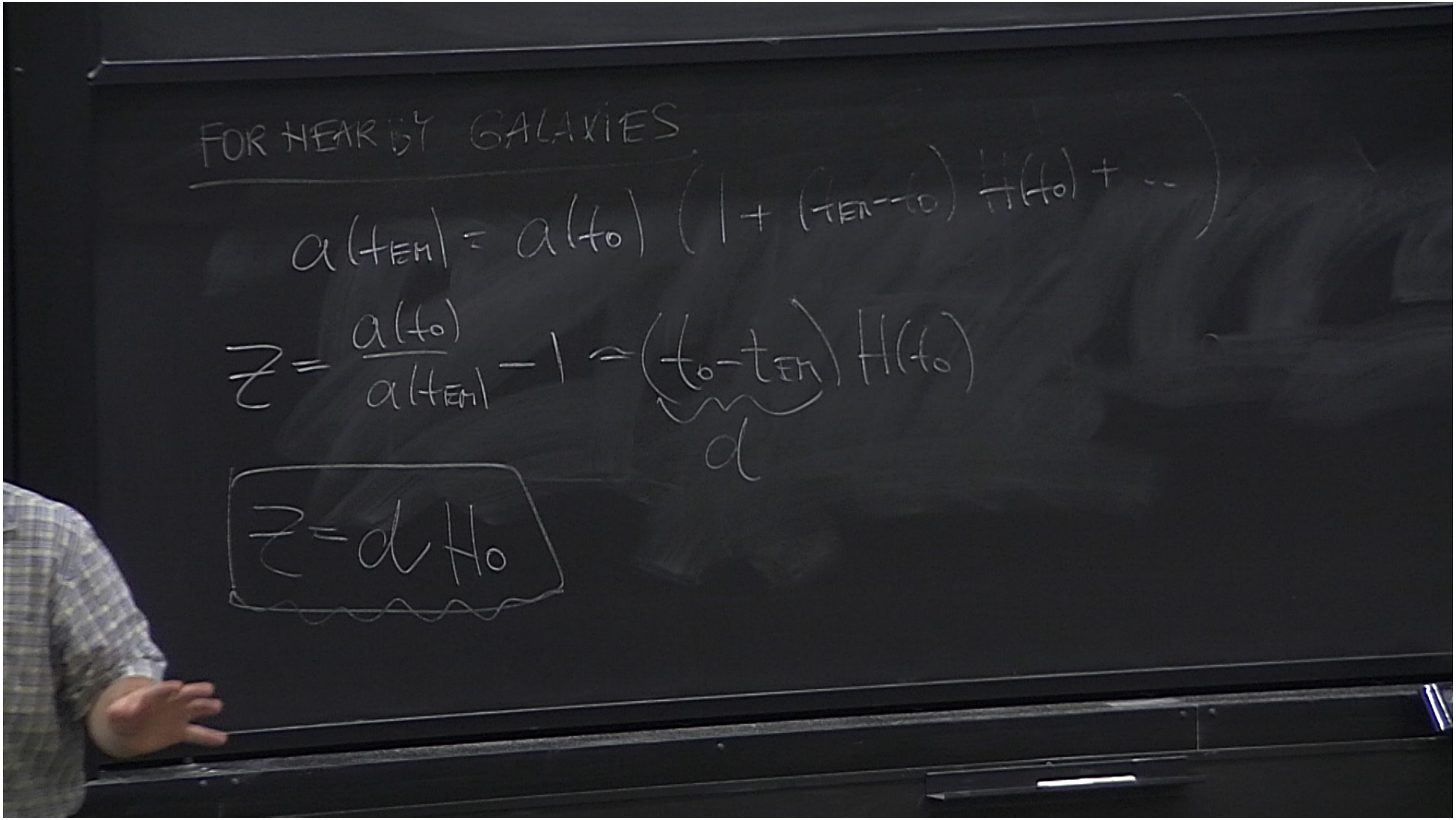
t: $\frac{\partial L}{\partial t} = \left(\frac{\partial L}{\partial t'}\right)' = a\dot{a} g_{ij} x^{i\prime} x^{j\prime} = HP^2 = -p^0{}'$

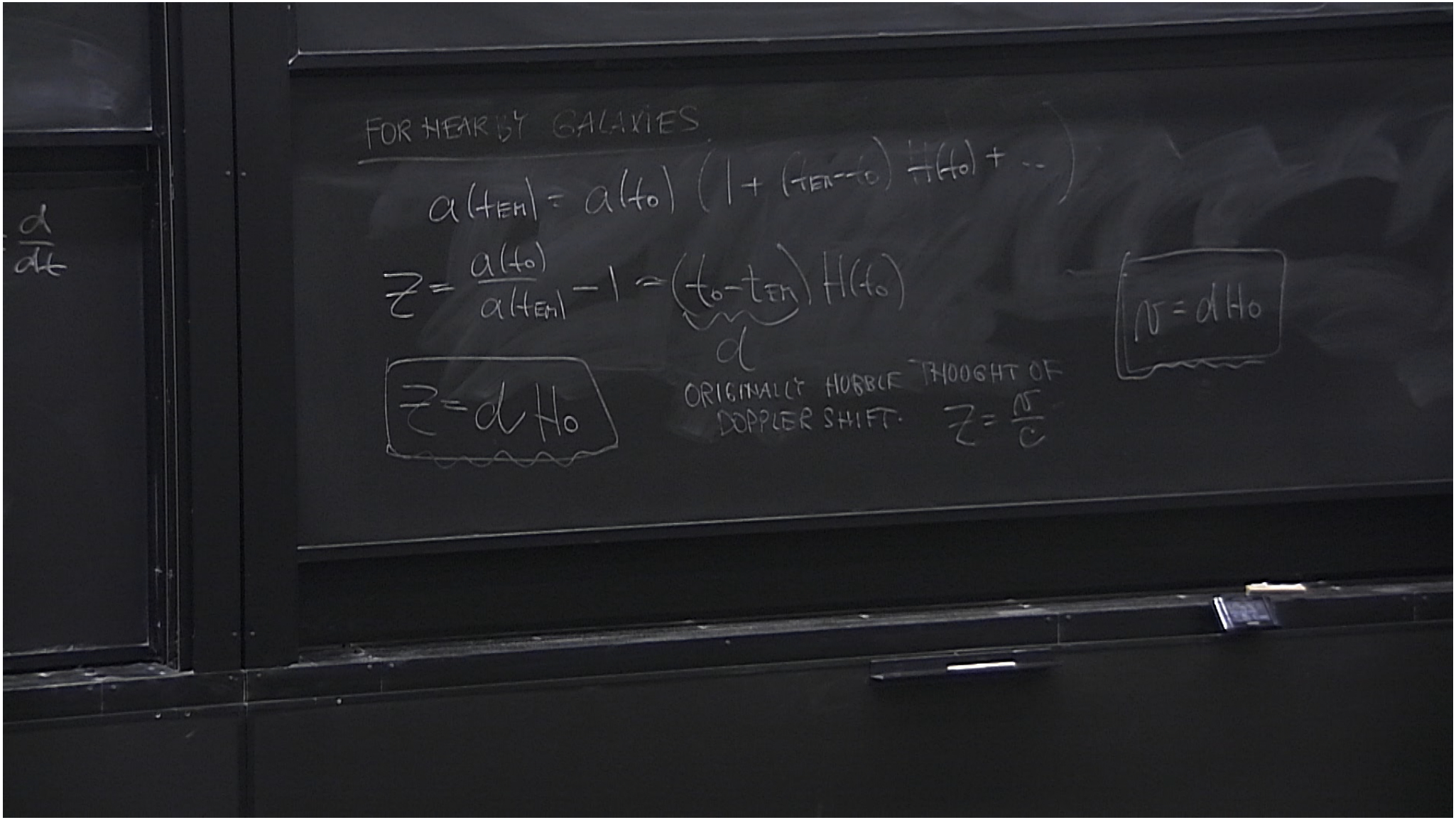
$-p^0 p^{0\prime} + pp' = 0$


$ds^2 = -dt^2 + a^2\left(dx^2 + S_K^2(x)\,d\Omega^2\right)$
C) CONFORMAL TIME η:
K=+1 EMBEDDING VIEW POINT
$-\dot{p}p = -\frac{pp'}{p^0} = -p^{0\cdot} = Hp^2 = \frac{\dot{a}}{a}p^2$
$\frac{\dot{p}}{p} = -\frac{\dot{a}}{a} \Rightarrow p \sim \frac{1}{a}$
• MOTION OF GALAXIES
$p \approx m v_{PEC}$


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HUBBLE DRAG
MOTION GRADUALLY COMES TO REST W.R.T. COMOVING OBSERVER.
• MOTION OF GALAXIES
$\rho \approx m\,v_{PEC} \Rightarrow v_{PEC} \sim \frac{1}{a}$


• RADIATION (MOTION OF LIGHT)
$P = p^0 = E \propto \omega$
$\omega \sim \frac{1}{a}$
COSMOLOGICAL REDSHIFT
$\lambda \sim a$
HUBBLE DRAG
MOTION GRADUALLY COMES TO REST.
W.R.T. COMOVING OBSERVER.
RANDOM OBJECT: $v_{pec} \simeq 200\,km/s$
REDSHIFT PARAM
$z = \frac{\lambda_0 - \lambda_{em}}{\lambda_{em}} = \frac{a(t_0)}{a(t_e)} - 1$
• OTHER COORDINATE SYSTEMS


FOR NEARBY GALAXIES
$a(t_{EM}) = a(t_0)\left(1 + (t_{EM} - t_0)H(t_0) + \cdots\right)$
$z = \frac{a(t_0)}{a(t_{EM})} - 1 \approx (t_0 - t_{EM})H(t_0)$
d
$z = dH_0$


FOR NEARBY GALAXIES
$a(t_{EM}) = a(t_0)\left(1 + (t_{EM} - t_0)H(t_0) + \cdots\right)$
$z = \frac{a(t_0)}{a(t_{EM})} - 1 = (t_0 - t_{EM})H(t_0)$
d
$z = d H_0$
ORIGINALLY HUBBLE THOUGHT OF DOPPLER SHIFT. $z = \frac{v}{c}$
$v = d H_0$
$\frac{d}{dt}$