Title: PSI 17/18 - Condensed Matter - Lecture 13

Date: Jan 19, 2018 11:30 AM

URL: http://pirsa.org/18010046

Abstract:

Equilibration is much weaker than Thermalization

$\bar{A} = \mathrm{Tr}\left[A \sum_n p_n \Pi_n\right]$

↑ populations

Equilibration is much weaker than Thermalization

$\bar{A} = \mathrm{Tr}\left[A \sum_n P_n \Pi_n\right]$

↑ populations of the initial state

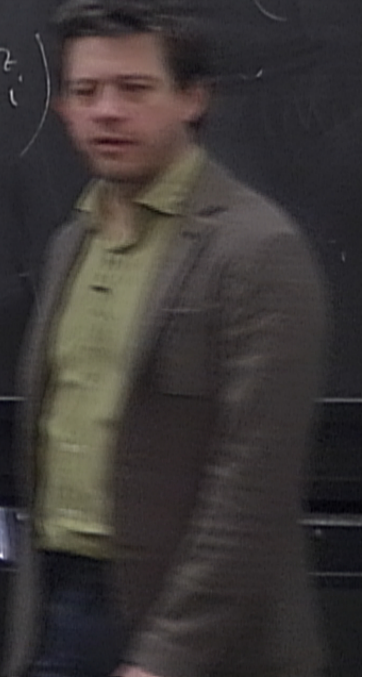
depends explicitly on the $\{P_n\}$

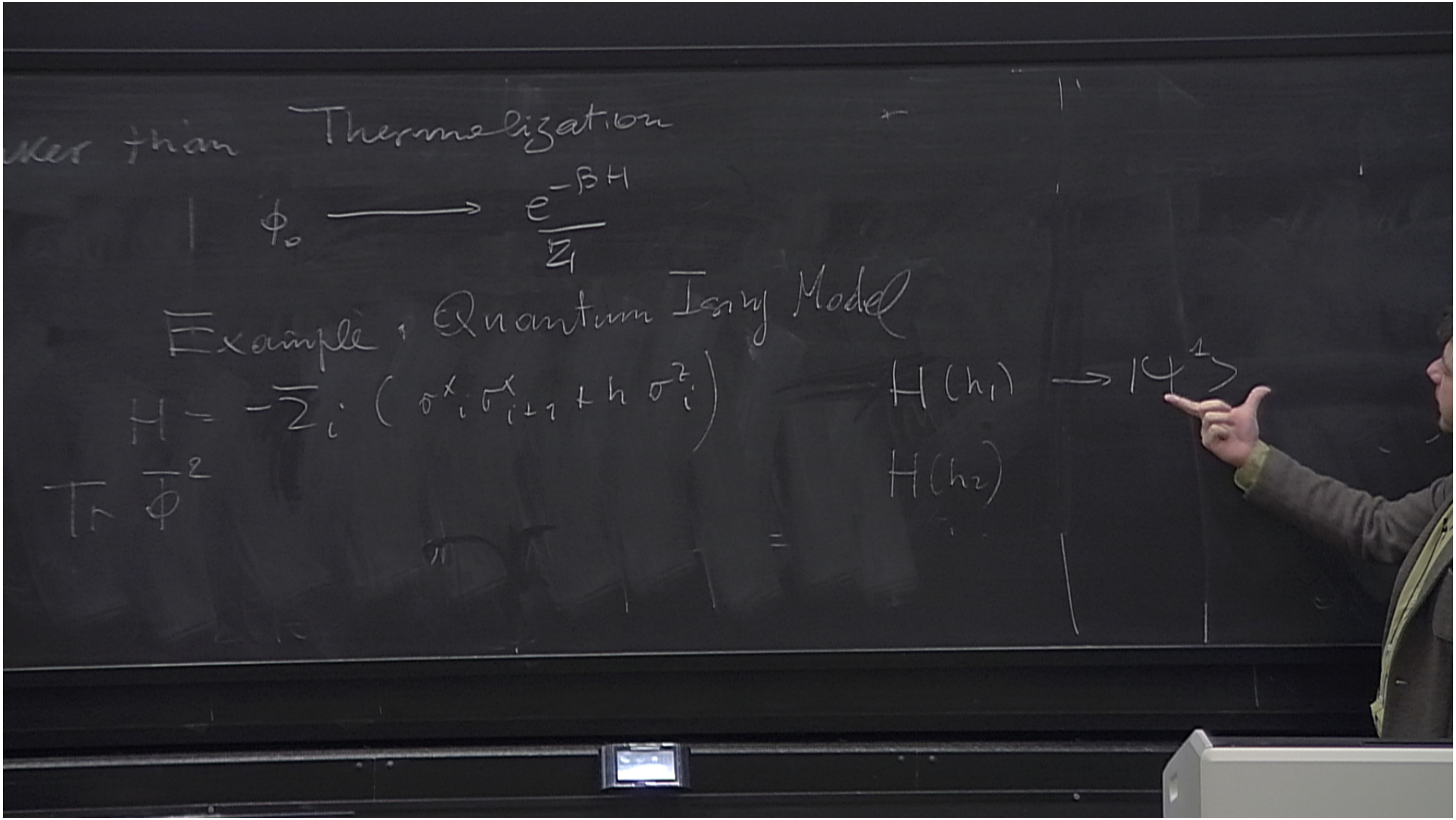
$|\phi_0\rangle \longrightarrow \frac{e^{-\beta H}}{Z}$

Example: Quantum Ising Model

$H = -\sum_i \left(\sigma^x_i \sigma^x_{i+1} + h\,\sigma^z_i\right)$

$\mathrm{Tr}\,\bar{\phi}^2$




ker than Thermalization
$\phi_0 \longrightarrow \frac{e^{-\beta H}}{Z}$
Example: Quantum Ising Model
$H = -\sum_i (\sigma^x_i \sigma^x_{i+1} + h\,\sigma^z_i)$
$H(h_1) \rightarrow |\psi^1\rangle$
$H(h_2)$
$\mathrm{Tr}\,\phi^2$

much weaker than Thermalization

$$\phi_0 \longrightarrow \frac{e^{-\beta H}}{Z}$$

lations of the initial state

Example : Quantum Ising Model

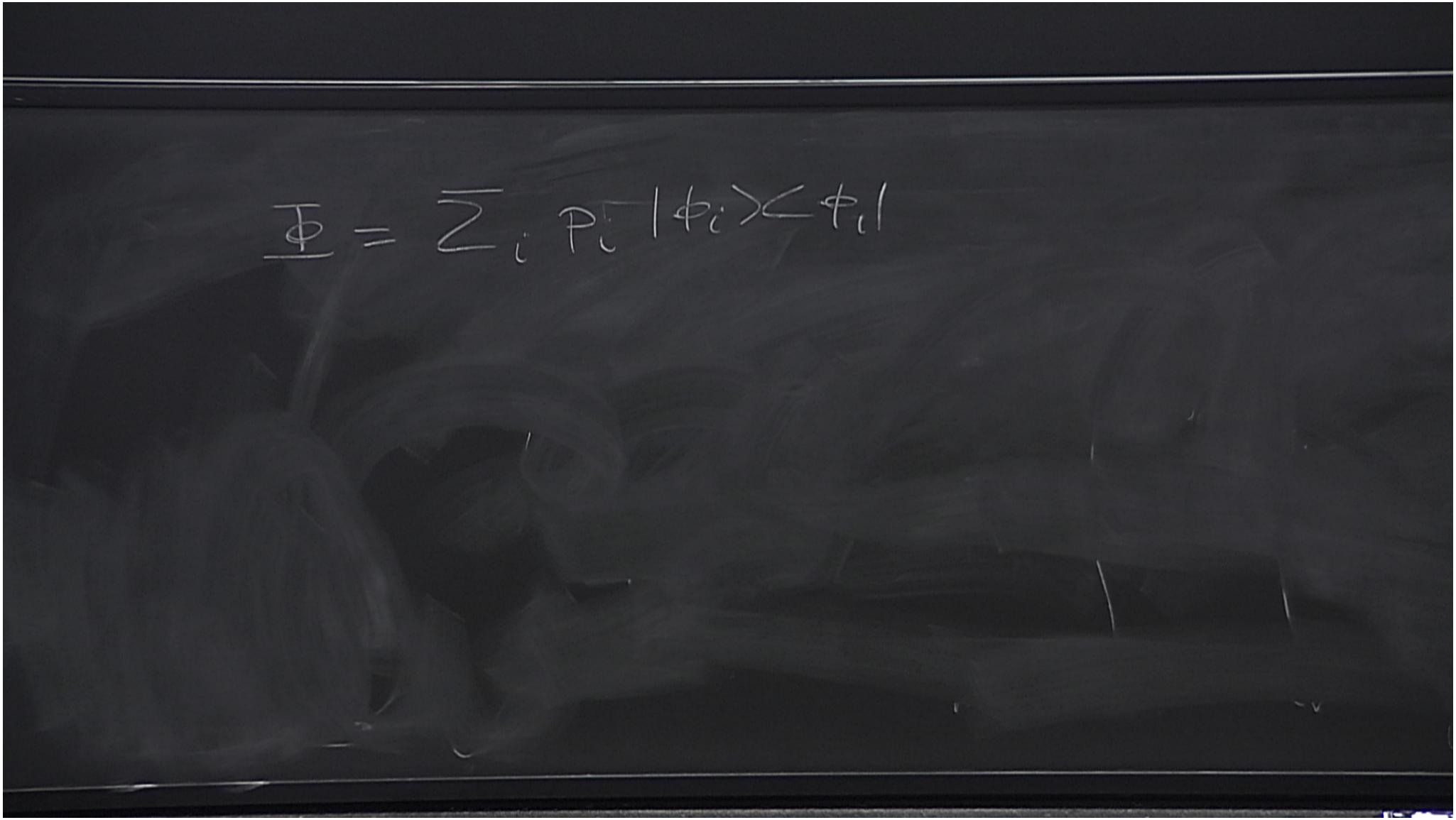
$$H = -\sum_i \left(\sigma^x_i \sigma^x_{i+1} + h\, \sigma^z_i \right)$$

$H(h_1) \longrightarrow$

$$\mathrm{Tr}\, \phi^2 = \prod_{k>0} \left(1 - \sin^2 \frac{\theta'_k - \theta_k}{2} \right)$$

$H(h_2)$

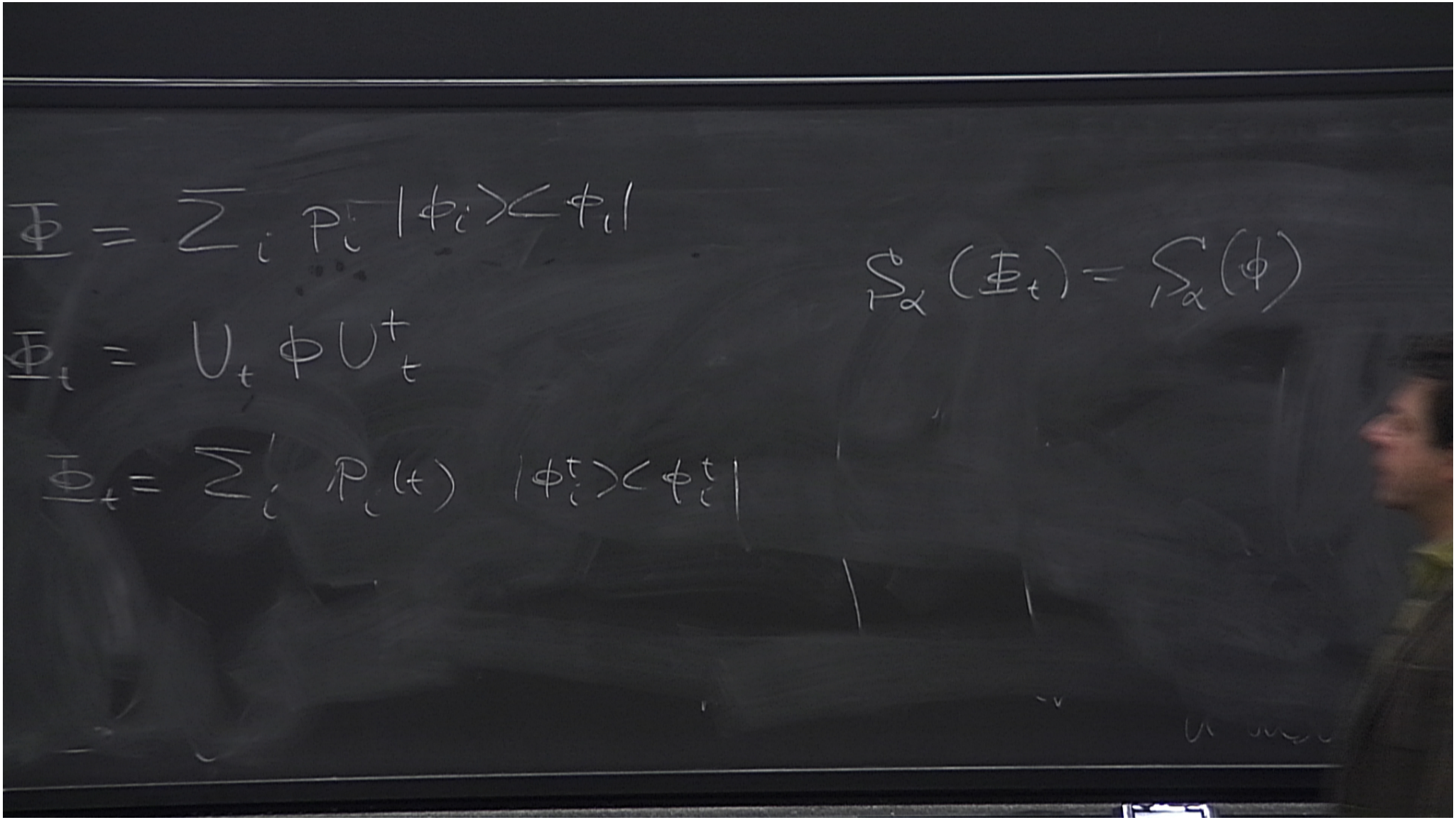

Model
H(h1)
H(h2)

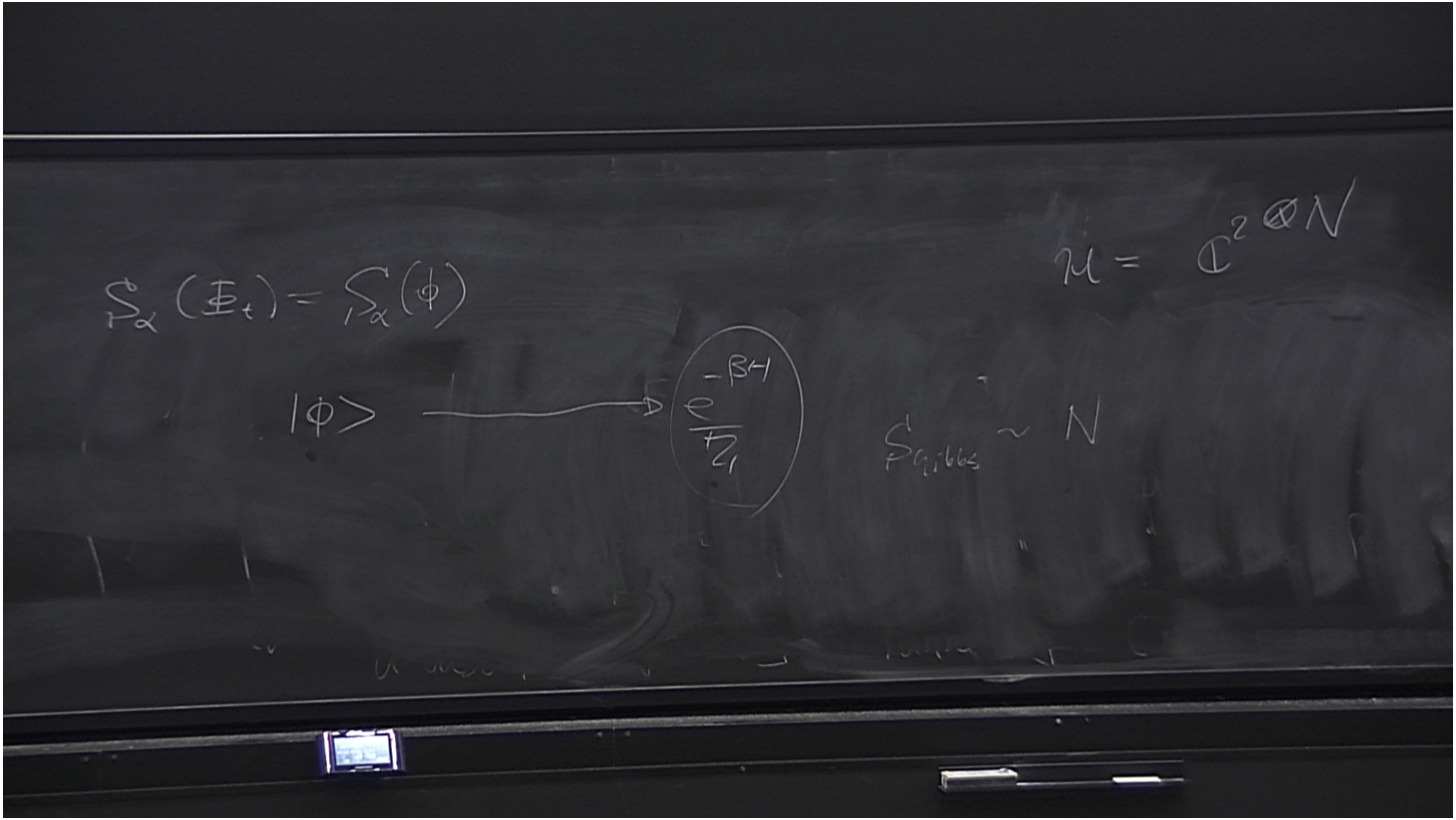

$\underline{\Phi} = \sum_i p_i |\phi_i\rangle\langle\phi_i|$

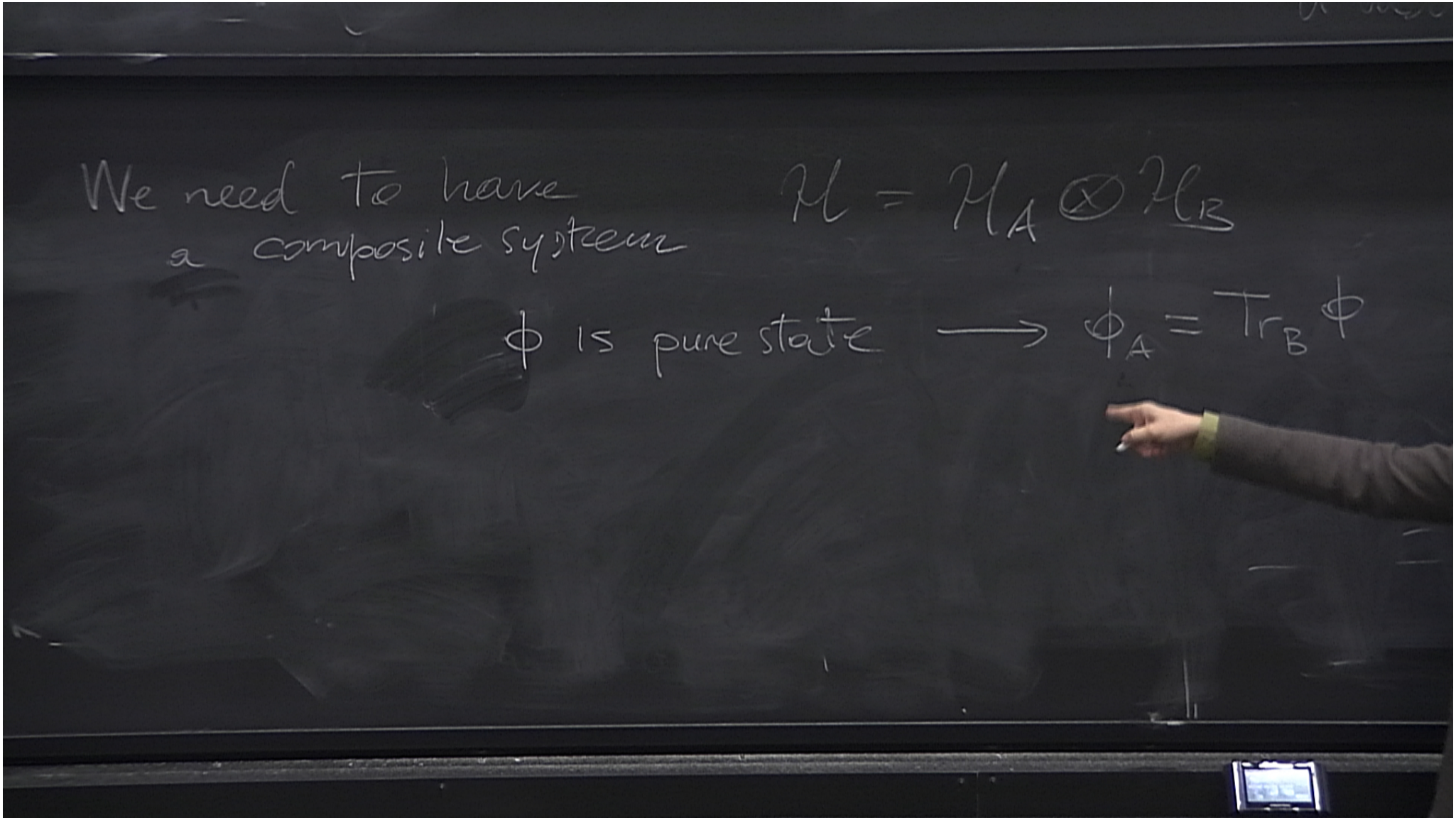
$$\underline{\Phi} = \sum_i p_i |\phi_i\rangle\langle\phi_i|$$

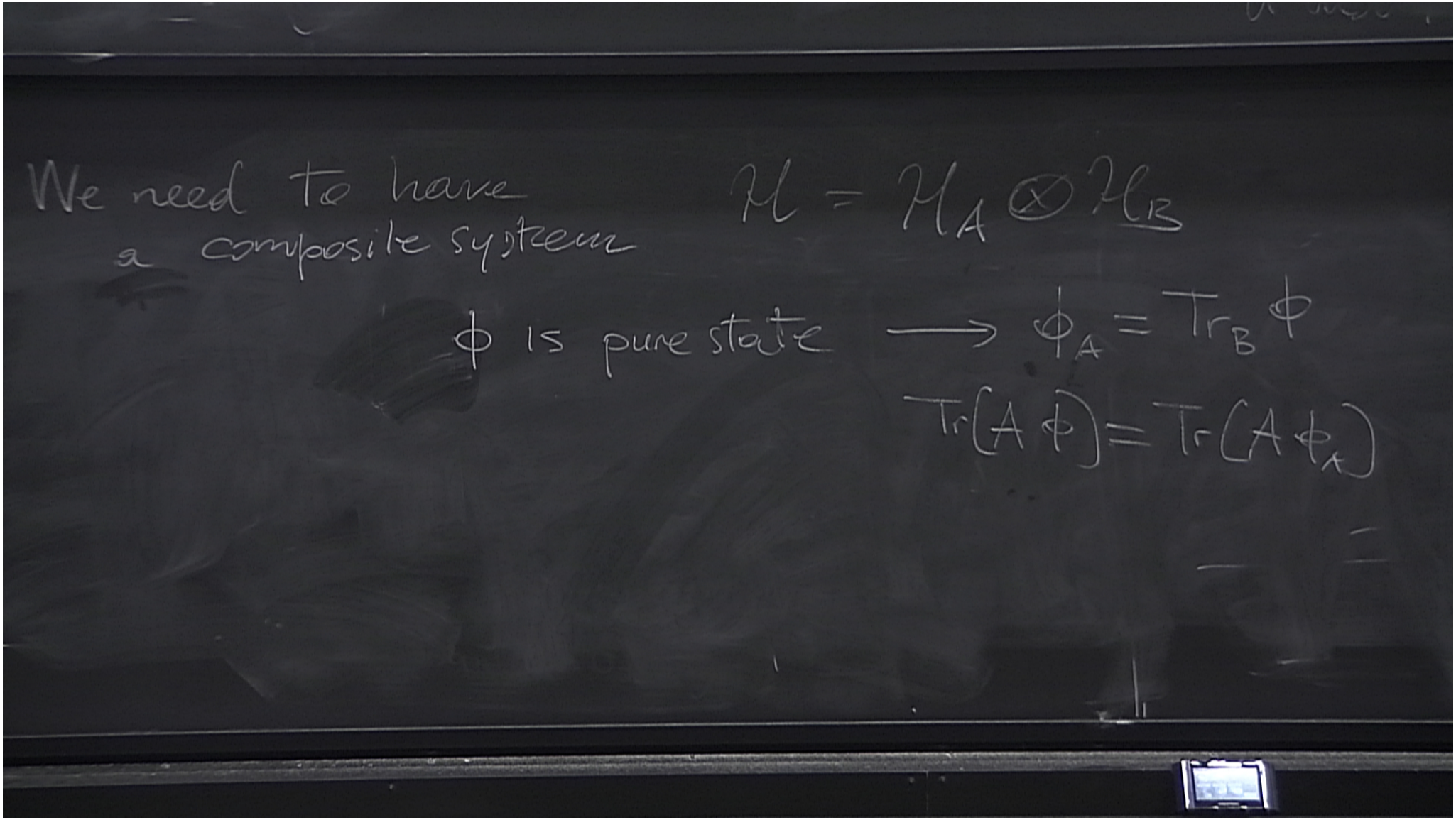
$$\underline{\Phi}_t = U_t \Phi U_t^\dagger$$

$$\underline{\Phi}_t = \sum_i' p_i(t) \; |\phi_i^t\rangle\langle\phi_i^t|$$


$\Phi = \sum_i p_i |\phi_i\rangle\langle\phi_i|$
$\Phi_t = U_t \Phi U_t^\dagger$
$\Phi_t = \sum_i p_i(t) |\phi_i^t\rangle\langle\phi_i^t|$
$S_\alpha(\Phi_t) = S_\alpha(\Phi)$


$S_\alpha(E_t) = S_\alpha(\phi)$
$\mathcal{H} = \mathbb{C}^2 \otimes N$
$|\phi\rangle$
$e^{-\beta H}$
$S_{Gibbs} \sim N$


We need to have a composite system
$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$
ϕ is pure state $\longrightarrow$ $\phi_A = Tr_B \phi$

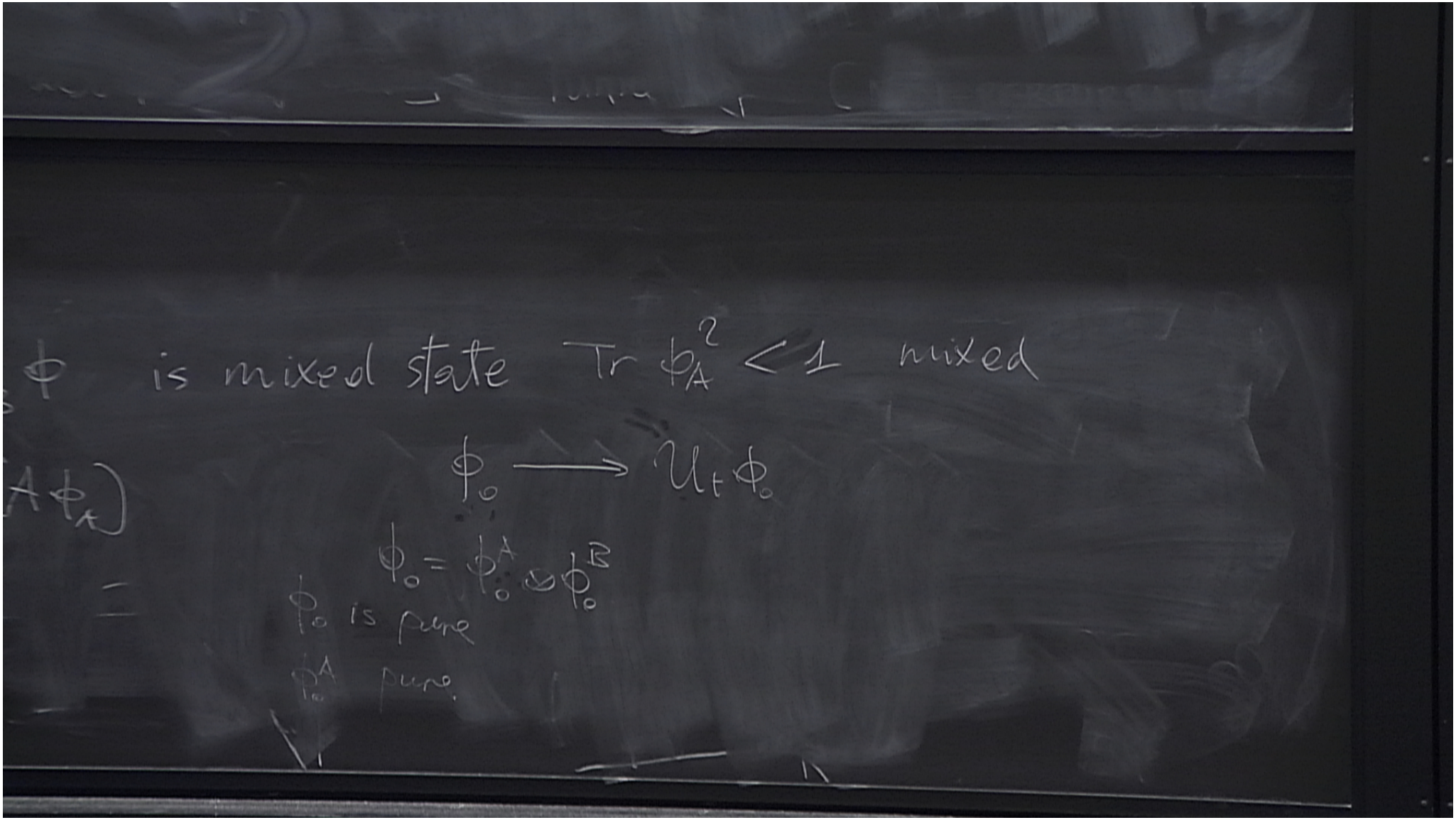

We need to have a composite system
$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$
ϕ is pure state $\longrightarrow \phi_A = \mathrm{Tr}_B \phi$
$\mathrm{Tr}(A\phi) = \mathrm{Tr}(A\phi_A)$

$\rho_t = \sum_i p_i(t) \, |\phi_i^t\rangle\langle\phi_i^t|$

We need to have a composite system $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$

ϕ is pure state $\longrightarrow$ $\phi_A = \mathrm{Tr}_B \phi$ is mixed state $\mathrm{Tr}\,\phi_A^2 < 1$ mixed

$\mathrm{Tr}\,\phi^2 = 1$ $\qquad$ $\mathrm{Tr}(A\phi) = \mathrm{Tr}(A\phi_A)$


ϕ is mixed state $\mathrm{Tr}\,\phi_A^2 < 1$ mixed
$\phi_0 \longrightarrow U_t \phi_0$
$\phi_0 = \phi_0^A \otimes \phi_0^B$
ϕ_0 is pure
ϕ_0^A pure

ϕ is mixed state $\mathrm{Tr}\,\phi_A^2 < 1$ mixed

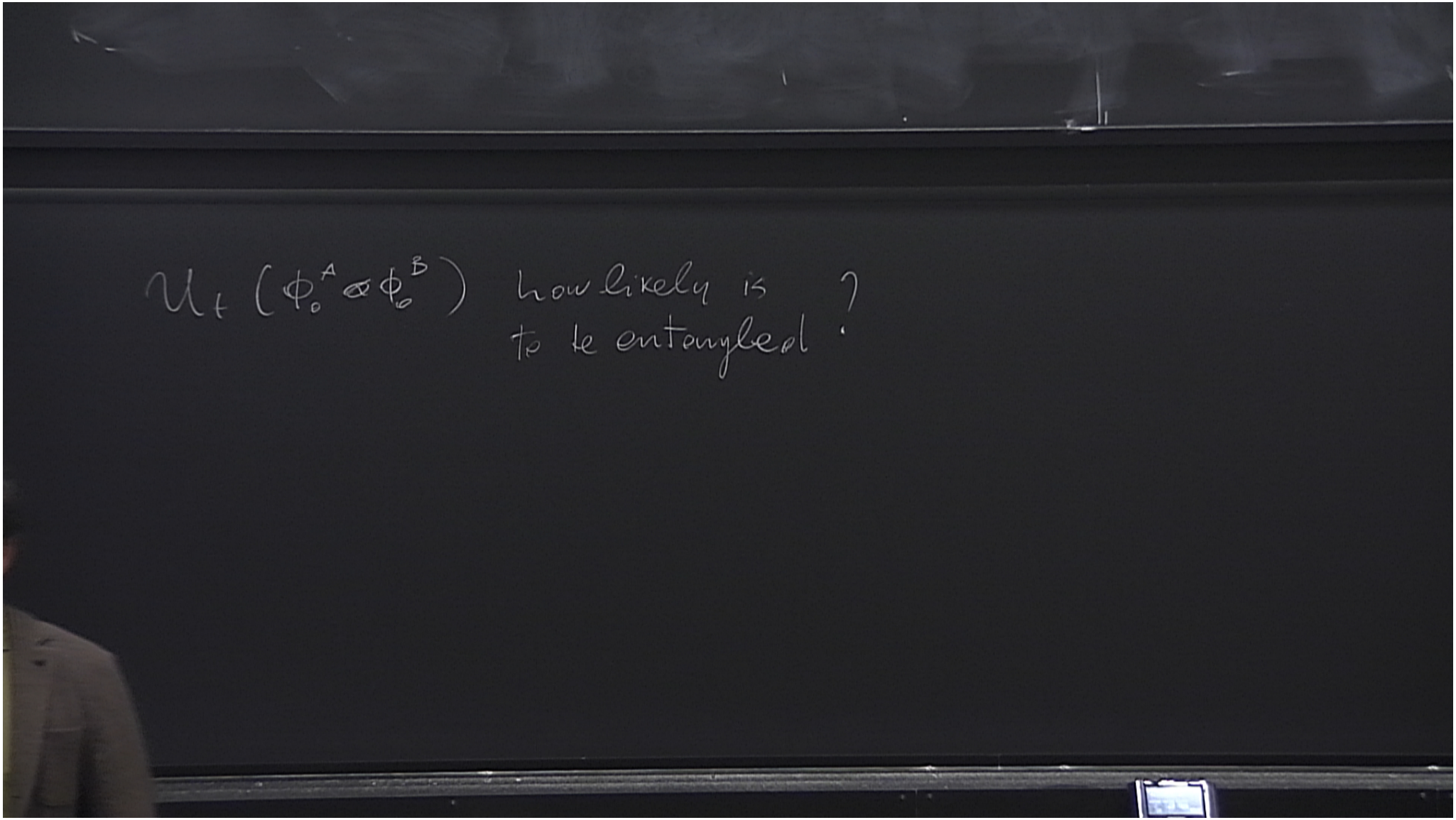
$\phi_0 \longrightarrow U_t \phi_0$

$\phi_0 = \phi_0^A \otimes \phi_0^B \xrightarrow{U_t} U_t(\phi_0^A \otimes \phi_0^B)$ still zero entropy

ϕ_0 is pure

ϕ_0^A pure

mixed $\phi_t^A = \mathrm{Tr}_B\, U_t(\phi_0^A \otimes \phi_0^B)$

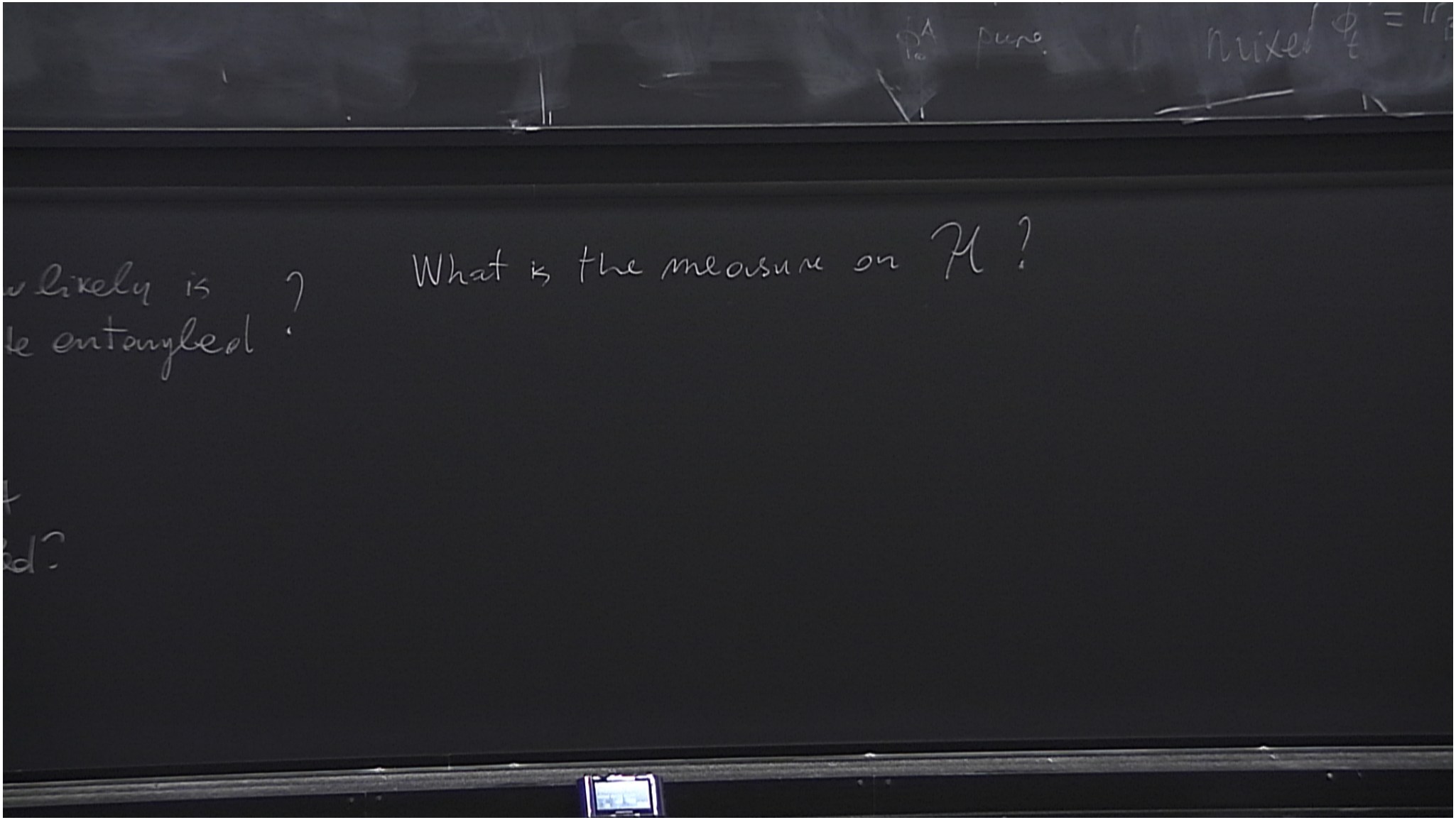

$\mathcal{U}_t(\phi_0^A \otimes \phi_0^B)$ how likely is
to be entangled ?

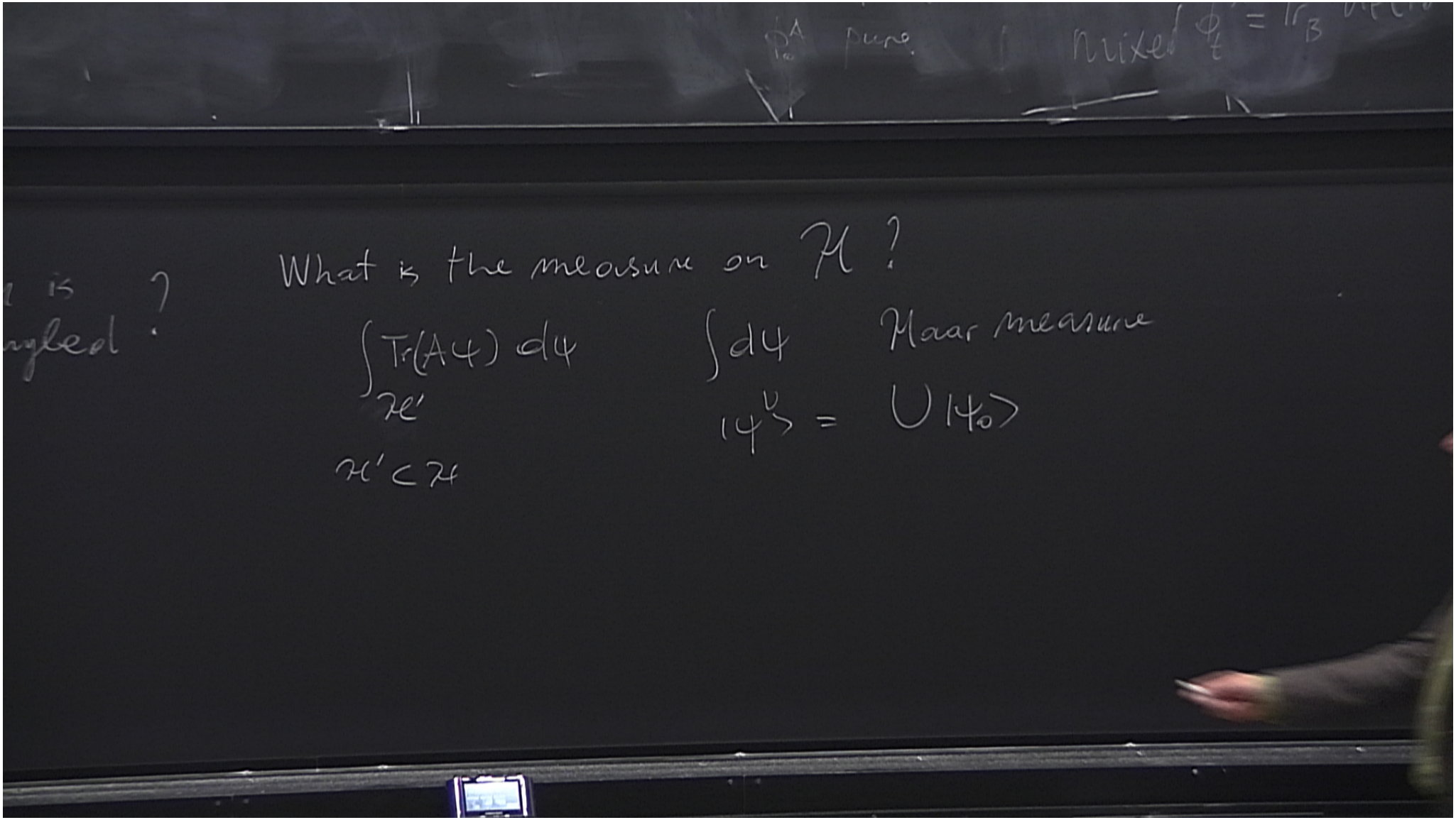
$U_t(\phi_0^A \otimes \phi_0^B)$ How likely is ? to be entangled!

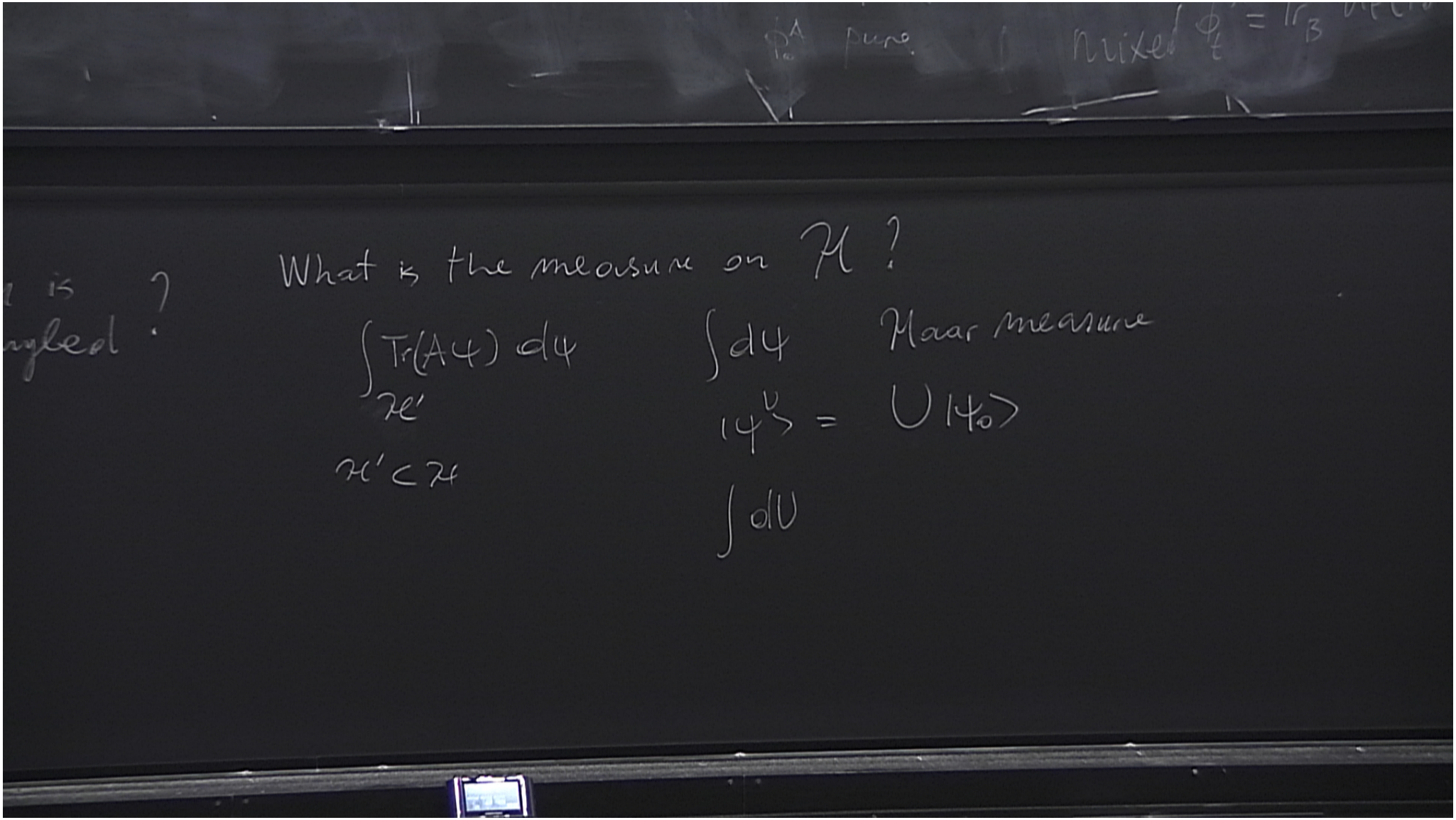
$\mathcal{H} = \mathcal{H}^A \otimes \mathcal{H}^B$

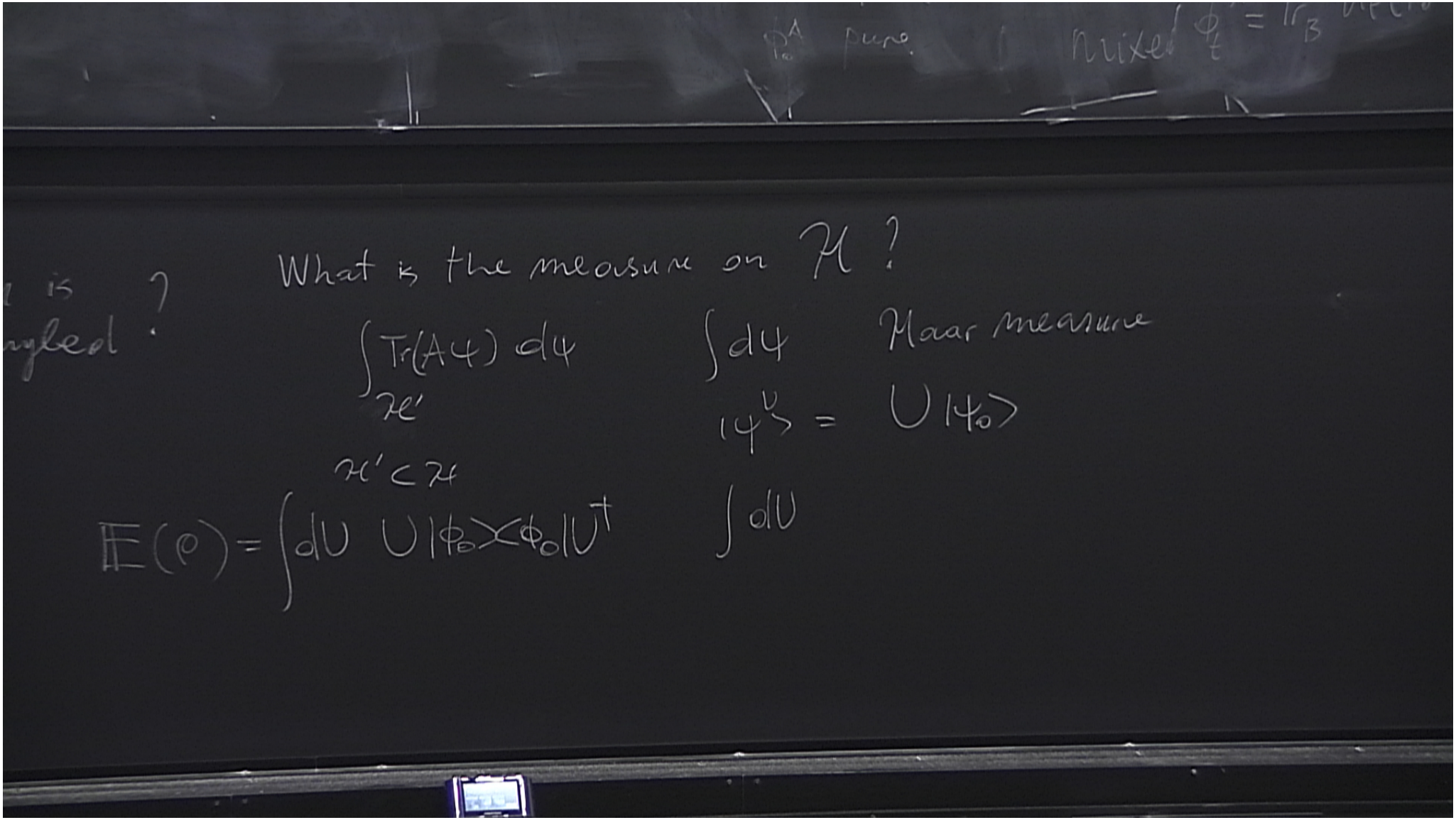
ψ

How likely is ψ to be entangled?


likely is
le entangled?
What is the measure on H?


pure
mixed
What is the measure on $\mathcal{H}$?
$\int_{\mathcal{H}'} \mathrm{Tr}(A\psi)\, d\psi$
$\mathcal{H}' \subset \mathcal{H}$
$\int d\psi$ Haar measure
$|\psi\rangle = U|\psi_0\rangle$


pure
mixed
What is the measure on $\mathcal{H}$?
$\int \mathrm{Tr}(A\rho)\, d\rho$
$\mathcal{H}'$
$\mathcal{H}' \subset \mathcal{H}$
$\int d\rho$
Haar measure
$|\psi^U\rangle = U|\psi_0\rangle$
$\int dU$


pure
mixed
What is the measure on $\mathcal{H}$?
$\int_{\mathcal{H}'} \mathrm{Tr}(A\rho)\, d\psi$
$\int d\psi$ Haar measure
$|\psi\rangle = U|\psi_0\rangle$
$\mathcal{H}' \subset \mathcal{H}$
$\mathbb{E}(\rho) = \int dU\, U|\phi_0\rangle\langle\phi_0|U^\dagger$
$\int dU$

pure mixed

What is the measure on $\mathcal{H}$?

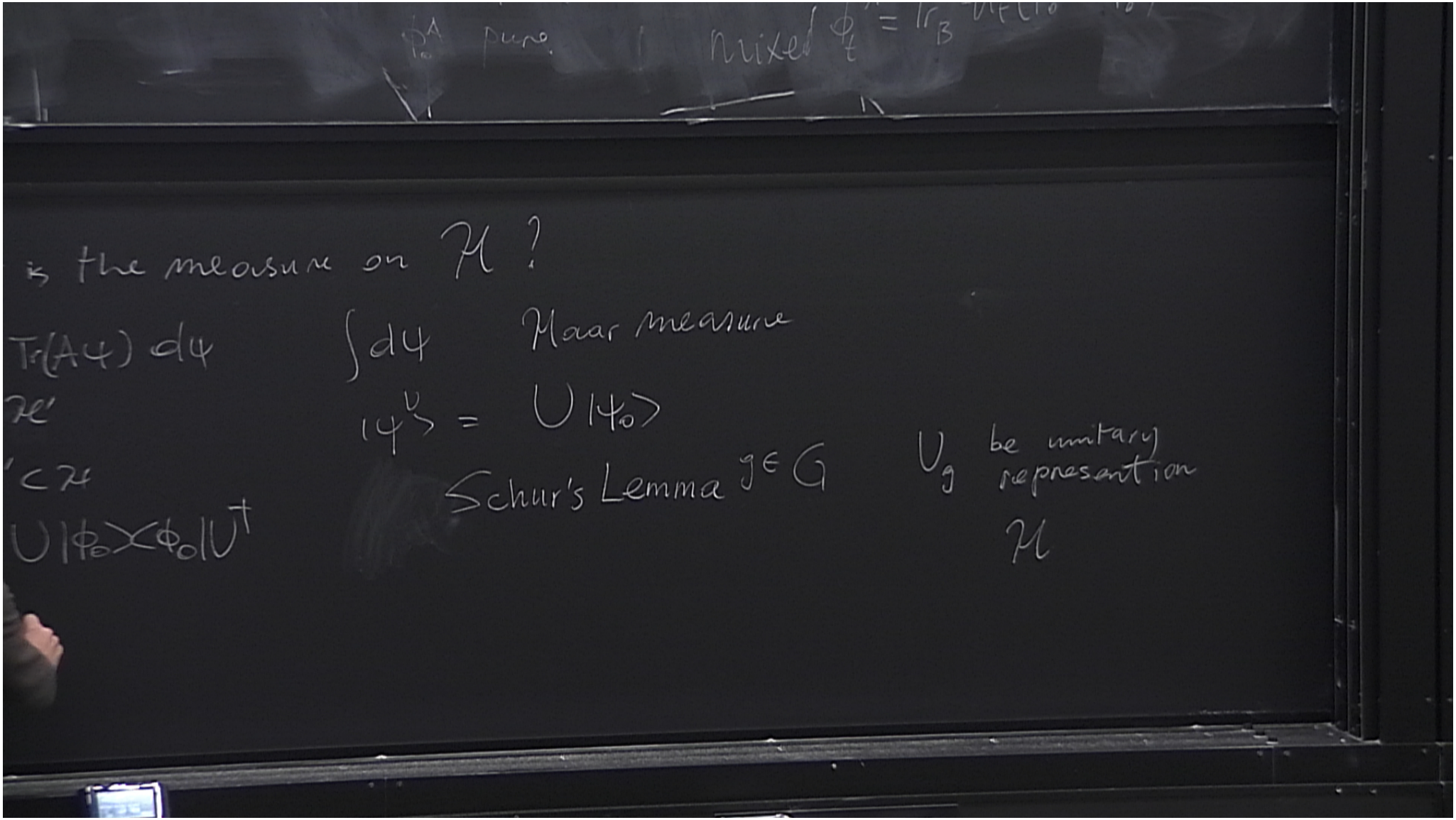
$\int_{\mathcal{H}'} \mathrm{Tr}(A\rho)\, d\rho$ $\int d\rho$ Haar measure

$|\psi\rangle = U|\psi_0\rangle$

$\mathcal{H}' \subset \mathcal{H}$

Schur's Lemma

$$\mathbb{E}(\rho) = \int dU\, U|\phi_0\rangle\langle\phi_0|U^\dagger$$


pure
mixed
is the measure on $\mathcal{H}$?
$\mathrm{Tr}(A\,d\psi)$
$\int d\psi$ Haar measure
$\mathcal{H}'$
$|\psi_0\rangle\langle\psi_0|$
$|\psi\rangle = U|\psi_0\rangle$
Schur's Lemma
$g \in G$
U_g be unitary representation
$\mathcal{H}$

$\rho_{t_0}^A$ pure mixed $\phi_t = Tr_B$

What is the measure on $\mathcal{H}$?

$\int Tr(A\psi)\, d\psi$ $\int d\psi$ Haar measure

$\mathcal{H}' \subset \mathcal{H}$

$|\psi_0^U\rangle = U|\psi_0\rangle$

Schur's Lemma $g \in G$ U_g be unitary representation

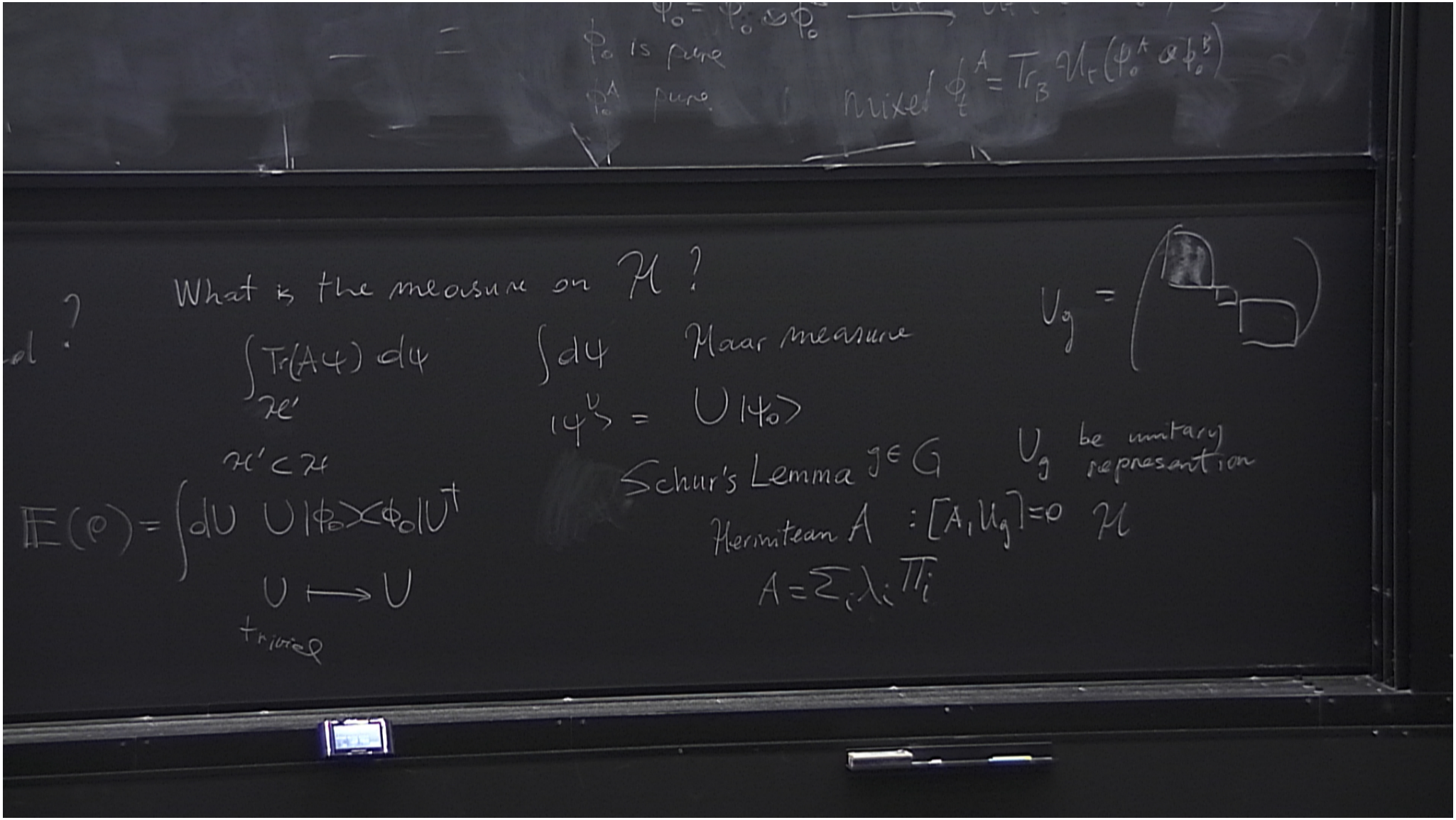
Hermitean A : $[A, U_g] = 0$ $\mathcal{H}$

$A = \sum_i \lambda_i \Pi_i$

$= \int dU\, U|\phi_0\rangle\langle\phi_0|U^\dagger$

$U \longmapsto U$

trivial


$\phi_0 = \phi_0 \otimes \phi_0$
ϕ_0 is pure
ϕ_0^A pure
mixed $\phi_t^A = Tr_B\, U_t(\phi_0^A \otimes \phi_0^B)$
What is the measure on $\mathcal{H}$?
$\int Tr(A\psi)\, d\psi$
$\mathcal{H}' \subset \mathcal{H}$
$\int d\psi$ Haar measure
$|\psi^U\rangle = U|\psi_0\rangle$
U_g be unitary representation
Schur's Lemma $g \in G$
Hermitean A : $[A, U_g] = 0$ $\mathcal{H}$
$A = \Sigma_i \lambda_i \Pi_i$
$\mathbb{E}(\rho) = \int dU\, U|\phi_0\rangle\langle\phi_0|U^\dagger$
$U \longmapsto U$
trivial

$\phi_0 = \psi_0 \otimes \psi_0$ ⟶

ψ_0 is pure

ψ_0^A pure

mixed $\phi_t^A = Tr_B\, U_t(\psi_0^A \otimes \psi_0^B)$

is gleal ?

What is the measure on $\mathcal{H}$?

$\int_{\mathcal{H}'} Tr(A\,\psi)\, d\psi$ $\int d\psi$ Haar measure

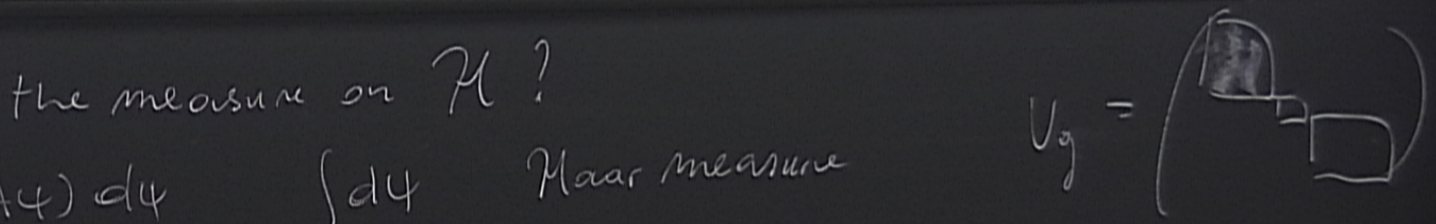
$|\psi_0\rangle = U|\psi_0\rangle$

$\mathcal{H}' \subset \mathcal{H}$ Schur's Lemma $g \in G$ U_g be unitary representation

$\mathbb{V}\,\mathbb{E}(\rho) = \int dU\; U|\phi_0\rangle\langle\phi_0|U^\dagger$ Hermitean $A : [A, U_g] = 0$ $\mathcal{H}$

$U \longmapsto U$ $A = \sum_i \lambda_i \Pi_i$ trivial $A = \lambda \mathbb{1}$

trivial



$\phi_0 = \phi_0^A \otimes \phi_0^B \xrightarrow{U_t}$ zero entropy

ρ_0 is pure

ρ_0^A pure. mixed $\phi_t^A = Tr_B\, U_t(\phi_0^A \otimes \phi_0^B)$

likely is entangled?

What is the measure on $\mathcal{H}$?

$\int_{\mathcal{H}'} Tr(A\psi)\, d\psi$ $\int d\psi$ Haar measure

$|\psi^U\rangle = U|\psi_0\rangle$

U_g = 

$\mathcal{H}' \subset \mathcal{H}$ $g \in G$ U_g be unitary representation

$\mathbb{V}\,\mathbb{E}(\rho) = \int dU \underbrace{VU}_{W} |\phi_0\rangle\langle\phi_0| \underbrace{U^\dagger V^\dagger}_{W^\dagger} V = \mathbb{E}(\rho) V$ Schur's Lemma

$\mathcal{U}$ $dU = dW = d(VU)$ Hermitean A : $[A, U_g] = 0$ $\mathcal{H}$

$A = \sum_i \lambda_i \Pi_i$ trivial $A = \lambda \mathbb{1}$

$\phi_0 = \phi_0^A \otimes \phi_0^B \xrightarrow{U_t}$ zero entropy

ϕ_0 is pure

ρ_0^A pure. mixed $\phi_t^A = Tr_B\, U_t(\phi_0^A \otimes \phi_0^B)$

likely is ? entangled.

What is the measure on $\mathcal{H}$?

$\int_{\mathcal{H}'} Tr(A\psi)\, d\psi$ $\int d\psi$ Haar measure

$|\psi^U\rangle = U|\psi_0\rangle$

$\mathcal{H}' \subset \mathcal{H}$

U_g be unitary representation $g \in G$ Schur's Lemma

$V\, \mathbb{E}(\rho) = \int dU\, \underbrace{VU}_{W} |\phi_0\rangle\langle\phi_0| \underbrace{U^\dagger V^\dagger}_{W^\dagger} V = \mathbb{E}(\rho) V$

Hermitean A : $[A, U_g] = 0$ $\mathcal{H}$ trivial $A = \lambda \mathbb{1}$

$A = \sum_i \lambda_i \Pi_i$

$\mathcal{U}$ $dU = dW = d(VU)$

$\phi_0 = \phi_0^A \otimes \phi_0^B \xrightarrow{U_t}$ zero entropy

ϕ_0 is pure

ϕ_0^A pure

mixed $\phi_t^A = Tr_B\, U_t(\phi_0^A \otimes \phi_0^B)$

likely is ? entangled

What is the measure on $\mathcal{H}$?

$Tr A\overline{\Psi}^{2t} = \int Tr(A\Psi)\, d\Psi$ $\int d\Psi$ Haar measure

$|\Psi^U\rangle = U|\Psi_0\rangle$

U_g = 

$g \in G$ U_g be unitary representation

$\mathcal{H}' \subset \mathcal{H}$

$\mathbb{V}\,\mathbb{E}(\rho) = \int dU\, \underbrace{VU}_{W} |\phi_0\rangle\langle\phi_0| \underbrace{U^\dagger V^\dagger}_{W^\dagger} V = \mathbb{E}(\rho) V$ Schur's Lemma

$\mathcal{U}$ $dU = dW = d(VU)$

Hermitean A : $[A, U_g] = 0$ $\mathcal{H}$

$A = \sum_i \lambda_i \Pi_i$ trivial $A = \lambda \mathbb{1}$

$\phi_0 = \phi_0^A \otimes \phi_0^B \xrightarrow{U_t}$ zero entropy

ϕ_0 is pure

ϕ_0^A pure. mixed $\phi_t^A = Tr_B\, U_t(\phi_0^A \otimes \phi_0^B)$

likely is ? What is the measure on $\mathcal{H}$?

entangled ! $\overline{Tr A\Psi^{\otimes 2}} = \int Tr(A\psi)\, d\psi$ $\int d\psi$ Haar measure

$|\psi^U\rangle = U|\psi_0\rangle$

U_g =  be unitary representation

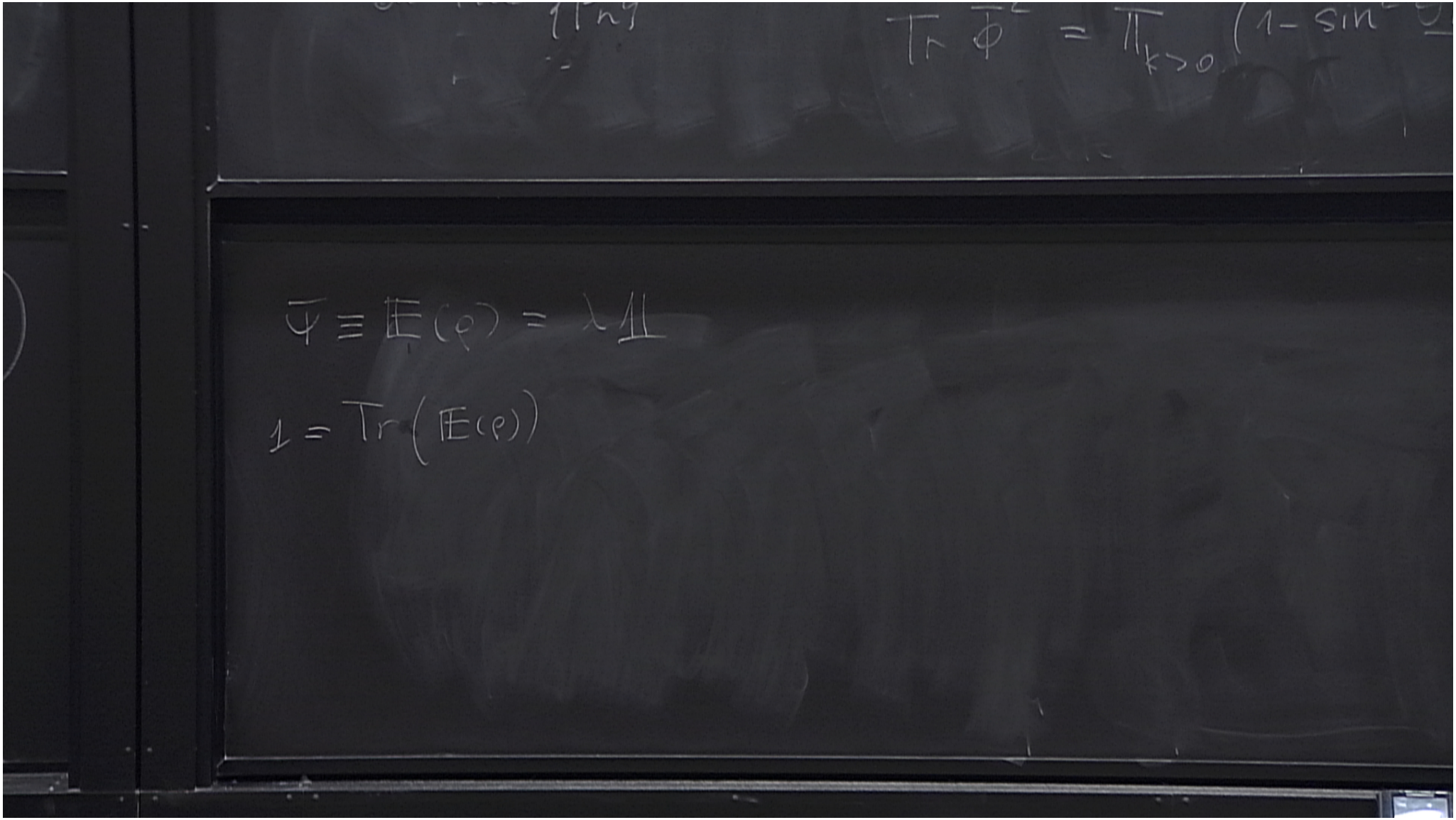
$\mathcal{H}' \subset \mathcal{H}$

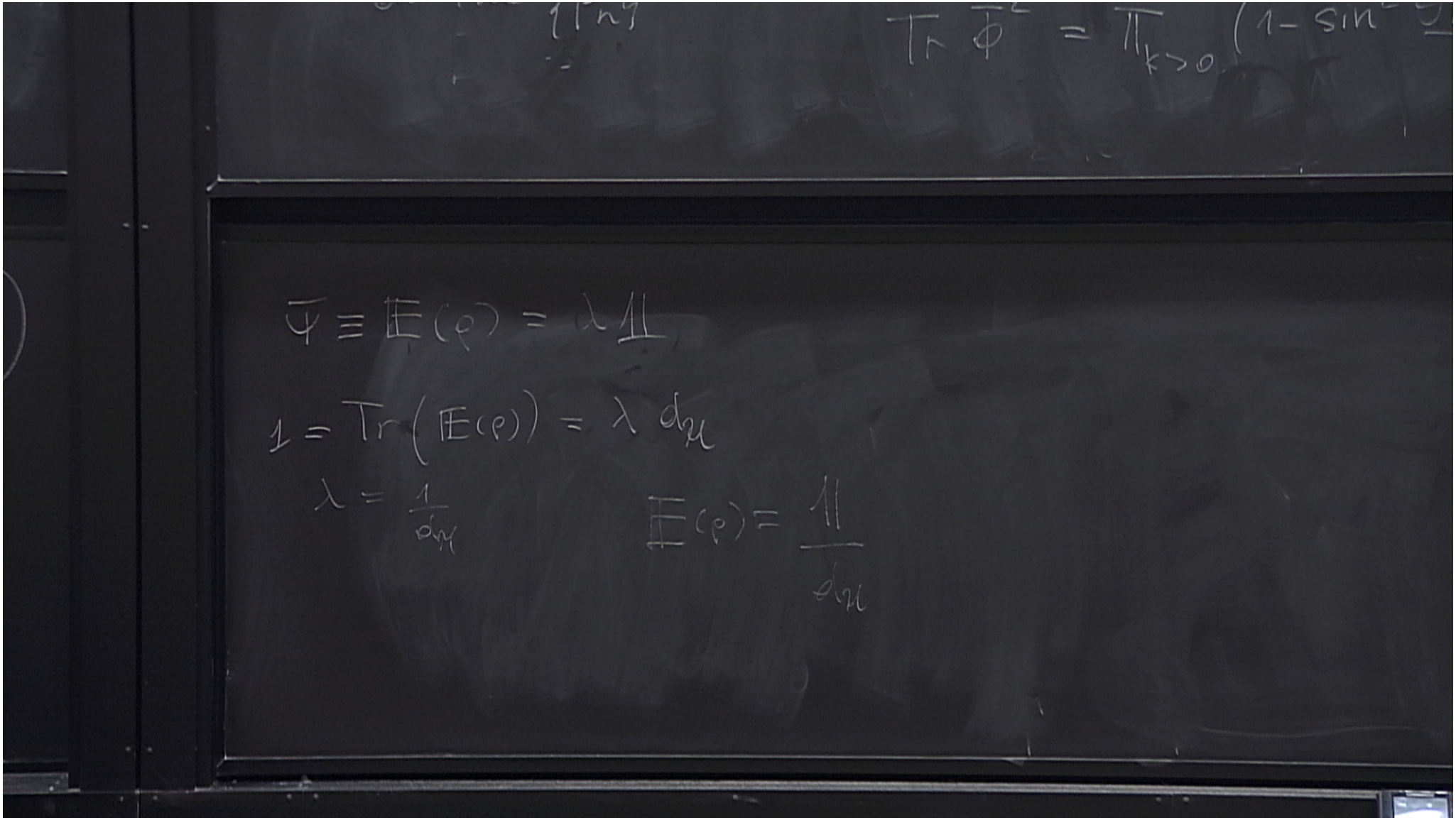
$\mathbb{V}\, \mathbb{E}(\rho) = \int dU\, \underbrace{VU}_{W} |\phi_0\rangle\langle\phi_0| \underbrace{U^\dagger V^\dagger}_{W^\dagger} V = \mathbb{E}(\rho) V$ Schur's Lemma $g \in G$

$\mathcal{U}$ $dU = dW = d(VU)$

Hermitean A : $[A, U_g] = 0$ $\mathcal{H}$

$A = \sum_i \lambda_i \Pi_i$ trivial $A = \lambda \mathbb{1}$


$\Psi \equiv \mathbb{E}(\rho) = \lambda \mathbb{1}$
$1 = \mathrm{Tr}\left(\mathbb{E}(\rho)\right)$


$\Psi \equiv \mathbb{E}(\rho) = \lambda \mathbb{1}$
$1 = \mathrm{Tr}(\mathbb{E}(\rho)) = \lambda\, d_{\mathcal{H}}$
$\lambda = \frac{1}{d_{\mathcal{H}}}$
$\mathbb{E}(\rho) = \frac{\mathbb{1}}{d_{\mathcal{H}}}$

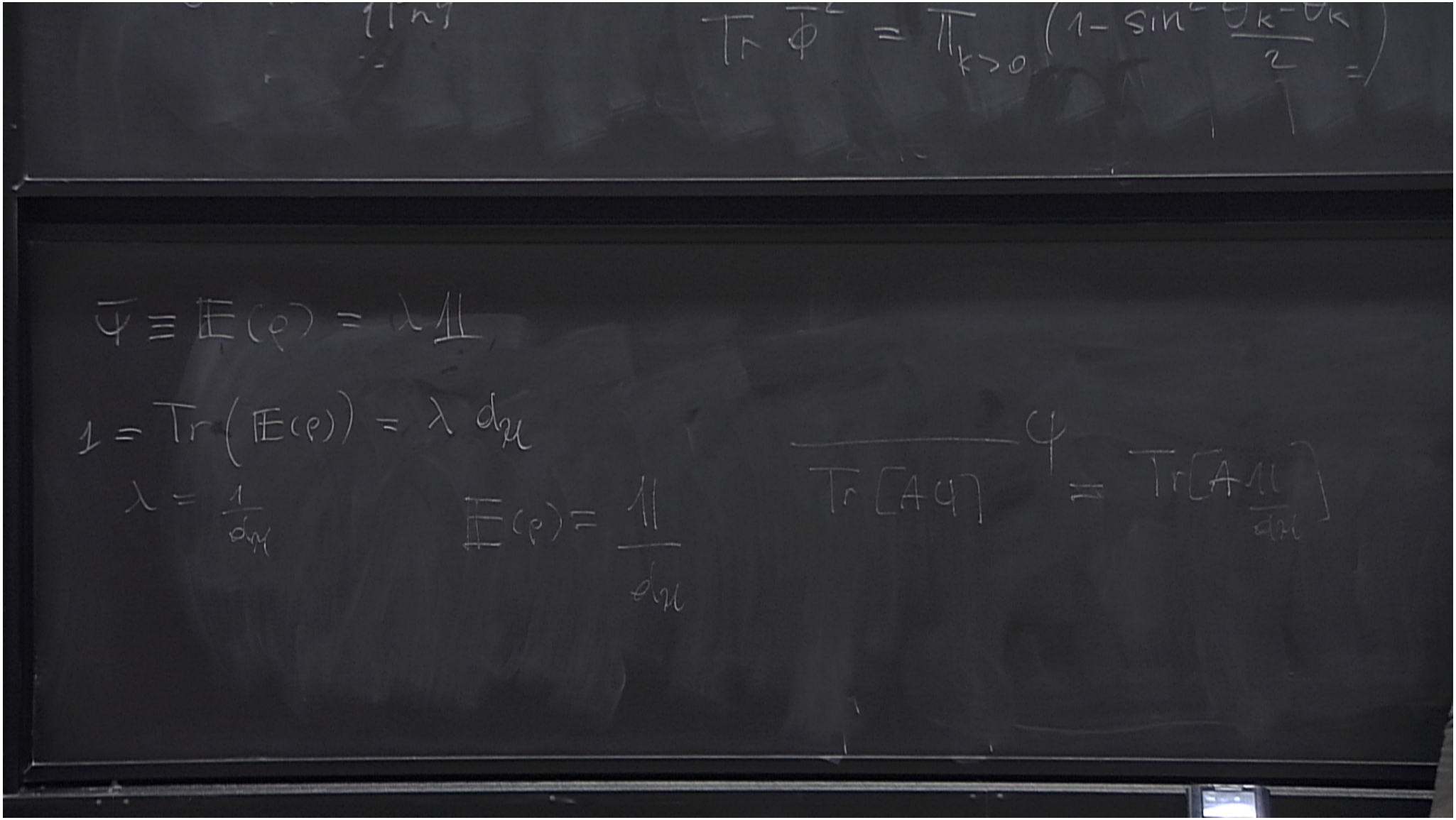
$\Psi \equiv \mathbb{E}(\rho) = \lambda \mathbb{1}$

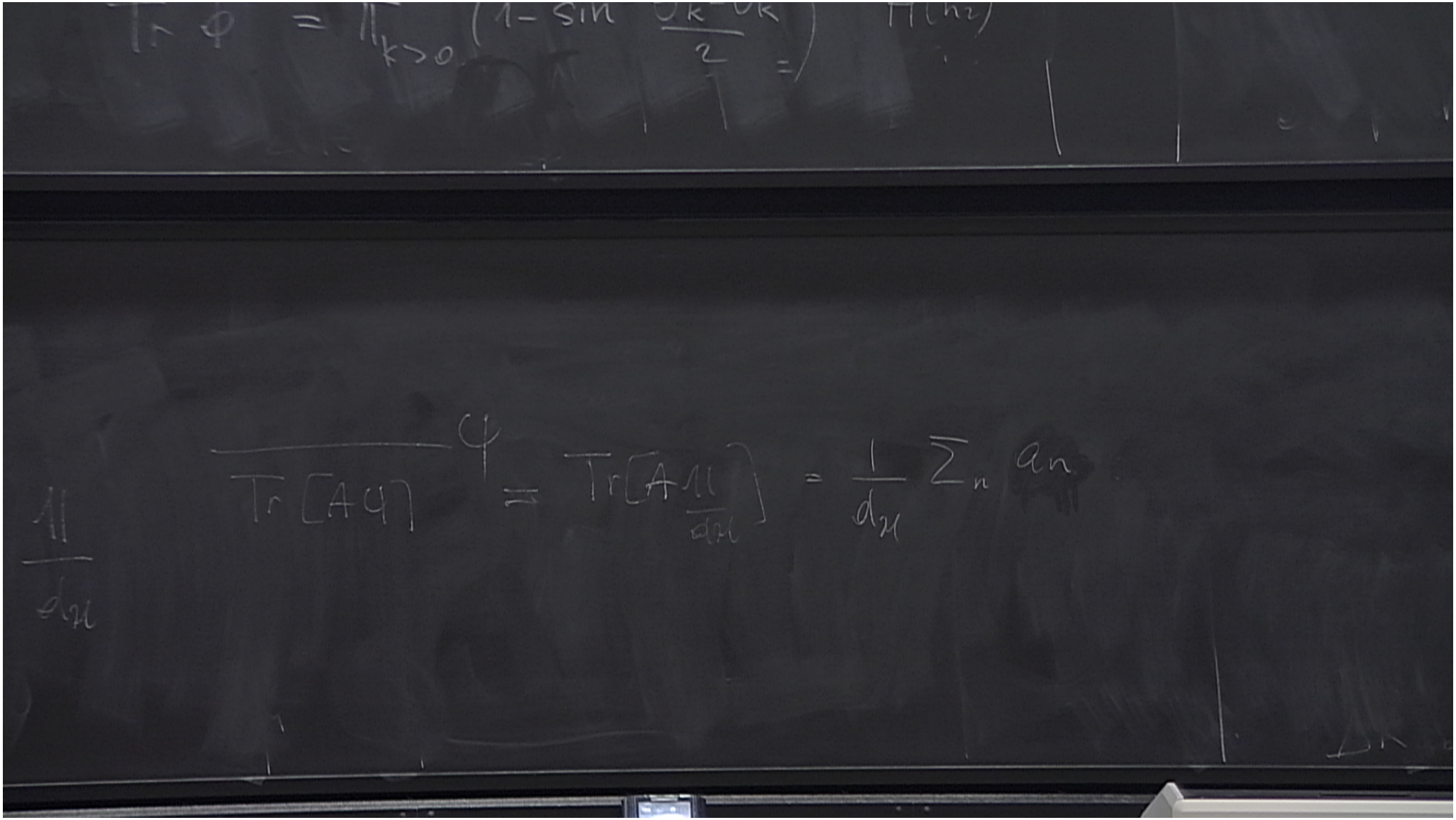
$1 = \mathrm{Tr}(\mathbb{E}(\rho)) = \lambda\, d_{\mathcal{H}}$

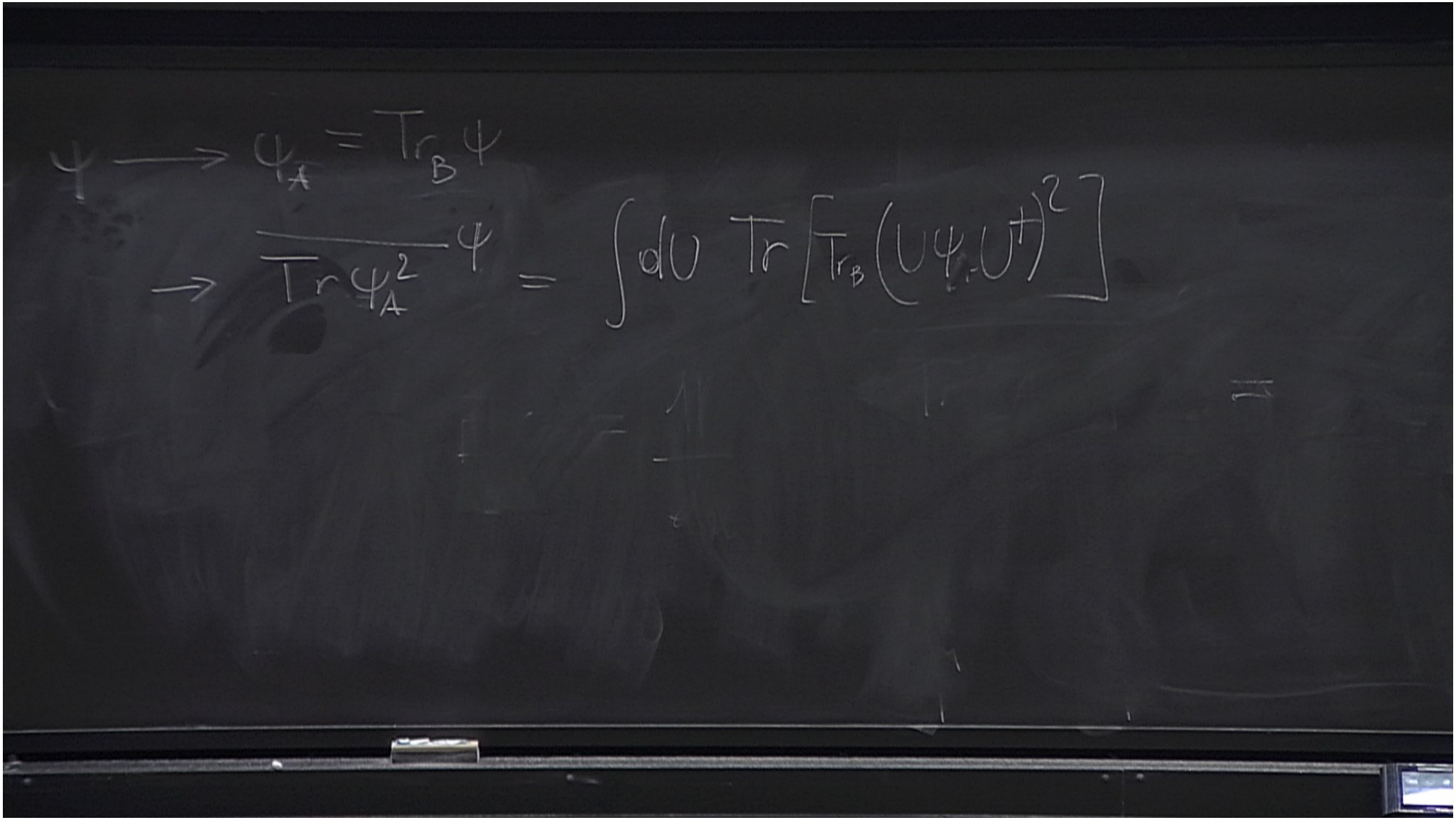
$\lambda = \frac{1}{d_{\mathcal{H}}}$ $\qquad \mathbb{E}(\rho) = \frac{\mathbb{1}}{d_{\mathcal{H}}}$

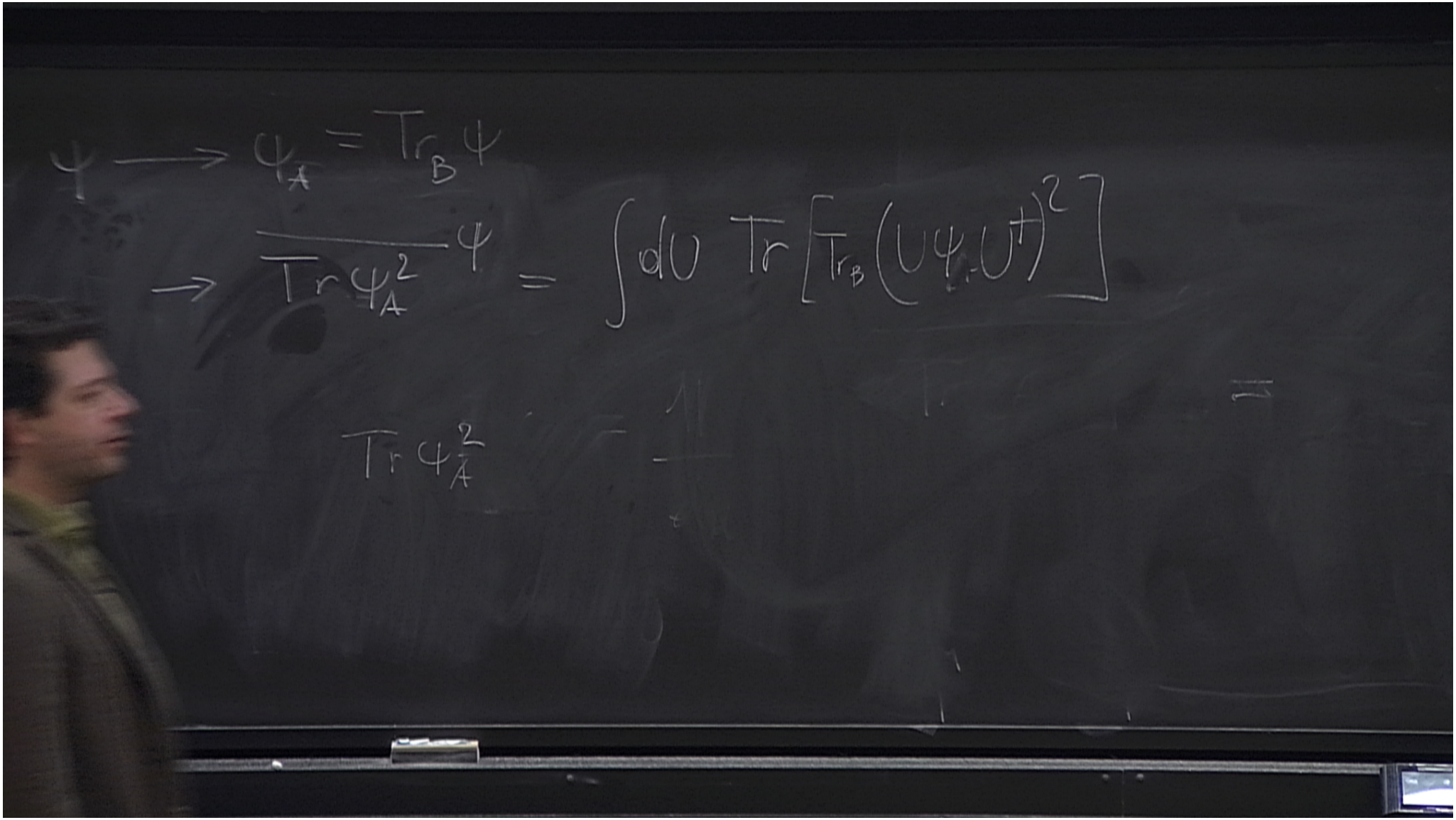
$\overline{\Psi}$

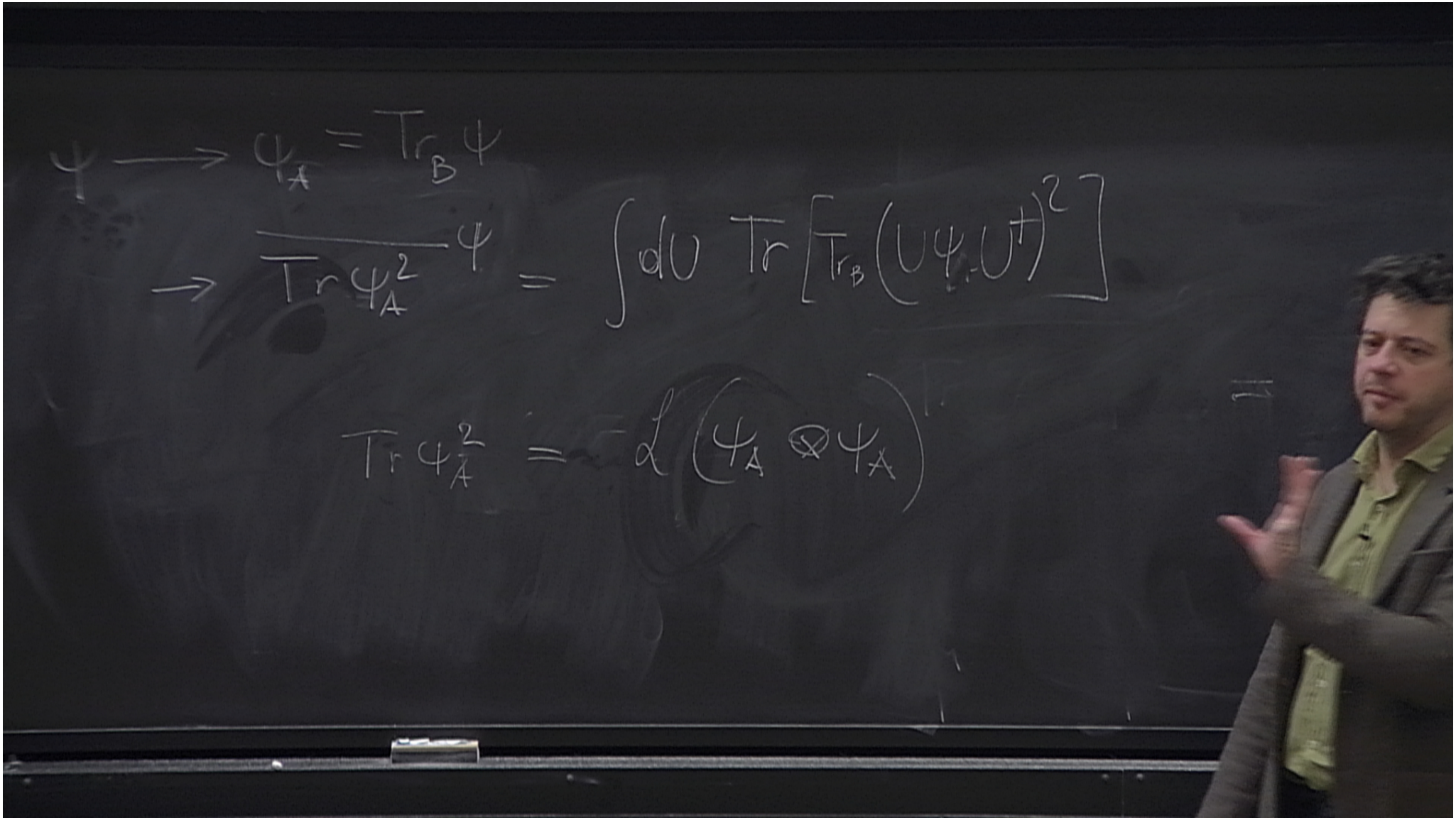
$\mathrm{Tr}[A\rho] = \mathrm{Tr}[A$

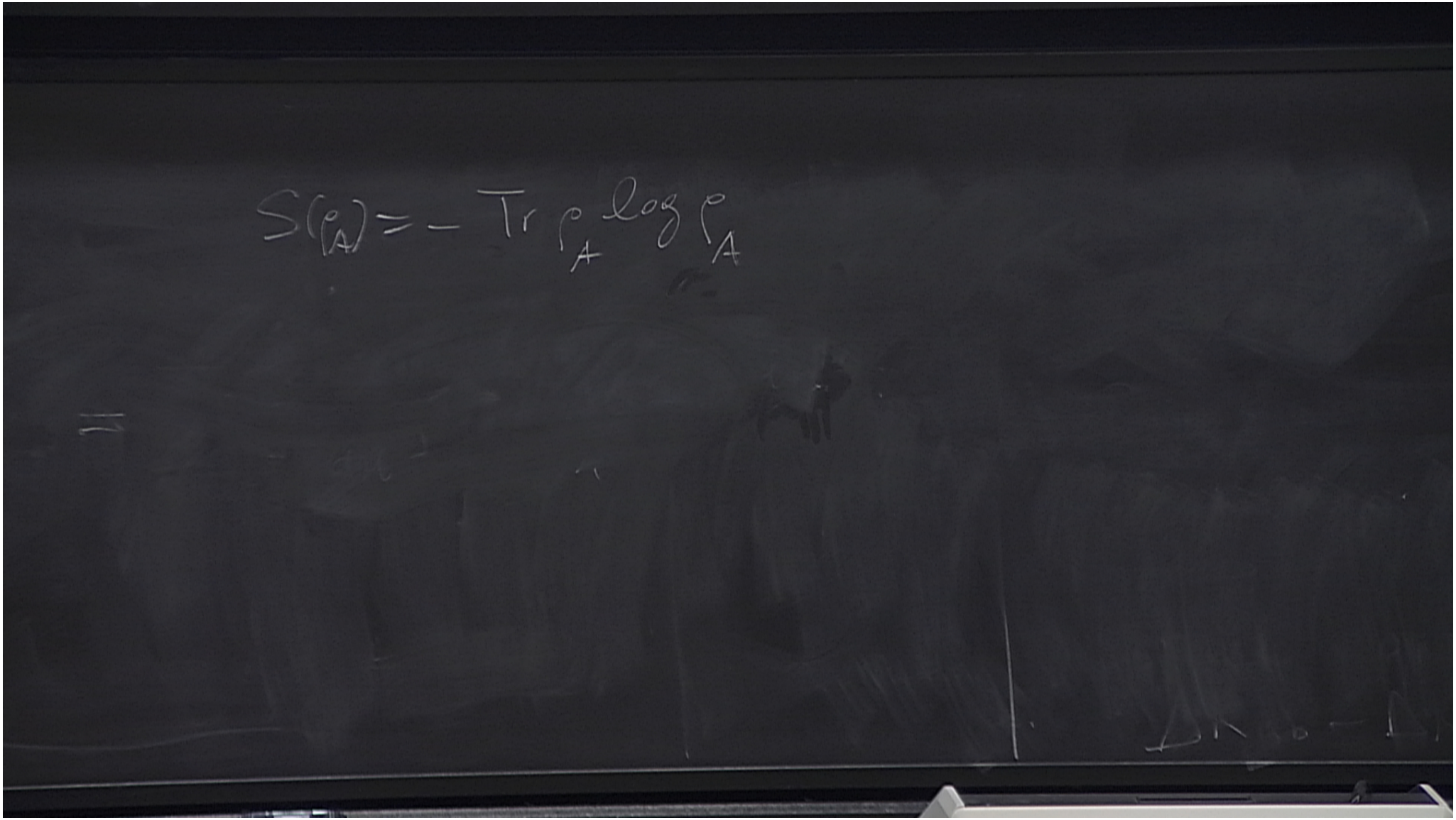

$\mathrm{Tr}\,\bar{\Phi}^2 = \prod_{k>0}\left(1-\sin^2\frac{\theta_k-\theta_{\bar{k}}}{2}\right)$
$\bar{\Psi} \equiv \mathbb{E}(\rho) = \lambda \mathbb{1}$
$1 = \mathrm{Tr}\left(\mathbb{E}(\rho)\right) = \lambda\, d_{\mathcal{H}}$
$\lambda = \frac{1}{d_{\mathcal{H}}}$
$\mathbb{E}(\rho) = \frac{\mathbb{1}}{d_{\mathcal{H}}}$
$\frac{\Psi}{\mathrm{Tr}[A\rho]} = \mathrm{Tr}\left[A\frac{\mathbb{1}}{d_{\mathcal{H}}}\right]$


$\Pi_{k>0}$
$\Pi(h_2)$
$\overline{\mathrm{Tr}[A\Psi]}^{\Psi} = \mathrm{Tr}[A\frac{\Pi}{d_\Pi}] = \frac{1}{d_\Pi}\sum_n a_n$
$\frac{\Pi}{d_\Pi}$


$\psi \longrightarrow \psi_A = \mathrm{Tr}_B \psi$
$\rightarrow \overline{\mathrm{Tr}\,\psi_A^2}^{\psi} = \int dU \, \mathrm{Tr}\left[\mathrm{Tr}_B\left(U \psi U^\dagger\right)^2\right]$


$\psi \longrightarrow \psi_A = \mathrm{Tr}_B \psi$
$\rightarrow \overline{\mathrm{Tr}\,\psi_A^2}\,\psi = \int dU\, \mathrm{Tr}\left[\mathrm{Tr}_B\left(U \psi U^\dagger\right)^2\right]$
$\mathrm{Tr}\,\psi_A^2$


$\psi \longrightarrow \psi_A = \mathrm{Tr}_B \psi$
$\overline{\mathrm{Tr}\,\psi_A^2}^{\psi} = \int dU \, \mathrm{Tr}\left[\mathrm{Tr}_B\left(U \psi U^\dagger\right)^2\right]$
$\mathrm{Tr}\,\psi_A^2 = \mathcal{L}\left(\psi_A \otimes \psi_A\right)$


$S(\rho_A) = -\mathrm{Tr}\, \rho_A \log \rho_A$