

Title: PSI 17/18 - Condensed Matter - Lecture 10

Date: Jan 16, 2018 11:30 AM

URL: <http://pirsa.org/18010043>

Abstract:

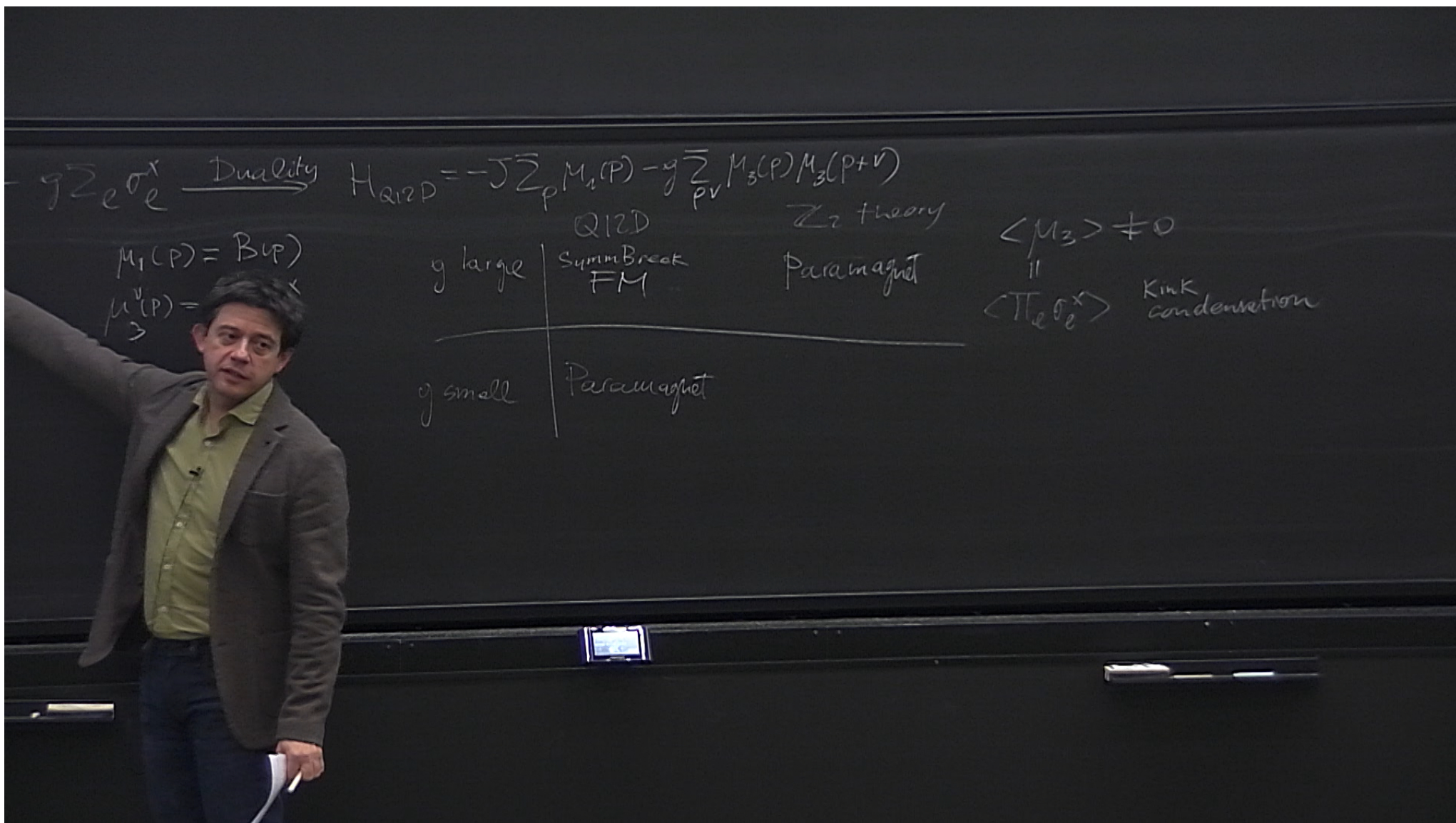
$$H_{\mathbb{Z}_2} = -J \sum_p B(p) - g \sum_e \sigma_e^x \xrightarrow{\text{Duality}} H_{Q12D} = -J \sum_p M_1(p) - g \sum_{p,v} M_3(p) M_3(p+v)$$

$$M_1(p) = B(p)$$

$$\mu_3^v(p) = \prod_{l=0}^p \sigma_l^x$$

g large

g small



$$g \sum_e \sigma_e^x \xrightarrow{\text{Duality}} H_{Q12D} = -J \sum_p M_1(p) - g \sum_{p,v} M_3(p) M_3(p+v)$$

$$M_1(p) = B(p)$$

$$\mu_3^v(p) = \prod_{e \in \text{path}} \sigma_e^x$$

	Q12D	$\mathbb{Z}_2$ theory
g large	Symm Break FM	Paramagnet
g small	Paramagnet	Topological Phase

$$\langle M_3 \rangle \neq 0$$

$$\parallel$$

$$\langle \prod_e \sigma_e^x \rangle \text{ Kink condensation}$$

$$g \sum_e \sigma_e^x \xrightarrow{\text{Duality}} H_{Q12D} = -J \sum_p M_1(p) - g \sum_{p,v} M_3(p) M_3(p+v)$$

$$M_1(p) = B(p)$$

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	Q12D	$\mathbb{Z}_2$ theory
g large	Symm Break FM	Paramagnet
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$$\langle M_3 \rangle \neq 0$$

$$\parallel$$

$$\langle \prod_e \sigma_e^x \rangle \text{ Kink condensation}$$

$$\langle M_3(p) M_3(p+v) M_3(q) M_3(q+v) \rangle$$

$$g \sum_e \sigma_e^x \xrightarrow{\text{Duality}} H_{Q12D} = -J \sum_p M_1(p) - g \sum_{p,v} M_3(p) M_3(p+v)$$

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	Q12D	$\mathbb{Z}_2$ theory
g large	Symm Break FM	Paramagnet
g small	Paramagnet	Topological Phase

$$\langle M_3 \rangle \neq 0$$

||

$$\langle \prod_e \sigma_e^x \rangle \quad \text{Kink condensation}$$

$$\langle \mu_3(p) \mu_3(p+v) \mu_3(q) \mu_3(q+v) \rangle$$

$$g \sum_e \sigma_e^x \xrightarrow{\text{Duality}} H_{Q12D} = -J \sum_p M_1(p) - g \sum_{p,v} M_3(p) M_3(p+v)$$

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	Q12D	$\mathbb{Z}_2$ theory
g large	Symm Break FM	Paramagnet
g small	Paramagnet	Topological Phase

$$\langle M_3 \rangle \neq 0$$

||

$$\langle \prod_e \sigma_e^x \rangle \text{ Kink condensation}$$

$$\langle \mu_3(p) \mu_3(p+v) \mu_3(q) \mu_3(q+v) \rangle$$

$$\sum_p M_1(p) - g \sum_{p,v} M_3(p) M_3(p+v)$$

Q1D

$\mathbb{Z}_2$ theory

Symm Break
FM

Paramagnet

$$\langle \mu_3 \rangle \neq 0$$

$$\langle \Pi_e \sigma_e^x \rangle \quad \text{Kink condensation}$$

Paramagnet

Topological
Phase

$$\langle \mu_3(p) \mu_3(p+v) \mu_3(q) \mu_3(q+v) \rangle \sim e^{-|p-q|/\xi}$$

$$H_{\mathbb{Z}_2} = -J \sum_p B(p) - g \sum_e \sigma_e^x \xrightarrow{\text{Duality}} H_{\text{Q12D}} = -J \sum_p M_1(p) - g \sum_{pv} M_3$$

$$M_1(p) = B(p)$$

$$M_3^v(p) = \prod_{l=0}^p \sigma_l^x$$

$$\begin{array}{ccccccc} \times & \times & \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times & \times & \times \end{array}$$

Q12D	
g large	Symm Break FM
g small	Param

$$\sum_p M_1(p) - g \sum_{p,v} M_3(p) M_3(p+v)$$

Q12D

Symm Breck
FM

$\mathbb{Z}_2$ theory

Paramagnet

$$\langle M_3 \rangle \neq 0$$

$$\langle \Pi_e \sigma_e^x \rangle \quad \text{Kink condensation}$$

Paramagnet

Topological
phase

$$\langle (M_3(p) M_3(p+v)) (M_3(q) M_3(q+v)) \rangle \sim e^{-|p-q|/\xi}$$

$$\langle \sigma_i^x \sigma_j^x \rangle$$

$$M_1(P) = B(P)$$

$$\mu^V(P) = \prod_{l=0}^P \sigma_l^x$$

genus

Degeneracy of GS = 2^{2g}

$$\begin{array}{c} \times \times \times \times \times \\ \diagup \quad \diagdown \end{array}$$

y large	Symm Break FM
y small	Paramagnet

$$\sum_p M_1(p) - g \sum_{p,v} M_3(p) M_3(p+v)$$

Q1D

Symm Break
FM

$\mathbb{Z}_2$ theory

Paramagnet

$$W_1^z W_2^z = +1$$

$$\langle M_3 \rangle \neq 0$$

$$\langle \Pi_e \sigma_e^x \rangle \quad \text{Kink condensation}$$

Para

Topological
phase

$$\langle (M_3(p) M_3(p+v)) (M_3(q) M_3(q+v)) \rangle \sim e^{-|p-q|/\xi}$$

$$\langle \sigma_i^x \sigma_j^x \rangle$$

$$H_{Z_2} = -J \sum_p B(p) - g \sum_e \sigma_e^x \xrightarrow{\text{Duality}} H_{Q12D} = -J \sum_p M_1(p) - g \sum_{pv} M_3(p)$$

$$M_1(p) = B(p)$$

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genus

$$\begin{array}{ccccccc} \times & \times & \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times & \times & \times \end{array}$$

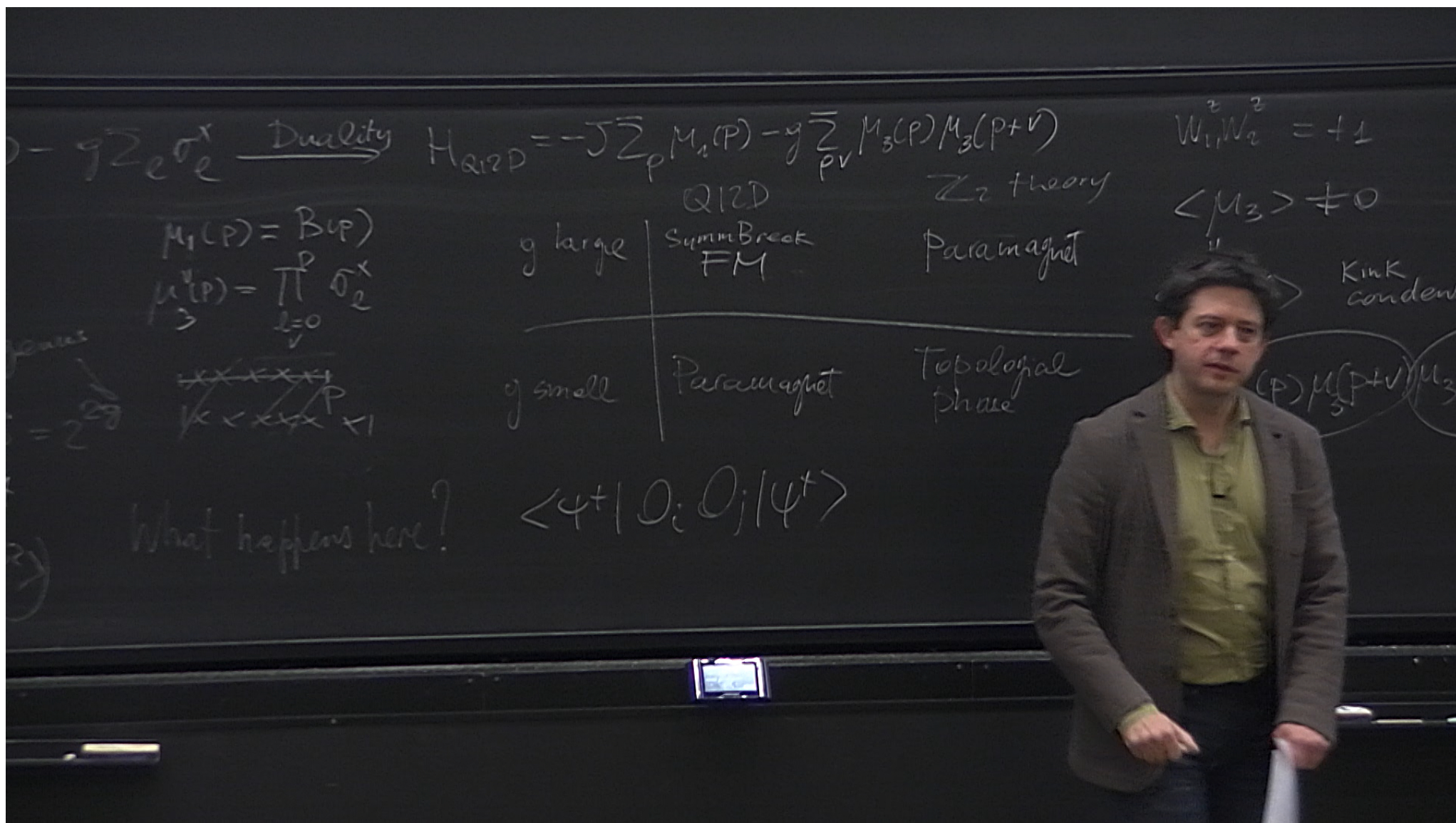
— Degeneracy of GS = 2^{2g}

$$\psi^1, \psi^2, \psi^3, \psi^4$$

$$|\psi^+\rangle = \frac{1}{\sqrt{2}}(|\psi^1\rangle + |\psi^2\rangle)$$

What happens here?

g large	Q12D Symm Break FM
g small	Paramagnet



$$\sum_p M_1(p) - g \sum_{p,v} M_3(p) M_3(p+v)$$

Q12D

Symm Break
FM

$\mathbb{Z}_2$ theory

Paramagnet

$$W_1^2 W_2^2 = +1$$

$$\langle M_3 \rangle \neq 0$$

$$\langle \prod_e \sigma_e^x \rangle \quad \text{Kink condensation}$$

Paramagnet

Topological
phase

$$\langle M_3(p) M_3(p+v) M_3(q) M_3(q+v) \rangle$$

$$e^{-|p-q|/\xi}$$

$$O_i O_j |\psi^+\rangle \approx \delta_{ij}^2$$

$$\langle \sigma_i^x \sigma_j^x \rangle$$

$$O_i =$$

$$\sum_p M_1(p) - g \sum_{p,v} M_3(p) M_3(p+v)$$

Q12D

Symm Break
FM

$\mathbb{Z}_2$ theory

Paramagnet

$$W_1^2 W_2^2 = +1$$

$$\langle M_3 \rangle \neq 0$$

$$\langle \prod_e \sigma_e^x \rangle \quad \text{Kink condensation}$$

Paramagnet

Topological
phase

$$\langle \mu_3(p) \mu_3(p+v) \mu_3(q) \mu_3(q+v) \rangle \sim e^{-|p-q|/\xi}$$

$$O_i O_j |\psi^+\rangle \simeq \alpha^2$$

$$\langle \sigma_i^x \sigma_j^x \rangle$$

$$O_i = \begin{pmatrix} \psi^1 & \psi^2 \\ 0_{11} & 2 \\ \alpha^* & 0_{12} \end{pmatrix} \psi^1$$

$$\sum_p M_1(p) - g \sum_{p,v} M_3(p) M_3(p+v)$$

Q12D

Symm Break
FM

$\mathbb{Z}_2$ theory

Paramagnet

$$W_1^z W_2^z = +1$$

$$\langle M_3 \rangle \neq 0$$

$$\langle \prod_e \sigma_e^x \rangle \quad \text{Kink condensation}$$

$$\sigma_i^z \frac{|0000\rangle + |1111\rangle}{\sqrt{2}}$$

$$\frac{|0000\rangle - |1111\rangle}{\sqrt{2}} \quad -|p-q|/3$$

Paramagnet

Topological
phase

$$\langle \mu_3(p) \mu_3(p+v) \mu_3(q) \mu_3(q+v) \rangle \sim e$$

$$\langle \sigma_i^x \sigma_j^x \rangle$$

$$O_i = \begin{pmatrix} \psi_1 & \psi_2 \\ 0_{11} & 2 \\ \alpha^* & 0_{12} \end{pmatrix} \psi'$$

$$O_i O_j |\psi^+\rangle \simeq \alpha^2$$

Q12D

Symm Break

FM

$\mathbb{Z}_2$ theory

Paramagnet

$\langle \mu_3 \rangle \neq 0$

$\langle \Pi_e \sigma_e^x \rangle$ Kink condensation

$\sigma_i \in \frac{|0000\rangle + |1111\rangle}{\sqrt{2}}$

$\frac{|000\rangle - |111\rangle}{\sqrt{2}} = -|p-q|/2$

Paramagnet

Topological phase

$\langle \mu_3(p) \mu_3(p+v) \mu_3(q) \mu_3(q+v) \rangle \sim e$

$\langle \sigma_i^x \sigma_j^x \rangle$

$O_i = \begin{pmatrix} \psi_1 & \psi_2 \\ 0_{11} & 2 \\ \psi_1^* & 0_{12} \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$

$O_i O_j |\psi^+\rangle \simeq \lambda^2$

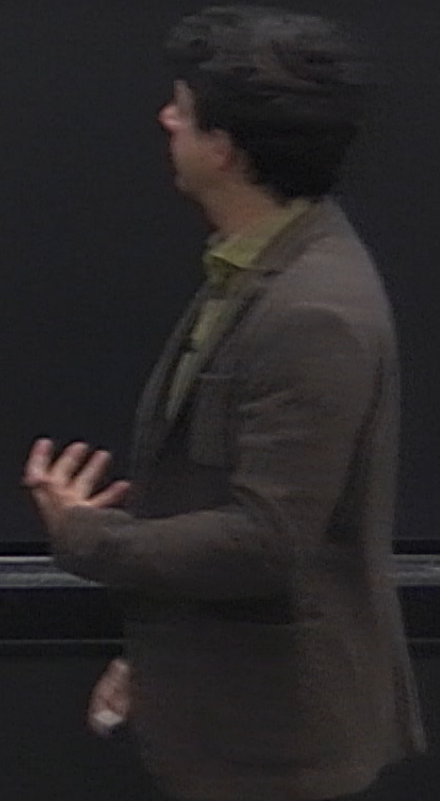
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$$\mathcal{H}_\Lambda = \bigotimes_{i \in \Lambda} \mathcal{H}_i$$

$$\mathcal{H}_\lambda = \bigotimes_{i \in \Lambda} \mathcal{H}_i$$

$$H =$$

$$-J \sum_p B(p) - g \sum_e \sigma_e^x$$



$$\mathcal{H}_\lambda = \bigotimes_{i \in \Lambda} \mathcal{H}_i$$

$$H = -U \sum_n A(n) - J \sum_p B(p) - g \sum_\ell \sigma_\ell^x$$

$$\mathcal{H}_\lambda = \bigotimes_{i \in \Lambda} \mathcal{H}_i$$

$$H = -U \sum_n A(n) - J \sum_p B(p) - g \sum_e \sigma_e^x$$

$$A(n)|4\rangle = |4\rangle$$

$$\mathcal{H}_\lambda = \bigotimes_{i \in \Lambda} \mathcal{H}_i$$

$$H = -U \sum_n A(n) - J \sum_p B(p) - g \sum_\ell \sigma_\ell^x$$

$$g=0 \quad E_0 = -L^2$$

$$A(n)|4\rangle = |4\rangle$$

$$\mathcal{H}_\lambda = \bigotimes_{i \in \Lambda} \mathcal{H}_i$$

$$H = -U \sum_n A(n) - J \sum_p B(p) - g \sum_\ell \sigma_\ell^x$$

$$g=0 \quad E_0 = -L^2(U+J)$$

$$A(n)|\psi\rangle = |\psi\rangle$$

$$\mathcal{H}_\Lambda = \bigotimes_{i \in \Lambda} \mathcal{H}_i$$

$$H = -U \sum_n A(n) - J \sum_p B(p) - g \sum_e \sigma_e^x$$

$$U \gg J, g \quad \mathcal{L}_{\text{low}} = \{ A(n) | \psi \rangle \equiv | \psi \rangle \}$$

$$g=0 \quad E_0 = -L^2(U + \dots)$$

$$\mathcal{H}_\Lambda = \bigotimes_{i \in \Lambda} \mathcal{H}_i$$

$$H = -U \sum_n A(n) - J \sum_p B(p) - g \sum_e \sigma_e^x$$

$$U \gg J, g$$

$$g=0 \quad E_0 = -L^2(U +$$

$$\mathcal{L}_{\text{low}} = \{ A(n) | \psi \rangle = | \psi \rangle \} = \mathcal{H}_{\text{gauge}}$$

$$H = -U \sum_n A(n) - J \sum_p B(p) - g \sum_e \sigma_e^z$$

$$g=0 \quad E_0 = -L^2(U+J)$$

$$A(n) |4\rangle = |4\rangle \} = \mathcal{H}_{\text{gauge}}$$

$$H_i: \quad H = -U \sum_n A(n) - J \sum_p B(p) - \lambda \sum_i O_i$$

$$g=0 \quad E_0 = -L^2(U+J)$$

$$A(n) |4\rangle = |4\rangle \quad \mathcal{H}_{\text{gauge}}$$

$$H_i: \quad H = -U \sum_n A(n) - J \sum_p B(p) - \lambda \sum_e V_e$$

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$$A(n) |4\rangle = |4\rangle \} = \mathcal{H}_{\text{gauge}}$$

$$H = -U \sum_n A(n) - J \sum_p B(p) - \lambda \sum_i V_i$$

$$g=0 \quad E_0 = -L^2(U+J)$$

$$\mathcal{Z} = \mathcal{H}_{\text{gauge}}$$

$$g=0$$

$$O_i O_j \neq W$$

$$H = -U \sum_n A(n) - J \sum_p B(p) - \lambda \sum_e V_e$$

$$g=0 \quad E_0 = -L^2(U+J)$$

$$\mathcal{Z} = \mathcal{H}_{\text{gauge}}$$

$$g=0$$

$$O_i O_j \neq W \rightarrow \alpha=0$$

$$H = -U \sum_n A(n) - J \sum_p B(p) - \lambda \sum_e V_e$$

$$g=0 \quad E_0 = -L^2(U+J)$$

$$\mathcal{Z} = \mathcal{H}_{\text{gauge}}$$

$$g=0$$

$$O_i O_j \neq W \rightarrow \alpha=0$$

$$\Delta E = 0$$

$$H = -U \sum_n A(n) - J \sum_p B(p) - \lambda \sum_i V_i$$

$$\lambda = 0 \quad E_0 = -L^2(U+J)$$

$$\mathcal{H} = \mathcal{H}_{\text{gauge}}$$

$$\lambda = 0$$

$$O_i O_j \neq W \rightarrow \alpha = 0$$

$$\Delta E = 0$$

$$\psi(\mathbf{r}) = \lambda \sum_i V_i$$

$$E_0 = -L^2(V+J)$$

$$O_i O_j \neq W \rightarrow \alpha = 0$$

$$\Delta E = 0$$

$$\Delta E \sim \langle \psi^2 | \lambda \sum_i V_i | \psi^2 \rangle$$

$$S(\psi) = \lambda \sum_i V_e$$

$$E_0 = -L^2(V+J)$$

$$O_i O_j \neq W \rightarrow \alpha = 0$$

$$\Delta E = 0$$

$$\Delta E \sim \langle \psi^2 | \lambda \sum_e V_e | \psi^2 \rangle$$

ψ^2

$$S(\psi) = \lambda \sum_i V_e$$

$$E_0 = -L^2(V+J)$$

$$O_i O_j \neq W \rightarrow \alpha = 0$$

$$\Delta E = 0$$

$$\Delta E \sim \langle \psi^2 | (\lambda \sum_e V_e)^2 | \psi^2 \rangle$$

$$S(\psi) = \lambda \sum_i V_e$$

$$E_0 = -L^2(V+J)$$

$$O_i O_j \neq W \rightarrow \alpha=0$$

$$\Delta E = 0$$

$$\Delta E \sim \langle \psi^2 | \lambda \sum_e V_e | \psi^2 \rangle$$

$$\lambda^2 \sum_{e,e'} \langle \psi^2 | V_e V_{e'} | \psi^2 \rangle$$

$$S(\psi) = \lambda \sum_e V_e$$

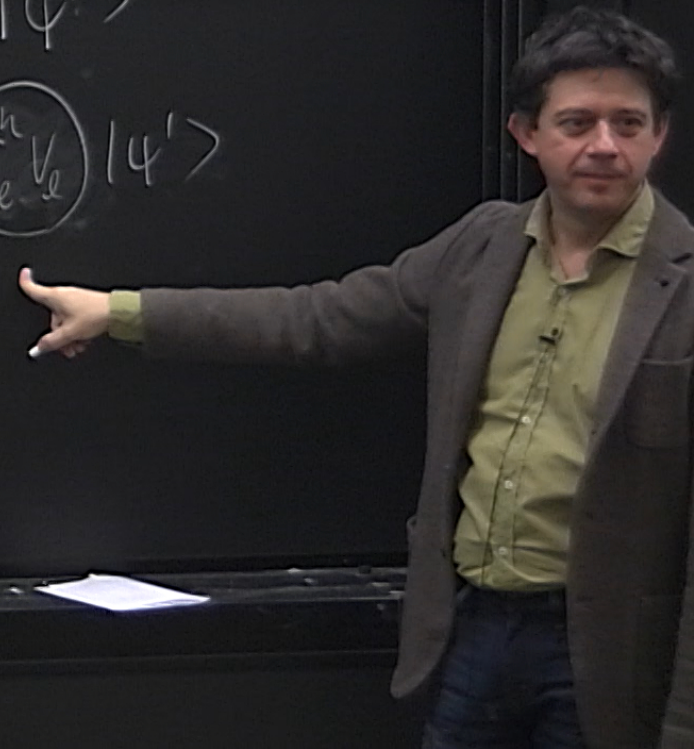
$$E_0 = -L^2(V+J)$$

$$O_i O_j \neq W \rightarrow \alpha = 0$$

$$\Delta E = 0$$

$$\Delta E \sim \langle \psi^2 | (\lambda \sum_e V_e)^2 | \psi^2 \rangle$$

$$\lambda^n \sum_{e_1, \dots, e_n} \langle \psi^2 | \prod_e V_{e_i} | \psi^2 \rangle$$



$$S(\psi) = \lambda \sum_i V_e$$

$$E_0 = -L^2(U+J)$$

$$O_i O_j \neq W \rightarrow \alpha = 0$$

$$\Delta E = 0$$

$$\Delta E \sim \langle \psi^2 | (\lambda \sum_e V_e) | \psi^2 \rangle$$

$$\lambda \sum_{e \in \dots}^n \langle \psi^2 | \left(\frac{1}{n} \sum_e V_e \right) | \psi^2 \rangle$$

↓
 nR

$$S(\psi) = \lambda \sum_e V_e$$

$$E_0 = -L^2(V+J)$$

$$O_i O_j \neq W \rightarrow \alpha = 0$$

$$\Delta E = 0$$

$$\Delta E \sim \langle \psi^2 | (\lambda \sum_e V_e)^2 | \psi^2 \rangle$$

$$\lambda \sum_{e \in \dots} \langle \psi^2 | \left(\frac{1}{n} \sum_e V_e \right)^n | \psi^2 \rangle$$

$$nR = L$$

$$S(\psi) = \lambda \sum_e V_e$$

$$E_0 = -L^2(U+J)$$

$$O_i O_j \neq W \rightarrow \alpha=0$$

$$\Delta E = 0$$

$$\Delta E \sim \frac{L}{\lambda R} \quad n = \frac{L}{R}$$

$$\Delta E \sim \langle \psi^2 | (\lambda \sum_e V_e)^2 | \psi^2 \rangle$$

$$\lambda^n \sum_{e \dots} \langle \psi^2 | \left(\frac{\pi_e^n V_e}{\pi_e} \right) | \psi^2 \rangle$$

$$nR = L$$

$$S(\psi) = \lambda \sum_e V_e$$

$$E_0 = -L^2(U+J)$$

$$O_i O_j \neq W \rightarrow \alpha = 0$$

$$\Delta E = 0$$

$$\Delta E \sim \langle \psi^2 | (\lambda \sum_e V_e)^2 | \psi^2 \rangle$$

$$\lambda^n \sum_{e_1, \dots, e_n} \langle \psi^2 | \prod_e V_e | \psi^2 \rangle$$

$$nR = L$$

$$\Delta E \sim \frac{L}{\lambda R} \sim e^{-L/R} \rightarrow 0$$

$$\mathcal{H}_\lambda = \bigotimes_{i \in \Lambda} \mathcal{H}_i$$

$$H = -U \sum_n A(n) - J \sum_p B(p) - \lambda \sum_i$$

$$U \gg J, g$$

$$\mathcal{L}_{\text{low}} = \{ A(n) | \psi \rangle = | \psi \rangle \} = \mathcal{H}_{\text{gauge}}$$

$$\lambda = 0 \quad E_0 = -L^2(114$$

$$\lambda = 0$$

$$O_i$$

$$\Delta E$$

$$|0000\rangle \longrightarrow |1111\rangle$$

$$\mathcal{H}_\lambda = \bigotimes_{i \in \Lambda} \mathcal{H}_i$$

$$H = -U \sum_n A(n) - J \sum_p B(p) - \lambda \sum_i$$

$$U \gg J, g$$

$$\mathcal{L}_{low} = \{ A(n) | \psi \rangle = | \psi \rangle \} = \mathcal{H}_{gauge}$$

$$\lambda = 0 \quad E_0 = -L^2 (U +$$

$$\lambda = 0 \quad \phi_i \phi_j =$$

$$|0000\rangle \longrightarrow |1111\rangle$$

$$\Delta E = 0$$

$$|000\rangle + |111\rangle \xrightarrow{\delta_i^x} |000\rangle - |111\rangle$$

$$34p) - \lambda \sum_i V_e$$

$$E_0 = -L^2(U+J)$$

$$O_i O_j \neq W \rightarrow \alpha = 0$$

$$\Delta E = 0$$

11)

$$\Delta E \sim \langle \psi^2 | (\lambda \sum_e V_e)^2 | \psi^2 \rangle$$

$$\lambda^n \sum_{e_1, \dots, e_n} \langle \psi^2 | \prod_e V_e | \psi' \rangle$$

$$nR = L$$

$$\psi' = N \psi^2$$

$$\Delta E \sim \frac{L}{\lambda R} \sim e \rightarrow 0$$

$$S(p) = \lambda \sum_e V_e$$

$$E_0 = -L^2(U+J)$$

$$O_i O_j \neq W \rightarrow \alpha = 0$$

$$\Delta E = 0$$

11)

$$\Delta E \sim \langle \psi^2 | (\lambda \sum_e V_e)^2 | \psi^2 \rangle$$

$$\lambda^n \sum_{e_1, \dots, e_n} \langle \psi^2 | \prod_e V_e | \psi^2 \rangle$$

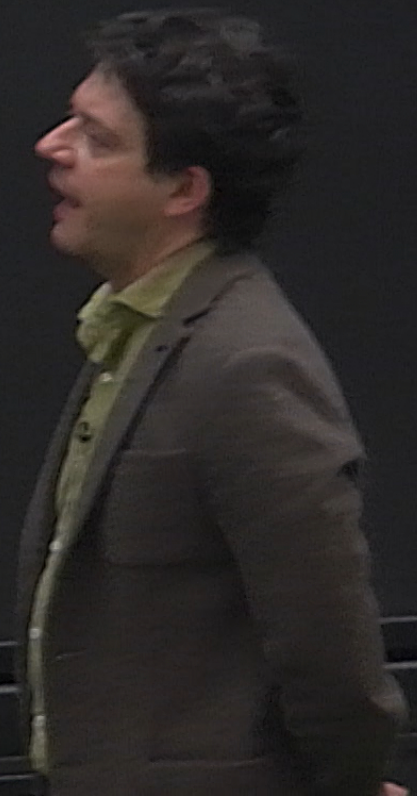
$$\vec{n} R = L$$

$$\psi' = N \psi^2$$

$$\psi^+ = N \psi^-$$

$$\Delta E \sim \frac{L}{\lambda R} \sim e \rightarrow 0$$

$$\lambda = 0$$



$$\boxed{\lambda=0}$$

$$E_0 = -\ell^2 (U+J)$$

$$|\psi_0\rangle = \text{span} \left\{ \begin{array}{c} \psi^1, \psi^2, \psi^3, \psi^4 \\ \parallel \quad \parallel \\ w_1 \psi^1 \quad w_2 \psi^2 \end{array} \right\} = w_1 w_2 \psi^1$$

$$|\psi^1\rangle = |A_i = +1/2, B_i = +1/2\rangle$$

$$|\psi_n\rangle$$

$$\boxed{\lambda=0}$$

$$E_0 = -c^2 (U+J)$$

$$|\psi_0\rangle = \text{span} \left\{ \begin{array}{c} \psi^1, \psi^2, \psi^3, \psi^4 \\ \parallel \quad \parallel \\ W_1 \psi^1 \quad W_2 \psi^2 \end{array} \right\} = \text{span} \psi^1$$

$$|\psi^1\rangle = |A_i = +1, B_i = +1\rangle$$

$$|\psi_n\rangle = |A_i = -1, A_j = -1, B_l = -1, B_k = -1\rangle$$

A excitations

4U

B excitations

4J

----- E_0

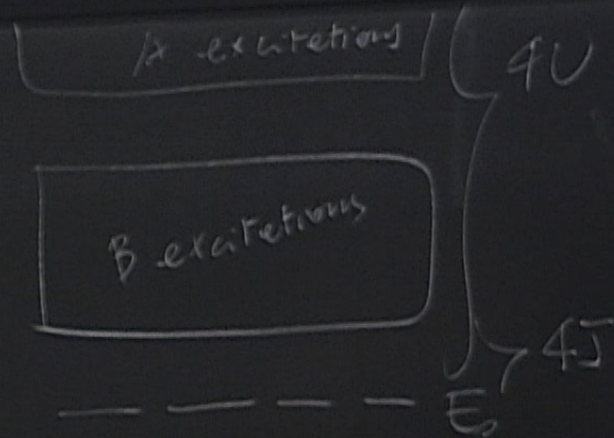
$$\boxed{\lambda=0}$$

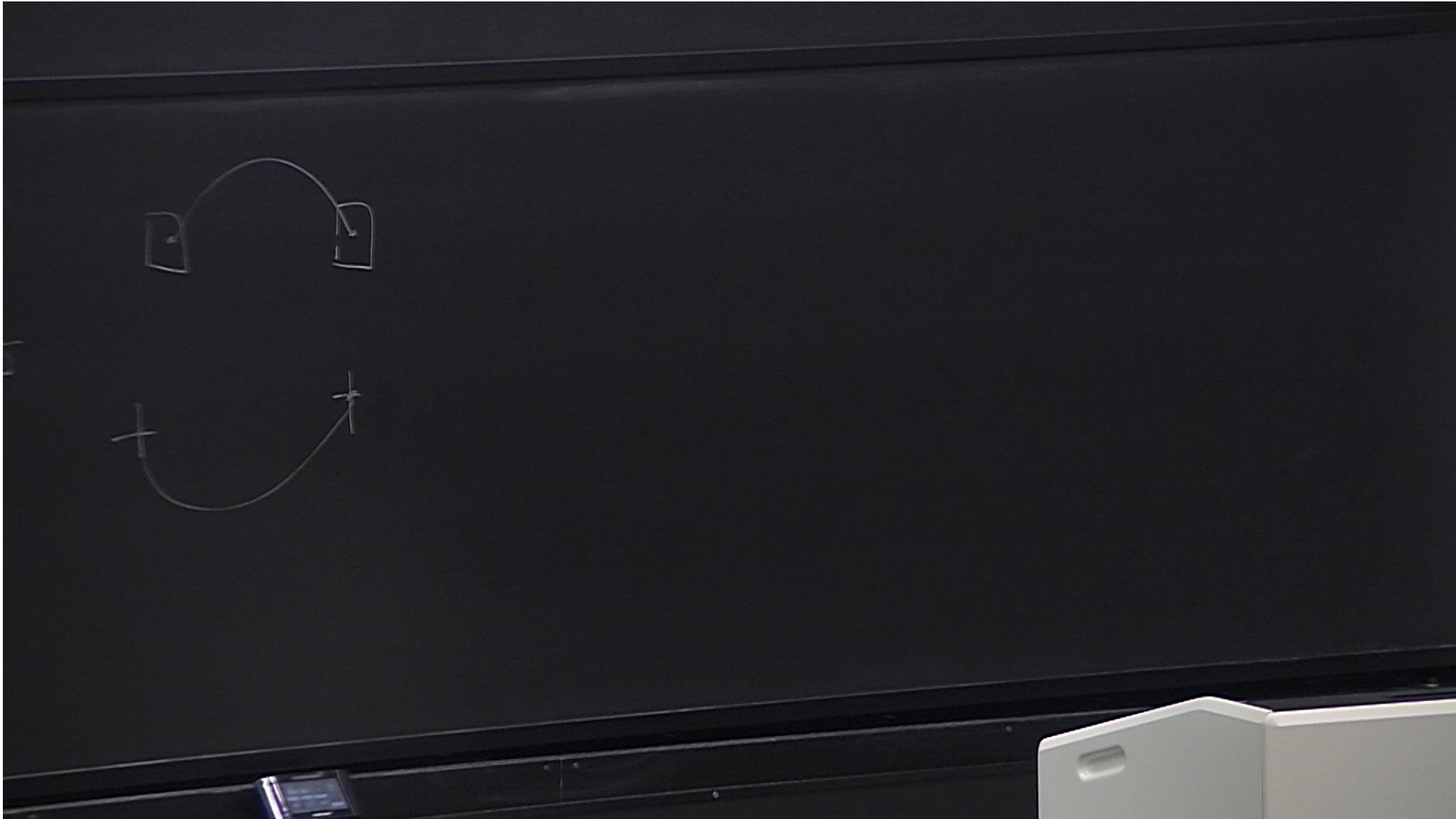
$$E_0 = -L^2 (U+J)$$

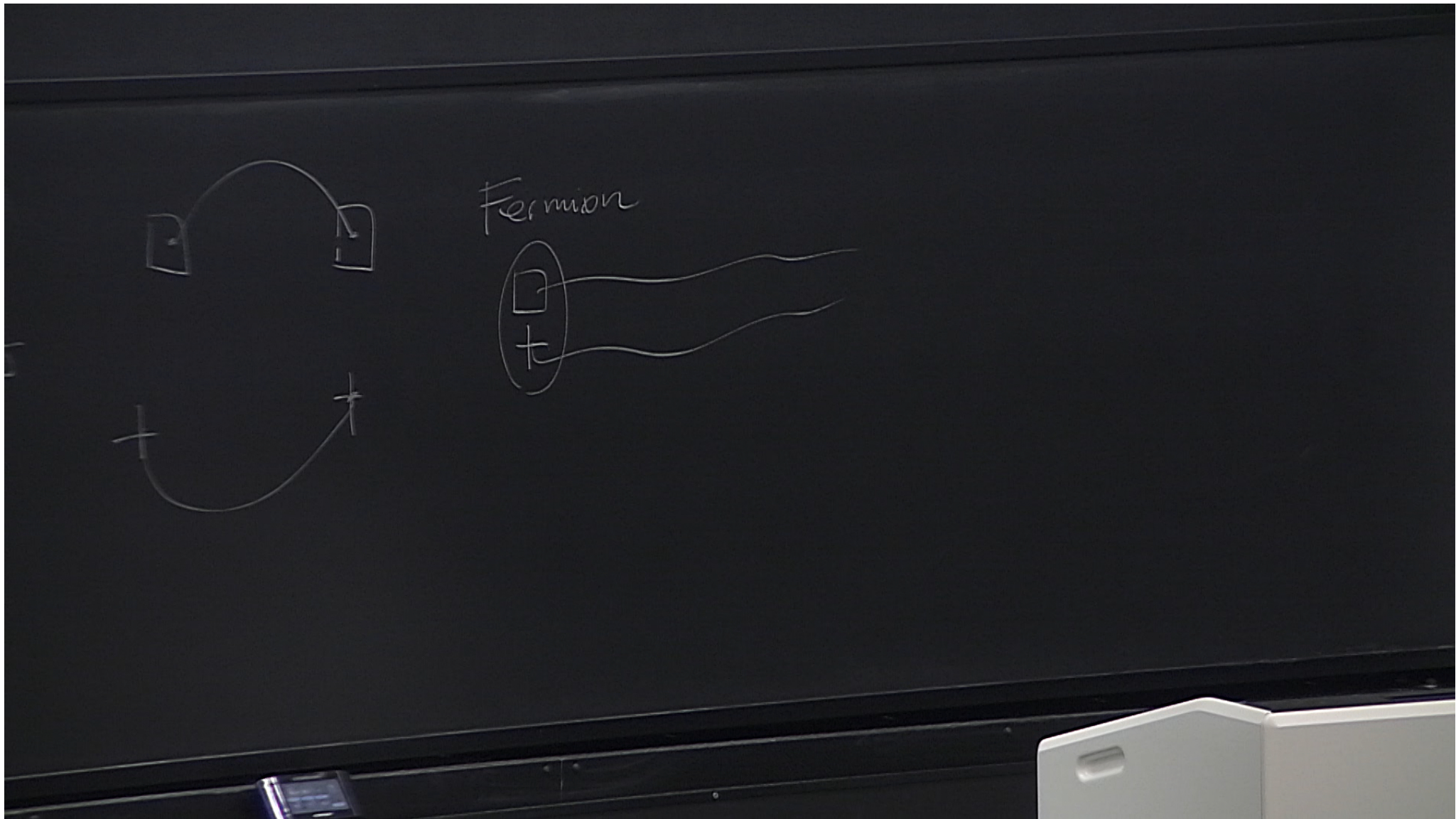
$$|\psi_0\rangle = \text{span} \left\{ \begin{array}{c} \psi^1, \psi^2, \psi^3, \psi^4 \\ \text{"} \quad \text{"} \quad \text{"} \quad \text{"} \\ w_1 \psi^1 \quad w_2 \psi^2 \end{array} \right\} = w_1 w_2 \psi^1$$

$$|\psi^1\rangle = |A_i = +1, B_i = +1\rangle$$

$$|\psi_n\rangle = |A_i = -1, A_j = -1, B_1 = -1, B_k = -1\rangle$$







$$\boxed{\lambda=0}$$

$$E_0 = -c^2 (U+J)$$

$$|\psi_0\rangle = \text{span} \left\{ \begin{array}{c} \psi^1, \psi^2, \psi^3, \psi^4 \\ \parallel \quad \parallel \\ w_1 \psi^1 \quad w_2 \psi^2 \end{array} \right\} = w_1 w_2 \psi^1$$

$$|\psi^1\rangle = |A_i = +1, B_i = +1\rangle$$

$$|\psi_n\rangle = |A_i = -1, A_j = -1, B_i = -1, B_k = -1\rangle$$

A excitations

4U

B excitations

4J

E_0

Semion

