Title: PSI 17/18 - Condensed Matter - Lecture 7

Date: Jan 11, 2018 11:30 AM

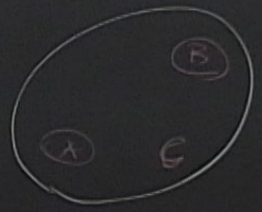
URL: http://pirsa.org/18010040

Abstract:

Applications of the LR bounds

$\| [A(t), B] \| \leq 2 \|A\| \|B\| \epsilon^{\mu(vt - \ell)}$

1. LR $\Rightarrow$ No signalling outside the light cone



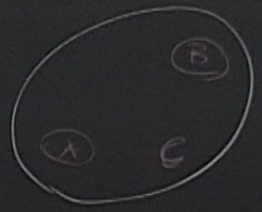

$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B \otimes \mathcal{H}_C$



Applications of the LR bounds

$\| [A(t), B] \| < 2 \|A\| \|B\| e^{\mu(vt - \ell)}$

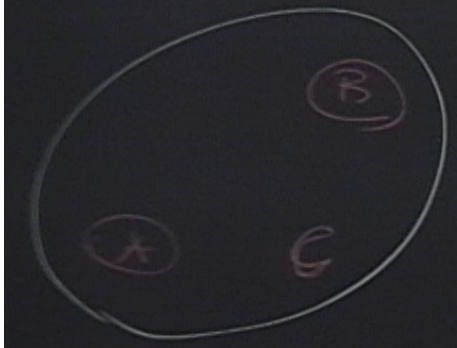
1. LR $\Rightarrow$ No signalling outside the light cone




$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B \otimes \mathcal{H}_C$

$\rho^0_{ABC}$ $\xrightarrow{\text{input}}$ $U^k_A \rho^0_{ABC} U^{k\dagger}_A$

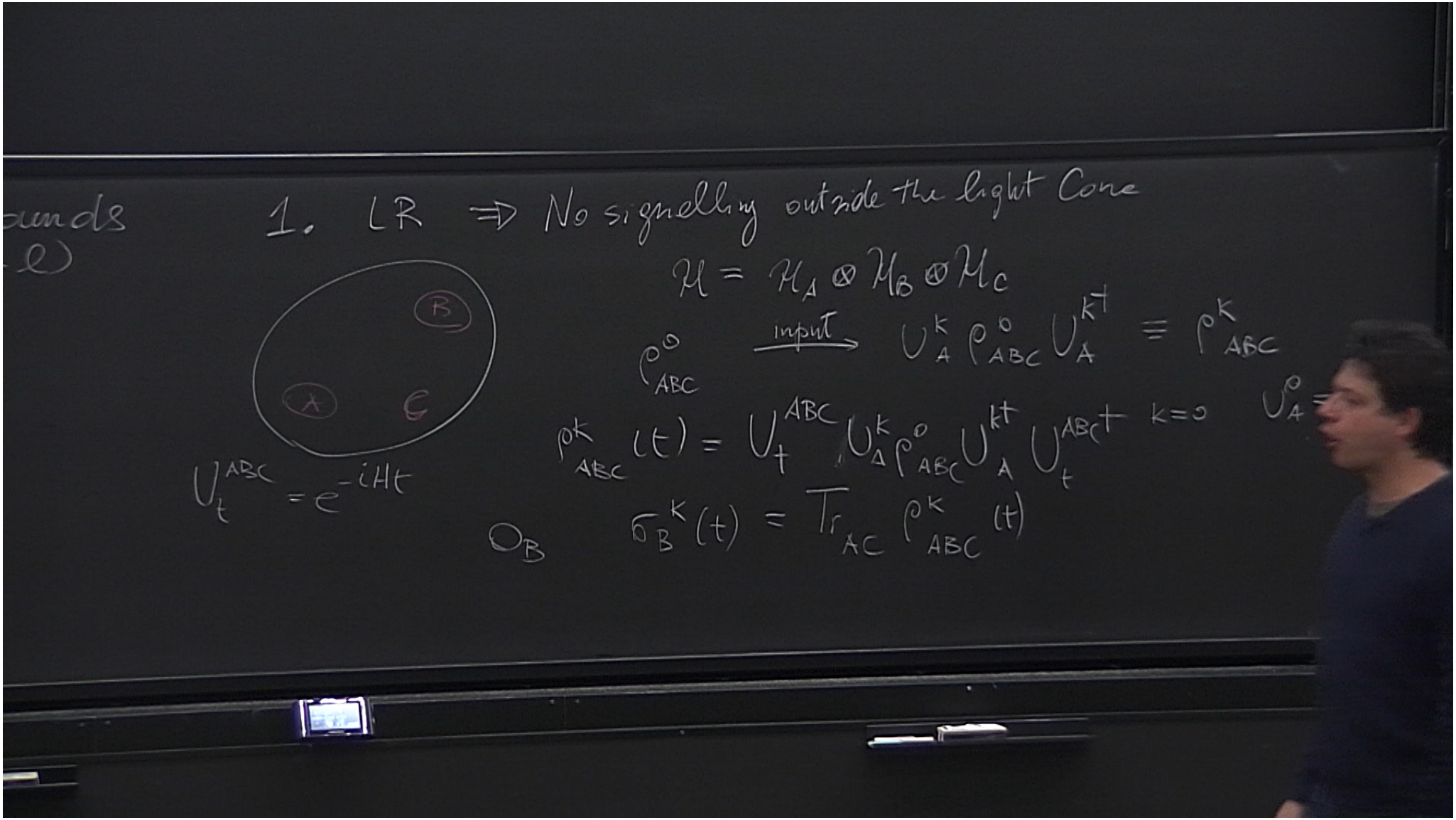
1. LR $\Rightarrow$ No signalling outside the light Cone



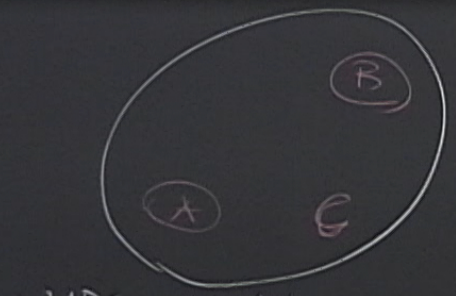

$$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B \otimes \mathcal{H}_C$$

$$\rho^0_{ABC} \xrightarrow{\text{input}} U^k_A \rho^0_{ABC} U^{k\dagger}_A \equiv \rho^k_{ABC}$$

$$\rho^k_{ABC}(t) = U^{ABC}_t U^k_A \rho^0_{ABC} U^{k\dagger}_A U^{ABC\dagger}_t \quad k=0 \quad U^0_A = 1\!\!1_A$$


1. LR ⇒ No signalling outside the light Cone
$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B \otimes \mathcal{H}_C$
$\rho^0_{ABC}$ input $U^k_A \rho^0_{ABC} U^{k\dagger}_A \equiv \rho^k_{ABC}$
$\rho^k_{ABC}(t) = U_t^{ABC} U^k_A \rho^0_{ABC} U^{kt}_A U_t^{ABC\dagger}$ k=0
$U_t^{ABC} = e^{-iHt}$
$\sigma_B^k(t) = Tr_{AC} \rho^k_{ABC}(t)$
$O_B$
A
B
C

1. LR ⇒ No signalling outside the light cone




$$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B \otimes \mathcal{H}_C$$

$$\rho^0_{ABC} \xrightarrow{\text{input}} U^k_A \rho^0_{ABC} U^{k\dagger}_A \equiv \rho^k_{ABC}$$

$$\rho^k_{ABC}(t) = U^{ABC}_t U^k_A \rho^0_{ABC} U^{k\dagger}_A U^{ABC\dagger}_t \qquad k=0 \quad U^0_A = \mathbb{1}_A$$

$$U^{ABC}_t = e^{-iHt}$$

$$O_B \qquad \sigma^k_B(t) = \mathrm{Tr}_{AC}\, \rho^k_{ABC}(t)$$

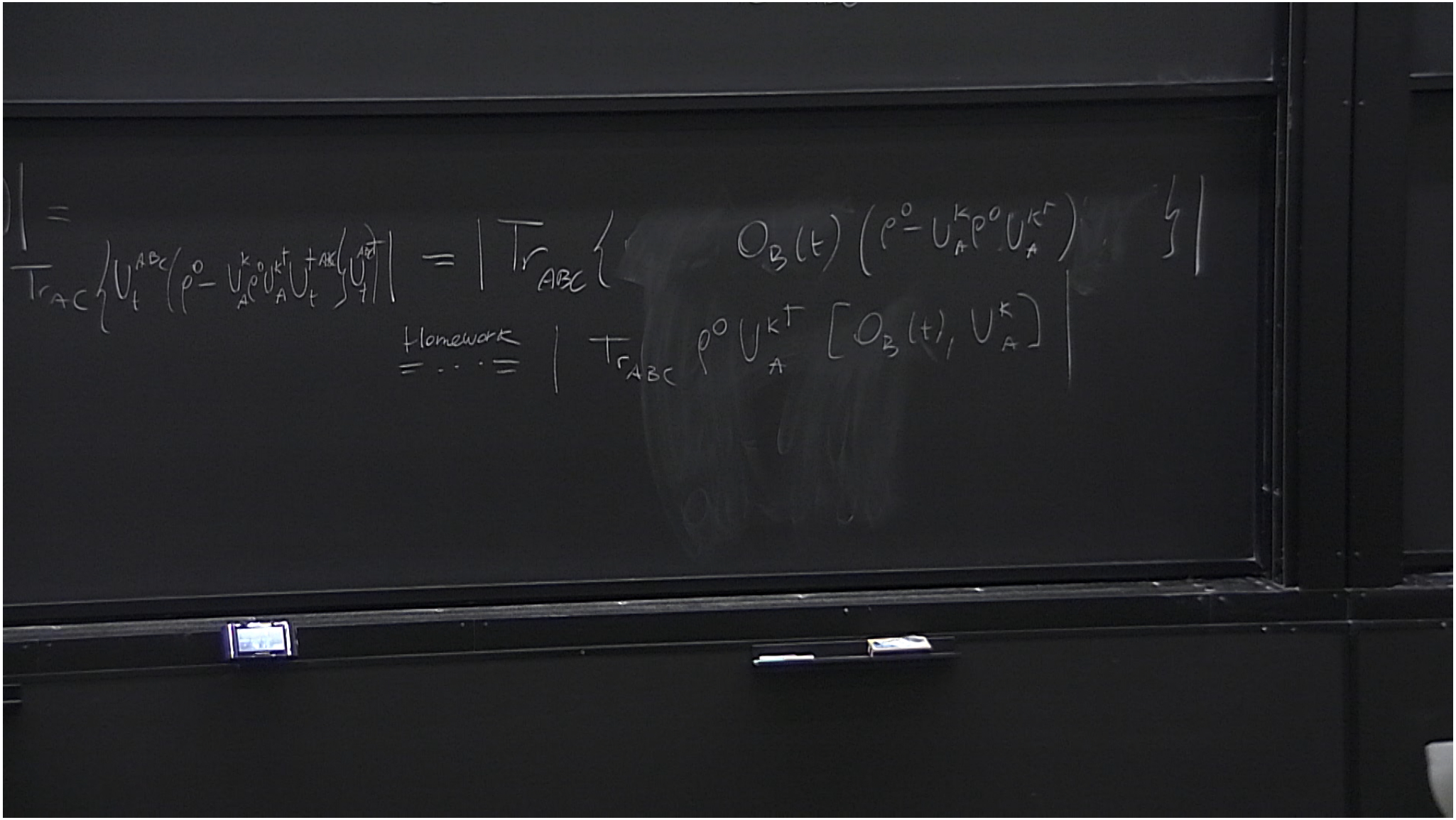
$$\left| Tr\left[ O_B \sigma_B^k(t) \right] - Tr\left[ O_B \sigma_B^0(t) \right] \right| =$$

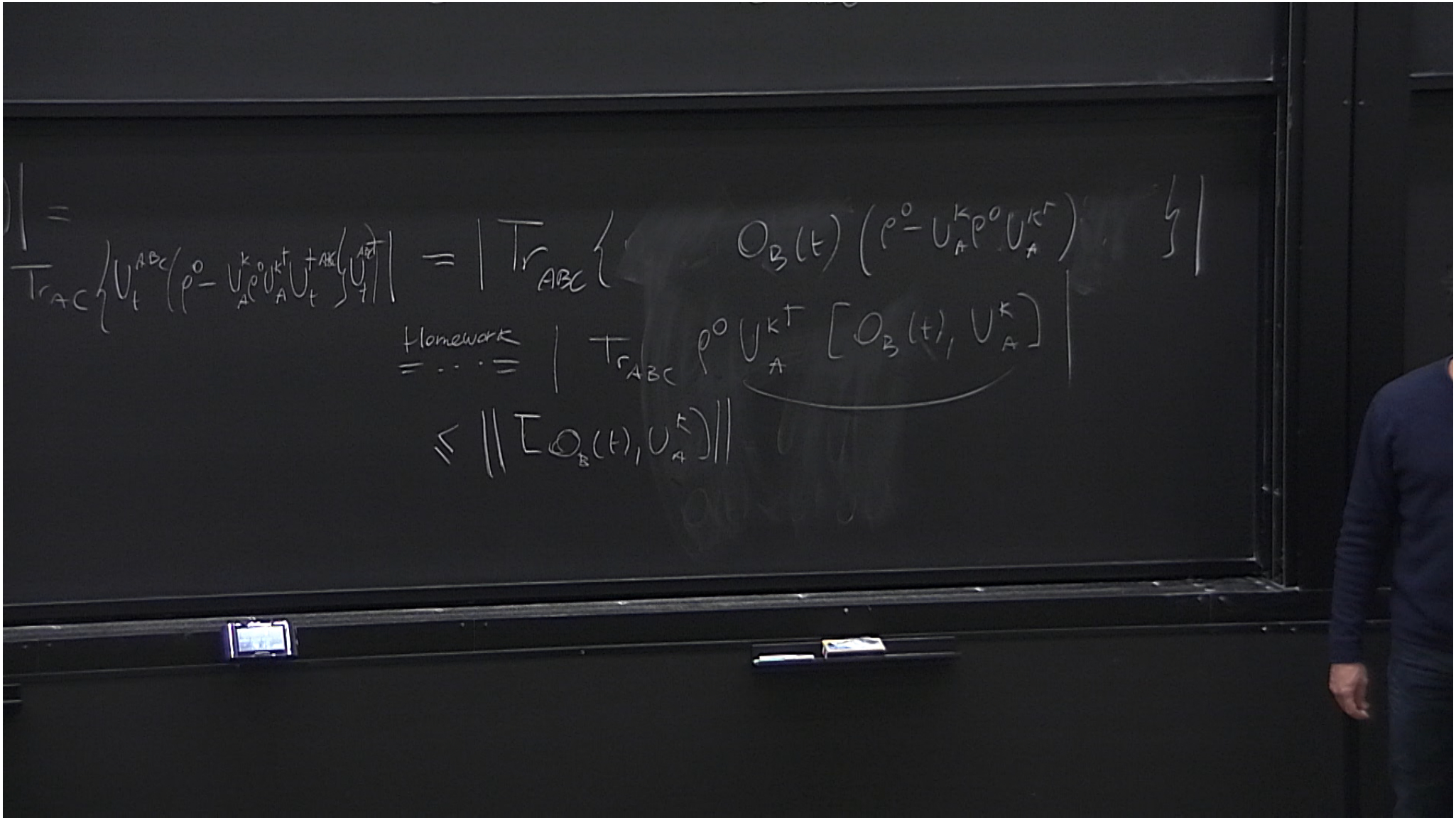
$$= \left| Tr_B \, O_B \left( \sigma_B^k(t) - \sigma_B^0(t) \right) \right| = \left| Tr_B \left( O_B \, Tr_{AC} \left\{ U_t^{ABC} \left( \rho^0 - U_A^k \rho^0 U_A^{k\dagger} \right) U_t^{\dagger ABC} \right\} \right) \right|$$

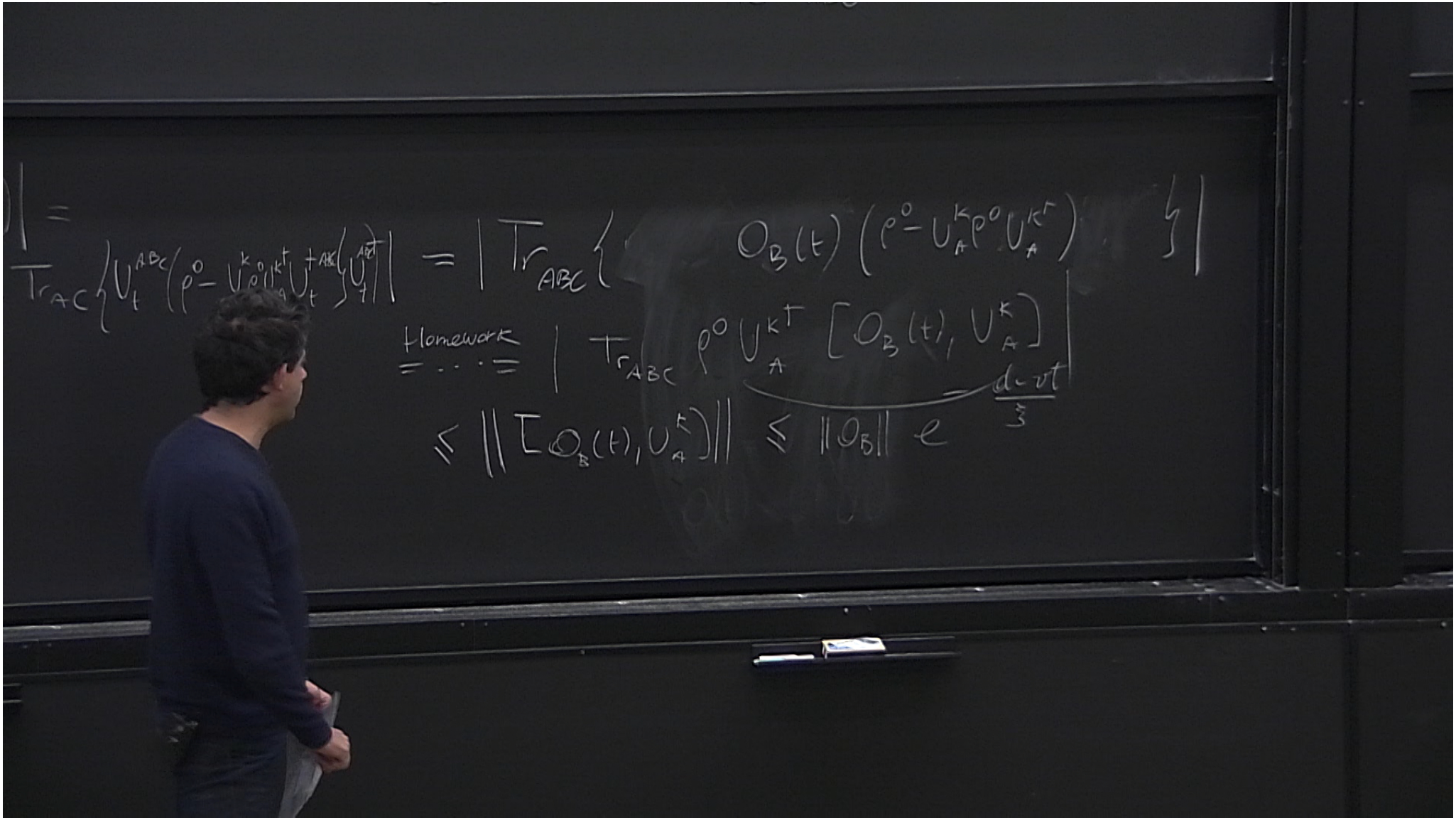
$$\left| Tr\left[ O_B \sigma_B^k(t) \right] - Tr\left[ O_B \sigma_B^0(t) \right] \right| =$$

$$= \left| Tr_B \; O_B \left( \sigma_B^k(t) - \sigma_B^0(t) \right) \right| = \left| Tr_B \left( O_B \; Tr_{AC} \left\{ U_t^{ABC} \left( \rho^0 - U_{AC}^k \rho^0 U_A^{k\dagger} U_t^{\dagger ABC} \right) \right\} \right| = \left| Tr \right.$$

$$Tr_a (X \, Tr_b \, Y) = Tr_{ab} (X \otimes 1_b \, Y)$$


Homework


Homework


Homework

Truncation of Time-evolved operators

$O_A$ is local

$O_A(t) = e^{iHt} O_A e^{-iHt}$ is everywhere

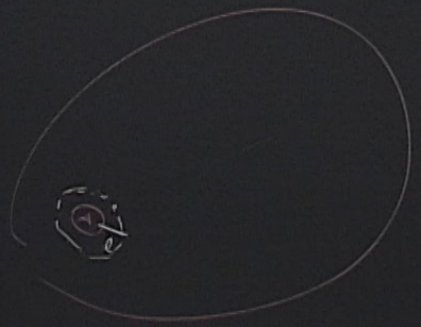


Truncation of Time-evolved operators

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$O_A(t) = e^{iHt} O_A e^{-iHt}$ is everywhere

$O_A^{\ell}(t) \quad O_A(t)$

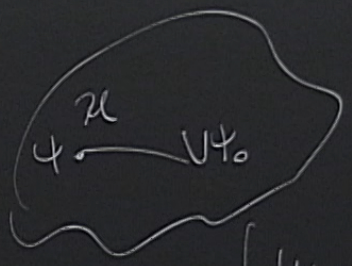


lved

$O_A$ is local

$O_A(t) = e^{iHt} O_A e^{-iHt}$ is everywhere

$O_A^{\ell}(t) \quad O_A(t)$




$\int dV = 1$

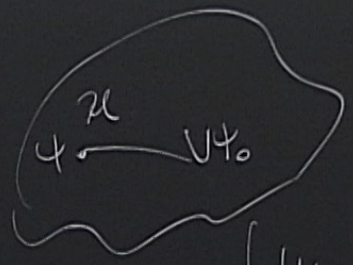
$\int dV \; V^{\dagger} O V$

$\hat{I}$ Haar integral

$=$

$e^{iHt} O_A e^{-iHt}$ is everywhere

$O_A(t)$



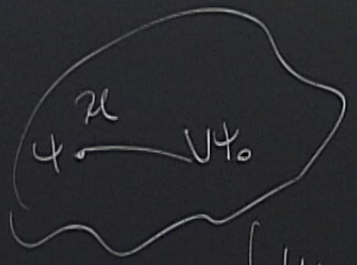

$\int dU = 1$

$\int dU\, U^\dagger O U$

↑ Haar integral

$= \sum_{irr} \frac{\mathrm{Tr}[\Pi_i O]}{\mathrm{Tr}\,\Pi_i} \Pi_i$

everywhere

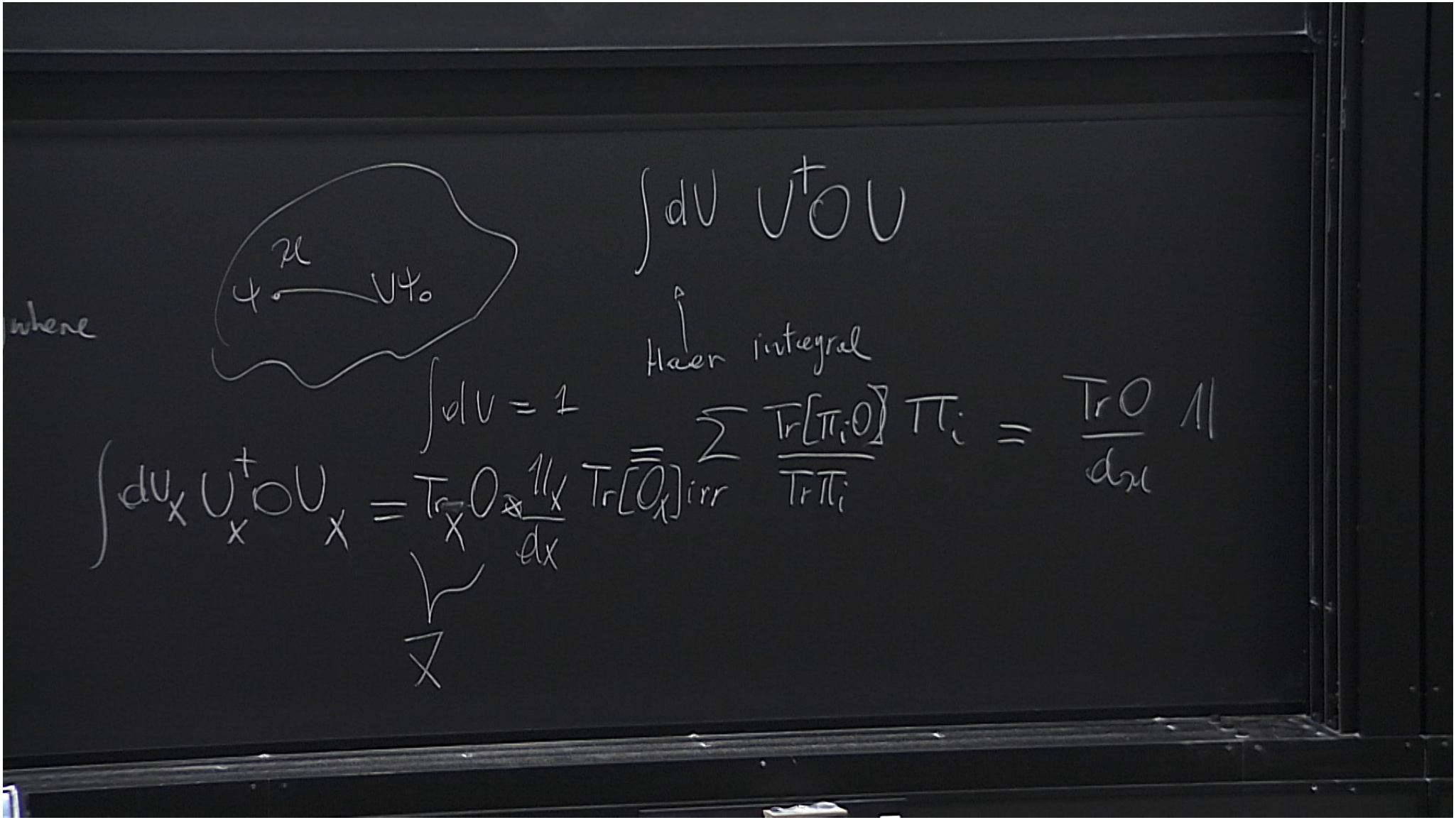


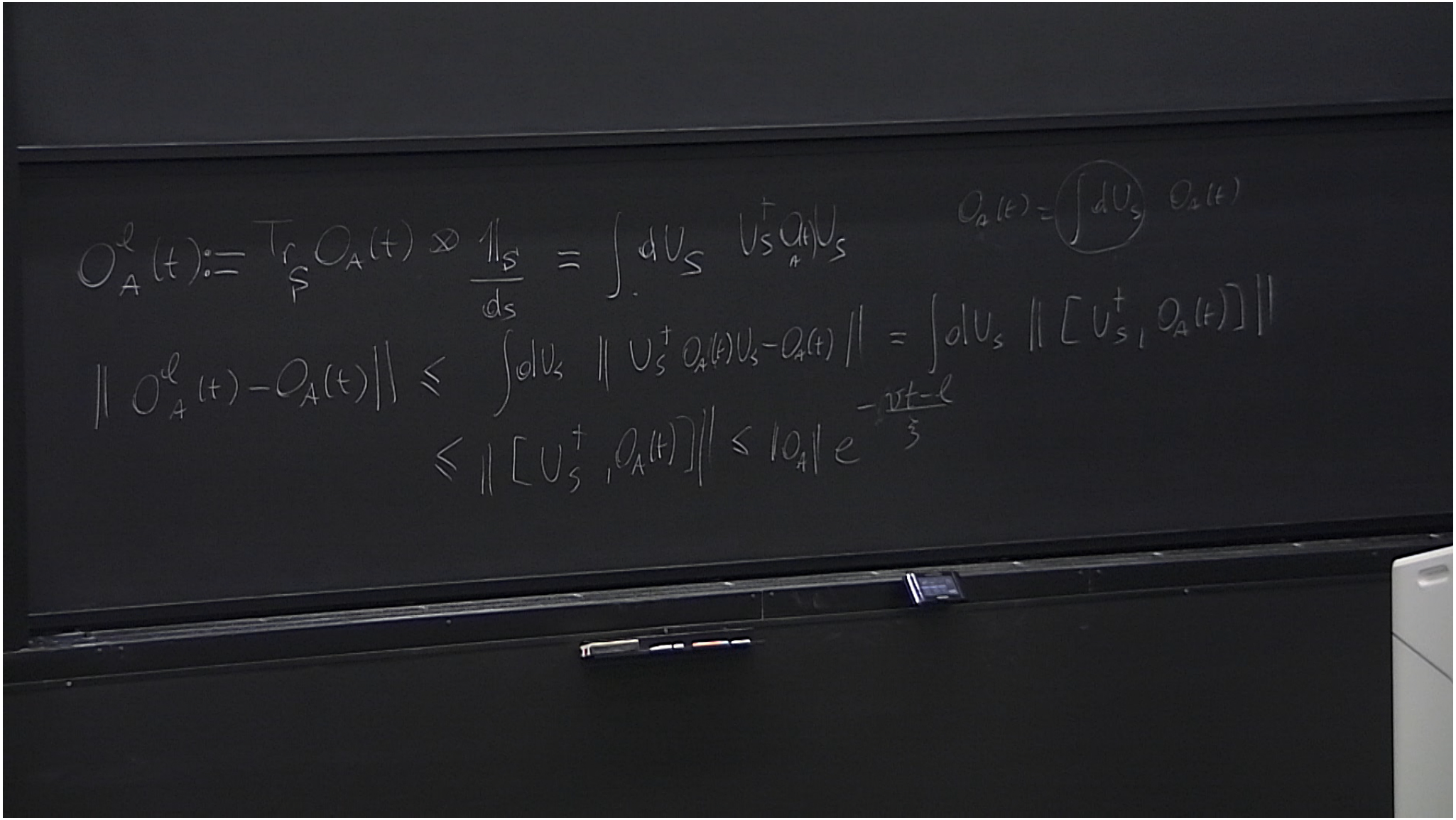

$$\int dU\, U^\dagger O U$$

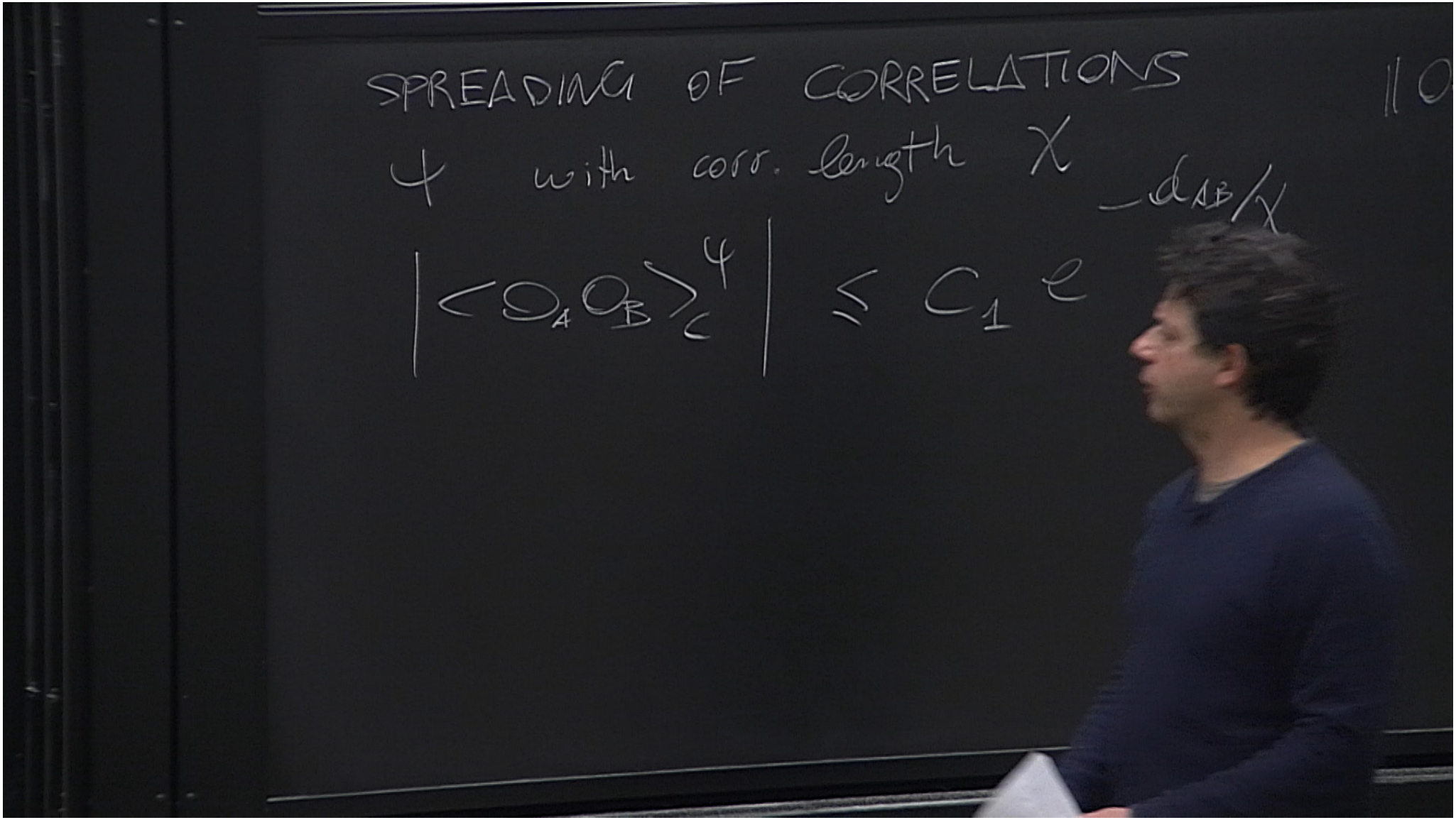
Haar integral

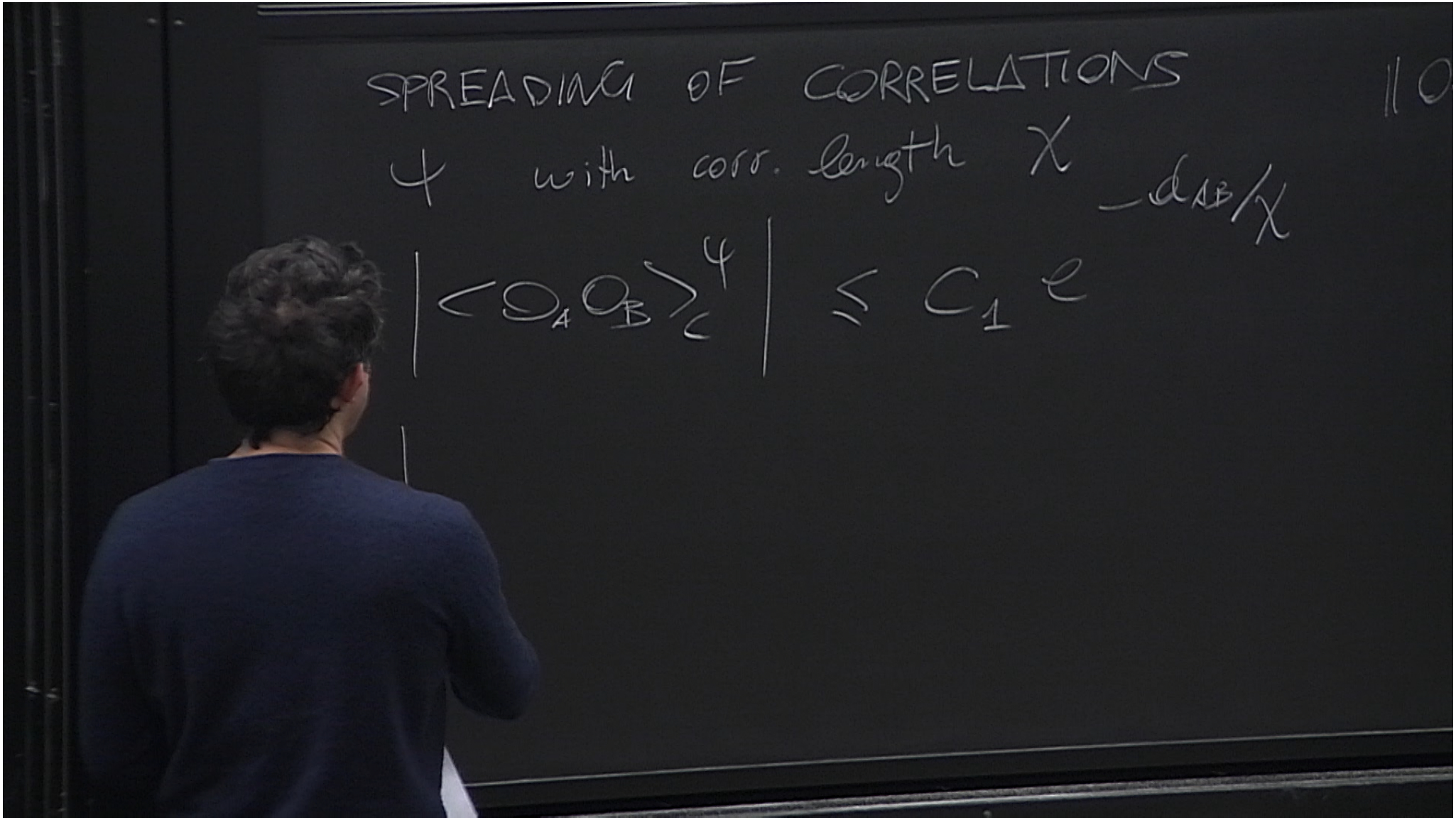
$$\int dU = 1$$

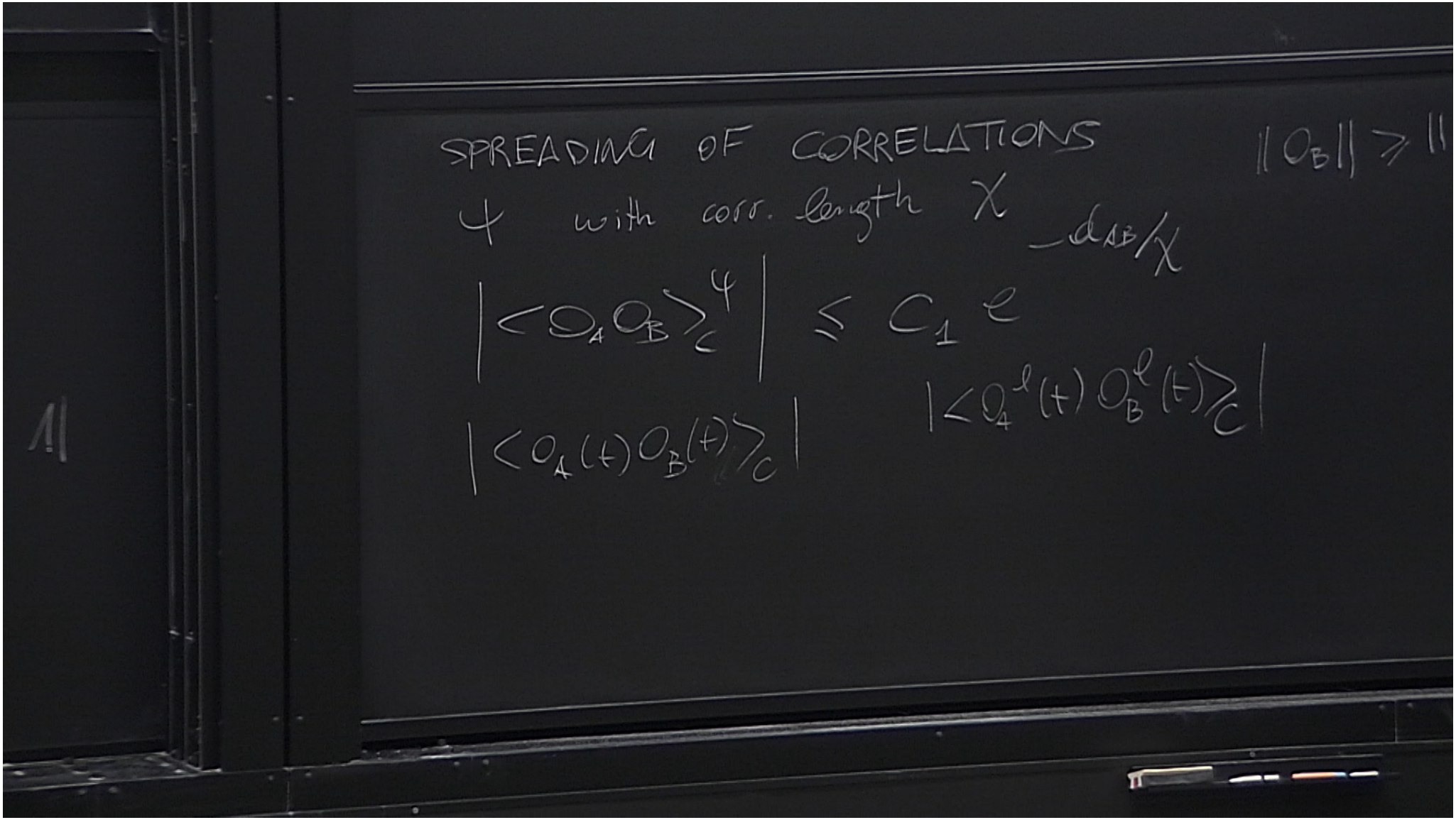
$$\int dU_x\, U_x^\dagger O U_x = \frac{\mathrm{Tr}\, O}{d_x} \mathbb{1}_x \quad \mathrm{Tr}[O_x]_{irr} \equiv \sum_i \frac{\mathrm{Tr}[\Pi_i O]}{\mathrm{Tr}\,\Pi_i} \Pi_i = \frac{\mathrm{Tr}\, O}{d_x} \mathbb{1}$$

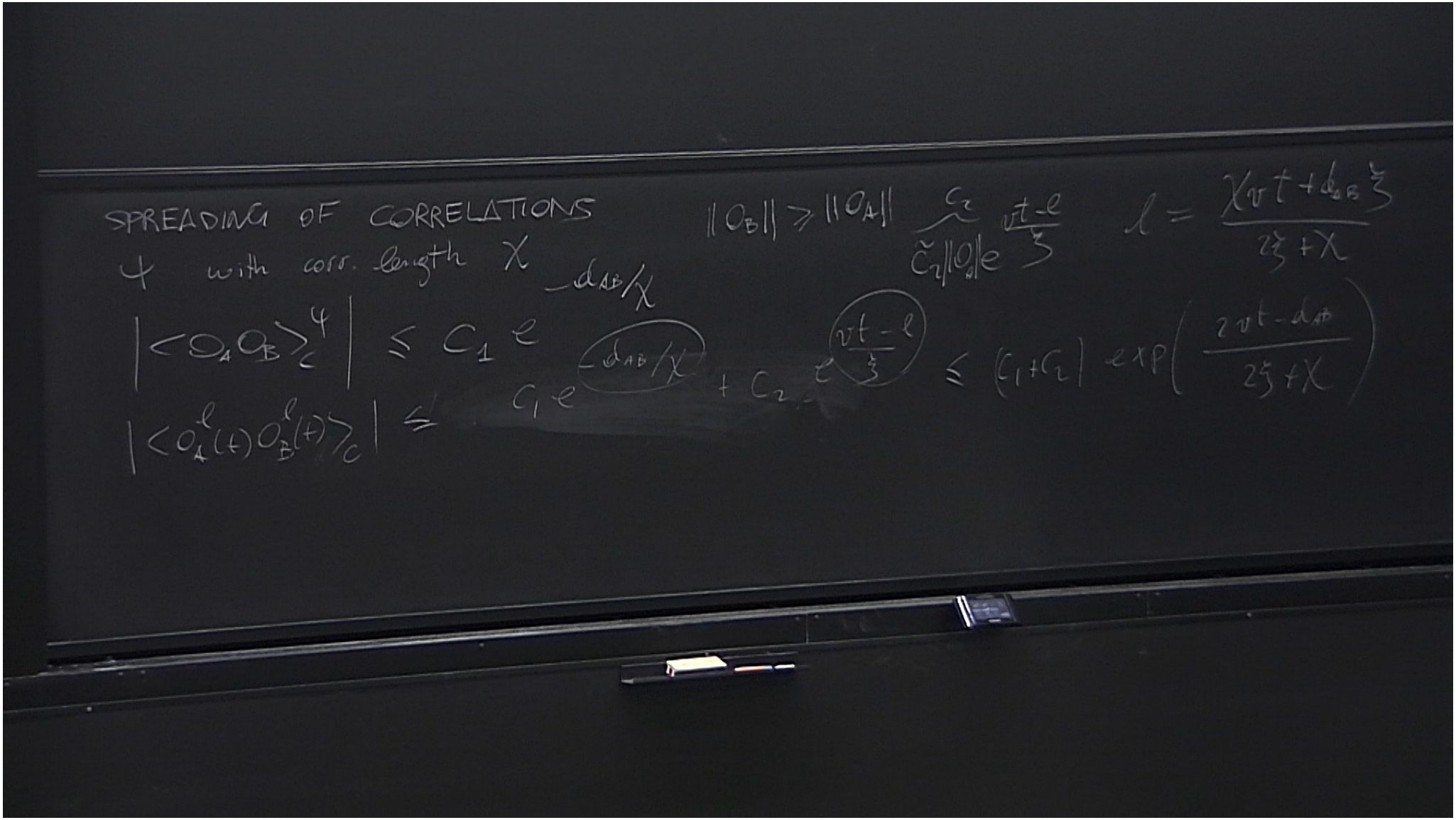

where
$\mathcal{U}$
$\psi$
$U\psi_0$
$\int dU\, U^\dagger O U$
Haar integral
$\int dU = 1$
$\int dU_x\, U_x^\dagger O U_x = \frac{\mathbb{1}_x}{d_x} \mathrm{Tr}[O_x]$
irr
$\sum \frac{\mathrm{Tr}[\Pi_i O]}{\mathrm{Tr}\Pi_i} \Pi_i = \frac{\mathrm{Tr} O}{d_{\mathcal{U}}} \mathbb{1}$

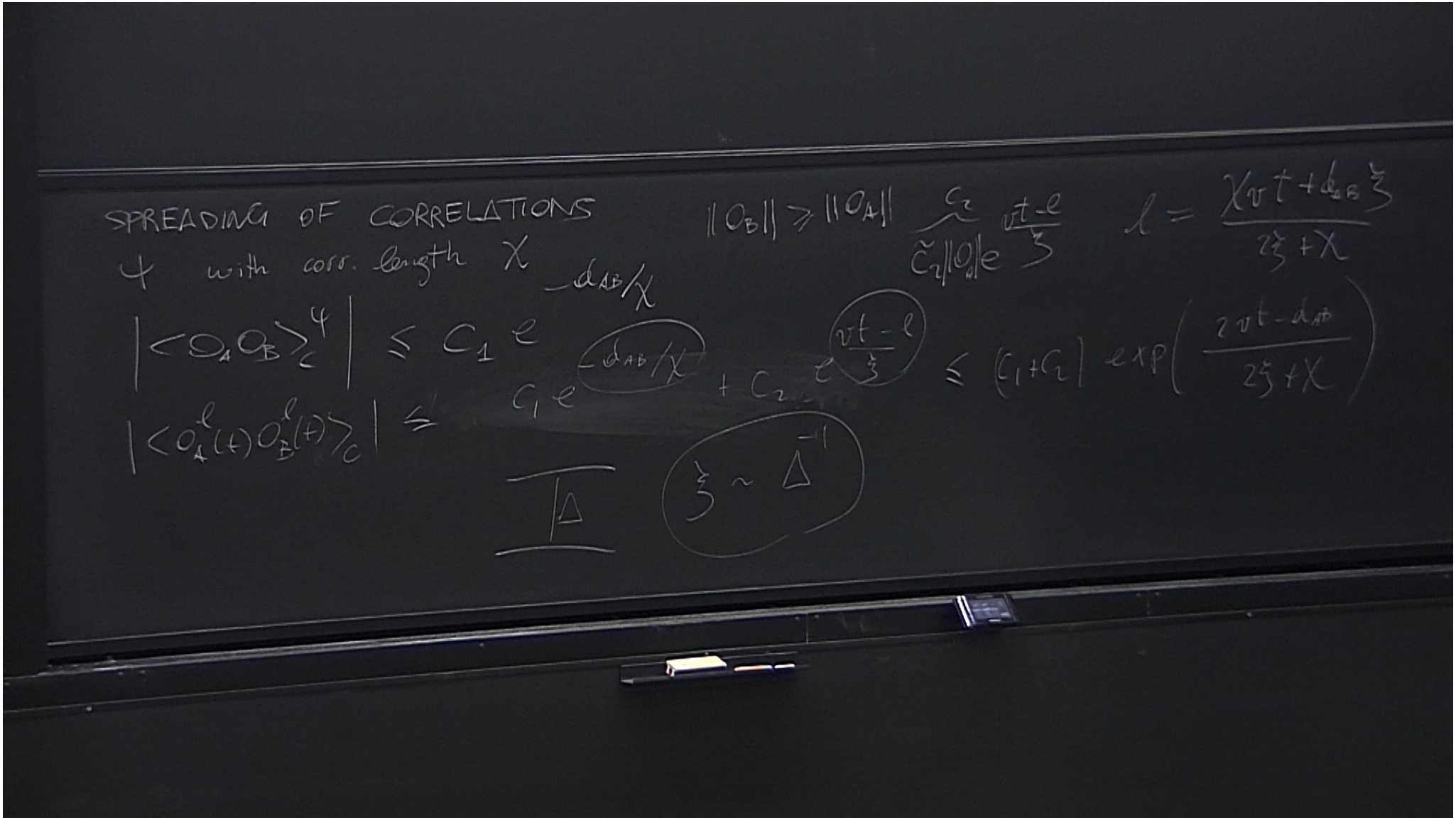

$O_A^\ell(t) := \mathrm{Tr}_S O_A(t) \otimes \frac{\mathbb{1}_S}{d_S} = \int dU_S \; U_S^\dagger O_A(t) U_S$
$O_A(t) = \int dU_S \; O_A(t)$
$\| O_A^\ell(t) - O_A(t) \| \leq \int dU_S \; \| U_S^\dagger O_A(t) U_S - O_A(t) \| = \int dU_S \; \| [U_S^\dagger, O_A(t)] \|$
$\leq \| [U_S^\dagger, O_A(t)] \| \leq |O_A| \, e^{-\frac{vt-\ell}{\xi}}$


SPREADING OF CORRELATIONS
$\psi$ with corr. length $\chi$
$|\langle O_A O_B \rangle_\psi^c| \leq C_1 e^{-d_{AB}/\chi}$


SPREADING OF CORRELATIONS
ψ with corr. length χ
$|\langle O_A O_B \rangle_c^\psi| \leq C_1 e^{-d_{AB}/\chi}$


SPREADING OF CORRELATIONS
$\|O_B\| = 1$
$\psi$ with corr. length $\chi$
$e^{-d_{AB}/\chi}$
$|\langle O_A O_B \rangle_\psi^C| \leq c_1 e$
$|\langle O_A(t) O_B(t) \rangle_C|$
$|\langle O_A^l(t) O_B^l(t) \rangle_C|$


SPREADING OF CORRELATIONS
ψ with corr. length χ
$\|O_B\| \geq \|O_A\|$
$\tilde{c}_2 \|O_B\| e^{\frac{vt-\ell}{\xi}}$
$\ell = \frac{\chi v t + d_{AB}\xi}{2\xi + \chi}$
$|\langle O_A O_B \rangle_c^\psi| \leq c_1 e^{-d_{AB}/\chi}$
$|\langle O_A^\ell(t) O_B^\ell(t) \rangle_c| \leq c_1 e^{-d_{AB}/\chi} + c_2 e^{\frac{vt-\ell}{\xi}} \leq (c_1+c_2) \exp\left(\frac{2vt - d_{AB}}{2\xi+\chi}\right)$


SPREADING OF CORRELATIONS
ψ with corr. length χ
$\|O_B\| \geq \|O_A\|$
$\tilde{c}_2 \|O_B\| e^{\frac{vt-\ell}{\xi}}$
$\ell = \frac{\chi v t + d_{AB}\xi}{2\xi + \chi}$
$|\langle O_A O_B \rangle_c^{\psi}| \leq c_1 e^{-d_{AB}/\chi} + c_2 e^{\frac{vt-\ell}{\xi}} \leq (c_1+c_2) \exp\left(\frac{2vt - d_{AB}}{2\xi+\chi}\right)$
$|\langle O_A^{\ell}(t) O_B^{\ell}(t) \rangle_c| \leq$
$c_1 e^{-d_{AB}/\chi}$
$\xi \sim \Delta^{-1}$
$\frac{1}{\Delta}$