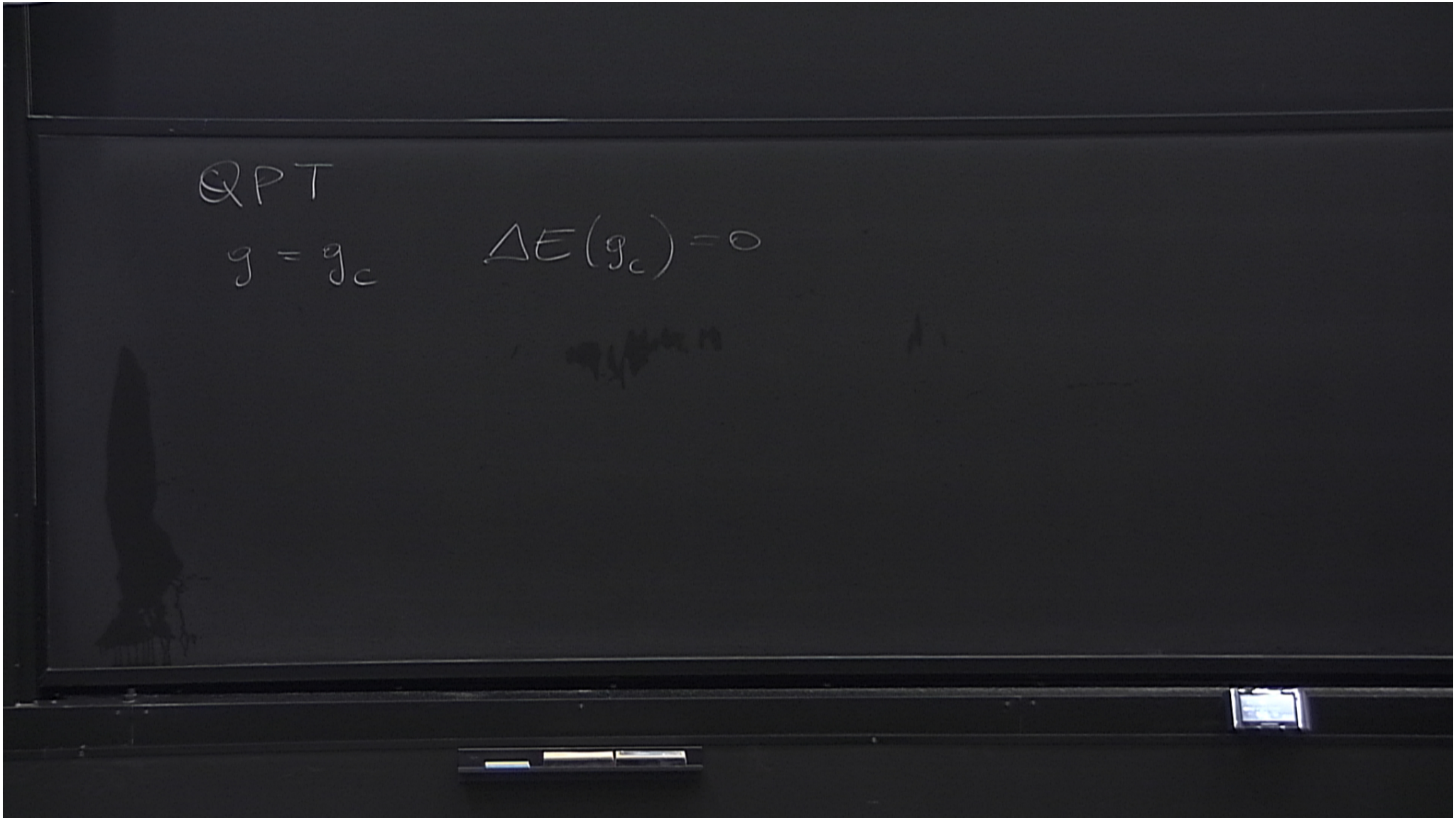
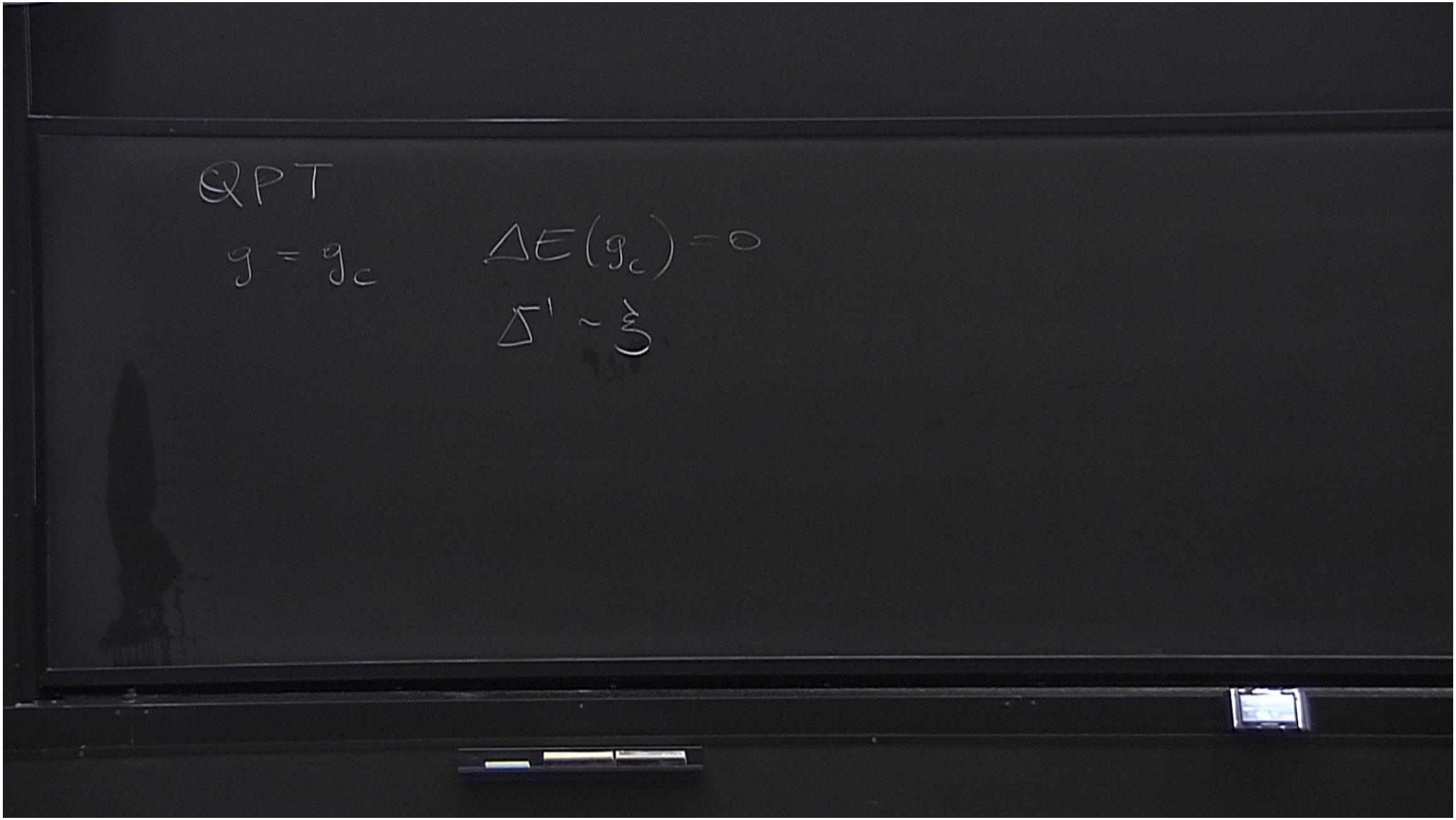
Title: PSI 17/18 - Condensed Matter - Lecture 5

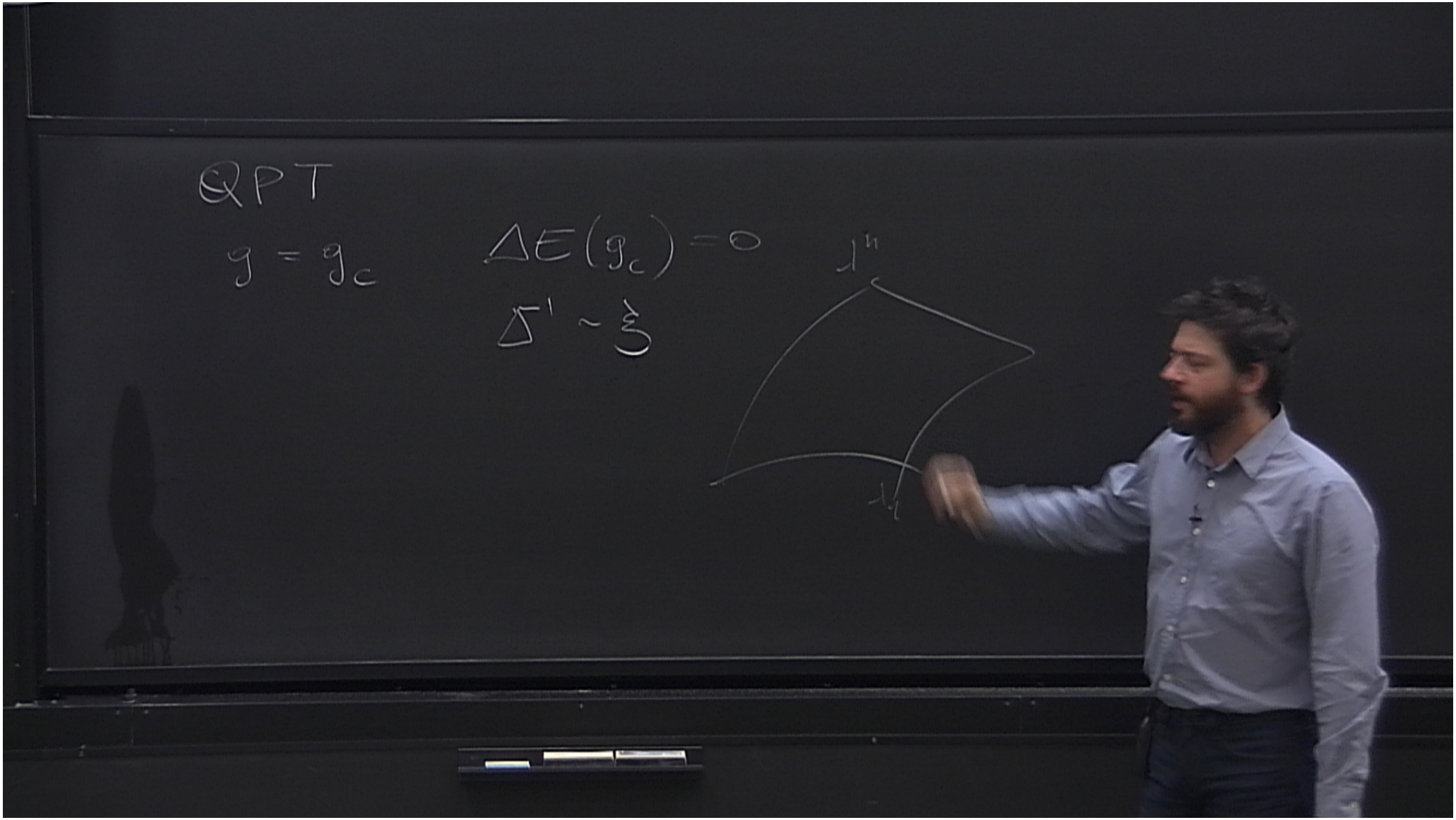
Date: Jan 09, 2018 11:30 AM

URL: http://pirsa.org/18010038

Abstract:


QPT
$g = g_c$
$\Delta E(g_c) = 0$

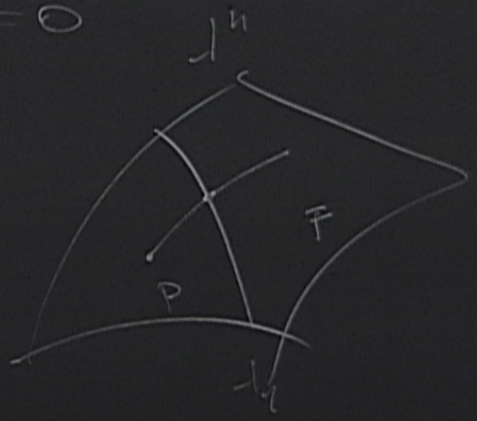

QPT
$g = g_c$
$\Delta E(g_c) = 0$
$\Delta^{-1} \sim \xi$


QPT
$g = g_c$
$\Delta E(g_c) = 0$
$\Delta^1 \sim \xi$
λ''
λ_1

QPT

$g = g_c$ $\quad \Delta E(g_c) = 0$

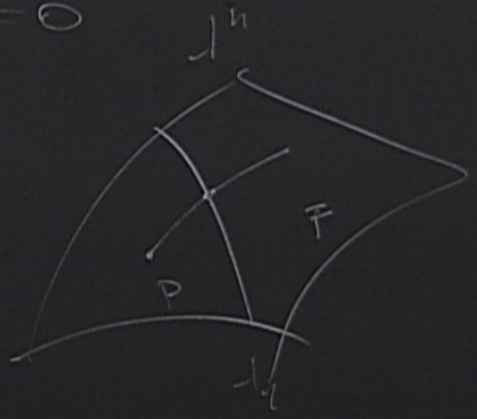
$\Delta' \sim \xi$

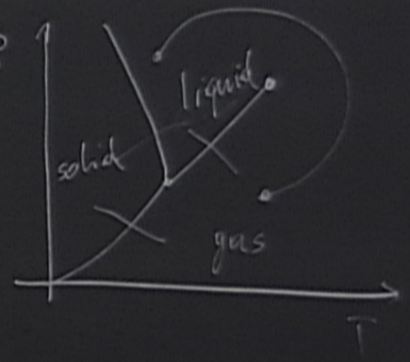


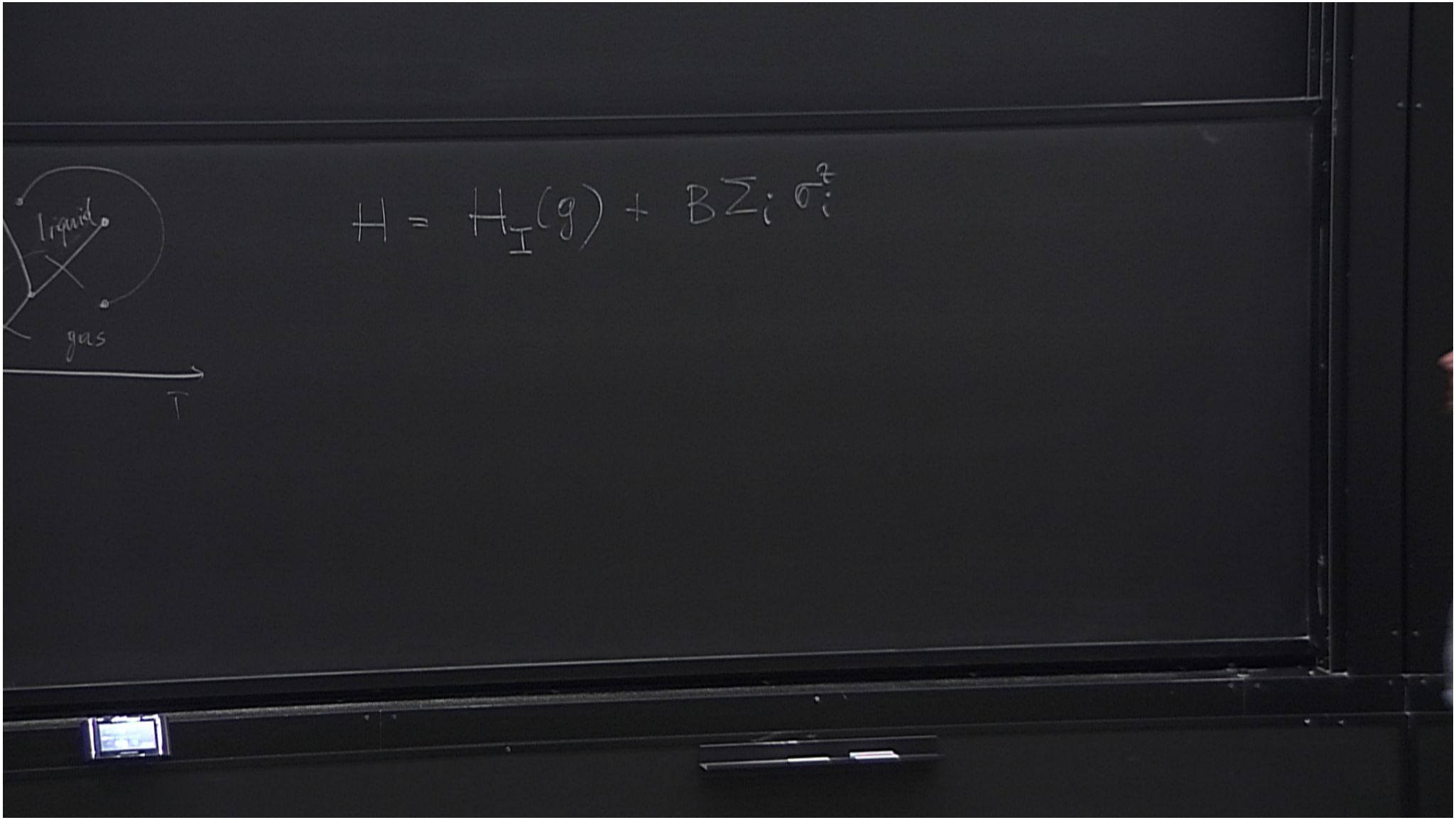
QPT

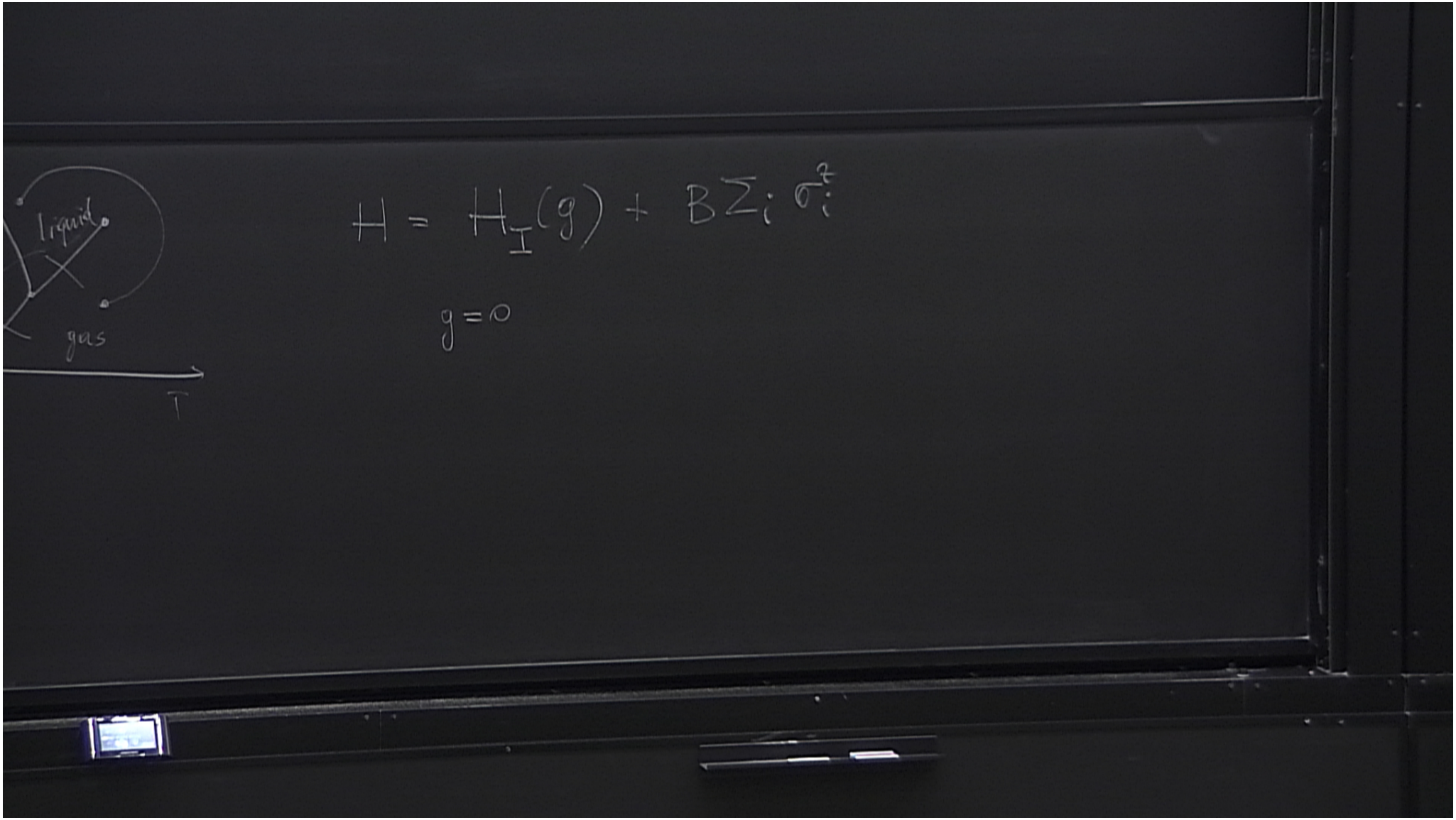
$g = g_c$ $\quad \Delta E(g_c) = 0$

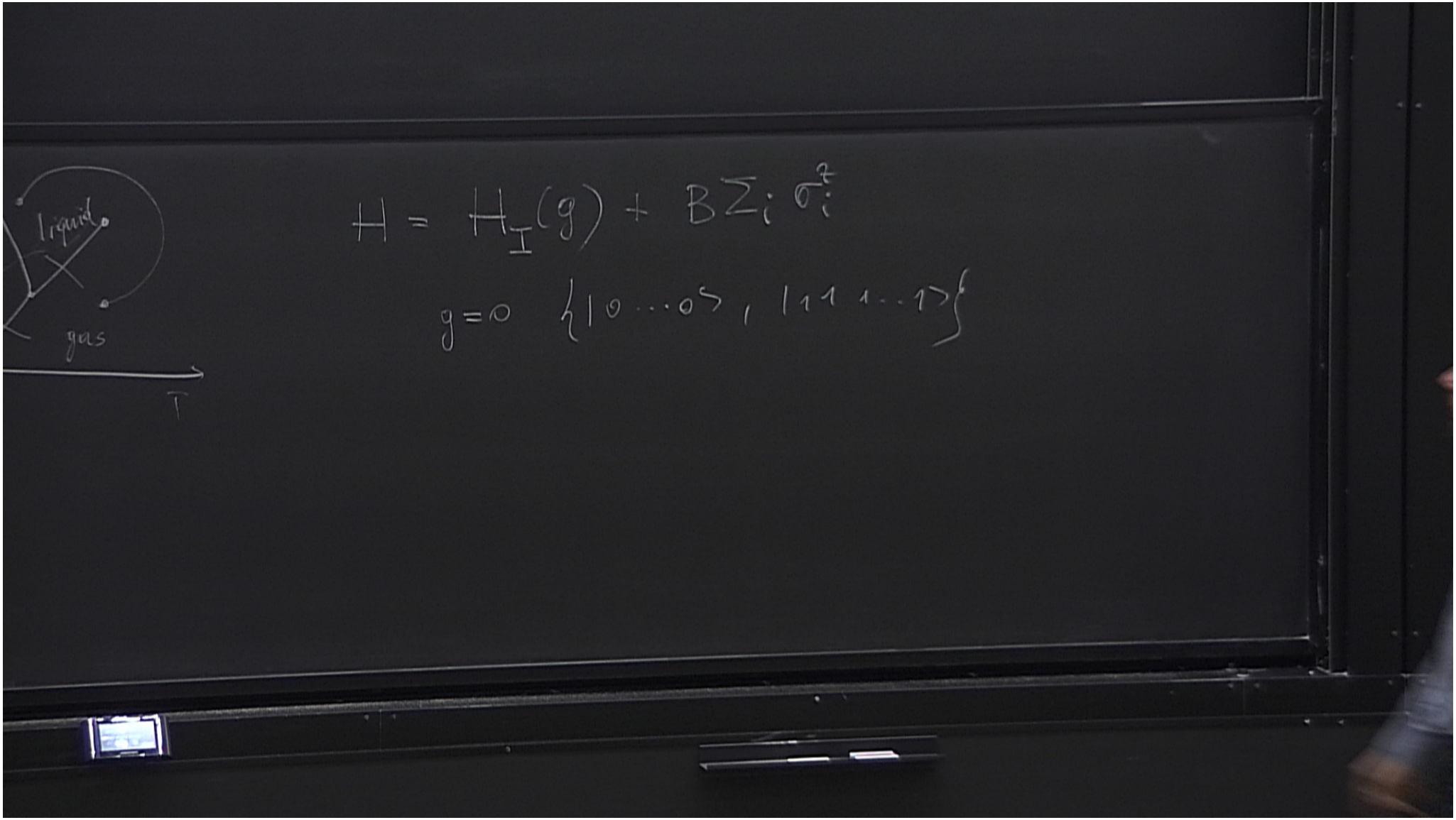
$\Delta' \sim \xi$

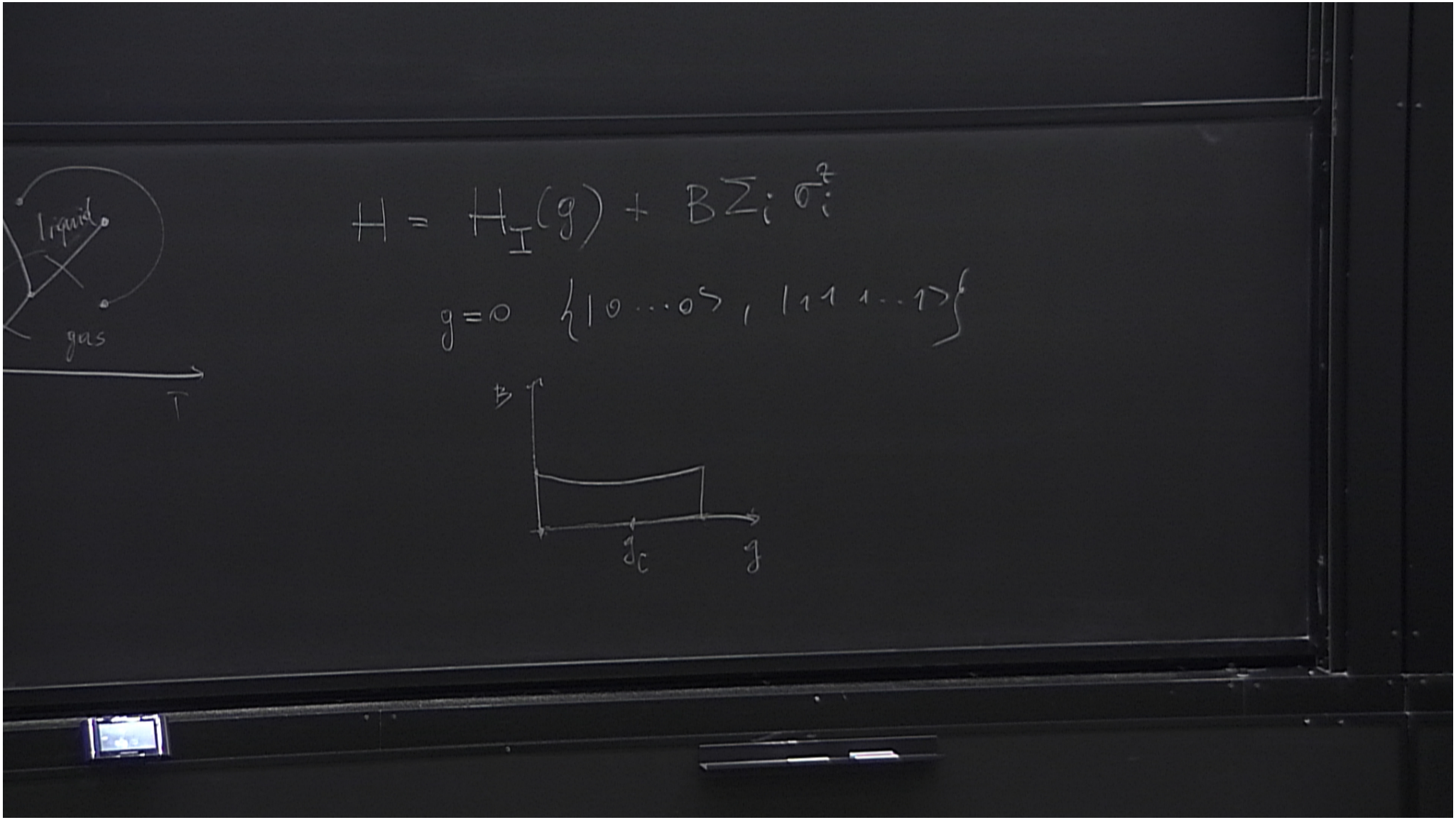


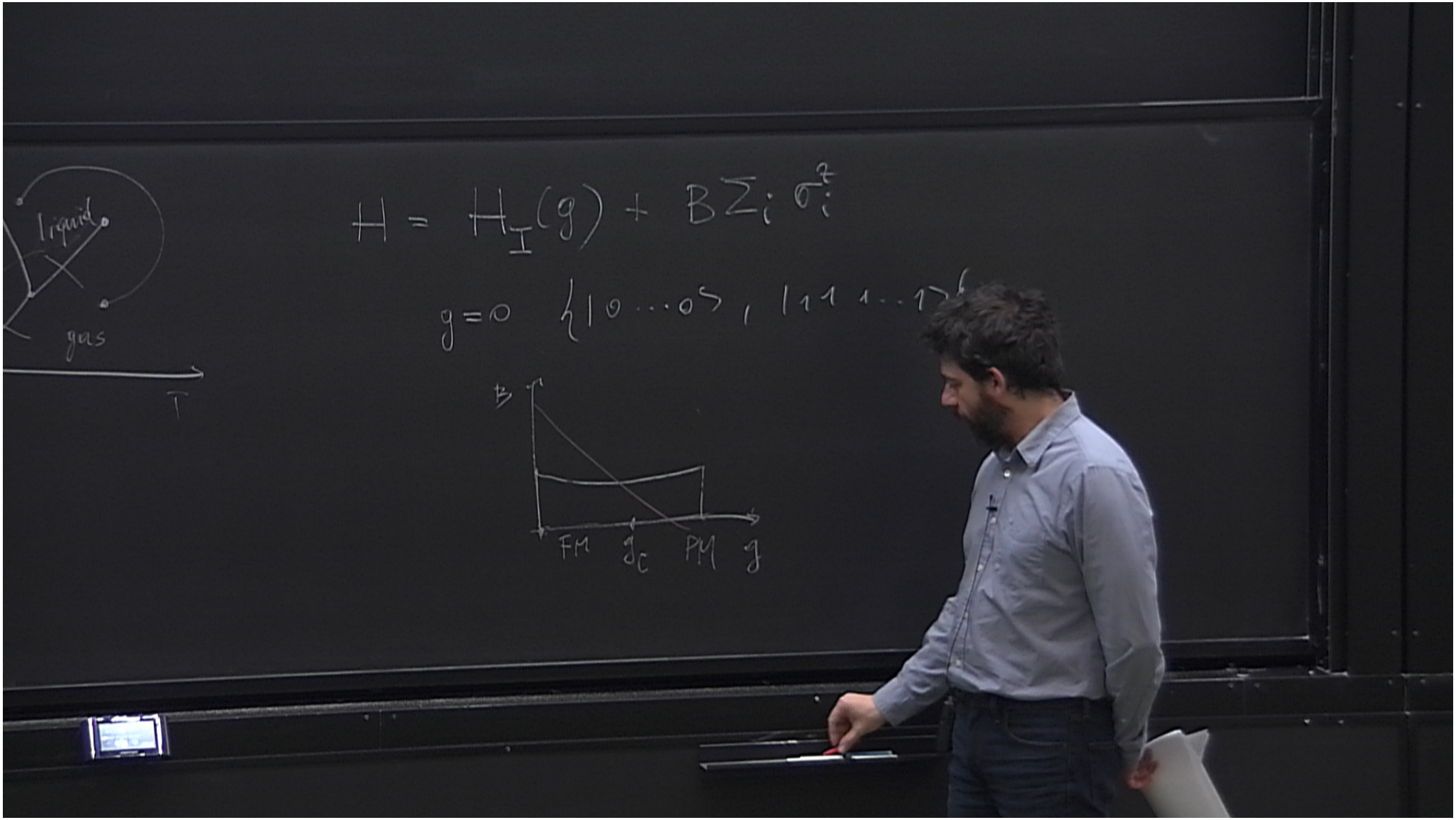


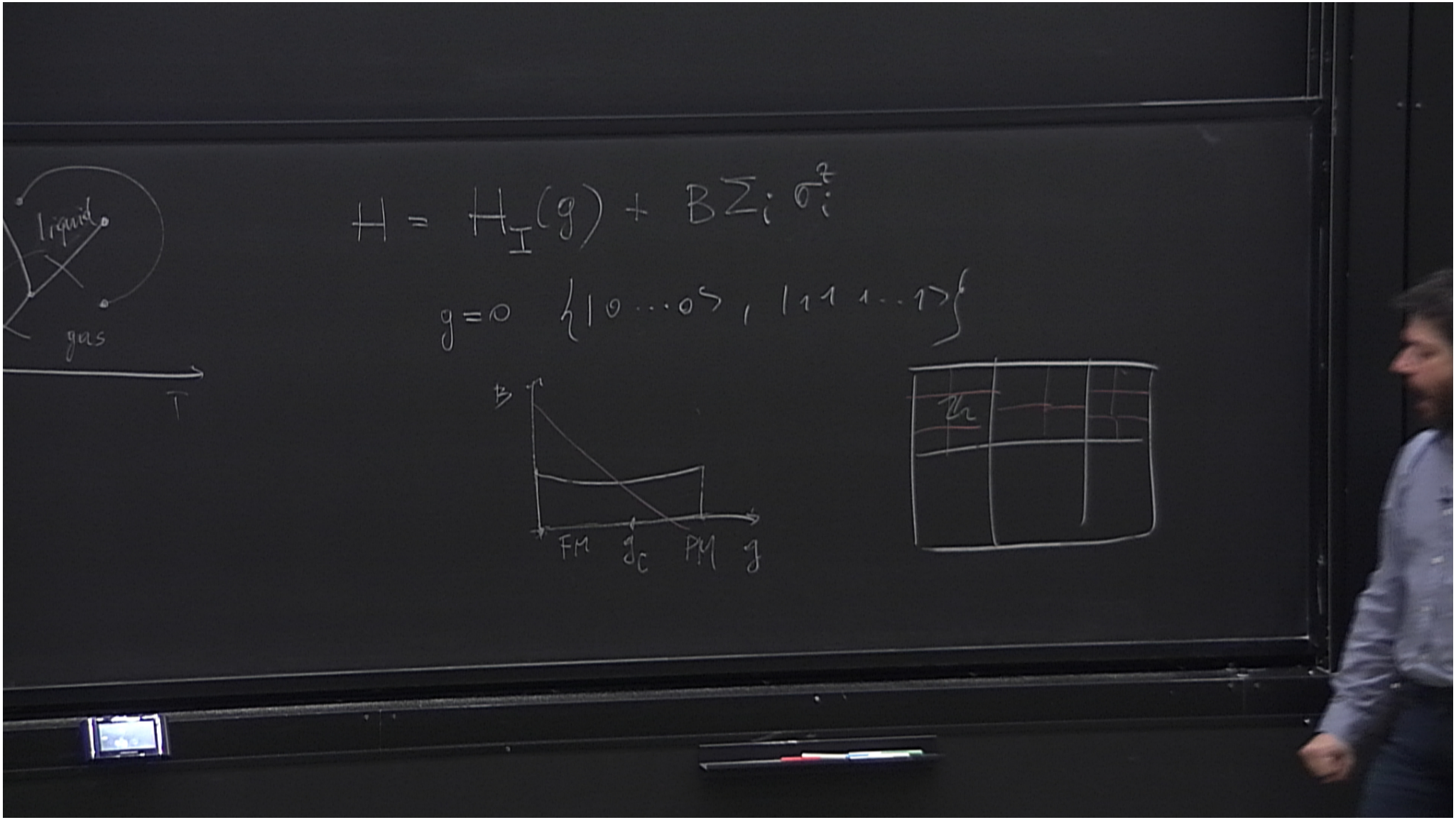

liquid
gas
T
$H = H_I(g) + B\sum_i \sigma_i^z$

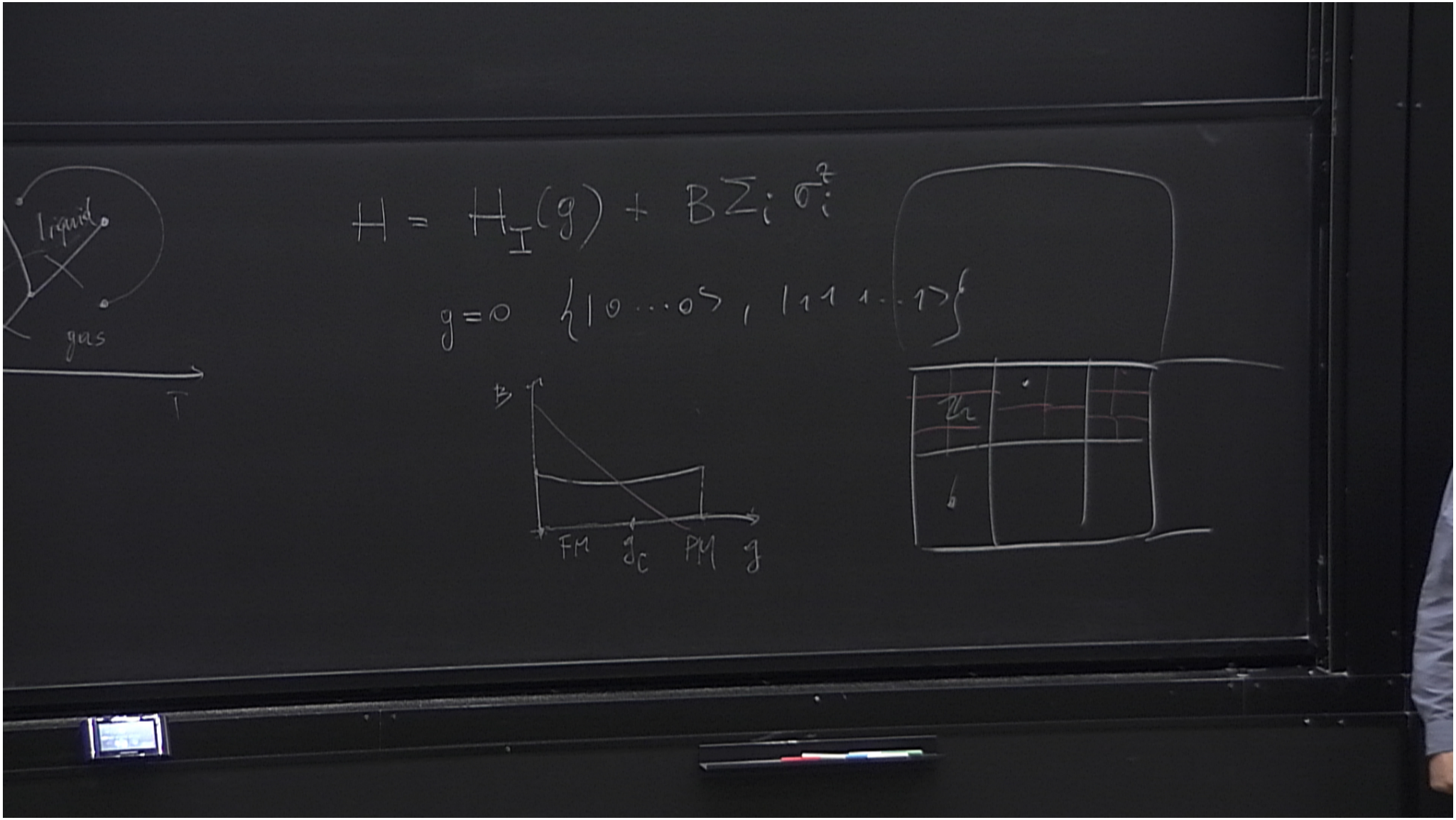

liquid
gas
T
$H = H_I(g) + B\Sigma_i \sigma_i^z$
$g = 0$

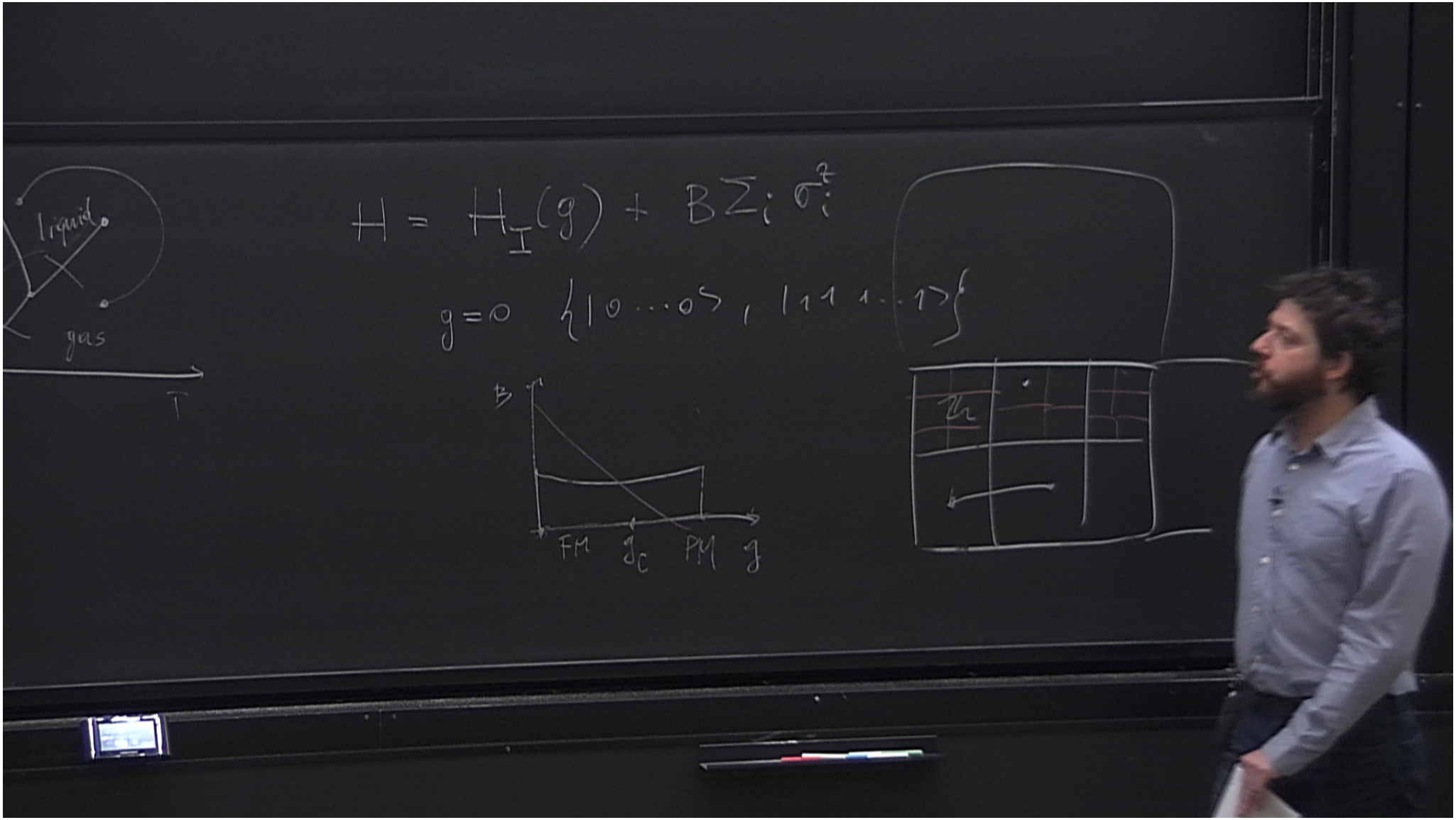

liquid
gas
T
$H = H_I(g) + B\sum_i \sigma_i^z$
$g=0$ $\{|0\ldots0\rangle, |1\,1\,1\ldots1\rangle\}$

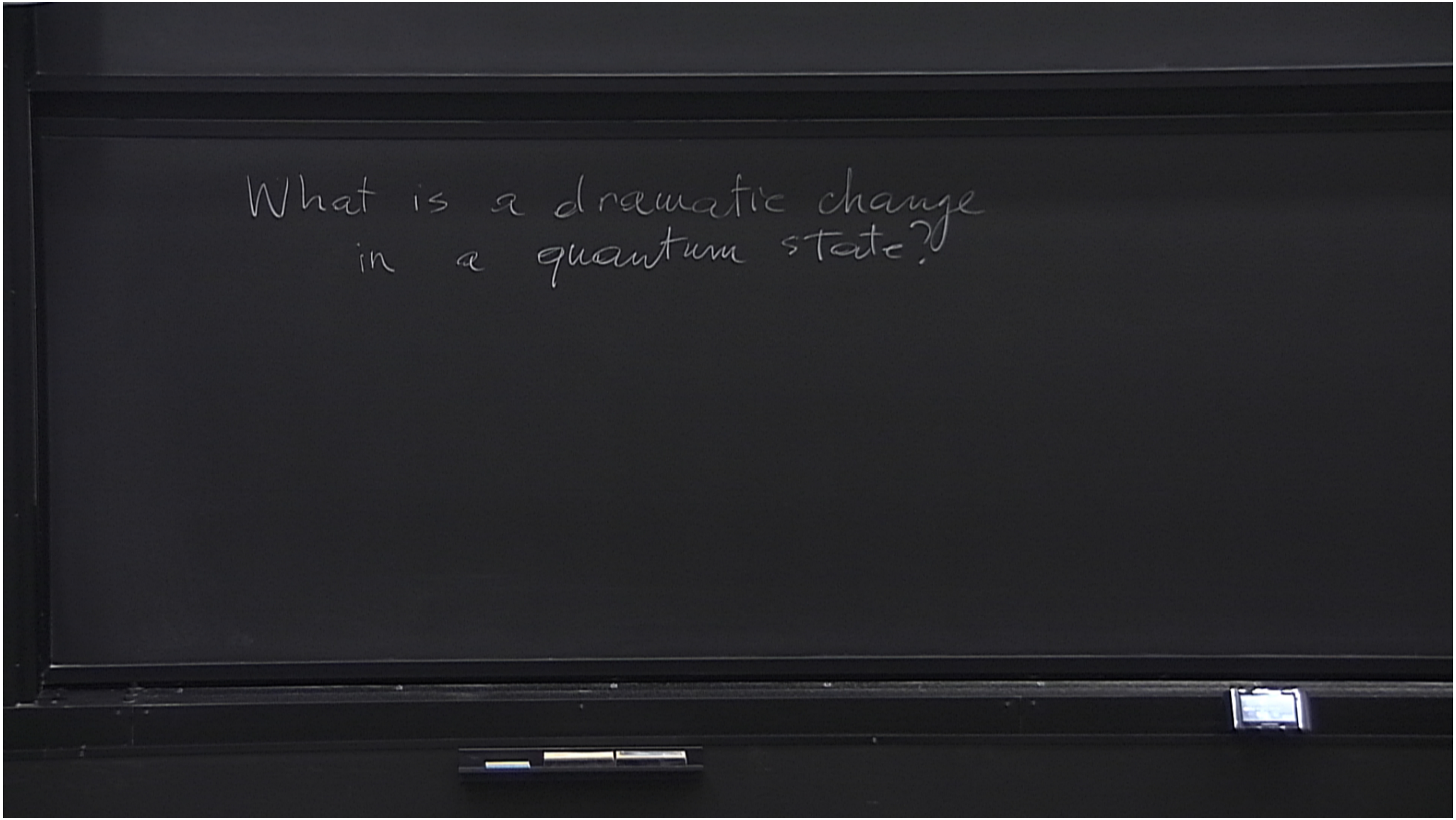

liquid
gas
T
$H = H_I(g) + B\sum_i \sigma_i^z$
$g=0$ $\{|0\ldots0\rangle, |1\,1\,1\ldots1\rangle\}$
B
g_c
g

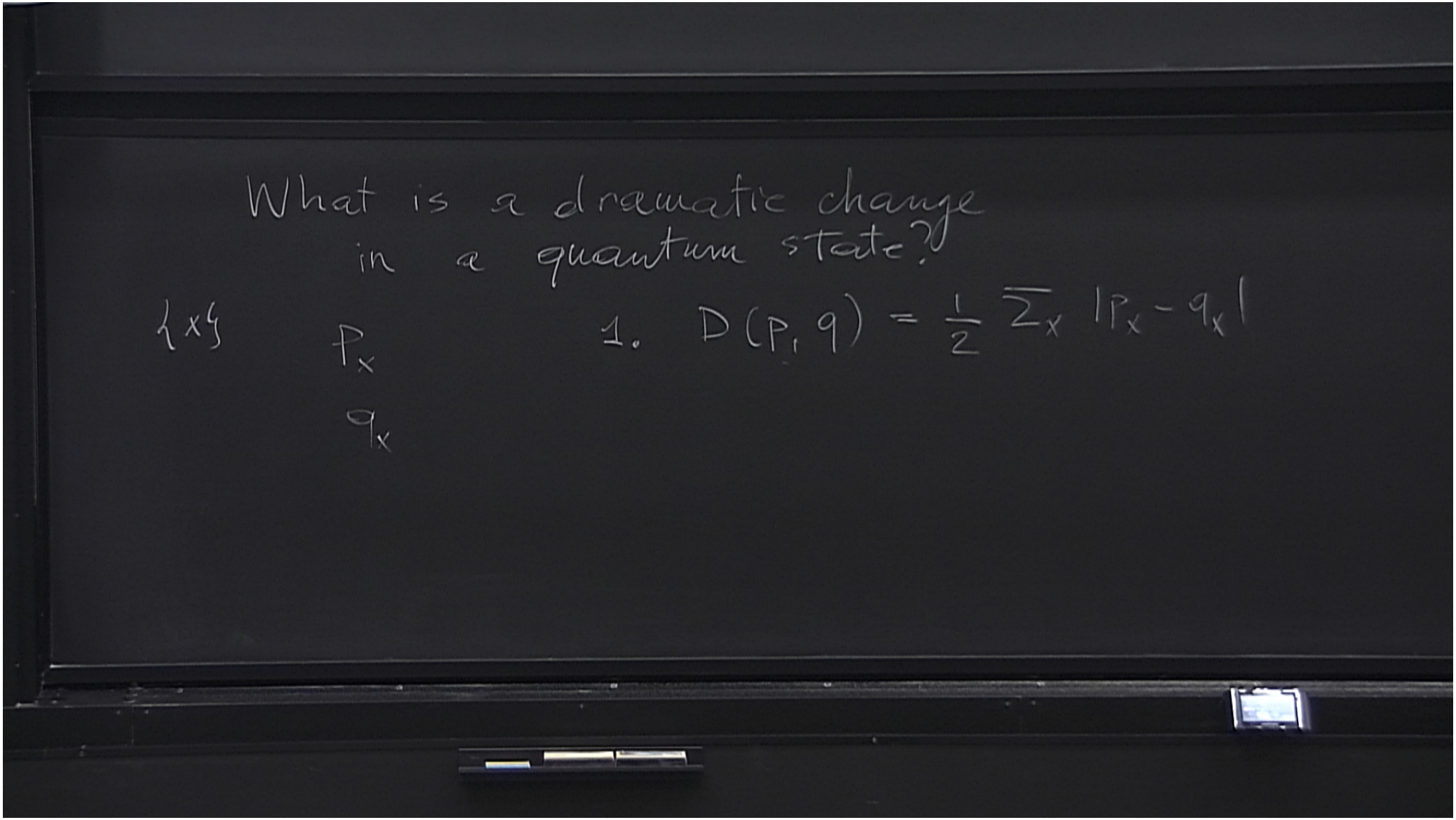

liquid
gas
T
$H = H_I(g) + B\sum_i \sigma_i^z$
$g=0$
FM
g_c
PM
g


liquid
gas
T
$H = H_I(g) + B\sum_i \sigma_i^z$
$g=0$ $\{|0\ldots0\rangle, |1\,1\,1\ldots1\rangle\}$
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FM
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B
FM
g_c
PM
g


What is a dramatic change
in a quantum state?


What is a dramatic change
in a quantum state?
{x}
P_x
q_x
1. $D(P,q) = \frac{1}{2}\sum_x |P_x - q_x|$

What is a dramatic change in a quantum state?

$\{x\}$ P_x q_x

1. $D(P,q) = \frac{1}{2}\sum_x |P_x - q_x|$

$D(P,q) = D(q,P)$

$D(P,q) \leq D(P,z) + D(z,q)$

What is a dramatic change in a quantum state?

$\{x\}$ P_x q_x $S \subset \{x\}$

1. $D(P,q) = \frac{1}{2} \sum_x |P_x - q_x| =$

$D(P,q) = D(q,P)$

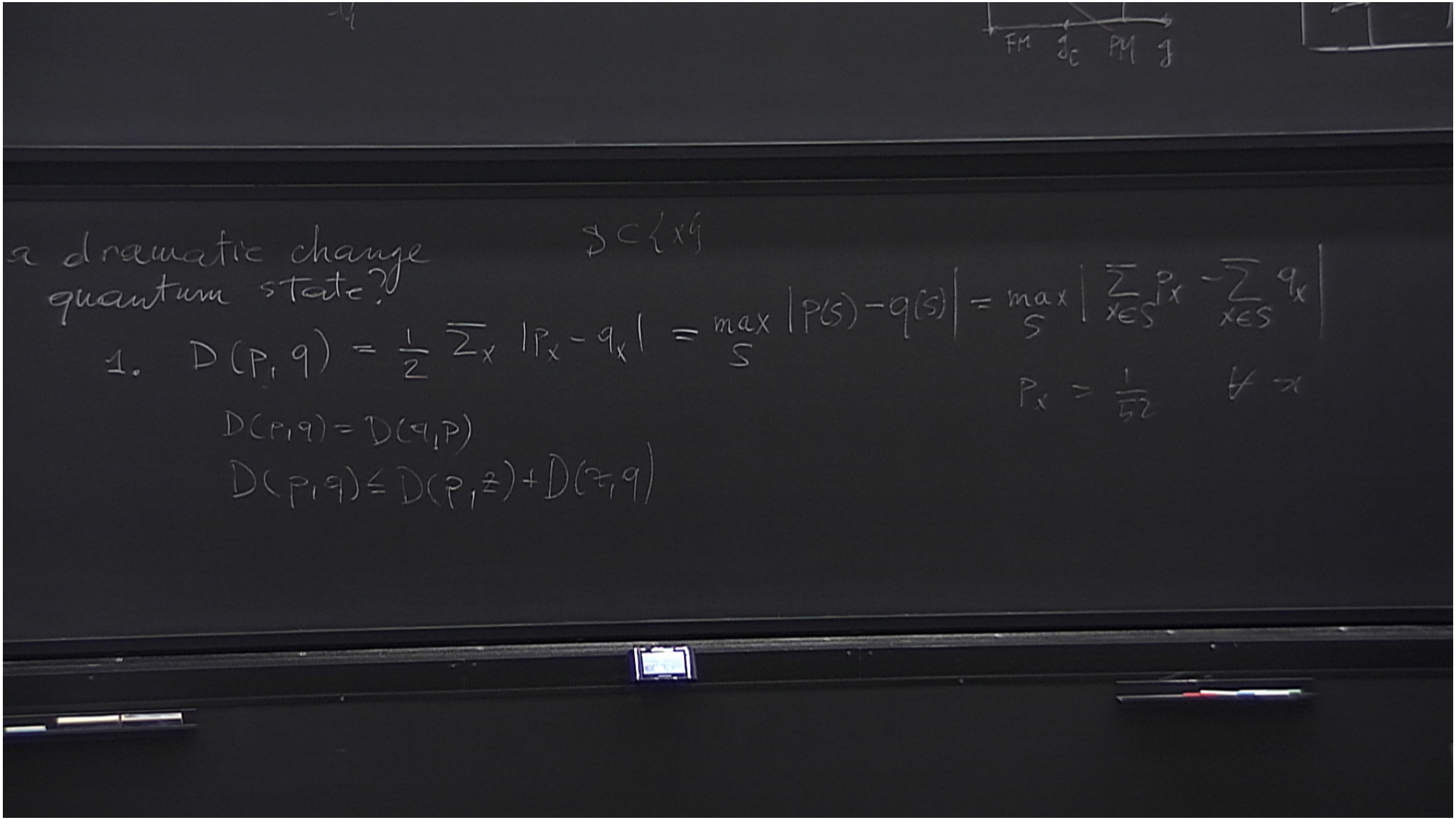
$D(P,q) \leq D(P,z) + D(z,q)$

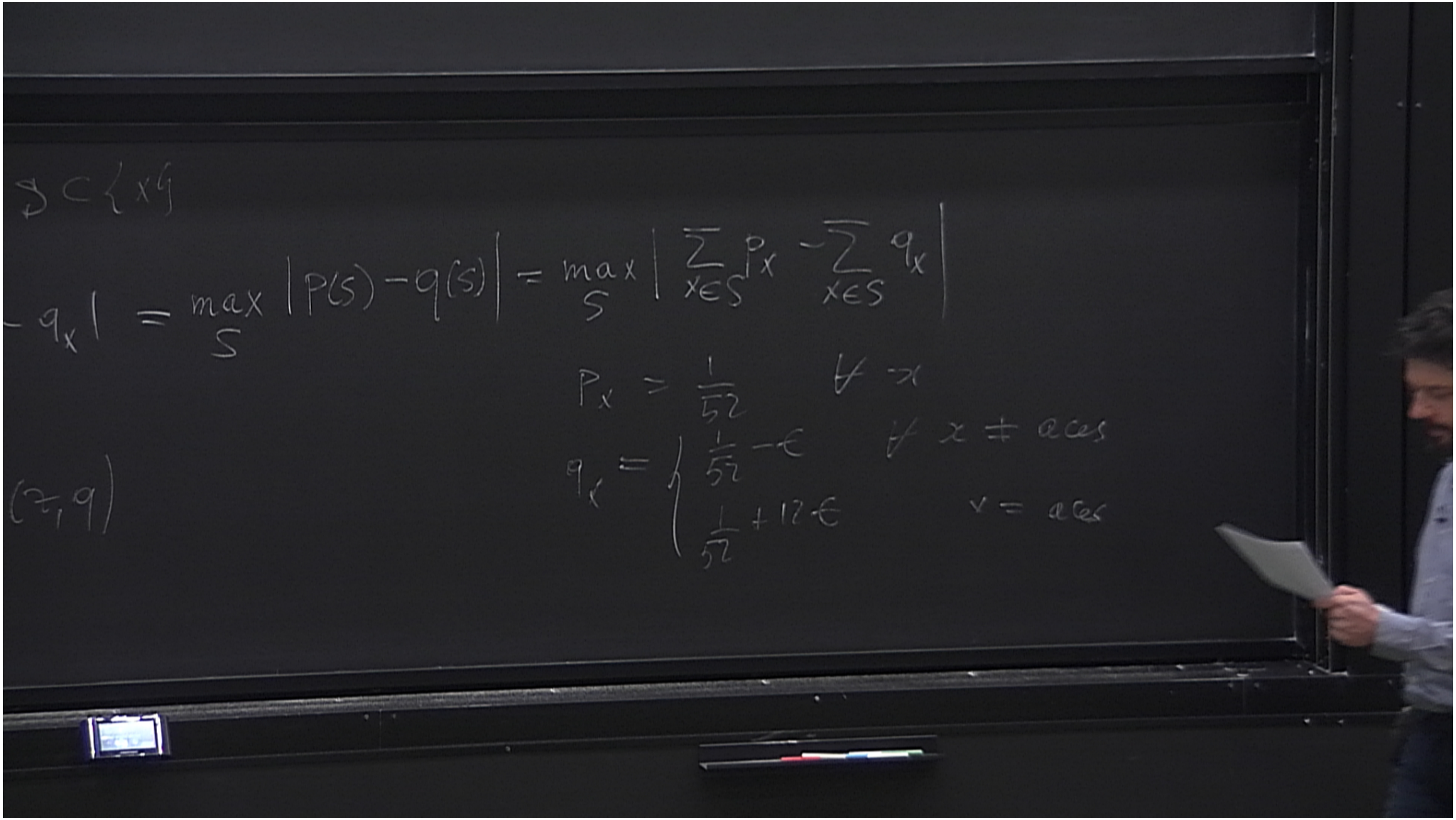
a dramatic change $\mathcal{X}$ quantum state?

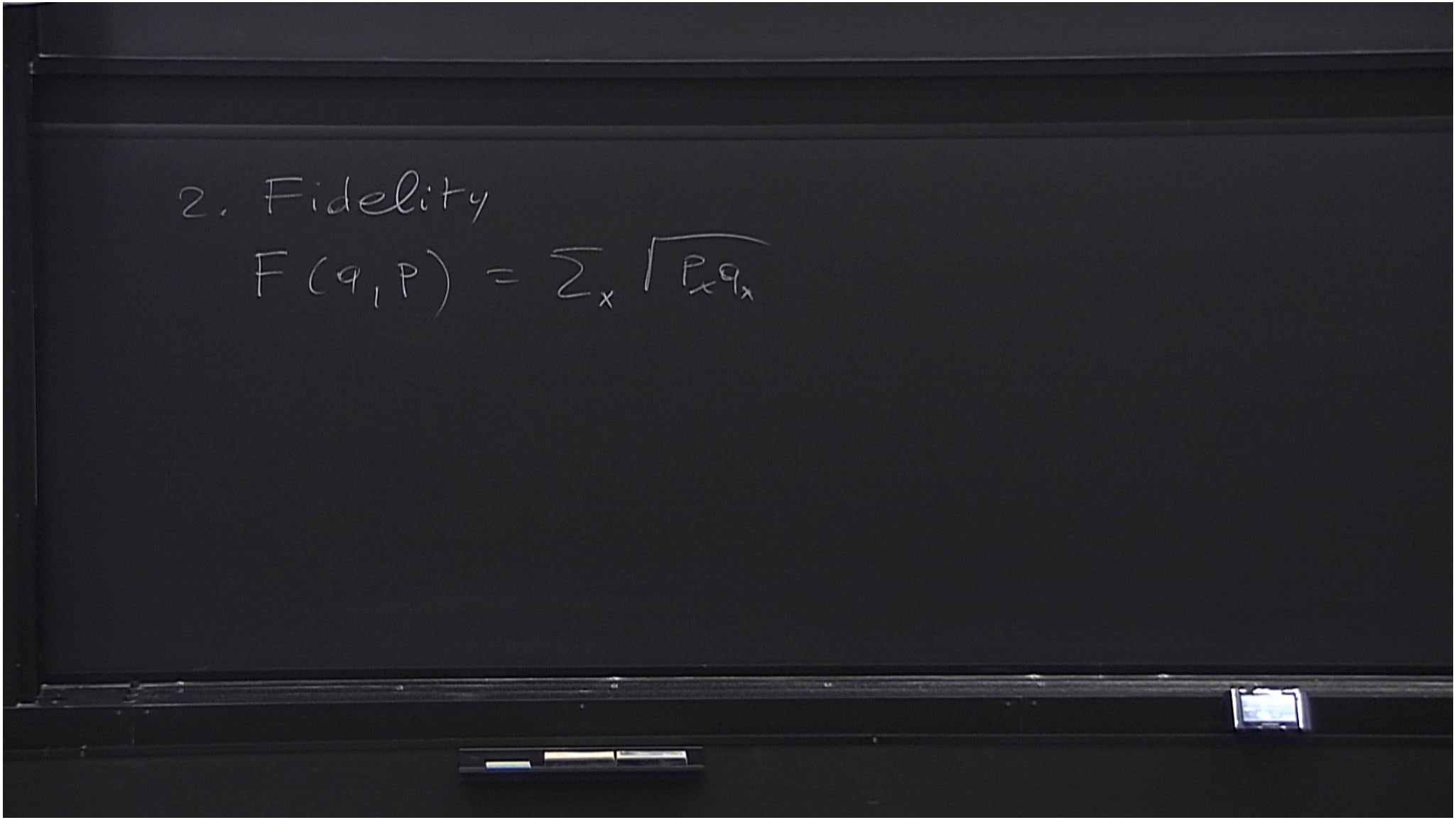
1. $D(p,q) = \frac{1}{2}\sum_x |p_x - q_x| = \max_S |p(S) - q(S)| = \max_S \left| \sum_{x\in S} p_x - \sum_{x\in S} q_x \right|$

$D(p,q) = D(q,p)$

$D(p,q) \leq D(p,z) + D(z,q)$


FM
g_c
PM
g
a dramatic change
quantum state?
$S \subset \{x\}$
1. $D(P,Q) = \frac{1}{2}\sum_x |P_x - Q_x| = \max_S |P(S) - Q(S)| = \max_S \left| \sum_{x\in S} P_x - \sum_{x\in S} Q_x \right|$
$P_x = \frac{1}{52}$ $\forall x$
$D(P,Q) = D(Q,P)$
$D(P,Q) \le D(P,Z) + D(Z,Q)$


$S \subset \{x\}$
$-q_x| = \max_S |P(S) - q(S)| = \max_S \left| \sum_{x \in S} p_x - \sum_{x \in S} q_x \right|$
(p, q)
$p_x = \frac{1}{52} \quad \forall x$
$q_x = \begin{cases} \frac{1}{52} - \epsilon & \forall x \neq \text{aces} \\ \frac{1}{52} + 12\epsilon & x = \text{aces} \end{cases}$


2. Fidelity
$F(q, p) = \sum_x \sqrt{p_x q_x}$

2. Fidelity

$$F(q,p) = \sum_x \sqrt{p_x q_x}$$

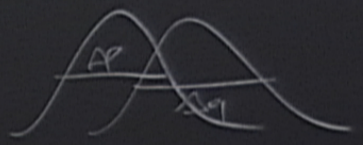
Coin 1 p, $1-p$ $\qquad \Delta p = \sqrt{\frac{p(1-p)}{n}}$

Coin 2 q, $1-q$

n = # trials

2. Fidelity

$F(q, p) = \sum_x \sqrt{p_x q_x}$



Coin 1 p, $1-p$

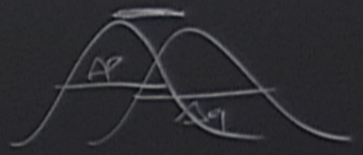
Coin 2 q, $1-q$

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2. Fidelity

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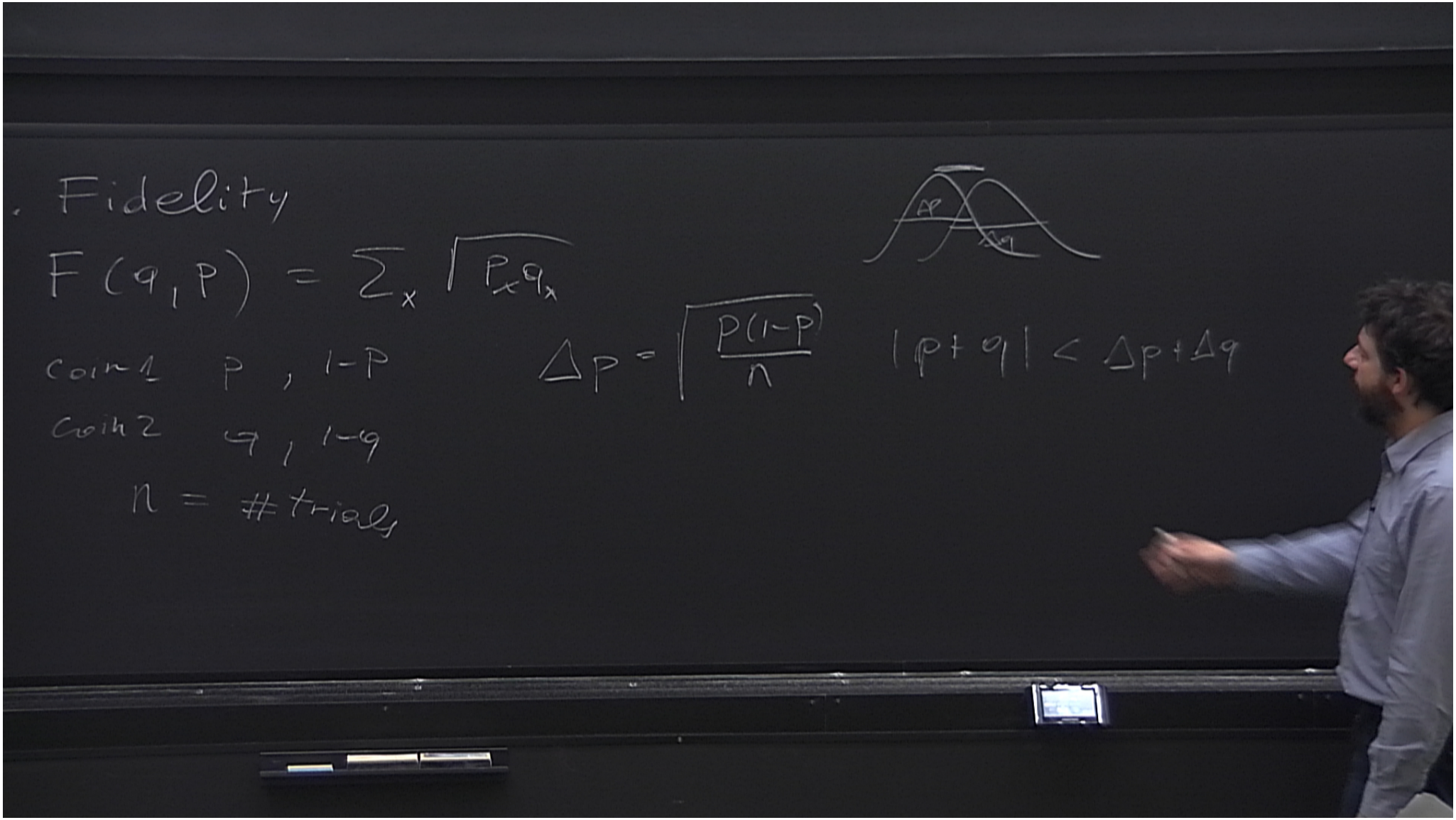


Coin 1 p, $1-p$

Coin 2 q, $1-q$

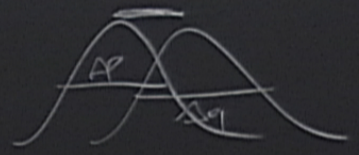
n = # trials

$$\Delta p = \sqrt{\frac{p(1-p)}{n}}$$


Fidelity
$F(q,p) = \sum_x \sqrt{p_x q_x}$
Coin 1 p, $1-p$
Coin 2 q, $1-q$
n = # trials
$\Delta p = \sqrt{\frac{p(1-p)}{n}}$
$|p+q| < \Delta p + \Delta q$

Fidelity

$F(q,p) = \sum_x \sqrt{p_x q_x}$

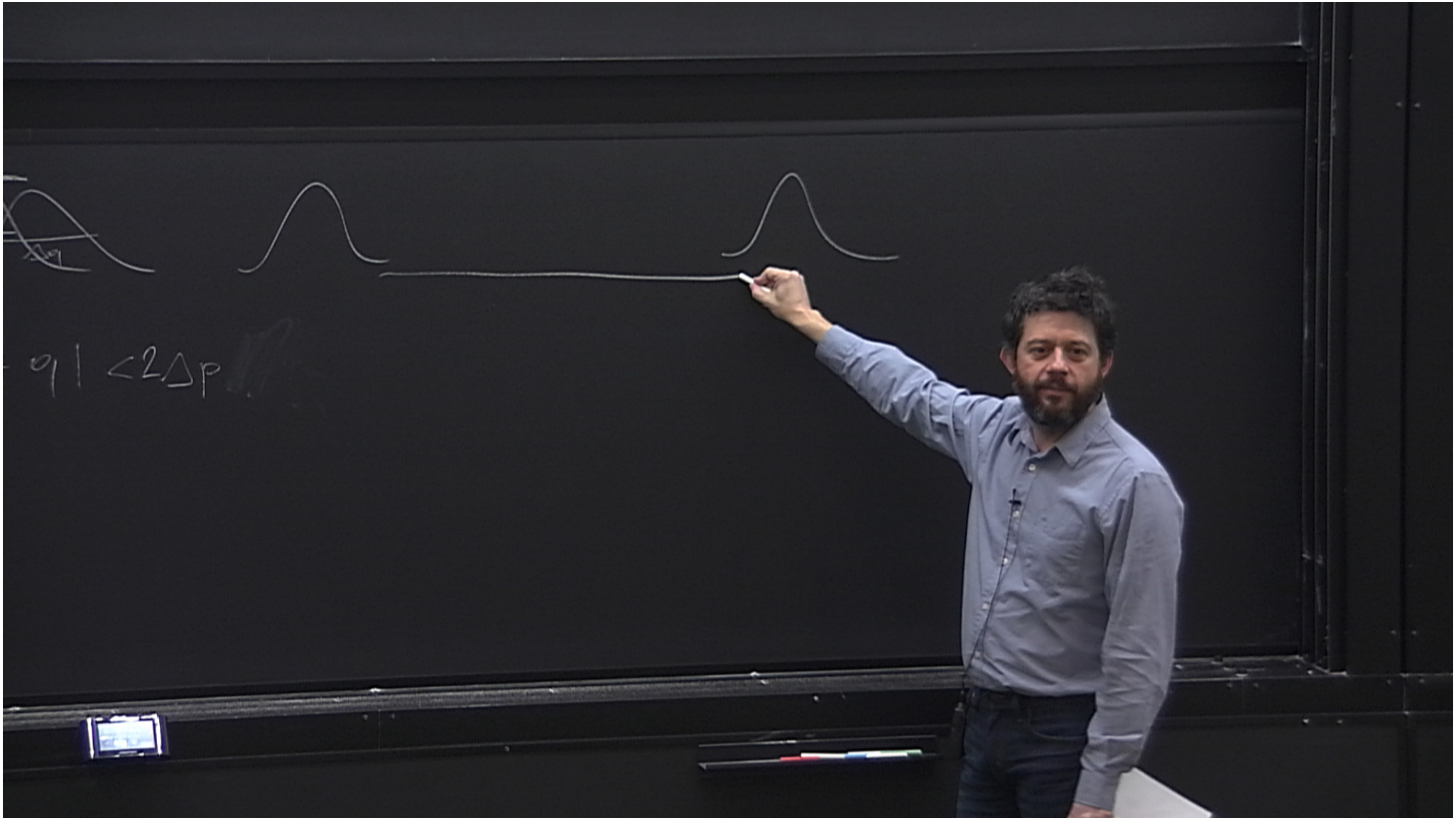


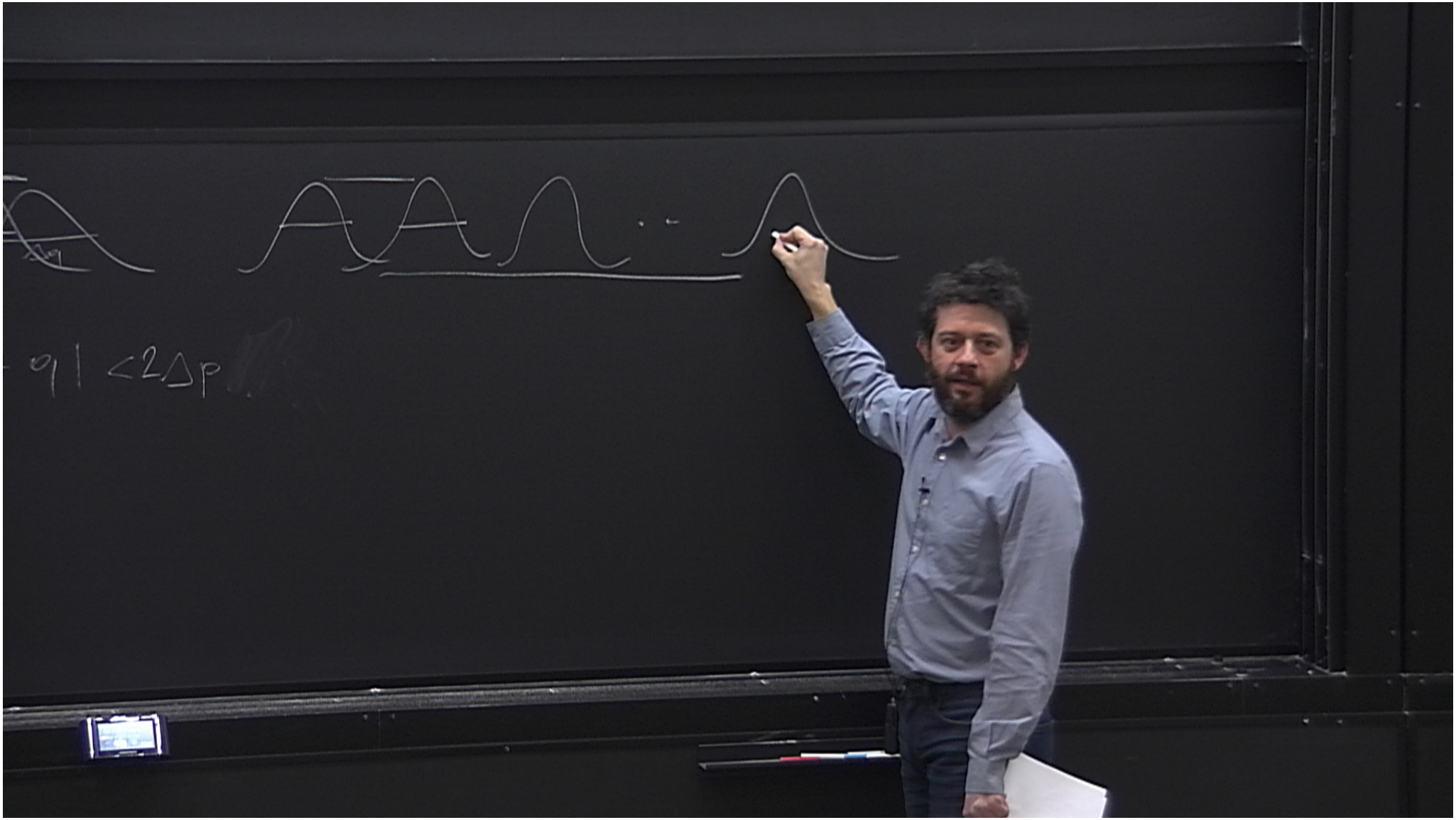
Coin 1 p, $1-p$

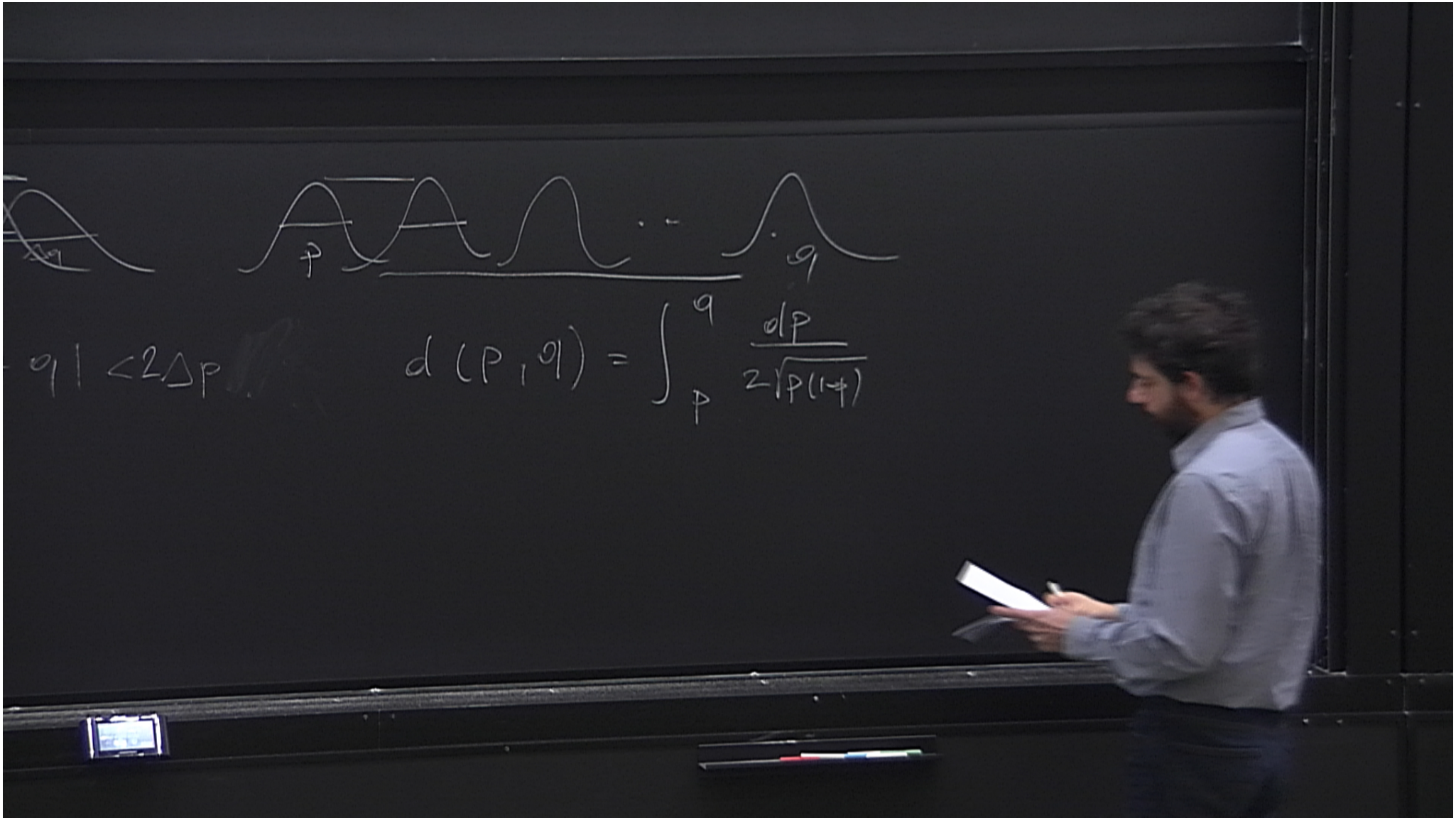
Coin 2 q, $1-q$

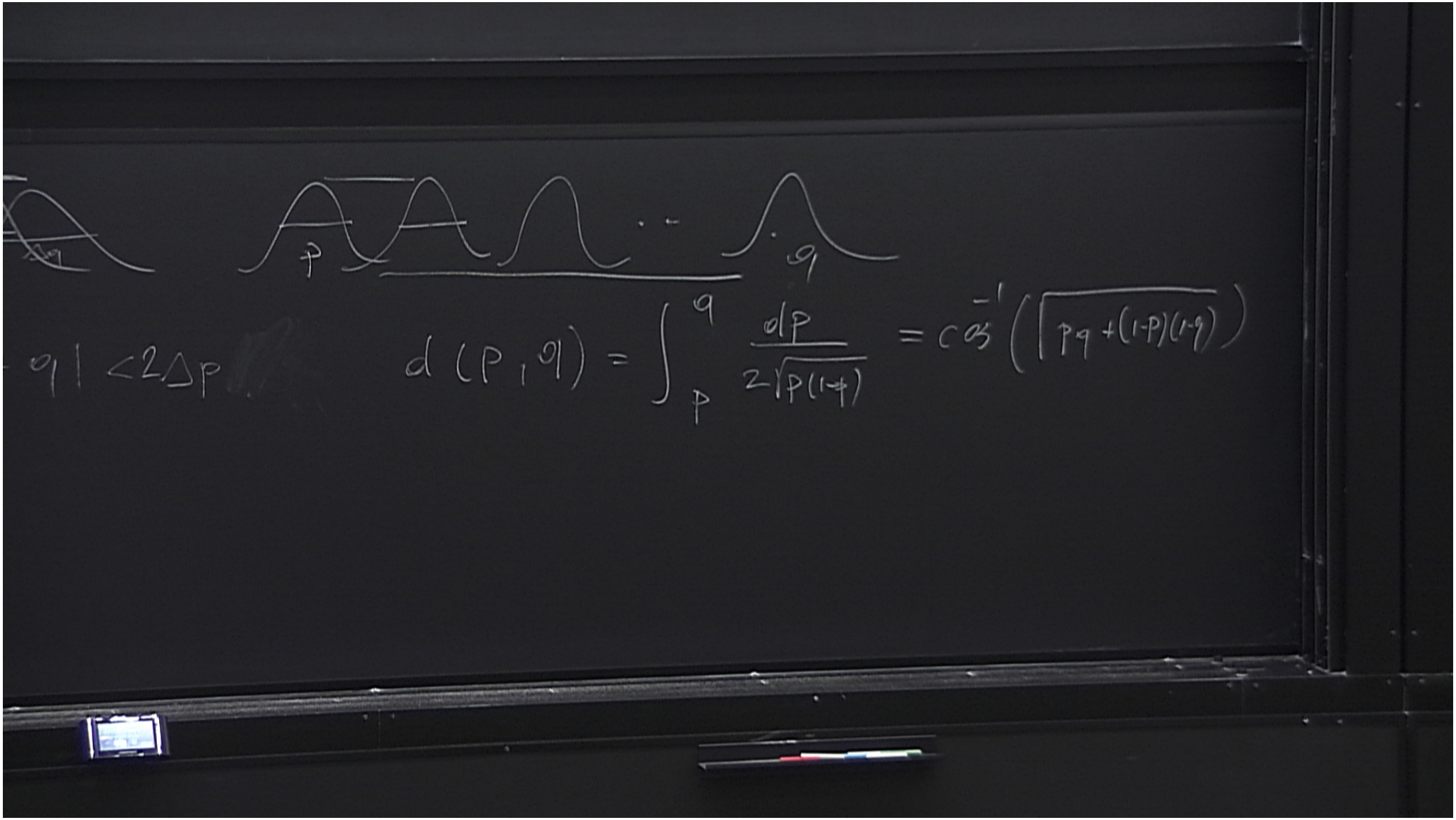
n = # trials

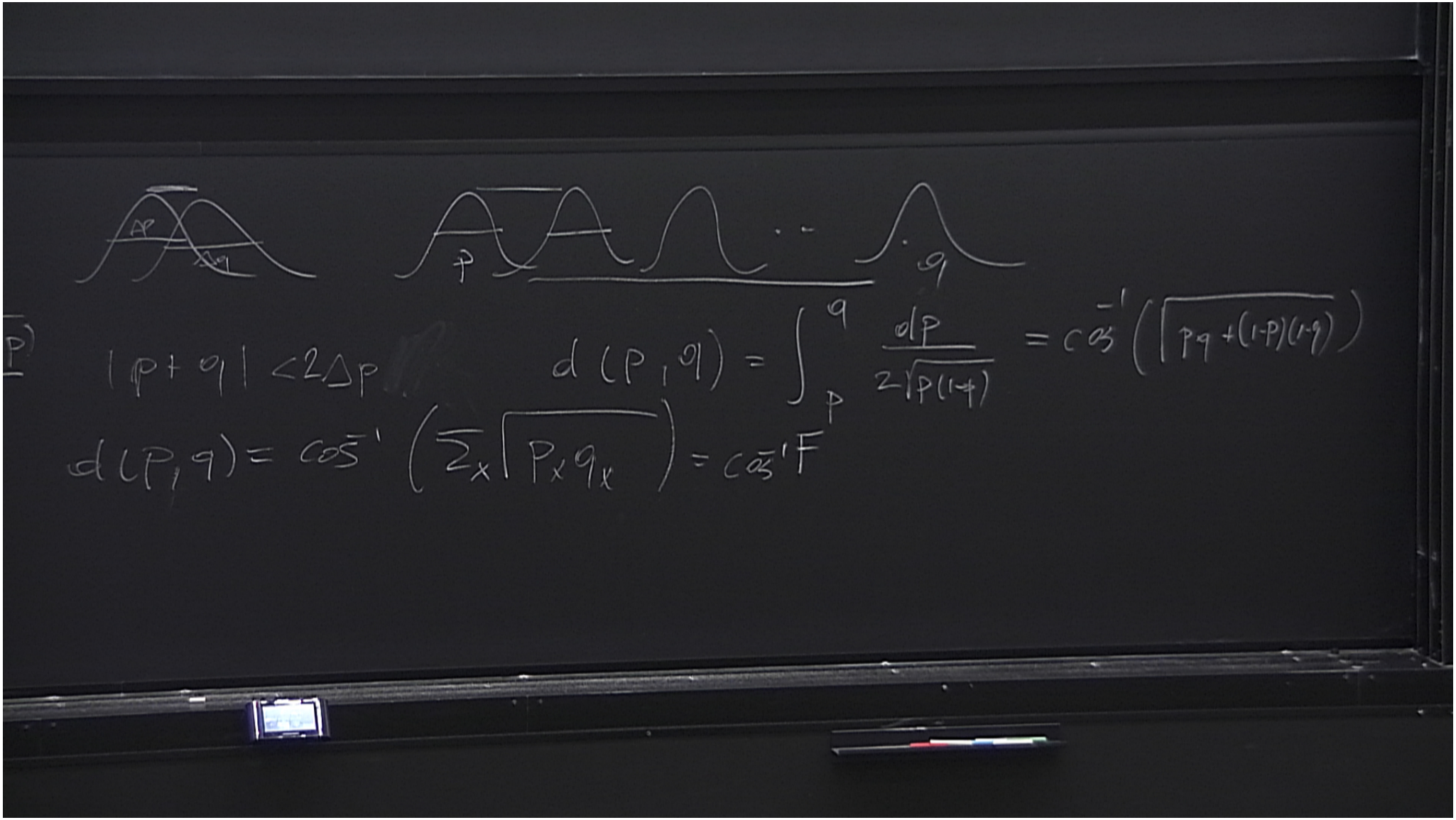
$\Delta p = \sqrt{\frac{p(1-p)}{n}}$ $\quad |p+q| < 2\Delta p$

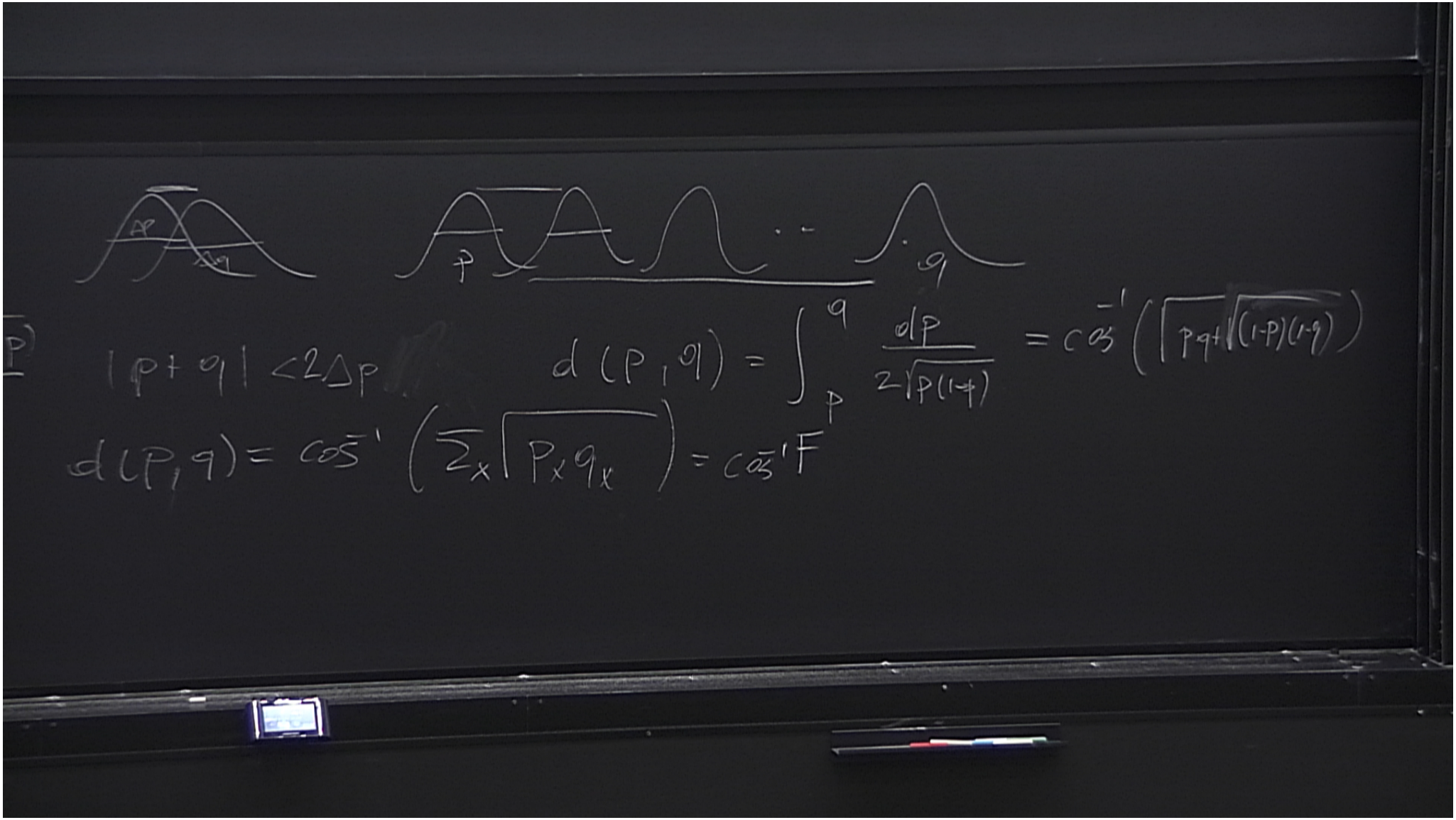

91 < 2Δp

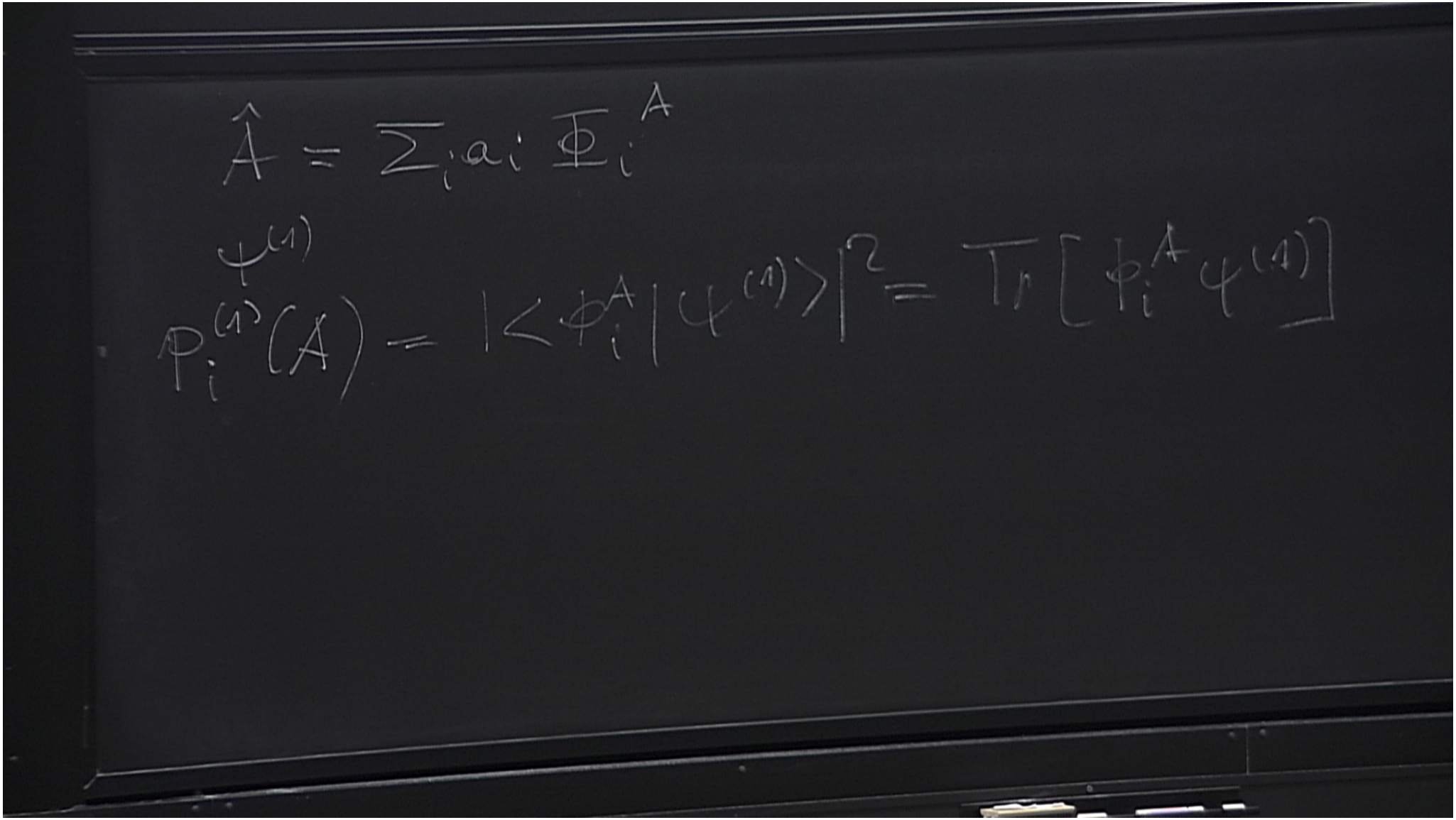

91 < 2Δp

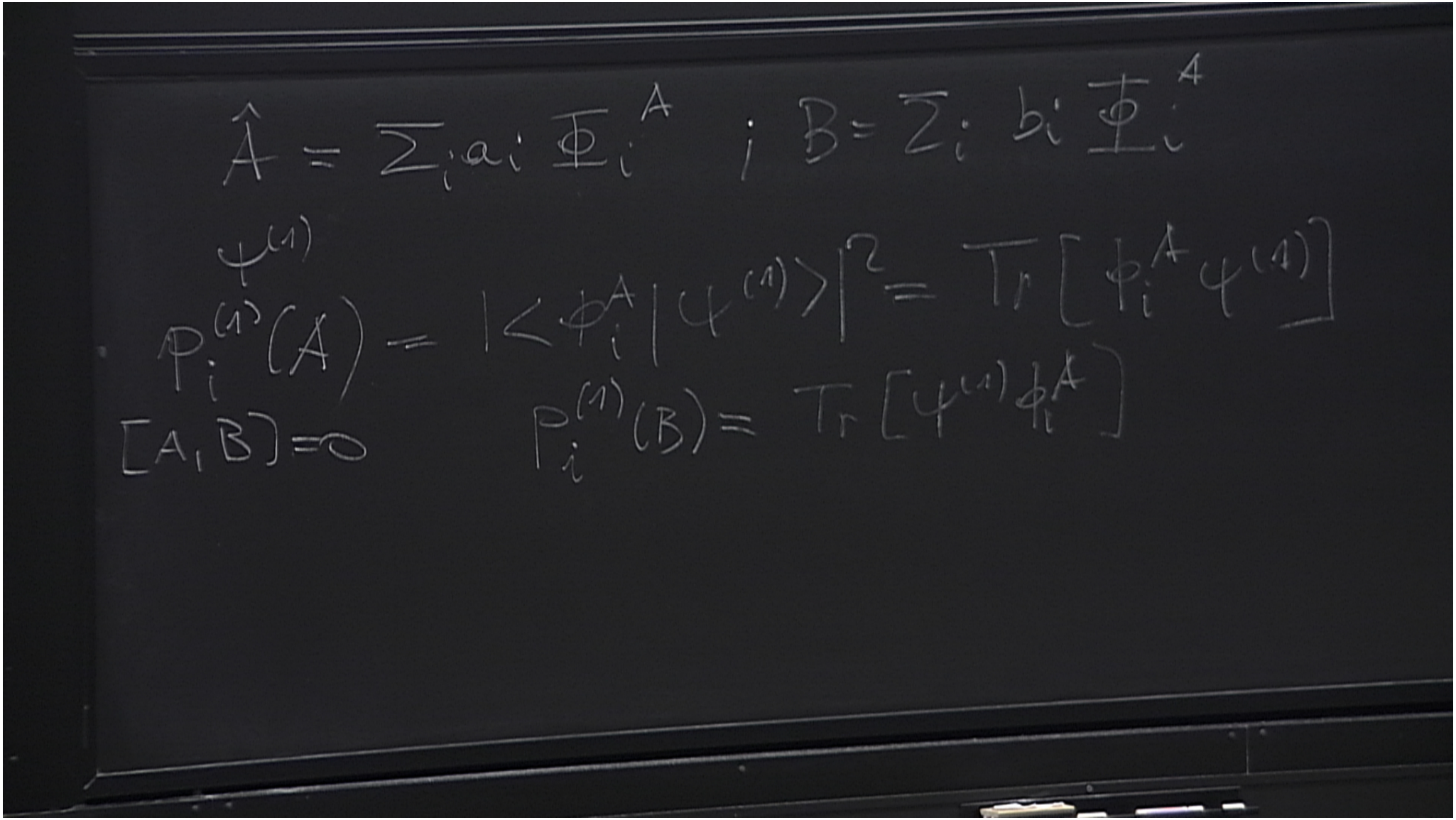

p
q
$|q| < 2\Delta p$
$d(p,q) = \int_p^q \frac{dp}{2\sqrt{p(1-p)}}$


p
q
$|q| < 2\Delta p$
$d(p,q) = \int_p^q \frac{dp}{2\sqrt{p(1-p)}} = \cos^{-1}\left(\sqrt{pq + (1-p)(1-q)}\right)$


p
q
|p+q| < 2Δp
d(p,q) = ∫_p^q dp / 2√(p(1-p))
= cos⁻¹(√(pq + (1-p)(1-q)))
d(p,q) = cos⁻¹ (Σ_x √(p_x q_x)) = cos⁻¹ F


p
q
|p+q| < 2Δp
d(p,q) = ∫_p^q dp / 2√(p(1-p))
= cos⁻¹(√(pq) + √((1-p)(1-q)))
d(p,q) = cos⁻¹ (Σ_x √(p_x q_x)) = cos⁻¹ F


$\hat{A} = \sum_i a_i \Phi_i^A$
$P_i^{(1)}(A) = |\langle \phi_i^A | \psi^{(1)} \rangle|^2 = Tr[\phi_i^A \psi^{(1)}]$

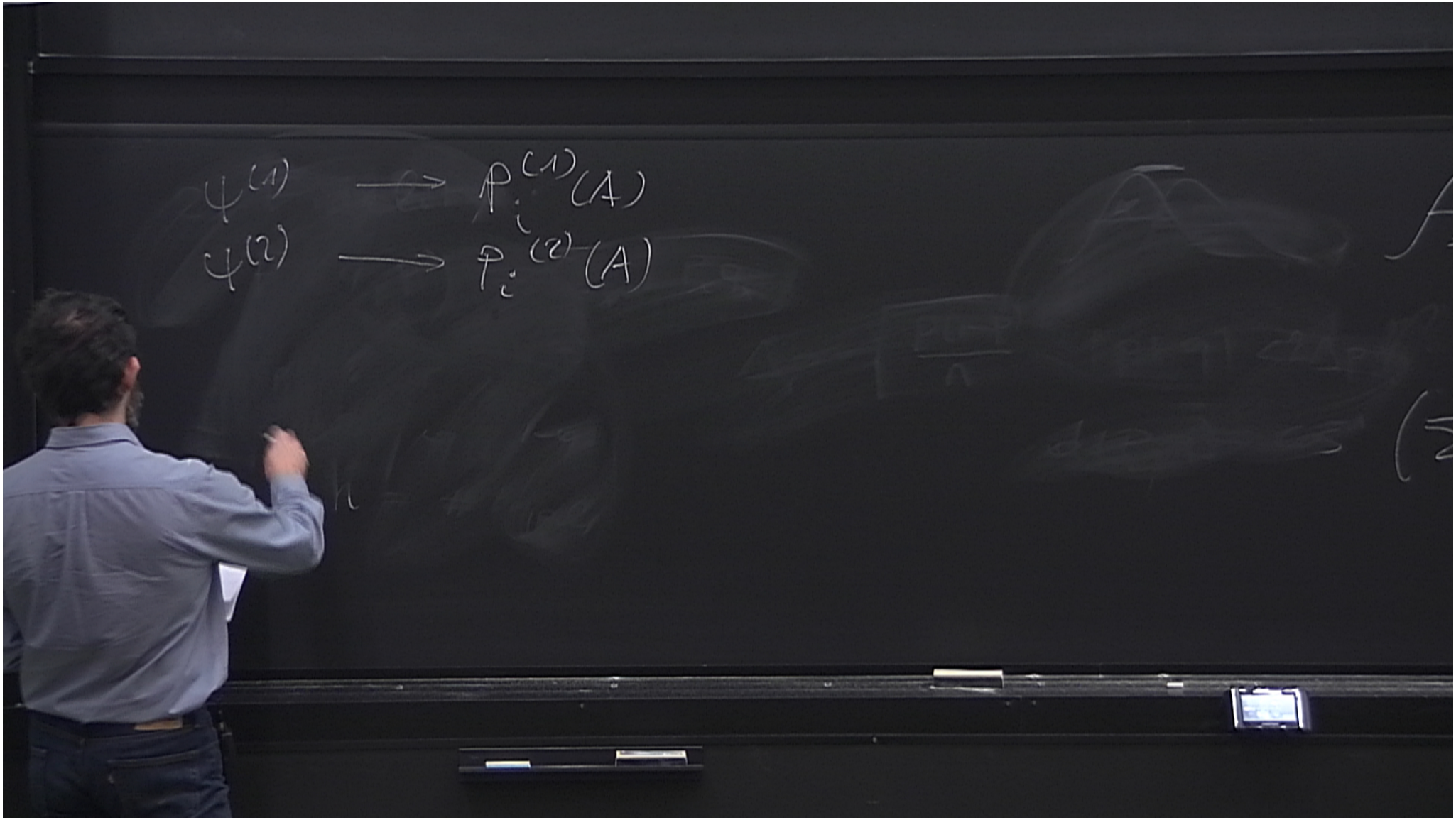

$\hat{A} = \sum_i a_i \Phi_i^A$; $B = \sum_i b_i \Phi_i^A$
$\psi^{(1)}$
$P_i^{(1)}(A) = |\langle \phi_i^A | \psi^{(1)} \rangle|^2 = Tr[\phi_i^A \psi^{(1)}]$
$[A, B] = 0$
$P_i^{(1)}(B) = Tr[\psi^{(1)} \phi_i^A]$

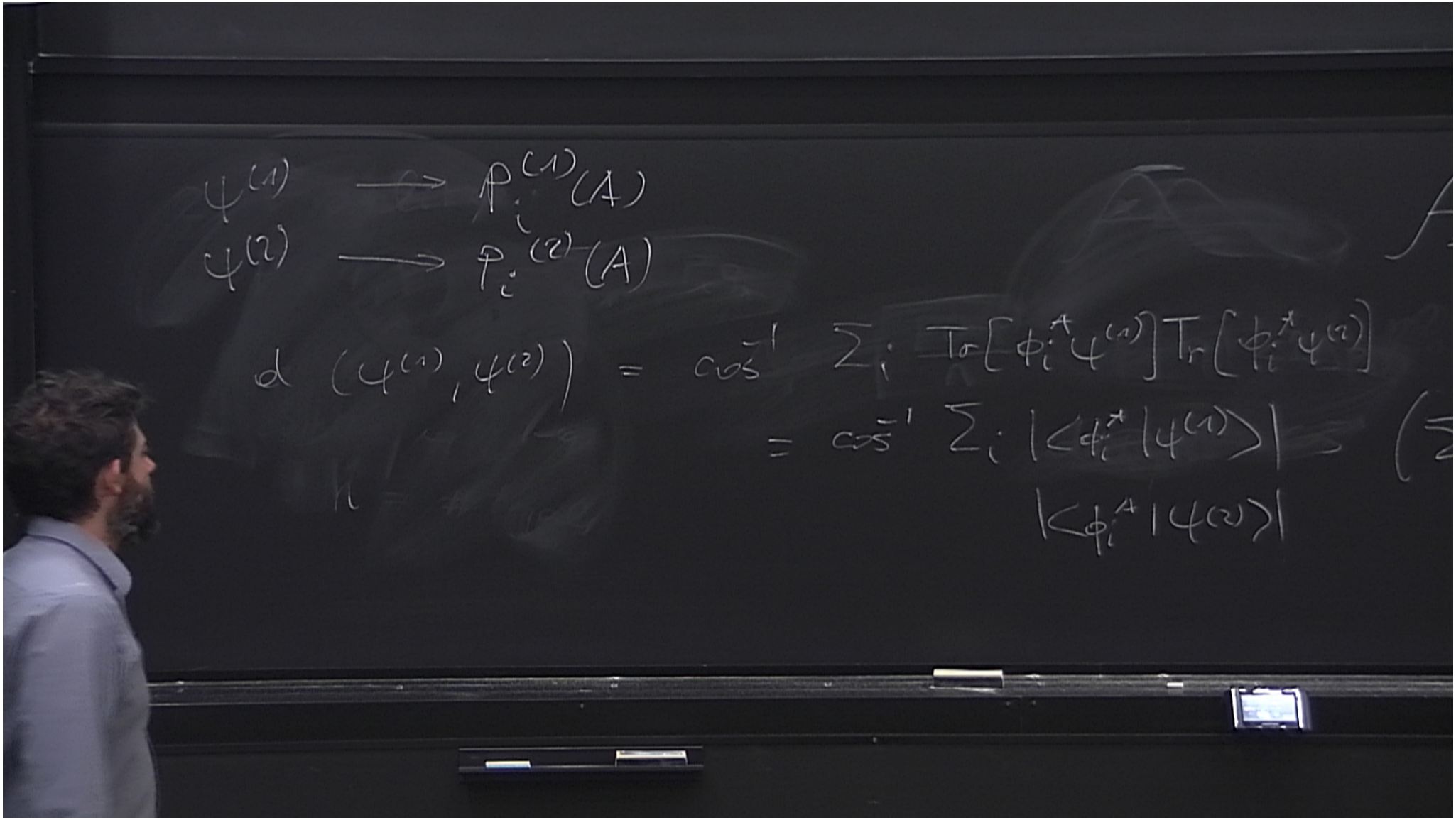
$\hat{A} = \sum_i a_i \Phi_i^A \quad ; \quad B = \sum_i b_i \Phi_i^B$

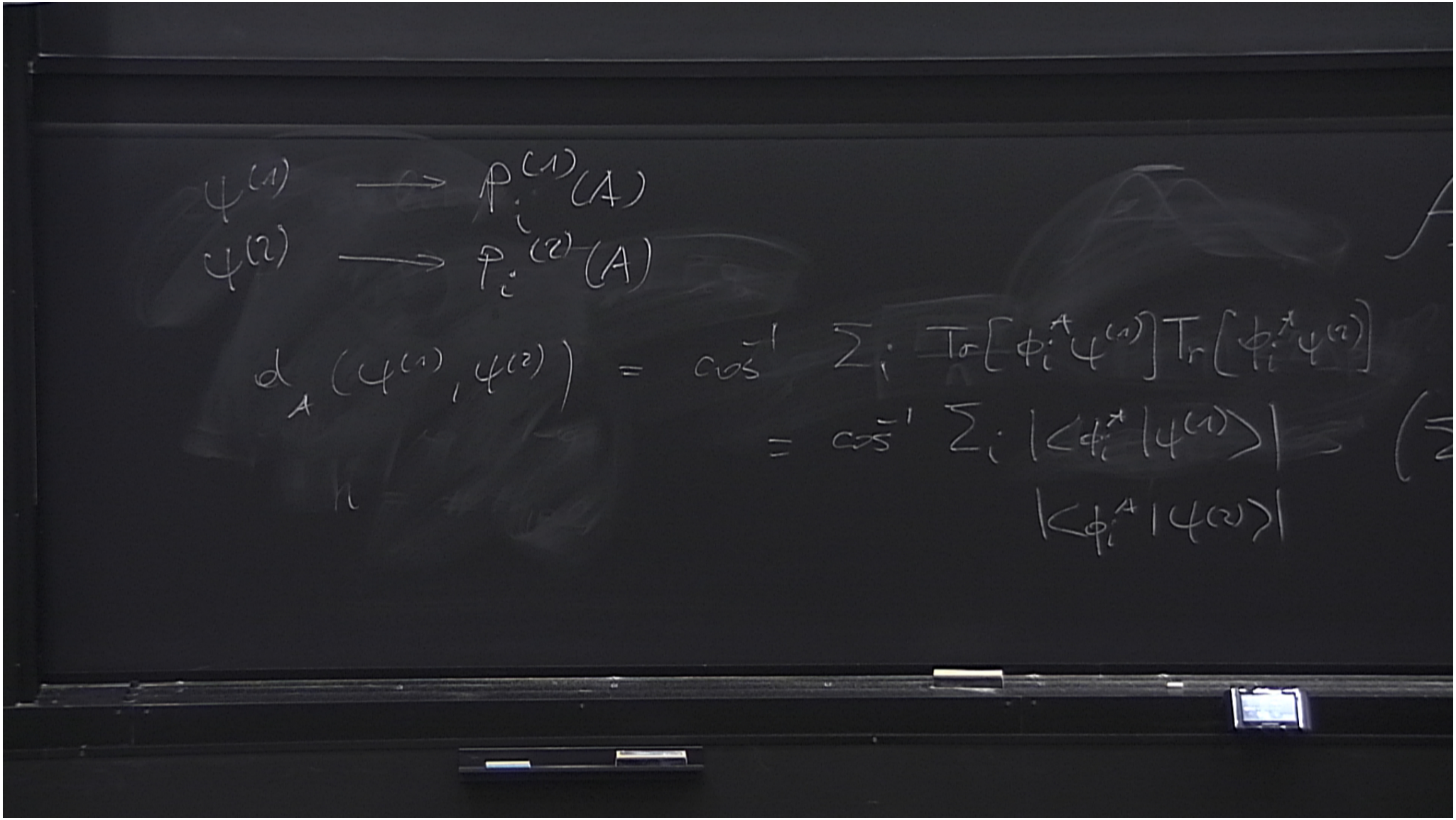
$\psi^{(1)}$

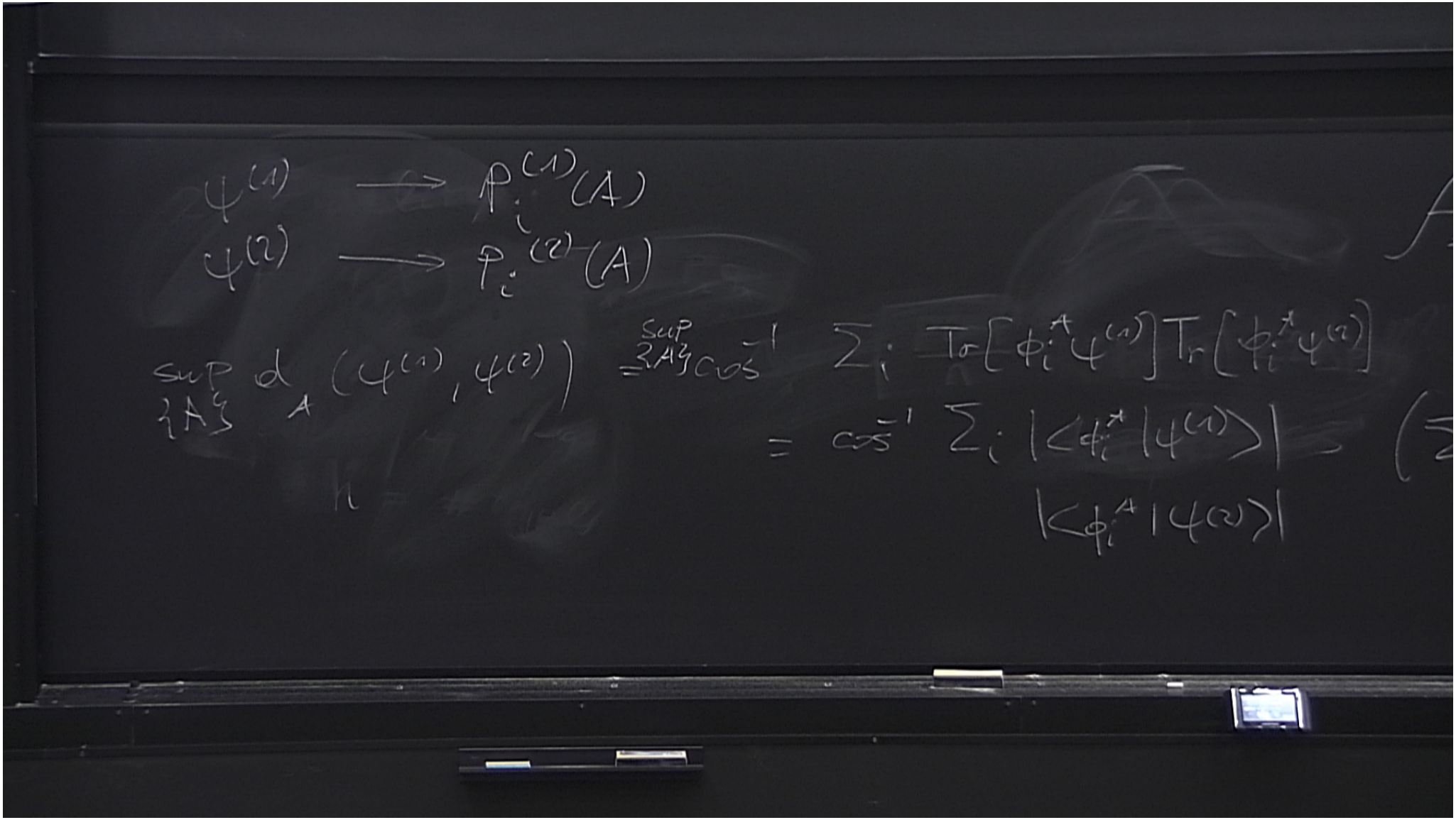
$P_i^{(1)}(A) = |\langle \Phi_i^A | \psi^{(1)} \rangle|^2 = Tr[\Phi_i^A \psi^{(1)}]$

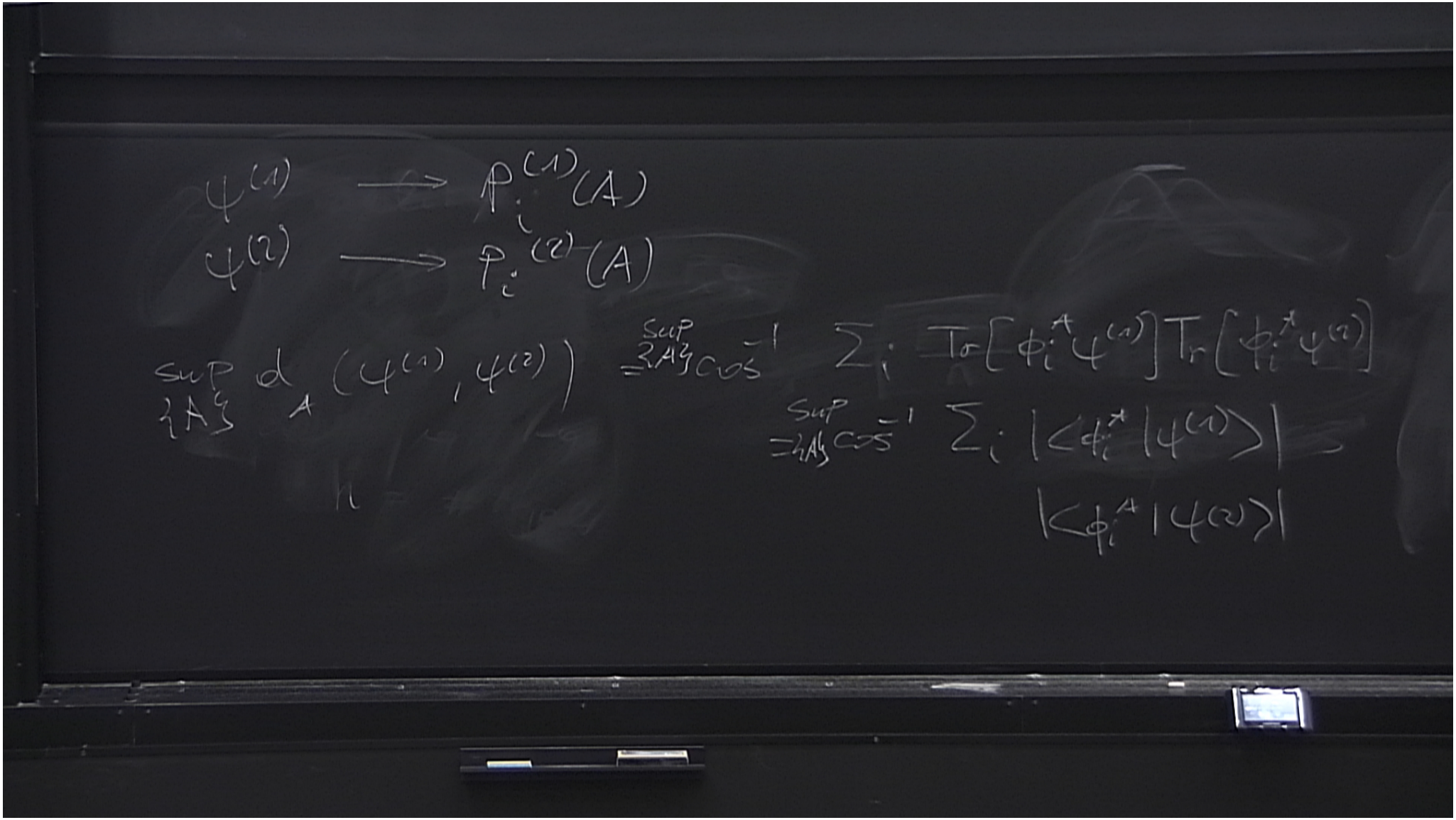
$[A, B] \neq 0 \qquad P_i^{(1)}(B) = Tr[\psi^{(1)} \Phi_i^B]$

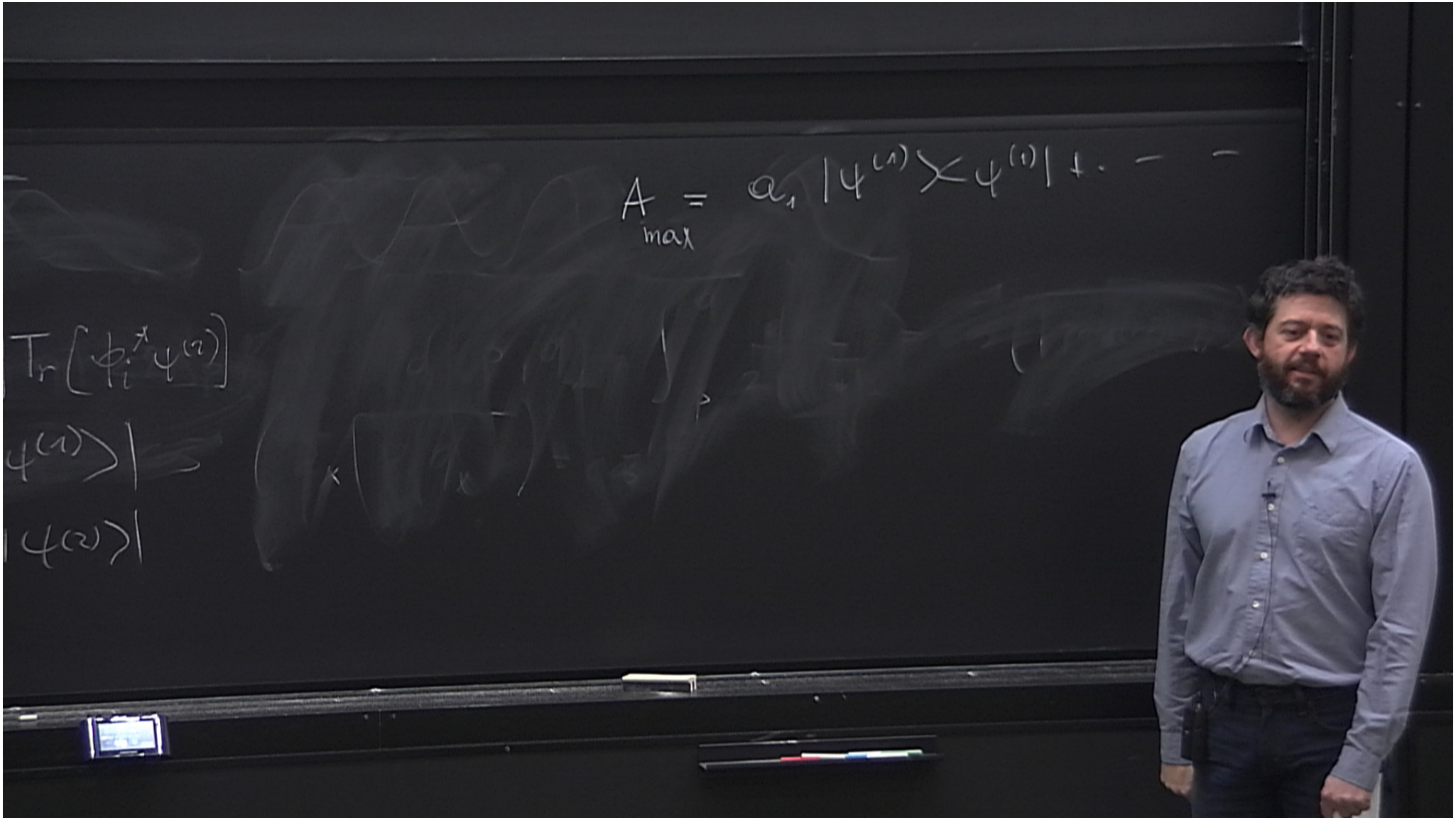

$\psi^{(1)} \longrightarrow P_i^{(1)}(A)$
$\psi^{(2)} \longrightarrow P_i^{(2)}(A)$

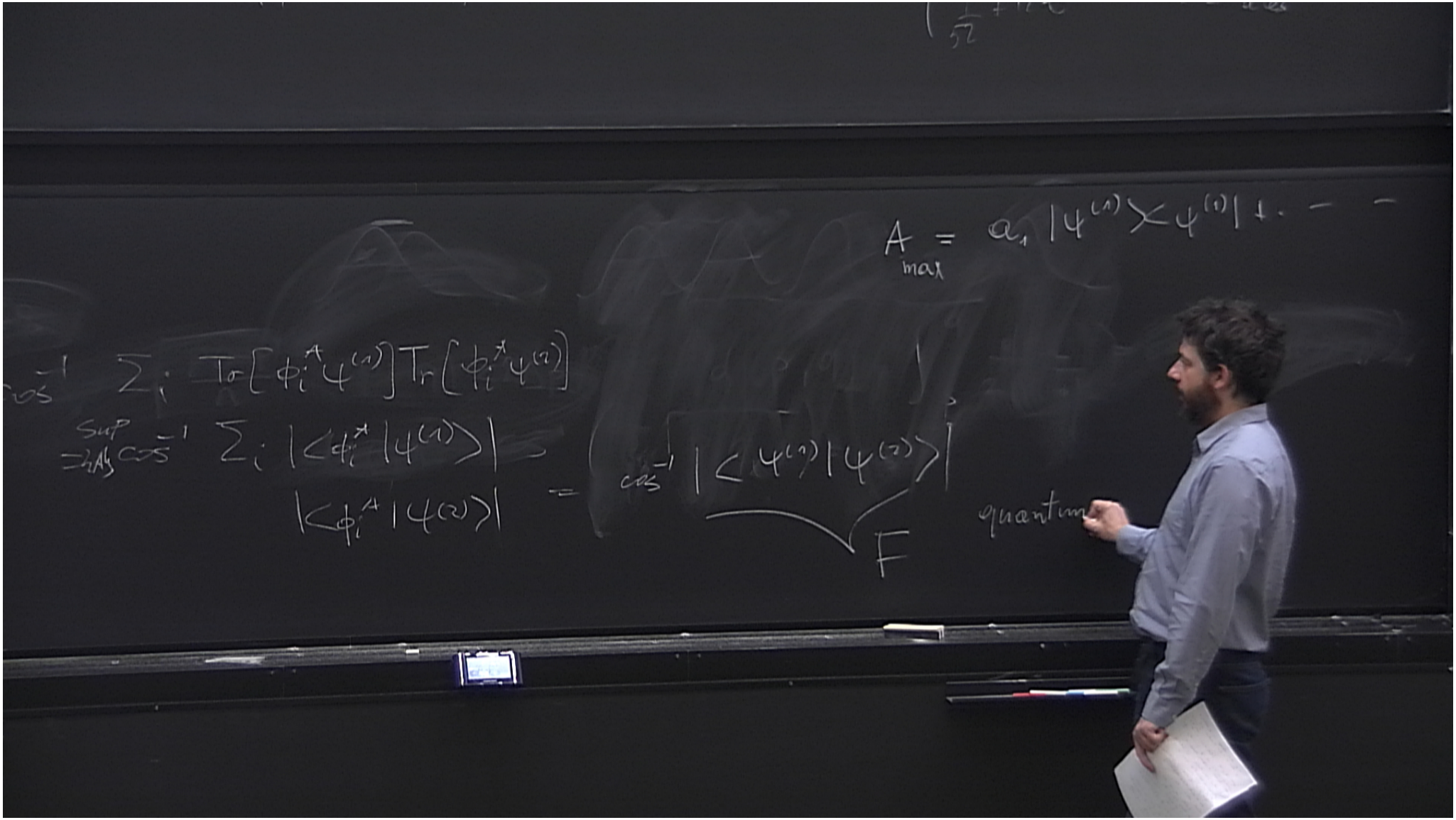

$\psi^{(1)} \longrightarrow p_i^{(1)}(A)$
$\psi^{(2)} \longrightarrow p_i^{(2)}(A)$
$d(\psi^{(1)}, \psi^{(2)}) = \cos^{-1} \sum_i \mathrm{Tr}[\phi_i^A \psi^{(1)}] \mathrm{Tr}[\phi_i^A \psi^{(2)}]$
$= \cos^{-1} \sum_i |\langle \phi_i^A | \psi^{(1)} \rangle| \; |\langle \phi_i^A | \psi^{(2)} \rangle|$

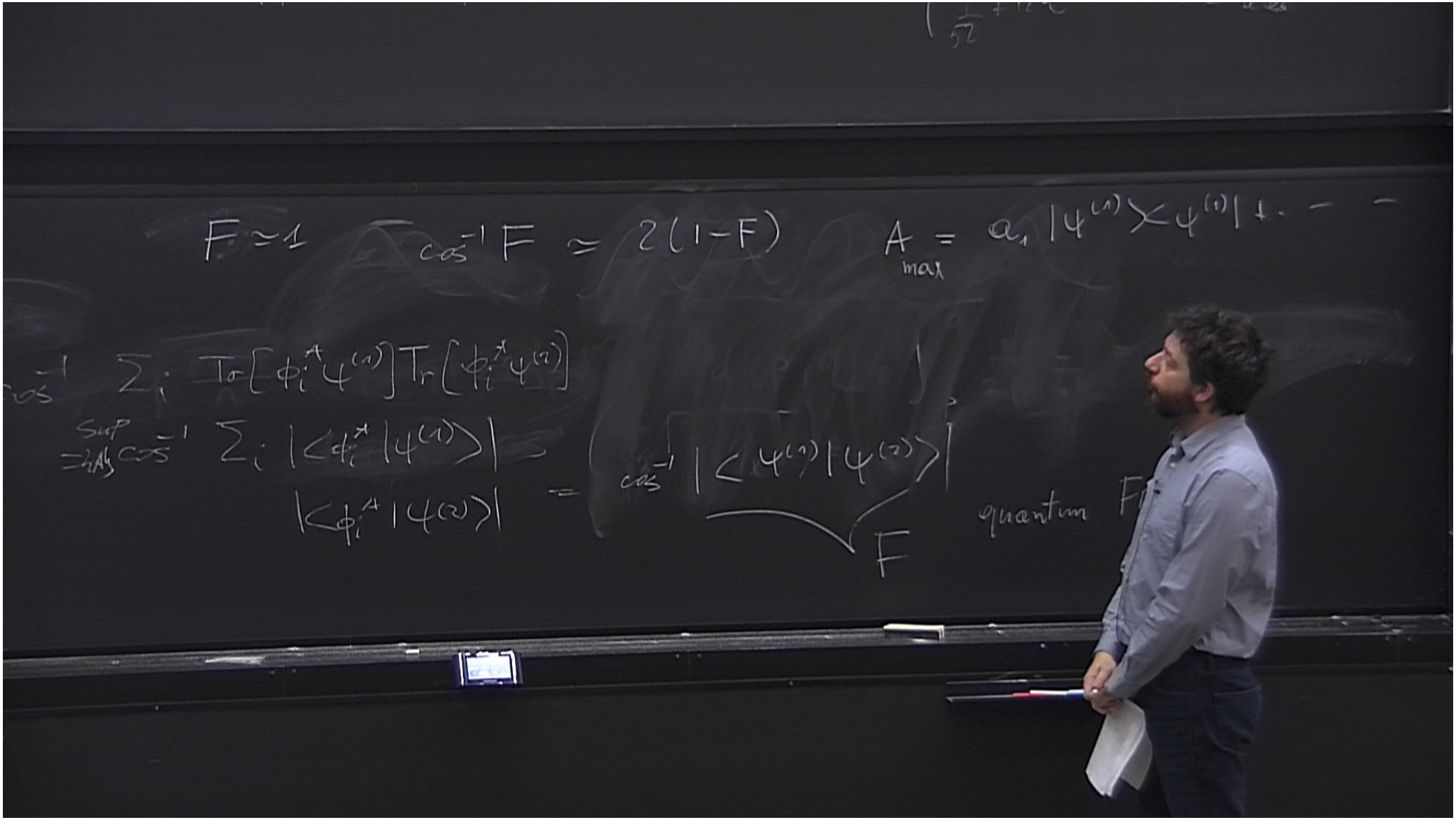

$\psi^{(1)} \longrightarrow P_i^{(1)}(A)$
$\psi^{(2)} \longrightarrow P_i^{(2)}(A)$
$d_A(\psi^{(1)}, \psi^{(2)}) = \cos^{-1} \sum_i \mathrm{Tr}[\phi_i^A \psi^{(1)}] \mathrm{Tr}[\phi_i^A \psi^{(2)}]$
$= \cos^{-1} \sum_i |\langle \phi_i^A | \psi^{(1)} \rangle| \; |\langle \phi_i^A | \psi^{(2)} \rangle|$

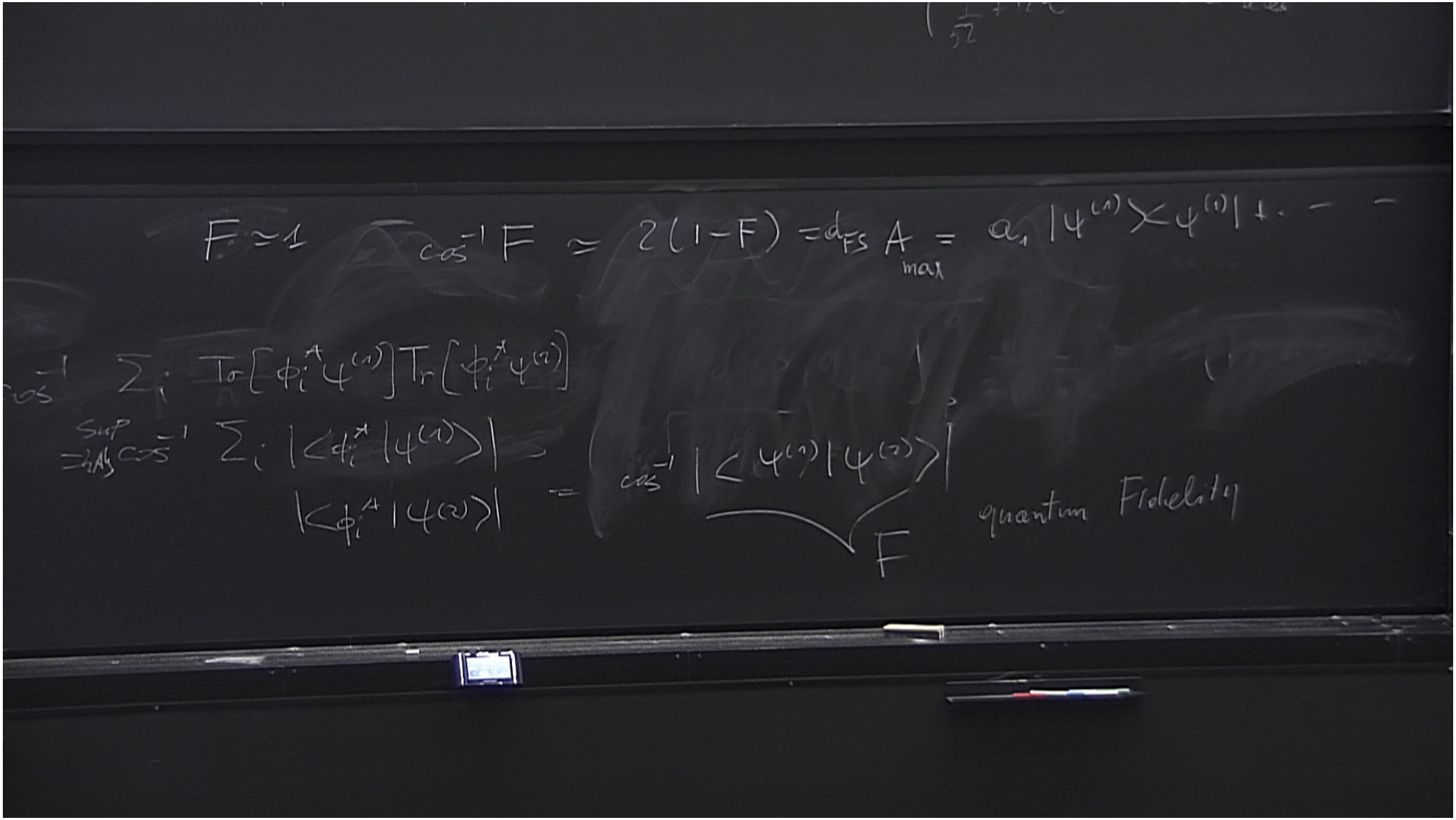

$\psi^{(1)} \longrightarrow P_i^{(1)}(A)$
$\psi^{(2)} \longrightarrow P_i^{(2)}(A)$
$\sup_{\{A\}} d_A(\psi^{(1)}, \psi^{(2)}) = \sup_{\{A\}} \cos^{-1} \sum_i \mathrm{Tr}[\Phi_i^A \psi^{(1)}] \mathrm{Tr}[\Phi_i^A \psi^{(2)}]$
$= \cos^{-1} \sum_i |\langle \Phi_i^A | \psi^{(1)} \rangle| \; |\langle \Phi_i^A | \psi^{(2)} \rangle|$

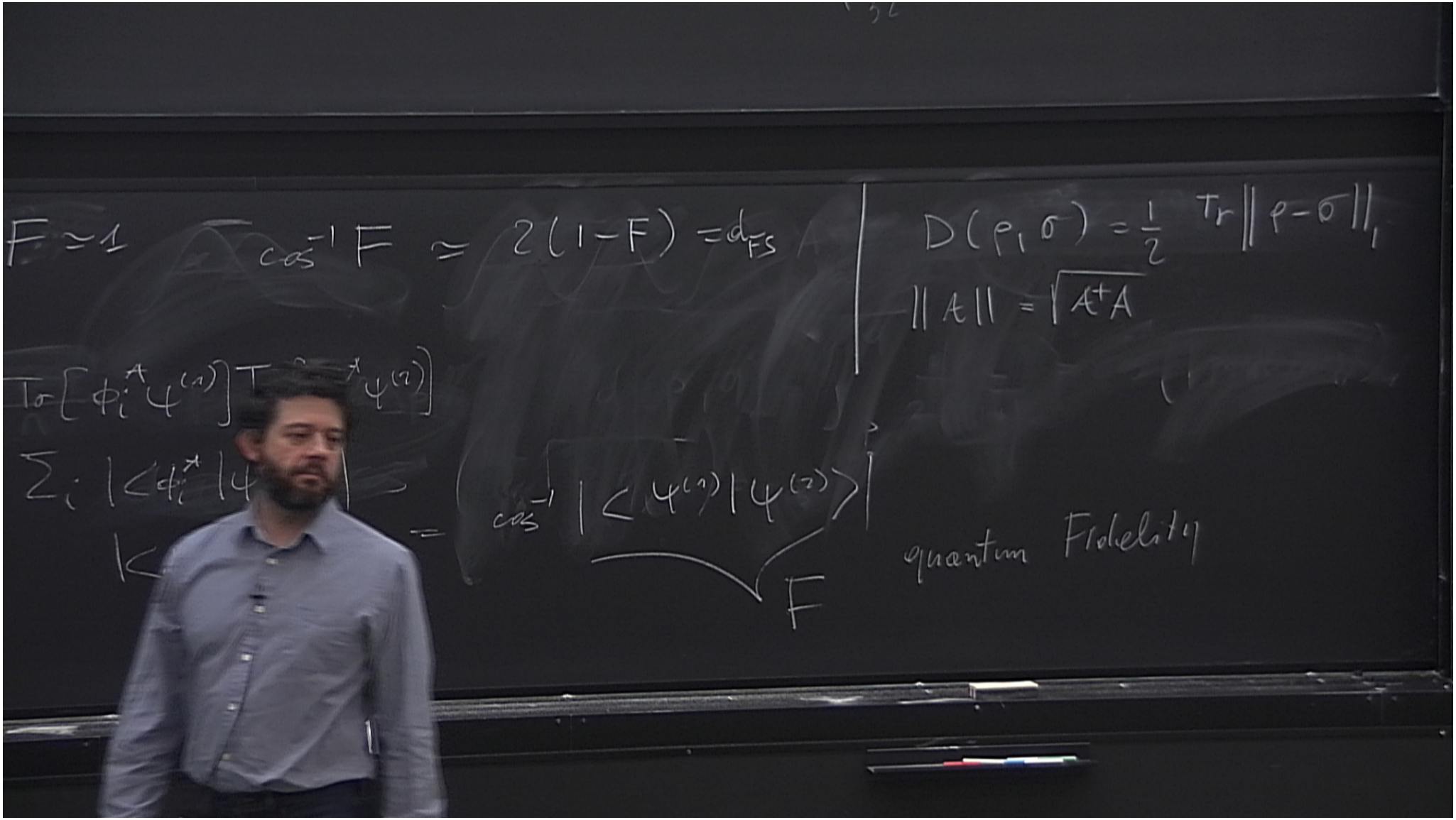

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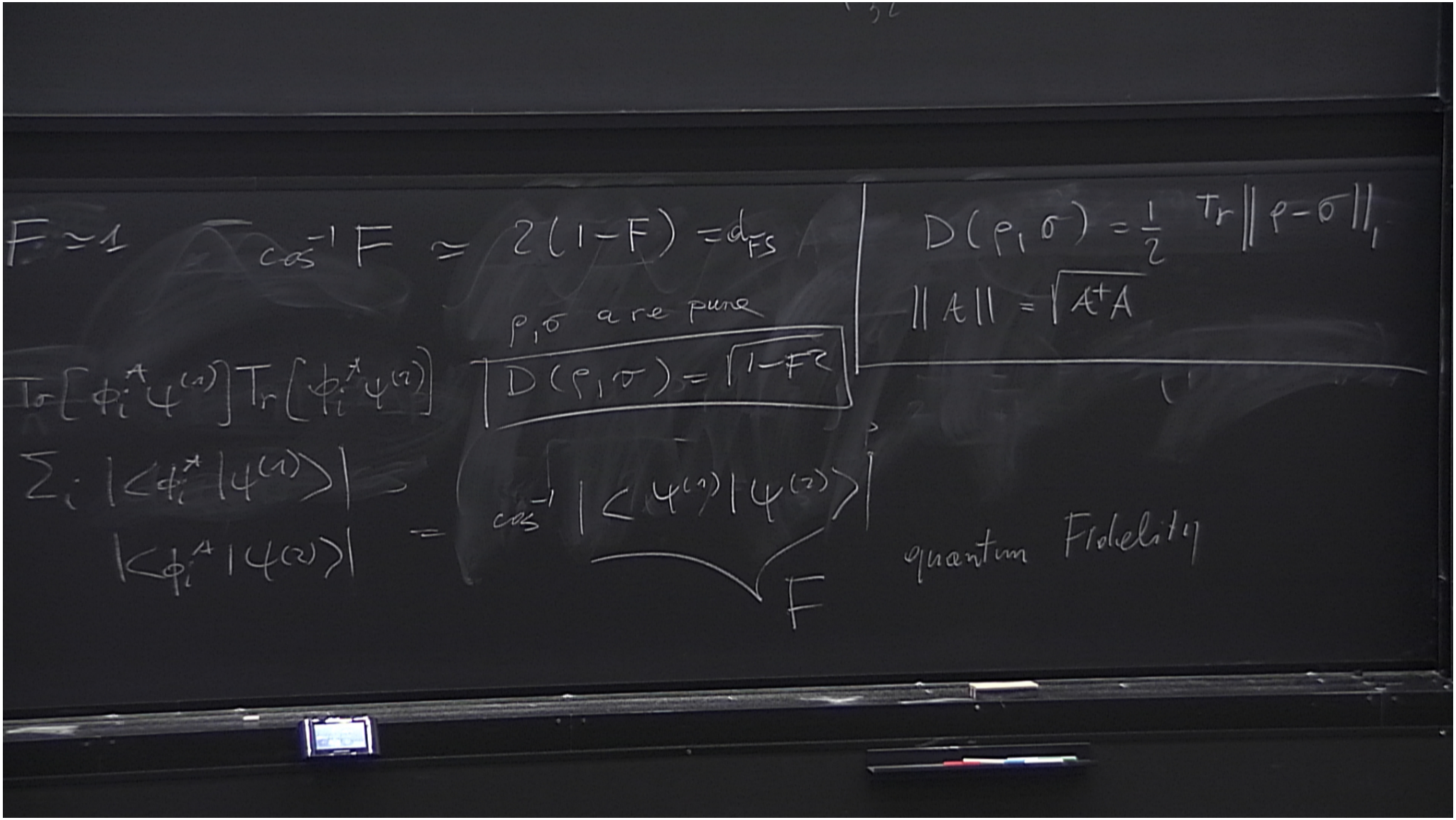

$A = a_1 |\psi^{(1)}\rangle\langle\psi^{(1)}| + \cdots$
max
$\mathrm{Tr}[\phi_i^* \psi^{(2)}]$
$|\psi^{(1)}\rangle$
$|\psi^{(2)}\rangle$

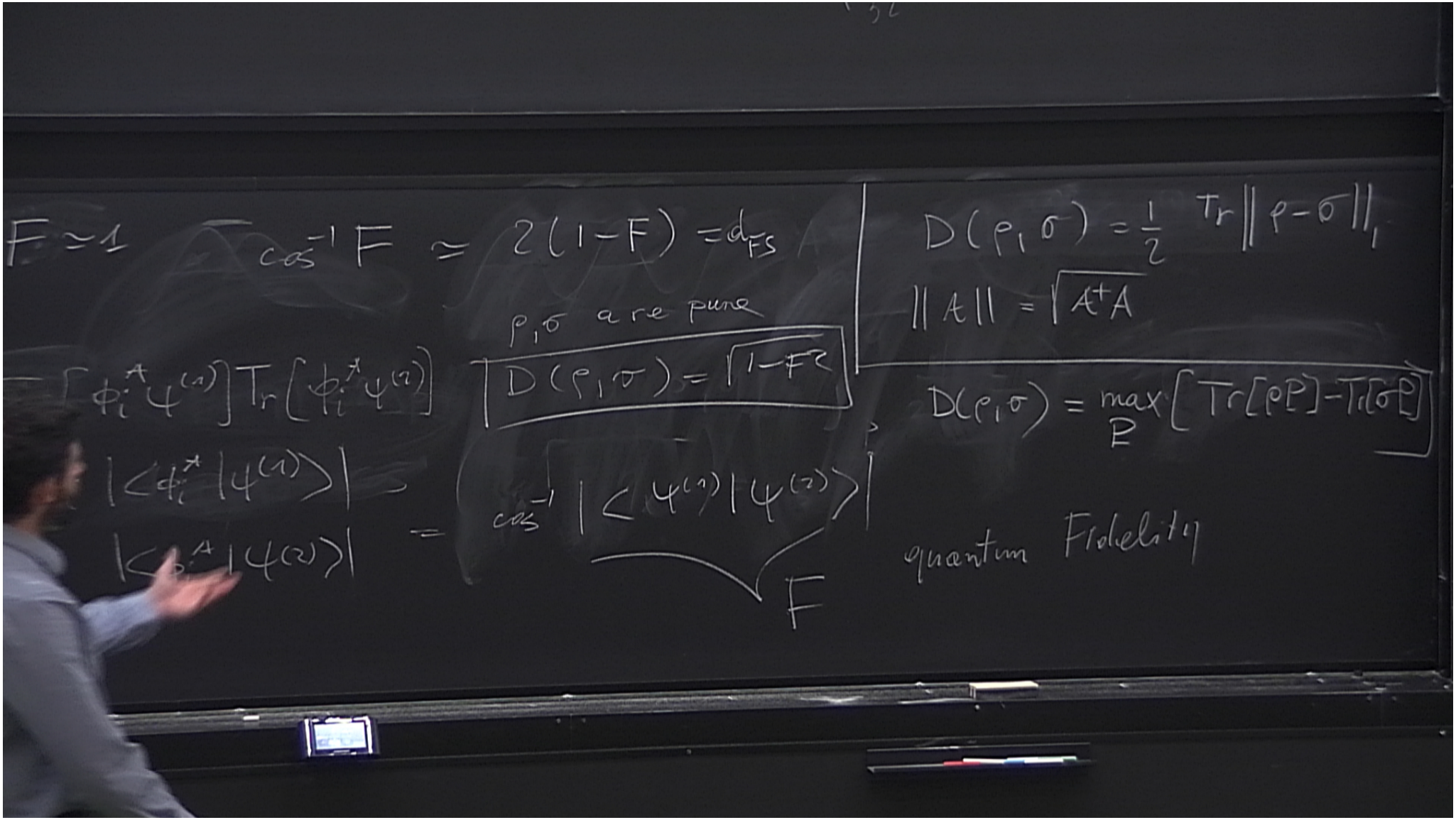

$A = \max a_1 |\psi^{(1)}\rangle\langle\psi^{(1)}| + \cdots$
$\cos^{-1} \sum_i \mathrm{Tr}[\phi_i^A \psi^{(1)}] \mathrm{Tr}[\phi_i^A \psi^{(2)}]$
$= \sup_A \cos^{-1} \sum_i |\langle \phi_i^A | \psi^{(1)} \rangle| \cdot |\langle \phi_i^A | \psi^{(2)} \rangle|$
$= \cos^{-1} |\langle \psi^{(1)} | \psi^{(2)} \rangle|$
F
quantum

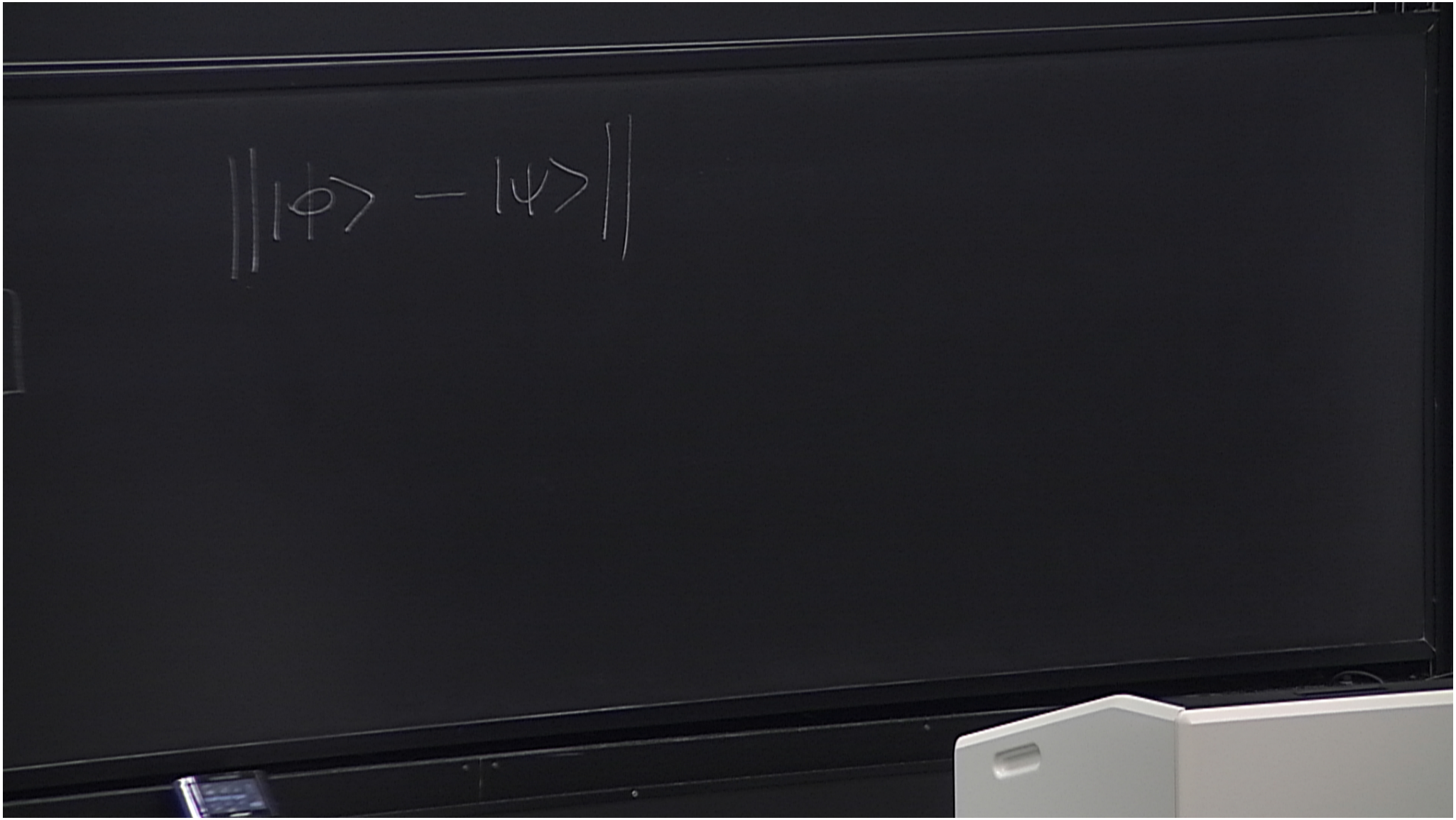

F ≃ 1
cos⁻¹ F ≃ 2(1−F)
A = a₁ |ψ⁽¹⁾⟩⟨ψ⁽¹⁾| + ...
max
Σᵢ Tr[φᵢ^A ψ⁽¹⁾] Tr[φᵢ^A ψ⁽²⁾]
sup cos⁻¹ Σᵢ |⟨φᵢ^A|ψ⁽¹⁾⟩|
|⟨φᵢ^A|ψ⁽²⁾⟩|
= cos⁻¹ |⟨ψ⁽¹⁾|ψ⁽²⁾⟩|
F
quantum F

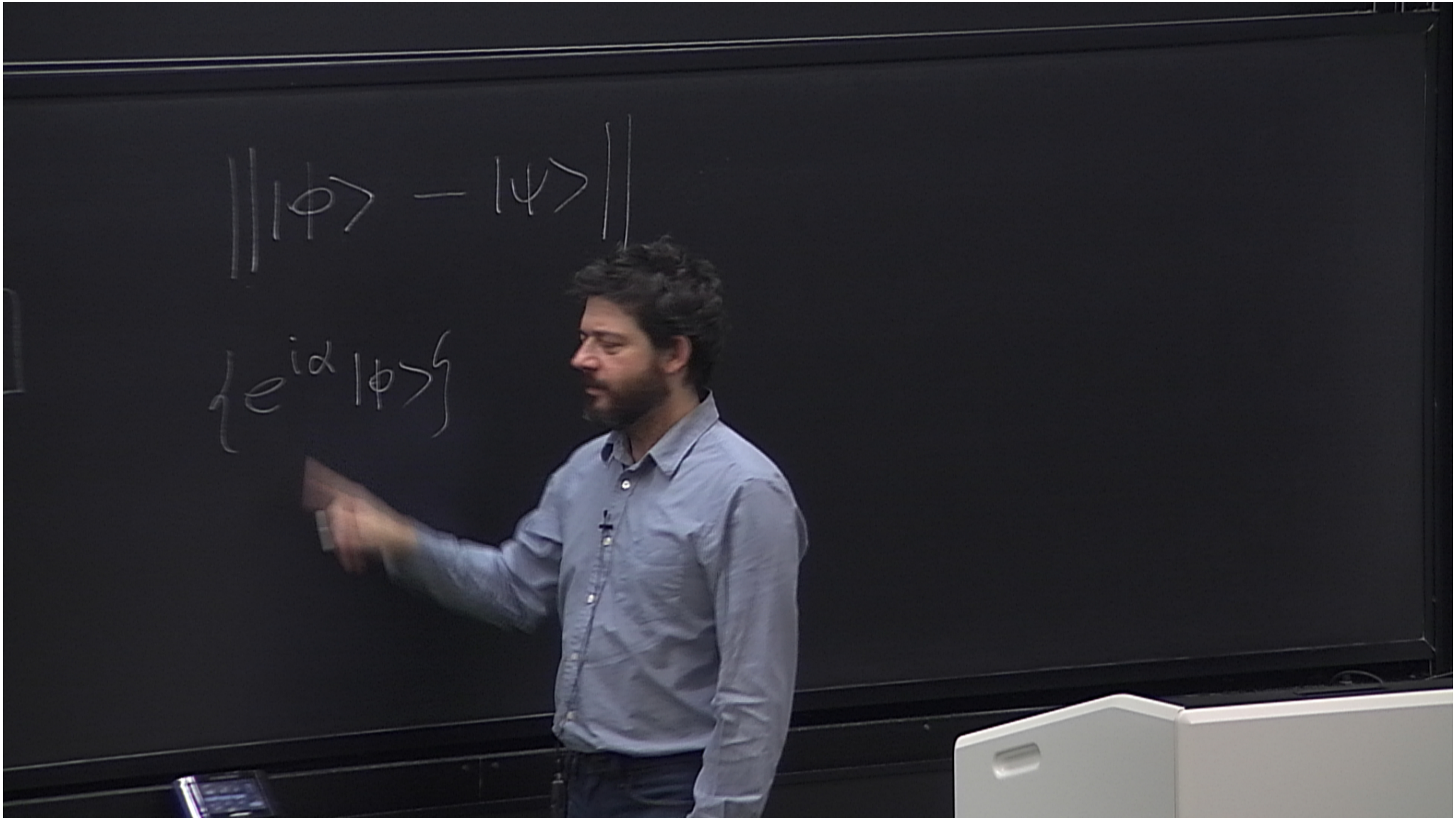

$F \simeq 1 \quad \cos^{-1} F \simeq 2(1-F) = d_{FS}$
$A = a_1 |\psi^{(1)}\rangle\langle\psi^{(1)}| + \dots$
$\max$
$\cos^{-1} \sum_i \mathrm{Tr}[\phi_i^A \psi^{(1)}] \mathrm{Tr}[\phi_i^A \psi^{(2)}]$
$\sup_{A} \cos^{-1} \sum_i |\langle \phi_i^A | \psi^{(1)} \rangle| =$
$|\langle \phi_i^A | \psi^{(2)} \rangle| = \cos^{-1} |\langle \psi^{(1)} | \psi^{(2)} \rangle|$
F
quantum Fidelity

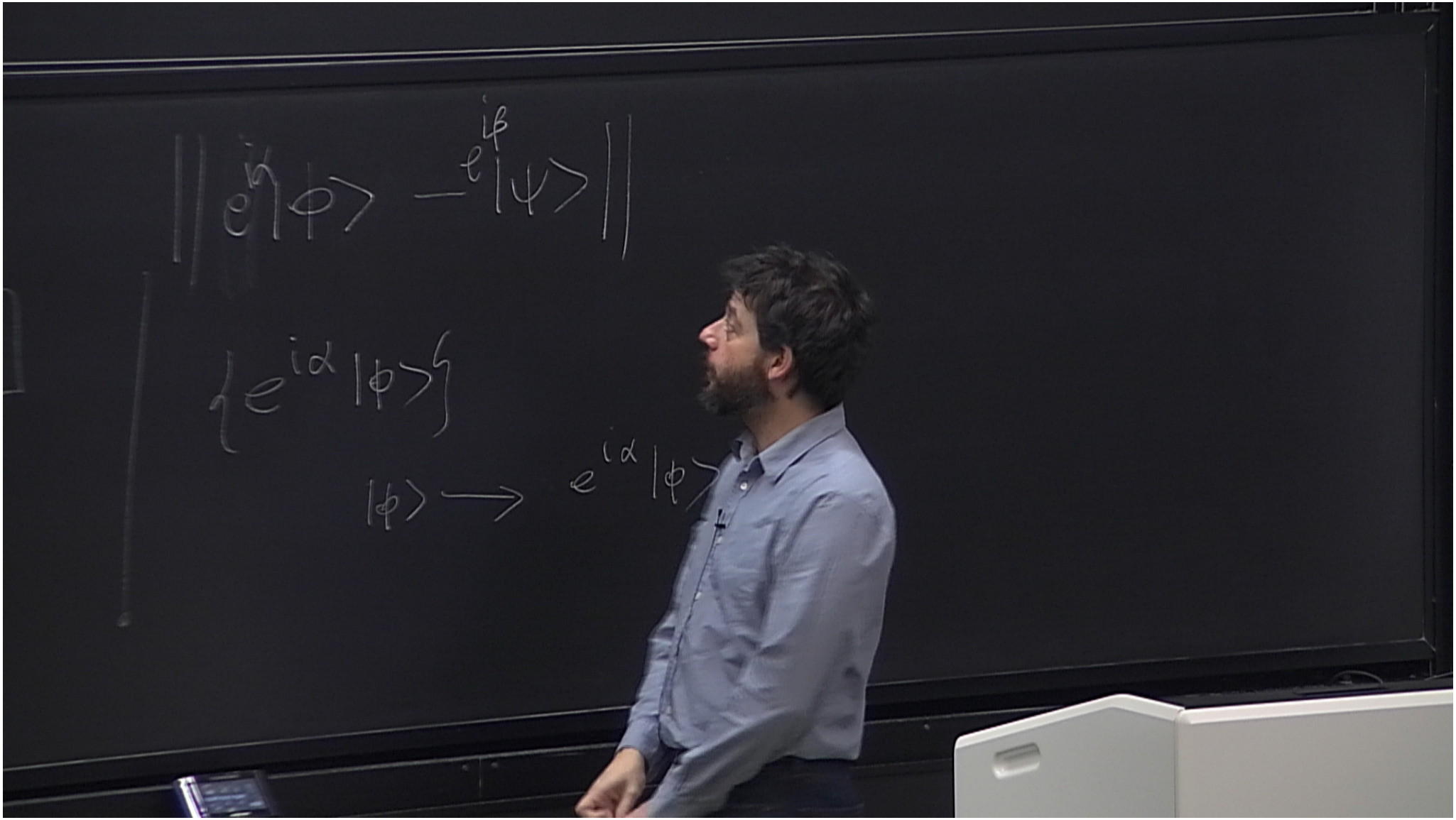

F ≃ 1
cos⁻¹ F ≃ 2(1−F) = d_FS
D(ρ,σ) = ½ Tr ‖ρ−σ‖₁
‖A‖ = √(A†A)
cos⁻¹ |⟨ψ(1)|ψ(2)⟩|
F
quantum Fidelity

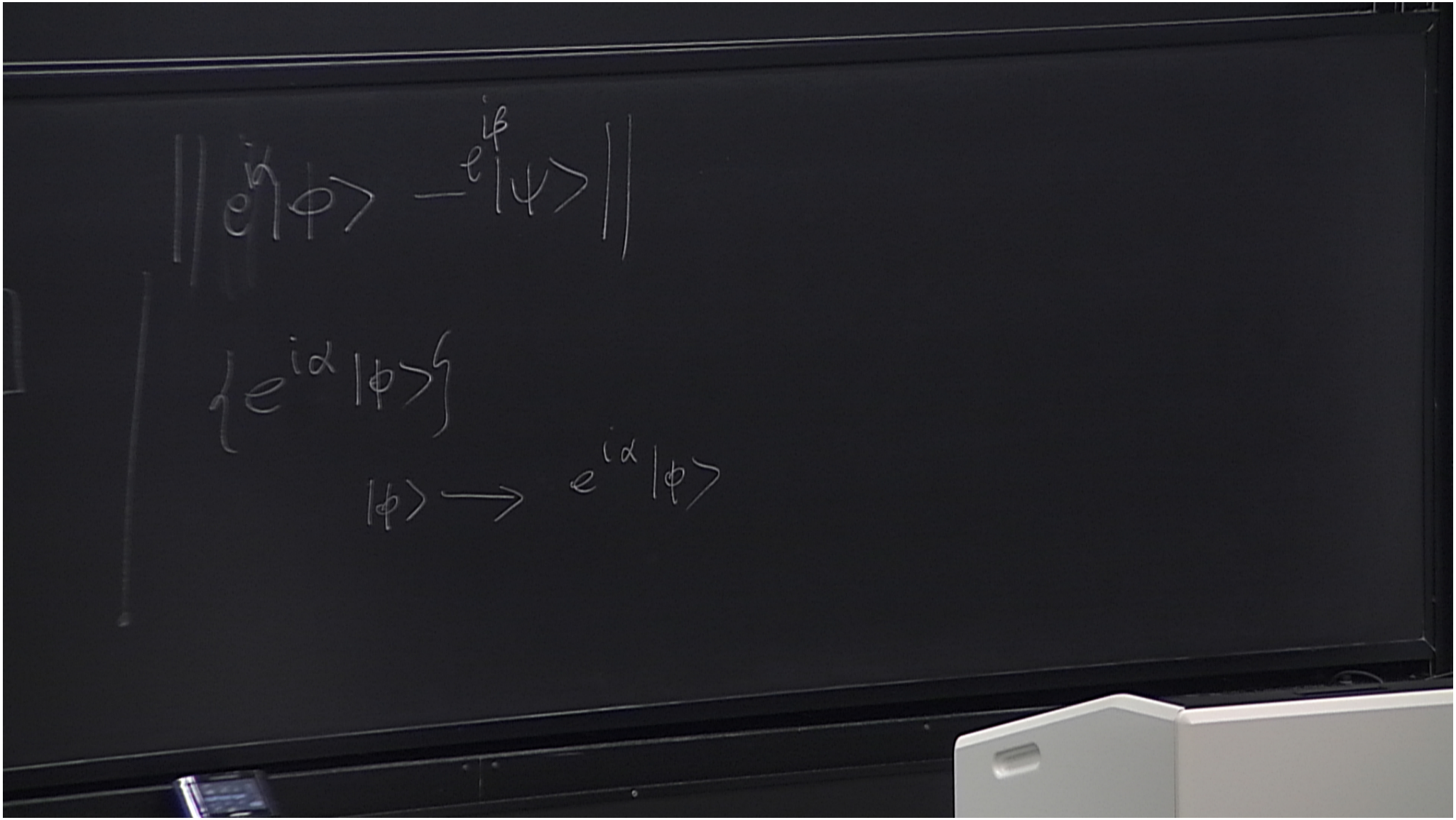

$F \simeq 1 \quad \cos^{-1} F \simeq 2(1-F) = d_{FS}$
$D(\rho,\sigma) = \frac{1}{2} \mathrm{Tr} \|\rho-\sigma\|_1$
$\|A\| = \sqrt{A^\dagger A}$
ρ, σ are pure
$D(\rho,\sigma) = \sqrt{1-F^2}$
$\mathrm{Tr}[\phi_i^A \psi^{(1)}]\,\mathrm{Tr}[\phi_i^A \psi^{(2)}]$
$\sum_i |\langle \phi_i^A | \psi^{(1)} \rangle| =$
$|\langle \phi_i^A | \psi^{(2)} \rangle| = \cos^{-1} |\langle \psi^{(1)} | \psi^{(2)} \rangle|$
F
quantum Fidelity

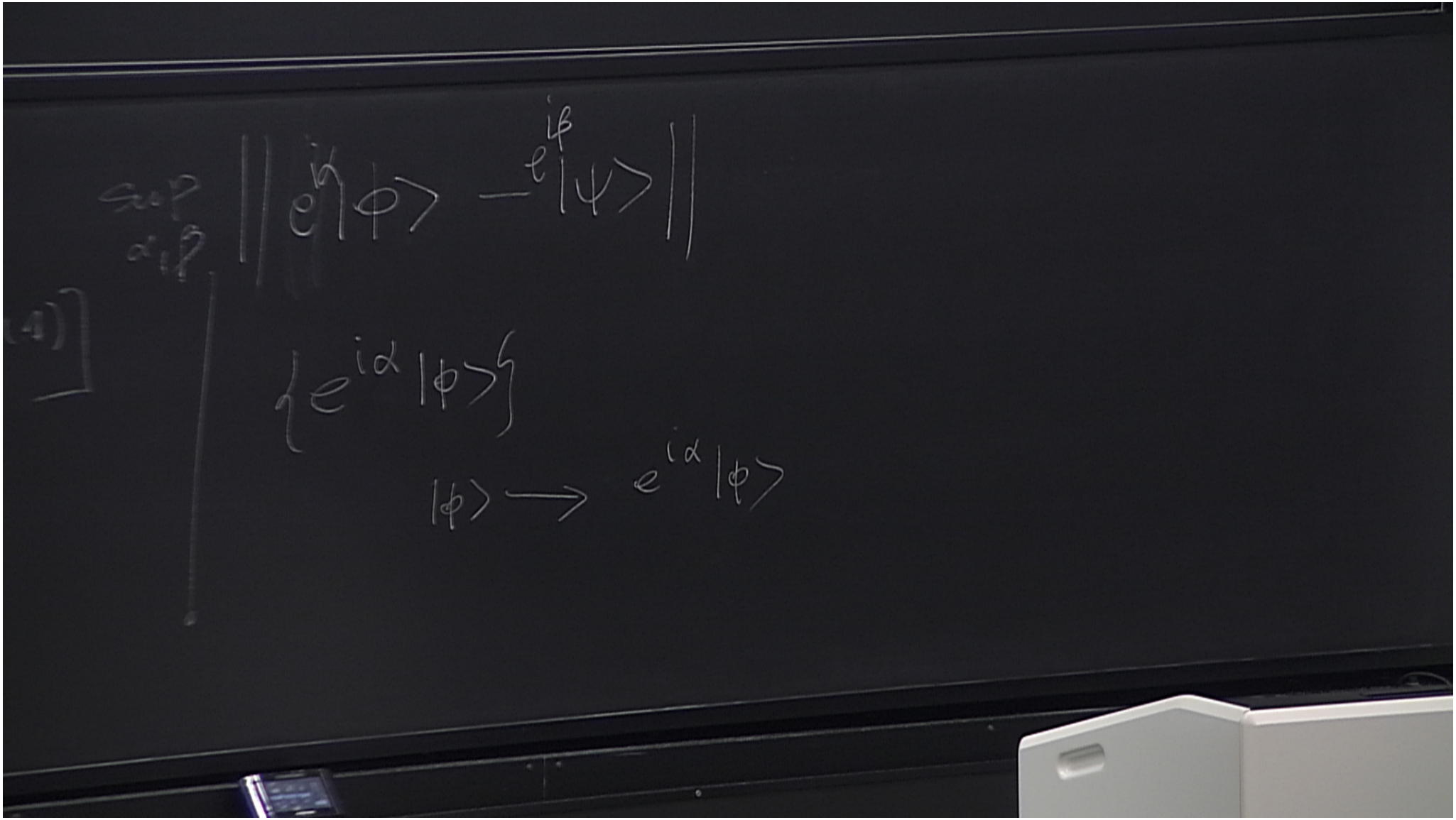

F ≃ 1
cos⁻¹ F ≃ 2(1−F) = d_FS
ρ, σ are pure
D(ρ,σ) = √(1−F²)
D(ρ,σ) = ½ Tr ‖ρ−σ‖₁
‖A‖ = √(A†A)
D(ρ,σ) = max_P [Tr[ρP] − Tr[σP]]
quantum Fidelity
F

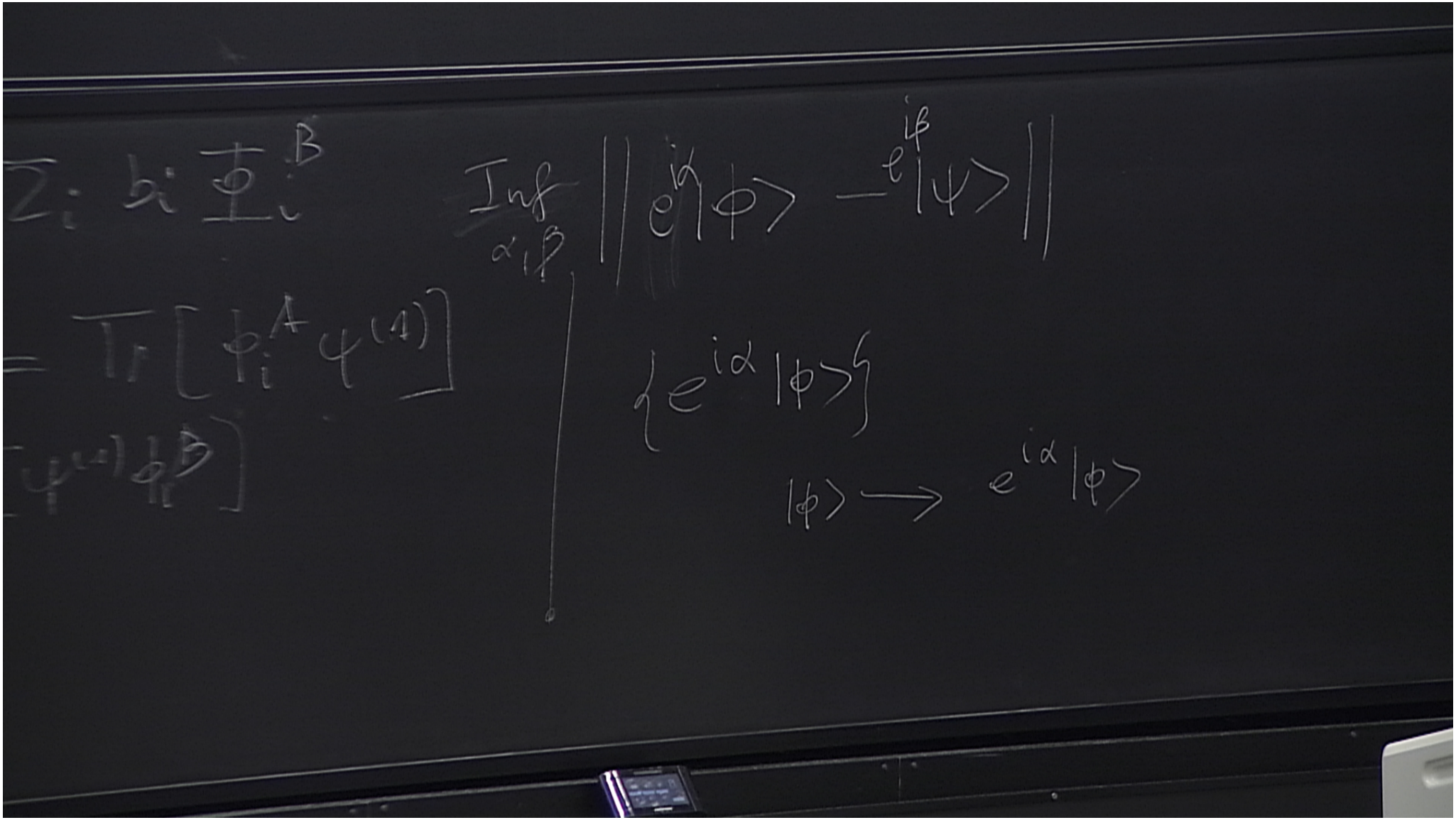

$\| |\psi\rangle - |\chi\rangle \|$


$\| |\phi\rangle - |\psi\rangle \|$
$\{e^{i\alpha}|\phi\rangle\}$

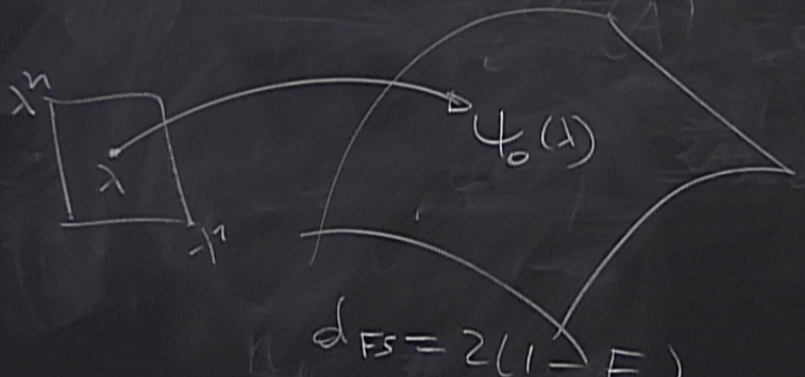

$\| e^{i\alpha}|\phi\rangle - e^{i\beta}|\psi\rangle \|$
$\{e^{i\alpha}|\phi\rangle\}$
$|\phi\rangle \longrightarrow e^{i\alpha}|\phi\rangle$


$\| e^{i\alpha}|\phi\rangle - e^{i\beta}|\psi\rangle \|$
$\{e^{i\alpha}|\phi\rangle\}$
$|\phi\rangle \longrightarrow e^{i\alpha}|\phi\rangle$


sup
α,β
$\| e^{i\alpha}|\phi\rangle - e^{i\beta}|\psi\rangle \|$
$\{e^{i\alpha}|\phi\rangle\}$
$|\phi\rangle \longrightarrow e^{i\alpha}|\phi\rangle$


$\sum_i \lambda_i \sigma_i^B$
$= \mathrm{Tr}[\phi_i^A \psi^{(1)}]$
$[\psi^{(1)} \phi_i^B]$
$\inf_{\alpha,\beta} \| e^{i\alpha}|\phi\rangle - e^{i\beta}|\psi\rangle \|$
$\{e^{i\alpha}|\phi\rangle\}$
$|\phi\rangle \longrightarrow e^{i\alpha}|\phi\rangle$

$H = H(\lambda)$ $\lambda = (\lambda^1, \ldots, \lambda^n)$



$F \simeq 1$ $\cos^{-1} F \simeq$

$\sum_i \mathrm{Tr}[\phi_i^\dagger \psi^{(n)}] \mathrm{Tr}[\phi_i^\dagger \psi^{(n)}]$ T

$\sup_{\mathrm{HAS}} \cos^{-1} \sum_i |\langle \phi_i^\dagger | \psi^{(n)} \rangle| =$

$|\langle \phi_i^\dagger | \psi^{(n)} \rangle| =$