Title: PSI 17/18 - Condensed Matter - Lecture 4

Date: Jan 08, 2018 11:30 AM

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Abstract:

Phases of Quantum Ising Model

$$H = -\sum_i \sigma_i^z \sigma_{i+1}^z - \lambda \sum_i \sigma_i^x$$

large λ — disordered or paramagnetic phase

$\langle \sigma_i^z \rangle = 0$

$\langle \sigma_i^z \sigma_j^z \rangle \sim e^{-|i-j|/\xi}$

$\xi = \Delta^{-1}$

non-symmetry breaking

Phases of Quantum Ising Model

$$H = -\sum_i \sigma_i^z \sigma_{i+1}^z - \lambda \sum_i \sigma_i^x \qquad \Delta = |1 - \lambda|$$

large λ — disordered or paramagnetic phase

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Phases of Quantum Ising Model

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large λ — disordered or paramagnetic phase — $\langle \sigma_i^z \rangle = 0$

$\lambda > 1$

$\langle \sigma_i^z \sigma_j^z \rangle \sim e^{-|i-j|/\xi}$

non-symmetry breaking

$\xi = \Delta^{-1}$

Phases of Quantum Ising Model

$$H = -\sum_i \sigma_i^z \sigma_{i+1}^z - \lambda \sum_i \sigma_i^x$$

$\Delta =$

large λ

$\lambda > 1$

disordered or paramagnetic phase

$\langle \sigma_i^z \rangle = 0$

$\langle \sigma_i^z \sigma_j^z \rangle \sim e^{-|i-j|/\xi}$

non-symmetry breaking

$\xi = \Delta^{-1}$

small

$\lambda < 1$

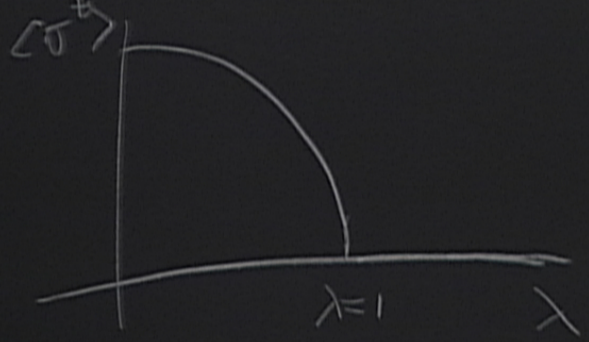
ferromagnetic; symmetry-breaking phase

$\Delta = |1 - \lambda|$

$\lambda < 1$ $\qquad$ $\lambda > 1$

symm breaking $\qquad$ 1 $\qquad$ paramagnet

$\langle \sigma^z \rangle \neq 0$



$\Delta \approx |1-\lambda|$

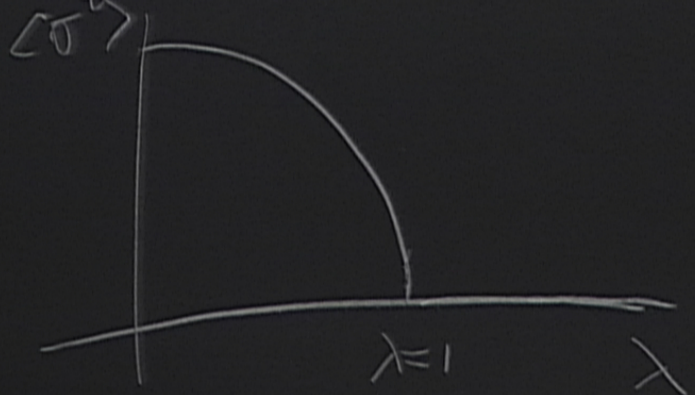
$\lambda < 1$ $\qquad$ $\lambda > 1$

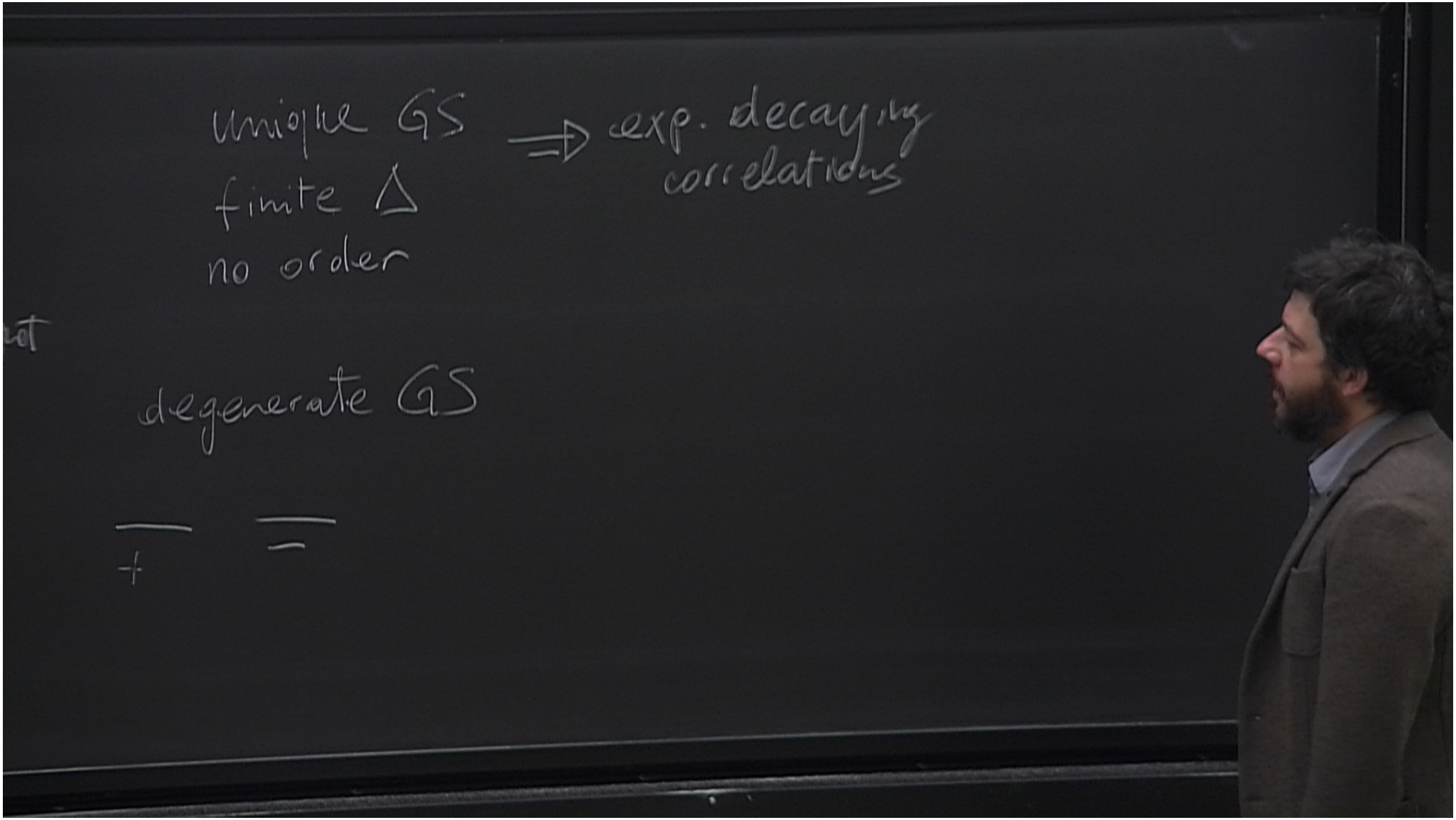
symm breaking $\qquad$ 1 $\qquad$ paramagnet

$\langle \sigma^z \rangle \neq 0$

$-|i-j|/\xi$

$= \Delta^{-1}$

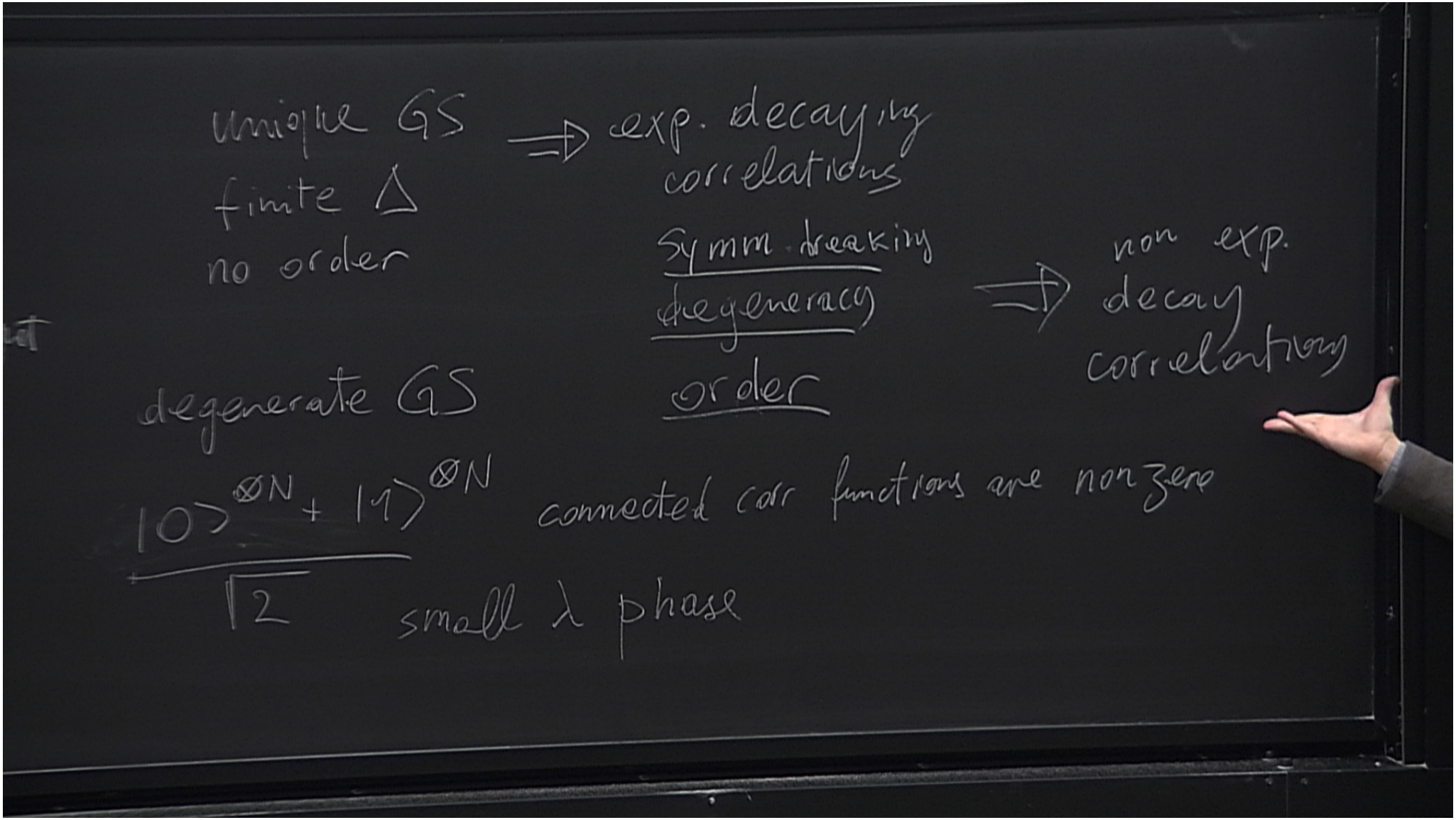


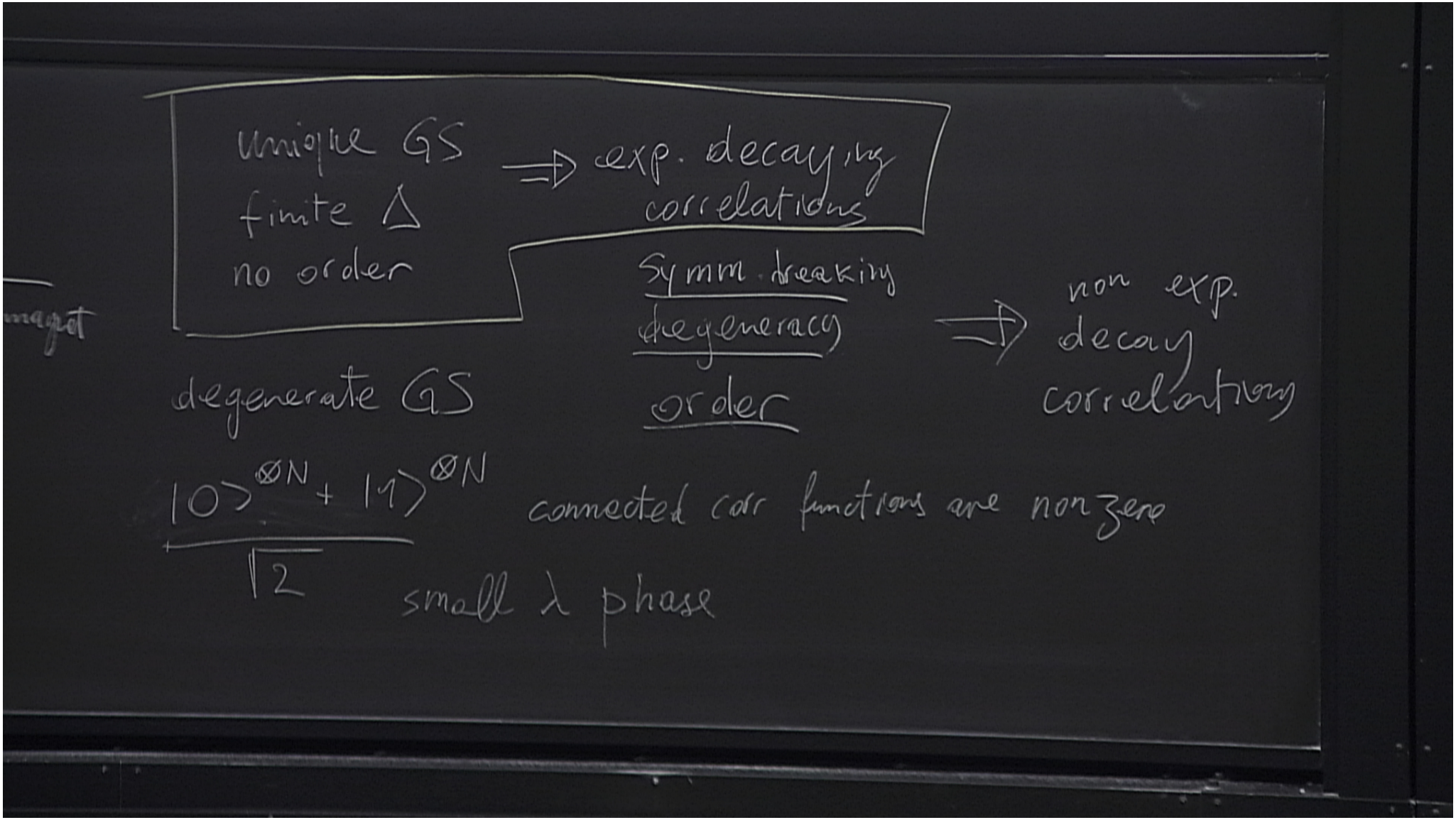

Unique GS
finite Δ
no order
⇒ exp. decaying correlations
degenerate GS

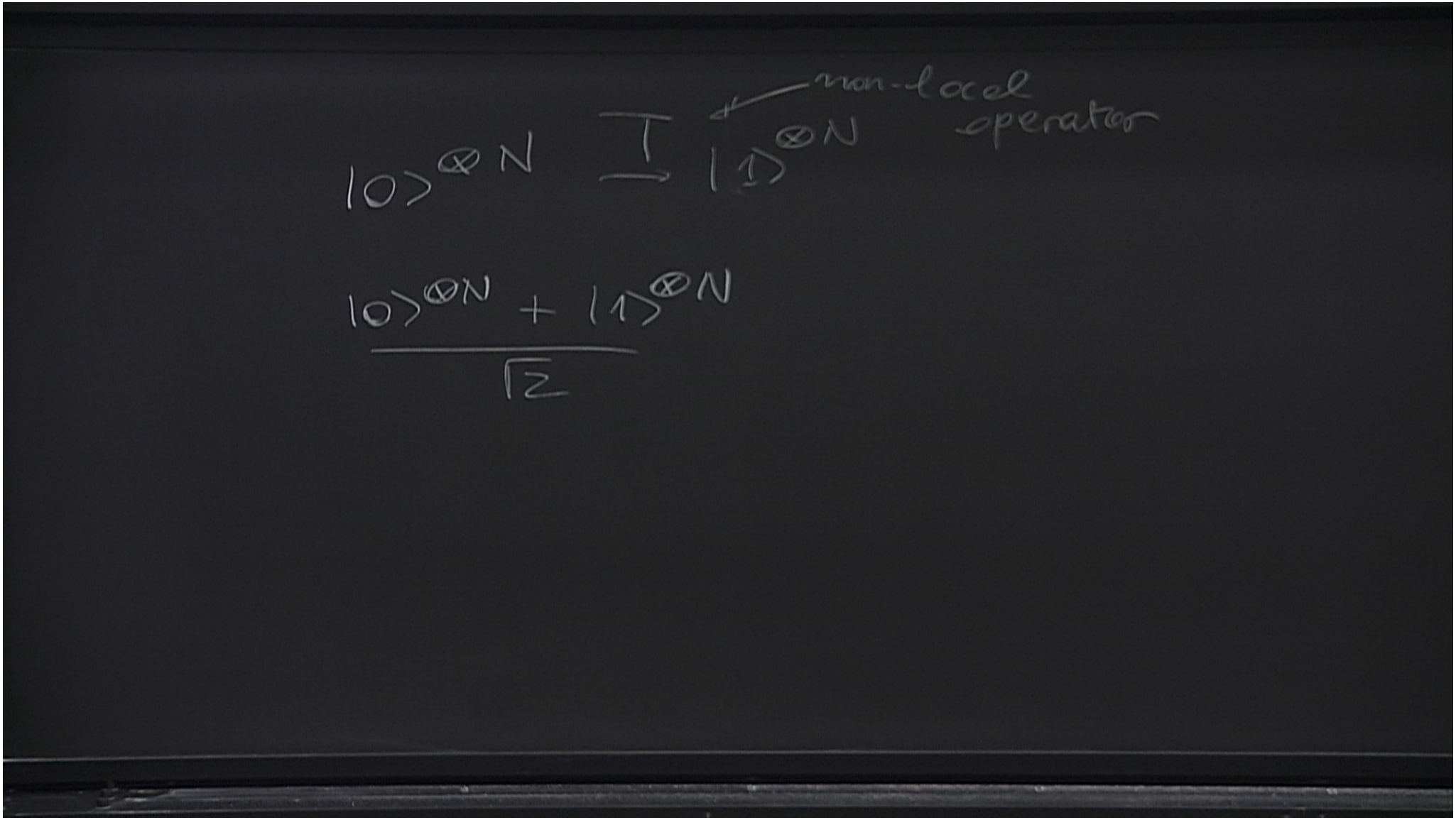
unique GS ⟹ exp. decaying correlations
finite Δ
no order

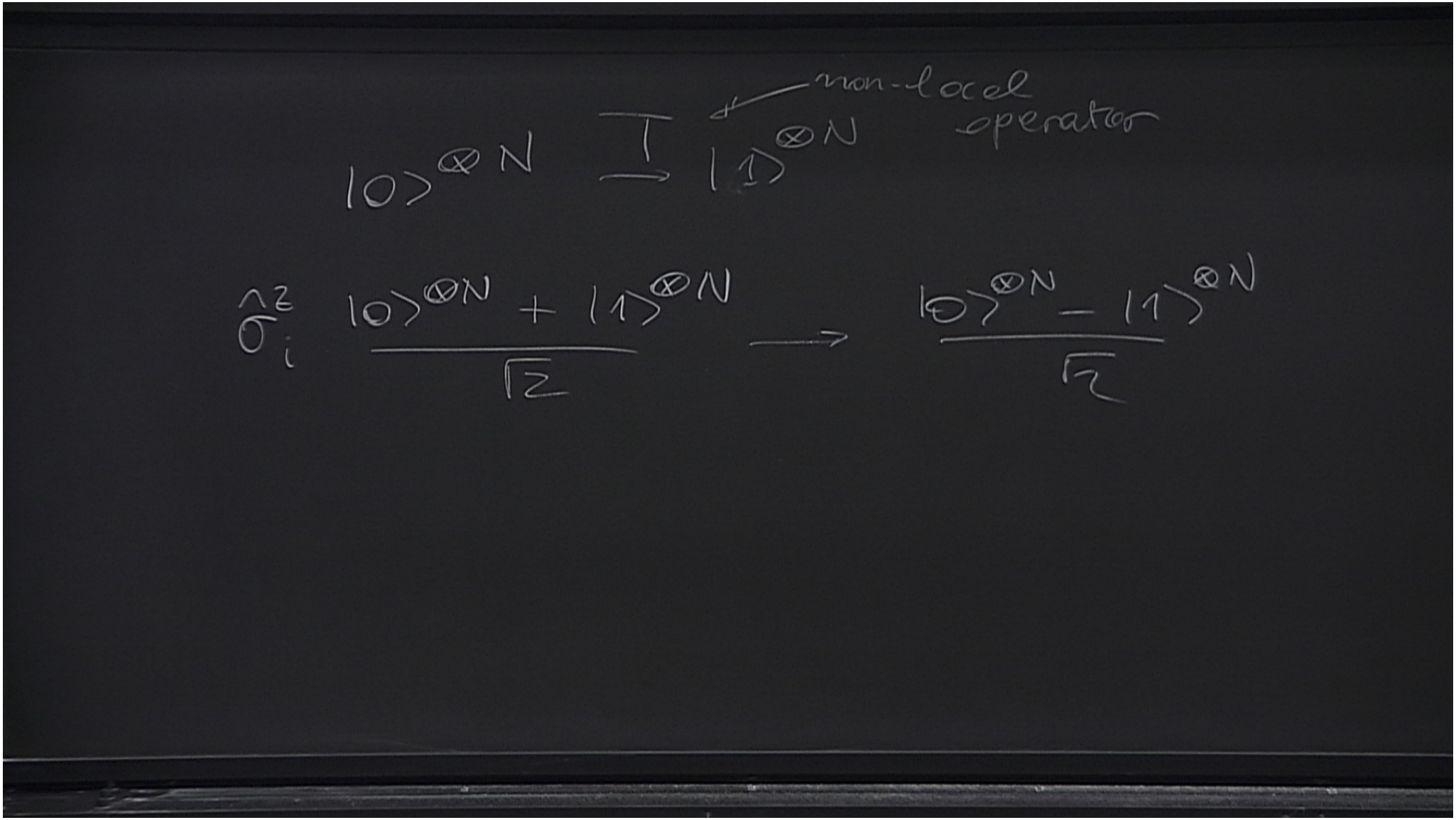
degenerate GS

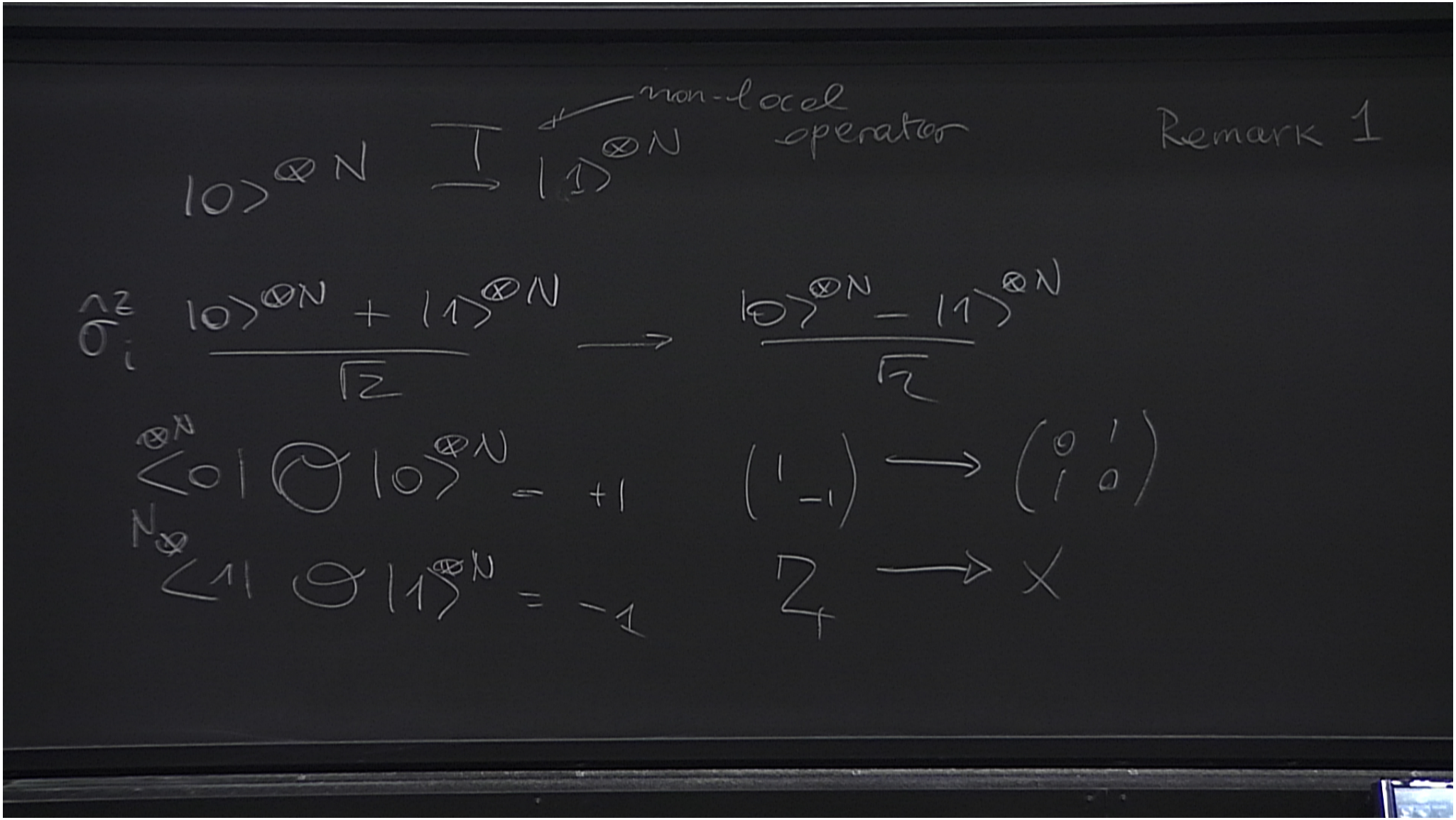
$\frac{|0\rangle^{\otimes N} + |1\rangle^{\otimes N}}{\sqrt{2}}$ connected corr functions are nonzero

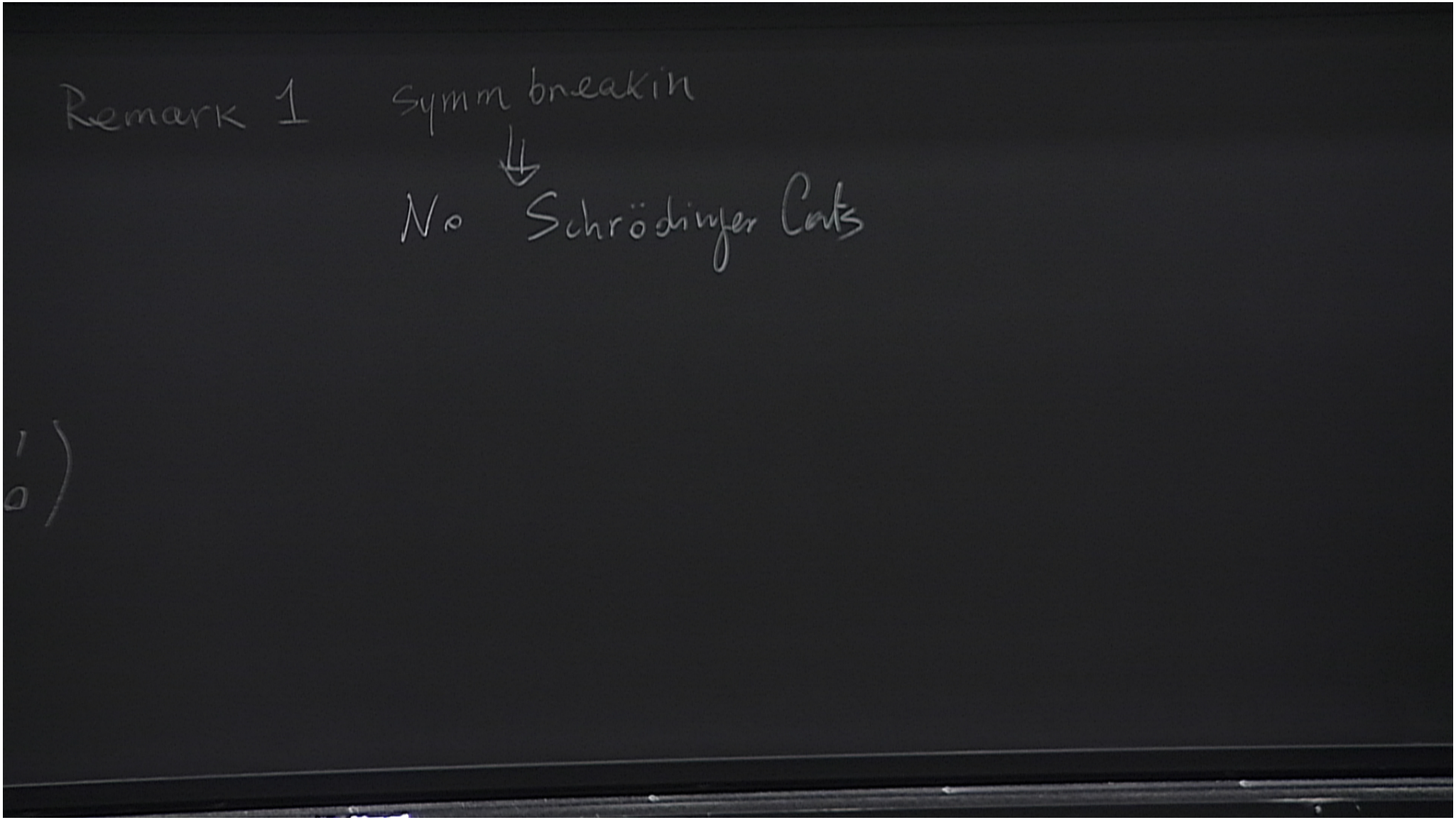

unique GS
finite Δ
no order
⇒ exp. decaying correlations
symm. breaking
degeneracy
order
⇒ non exp. decay correlations
degenerate GS
$\frac{|0\rangle^{\otimes N} + |1\rangle^{\otimes N}}{\sqrt{2}}$
connected corr functions are non zero
small λ phase

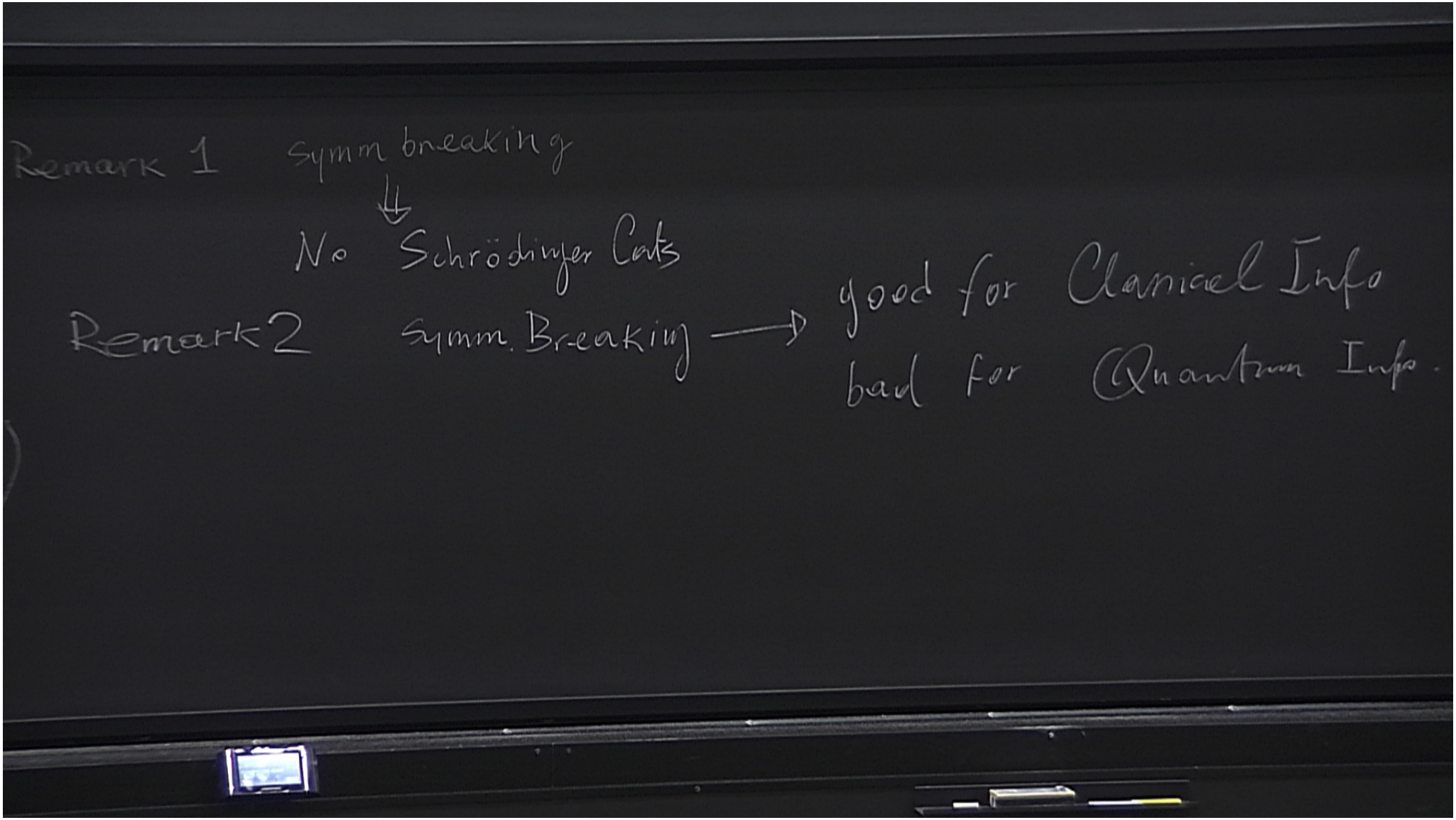

Unique GS
finite Δ
no order
⇒ exp. decaying correlations
Symm. breaking
degeneracy
order
⇒ non exp. decay correlations
magnet
degenerate GS
$\frac{|0\rangle^{\otimes N} + |1\rangle^{\otimes N}}{\sqrt{2}}$
connected corr functions are non zero
small λ phase


non-local
operator
$|0\rangle^{\otimes N}$
I
$|1\rangle^{\otimes N}$
$\frac{|0\rangle^{\otimes N} + |1\rangle^{\otimes N}}{\sqrt{2}}$


$|0\rangle^{\otimes N}$ $\xrightarrow{T}$ $|1\rangle^{\otimes N}$
non-local operator
$\hat{\sigma}_i^z$ $\frac{|0\rangle^{\otimes N} + |1\rangle^{\otimes N}}{\sqrt{2}}$ $\longrightarrow$ $\frac{|0\rangle^{\otimes N} - |1\rangle^{\otimes N}}{\sqrt{2}}$


non-local operator
Remark 1
$|0\rangle^{\otimes N} \xrightarrow{I} |1\rangle^{\otimes N}$
$\hat{\sigma}_i^z \quad \frac{|0\rangle^{\otimes N} + |1\rangle^{\otimes N}}{\sqrt{2}} \longrightarrow \frac{|0\rangle^{\otimes N} - |1\rangle^{\otimes N}}{\sqrt{2}}$
$\langle 0|^{\otimes N} \bigcirc |0\rangle^{\otimes N} = +1$
$\langle 1|^{\otimes N} \bigcirc |1\rangle^{\otimes N} = -1$
$\begin{pmatrix} 1 & \\ & -1 \end{pmatrix} \longrightarrow \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
$Z \longrightarrow X$

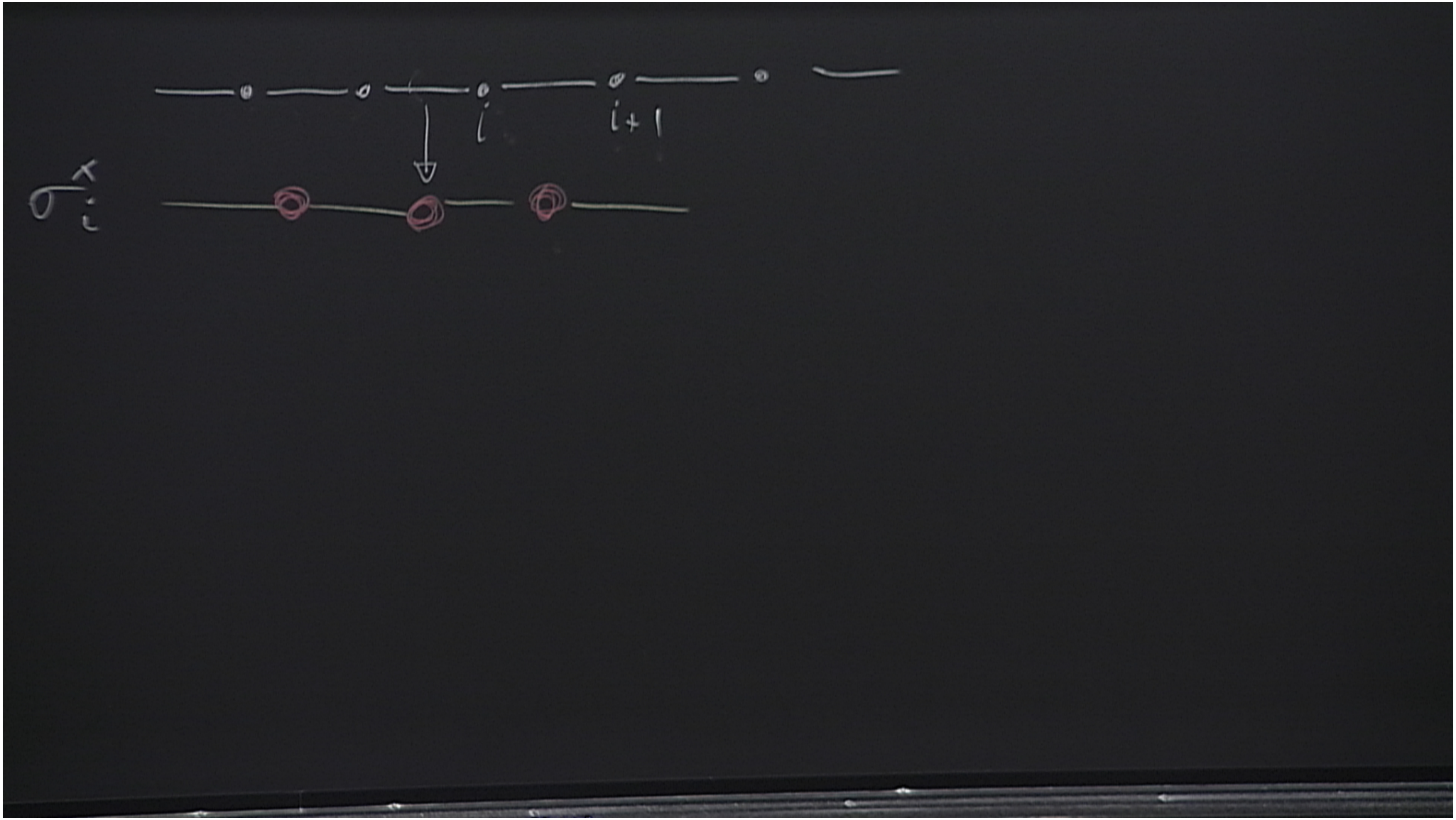

Remark 1
Symm breakin
No Schrödinger Cats

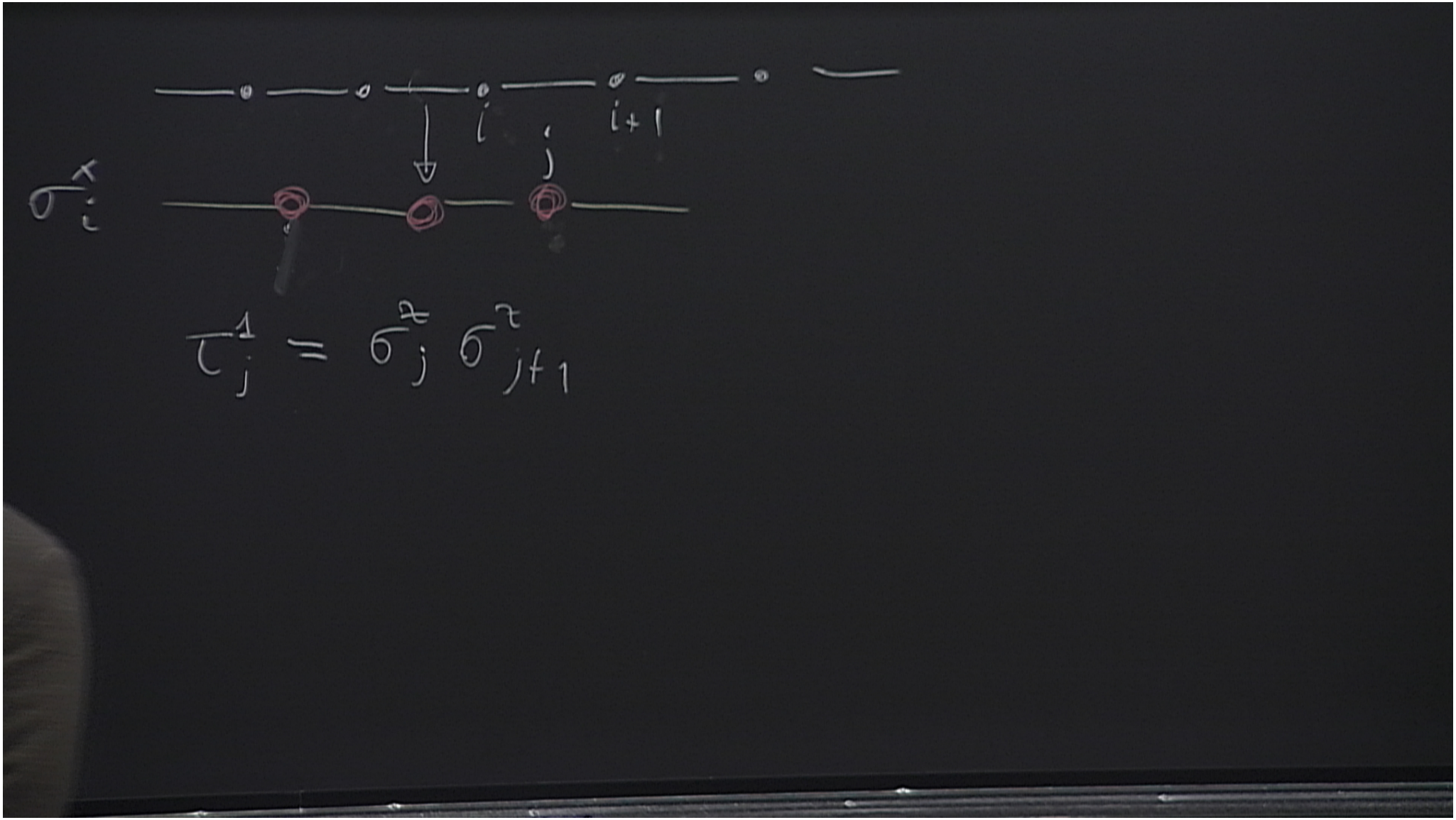

Remark 1 symm breaking
No Schrödinger Cats
Remark 2 symm. Breaking
good for Clanical Info
bad for Quantum Info.

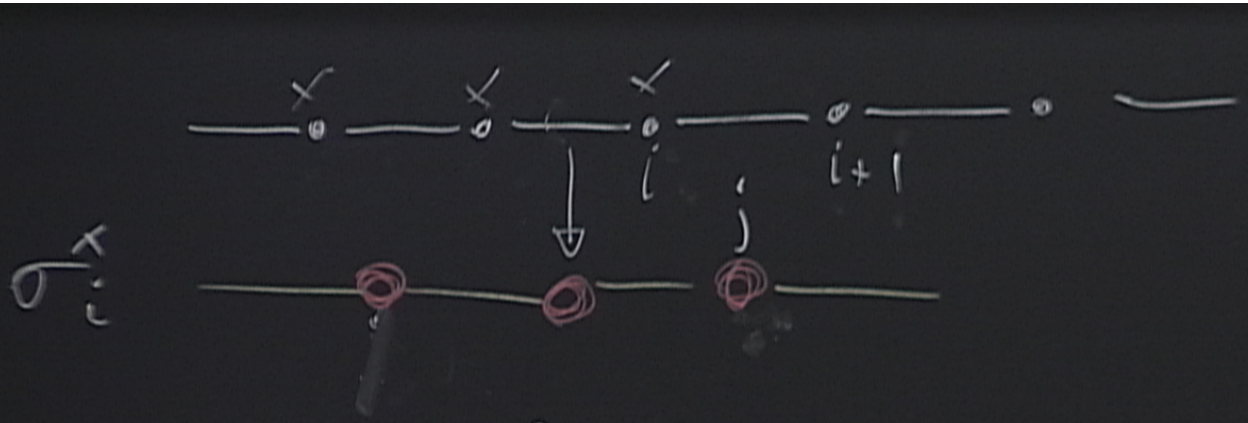
$\lambda_c = 1$

Duality

$$H = -\sum_i \sigma^z_i \sigma^z_{i+1} - \lambda \sum_i \sigma^x_i$$

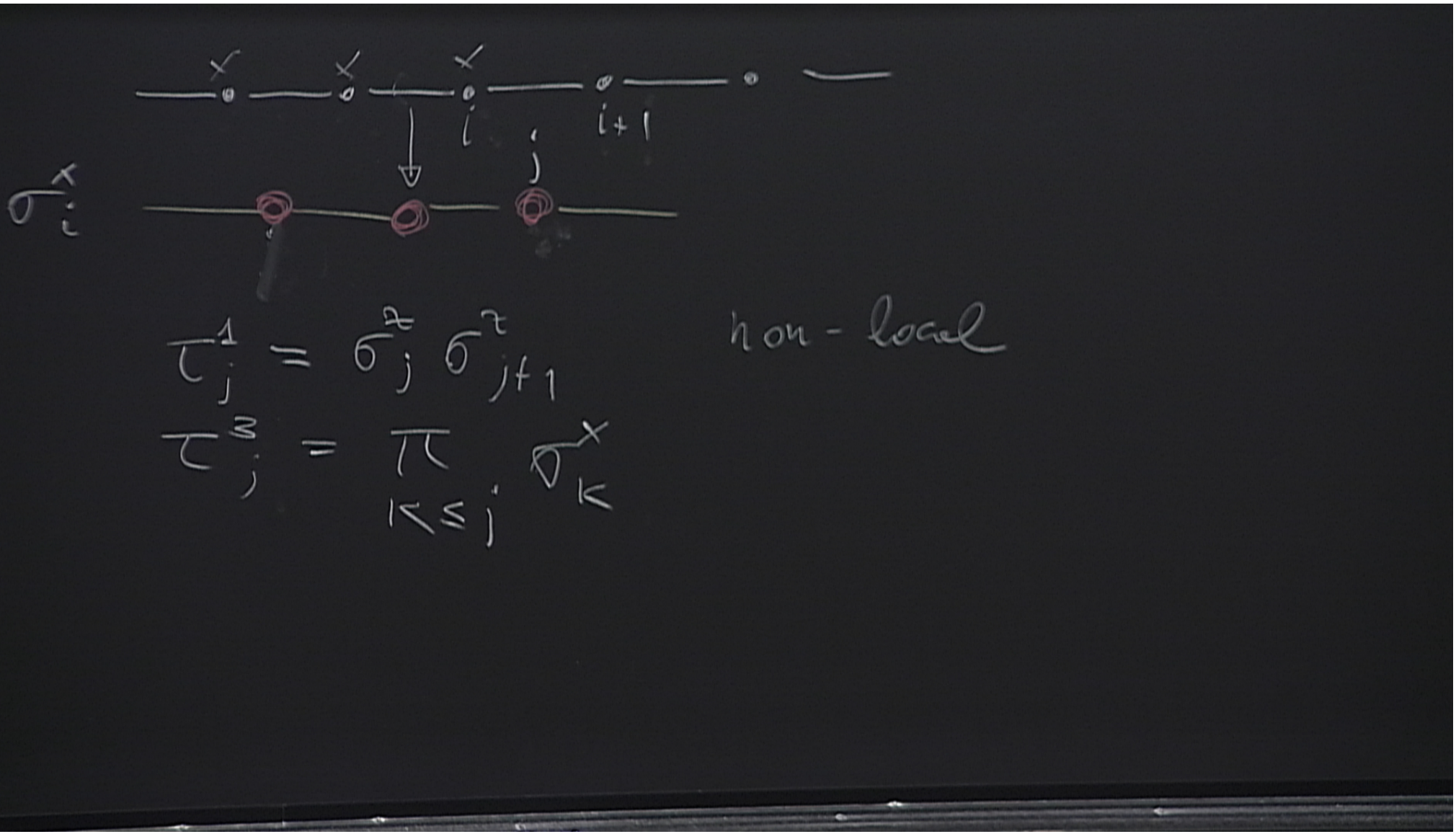

i
i+1
σ_i


i
i+1
j
σ_i^x
$\tau_j^1 = \sigma_j^z \sigma_{j+1}^z$



$$\tau_j^1 = \sigma_j^2 \sigma_{j+1}^2$$

$$\tau_j^3 = \prod_{k \le j} \sigma_k^2$$


i
i+1
j
σ^x_i
$\tau^1_j = \sigma^z_j \sigma^z_{j+1}$
$\tau^3_j = \prod_{k \le j} \sigma^x_k$
non-local

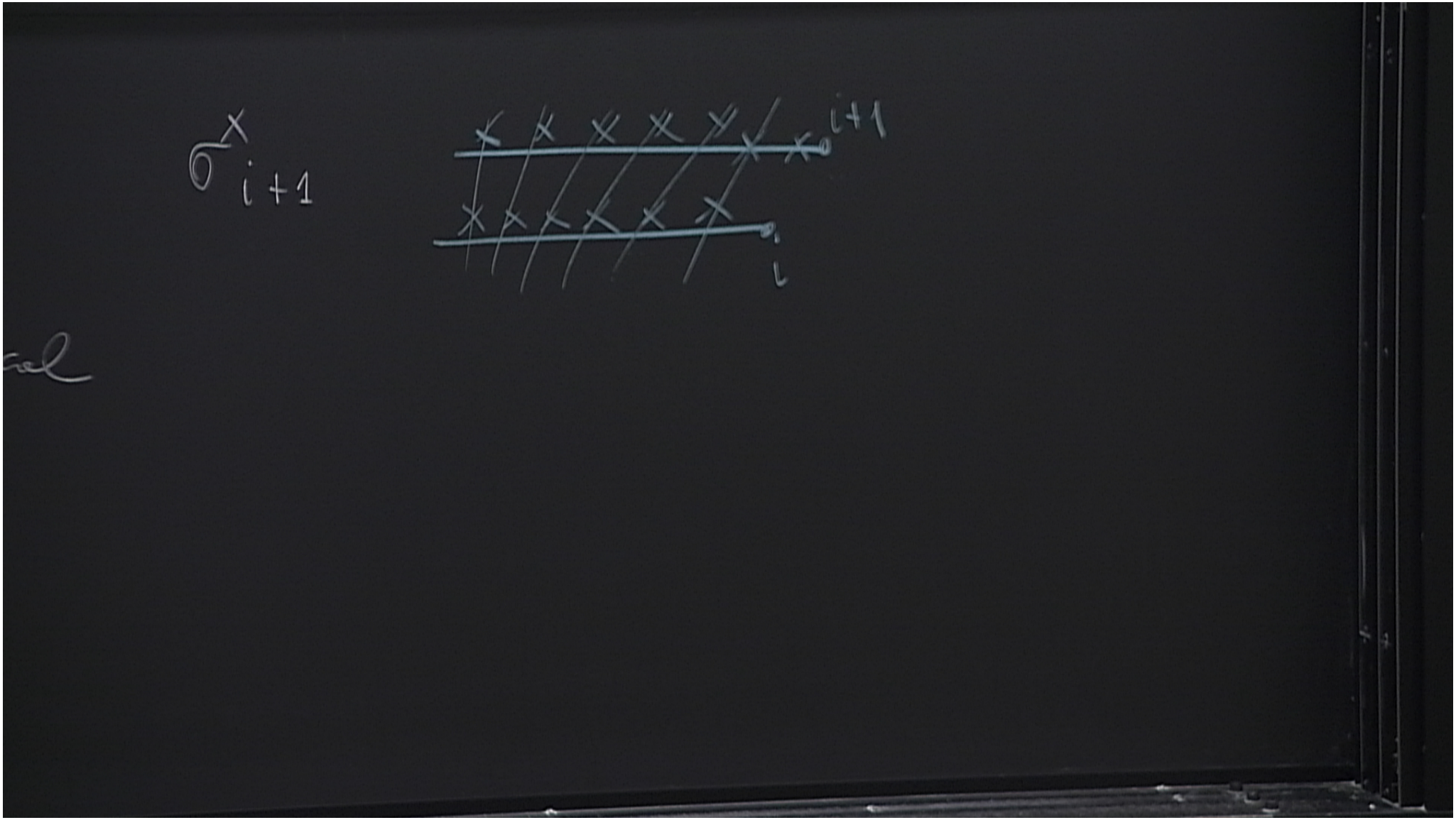
$\lambda_c = 1$

Duality

$$H_\sigma = -\sum_i \sigma^z_i \sigma^z_{i+1} - \lambda \sum_i \sigma^x_i$$

$\downarrow$ duality

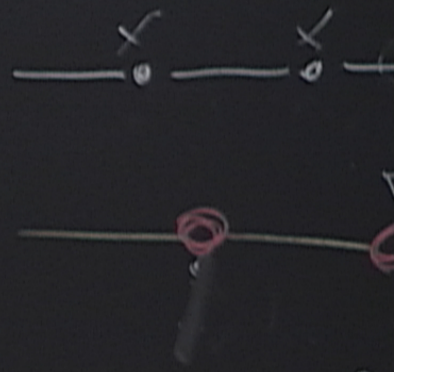
$$H_\tau = -\sum_j \tau^1_j - \lambda$$


σ_{i+1}
i+1
i

$c = 1$

Duality

$$H_\sigma = -\sum_i \sigma^z_i \sigma^z_{i+1} - \lambda \sum_i \sigma^x_i$$



$\downarrow$ duality

$$\tau^1_j = \sigma^z$$

$$\tau^3_j = \tau$$

$$H_\tau = -\sum_j \tau^1_j - \lambda \sum_j \tau^3_j \tau^3_{j+1}$$

$c = 1$

Duality

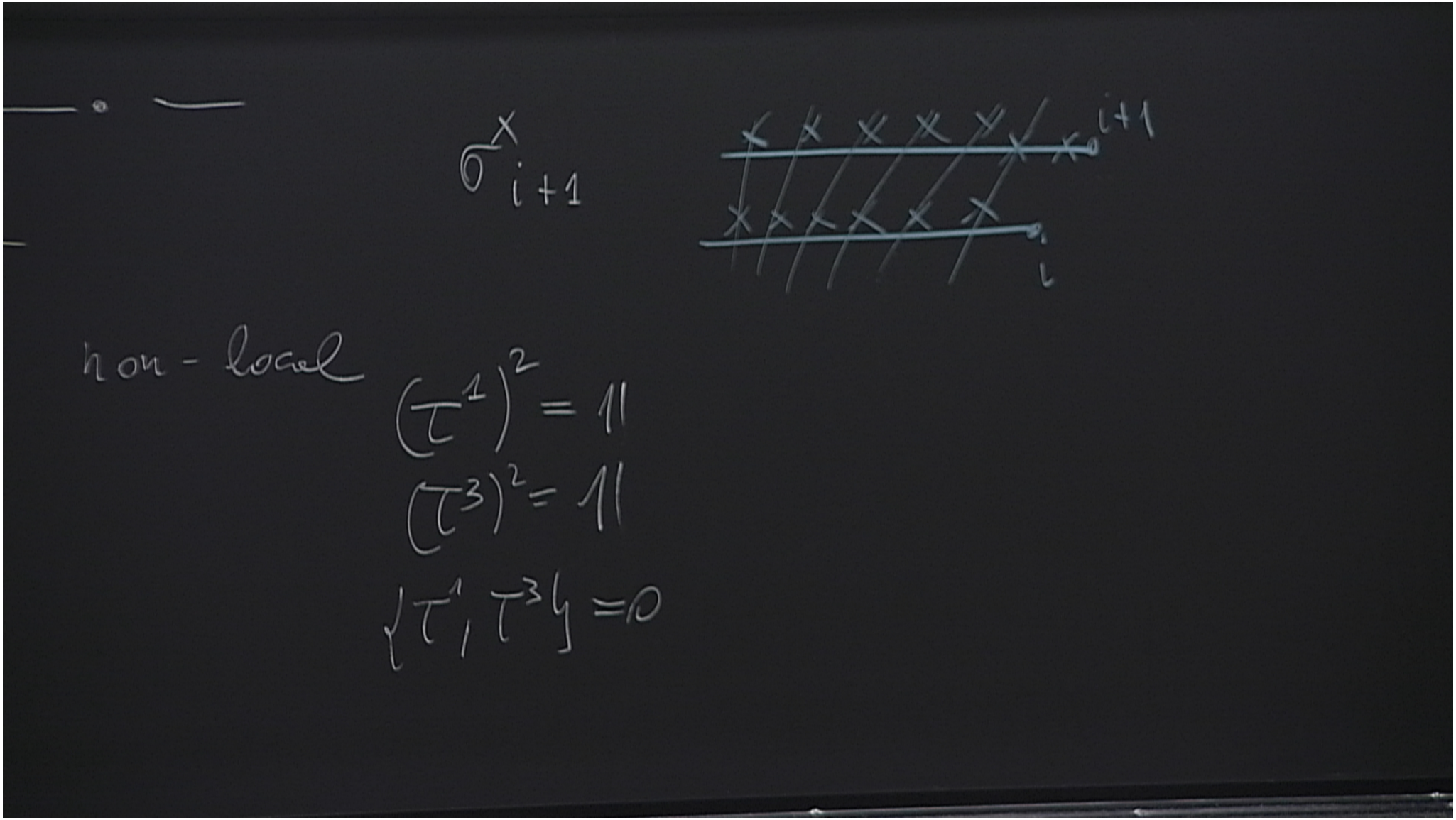
$$H_\sigma = -\sum_i \sigma^z_i \sigma^z_{i+1} - \lambda \sum_i \sigma^x_i$$

$\downarrow$ duality

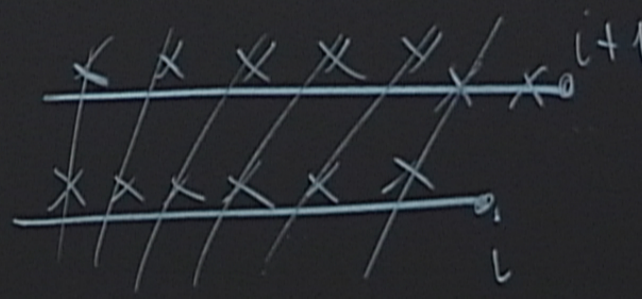
$$\tau^1_j = \sigma^z_j$$

$$\tau^3_j = \prod_{i \le j} \sigma^x_i$$

$$H_\tau = \lambda \left(-\frac{1}{\lambda} \sum_j \tau^1_j - \sum_j \tau^3_j \tau^3_{j+1} \right)$$


σ^x_{i+1}
i+1
i
non-local
$(\tau^1)^2 = 11$
$(\tau^3)^2 = 11$
$\{\tau^1, \tau^3\} = 0$

σ^x_{i+1}



non-local

$(\tau^1)^2 = 11$

$(\tau^3)^2 = 11$

$\{\tau^1, \tau^3\} = 0$



$\lambda_c = 1$

Duality



$$H_0 = -\sum_i \sigma^z_i \sigma^z_{i+1} - \lambda \sum_i \sigma^x_i$$



$$\tau^1_j = \prod_{i \le j} \sigma^x_i$$

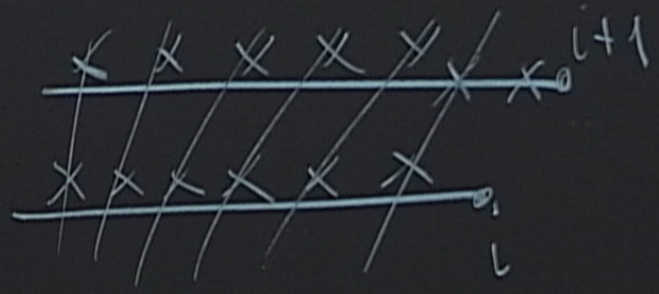
$$\tau^3_j =$$

$$\downarrow \text{duality}$$

$$H_t = \lambda \left(-\sum_j \frac{1}{\lambda} \tau^1_j - \sum_j \tau^3_j \tau^3_{j+1} \right)$$

$$= \lambda H_0 \left(\frac{1}{\lambda} \right)$$

$\sigma_{i+1}^{\times}$



local

$(\tau^1)^2 = 1\!\!1$

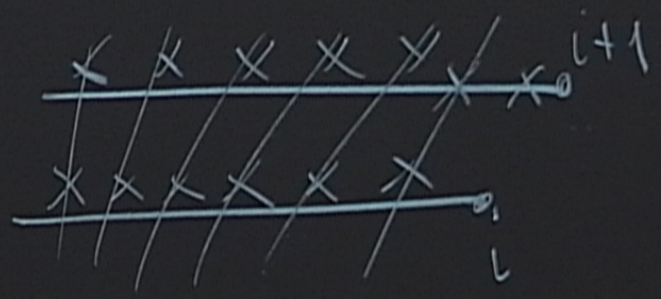
$(\tau^3)^2 = 1\!\!1$

$\{\tau^1, \tau^3\} = 0$

$\times \quad \times \quad \vee \quad \times \quad (\tau^1 \tau^3) = z$

$E(\lambda) = \lambda E\left(\frac{1}{\lambda}\right)$

$\sigma^{\times}_{i+1}$

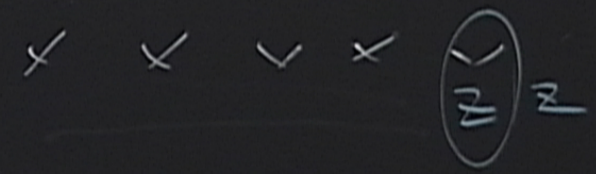


local

$(\tau^1)^2 = 11$

$(\tau^3)^2 = 11$

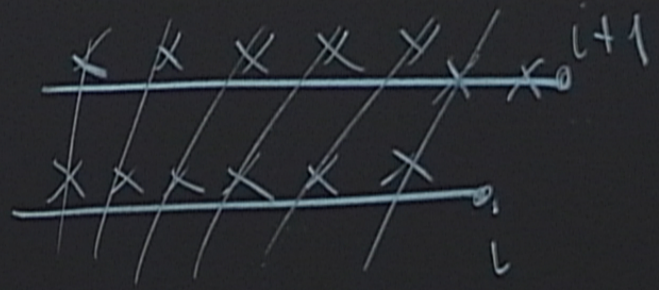
$\{\tau^1, \tau^3\} = 0$



$E(\lambda) = \lambda E(\frac{1}{\lambda})$

$\Delta E(\lambda) = \lambda \Delta E(\frac{1}{\lambda})$

$\sigma^{\times}_{i+1}$



local

$(\tau^1)^2 = 1\!\!1$

$(\tau^3)^2 = 1\!\!1$

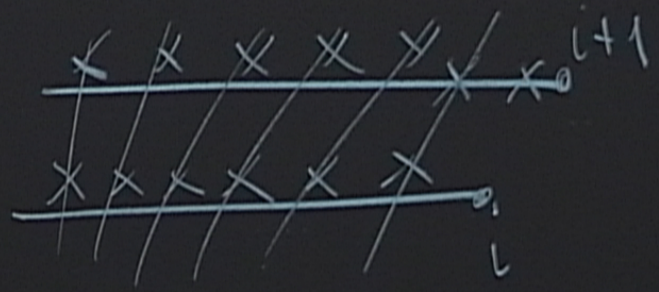
$\{\tau^1, \tau^3\} = 0$

$\times \quad \times \quad \vee \quad \times \quad \overset{\vee}{z} \quad z$

$E(\lambda) = \lambda E(\frac{1}{\lambda})$

$0 = \Delta E(\lambda) = \lambda \Delta E(\frac{1}{\lambda})$

$\sigma^{\times}_{i+1}$



local

$(\tau^1)^2 = 1\!\!1$

$(\tau^3)^2 = 1\!\!1$

$\{\tau^1, \tau^3\} = 0$

$\times \quad \times \quad \vee \quad \times \quad \overset{\vee}{z} \quad z$

$E(\lambda) = \lambda E(\frac{1}{\lambda})$

$0 = \Delta E(\lambda) = \lambda \Delta E(\frac{1}{\lambda})$

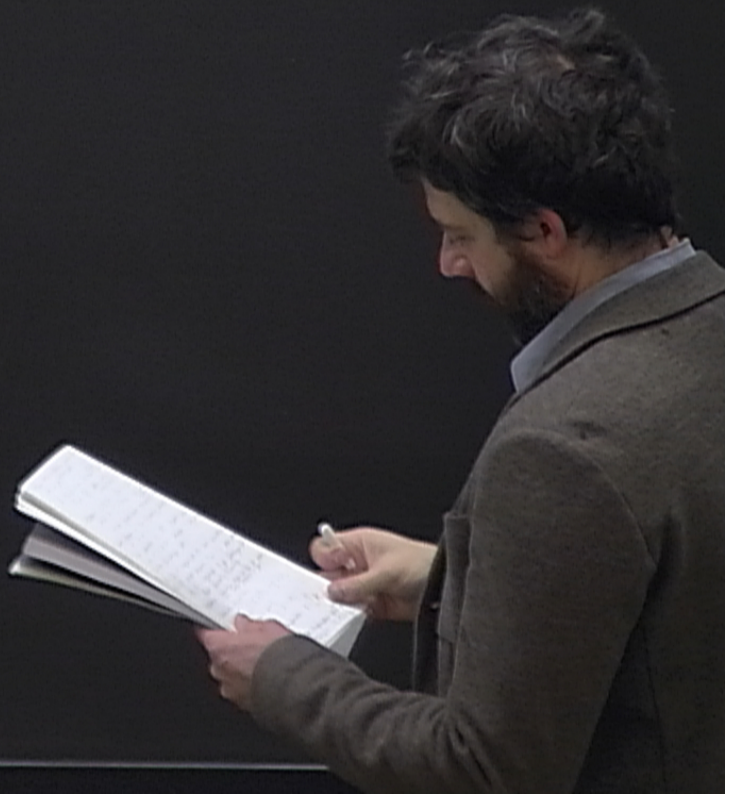
$\lambda = 1$

Diagonalising the Model

Integrable

$$H = \sum_{i=-M}^{M} \hat{\sigma}_i^x \hat{\sigma}_{i+1}^x$$

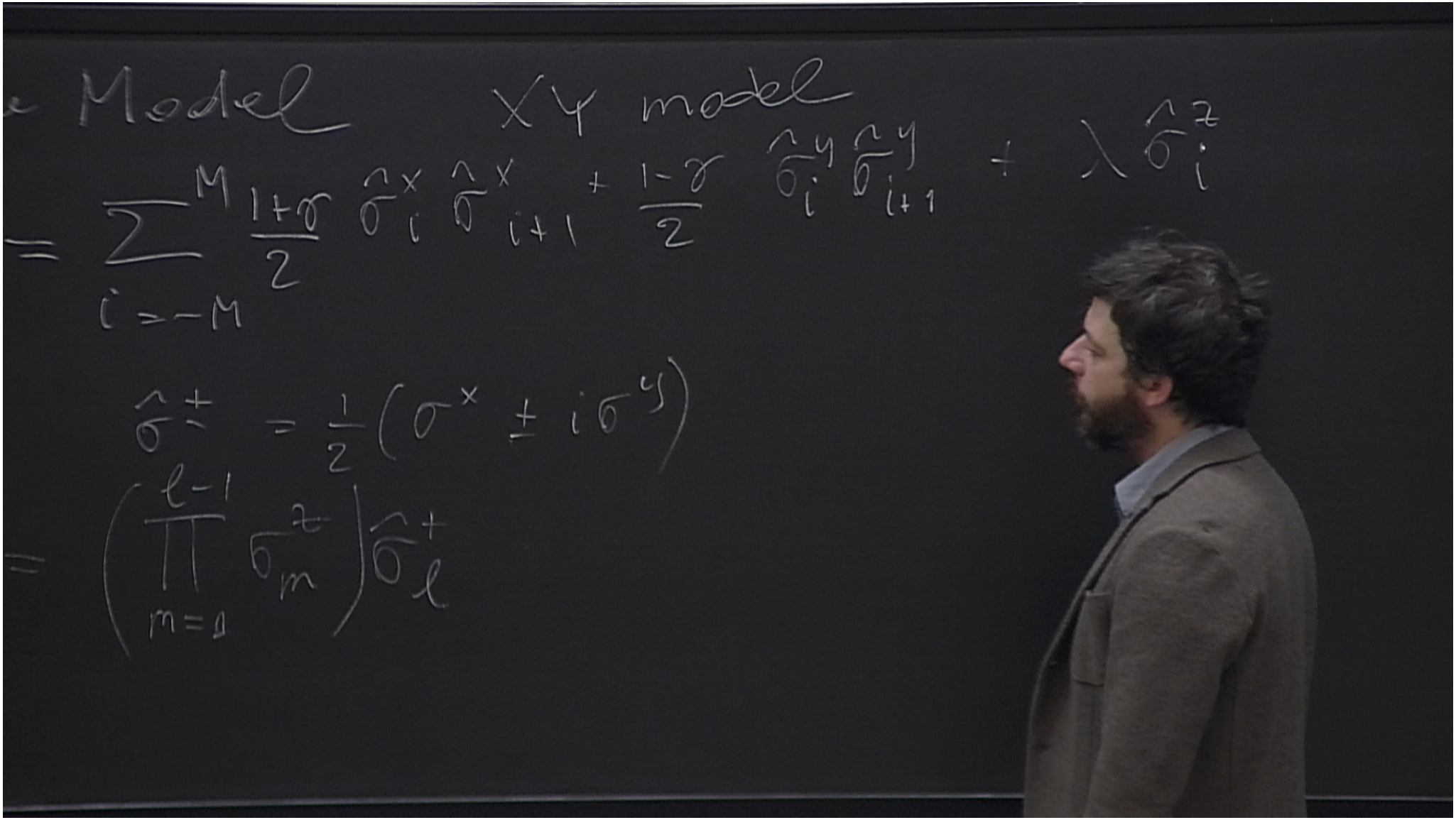
$$N = 2M+1$$



Model XY model

$$= \sum_{i=-M}^{M} \frac{1+\gamma}{2} \hat{\sigma}_i^x \hat{\sigma}_{i+1}^x + \frac{1-\gamma}{2} \hat{\sigma}_i^y \hat{\sigma}_{i+1}^y + \lambda \hat{\sigma}_i^z$$

$$\hat{\sigma}^{\pm} = \frac{1}{2}(\sigma^x \pm i\sigma^y)$$


Model
XY model
$= \sum_{i=-M}^{M} \frac{1+\gamma}{2} \hat{\sigma}_i^x \hat{\sigma}_{i+1}^x + \frac{1-\gamma}{2} \hat{\sigma}_i^y \hat{\sigma}_{i+1}^y + \lambda \hat{\sigma}_i^z$
$\hat{\sigma}^{\pm} = \frac{1}{2}(\sigma^x \pm i\sigma^y)$
$= \left(\prod_{m=1}^{l-1} \sigma_m^z \right) \hat{\sigma}_l^+$

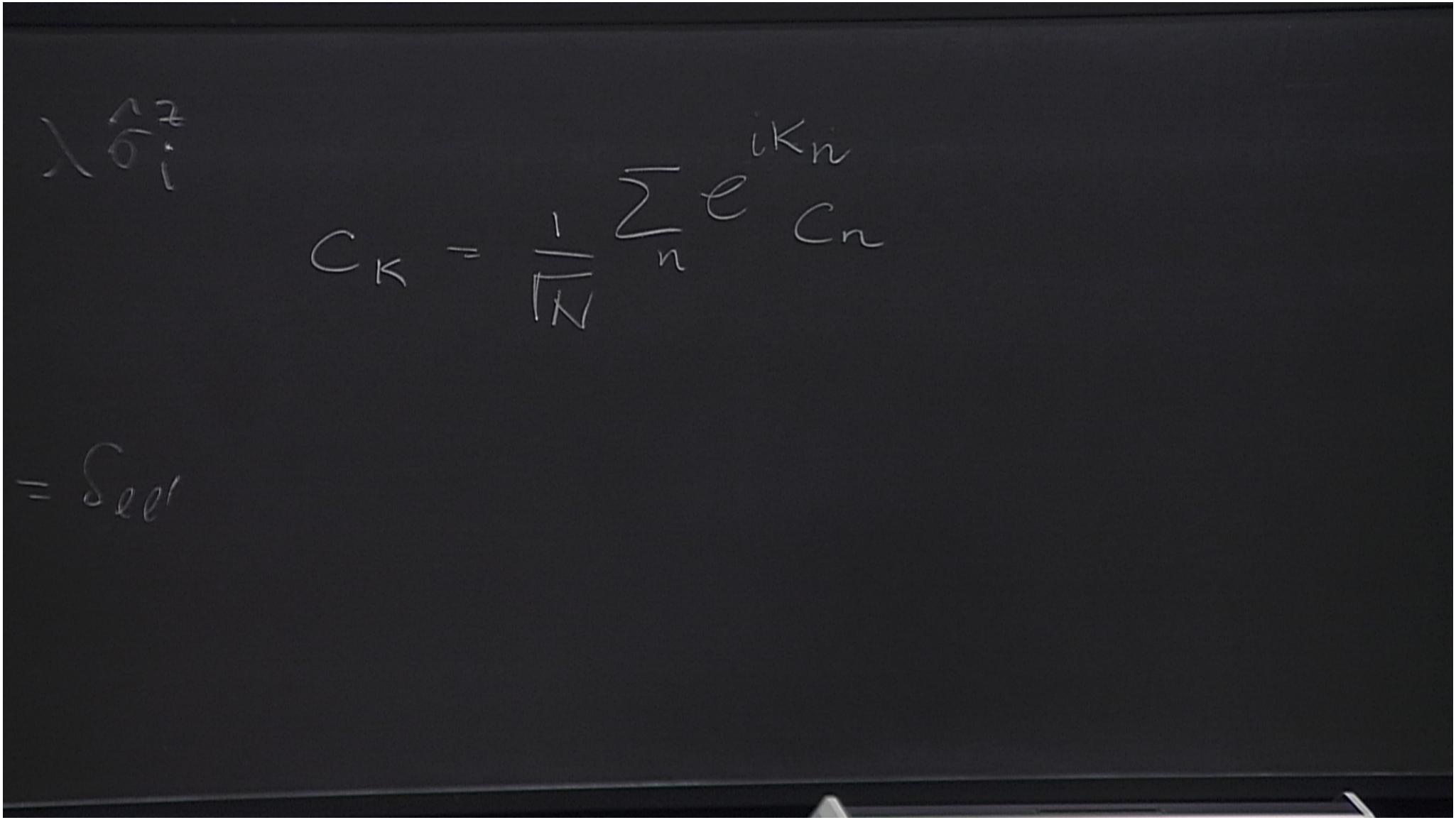
XY model

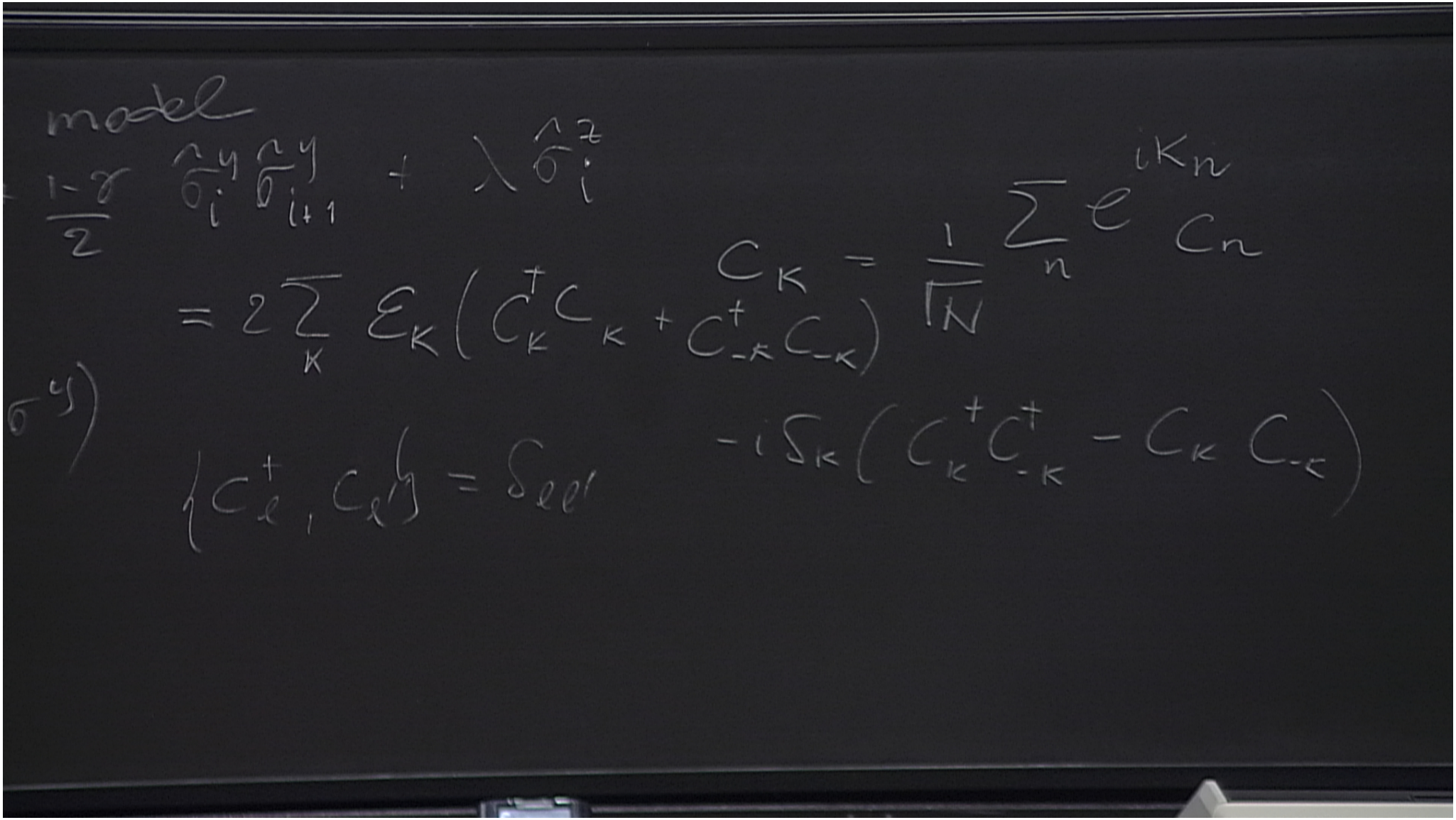
$$\hat{\sigma}_i^x \hat{\sigma}_{i+1}^x + \frac{1-\gamma}{2} \hat{\sigma}_i^y \hat{\sigma}_{i+1}^y + \lambda \hat{\sigma}_i^z$$

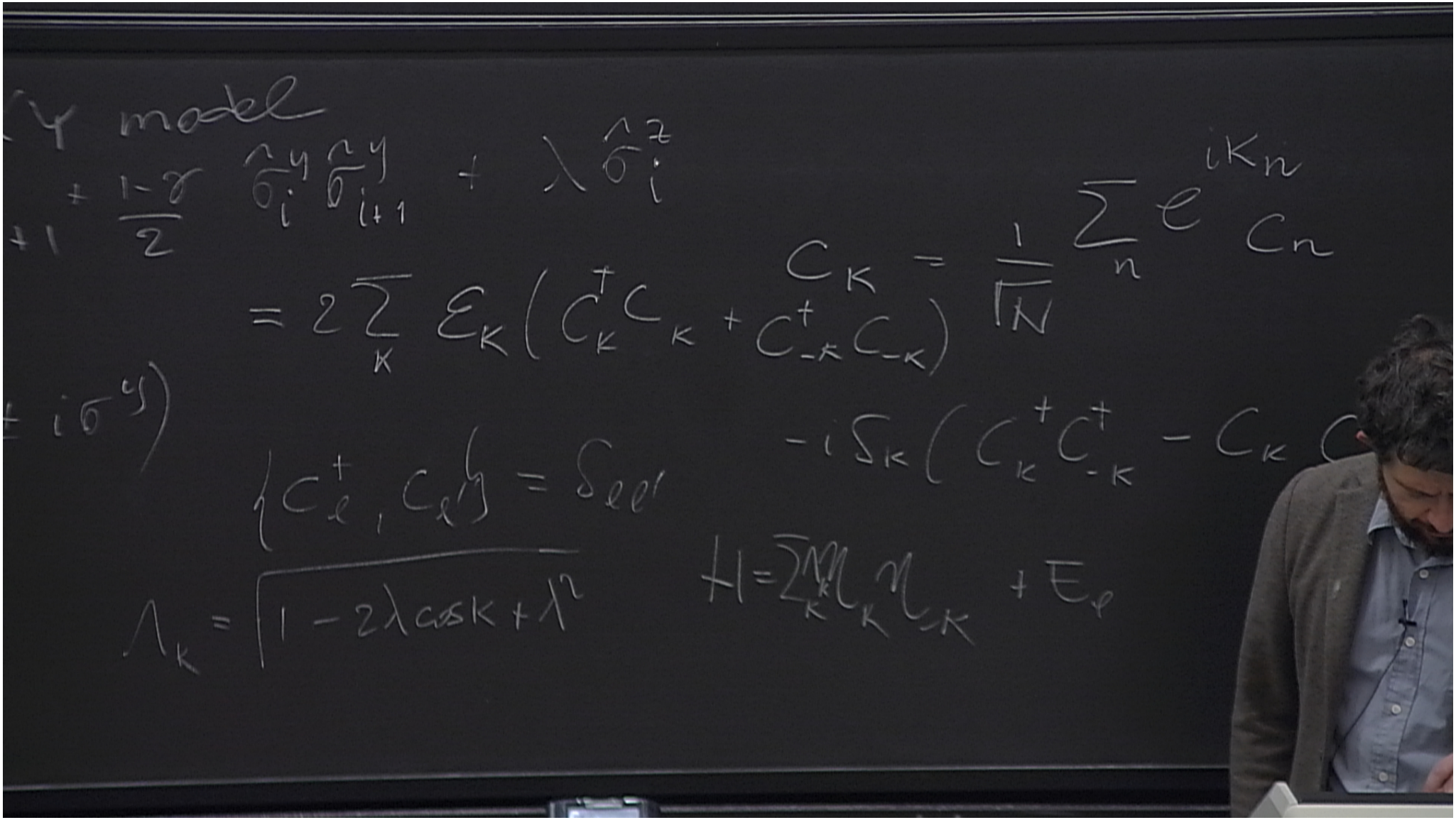
$$\frac{1}{2}(\sigma^x \pm i\sigma^y)$$

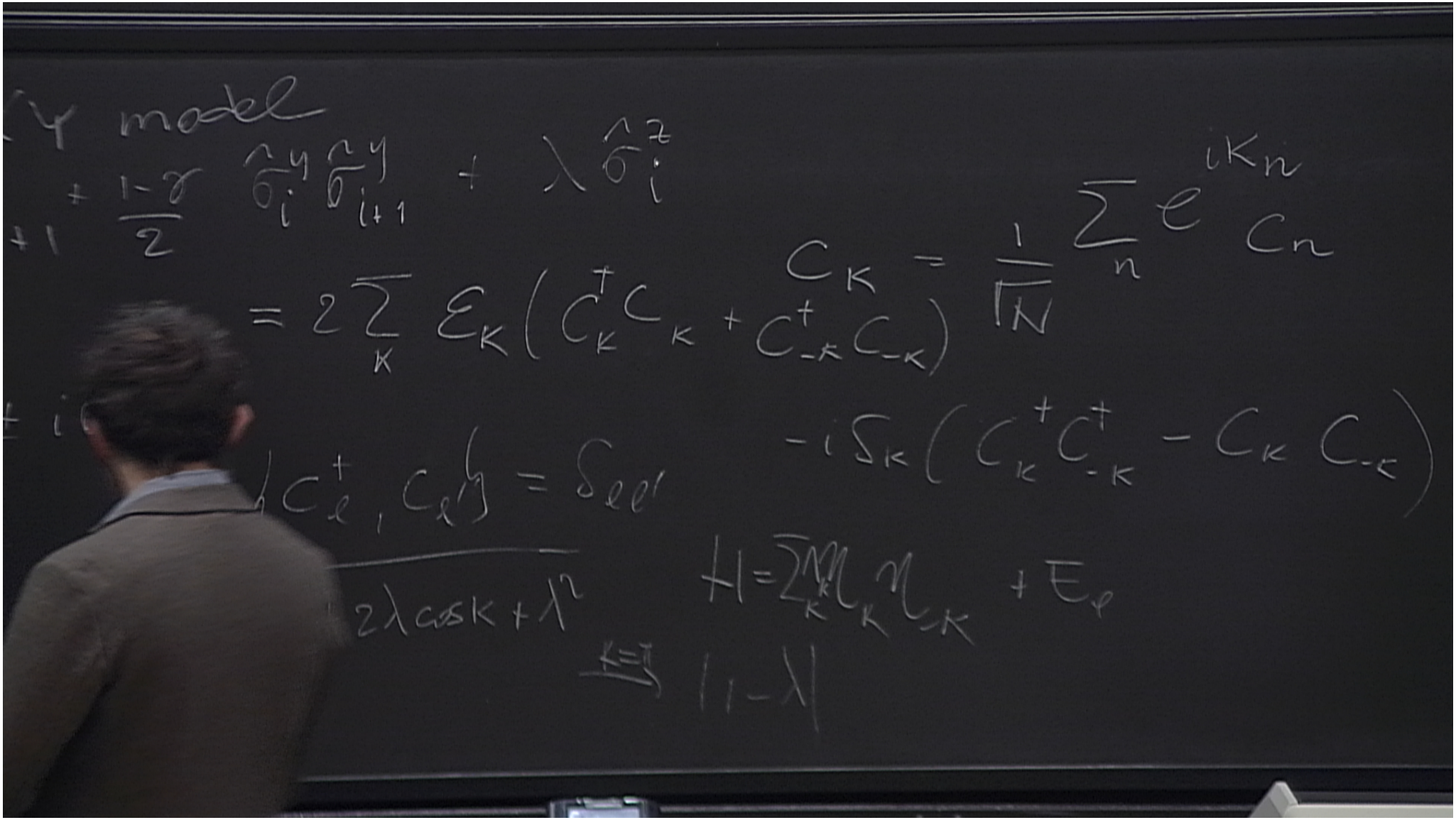
$$\hat{\sigma}_l^+$$

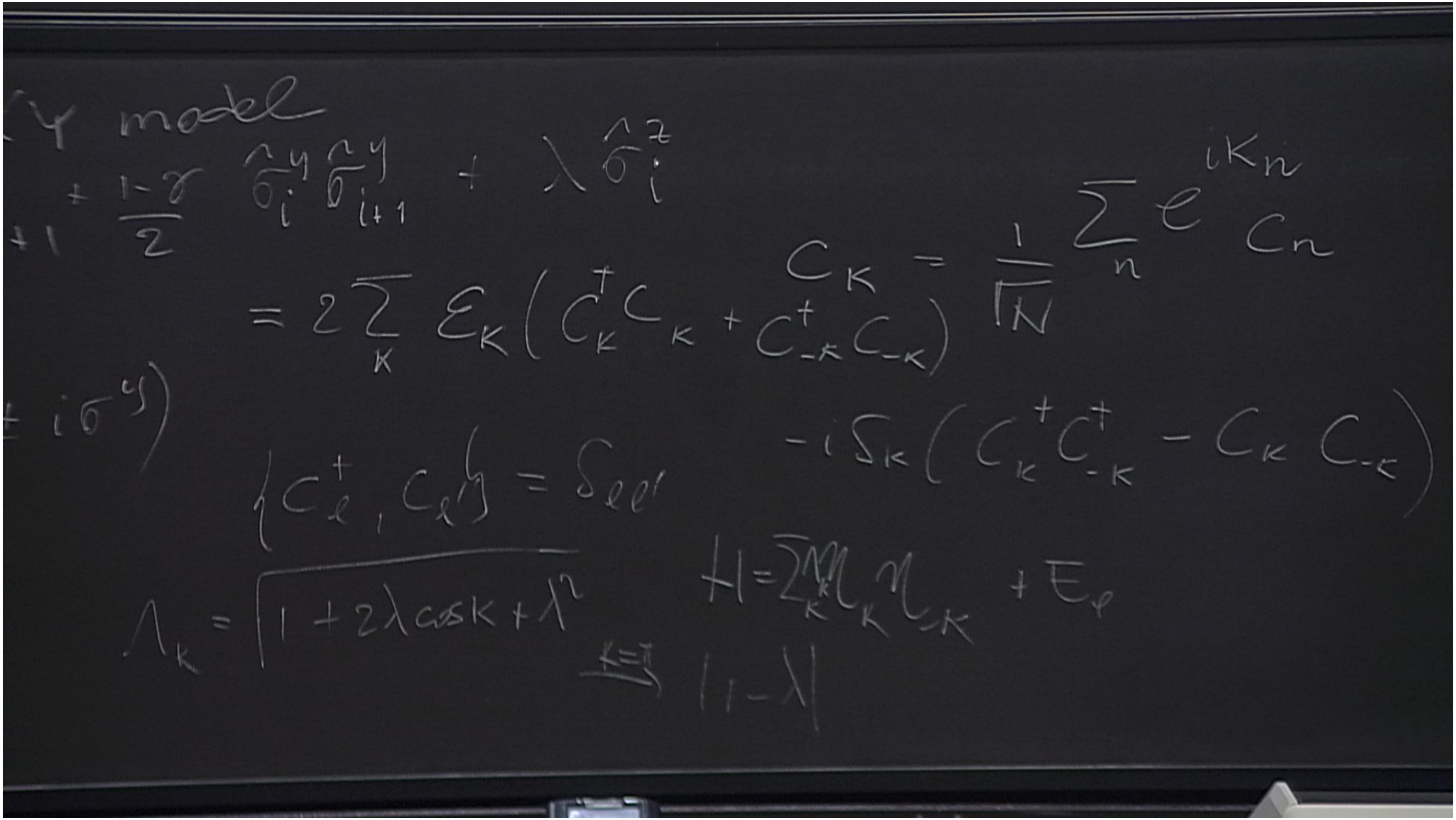
$$\{c_l^+, c_{l'}\} = \delta_{ll'}$$


$\lambda \hat{\sigma}_i^z$
$c_K = \frac{1}{\sqrt{N}} \sum_n e^{iK_n} c_n$
$= \delta_{ll'}$


model
$\frac{1-\gamma}{2}\hat{\sigma}_i^y\hat{\sigma}_{i+1}^y + \lambda\hat{\sigma}_i^z$
$= 2\sum_k \varepsilon_k\left(c_k^\dagger c_k + c_{-k}^\dagger c_{-k}\right)$
$c_k = \frac{1}{\sqrt{N}}\sum_n e^{ikn} c_n$
σ^y)
$\{c_\ell^\dagger, c_{\ell'}\} = \delta_{\ell\ell'}$
$-i\delta_k\left(c_k^\dagger c_{-k}^\dagger - c_k c_{-k}\right)$


Y model
$\pm \frac{1-\gamma}{2} \hat{\sigma}_i^y \hat{\sigma}_{i+1}^y + \lambda \hat{\sigma}_i^z$
$= 2 \sum_k \varepsilon_k \left(c_k^\dagger c_k + c_{-k}^\dagger c_{-k} \right)$
$\pm i\sigma^y)$
$\{ c_\ell^\dagger, c_{\ell'} \} = \delta_{\ell\ell'}$
$\Lambda_k = \sqrt{1 - 2\lambda \cos k + \lambda^2}$
$c_k = \frac{1}{\sqrt{N}} \sum_n e^{ikn} c_n$
$- i \delta_k \left(c_k^\dagger c_{-k}^\dagger - c_k c \right.$
$H = \sum_k \Lambda_k \eta_k^\dagger \eta_k + E_0$


model
$+\frac{1-\gamma}{2}\hat{\sigma}_i^y\hat{\sigma}_{i+1}^y + \lambda\hat{\sigma}_i^z$
$= 2\sum_k \varepsilon_k\left(c_k^\dagger c_k + c_{-k}^\dagger c_{-k}\right)$
$c_K = \frac{1}{\sqrt{N}}\sum_n e^{iKn} c_n$
$-i\delta_k\left(c_k^\dagger c_{-k}^\dagger - c_k c_{-k}\right)$
$\{c_\ell^\dagger, c_{\ell'}\} = \delta_{\ell\ell'}$
$2\lambda\cos k + \lambda^2$
$H = \sum_k \Lambda_k \eta_k^\dagger \eta_k + E_0$
$\Leftrightarrow |1-\lambda|$


Y model
$\pm \frac{1-\gamma}{2} \hat{\sigma}_i^y \hat{\sigma}_{i+1}^y + \lambda \hat{\sigma}_i^z$
$\pm i\sigma^y)$
$= 2\sum_k \varepsilon_k (c_k^\dagger c_k + c_{-k}^\dagger c_{-k})$
$c_k = \frac{1}{\sqrt{N}} \sum_n e^{ikn} c_n$
$-i\delta_k (c_k^\dagger c_{-k}^\dagger - c_k c_{-k})$
$\{c_\ell^\dagger, c_{\ell'}\} = \delta_{\ell\ell'}$
$\Lambda_k = \sqrt{1 + 2\lambda\cos k + \lambda^2}$
$H = \sum_k \Lambda_k \eta_k^\dagger \eta_k + E_0$
$k=\pi \Rightarrow |1-\lambda|$

Diagonalising the Model XY mod

Integrable

$$H = \sum_{i=-M}^{M} \frac{1+\gamma}{2} \hat{\sigma}_i^x \hat{\sigma}_{i+1}^x + \frac{1-\gamma}{2}$$

$$N = 2M+1$$

$$\hat{\sigma}_i^{\pm} = \frac{1}{2}(\sigma^x \pm i\sigma^y)$$

$$k = \frac{2\pi n}{N} + 1$$

$$c_l^+ = \left(\prod_{m=1}^{l-1} \sigma_m^z\right) \hat{\sigma}_l^+$$

$$\Lambda_k =$$