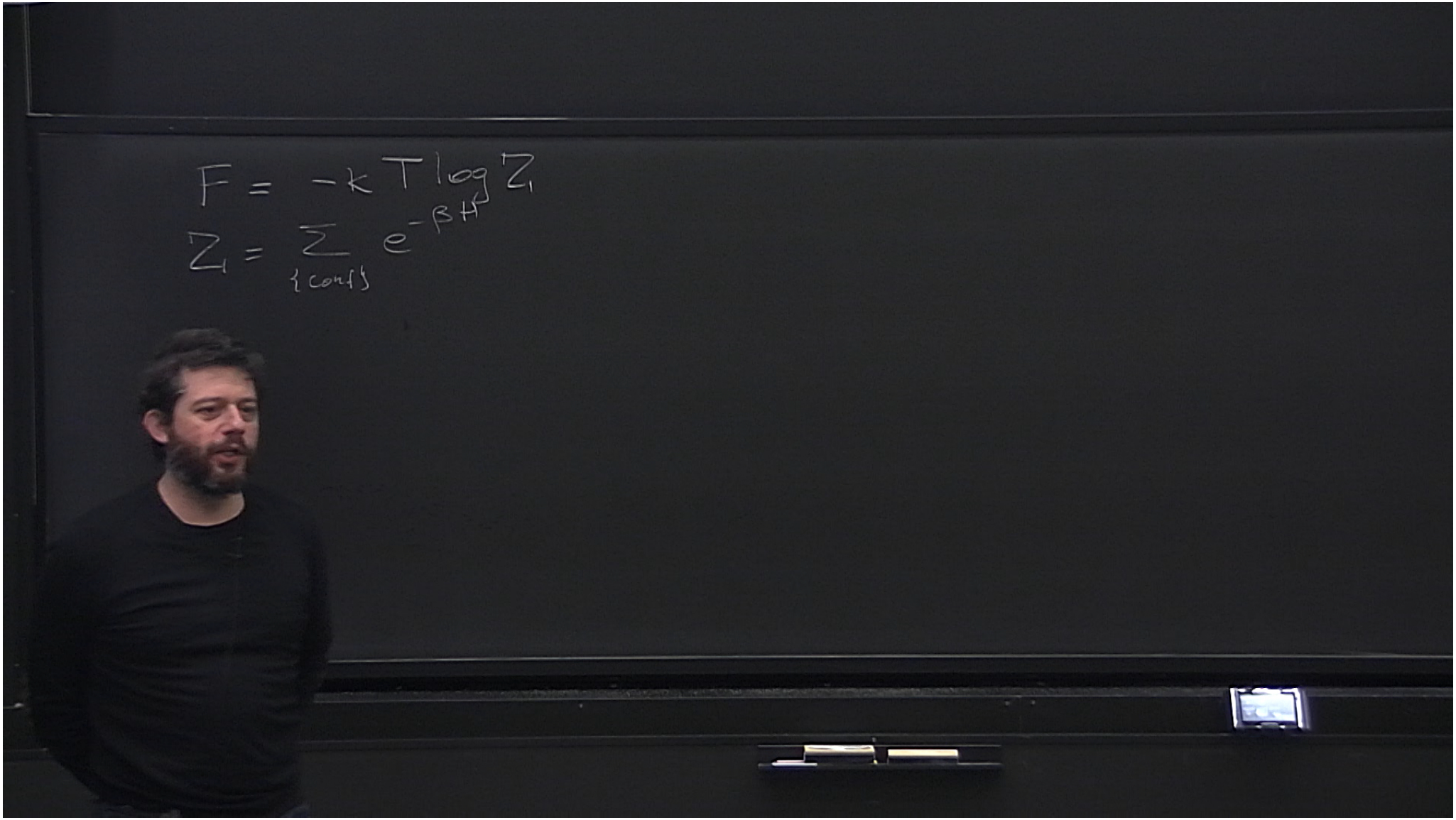
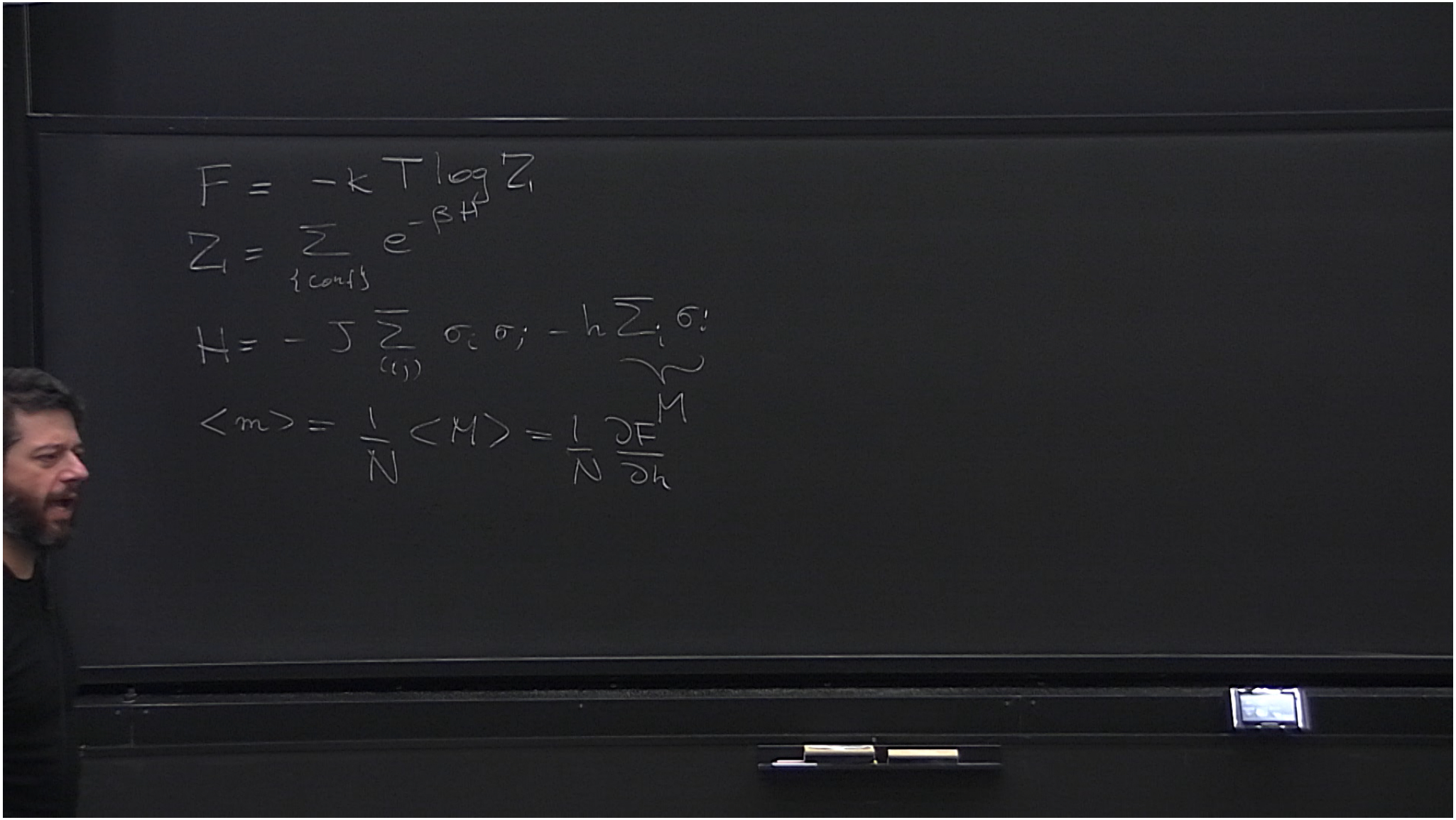
Title: PSI 17/18 - Condensed Matter - Lecture 3

Date: Jan 05, 2018 11:30 AM

URL: http://pirsa.org/18010036

Abstract:


F = −kT log Z
Z = Σ_{confs} e^{−βH}


$F = -kT \log Z_1$
$Z_1 = \sum_{\{\text{confs}\}} e^{-\beta H}$
$H = -J \sum_{(ij)} \sigma_i \sigma_j - h \sum_i \sigma_i$
$\langle m \rangle = \frac{1}{N} \langle M \rangle = \frac{1}{N} \frac{\partial F}{\partial h}$
M

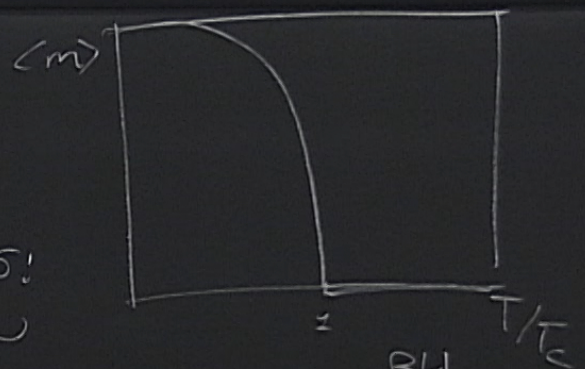
$$F = -kT \log Z$$

$$Z = \sum_{\{conf\}} e^{-\beta H}$$

$$H = -J \sum_{(ij)} \sigma_i \sigma_j - h \sum_i \sigma_i$$

$$\sim M$$

$$\langle m \rangle = \frac{1}{N} \langle M \rangle = \frac{1}{N} \frac{\partial F}{\partial h} = \frac{1}{N} \sum_{\{\sigma_i\}} M \frac{e^{-\beta H}}{Z}$$



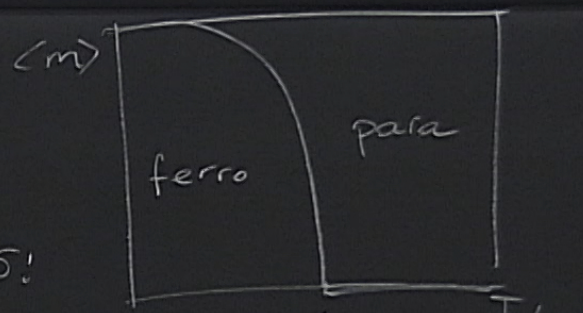
$$F = -kT \log Z$$

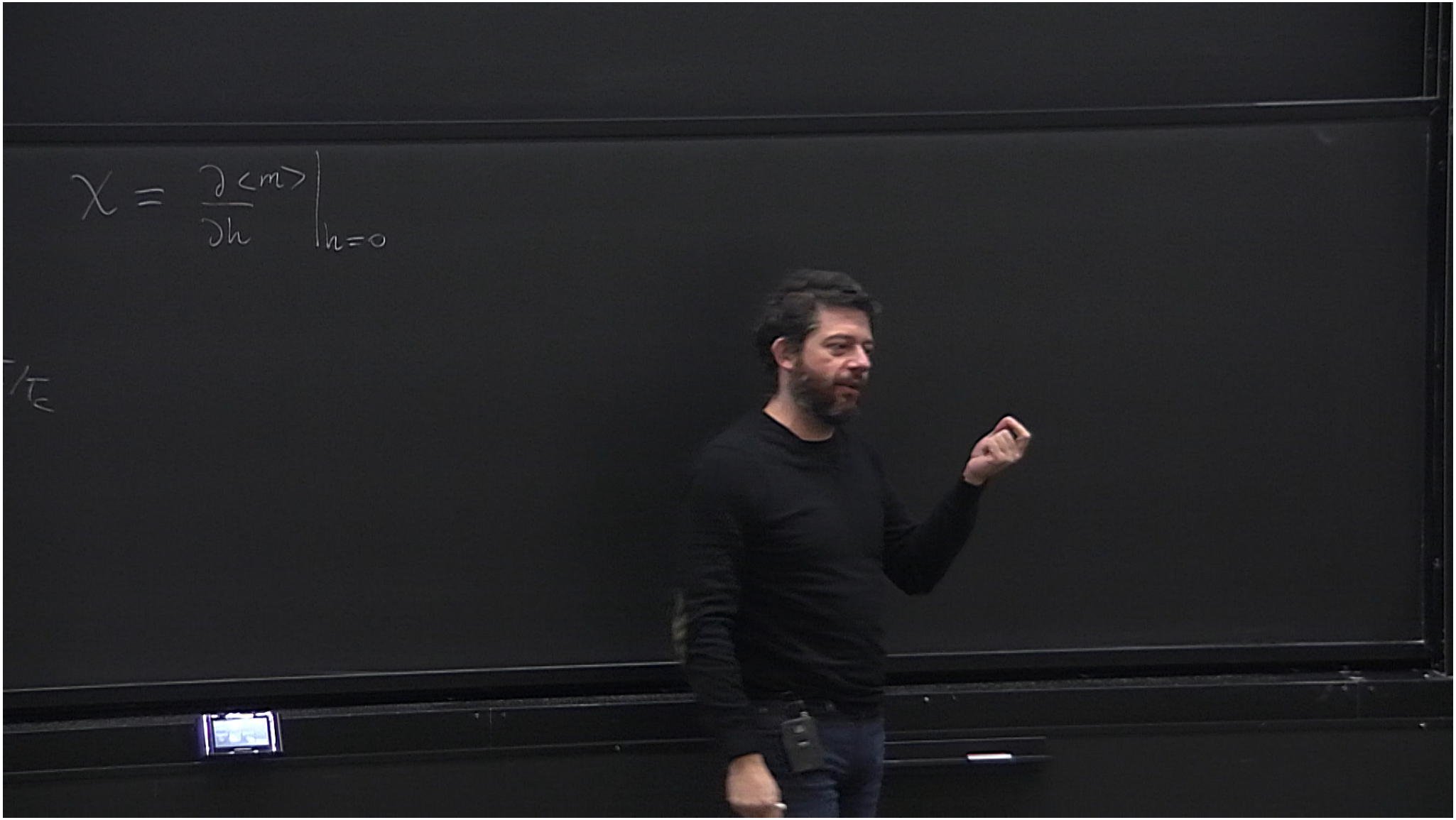
$$Z = \sum_{\{\text{conf}\}} e^{-\beta H}$$

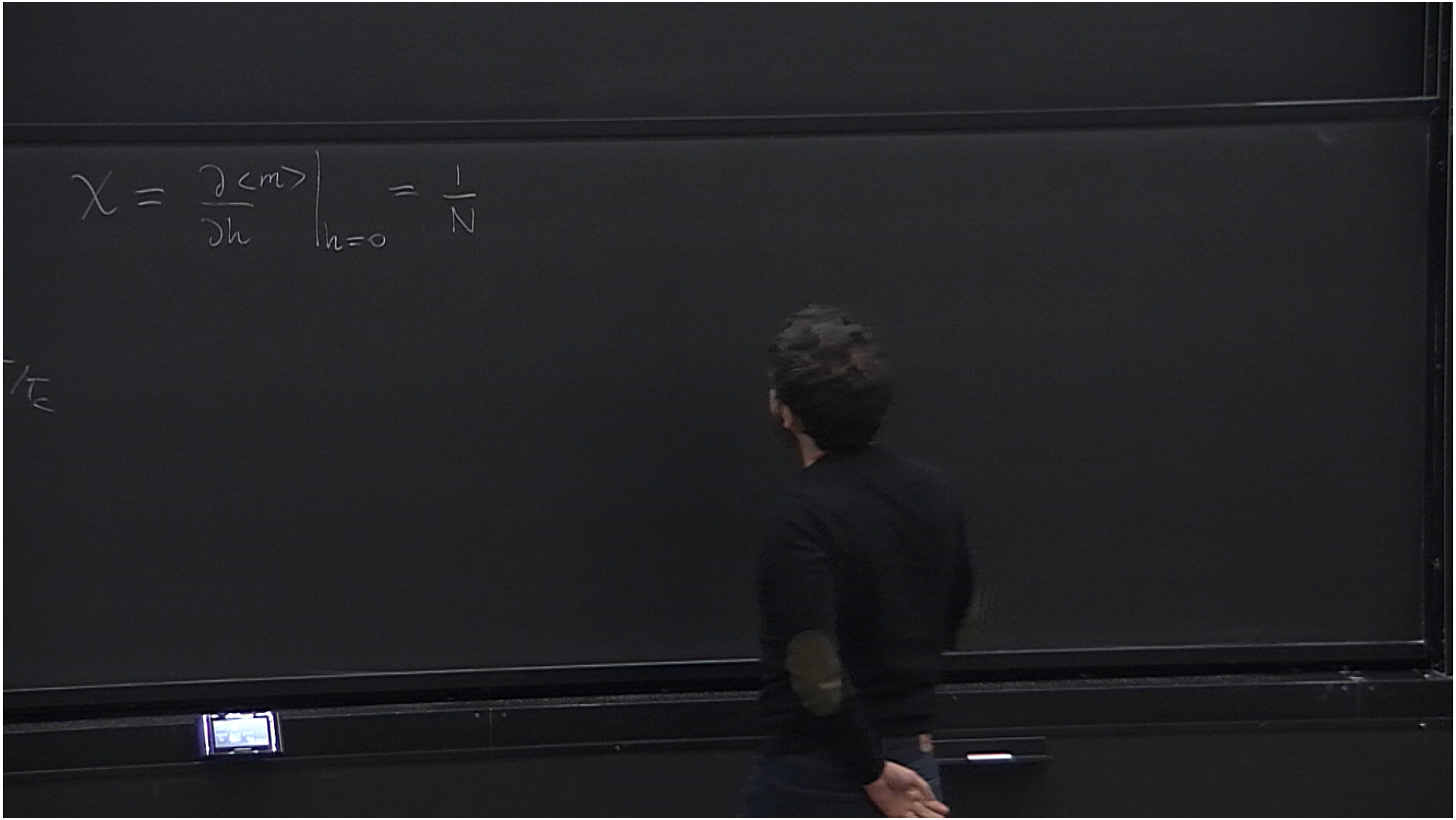
$$H = -J \sum_{(ij)} \sigma_i \sigma_j - h \sum_i \sigma_i$$

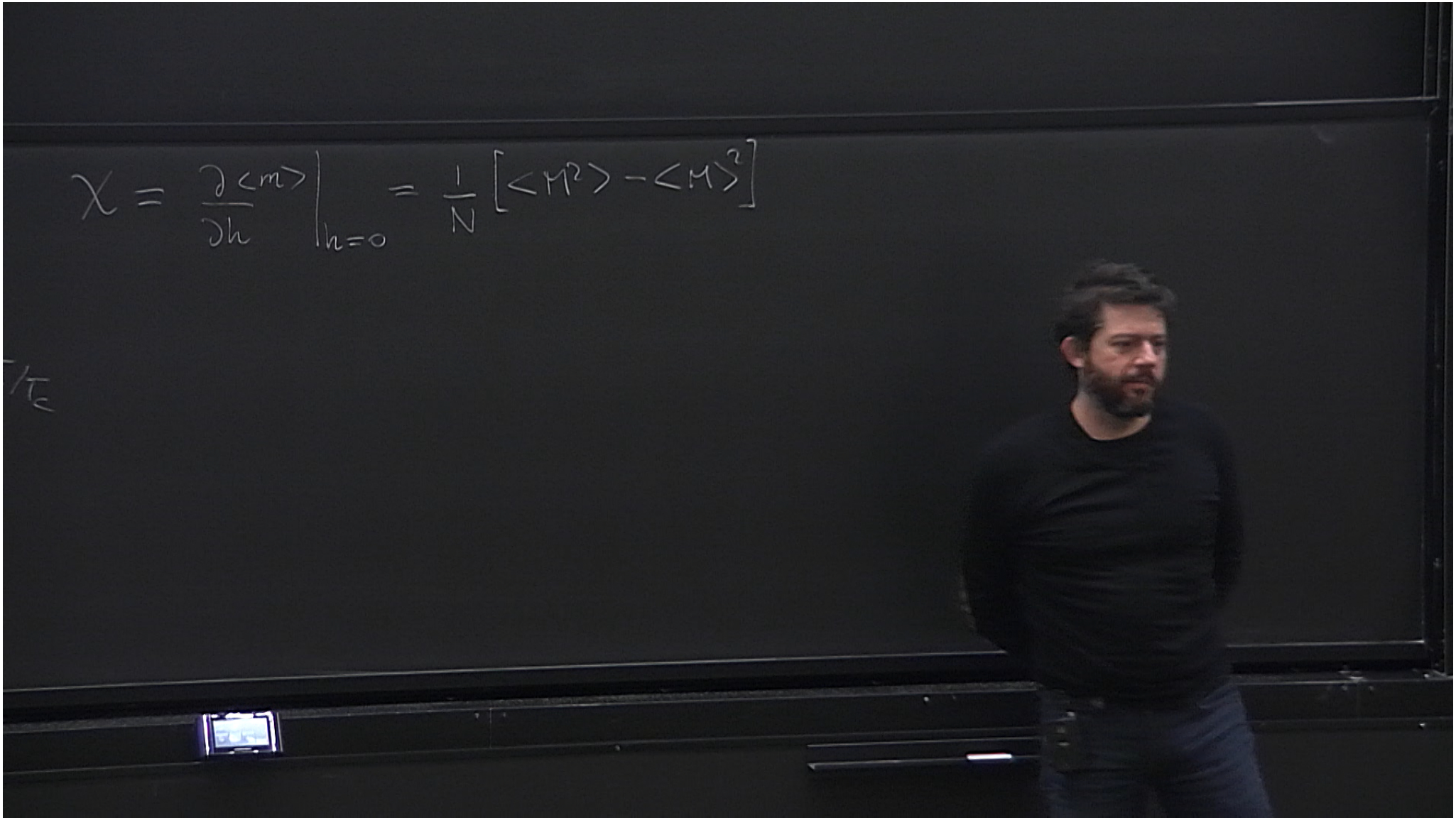
$$\sim M$$

$$\langle m \rangle = \frac{1}{N} \langle M \rangle = \frac{1}{N} \frac{\partial F}{\partial h} = \frac{1}{N} \sum_{\{\sigma_i\}} M \frac{e^{-\beta H}}{Z}$$



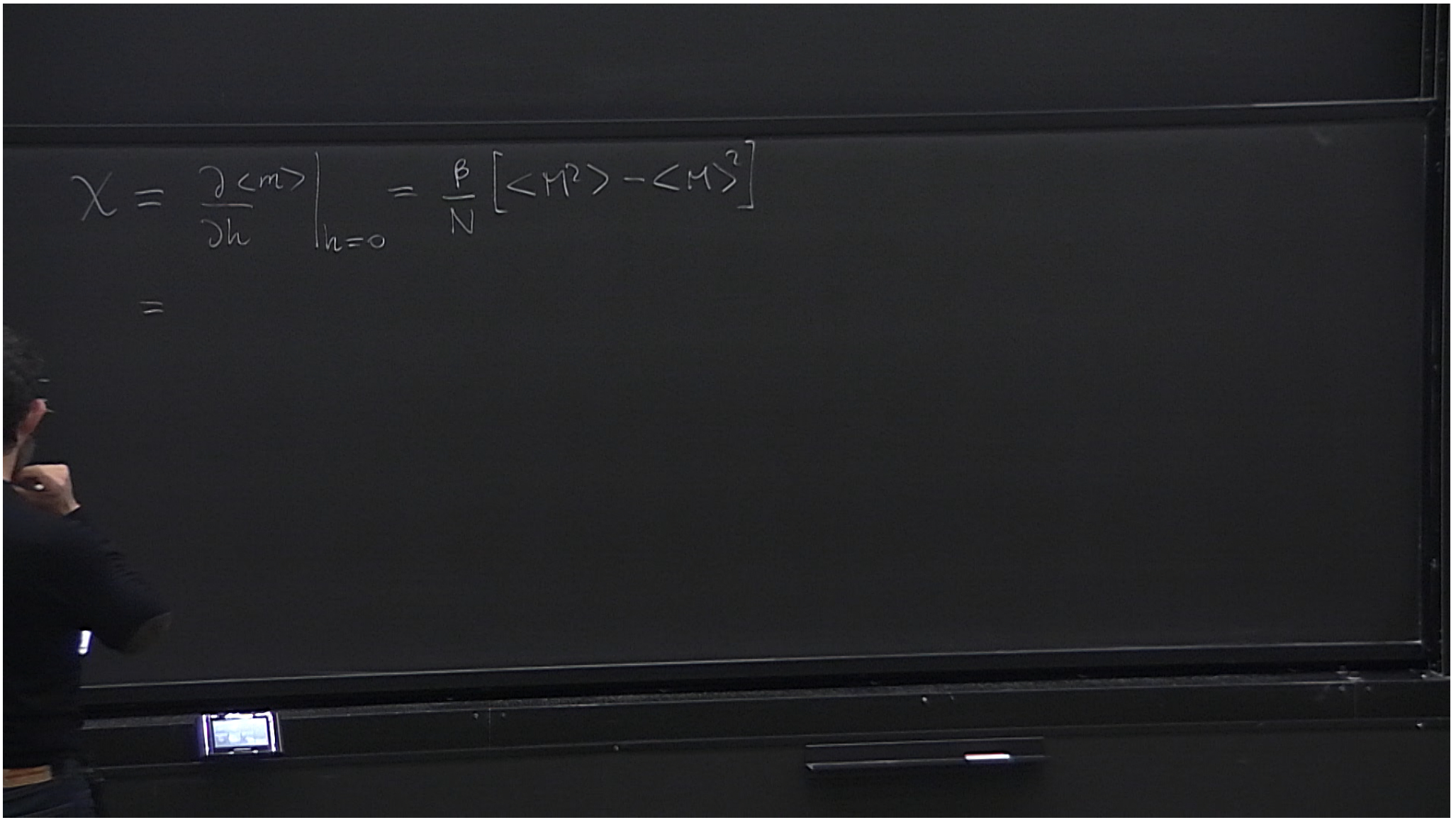

$\chi = \frac{\partial \langle m \rangle}{\partial h}\Big|_{h=0}$


$\chi = \frac{\partial \langle m \rangle}{\partial h}\Big|_{h=0} = \frac{1}{N}$


$\chi = \frac{\partial \langle m \rangle}{\partial h}\Big|_{h=0} = \frac{1}{N}\left[\langle M^2 \rangle - \langle M \rangle^2\right]$

$$\chi = \left.\frac{\partial \langle m \rangle}{\partial h}\right|_{h=0} = \frac{\beta}{N}\left[\langle M^2 \rangle - \langle M \rangle^2\right]$$

$=$



$$\chi = \left.\frac{\partial \langle m \rangle}{\partial h}\right|_{h=0} = \frac{\beta}{N}\left[\langle M^2 \rangle - \langle M \rangle^2\right]$$

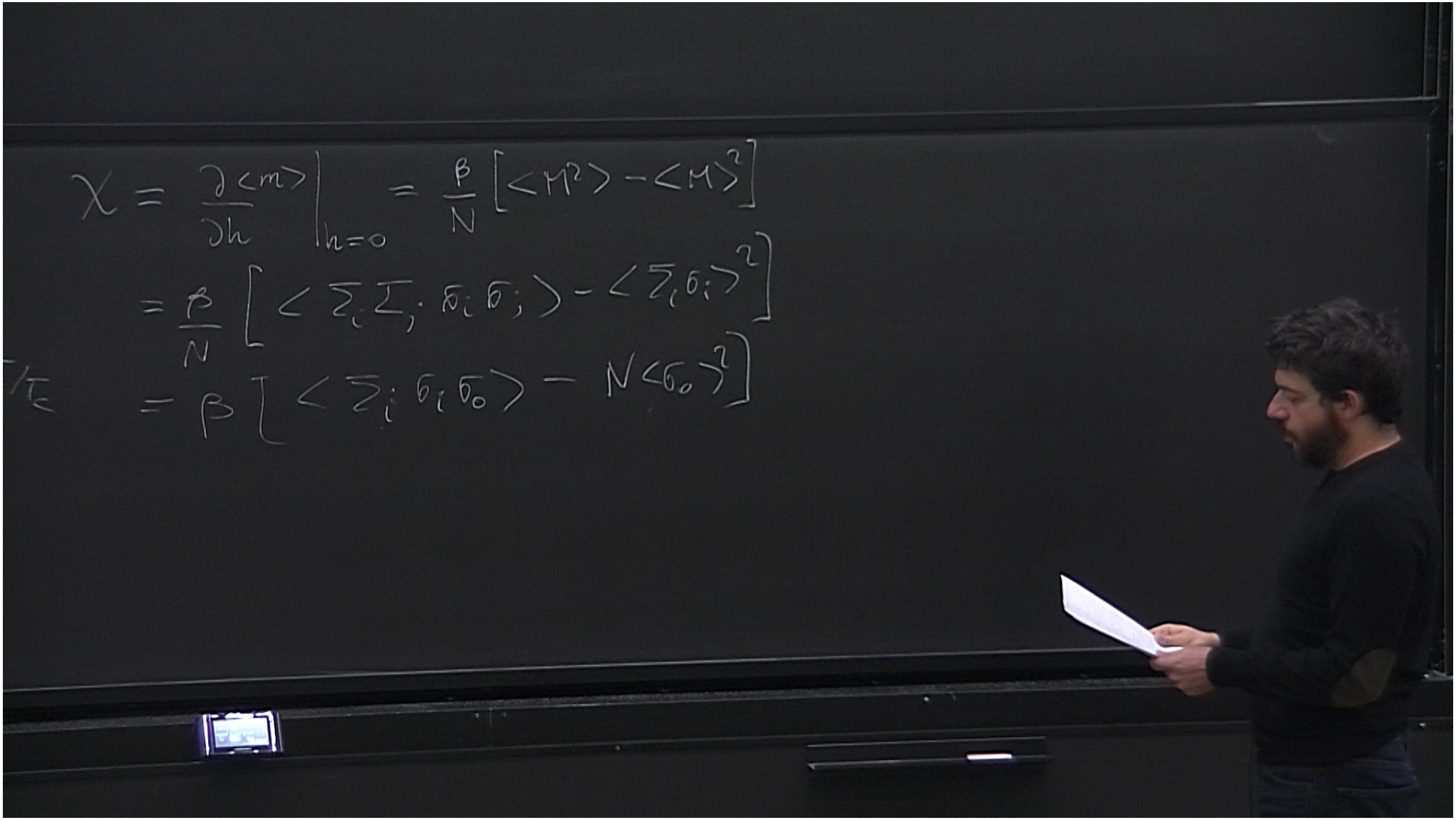
$$= \frac{\beta}{N}\left[\langle \Sigma_i \Sigma_j \cdot \sigma_i \sigma_j \rangle - \langle \Sigma_i \sigma_i \rangle^2\right]$$

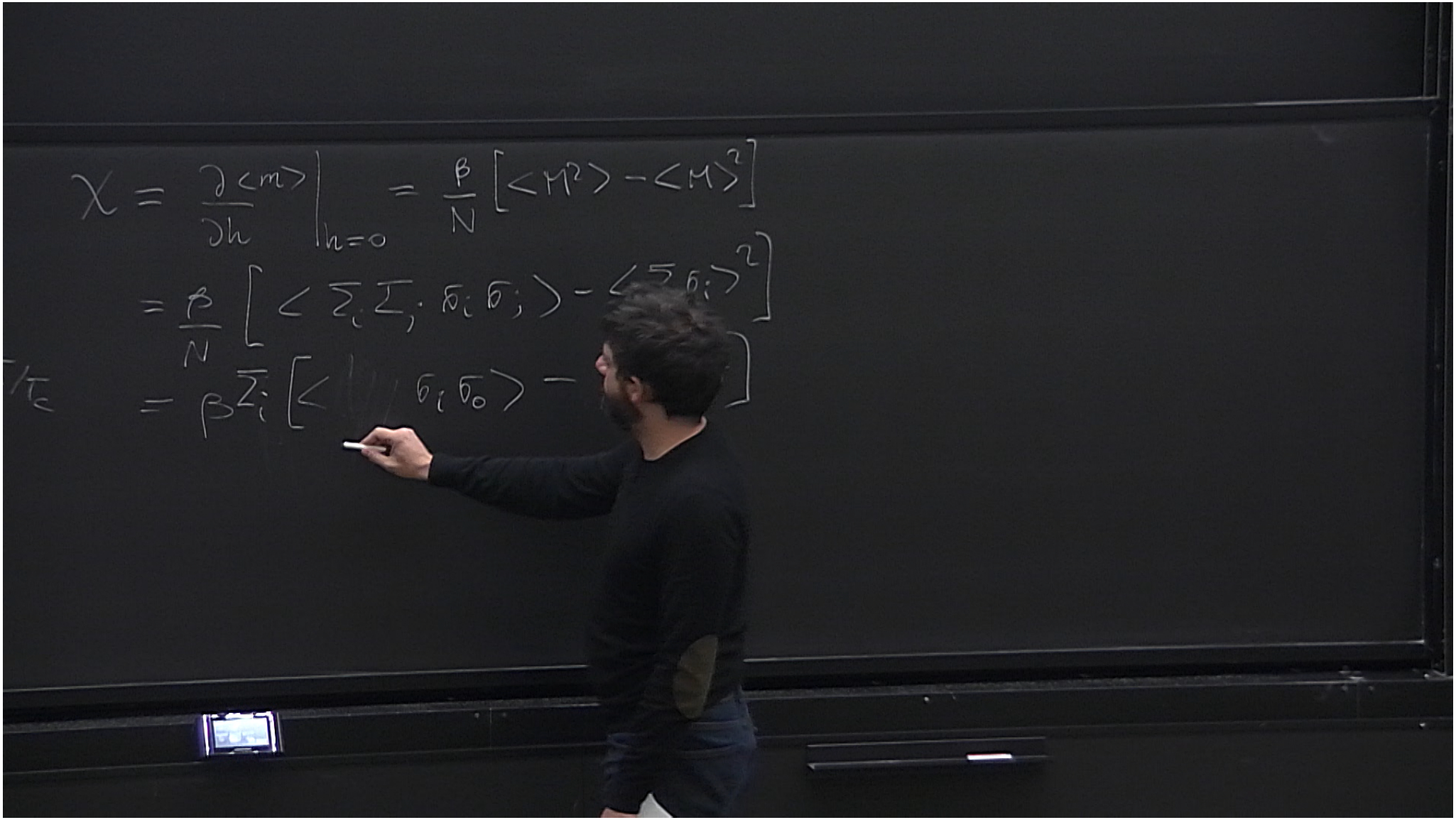
T/T_c

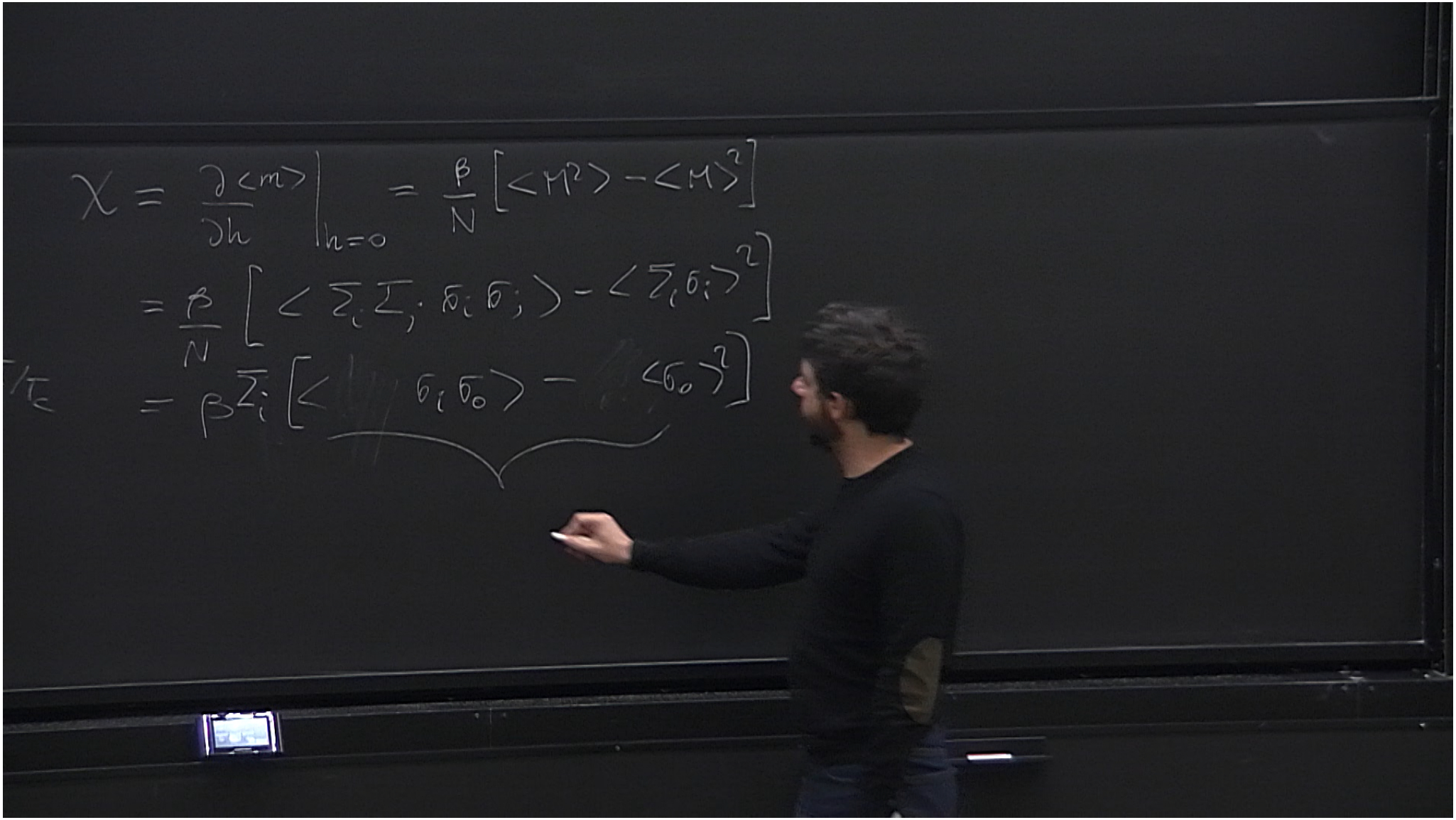
$$\chi = \left.\frac{\partial \langle m \rangle}{\partial h}\right|_{h=0} = \frac{\beta}{N}\left[\langle M^2 \rangle - \langle M \rangle^2\right]$$

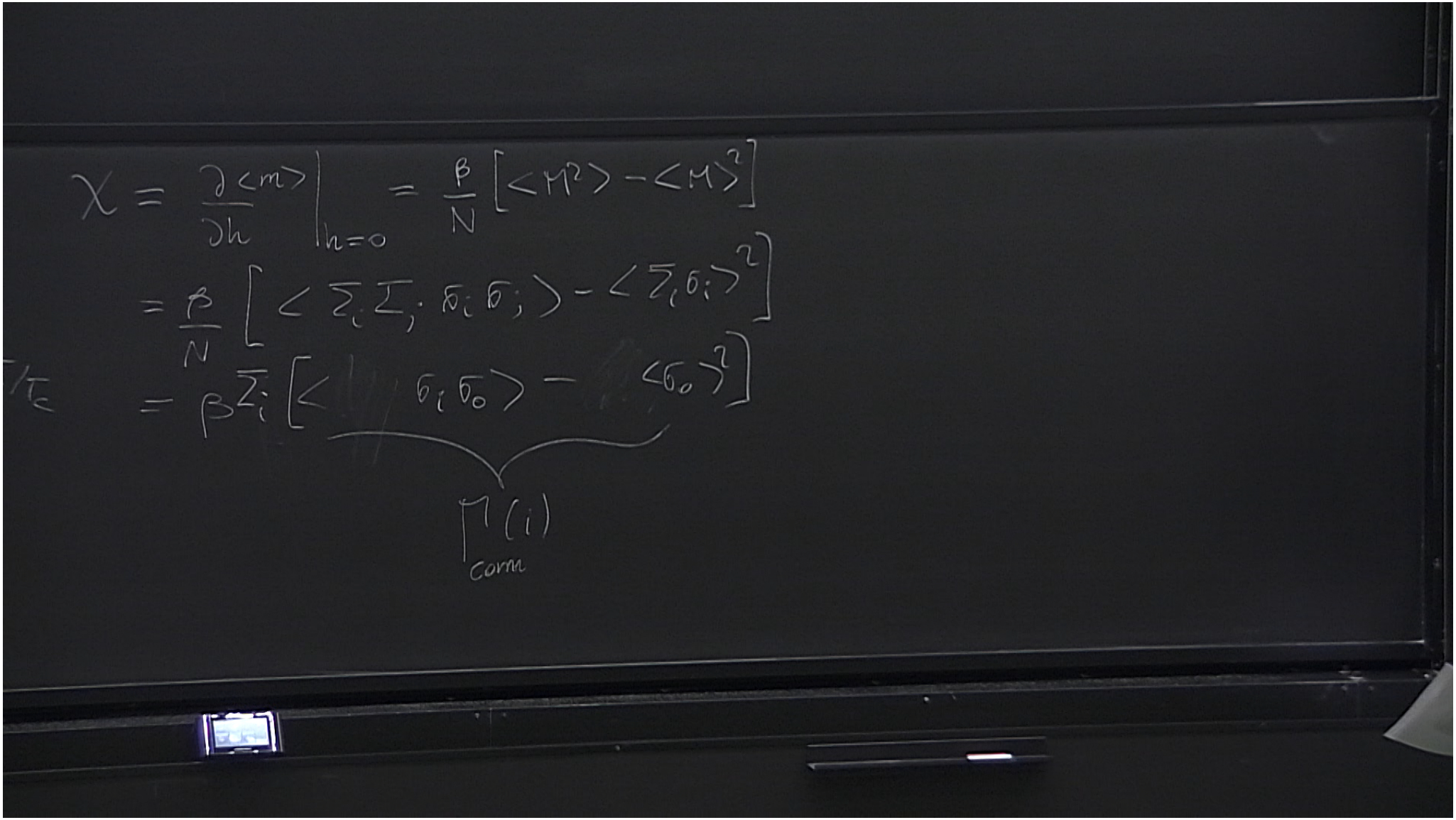
$$= \frac{\beta}{N}\left[\langle \Sigma_i \Sigma_j \cdot \sigma_i \sigma_j \rangle - \langle \Sigma_i \sigma_i \rangle^2\right]$$

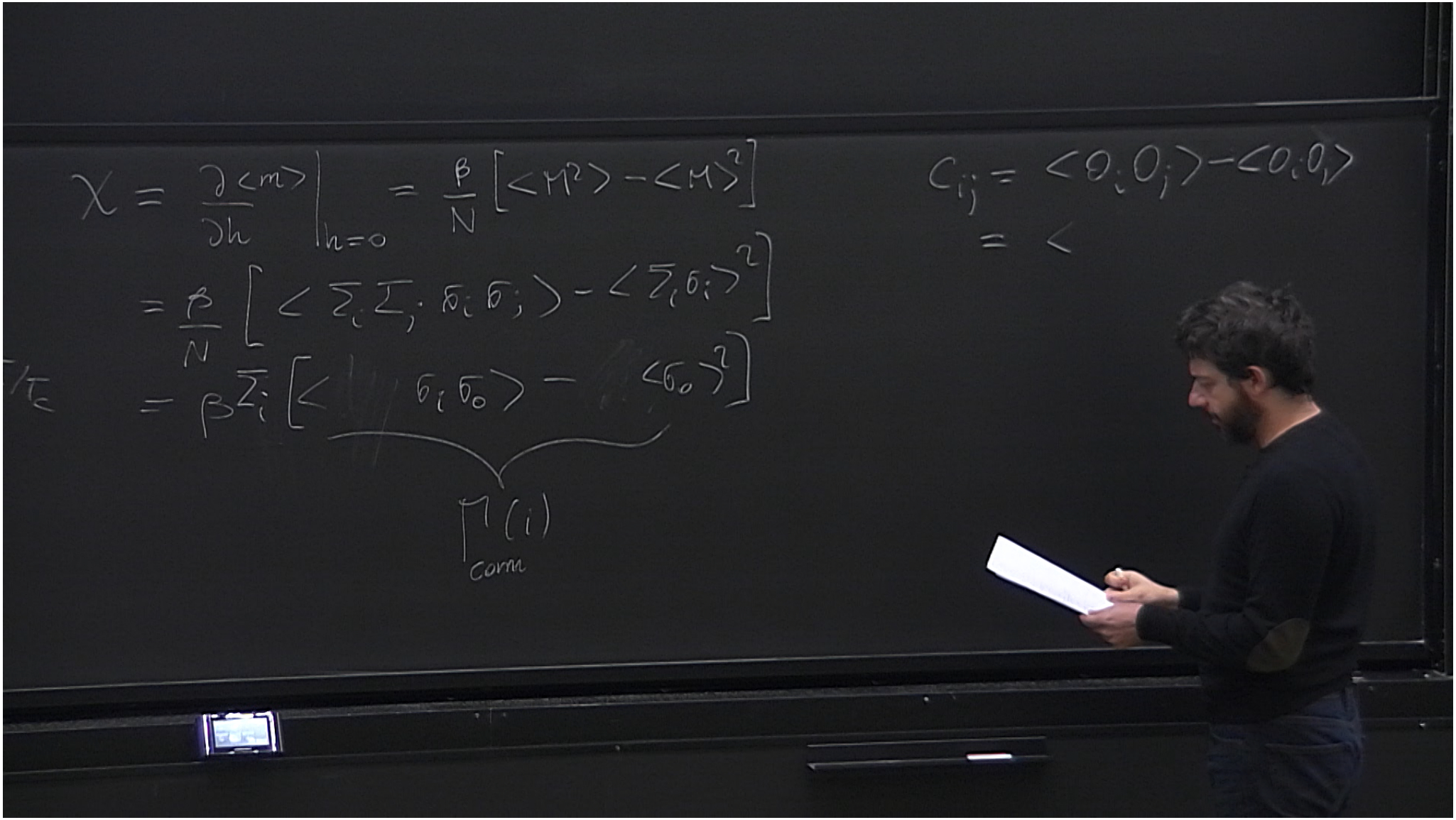
$$= \beta\left[\langle \Sigma_i \sigma_i \sigma_0 \rangle - N \langle \sigma_0 \rangle^2\right]$$

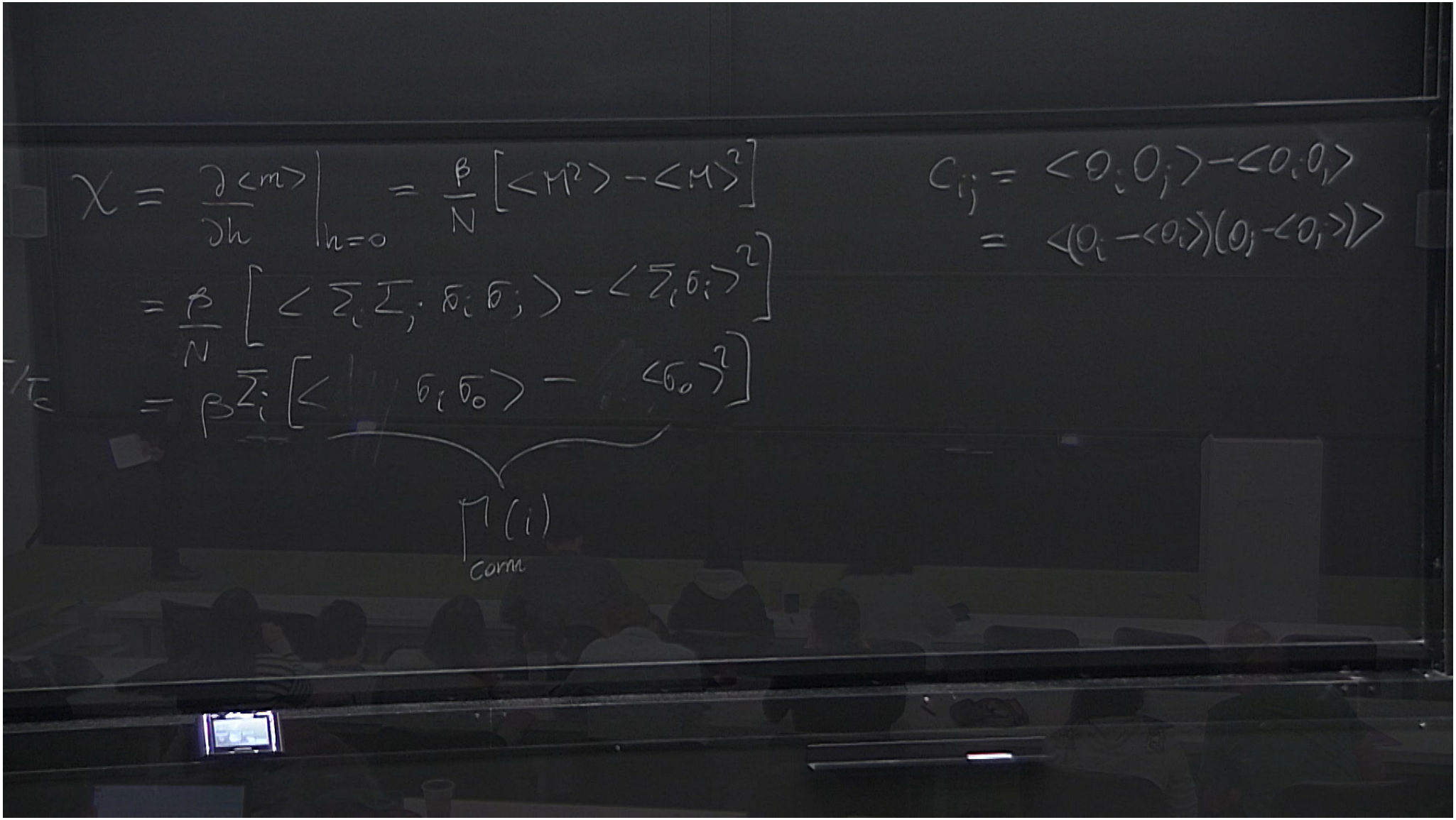



$\chi = \frac{\partial \langle m \rangle}{\partial h}\Big|_{h=0} = \frac{\beta}{N}\left[\langle M^2 \rangle - \langle M \rangle^2\right]$
$= \frac{\beta}{N}\left[\langle \Sigma_i \Sigma_j \cdot \sigma_i \sigma_j \rangle - \langle \Sigma \sigma_i \rangle^2\right]$
$= \beta \Sigma_i \left[\langle \sigma_i \sigma_0 \rangle -$


$\chi = \frac{\partial \langle m \rangle}{\partial h}\Big|_{h=0} = \frac{\beta}{N}\left[\langle M^2 \rangle - \langle M \rangle^2\right]$
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$\chi = \frac{\partial \langle m \rangle}{\partial h}\Big|_{h=0} = \frac{\beta}{N}\left[\langle M^2 \rangle - \langle M \rangle^2\right]$
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$\Gamma(i)$
Corra


$\chi = \frac{\partial \langle m \rangle}{\partial h}\Big|_{h=0} = \frac{\beta}{N}\left[\langle M^2 \rangle - \langle M \rangle^2\right]$
$= \frac{\beta}{N}\left[\langle \Sigma_i \Sigma_j \cdot \sigma_i \sigma_j \rangle - \langle \Sigma_i \sigma_i \rangle^2\right]$
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$\Gamma(i)$
Corrn
$C_{ij} = \langle \sigma_i \sigma_j \rangle - \langle \sigma_i \sigma_j \rangle$
$= \langle$


$\chi = \frac{\partial \langle m \rangle}{\partial h}\Big|_{h=0} = \frac{\beta}{N}\left[\langle M^2 \rangle - \langle M \rangle^2\right]$
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$= \beta \Sigma_i \left[\langle \sigma_i \sigma_0 \rangle - \langle \sigma_0 \rangle^2\right]$
$\Gamma(i)$
Corrn
$C_{ij} = \langle O_i O_j \rangle - \langle O_i O_j \rangle$
$= \langle (O_i - \langle O_i \rangle)(O_j - \langle O_j \rangle) \rangle$

$$= \frac{\partial \langle m \rangle}{\partial h}\bigg|_{h=0} = \frac{\beta}{N}\left[\langle M^2 \rangle - \langle M \rangle^2\right]$$

$$= \frac{\beta}{N}\left[\langle \Sigma_i \Sigma_j \sigma_i \sigma_j \rangle - \langle \Sigma_i \sigma_i \rangle^2\right]$$

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$\Gamma(i)$

Corrn

$$C_{ij} = \langle \sigma_i \sigma_j \rangle - \langle \sigma_i \sigma_j \rangle$$
$$= \langle (\sigma_i - \langle \sigma_i \rangle)(\sigma_j - \langle \sigma_j \rangle) \rangle$$

$$\langle m \rangle \sim |T - T_c|^{-\beta}$$

$$F = -kT \log Z$$

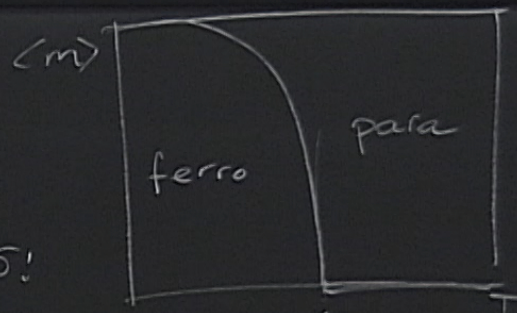
$$Z = \sum_{\{\text{conf}\}} e^{-\beta H}$$

$$H = -J \sum_{(ij)} \sigma_i \sigma_j - h \sum_i \sigma_i$$

$$\langle m \rangle = \frac{1}{N} \langle M \rangle = \frac{1}{N} \frac{\partial F}{\partial h} = \frac{1}{N} \sum_{\{\sigma_i\}} M \frac{e^{-\beta H}}{Z}$$

critical exp. Ising 2D

$$\beta = \frac{1}{8}$$



$$\chi = \left. \frac{\partial \langle m \rangle}{\partial h} \right|_{h=0} =$$

$$= \frac{\beta}{N} \left[\langle \Sigma_i \Sigma_j$$

$$= \beta \Sigma_i \left[\langle$$

$$= \frac{\partial \langle m \rangle}{\partial h}\bigg|_{h=0} = \frac{\beta}{N}\left[\langle M^2 \rangle - \langle M \rangle^2\right]$$

$$= \frac{\beta}{N}\left[\langle \Sigma_i \Sigma_j \sigma_i \sigma_j \rangle - \langle \Sigma_i \sigma_i \rangle^2\right]$$

$$= \beta \Sigma_i \left[\langle \sigma_i \sigma_0 \rangle - \langle \sigma_0 \rangle^2\right]$$

$\Gamma(i)$

corrn

$$C_{ij} = \langle \sigma_i \sigma_j \rangle - \langle \sigma_i \sigma_j \rangle$$

$$= \langle (\sigma_i - \langle \sigma_i \rangle)(\sigma_j - \langle \sigma_j \rangle) \rangle$$

$$\langle m \rangle \sim |T - T_c|^{-\beta}$$

$$\Gamma(i) = \langle \sigma_i \sigma_0 \rangle$$

$$= \frac{\partial \langle m \rangle}{\partial h}\bigg|_{h=0} = \frac{\beta}{N}\left[\langle M^2 \rangle - \langle M \rangle^2\right]$$

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$\Gamma(i)$

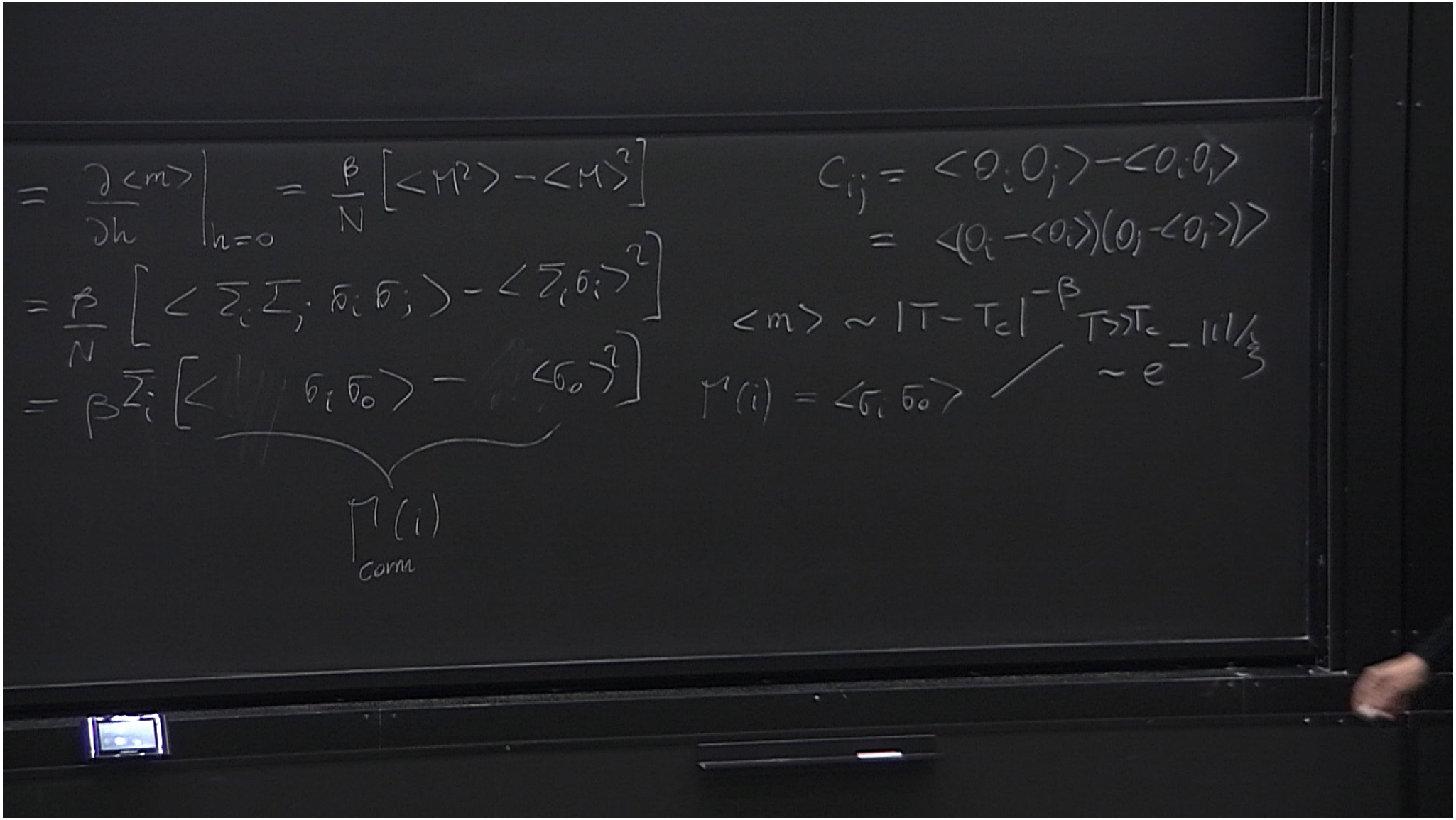
Corrn

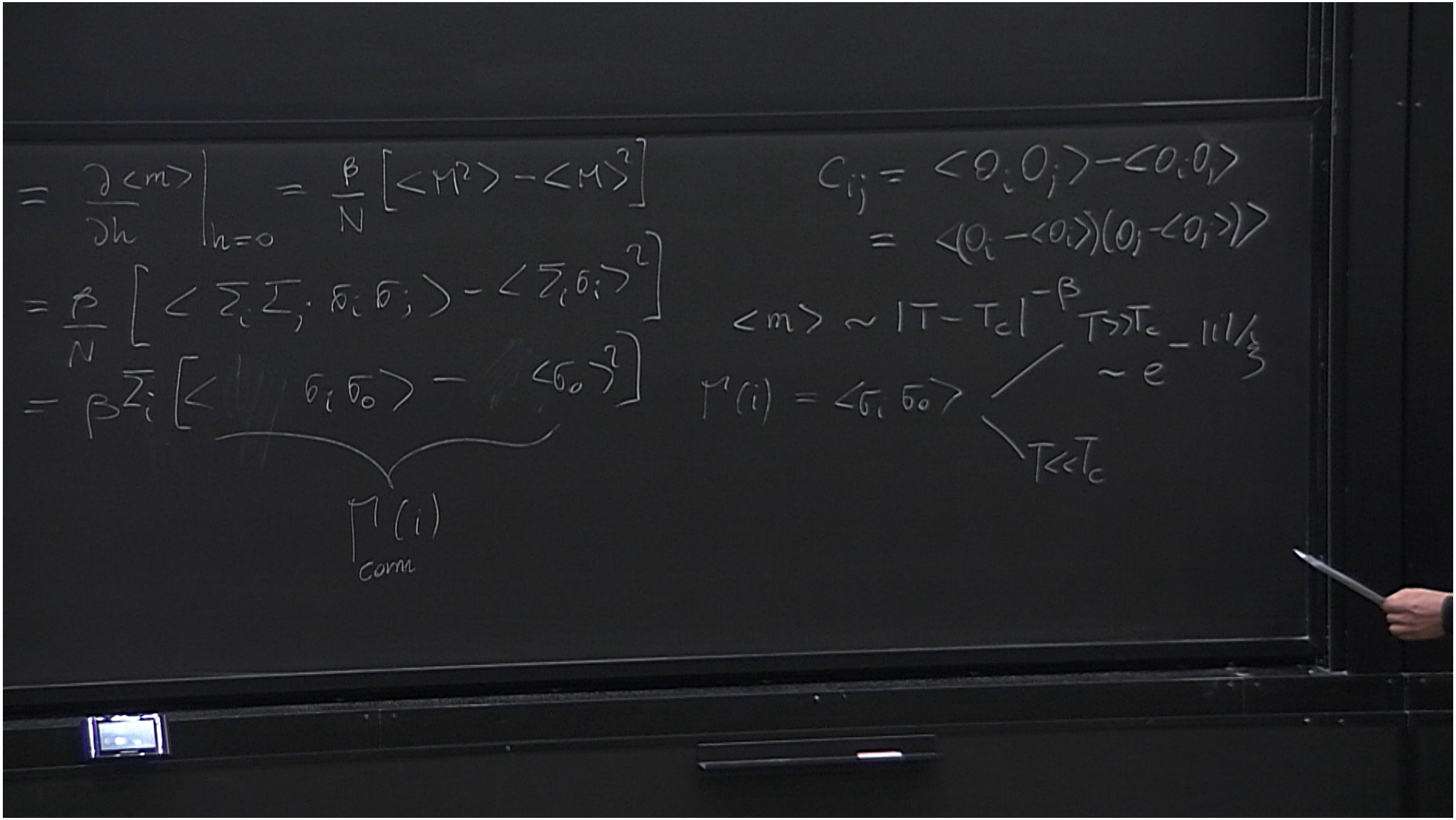
$$C_{ij} = \langle \sigma_i \sigma_j \rangle - \langle \sigma_i \sigma_j \rangle$$

$$= \langle (\sigma_i - \langle \sigma_i \rangle)(\sigma_j - \langle \sigma_j \rangle) \rangle$$

$$\langle m \rangle \sim |T - T_c|^{-\beta} \quad T \gg T_c$$

$$\Gamma(i) = \langle \sigma_i \sigma_0 \rangle$$


$= \frac{\partial \langle m \rangle}{\partial h}\Big|_{h=0} = \frac{\beta}{N}\left[\langle M^2 \rangle - \langle M \rangle^2\right]$
$= \frac{\beta}{N}\left[\langle \Sigma_i \Sigma_j \cdot \sigma_i \sigma_j \rangle - \langle \Sigma_i \sigma_i \rangle^2\right]$
$= \beta \Sigma_i \left[\langle \sigma_i \sigma_0 \rangle - \langle \sigma_0 \rangle^2\right]$
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$\langle m \rangle \sim |T - T_c|^{-\beta}$
$\Gamma(i) = \langle \sigma_i \sigma_0 \rangle$
$T \gg T_c$
$\sim e^{-i/\xi}$


$= \frac{\partial \langle m \rangle}{\partial h}\Big|_{h=0} = \frac{\beta}{N}\left[\langle M^2 \rangle - \langle M \rangle^2\right]$
$= \frac{\beta}{N}\left[\langle \sum_i \sum_j \sigma_i \sigma_j \rangle - \langle \sum_i \sigma_i \rangle^2\right]$
$= \beta \sum_i \left[\langle \sigma_i \sigma_0 \rangle - \langle \sigma_0 \rangle^2\right]$
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$T \ll T_c$

$$= \frac{\partial \langle m \rangle}{\partial h}\bigg|_{h=0} = \frac{\beta}{N}\left[\langle M^2 \rangle - \langle M \rangle^2\right]$$

$$= \frac{\beta}{N}\left[\langle \Sigma_i \Sigma_j \cdot \sigma_i \sigma_j \rangle - \langle \Sigma_i \sigma_i \rangle^2\right]$$

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$\Gamma(i)$ corrn

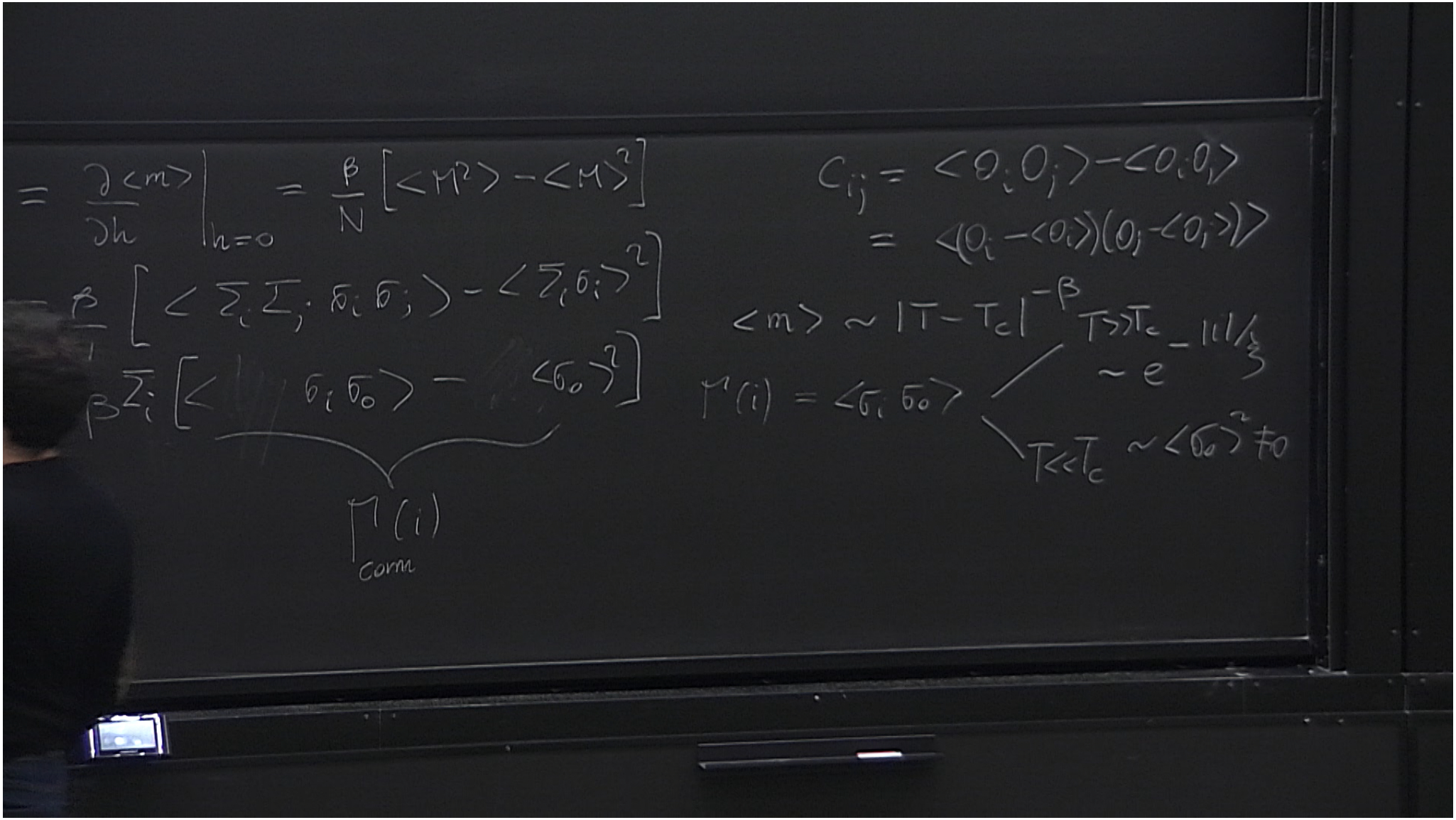
$$C_{ij} = \langle \sigma_i \sigma_j \rangle - \langle \sigma_i \sigma_j \rangle$$
$$= \langle (\sigma_i - \langle \sigma_i \rangle)(\sigma_j - \langle \sigma_j \rangle) \rangle$$

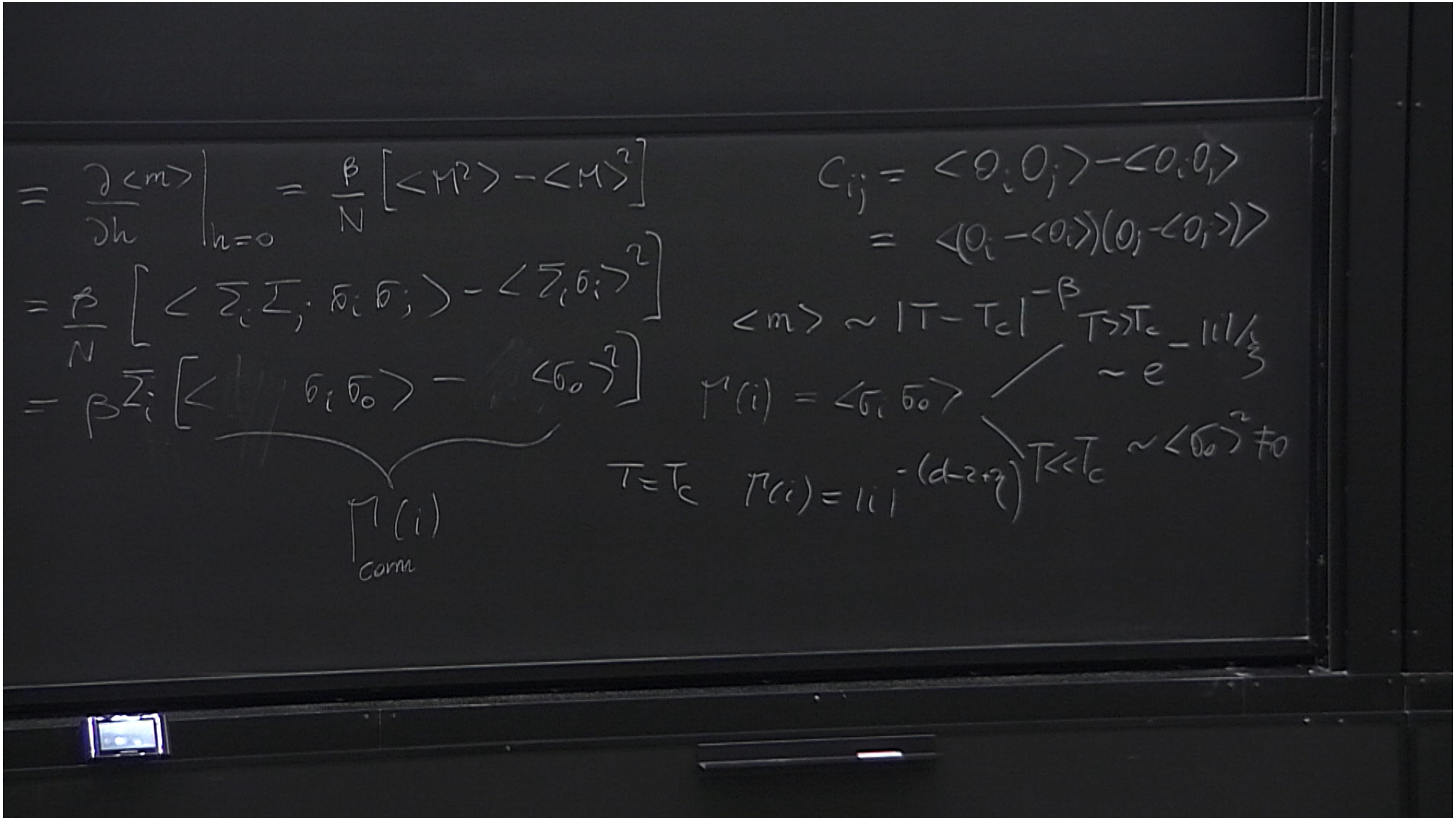
$$\langle m \rangle \sim |T - T_c|^{-\beta}$$

$$\Gamma(i) = \langle \sigma_i \sigma_0 \rangle$$

$T \gg T_c$: $\sim e^{-i/\xi}$

$T \ll T_c$: $\sim \langle \sigma_0 \rangle^2$


$= \frac{\partial \langle m \rangle}{\partial h}\Big|_{h=0} = \frac{\beta}{N}\left[\langle M^2 \rangle - \langle M \rangle^2\right]$
$\beta \left[\langle \Sigma_i \Sigma_j \cdot \sigma_i \sigma_j \rangle - \langle \Sigma_i \sigma_i \rangle^2 \right]$
$\beta \Sigma_i \left[\langle \sigma_i \sigma_0 \rangle - \langle \sigma_0 \rangle^2 \right]$
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$C_{ij} = \langle \sigma_i \sigma_j \rangle - \langle \sigma_i \sigma_j \rangle$
$= \langle (\sigma_i - \langle \sigma_i \rangle)(\sigma_j - \langle \sigma_j \rangle) \rangle$
$\langle m \rangle \sim |T - T_c|^{-\beta}$
$\Gamma(i) = \langle \sigma_i \sigma_0 \rangle$
$T \gg T_c$ $\sim e^{-i/\xi}$
$T \ll T_c$ $\sim \langle \sigma_0 \rangle^2 \neq 0$

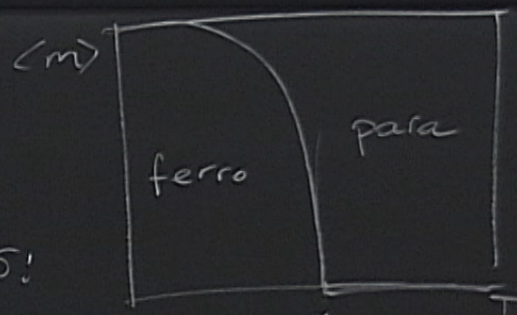

$= \frac{\partial \langle m \rangle}{\partial h}\Big|_{h=0} = \frac{\beta}{N}\left[\langle M^2 \rangle - \langle M \rangle^2\right]$
$= \frac{\beta}{N}\left[\langle \Sigma_i \Sigma_j \sigma_i \sigma_j \rangle - \langle \Sigma_i \sigma_i \rangle^2\right]$
$= \beta \Sigma_i \left[\langle \sigma_i \sigma_0 \rangle - \langle \sigma_0 \rangle^2\right]$
$\Gamma(i)$
Corm
$C_{ij} = \langle \sigma_i \sigma_j \rangle - \langle \sigma_i \sigma_j \rangle$
$= \langle (\sigma_i - \langle \sigma_i \rangle)(\sigma_j - \langle \sigma_i \rangle) \rangle$
$\langle m \rangle \sim |T - T_c|^{-\beta}$
$\Gamma(i) = \langle \sigma_i \sigma_0 \rangle$
$T \gg T_c$ $\sim e^{-i/\xi}$
$T \ll T_c$ $\sim \langle \sigma_0 \rangle^2 \neq 0$
$T = T_c$ $\Gamma(i) = |i|^{-(d-2+\eta)}$

$$F = -kT \log Z_1$$

$$Z_1 = \sum_{\{\text{confs}\}} e^{-\beta H}$$

$$H = -J \sum_{(ij)} \sigma_i \sigma_j - h \sum_i \sigma_i$$

$$\langle m \rangle = \frac{1}{N} \langle M \rangle = \frac{1}{N} \frac{\partial F}{\partial h} = \frac{1}{N} \sum_{\{\sigma_i\}} M \frac{e^{-\beta H}}{Z_1}$$



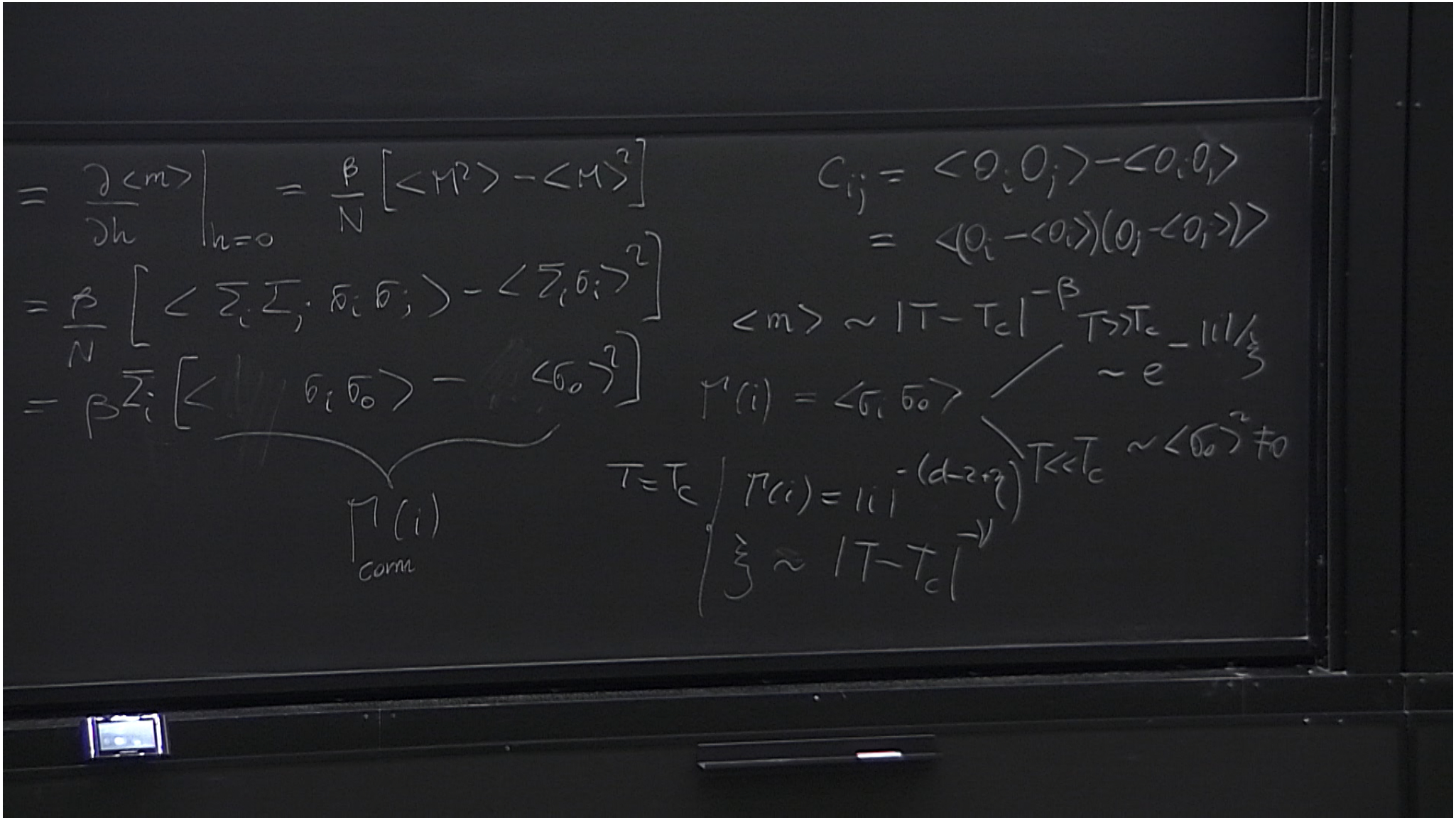
$$\chi = \left. \frac{\partial \langle m \rangle}{\partial h} \right|_{h=0} =$$

$$= \frac{\beta}{N} \left[\langle \Sigma_i \Sigma_j$$

$$= \beta \sum_i \left[\langle$$

critical exp. Ising 2D

$\beta = \frac{1}{8}$; $\eta = \frac{1}{4}$


$= \frac{\partial \langle m \rangle}{\partial h}\Big|_{h=0} = \frac{\beta}{N}\left[\langle M^2 \rangle - \langle M \rangle^2\right]$
$= \frac{\beta}{N}\left[\langle \Sigma_i \Sigma_j \sigma_i \sigma_j \rangle - \langle \Sigma_i \sigma_i \rangle^2\right]$
$= \beta \Sigma_i \left[\langle \sigma_i \sigma_0 \rangle - \langle \sigma_0 \rangle^2\right]$
$\Gamma(i)$
corrn
$C_{ij} = \langle \sigma_i \sigma_j \rangle - \langle \sigma_i \sigma_j \rangle$
$= \langle (\sigma_i - \langle \sigma_i \rangle)(\sigma_j - \langle \sigma_i \rangle) \rangle$
$\langle m \rangle \sim |T - T_c|^{-\beta}$
$\Gamma(i) = \langle \sigma_i \sigma_0 \rangle$
$T \gg T_c$
$\sim e^{-|i|/\xi}$
$T \ll T_c$
$\sim \langle \sigma_0 \rangle^2 \neq 0$
$T = T_c$
$\Gamma(i) = |i|^{-(d-2+\eta)}$
$\xi \sim |T - T_c|^{-\nu}$

$$F = -kT \log Z_1$$

$$Z_1 = \sum_{\{\text{confs}\}} e^{-\beta H}$$

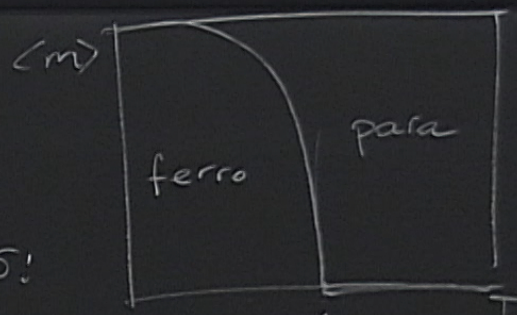
$$H = -J \sum_{(ij)} \sigma_i \sigma_j - h \sum_i \sigma_i$$

$$\sim M$$

$$\langle m \rangle = \frac{1}{N} \langle M \rangle = \frac{1}{N} \frac{\partial F}{\partial h} = \frac{1}{N} \sum_{\{\sigma_i\}} M \frac{e^{-\beta H}}{Z_1}$$

critical exp. Ising 2D

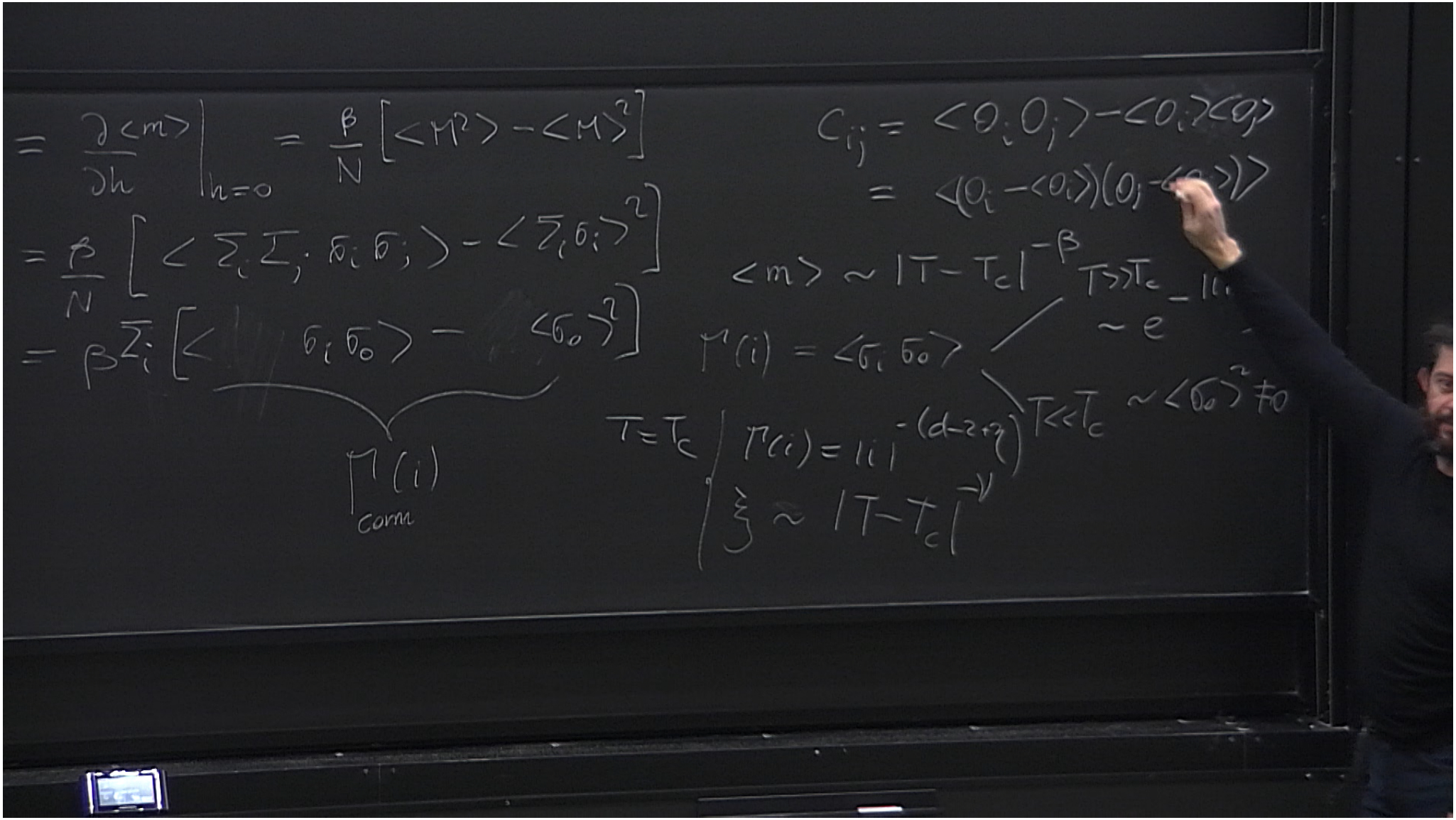
$\beta = \frac{1}{8}$; $\eta = \frac{1}{4}$, $\gamma = 2$

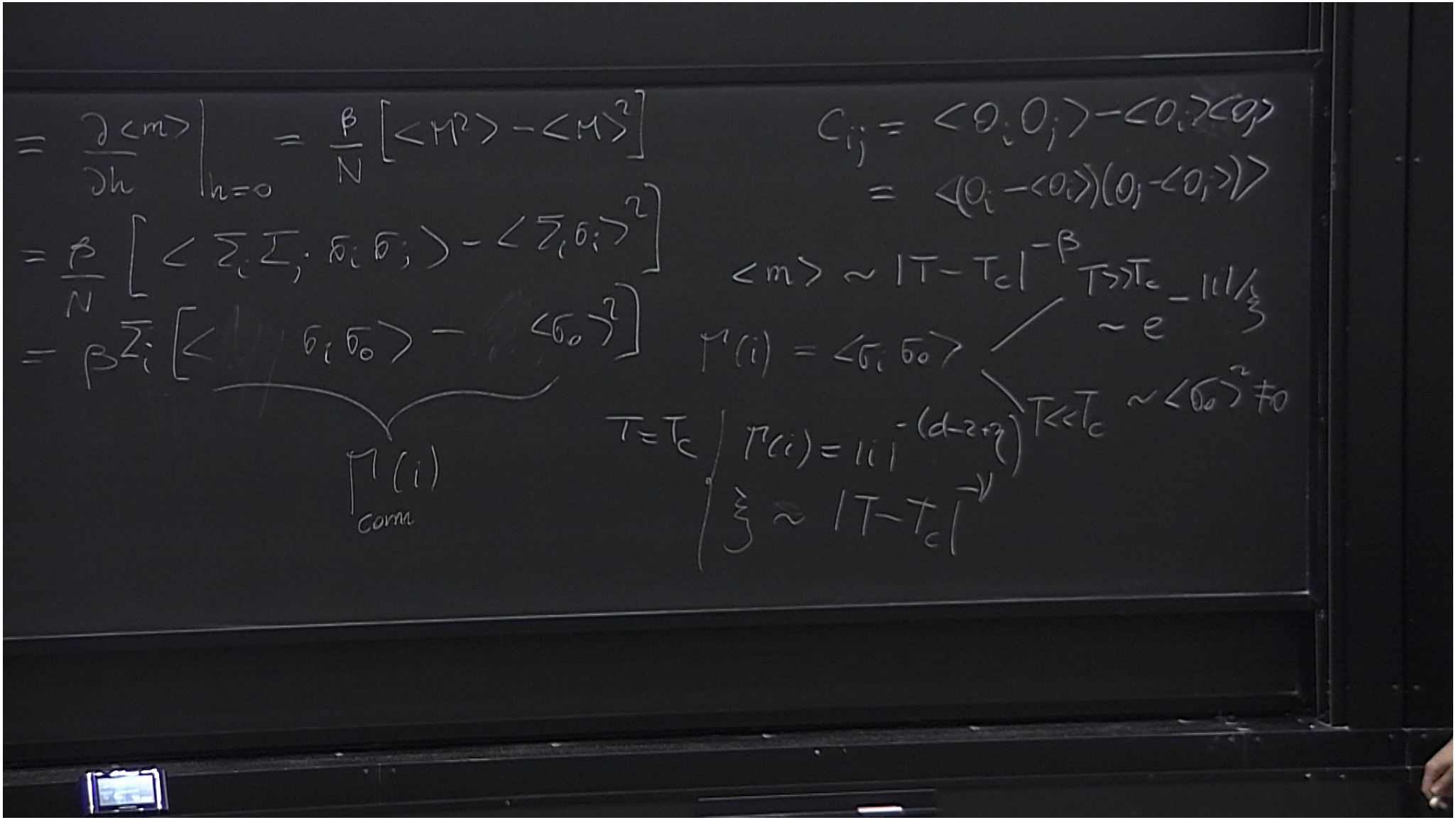


$$\chi = \left. \frac{\partial \langle m \rangle}{\partial h} \right|_{h=0} =$$

$$= \frac{\beta}{N} \left[\langle \Sigma_i \Sigma_j$$

$$= \beta \Sigma_i$$


$= \frac{\partial \langle m \rangle}{\partial h}\Big|_{h=0} = \frac{\beta}{N}\left[\langle M^2 \rangle - \langle M \rangle^2\right]$
$= \frac{\beta}{N}\left[\langle \Sigma_i \Sigma_j \sigma_i \sigma_j \rangle - \langle \Sigma_i \sigma_i \rangle^2\right]$
$= \beta \Sigma_i \left[\langle \sigma_i \sigma_0 \rangle - \langle \sigma_0 \rangle^2\right]$
$\Gamma(i)$
corr
$C_{ij} = \langle \sigma_i \sigma_j \rangle - \langle \sigma_i \rangle \langle \sigma_j \rangle$
$= \langle (\sigma_i - \langle \sigma_i \rangle)(\sigma_j - \langle \sigma_j \rangle) \rangle$
$\langle m \rangle \sim |T - T_c|^{-\beta}$
$\Gamma(i) = \langle \sigma_i \sigma_0 \rangle$
$T \gg T_c$
$\sim e^{-}$
$T \ll T_c$ $\sim \langle \sigma_0 \rangle^2 \neq 0$
$T = T_c$
$\Gamma(i) = |i|^{-(d-2+\eta)}$
$\xi \sim |T - T_c|^{-\nu}$


$= \frac{\partial \langle m \rangle}{\partial h}\Big|_{h=0} = \frac{\beta}{N}\left[\langle M^2 \rangle - \langle M \rangle^2\right]$
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$T = T_c$ $\Gamma(i) = |i|^{-(d-2+\eta)}$
$\xi \sim |T - T_c|^{-\nu}$

$$F = -kT \log Z$$

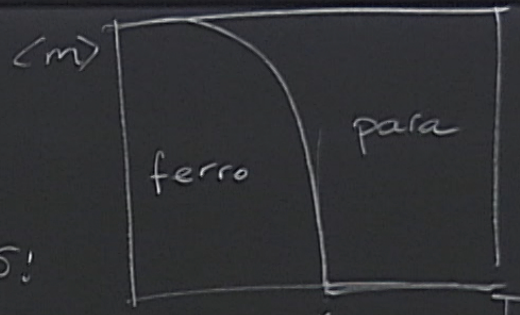
$$Z = \sum_{\{conf\}} e^{-\beta H}$$

$$H = -J \sum_{(ij)} \sigma_i \sigma_j - h \sum_i \sigma_i$$

$$\langle m \rangle = \frac{1}{N} \langle M \rangle = \frac{1}{N} \frac{\partial F}{\partial h} = \frac{1}{N} \sum_{\{\sigma_i\}} M \frac{e^{-\beta H}}{Z}$$

critical exp. Ising 2D

$\beta = \frac{1}{8}$; $\alpha = \frac{1}{4}$, $\gamma = 2$



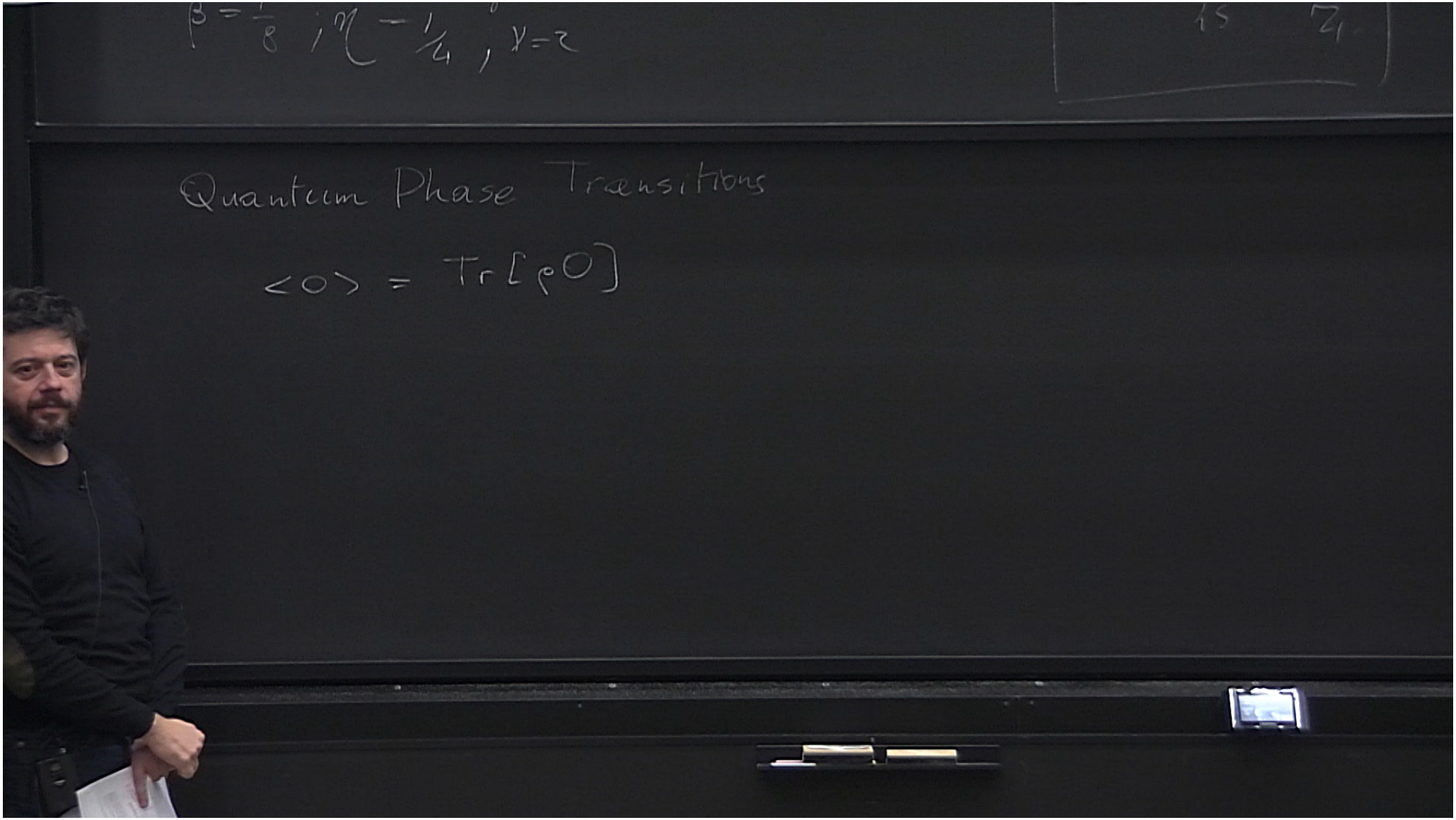
$$\chi = \left. \frac{\partial \langle m \rangle}{\partial h} \right|_{h=0} = \frac{\beta}{N} \left[\right.$$

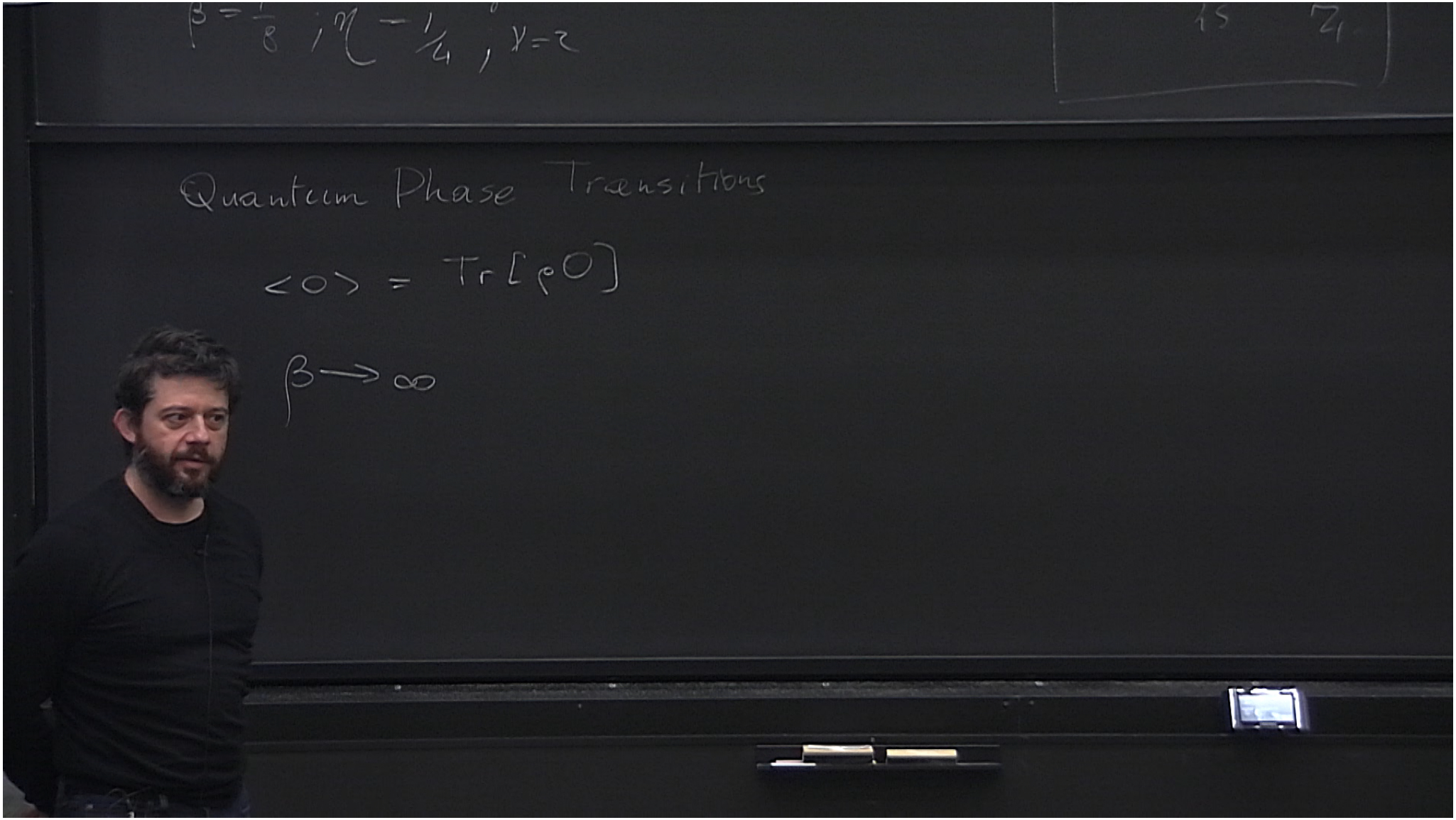
$$= \frac{\beta}{N} \left[\langle \sum_i \sum_j \sigma_i \sigma \right.$$

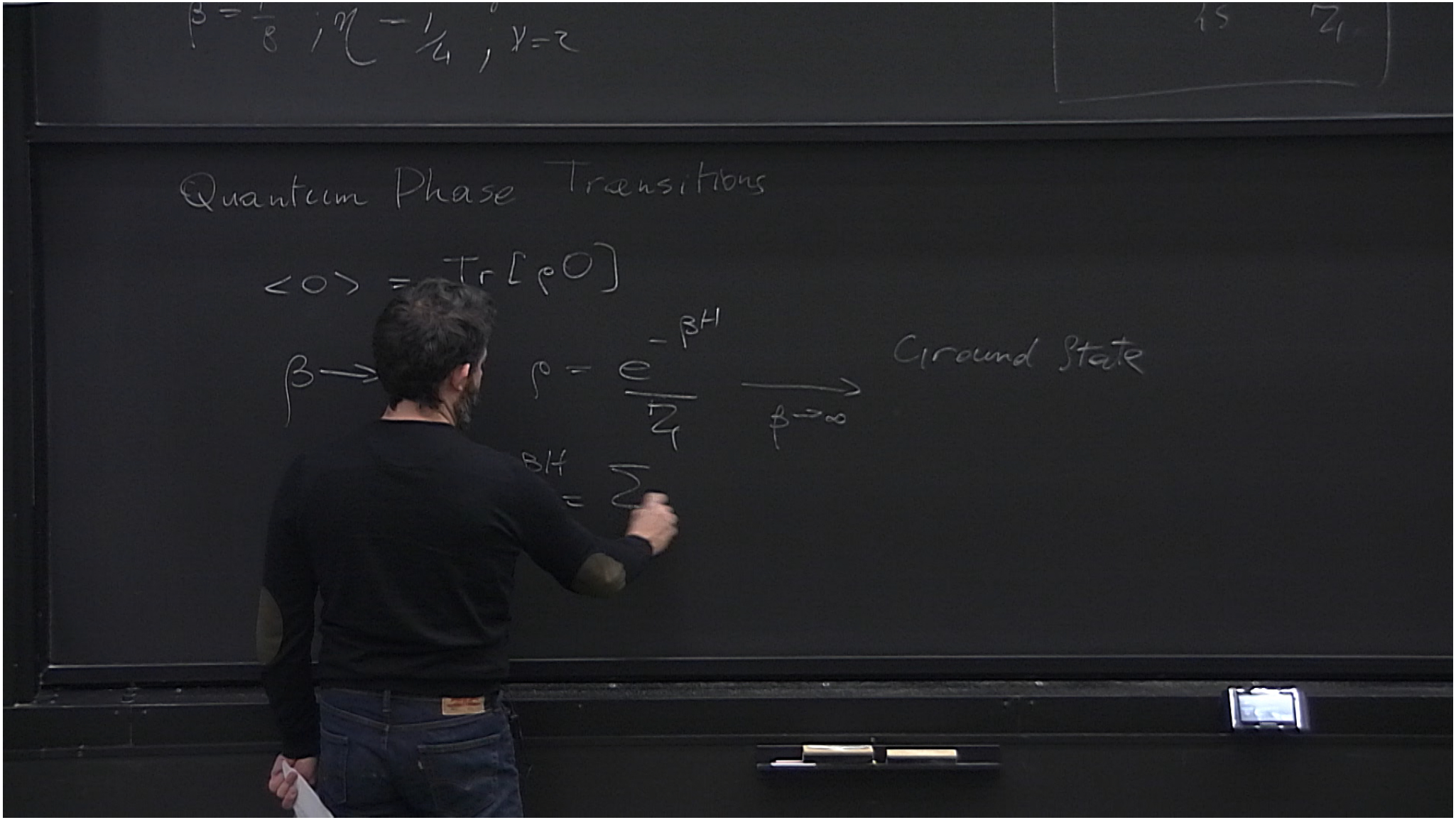
$$= \beta \sum_i \left[\langle \quad \sigma_i \sigma_0 \rangle \right.$$

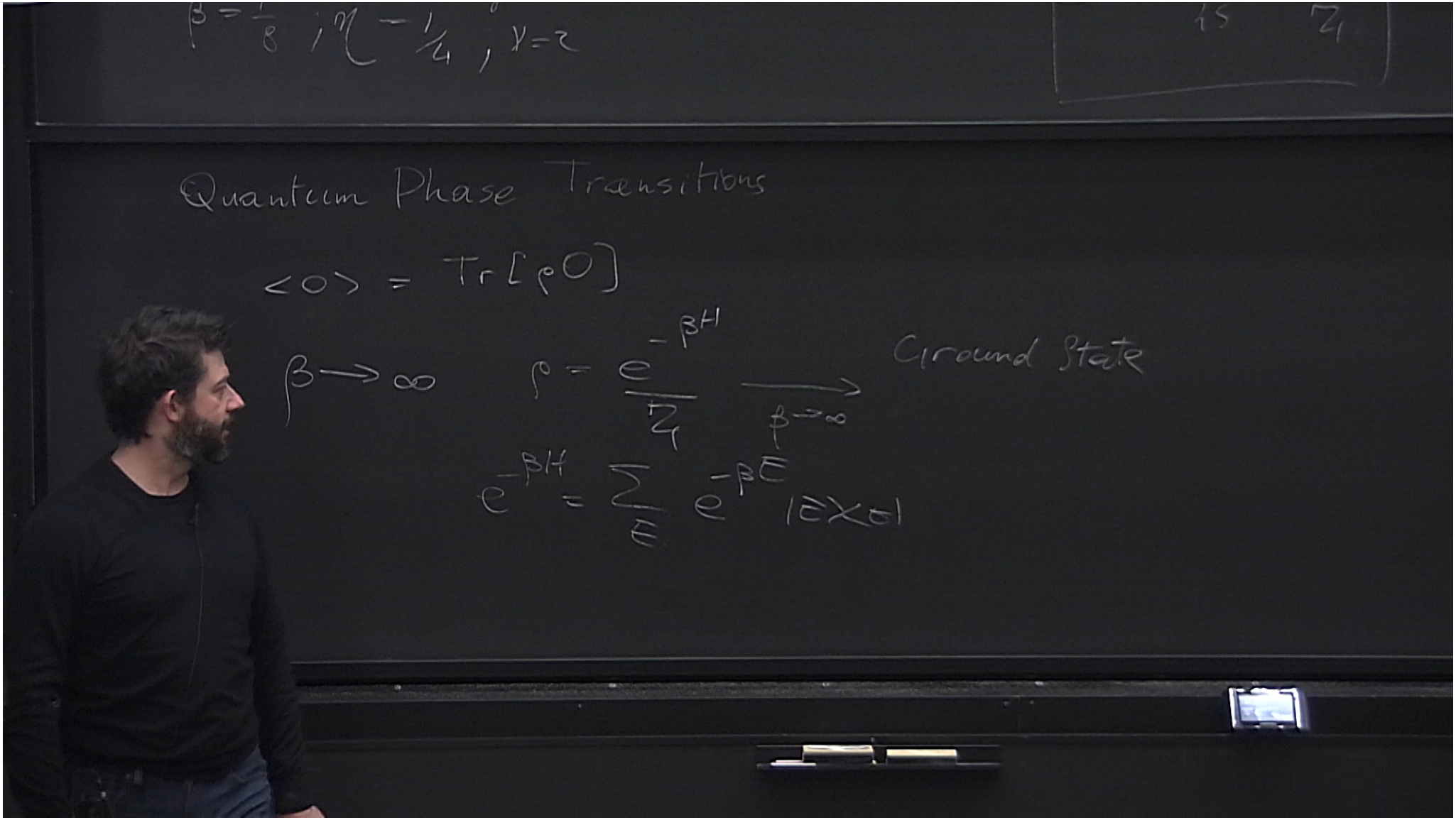
$$\langle \dots \rangle = \sum_{\{\sigma_i\}} \dots \frac{e^{-\beta H}}{Z}$$

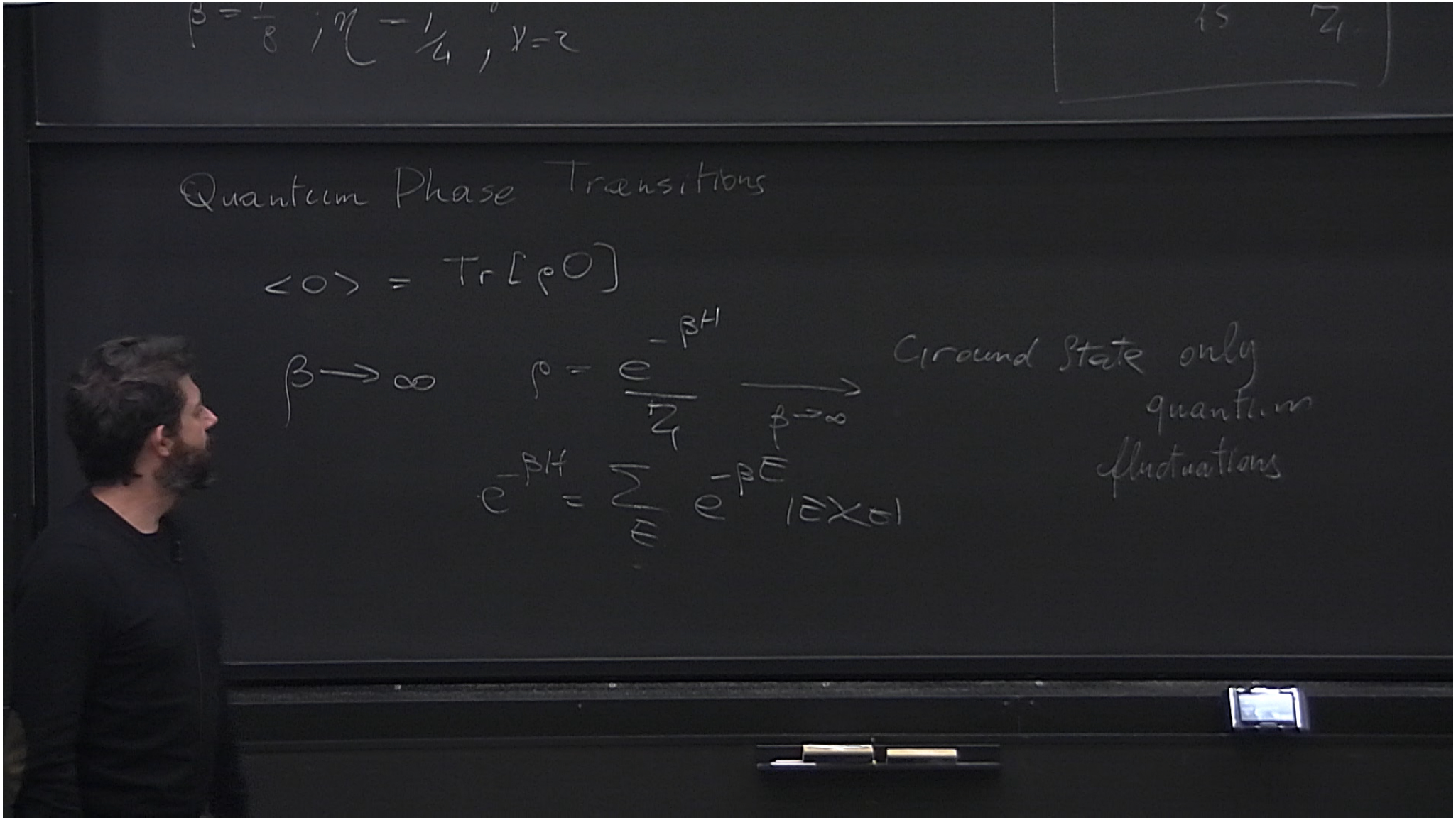
Corre

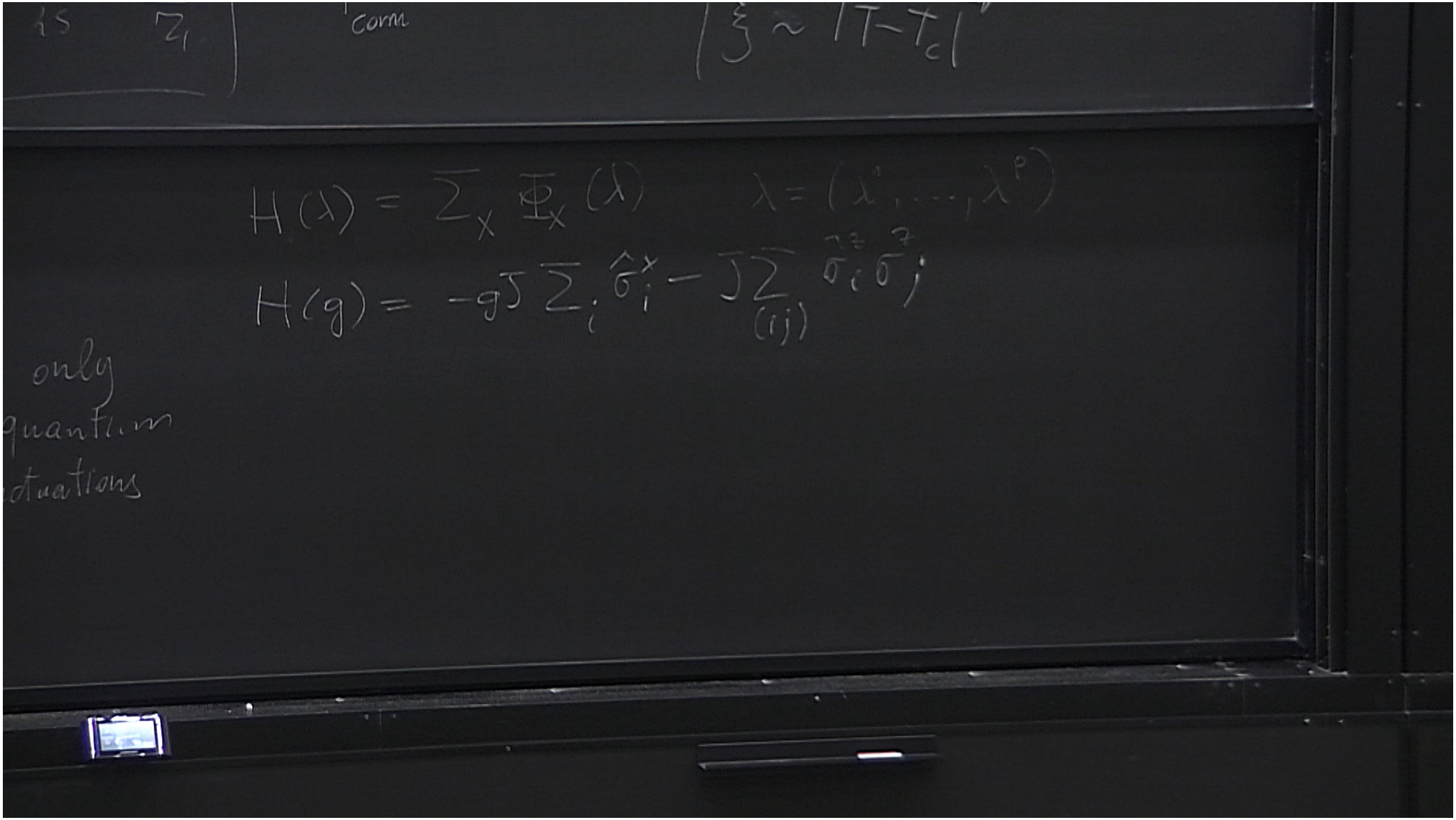

β = 1/8 ; η = −1/4 , γ = 2
Quantum Phase Transitions
⟨O⟩ = Tr[ρO]


Quantum Phase Transitions
$\langle O \rangle = Tr[\rho O]$
$\beta \to \infty$


Quantum Phase Transitions
$\langle O \rangle = Tr[\rho O]$
$\beta \to \infty$
$\rho = \frac{e^{-\beta H}}{Z}$
$\beta \to \infty$
Ground State
$Z = \sum e^{-\beta H}$


Quantum Phase Transitions
$\langle O \rangle = Tr[\rho O]$
$\beta \to \infty$
$\rho = \frac{e^{-\beta H}}{Z}$
$\beta \to \infty$
Ground State
$e^{-\beta H} = \sum_E e^{-\beta E} |E\rangle\langle E|$


Quantum Phase Transitions
$\langle O \rangle = Tr[\rho O]$
$\beta \to \infty$
$\rho = \frac{e^{-\beta H}}{Z}$
$\beta \to \infty$
Ground State only
quantum
fluctuations
$e^{-\beta H} = \sum_E e^{-\beta E} |E\rangle\langle E|$


$\xi \sim |T - T_c|^{\nu}$
$H(\lambda) = \sum_x \Phi_x(\lambda) \qquad \lambda = (\lambda^1, \ldots, \lambda^p)$
$H(g) = -gJ \sum_i \hat{\sigma}_i^x - J \sum_{(ij)} \hat{\sigma}_i^z \hat{\sigma}_j^z$
only
quantum
fluctuations

$\xi \sim |T - T_c|^{-\nu}$

ξ_{corr}

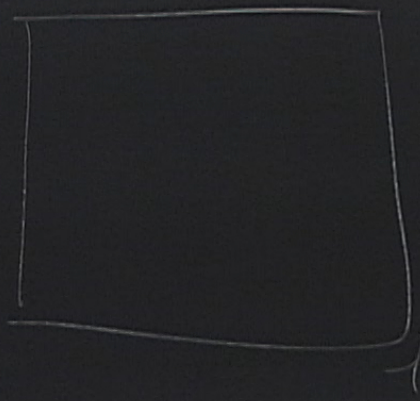
Z_1

$H(\lambda) = \sum_x \Phi_x(\lambda) \qquad \lambda = (\lambda^1, \ldots, \lambda^p)$

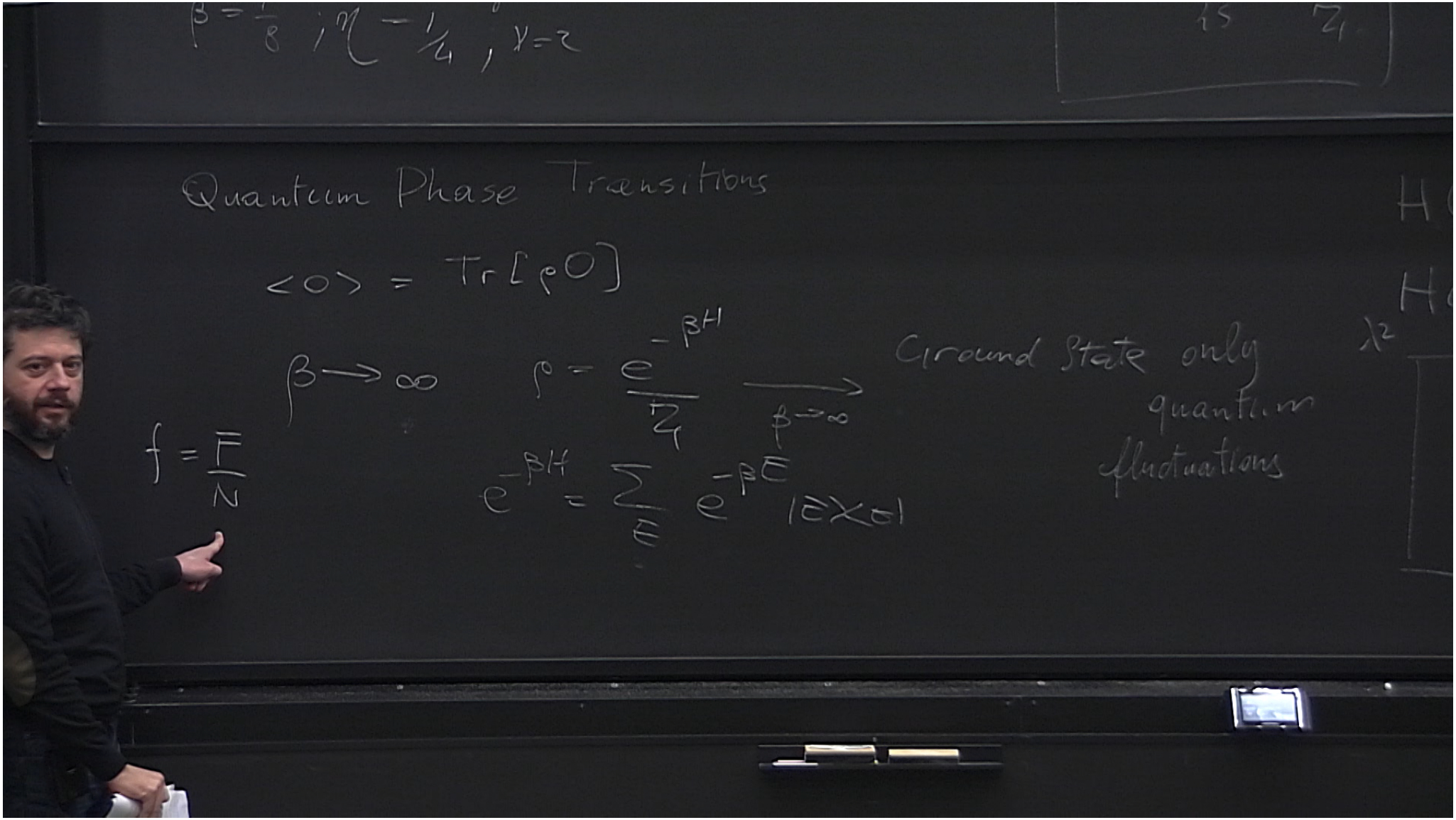
$H(g) = -gJ \sum_i \hat{\sigma}_i^x - J \sum_{(ij)} \hat{\sigma}_i^z \hat{\sigma}_j^z$

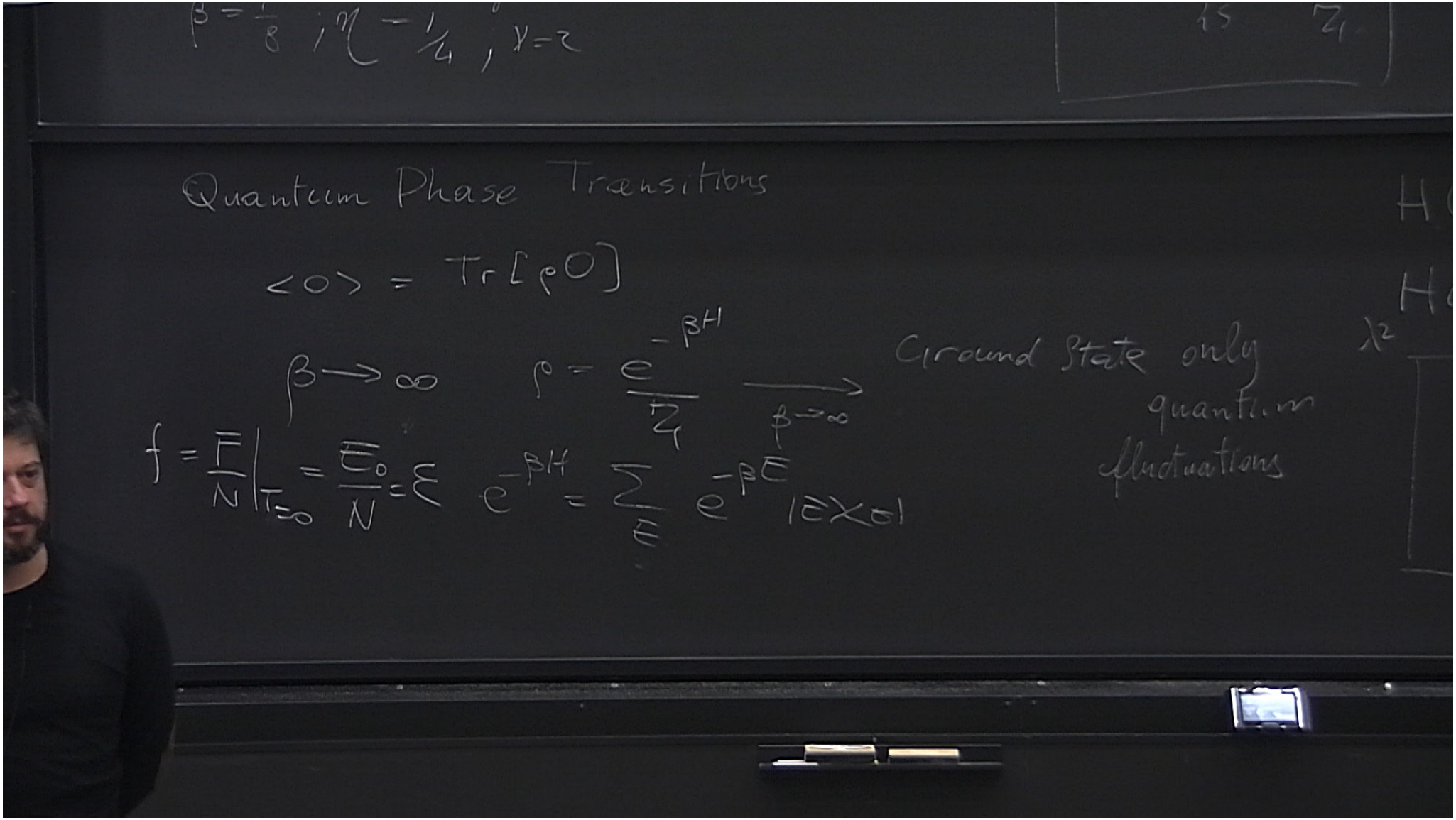
only quantum fluctuations

λ^2



λ^1


Quantum Phase Transitions
$\langle O \rangle = Tr[\rho O]$
$\beta \to \infty$
$\rho = \frac{e^{-\beta H}}{Z}$
$\beta \to \infty$
Ground State only
quantum
fluctuations
$f = \frac{E}{N}$
$e^{-\beta H} = \sum_E e^{-\beta E}$
$|E\rangle\langle E|$


Quantum Phase Transitions
$\langle O \rangle = Tr[\rho O]$
$\beta \to \infty \quad \rho = \frac{e^{-\beta H}}{Z} \xrightarrow{\beta \to \infty}$
Ground State only
quantum
fluctuations
$f = \frac{F}{N}\Big|_{T=0} = \frac{E_0}{N} = \mathcal{E} \quad e^{-\beta H} = \sum_E e^{-\beta E} |E\rangle\langle E|$

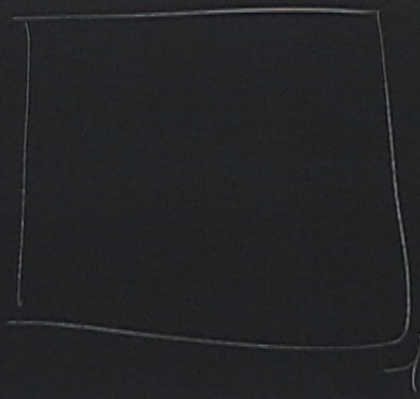
$\xi \sim |T - T_c|^{\nu}$

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$H(g) = -gJ \sum_i \hat{\sigma}_i^x - J \sum_{(ij)} \hat{\sigma}_i^z \hat{\sigma}_j^z$

λ^2

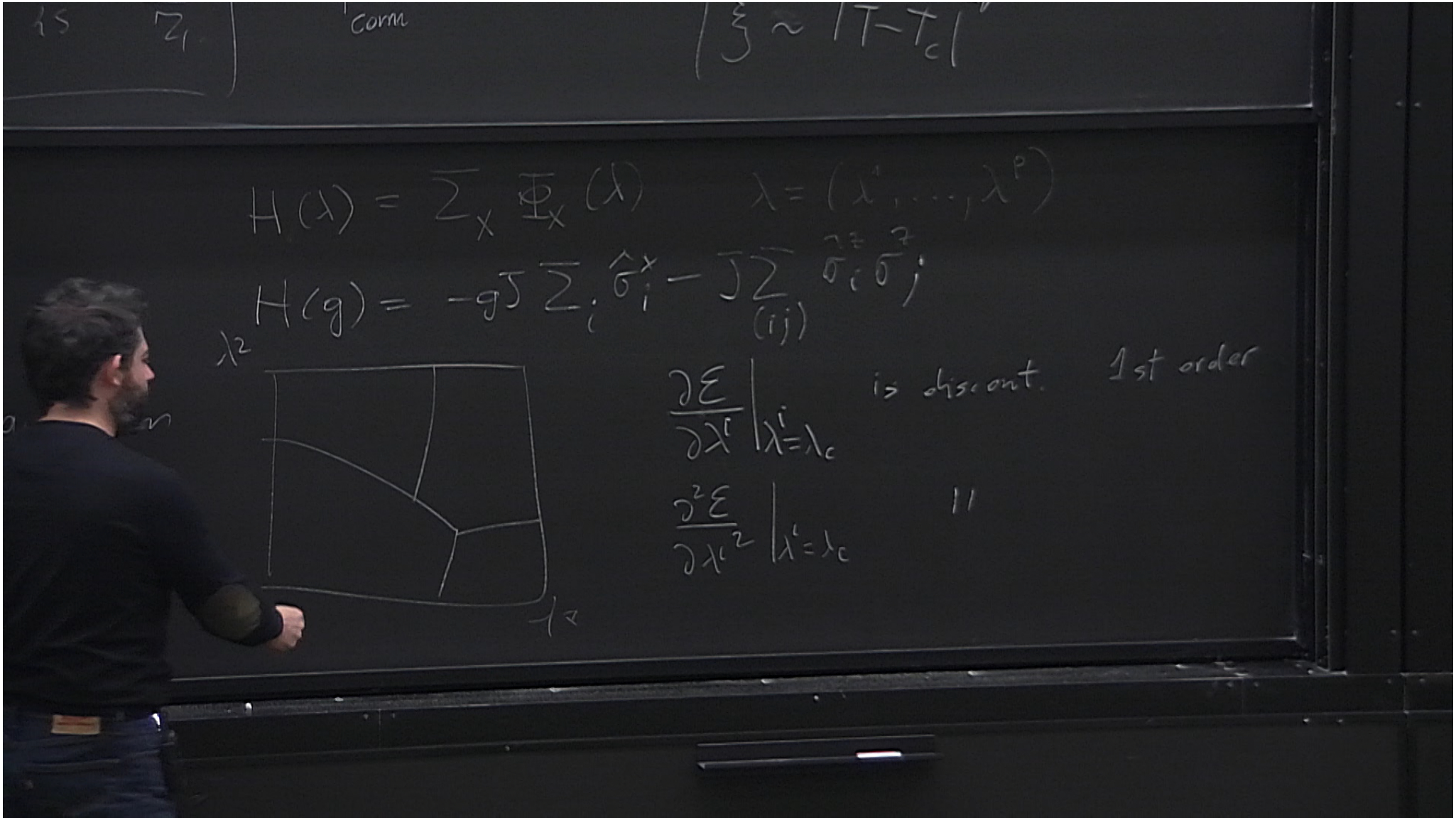
only quantum fluctuations



λ^1

$\left.\frac{\partial \mathcal{E}}{\partial \lambda^i}\right|_{\lambda^i = \lambda_c}$ is discont.

$\left.\frac{\partial^2 \mathcal{E}}{\partial \lambda^{i2}}\right|_{\lambda^i = \lambda_c}$ //

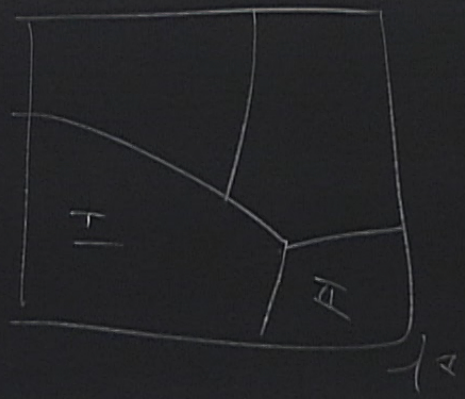

$\xi \sim |T - T_c|^{\nu}$
$H(\lambda) = \sum_x \Phi_x(\lambda)$
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$H(g) = -gJ \sum_i \hat{\sigma}_i^x - J \sum_{(ij)} \hat{\sigma}_i^z \hat{\sigma}_j^z$
λ^2
λ^1
$\frac{\partial \mathcal{E}}{\partial \lambda^i}\Big|_{\lambda^i = \lambda_c}$ is discont. 1st order
$\frac{\partial^2 \mathcal{E}}{\partial \lambda^{i2}}\Big|_{\lambda^i = \lambda_c}$ "

$\xi \sim |T - T_c|^{\nu}$

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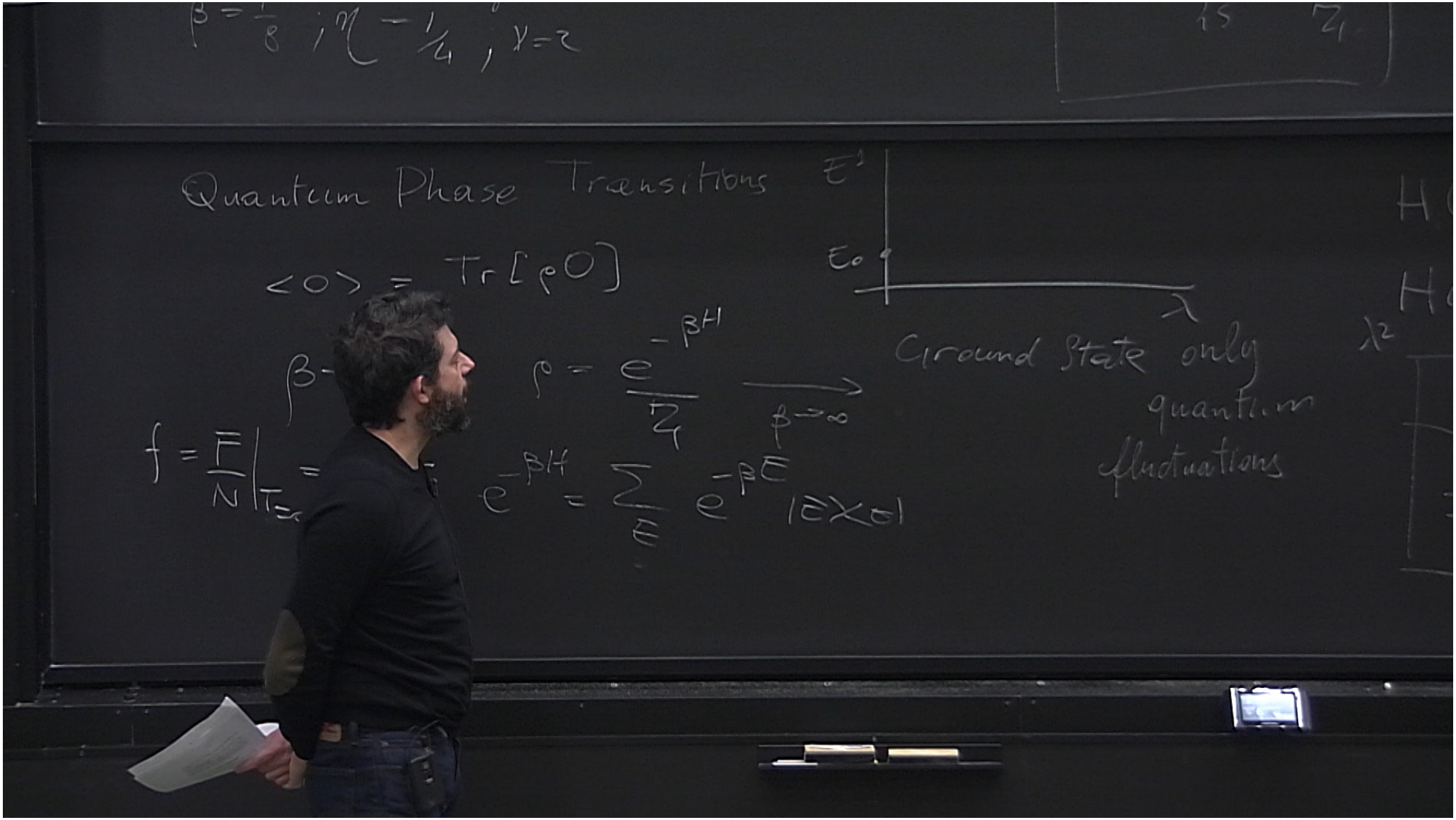
$H(g) = -gJ \sum_i \hat{\sigma}_i^x - J \sum_{(ij)} \hat{\sigma}_i^z \hat{\sigma}_j^z$

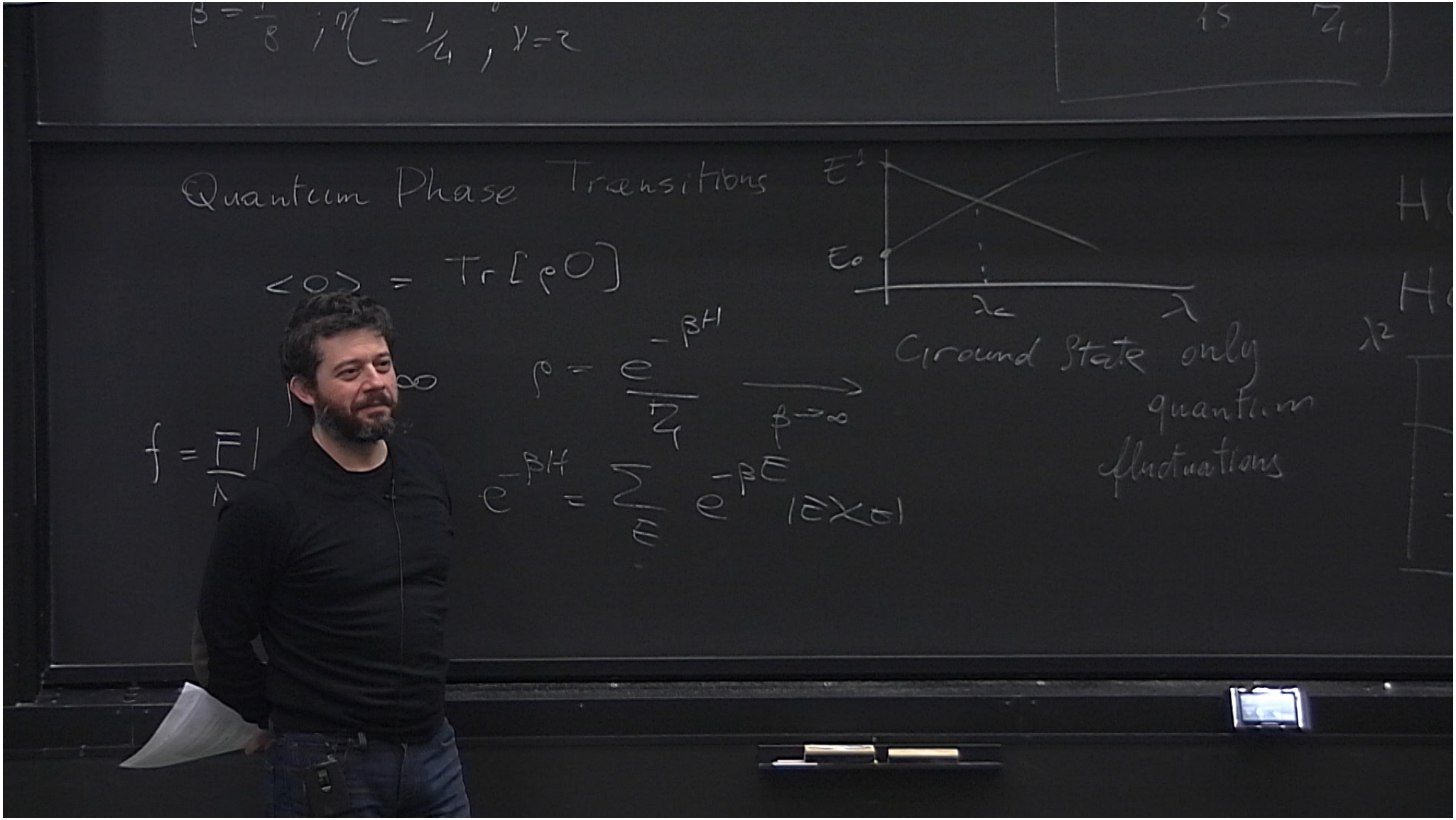
only quantum fluctuations

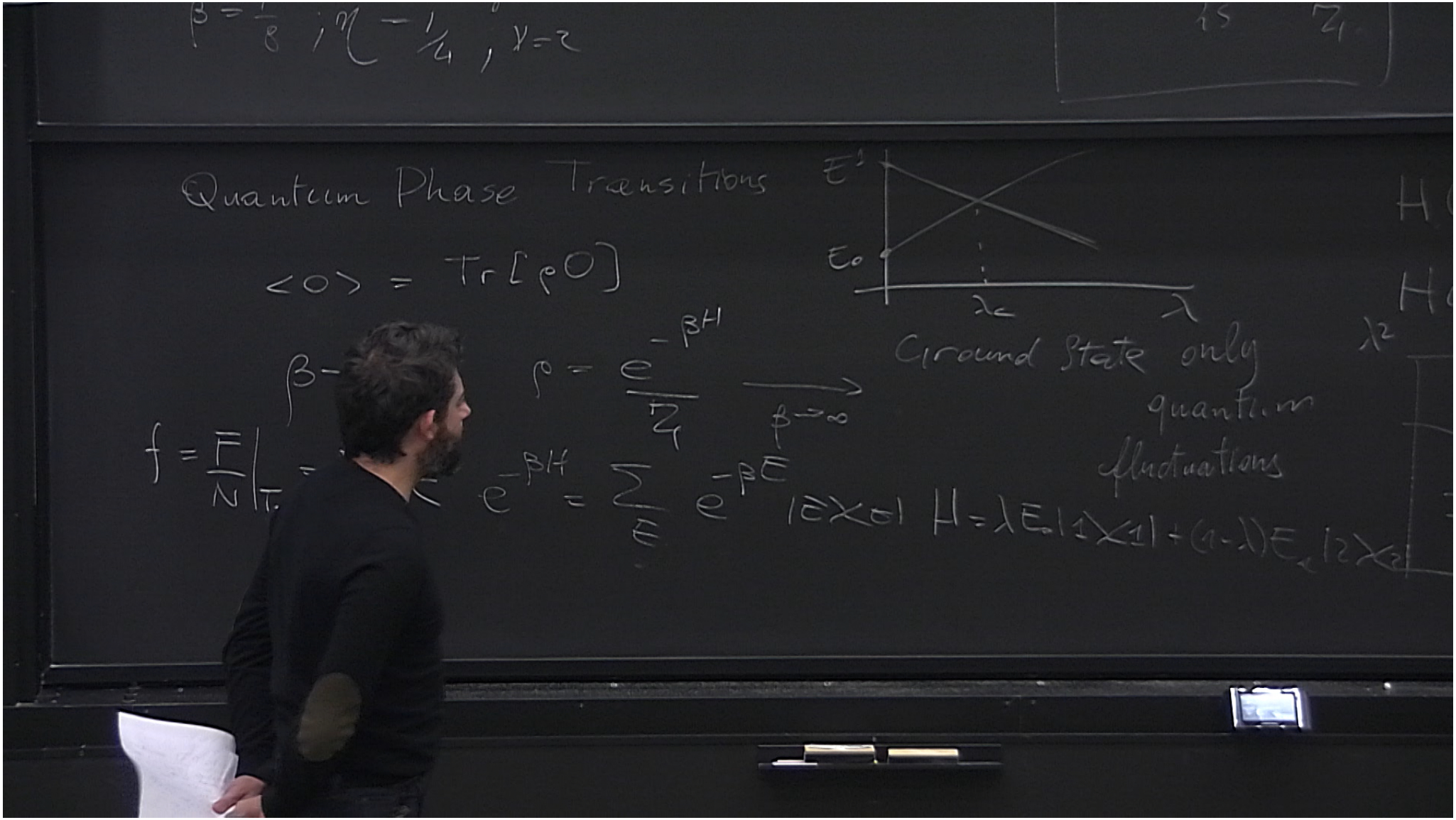


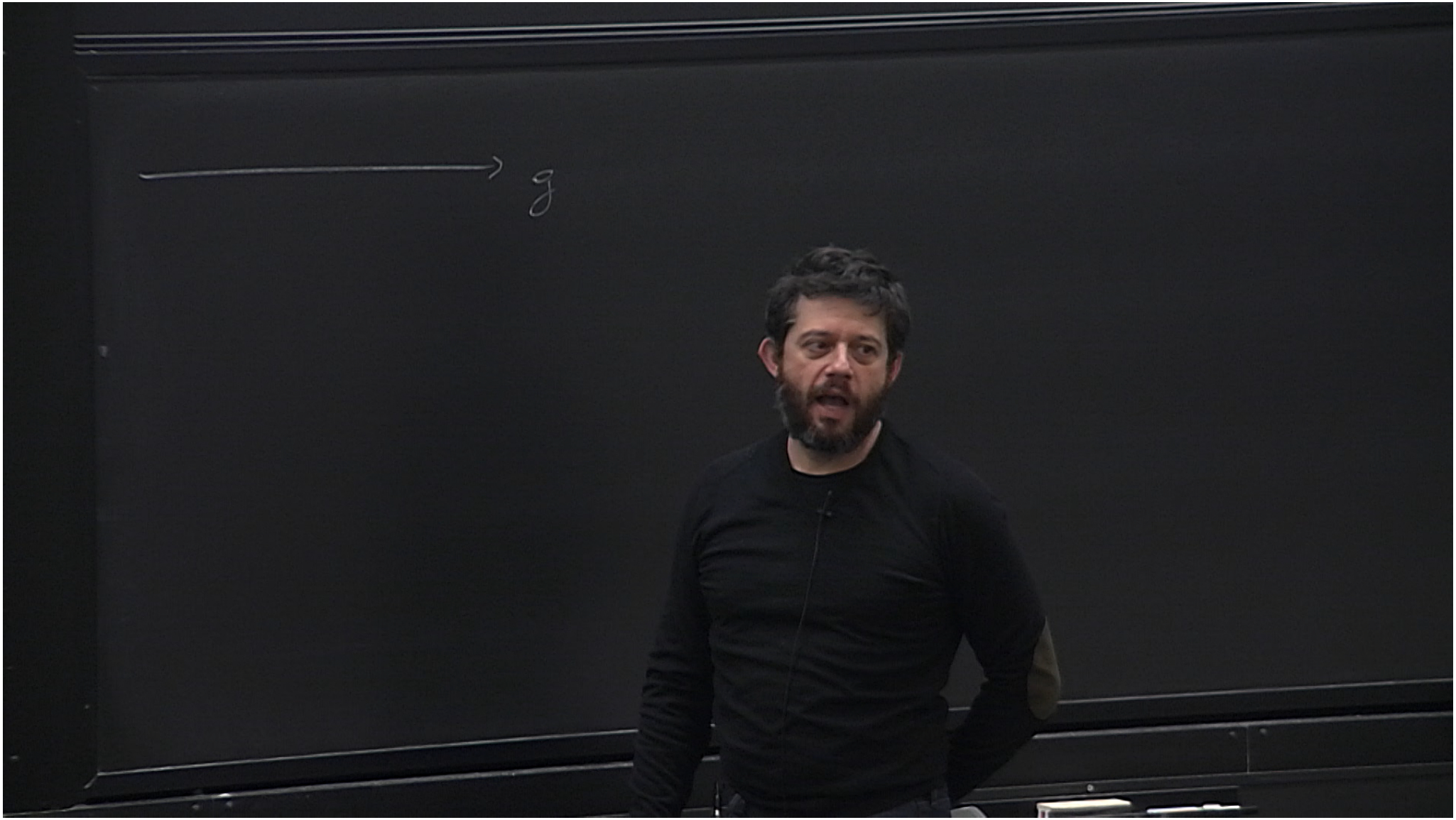
$\left.\frac{\partial \mathcal{E}}{\partial \lambda^i}\right|_{\lambda^i = \lambda_c}$ is discont. 1st order

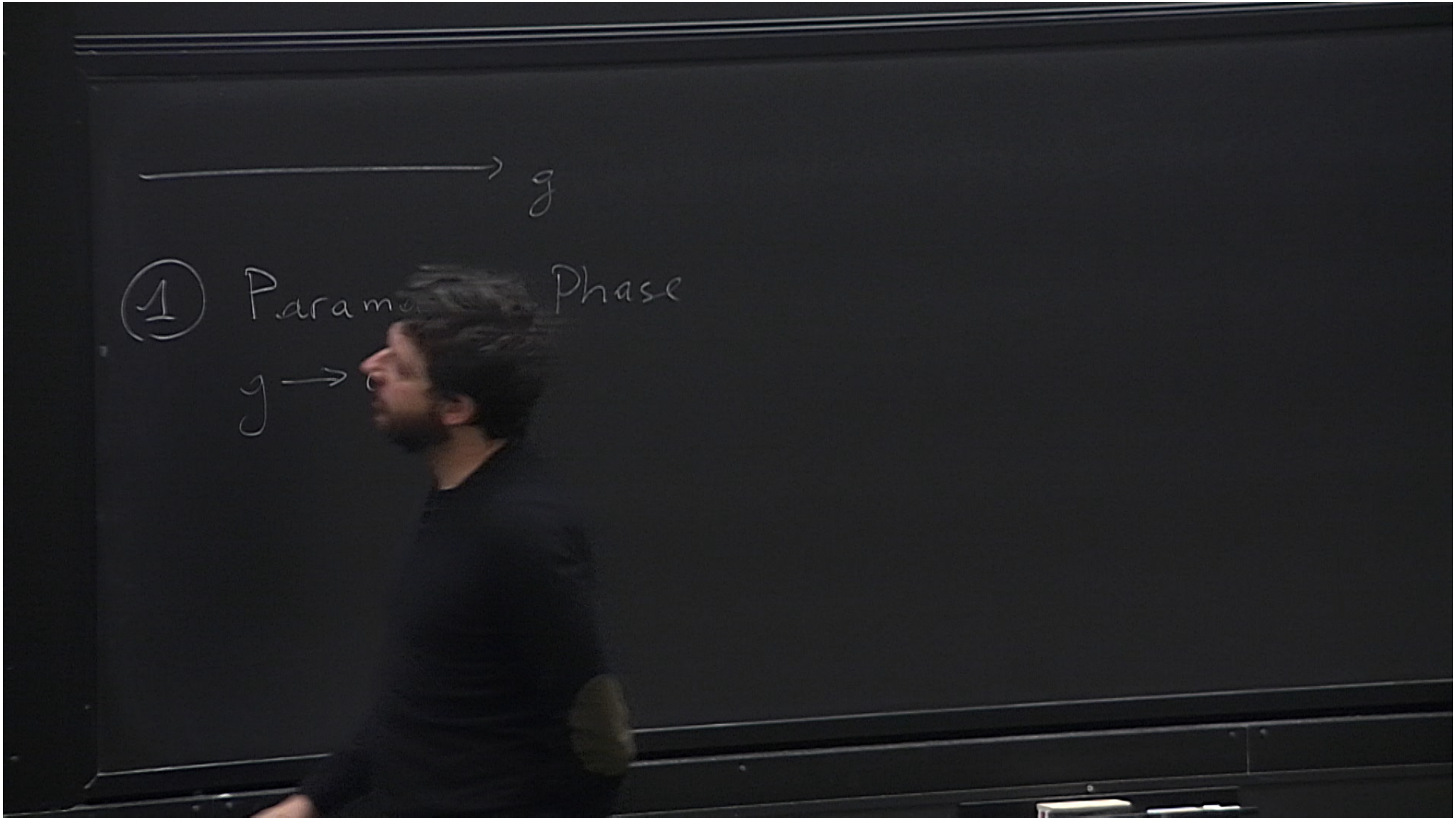
$\left.\frac{\partial^2 \mathcal{E}}{\partial \lambda^{i2}}\right|_{\lambda^i = \lambda_c}$ "

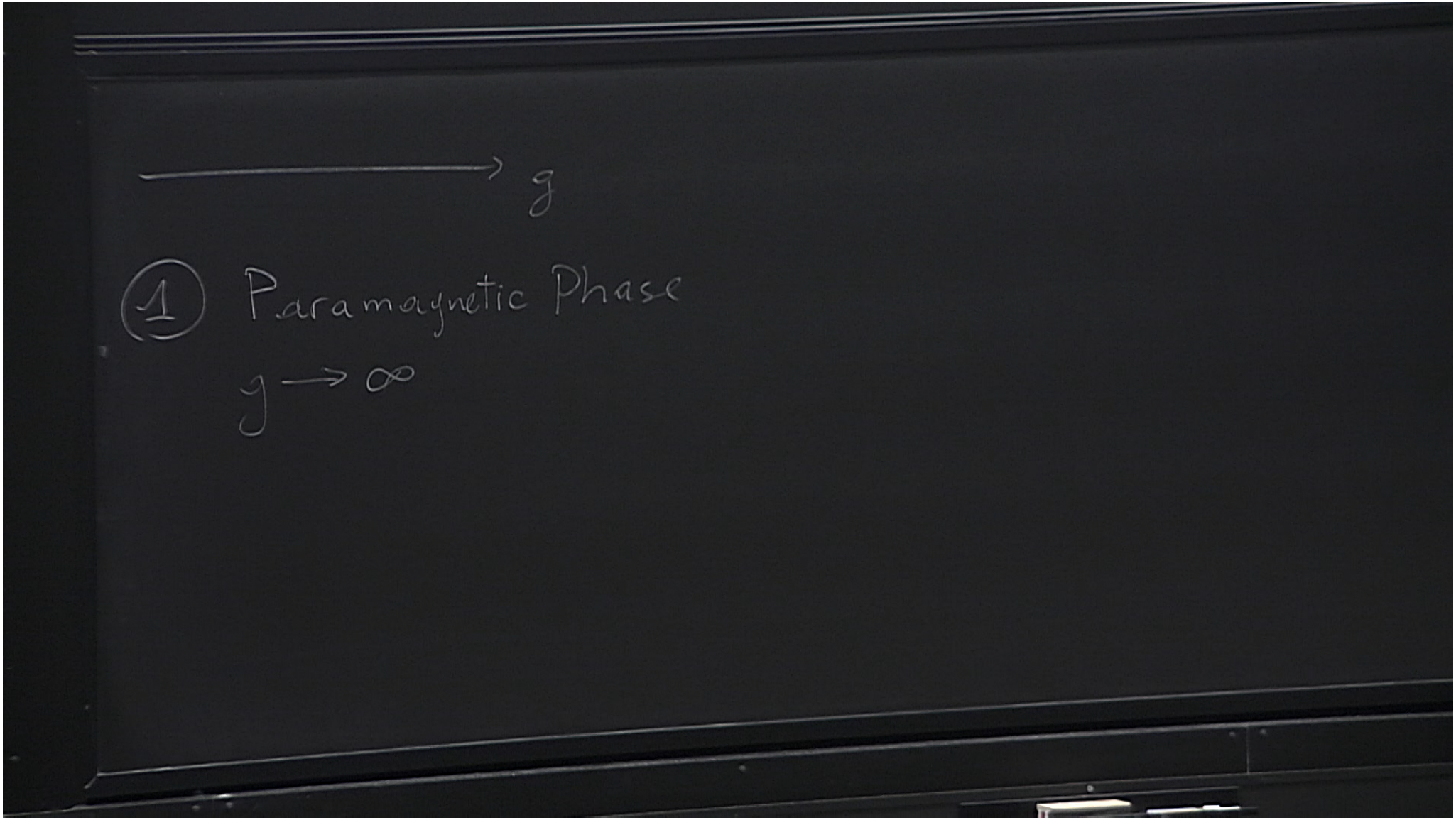

Quantum Phase Transitions
$\langle O \rangle = Tr[\rho O]$
$\rho = \frac{e^{-\beta H}}{Z}$
$\beta \to \infty$
$f = \frac{F}{N}$
$e^{-\beta H} = \sum_E e^{-\beta E} |E\rangle\langle E|$
E_0
λ
Ground State only
quantum
fluctuations


Quantum Phase Transitions
$\langle O \rangle = Tr[\rho O]$
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$f = \frac{F}{\Lambda}$
E_0
λ_c
λ
Ground State only
quantum
fluctuations


Quantum Phase Transitions
$\langle O \rangle = Tr[\rho O]$
$\rho = \frac{e^{-\beta H}}{Z}$
$\beta \to \infty$
$f = \frac{F}{N}\Big|_{T=0}$
$e^{-\beta H} = \sum_E e^{-\beta E}$
E_0
λ_c
λ
Ground State only
quantum
fluctuations
$H = \lambda E_0 |1\rangle\langle 1| + (1-\lambda) E_1 |2\rangle\langle 2|$


g


g
1
Phase


g
1 Paramagnetic Phase
g → ∞

Corm

$\xi \sim |T - T_c|^{-\nu}$

$H(\lambda) = \sum_x \Phi_x(\lambda)$ $\quad \lambda = (\lambda^1, \ldots, \lambda^p)$

$H(g) = -gJ \sum_i \hat{\sigma}_i^x - J \sum_{(ij)} \hat{\sigma}_i^z \hat{\sigma}_j^z$

$\sigma^z|0\rangle = |0\rangle$
$\sigma^z|1\rangle = -|1\rangle$

only
antum
uations

$(1-\lambda)E_x|2\rangle\langle 2|$

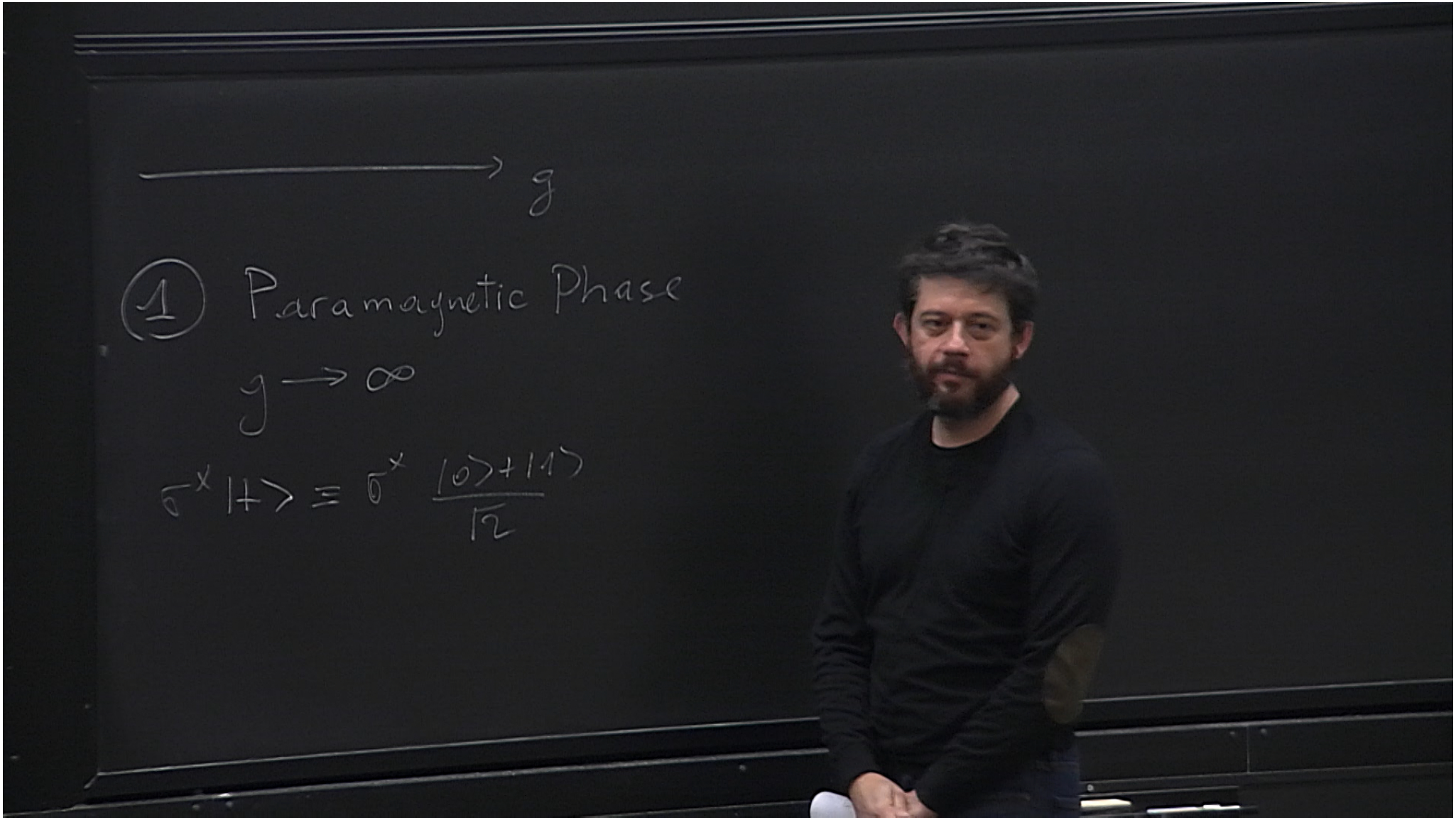
λ^2

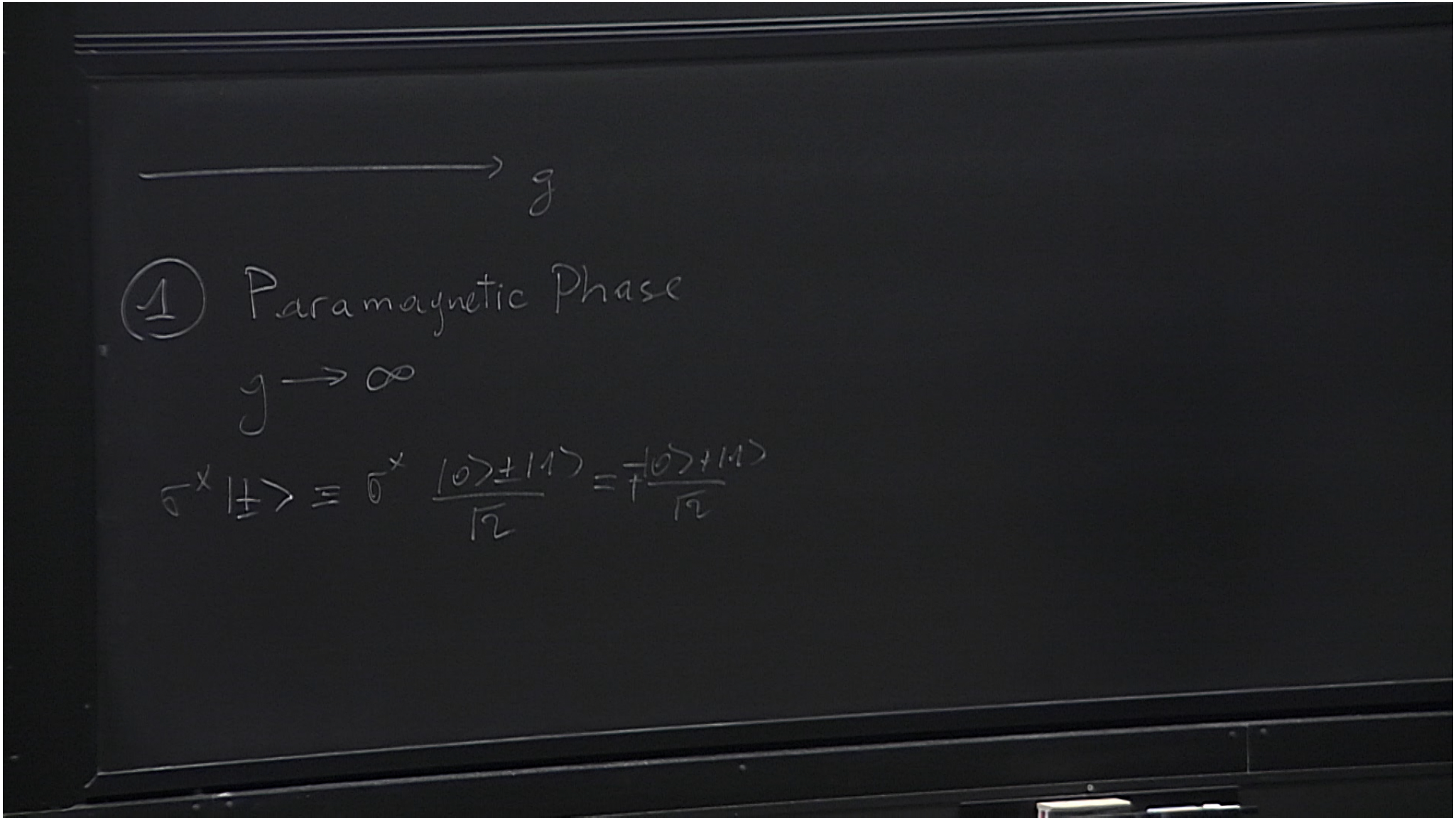


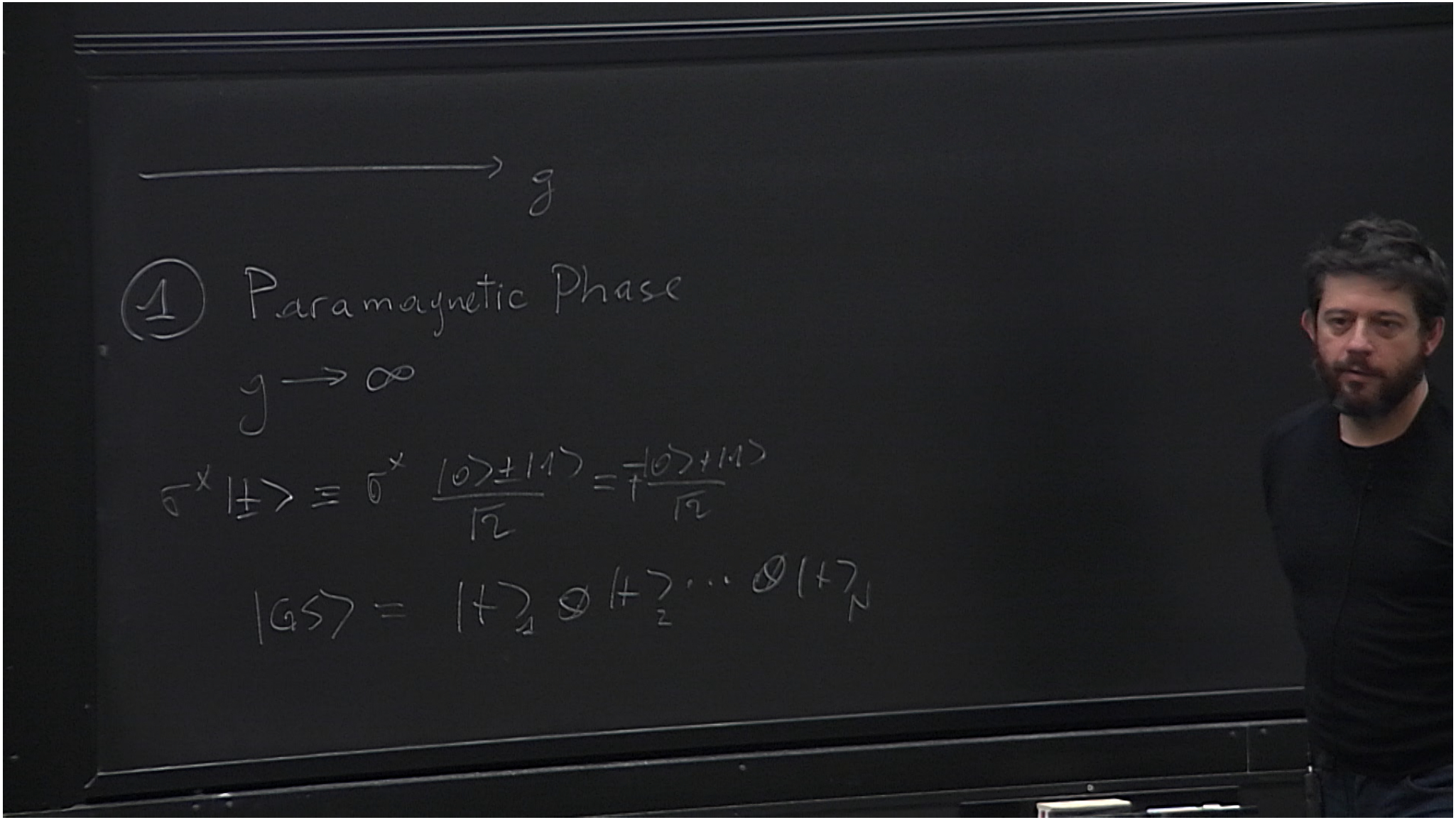
λ_c

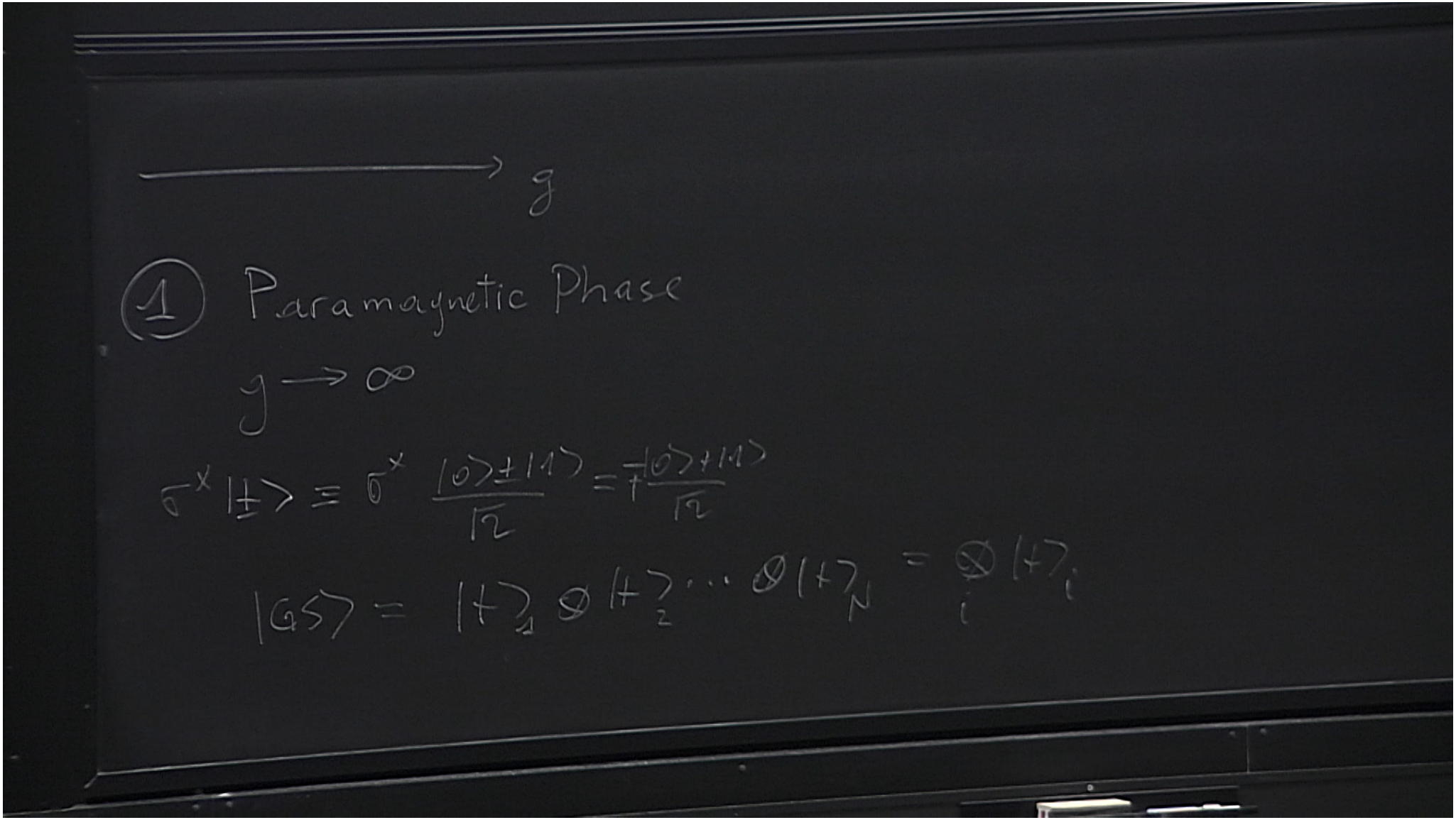
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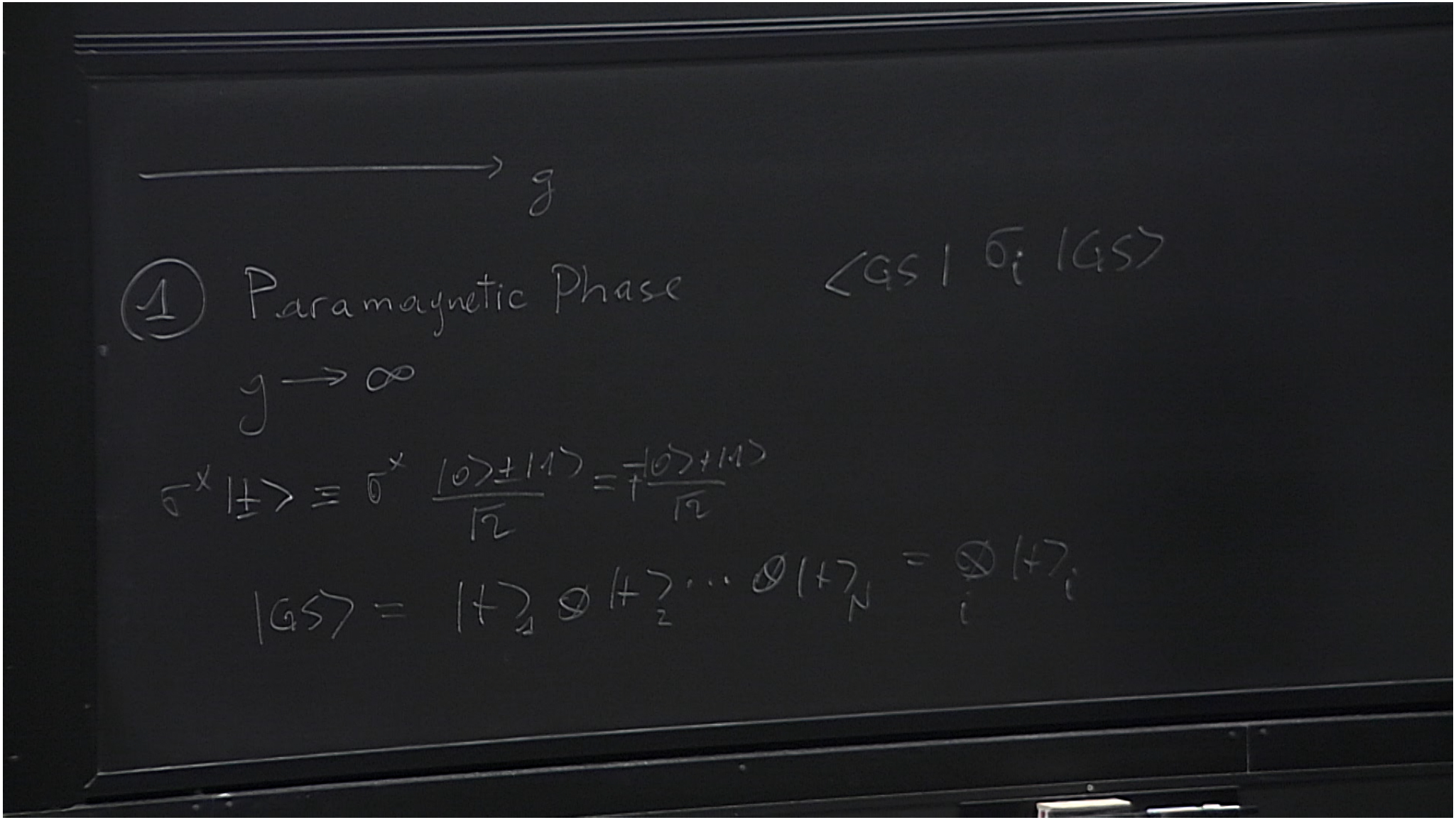
$\left.\frac{\partial^2 \mathcal{E}}{\partial \lambda^{i2}}\right|_{\lambda^i = \lambda_c}$ 11

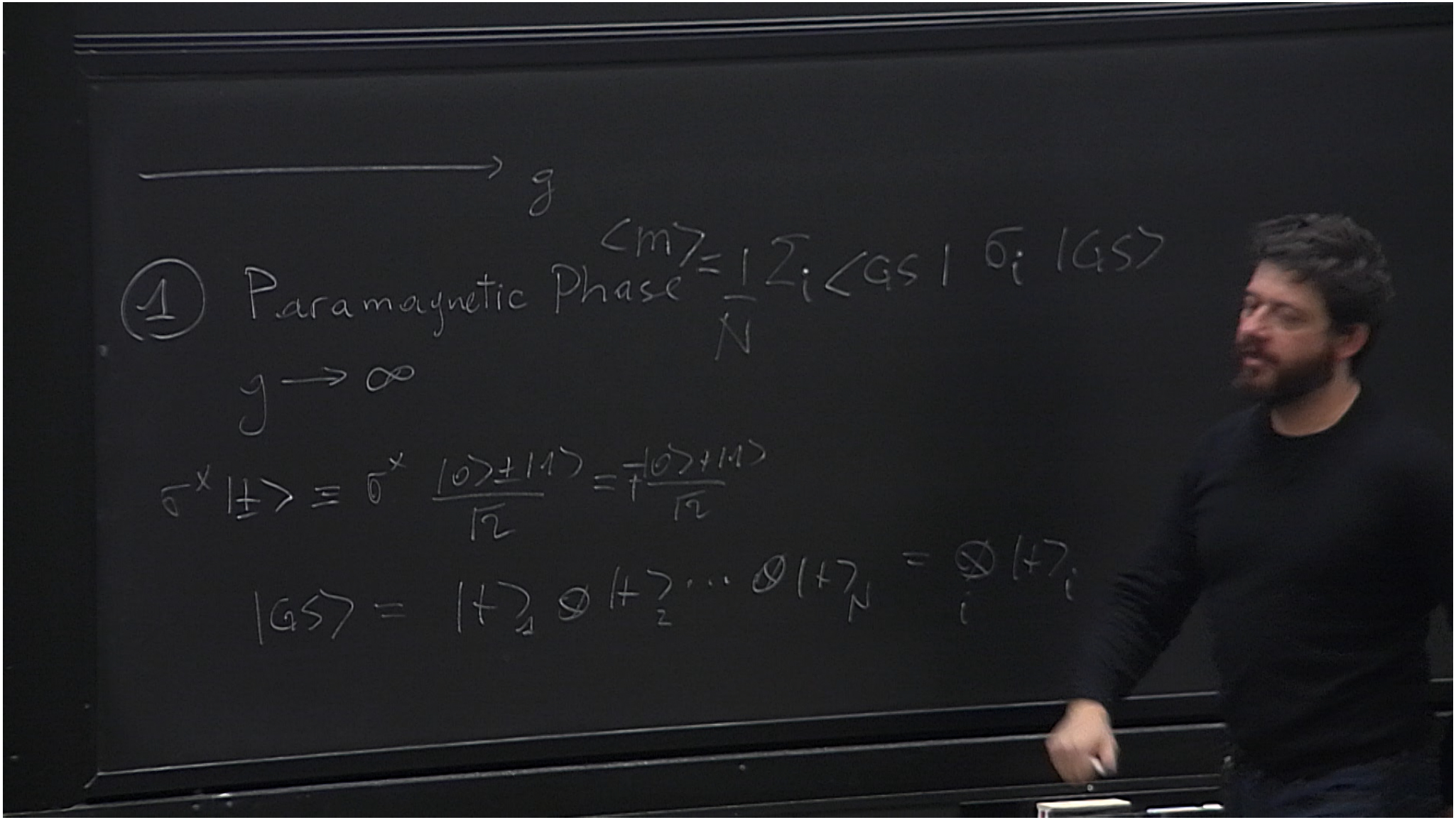

g
1 Paramagnetic Phase
g → ∞
σ^x |+> ≡ σ^x (|0>+|1>)/√2


g
1 Paramagnetic Phase
g → ∞
σ^x |±> ≡ σ^x (|0> ± |1>)/√2 = ∓(|0> + |1>)/√2


g
1 Paramagnetic Phase
g → ∞
σ^x |±> ≡ σ^x (|0> ± |1>)/√2 = ∓(|0> + |1>)/√2
|GS> = |+>_1 ⊗ |+>_2 ⋯ ⊗ |+>_N


g
(1) Paramagnetic Phase
g → ∞
σ^x |±> ≡ σ^x (|0> ± |1>)/√2 = ∓(|0> + |1>)/√2
|GS> = |+>_1 ⊗ |+>_2 ⋯ ⊗ |+>_N = ⊗_i |+>_i


g
(1) Paramagnetic Phase
⟨GS| σ_i |GS⟩
g → ∞
σ^x |±⟩ ≡ σ^x (|0⟩ ± |1⟩)/√2 = ∓(|0⟩ + |1⟩)/√2
|GS⟩ = |+⟩_1 ⊗ |+⟩_2 ⋯ ⊗ |+⟩_N = ⊗_i |+⟩_i


g
(1) Paramagnetic Phase
$\langle m \rangle = \frac{1}{N} \sum_i \langle GS | \sigma_i | GS \rangle$
$g \to \infty$
$\sigma^x |\pm\rangle \equiv \sigma^x \frac{|0\rangle \pm |1\rangle}{\sqrt{2}} = \pm \frac{|0\rangle + |1\rangle}{\sqrt{2}}$
$|GS\rangle = |+\rangle_1 \otimes |+\rangle_2 \cdots \otimes |+\rangle_N = \bigotimes_i |+\rangle_i$

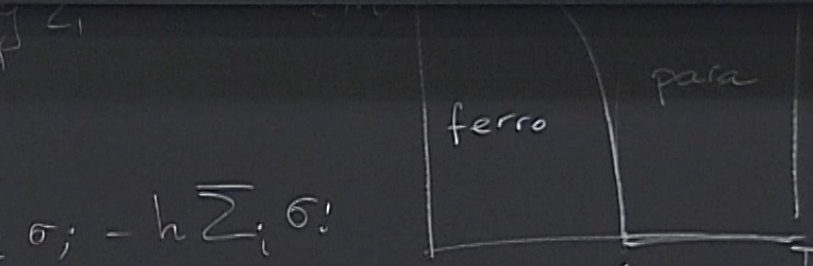
$$Z_i = \sum_{\{\text{conf}\}_i} e^{-\beta H_i}$$

$$H = -J \sum_{(i,j)} \sigma_i \sigma_j - h \sum_i \sigma_i$$

$$\langle m \rangle = \frac{1}{N} \langle M \rangle = \frac{1}{N} \frac{\partial F}{\partial h}\Big|_{h=0} = \frac{1}{N} \sum_{\{\sigma_i\}} M \frac{e^{-\beta H}}{Z_i}$$

critical exp. Ising 2D

$\beta = \frac{1}{8}$; $\eta = \frac{1}{4}$, $\gamma = 2$



$$\chi = \frac{\partial}{\partial h}\Big|_{h=0}$$

$$= \frac{\beta}{N} \left[\langle \Sigma_i \Sigma_j$$

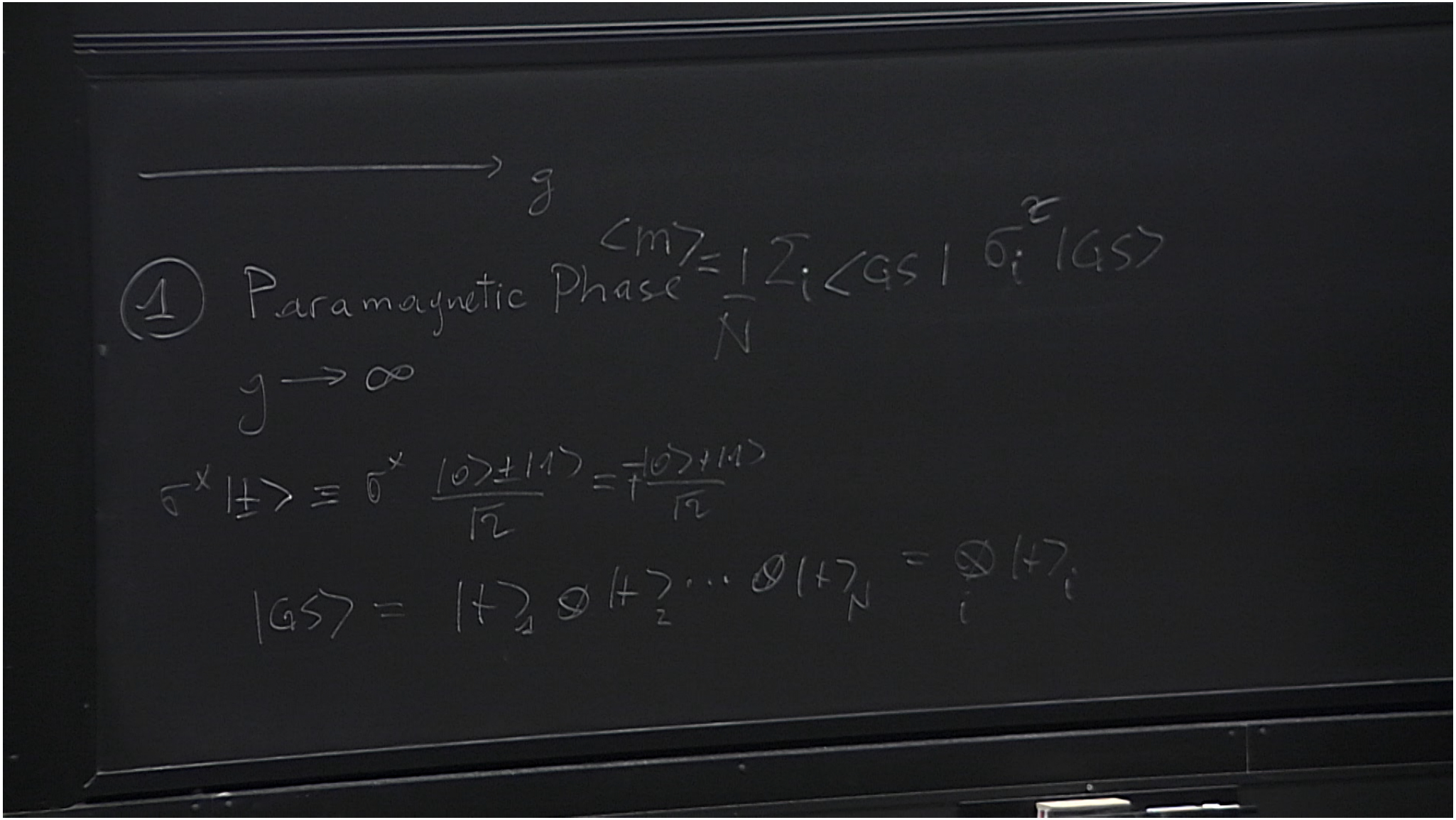
$$= \beta \sum_i \left[\langle$$

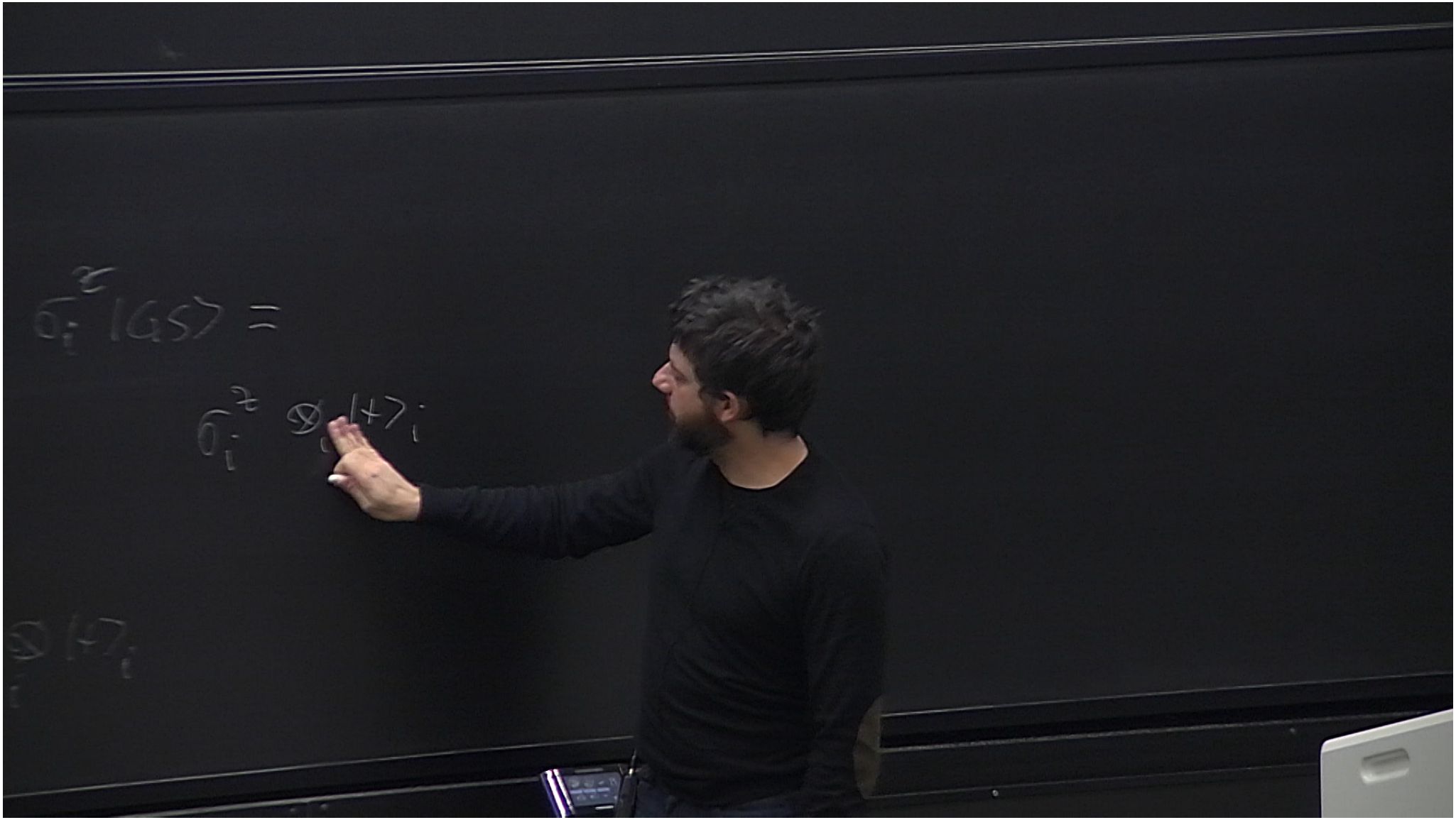
$$\langle \cdots \rangle = \sum_{\{S\}} \cdots \frac{e^{-\beta H}}{Z_i}$$

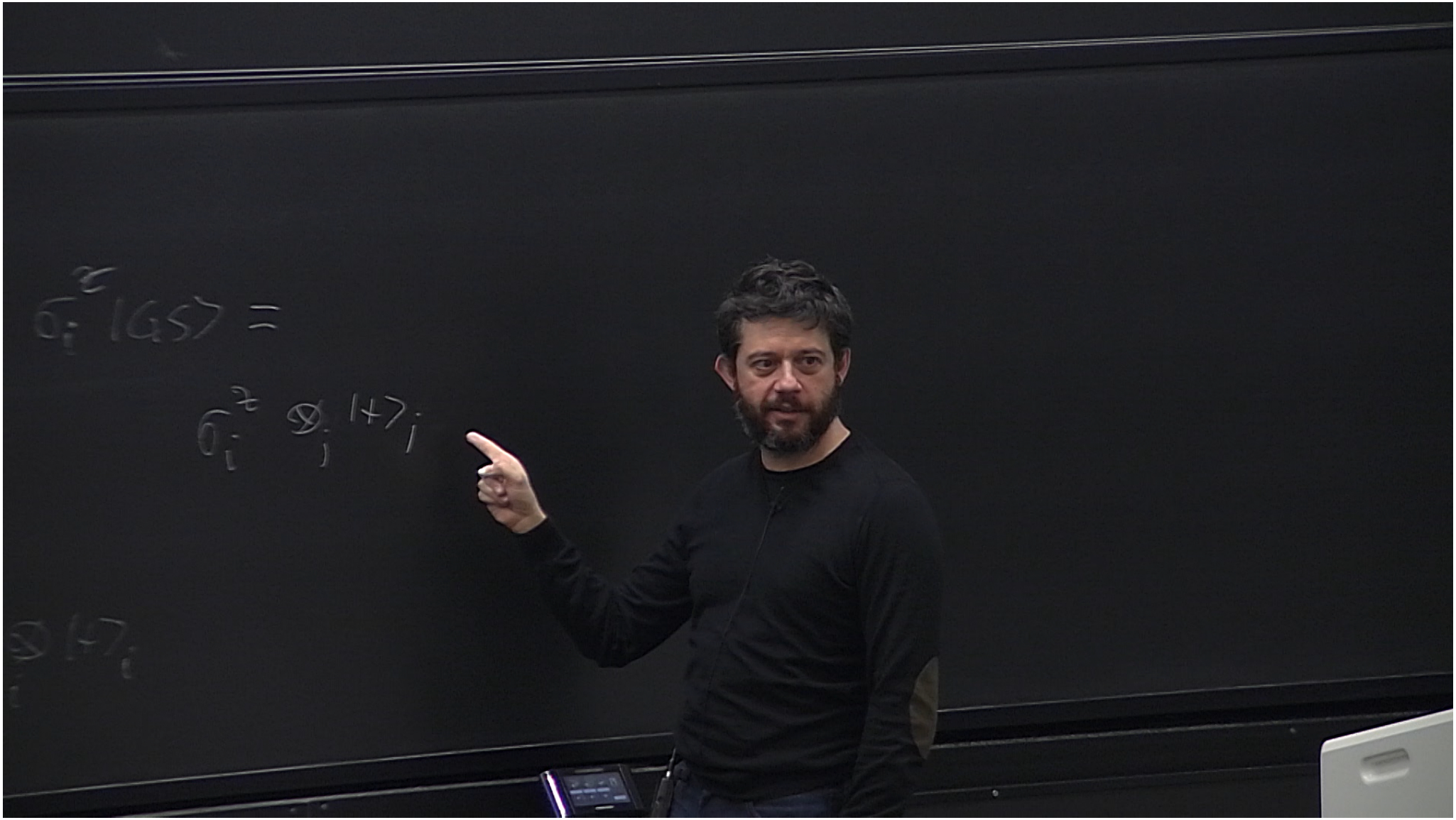
Quantum Phase Transitions

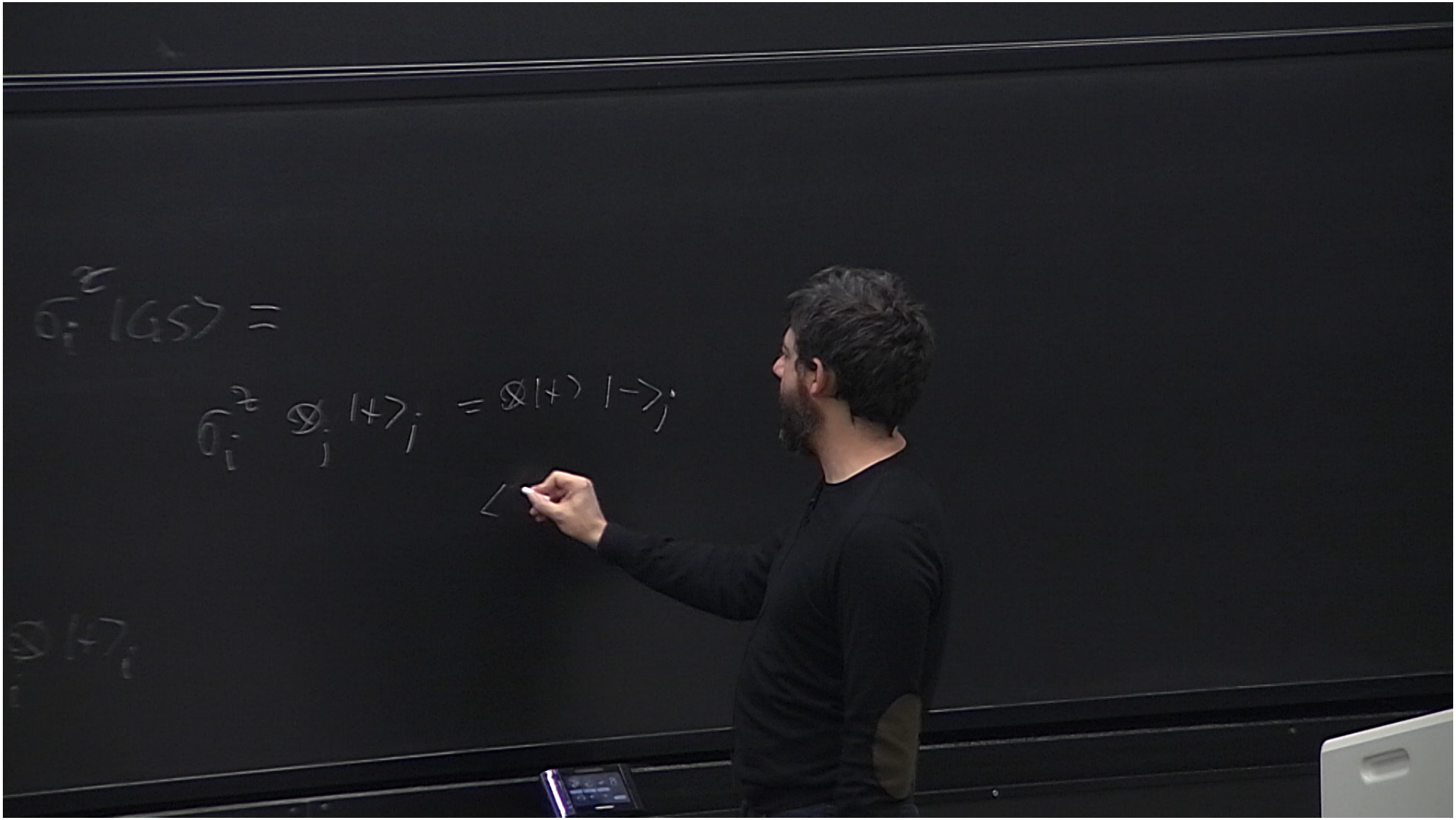


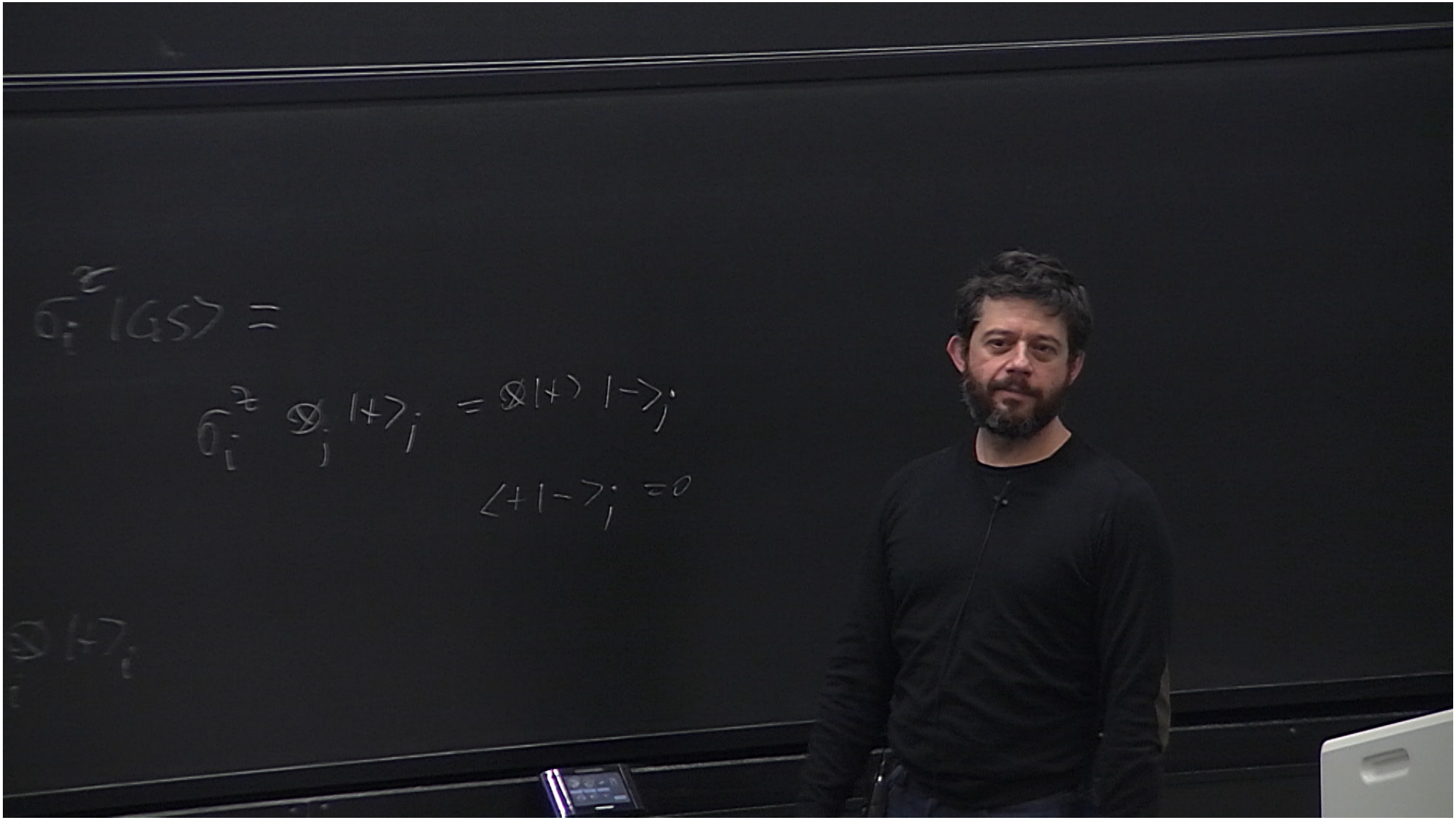
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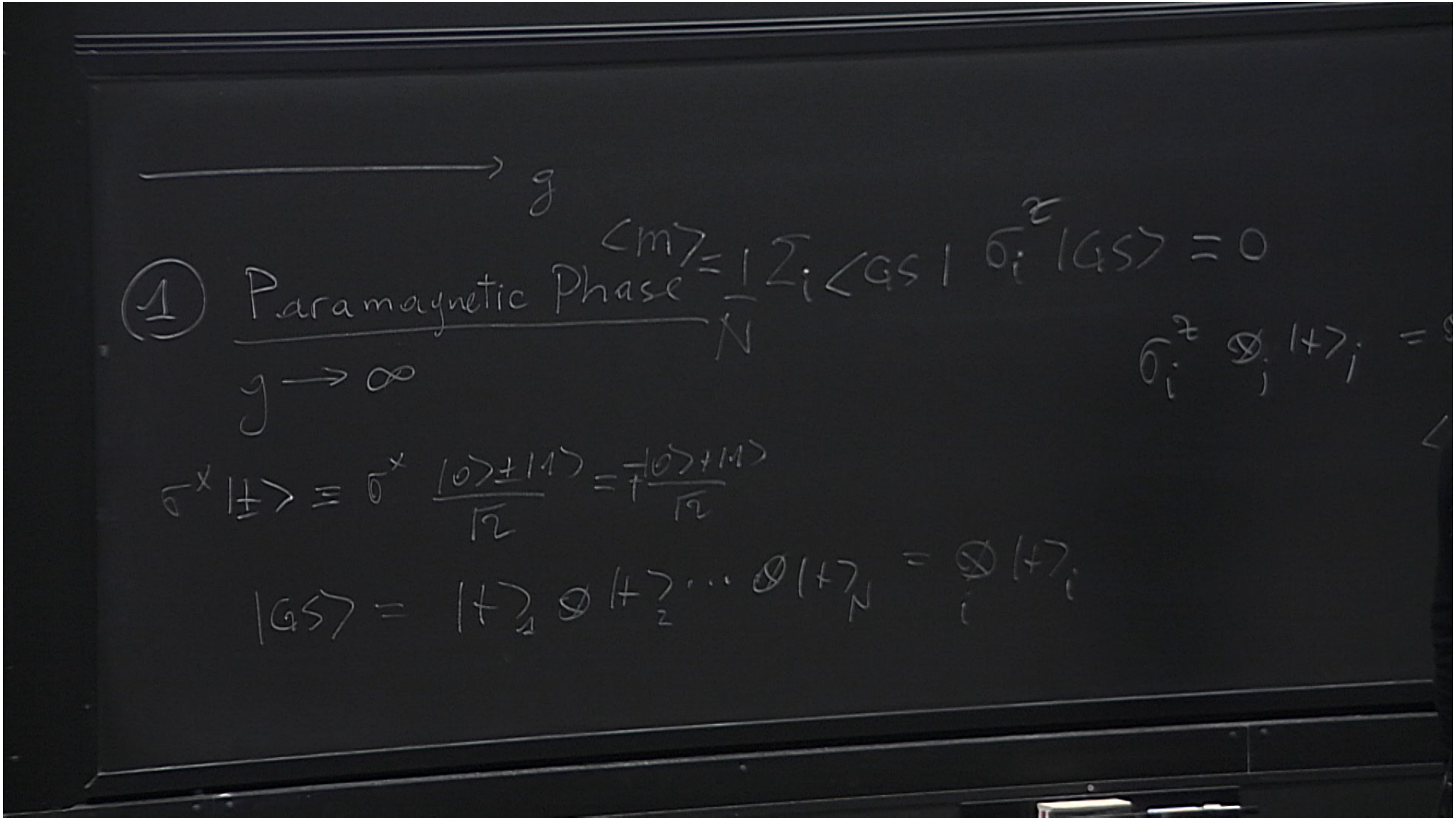

g
(1) Paramagnetic Phase
$\langle m \rangle = \frac{1}{N} \sum_i \langle GS | \sigma_i^z | GS \rangle$
$g \to \infty$
$\sigma^x |\pm\rangle \equiv \sigma^x \frac{|0\rangle \pm |1\rangle}{\sqrt{2}} = \pm \frac{|0\rangle + |1\rangle}{\sqrt{2}}$
$|GS\rangle = |+\rangle_1 \otimes |+\rangle_2 \cdots \otimes |+\rangle_N = \bigotimes_i |+\rangle_i$

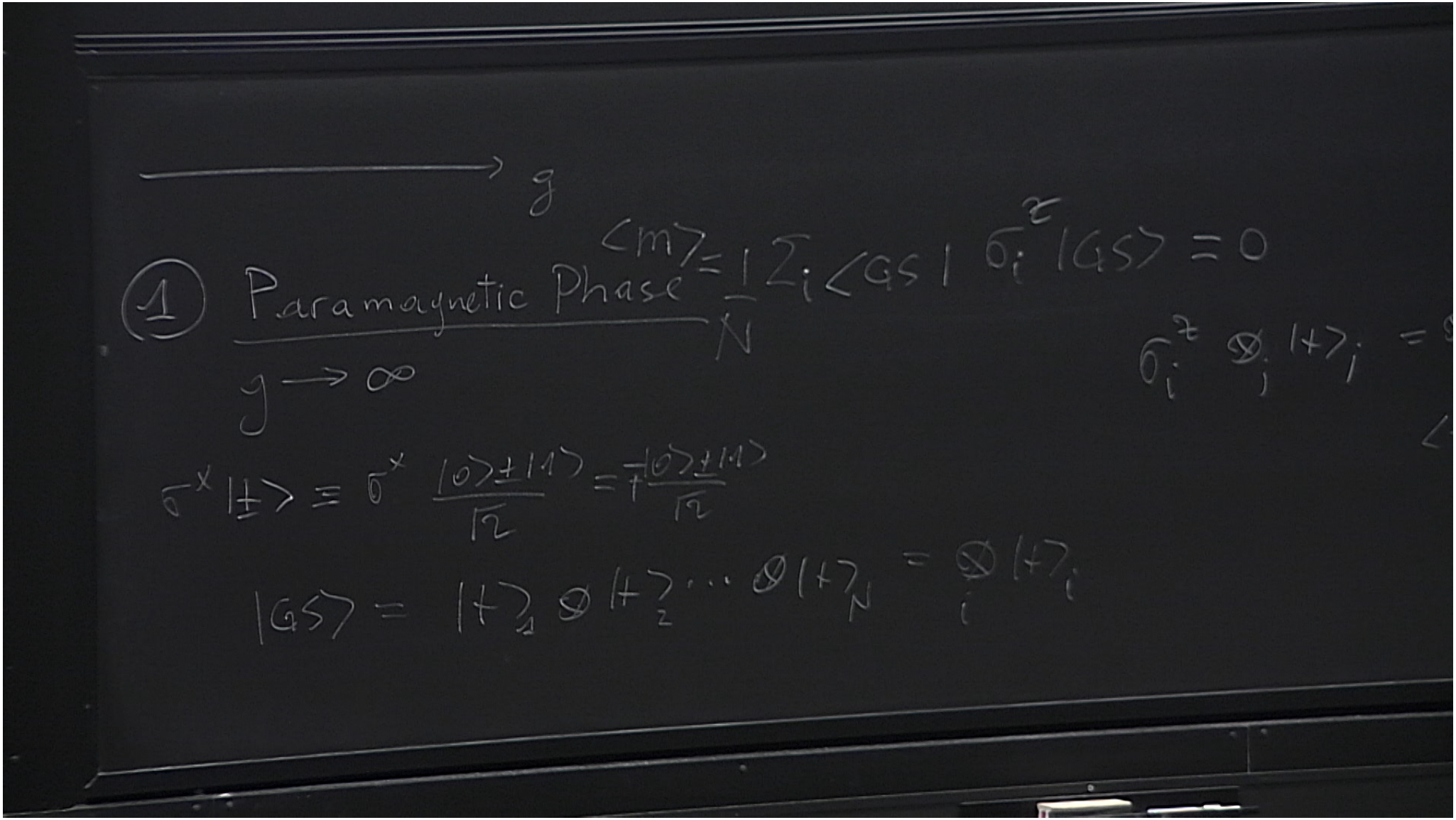

$\sigma_i^z |GS\rangle =$
σ_i^z
$|+\rangle_i$
$|\rangle_i$


$\sigma_i^z |GS\rangle =$
$\sigma_i^z \otimes_j |+\rangle_j$


$\sigma_i^z |GS\rangle =$
$\sigma_i^z \bigotimes_j |+\rangle_j = \bigotimes |+\rangle |-\rangle_j$

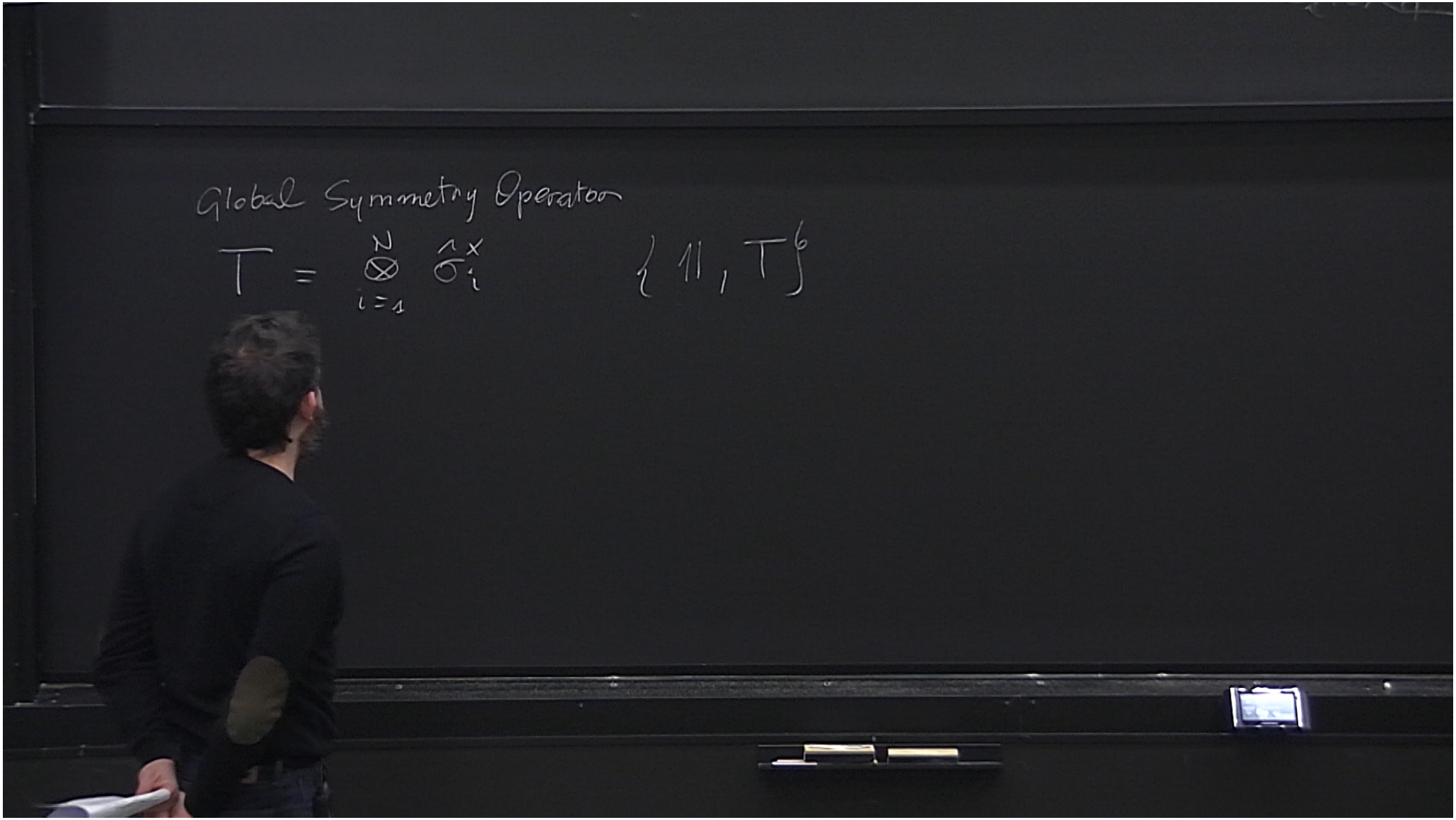

$\sigma_i^z |GS\rangle =$
$\sigma_i^z \bigotimes_j |+\rangle_j = \bigotimes |+\rangle |-\rangle_i$
$\langle + | - \rangle_i = 0$
$\bigotimes |+\rangle_i$

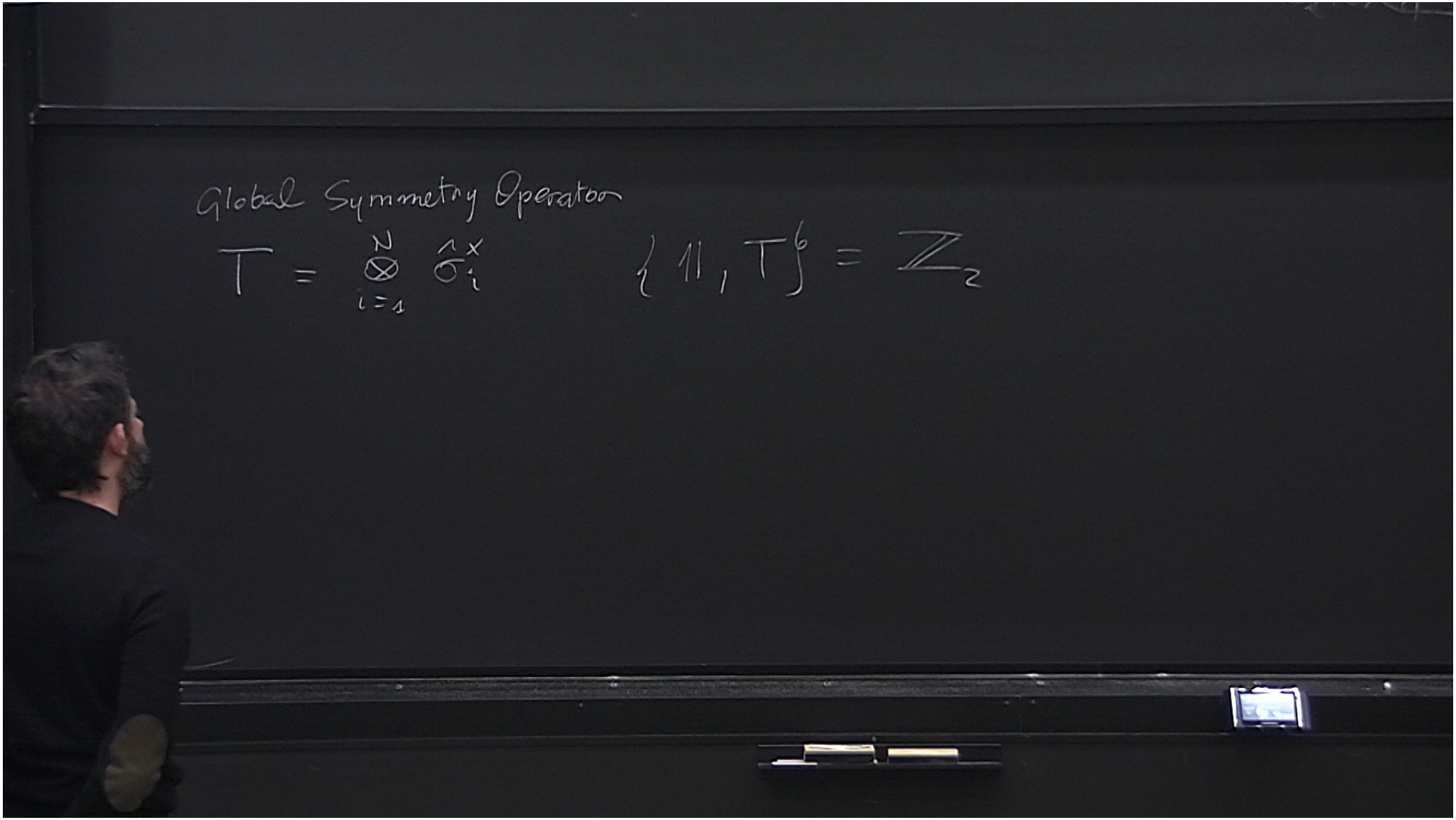

g
(1) Paramagnetic Phase
$\langle m \rangle = \frac{1}{N} \sum_i \langle GS | \sigma_i^z | GS \rangle = 0$
$g \to \infty$
$\sigma_i^z \otimes_j |+\rangle_j =$
$\sigma^x |\pm\rangle \equiv \sigma^x \frac{|0\rangle \pm |1\rangle}{\sqrt{2}} = \pm \frac{|0\rangle + |1\rangle}{\sqrt{2}}$
$|GS\rangle = |+\rangle_1 \otimes |+\rangle_2 \cdots \otimes |+\rangle_N = \bigotimes_i |+\rangle_i$

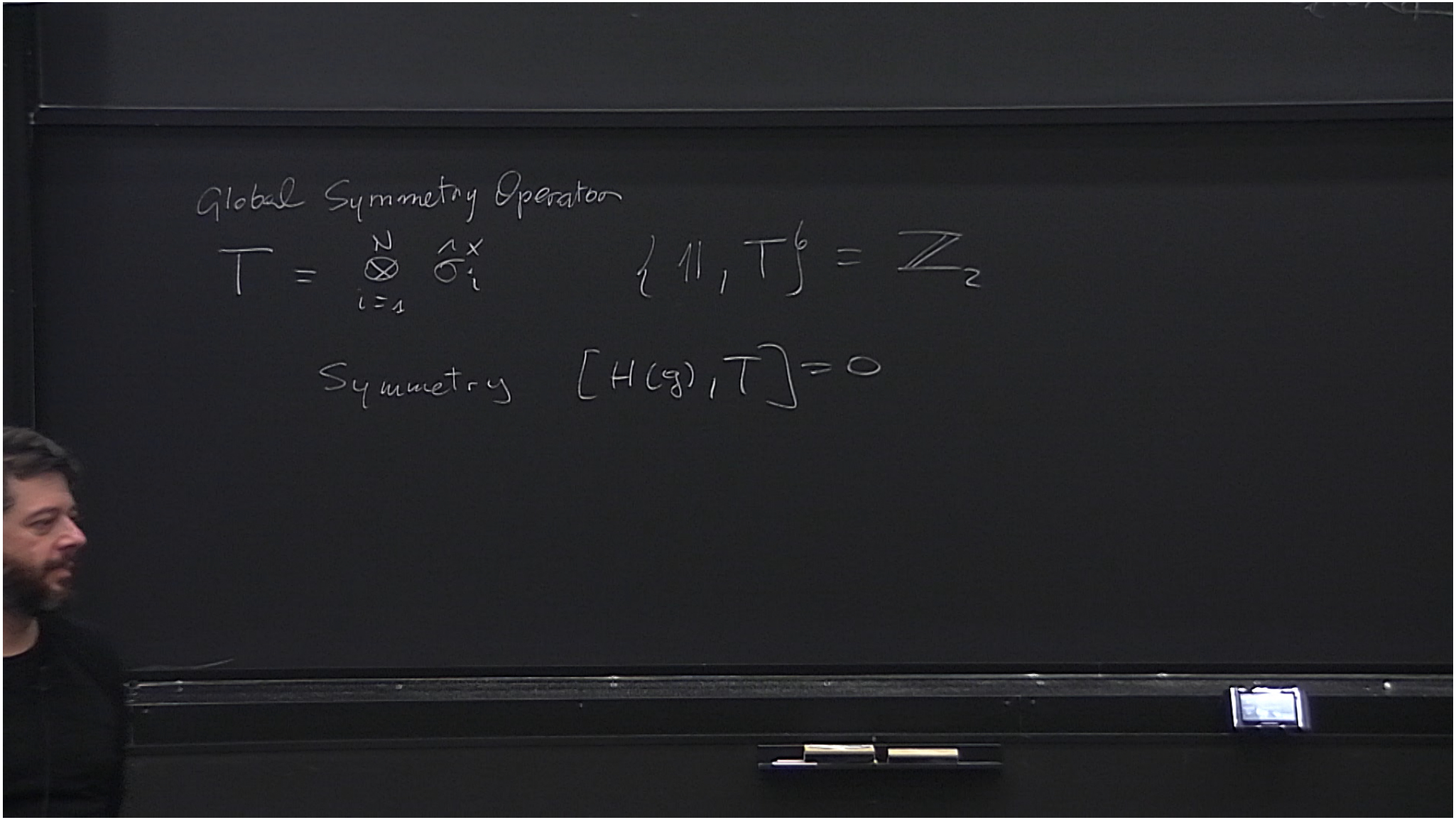

g
(1) Paramagnetic Phase
$\langle m \rangle = \frac{1}{N} \sum_i \langle GS | \sigma_i^z | GS \rangle = 0$
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$|GS\rangle = |+\rangle_1 \otimes |+\rangle_2 \cdots \otimes |+\rangle_N = \bigotimes_i |+\rangle_i$

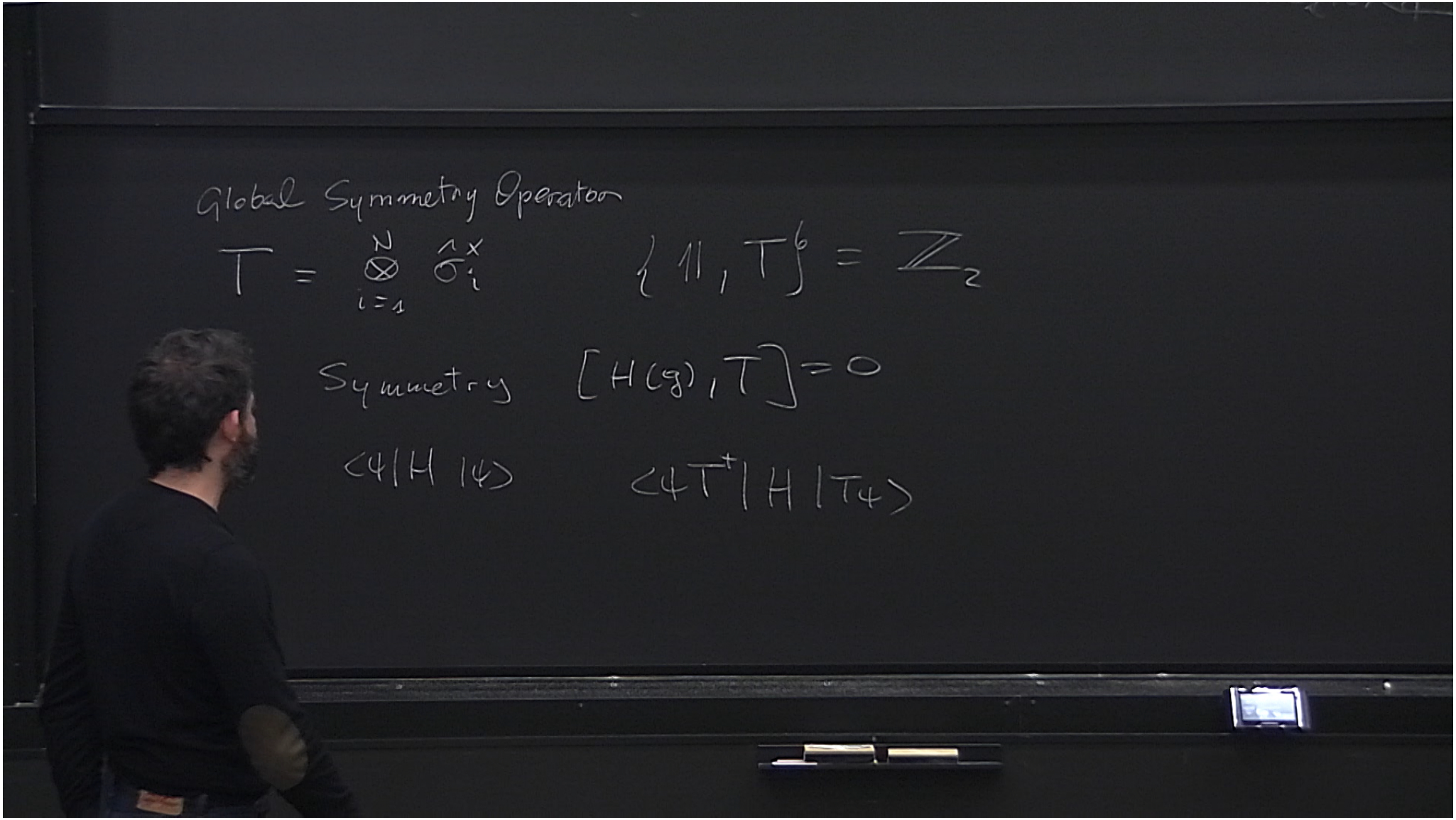
Global Symmetry Operation

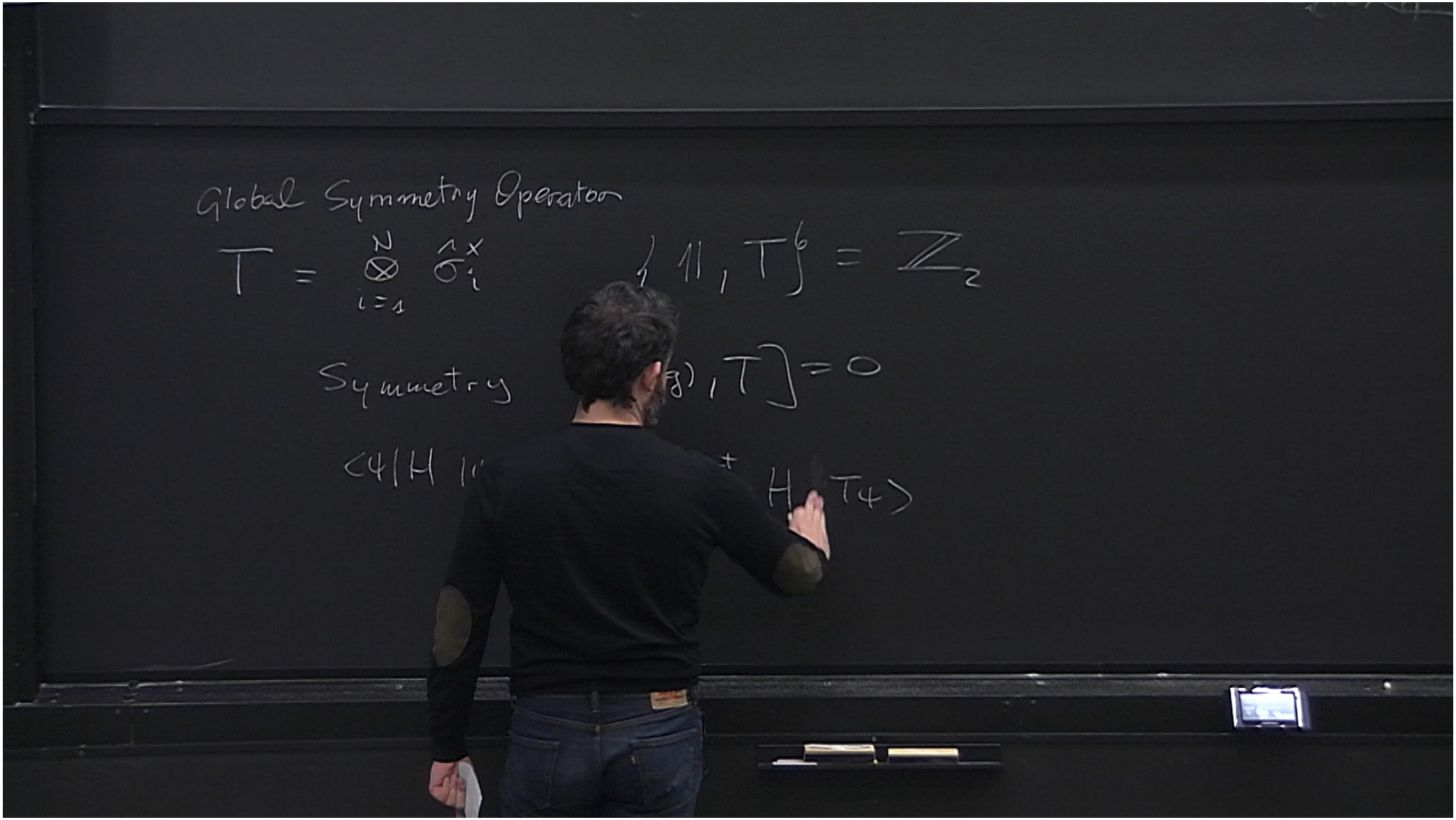
$$T = \bigotimes_{i=1}^{N} \hat{\sigma}_i^x$$

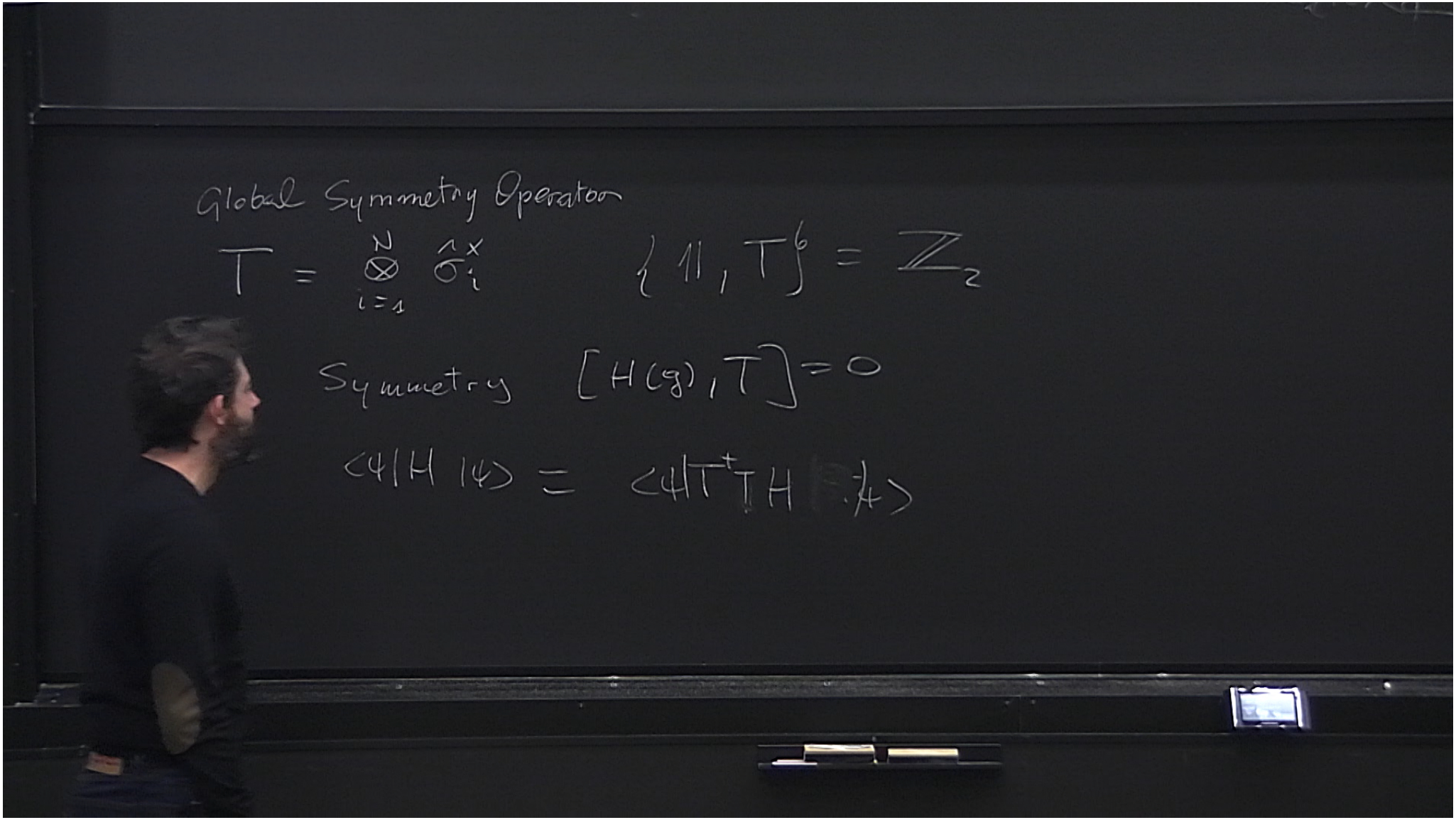

Global Symmetry Operator
$T = \bigotimes_{i=1}^{N} \hat{\sigma}_i^x$
$\{11, T\}$

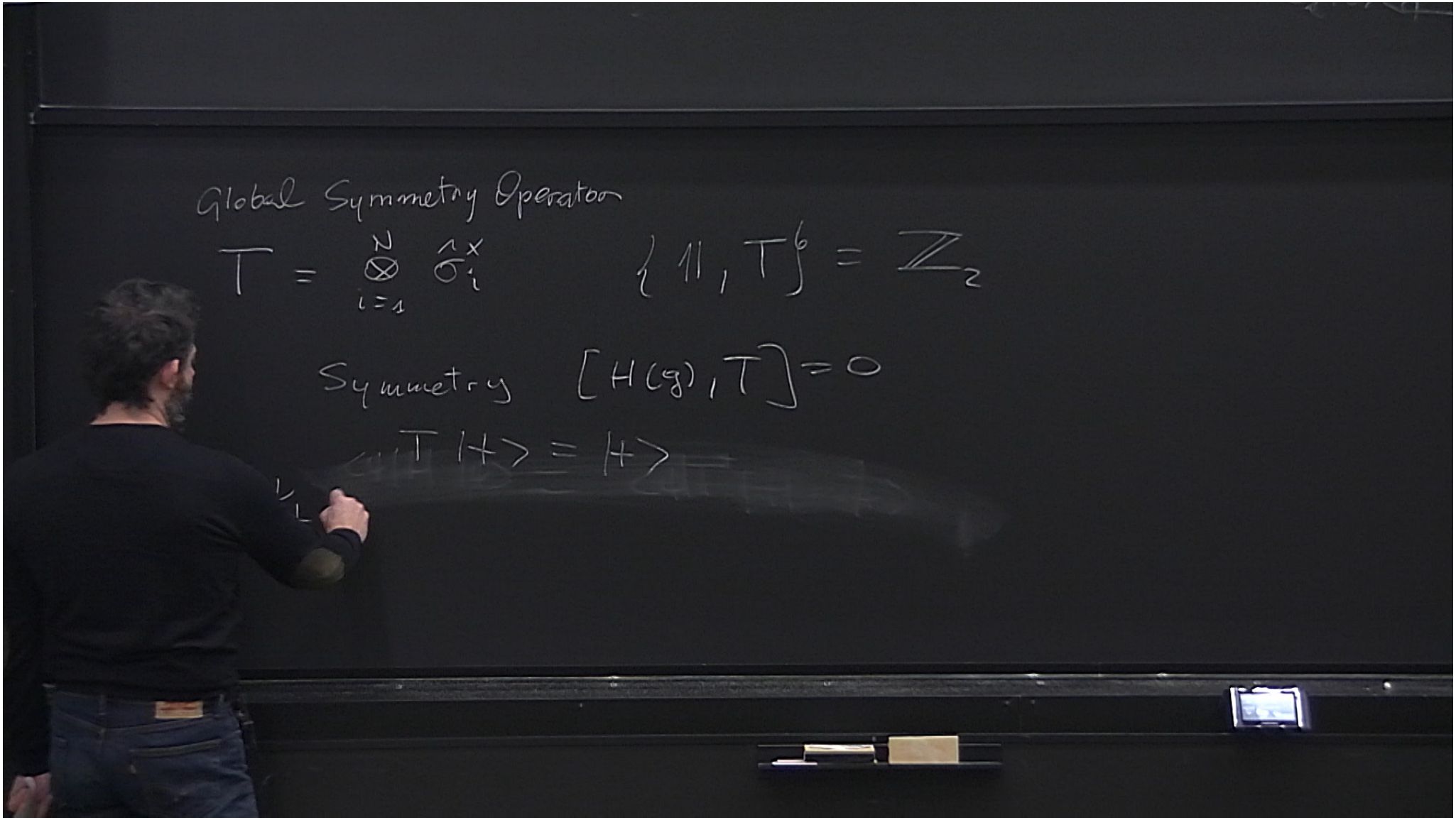

Global Symmetry Operator
$T = \bigotimes_{i=1}^{N} \hat{\sigma}_i^x$
$\{1\!\!1, T\} = \mathbb{Z}_2$

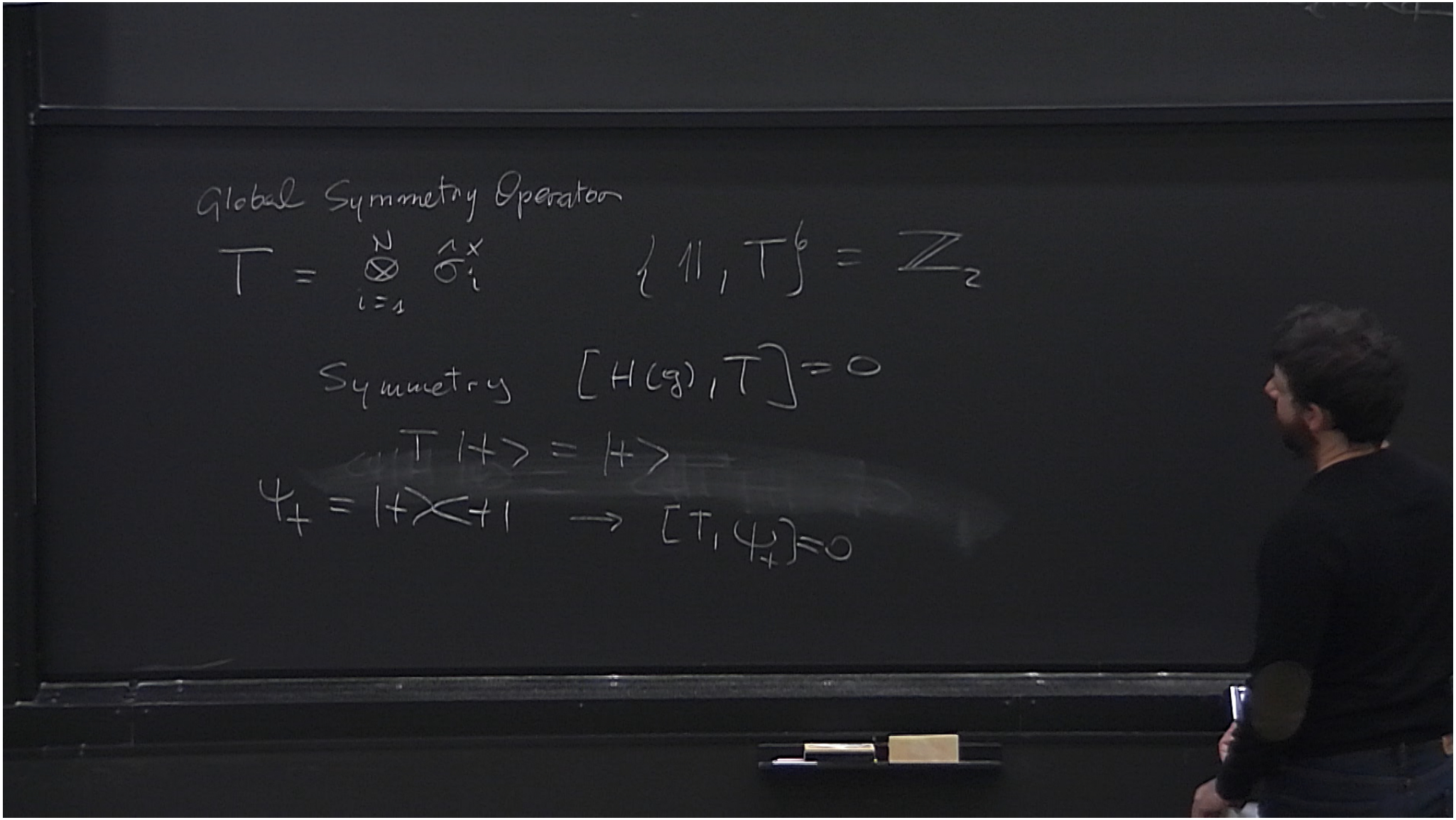

Global Symmetry Operator
$T = \bigotimes_{i=1}^{N} \hat{\sigma}_i^x$
$\{\mathbb{1}, T\} = \mathbb{Z}_2$
Symmetry $[H(g), T] = 0$

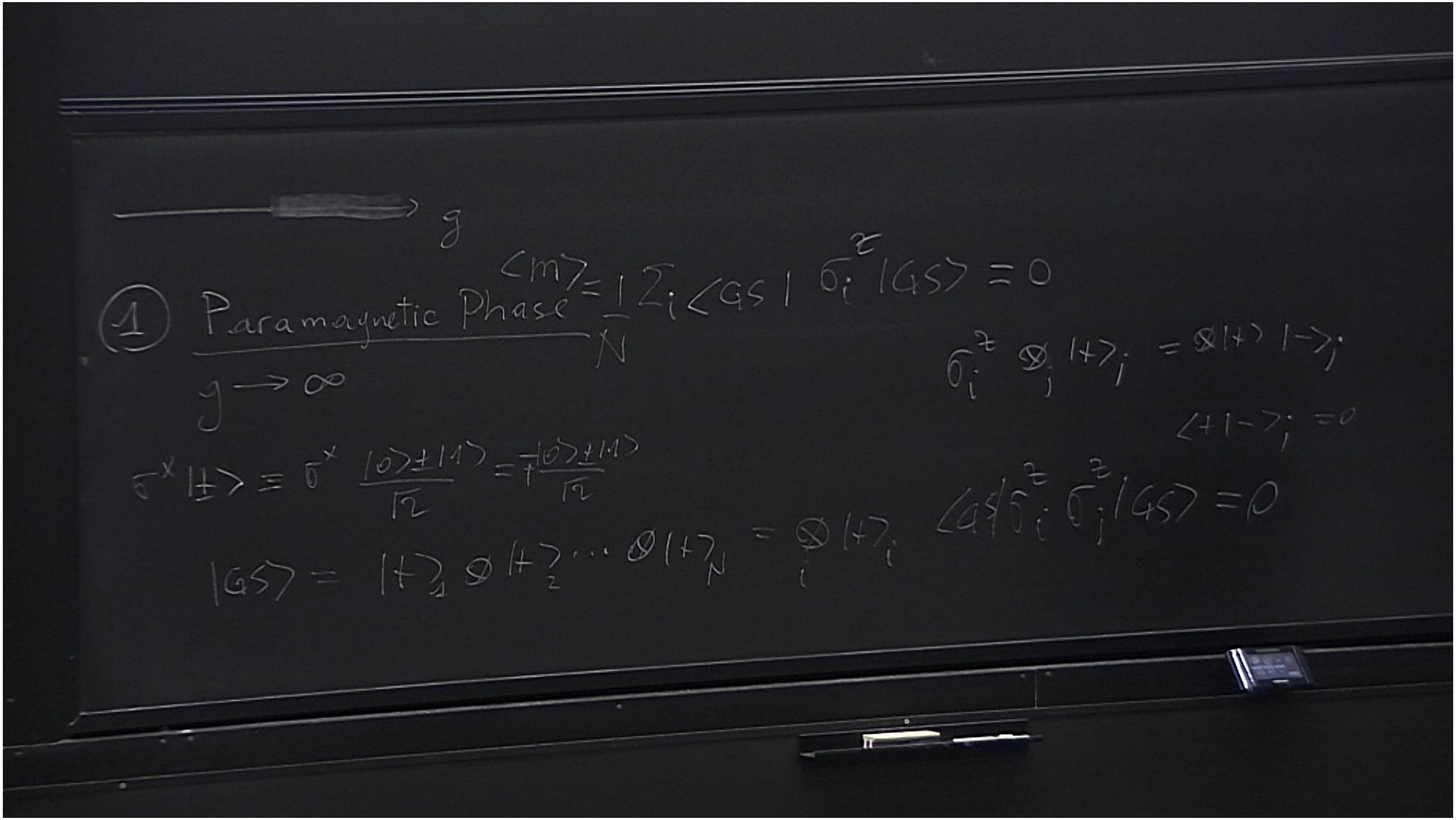

Global Symmetry Operator
$T = \bigotimes_{i=1}^{N} \hat{\sigma}_i^x$
$\{11, T\} = \mathbb{Z}_2$
Symmetry $[H(g), T] = 0$
$\langle\psi|H|\psi\rangle$
$\langle\psi T^\dagger|H|T\psi\rangle$

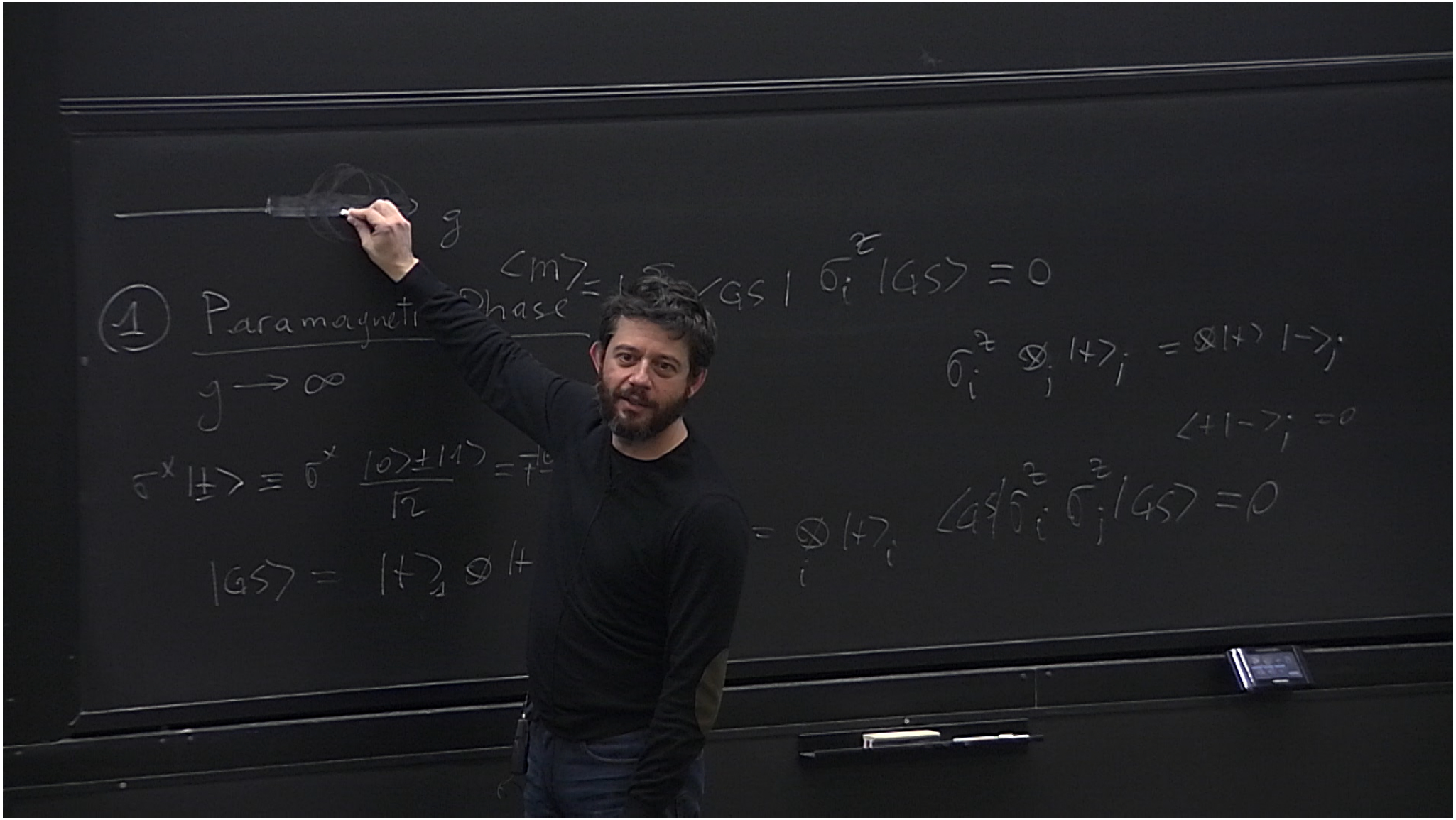

Global Symmetry Operator
$T = \bigotimes_{i=1}^{N} \hat{\sigma}_i^x$
$[H, T] = Z_2$
Symmetry
$[g), T] = 0$
$\langle 4|H$
$H, T4\rangle$

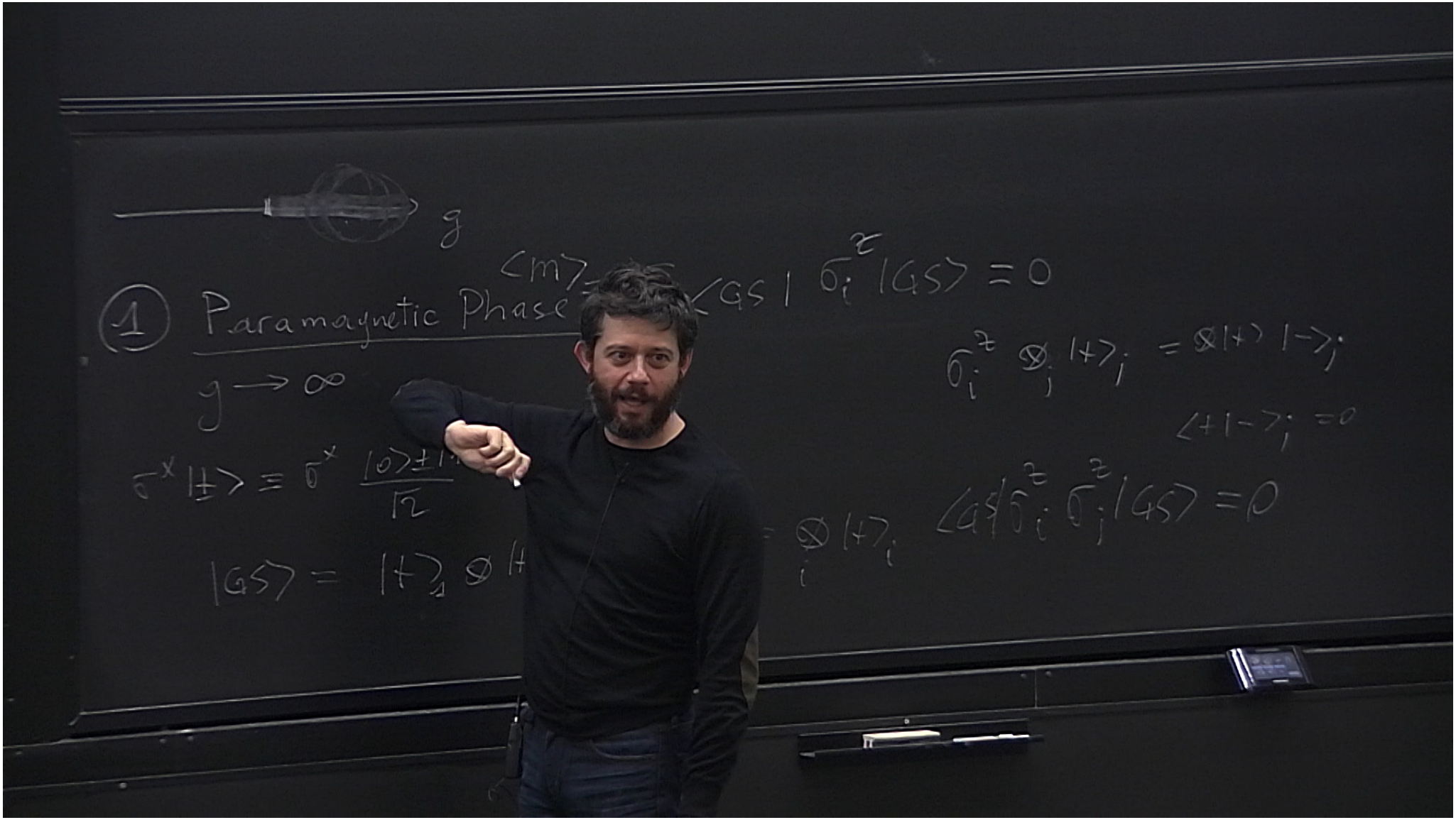

Global Symmetry Operator
$T = \bigotimes_{i=1}^{N} \hat{\sigma}_i^x$
$\{1\!\!1, T\} = \mathbb{Z}_2$
Symmetry $[H(g), T] = 0$
$\langle\psi|H|\psi\rangle = \langle\psi|T^\dagger T H$

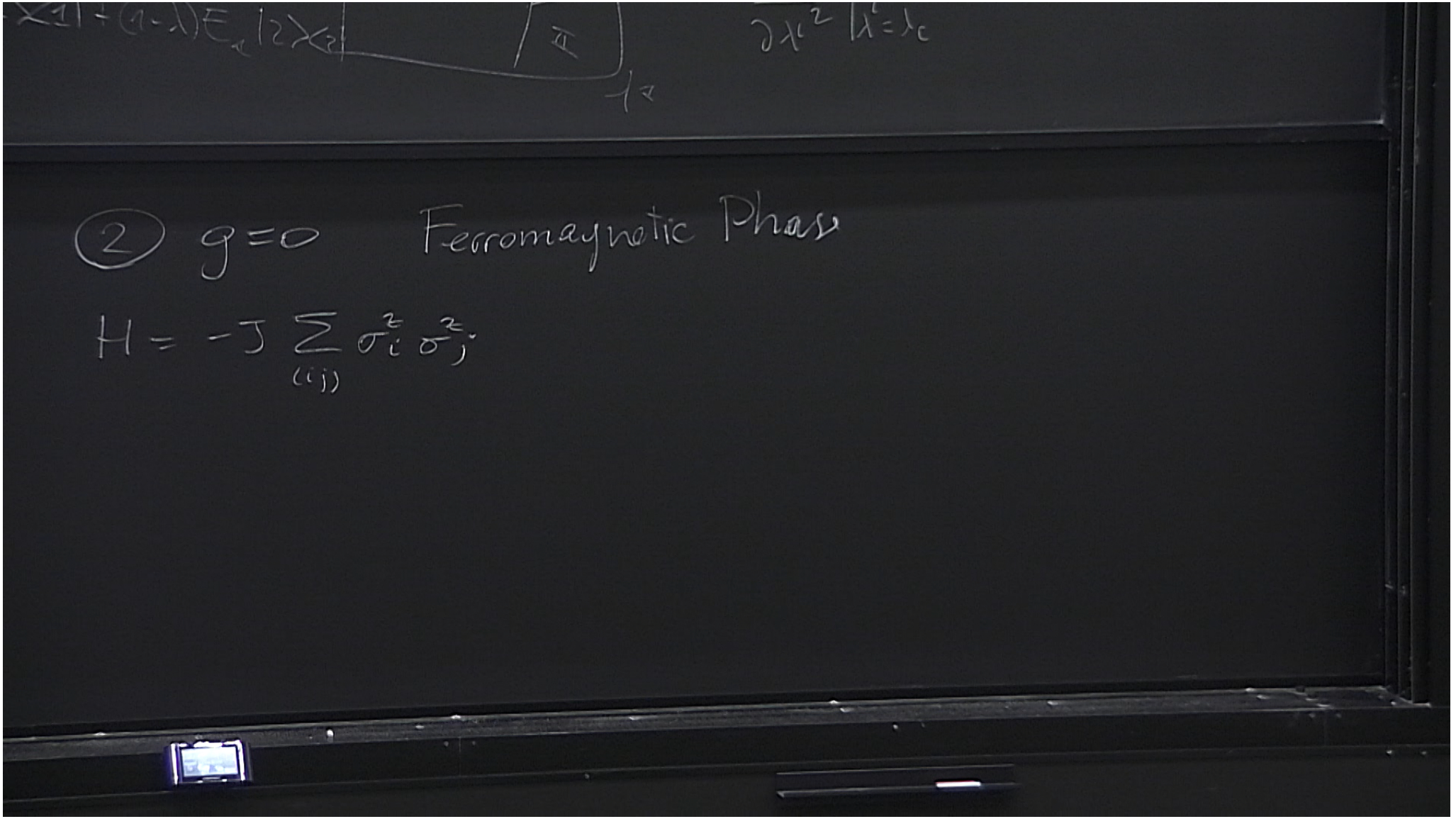

Global Symmetry Operator
$T = \bigotimes_{i=1}^{N} \hat{\sigma}_i^x$
$\{1\!\!1, T\} = \mathbb{Z}_2$
Symmetry $[H(g), T] = 0$
$T|+\rangle = |+\rangle$

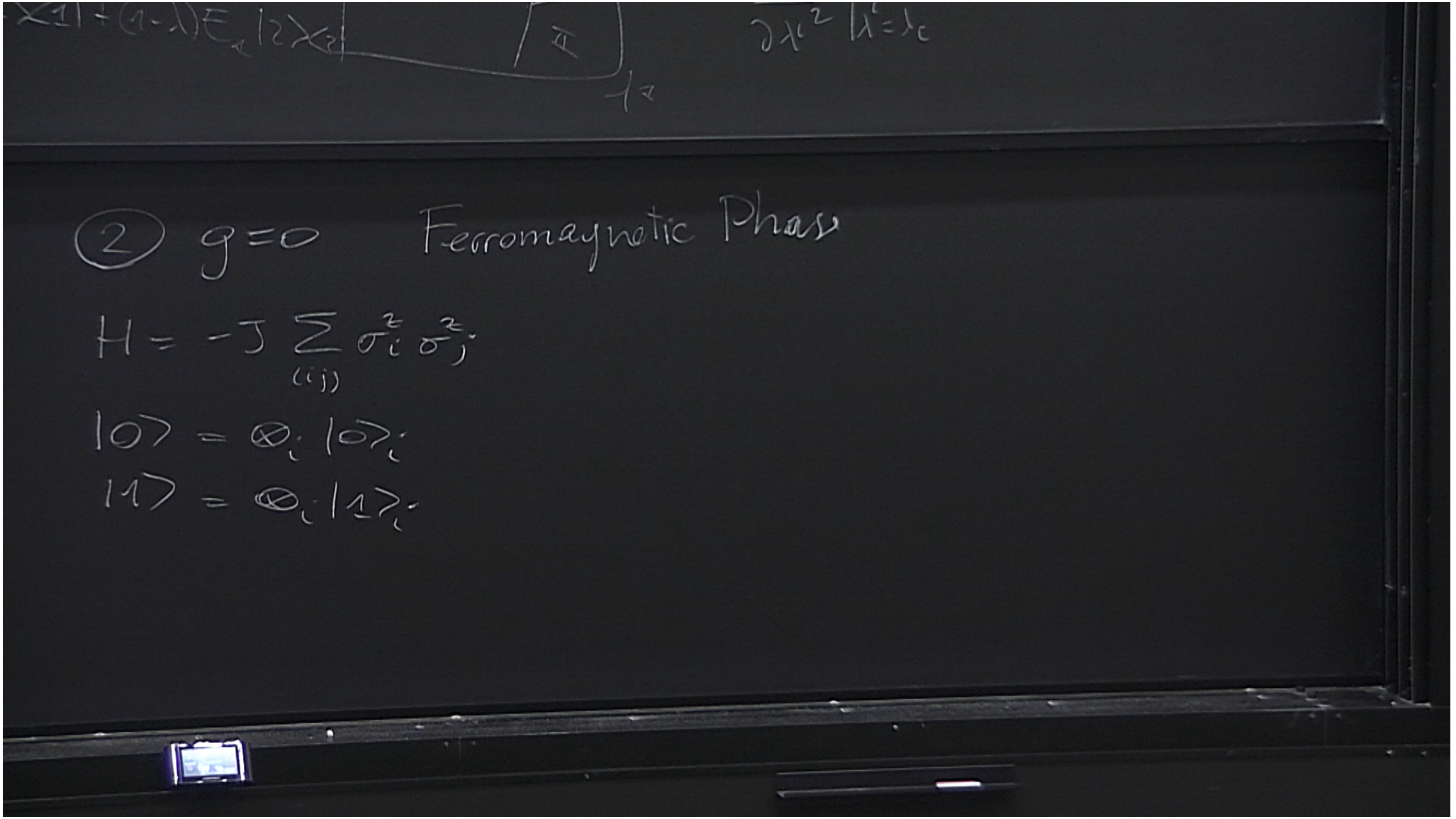

Global Symmetry Operator
$T = \bigotimes_{i=1}^{N} \hat{\sigma}_i^x$
$\{11, T\} = Z_2$
Symmetry $[H(g), T] = 0$
$T|+\rangle = |+\rangle$
$\psi_+ = |+\rangle\langle+| \rightarrow [T, \psi_+] = 0$

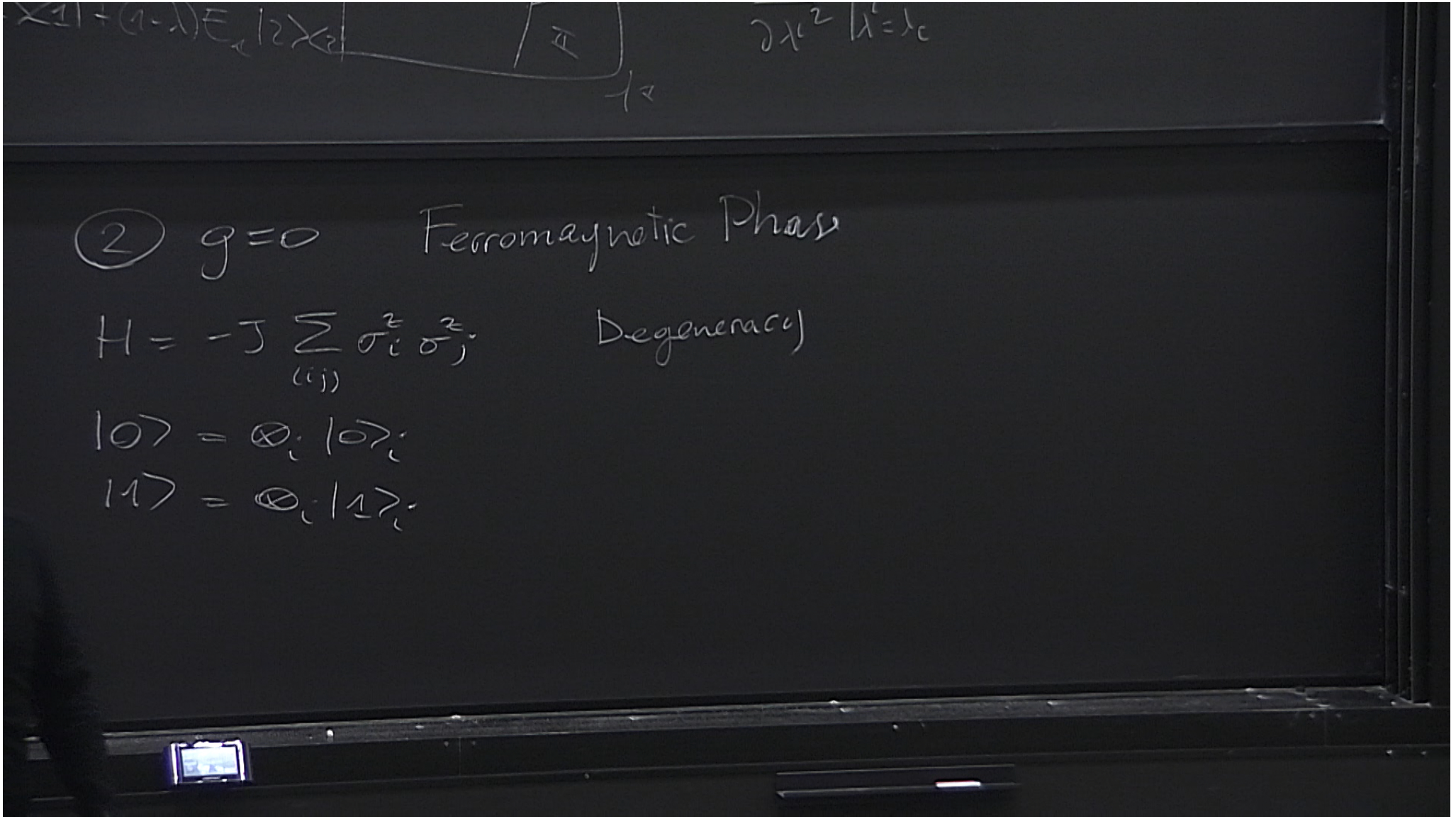

g
(1) Paramagnetic Phase
$\langle m \rangle = \frac{1}{N} \sum_i \langle GS | \sigma_i^z | GS \rangle = 0$
$g \to \infty$
$\sigma^x |\pm\rangle = \sigma^x \frac{|0\rangle \pm |1\rangle}{\sqrt{2}} = \pm \frac{|0\rangle \pm |1\rangle}{\sqrt{2}}$
$|GS\rangle = |+\rangle_1 \otimes |+\rangle_2 \cdots \otimes |+\rangle_N = \bigotimes_i |+\rangle_i$
$\sigma_i^z \bigotimes_j |+\rangle_j = \bigotimes |+\rangle |-\rangle_j$
$\langle + | - \rangle_j = 0$
$\langle GS | \sigma_i^z \sigma_j^z | GS \rangle = 0$

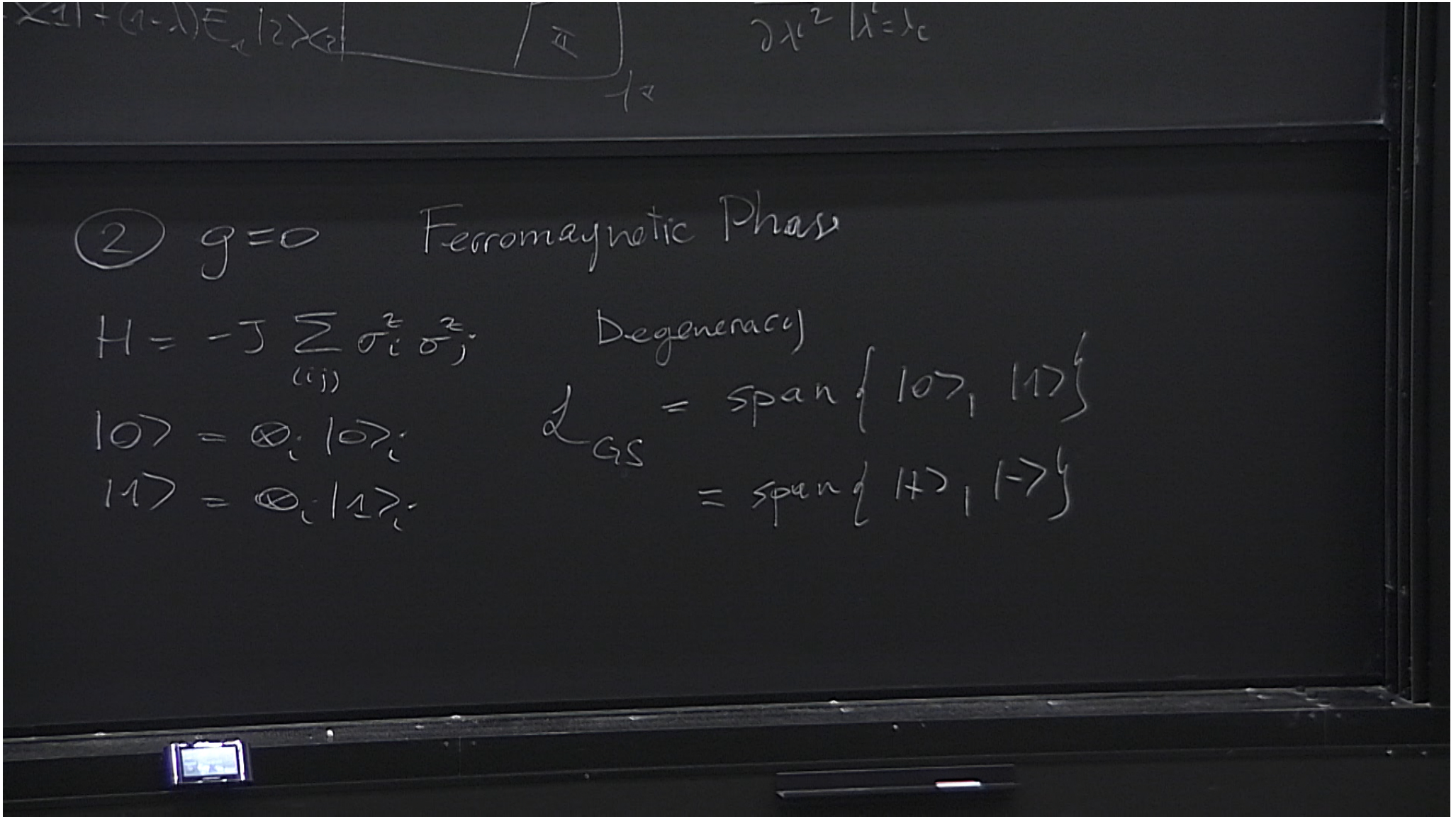

g
1 Paramagneti
Phase
g → ∞
|GS⟩ = |+⟩ ⊗ |+
⟨+|−⟩ = 0

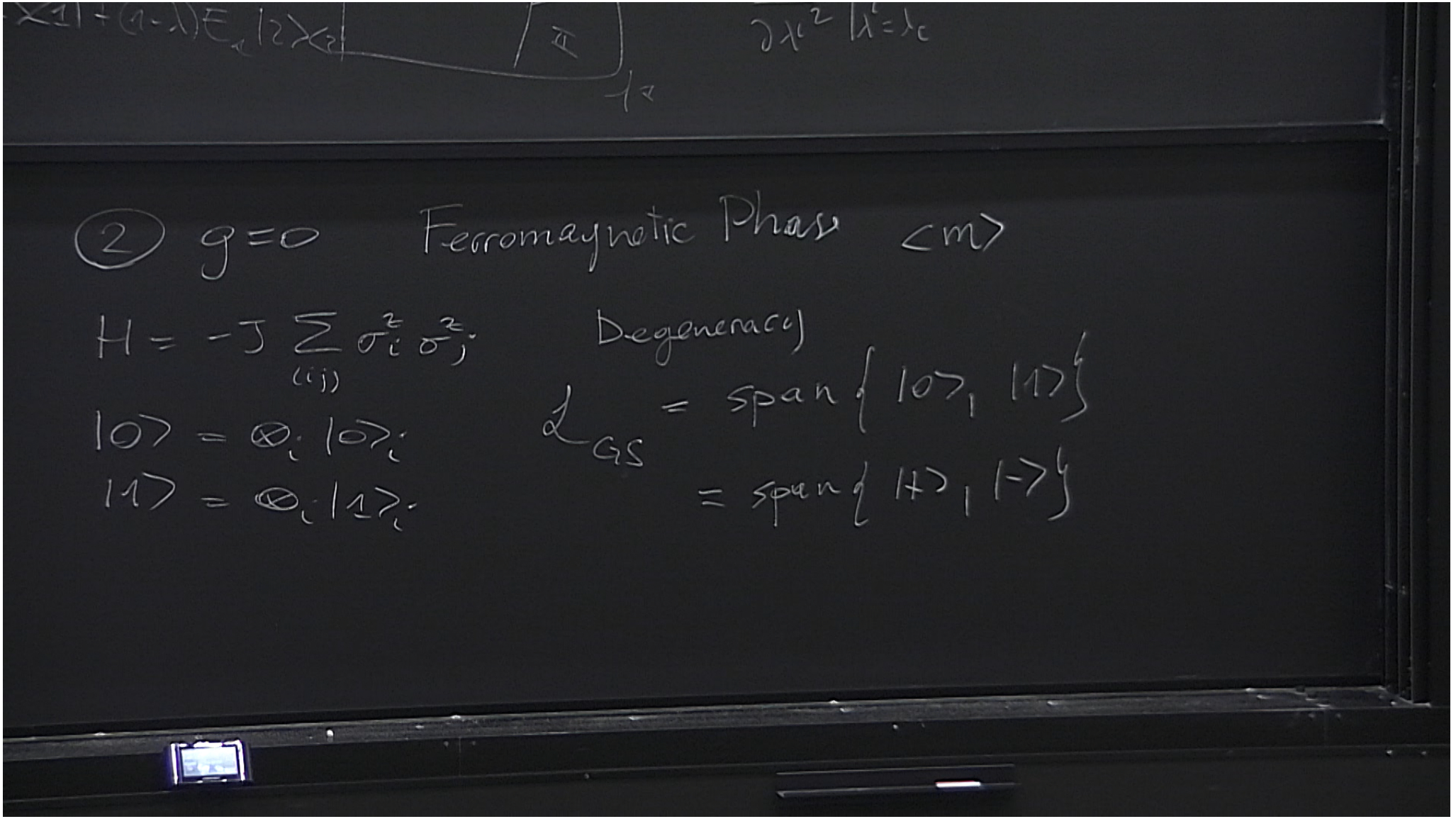

g
(1) Paramagnetic Phase
g → ∞
|GS⟩ = |+⟩ ⊗ |+⟩ ⊗ |+⟩

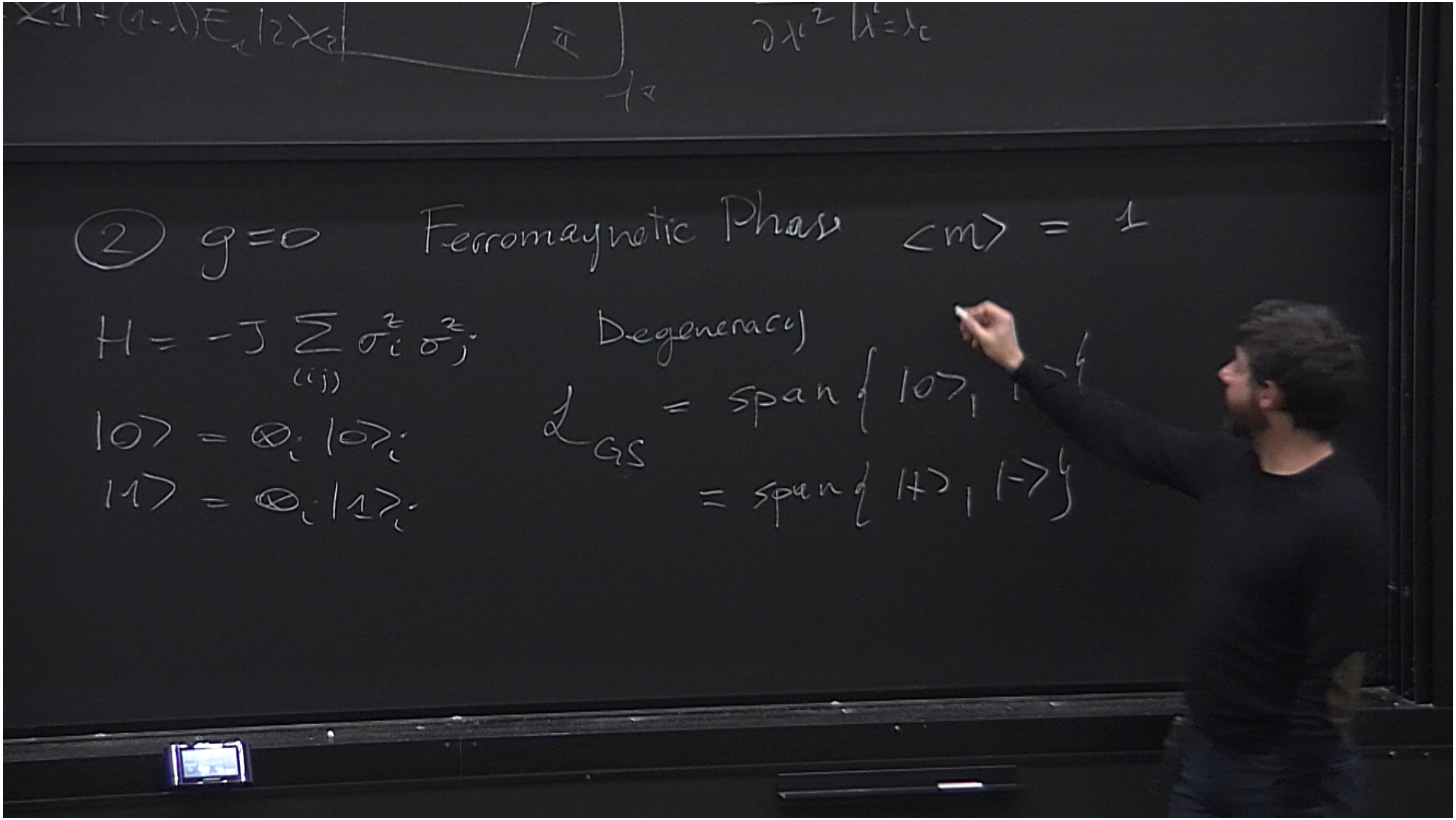

② g=0 Ferromagnetic Phase
H = -J Σ_(⟨ij⟩) σ_i^z σ_j^z

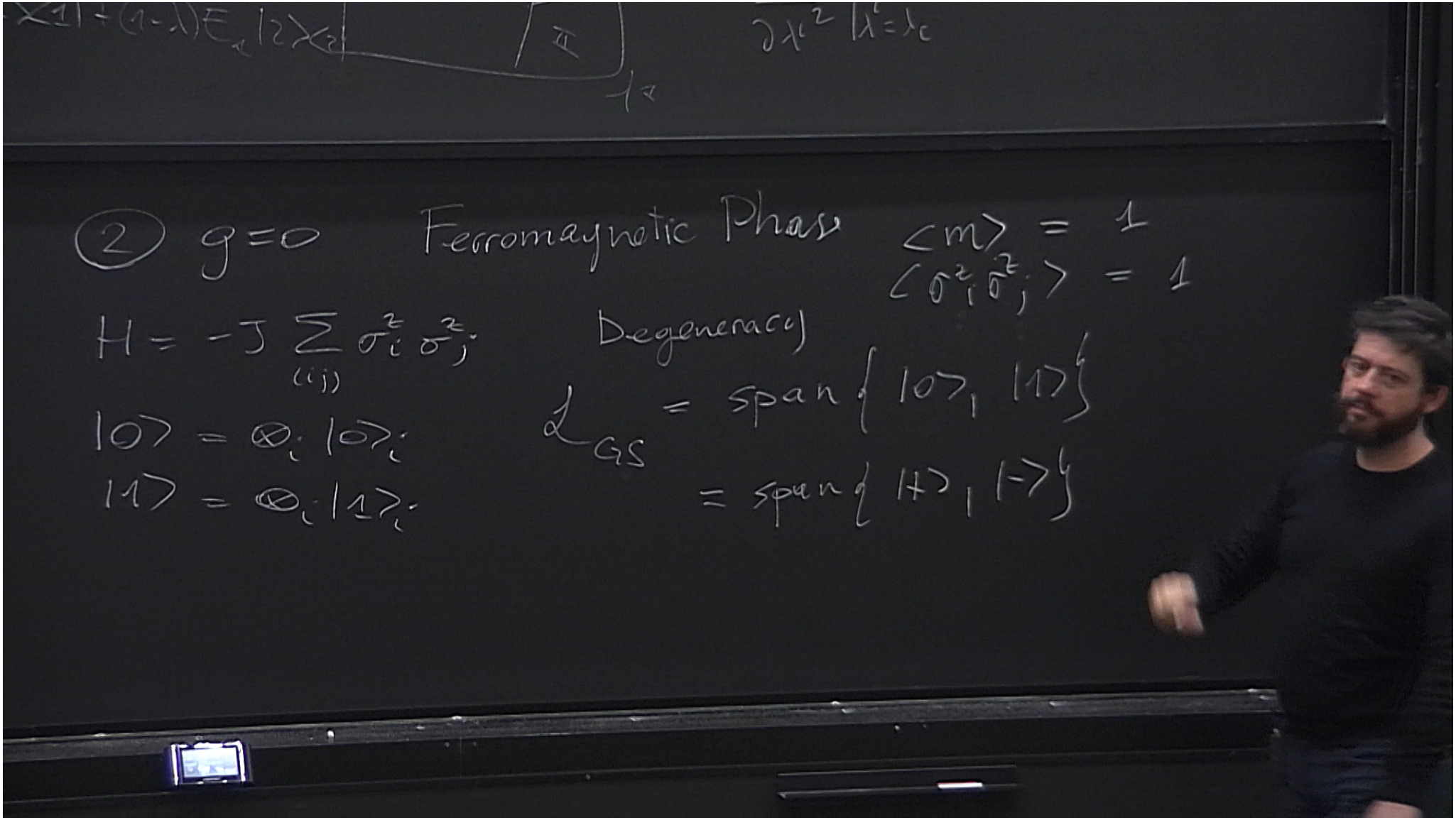

(2) g=0 Ferromagnetic Phase
$H = -J \sum_{\langle ij \rangle} \sigma_i^z \sigma_j^z$
$|0\rangle = \otimes_i |0\rangle_i$
$|1\rangle = \otimes_i |1\rangle_i$

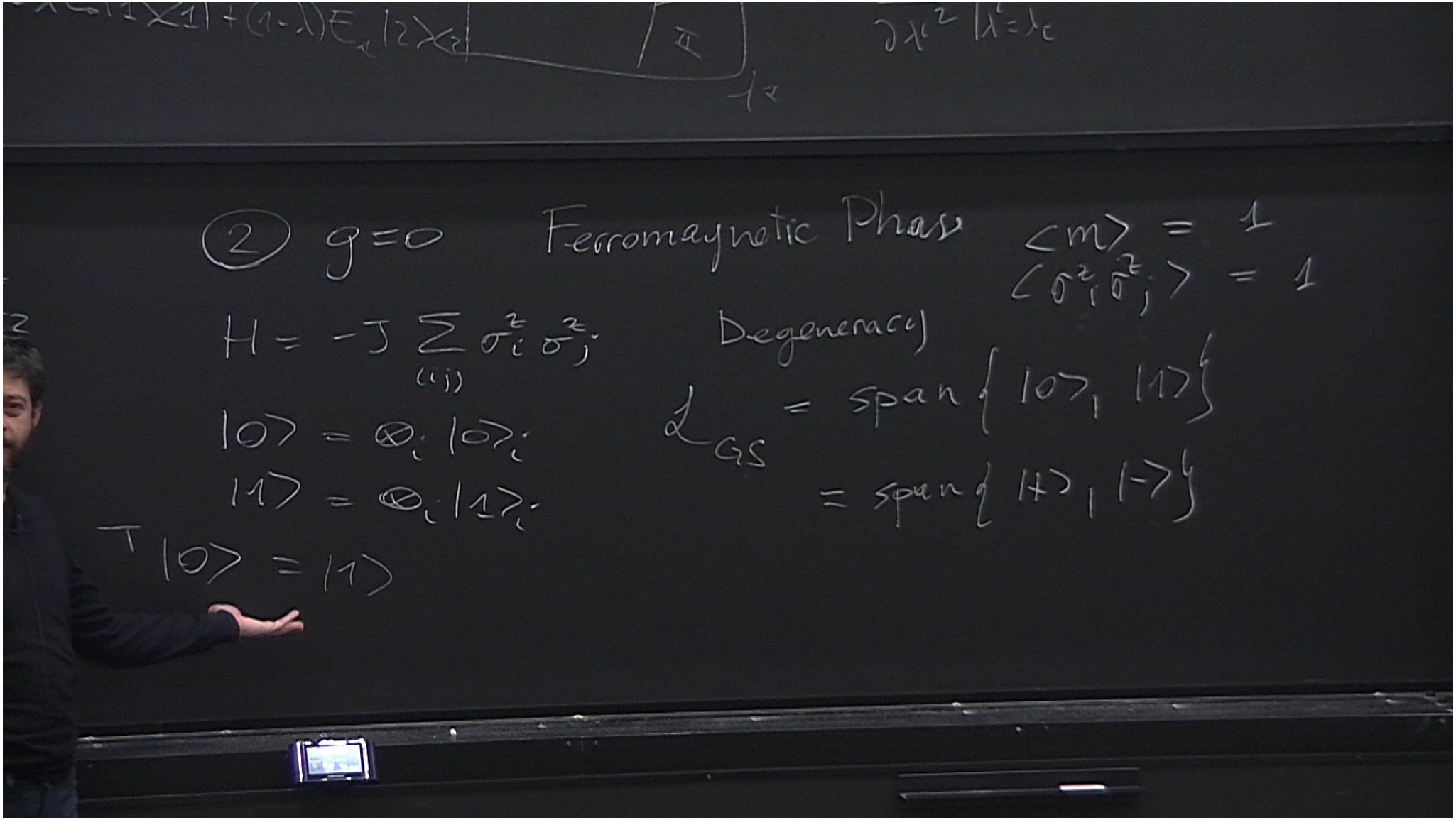

(2) g=0 Ferromagnetic Phase
$H = -J \sum_{\langle ij \rangle} \sigma_i^z \sigma_j^z$
Degeneracy
$|0\rangle = \otimes_i |0\rangle_i$
$|1\rangle = \otimes_i |1\rangle_i$

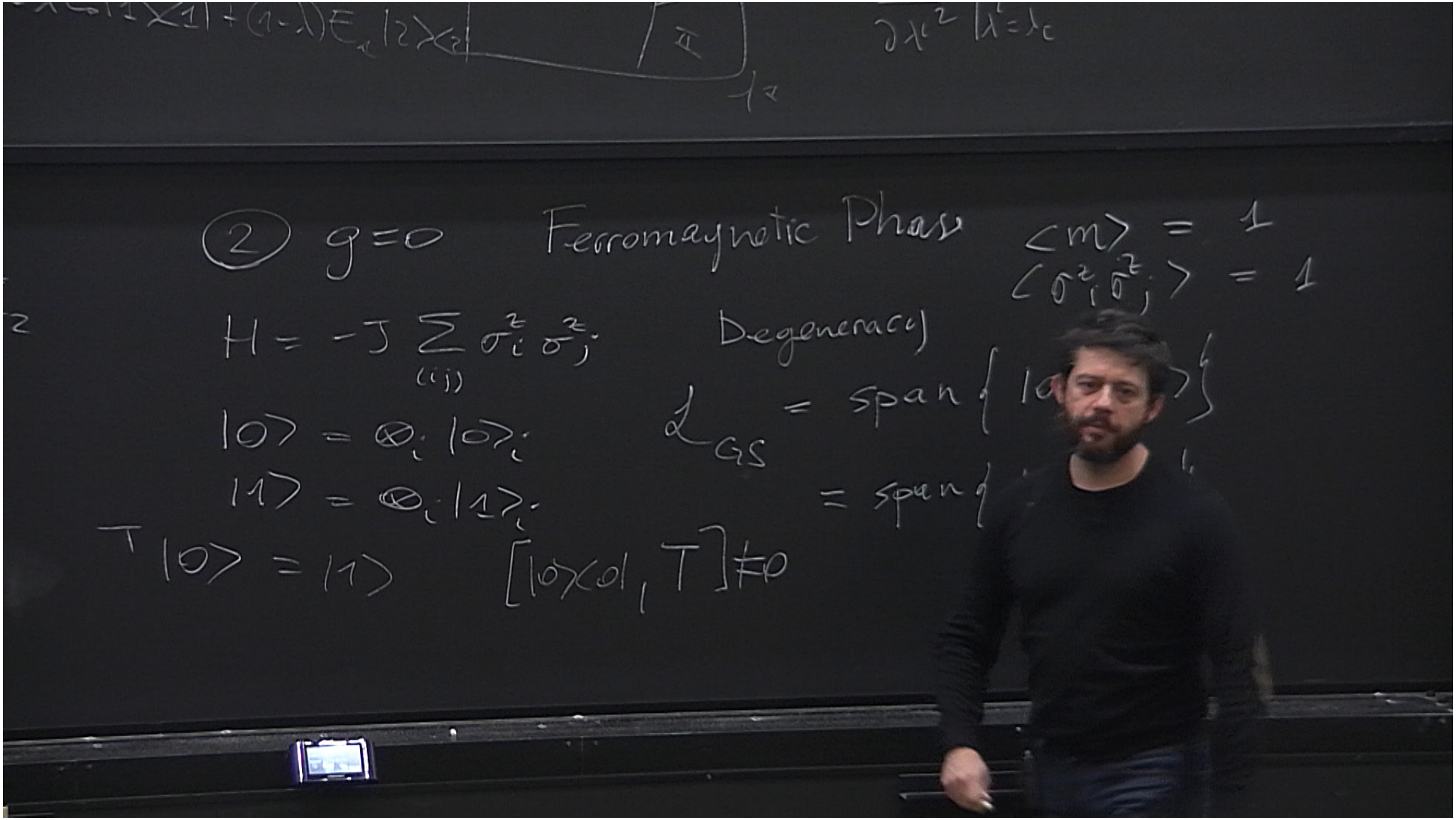

$\partial\lambda^2 |\lambda=\lambda_c$
π
$\frac{1}{2}$
② $g=0$ Ferromagnetic Phase
$H = -J \sum_{\langle ij \rangle} \sigma_i^z \sigma_j^z$
$|0\rangle = \otimes_i |0\rangle_i$
$|1\rangle = \otimes_i |1\rangle_i$
Degeneracy
$\mathcal{L}_{GS} = \text{span}\{|0\rangle, |1\rangle\}$
$= \text{span}\{|+\rangle, |-\rangle\}$

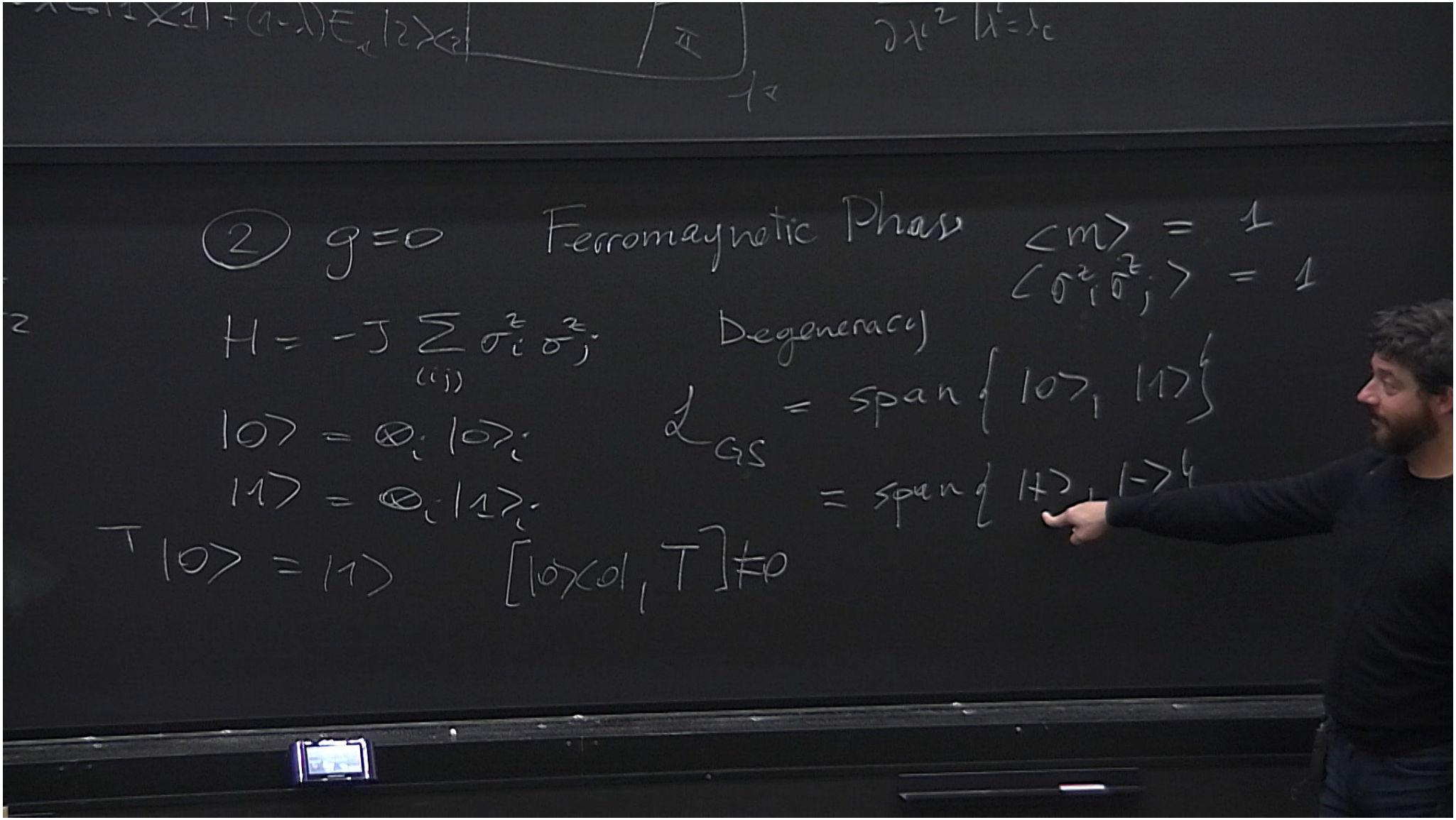

② g=0 Ferromagnetic Phase <m>
H = -J Σ_(⟨ij⟩) σ_i^z σ_j^z
|0⟩ = ⊗_i |0⟩_i
|1⟩ = ⊗_i |1⟩_i
Degeneracy
L_GS = span{|0⟩, |1⟩}
= span{|+⟩, |-⟩}

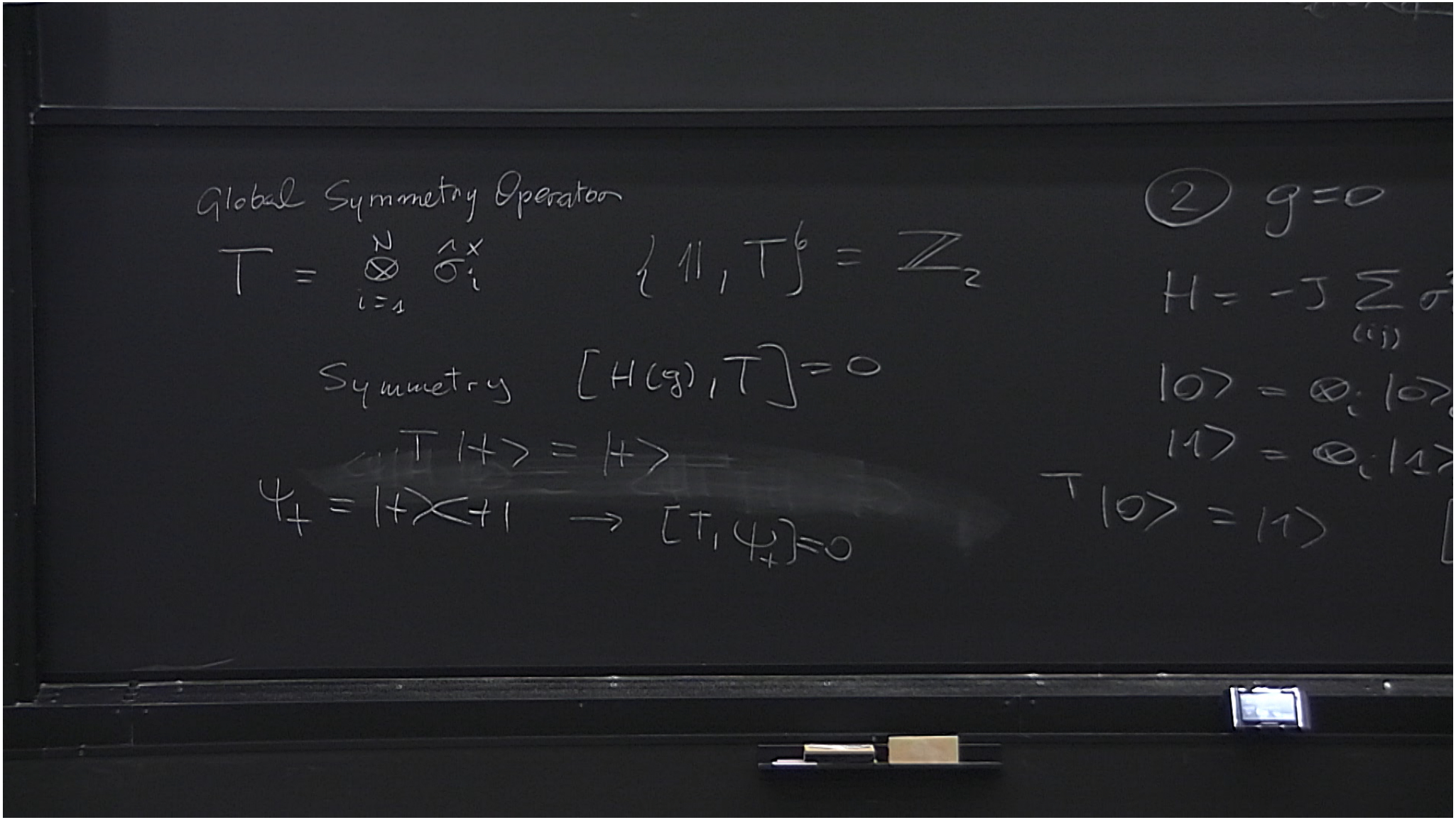

② g=0 Ferromagnetic Phase ⟨m⟩ = 1
H = -J Σ_⟨ij⟩ σ^z_i σ^z_j
|0⟩ = ⊗_i |0⟩_i
|1⟩ = ⊗_i |1⟩_i
Degeneracy
L_GS = span{|0⟩, |1⟩}
= span{|+⟩, |-⟩}

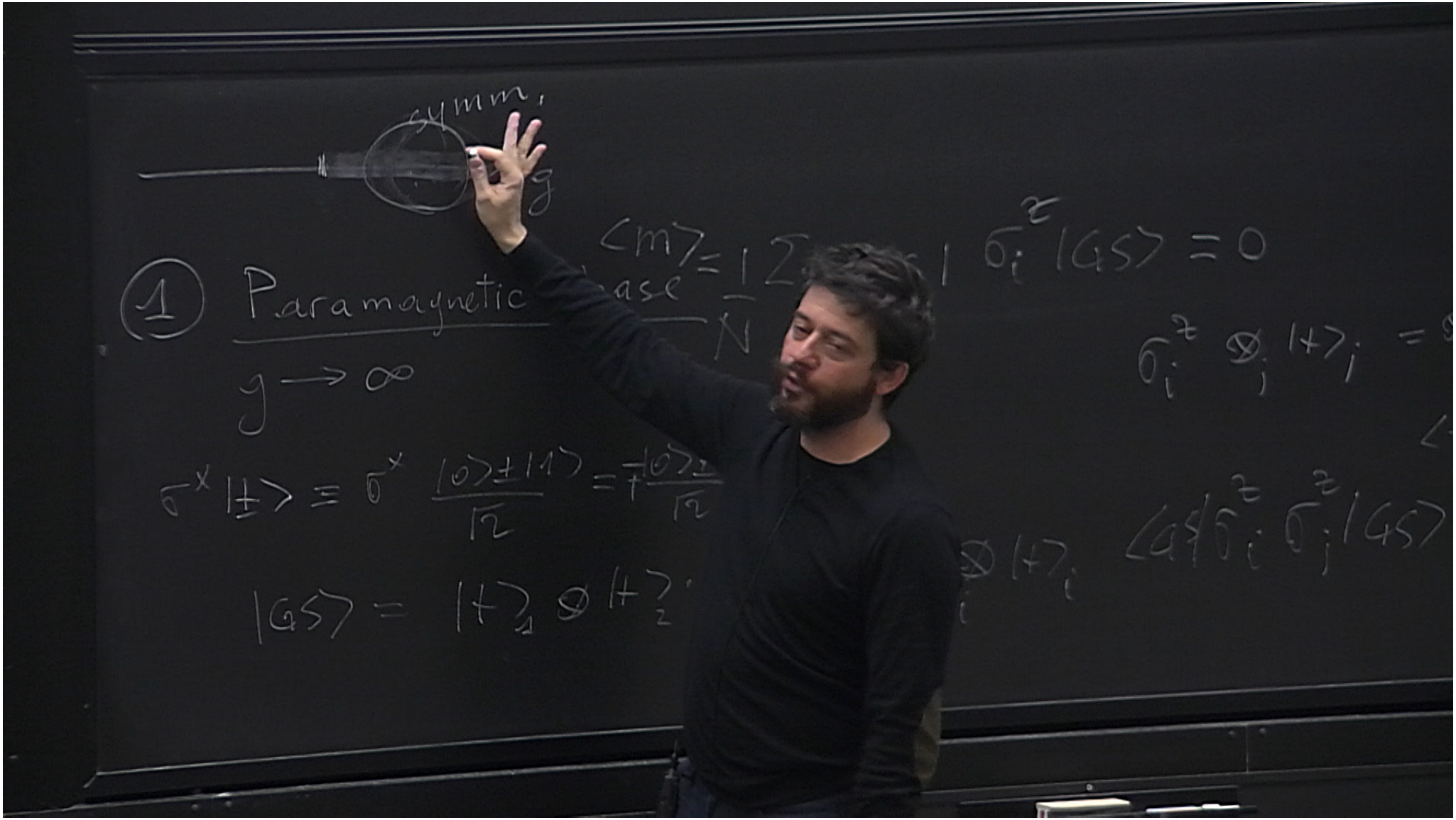

② g=0 Ferromagnetic Phase
⟨m⟩ = 1
⟨σ_i^z σ_j^z⟩ = 1
H = −J Σ_⟨i,j⟩ σ_i^z σ_j^z
Degeneracy
|0⟩ = ⊗_i |0⟩_i
|1⟩ = ⊗_i |1⟩_i
L_GS = span{|0⟩, |1⟩}
= span{|+⟩, |−⟩}

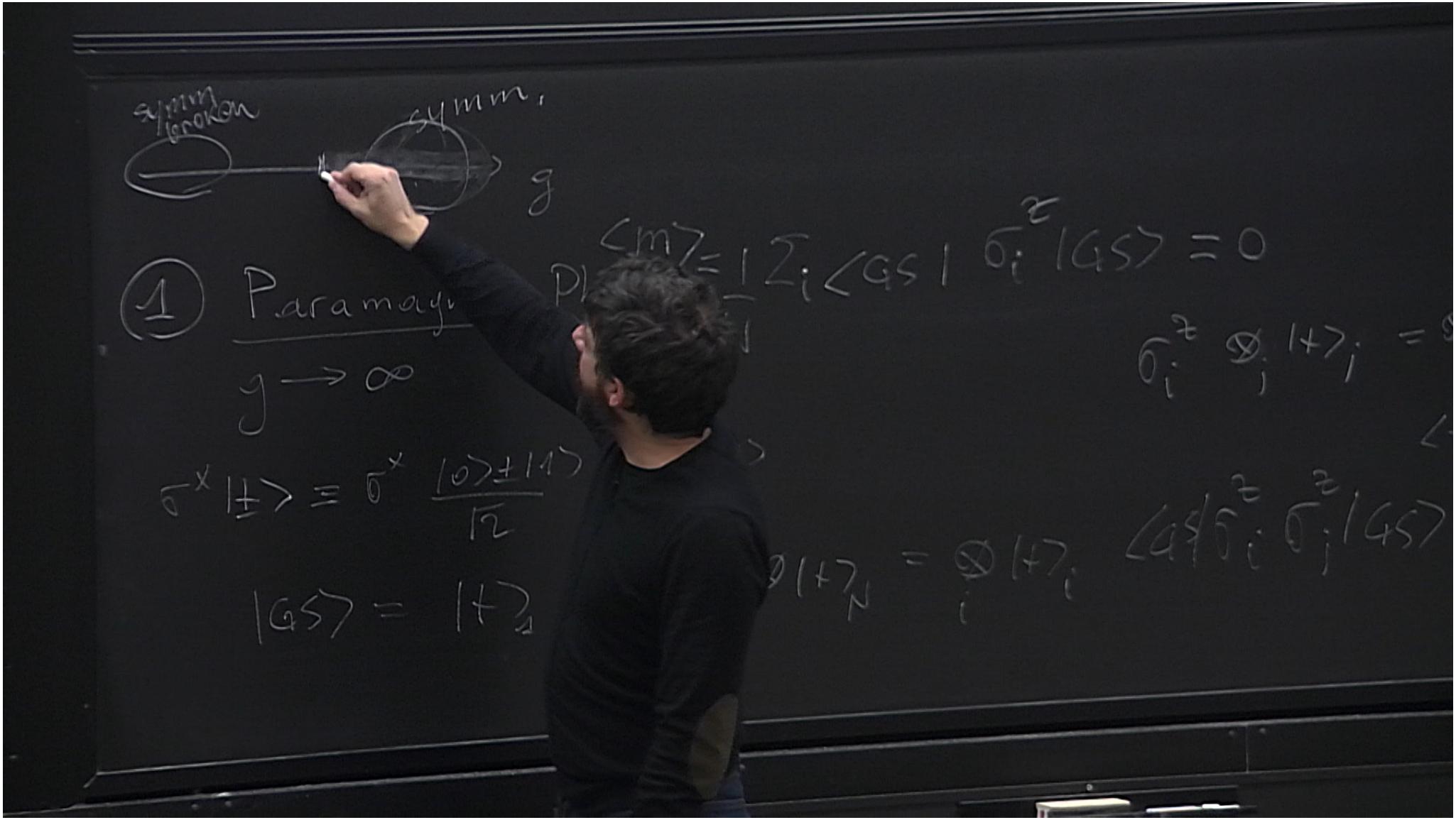

② g=0 Ferromagnetic Phase
$\langle m \rangle = 1$
$\langle \sigma_i^z \sigma_j^z \rangle = 1$
$H = -J \sum_{\langle ij \rangle} \sigma_i^z \sigma_j^z$
Degeneracy
$|0\rangle = \otimes_i |0\rangle_i$
$|1\rangle = \otimes_i |1\rangle_i$
$\mathcal{L}_{GS} = \text{span}\{|0\rangle, |1\rangle\}$
$= \text{span}\{|+\rangle, |-\rangle\}$
$T|0\rangle = |1\rangle$


② g=0 Ferromagnetic Phase
$\langle m \rangle = 1$
$\langle \sigma_i^z \sigma_j^z \rangle = 1$
$H = -J \sum_{\langle ij \rangle} \sigma_i^z \sigma_j^z$
Degeneracy
$|0\rangle = \otimes_i |0\rangle_i$
$|1\rangle = \otimes_i |1\rangle_i$
$T|0\rangle = |1\rangle$
$[|0\rangle\langle 0|, T] \neq 0$


② g=0 Ferromagnetic Phase
$\langle m \rangle = 1$
$\langle \sigma_i^z \sigma_j^z \rangle = 1$
$H = -J \sum_{\langle ij \rangle} \sigma_i^z \sigma_j^z$
Degeneracy
$|0\rangle = \otimes_i |0\rangle_i$
$|1\rangle = \otimes_i |1\rangle_i$
$\mathcal{L}_{GS} = \text{span}\{|0\rangle, |1\rangle\}$
$= \text{span}\{|+\rangle, |-\rangle\}$
$T|0\rangle = |1\rangle$
$[|0\rangle\langle 0|, T] \neq 0$


Global Symmetry Operator
$T = \bigotimes_{i=1}^{N} \hat{\sigma}_i^x$
$\{11, T\} = \mathbb{Z}_2$
Symmetry $[H(g), T] = 0$
$T|+\rangle = |+\rangle$
$\psi_+ = |+\rangle\langle+| \rightarrow [T, \psi_+] = 0$
② $g = 0$
$H = -J \sum_{\langle i,j \rangle}$
$|0\rangle = \otimes_i |0\rangle$
$|1\rangle = \otimes_i |1\rangle$
$T|0\rangle = |1\rangle$


symm.
g
(1) Paramagnetic
g → ∞
|GS⟩ = |+⟩₁ ⊗ |+⟩₂


symm. broken
symm.
g
(1) Paramag.
g → ∞
|GS⟩ = |+⟩