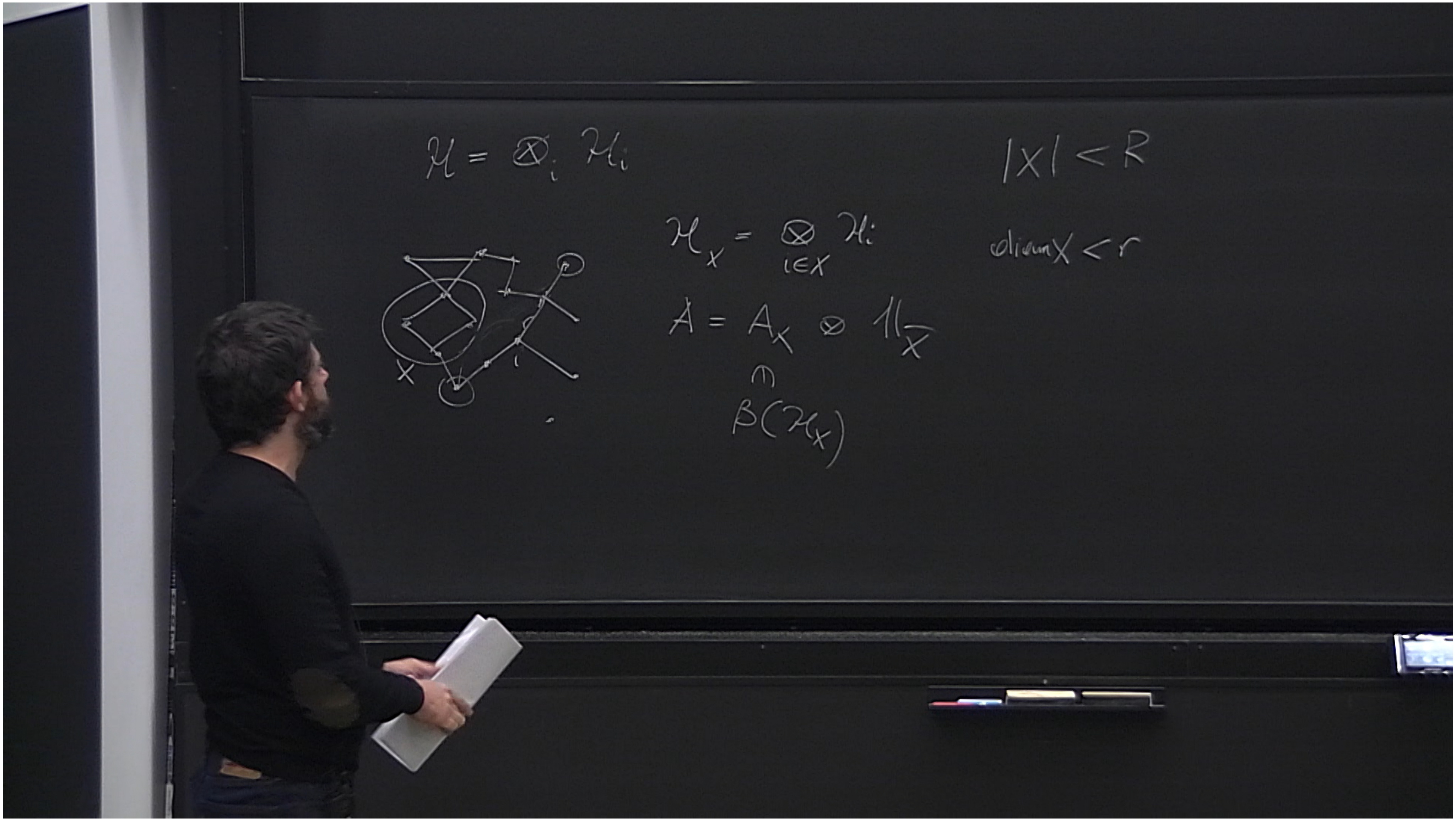
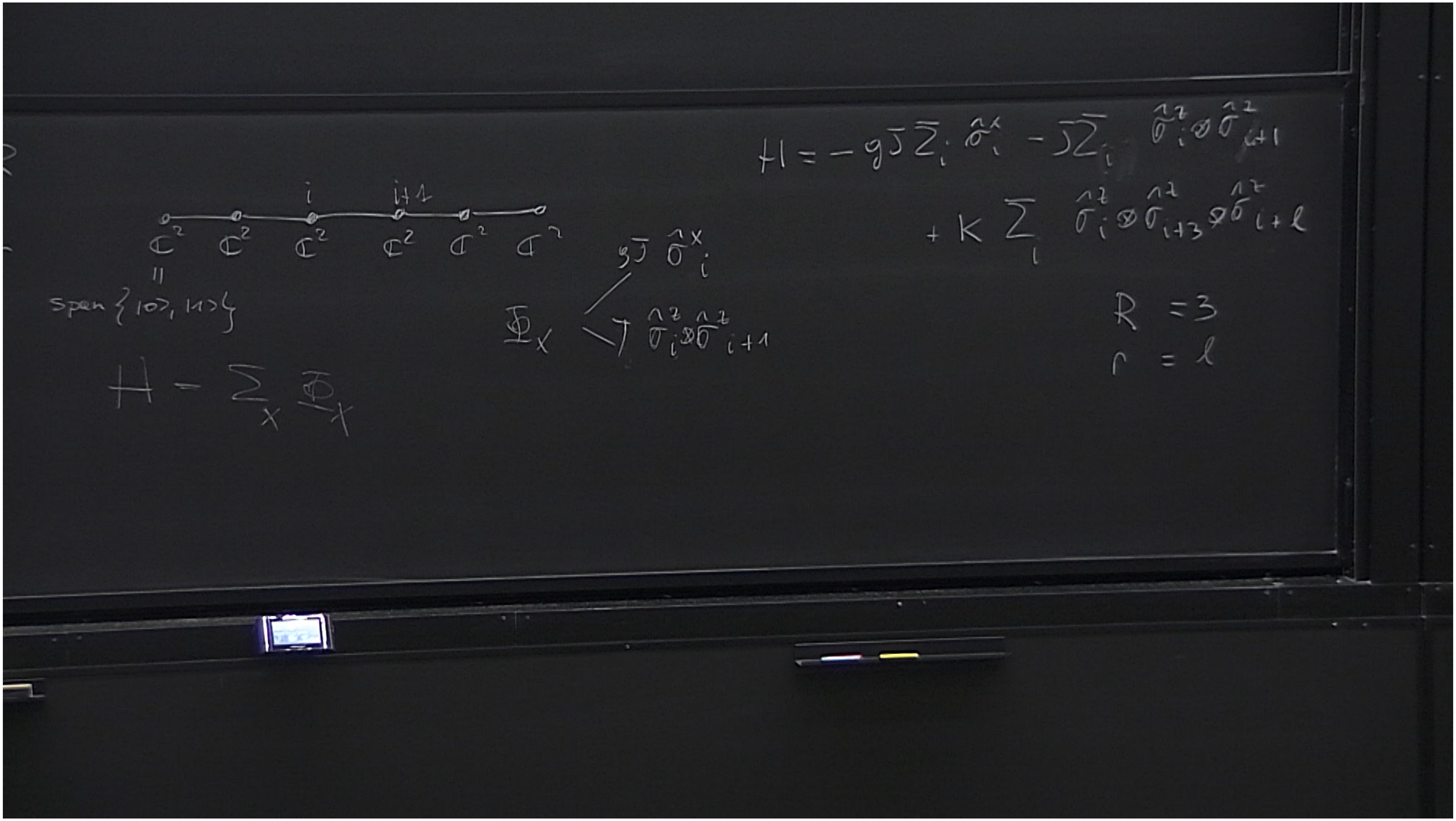
Title: PSI 17/18 - Condensed Matter - Lecture 2

Date: Jan 04, 2018 11:30 AM

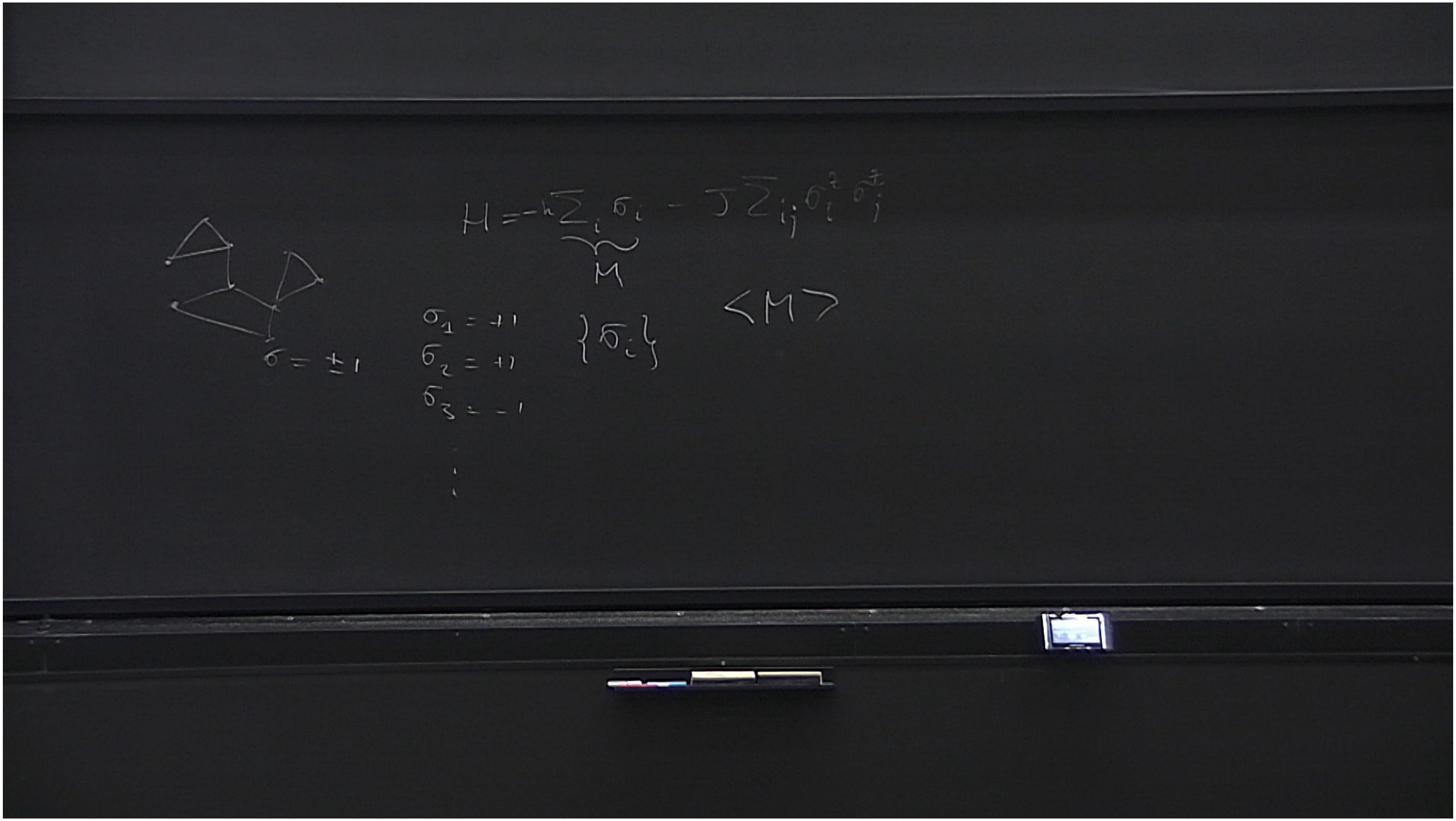
URL: http://pirsa.org/18010035

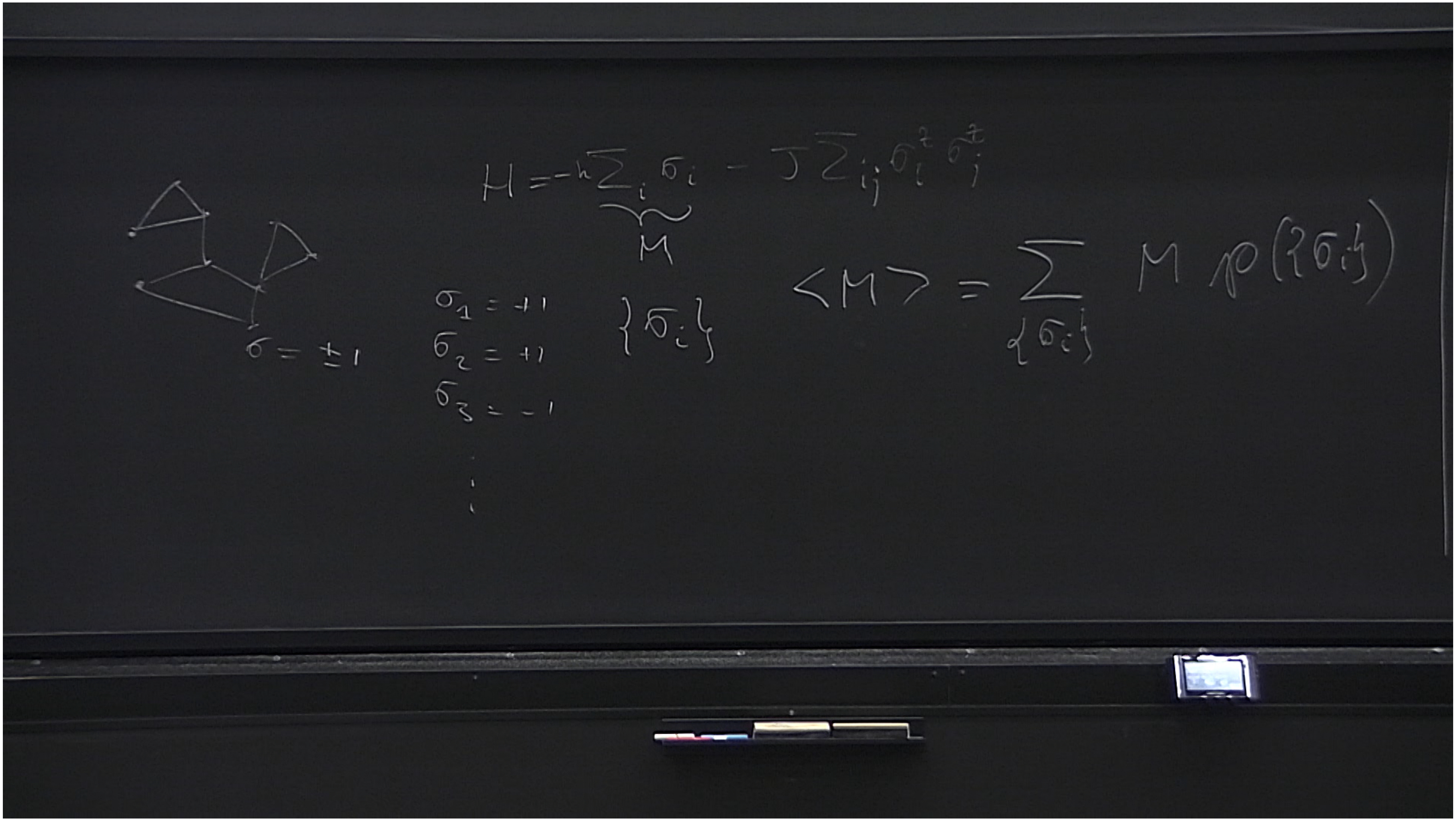
Abstract:


$\mathcal{H} = \otimes_i \mathcal{H}_i$
$|X| < R$
$\mathcal{H}_X = \bigotimes_{i \in X} \mathcal{H}_i$
$\mathrm{diam}\, X < r$
$A = A_X \otimes \mathbb{1}_{\bar{X}}$
$\cap$
$\mathcal{B}(\mathcal{H}_X)$
X
i



i $i+1$
$\mathbb{C}^2$ $\mathbb{C}^2$ $\mathbb{C}^2$ $\mathbb{C}^2$ $\mathbb{C}^2$ $\mathbb{C}^2$
$\mathrm{span}\{|0\rangle, |1\rangle\}$
$H - \sum_x \Phi_x$
Φ_x
$gJ\,\hat{\sigma}^x_i$
$\hat{\sigma}^z_i \otimes \hat{\sigma}^z_{i+1}$
$H = -gJ\sum_i \hat{\sigma}^x_i - J\sum_i \hat{\sigma}^z_i \otimes \hat{\sigma}^z_{i+1}$
$+ K\sum_i \hat{\sigma}^z_i \otimes \hat{\sigma}^z_{i+3} \otimes \hat{\sigma}^z_{i+l}$
$R = 3$
$r = l$


$H = -h \underbrace{\sum_i \sigma_i}_{M} - J \sum_{ij} \sigma_i^z \sigma_j^z$
$\sigma = \pm 1$
$\sigma_1 = +1$
$\sigma_2 = +1$
$\sigma_3 = -1$
$\{\sigma_i\}$
$\langle M \rangle$


$H = -h\sum_i \sigma_i - J\sum_{ij} \sigma_i^z \sigma_j^z$
M
$\sigma = \pm 1$
$\sigma_1 = +1$
$\sigma_2 = +1$
$\sigma_3 = -1$
$\{\sigma_i\}$
$\langle M \rangle = \sum_{\{\sigma_i\}} M \, p(\{\sigma_i\})$

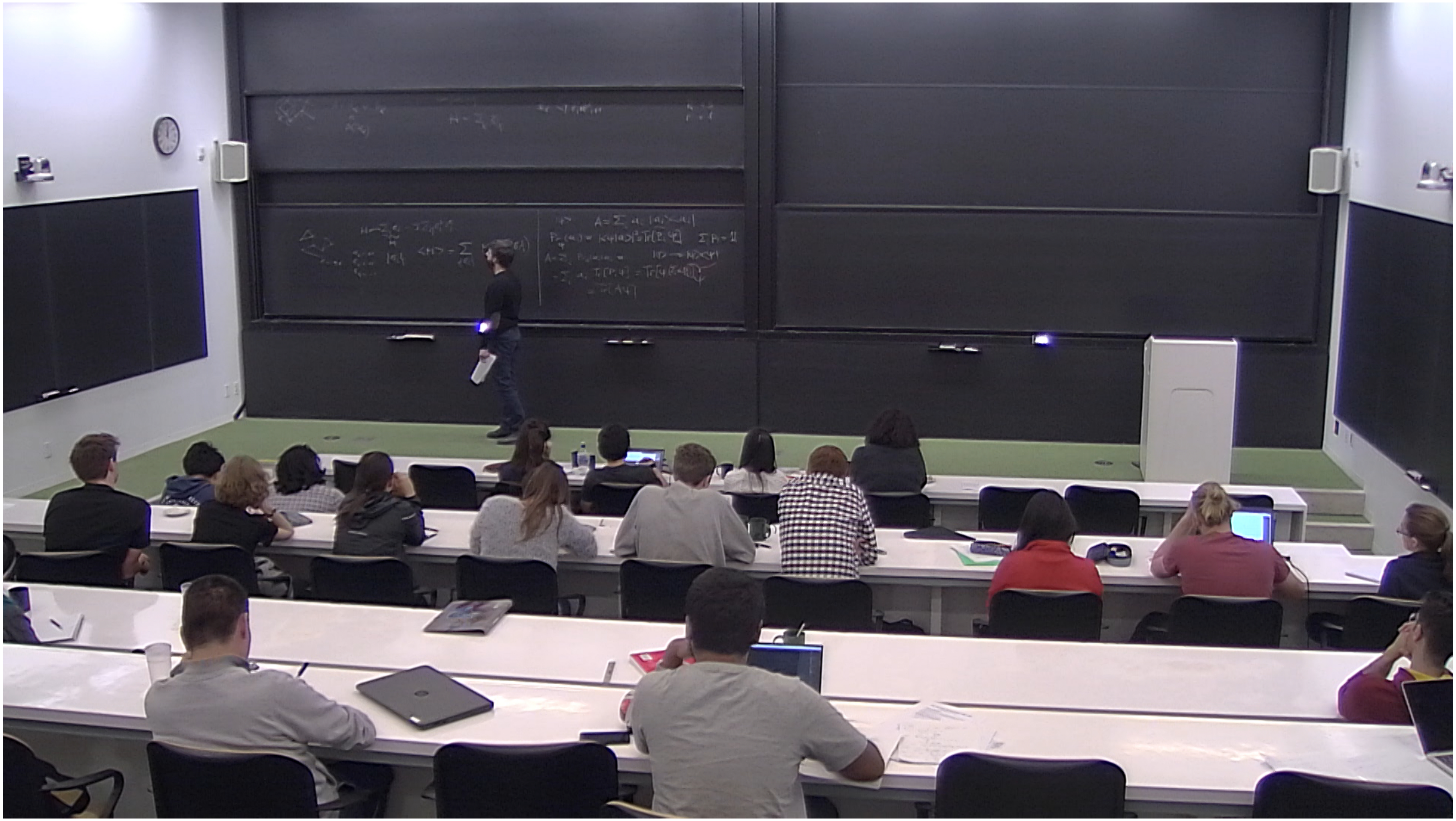
$\sigma_{ij})$

$|\psi\rangle \qquad A = \sum_i a_i |a_i\rangle\langle a_i|$

$\Pr_\psi(a_i) = |\langle\psi|a_i\rangle|^2 = \mathrm{Tr}[P_i \psi] \qquad \sum_i P_i = \mathbb{1}$

$|\psi\rangle \longrightarrow \underbrace{|\psi\rangle\langle\psi|}_{\psi}$

$A = \sum_i \Pr_\psi(a_i)\, a_i$



$$|\psi\rangle \qquad A = \sum_i a_i |a_i\rangle\langle a_i|$$

$$\Pr_\psi(a_i) = |\langle\psi|a_i\rangle|^2 = \mathrm{Tr}[P_i \psi] \qquad \sum_i P_i = \mathbb{1}$$

$$|\psi\rangle \longrightarrow \underbrace{|\psi\rangle\langle\psi|}_{\psi}$$

$$A = \sum_i \Pr_\psi(a_i)\, a_i = $$

$$= \sum_i a_i \mathrm{Tr}[P_i \psi] = \mathrm{Tr}\Big[\psi \Big(\sum_i a_i P_i\Big)\Big]$$

$$= \mathrm{Tr}[A\psi]$$

$|\psi_j\rangle \qquad q_j$

$\uparrow$ probability

$$\Pr[a_i] = \sum_j q_j \,\mathrm{Tr}[P_i \psi_j] = \mathrm{Tr}\Big[P_i \big(\underbrace{\textstyle\sum_j q_j \psi_j}_{\rho}\big)\Big]$$

$\langle \psi | A | \psi \rangle$

$|\psi\rangle\langle\psi| \neq q\,|\phi_1\rangle\langle\phi_1| + (1-q)\,|\phi_2\rangle\langle\phi_2|$

$\mathrm{Tr}[A\psi]$

$$|\psi\rangle\langle\psi| \downarrow \begin{pmatrix} 1 & & & & \\ & 0 & & & \\ & & 0 & & \\ & & & 0 & \\ & & & & \ddots \\ & & & & & 0 \end{pmatrix} \qquad \begin{pmatrix} q & & & & & \\ & 1-q & & & & \\ & & 0 & & & \\ & & & 0 & & \\ & & & & 0 & \\ & & & & & 0 \end{pmatrix}$$

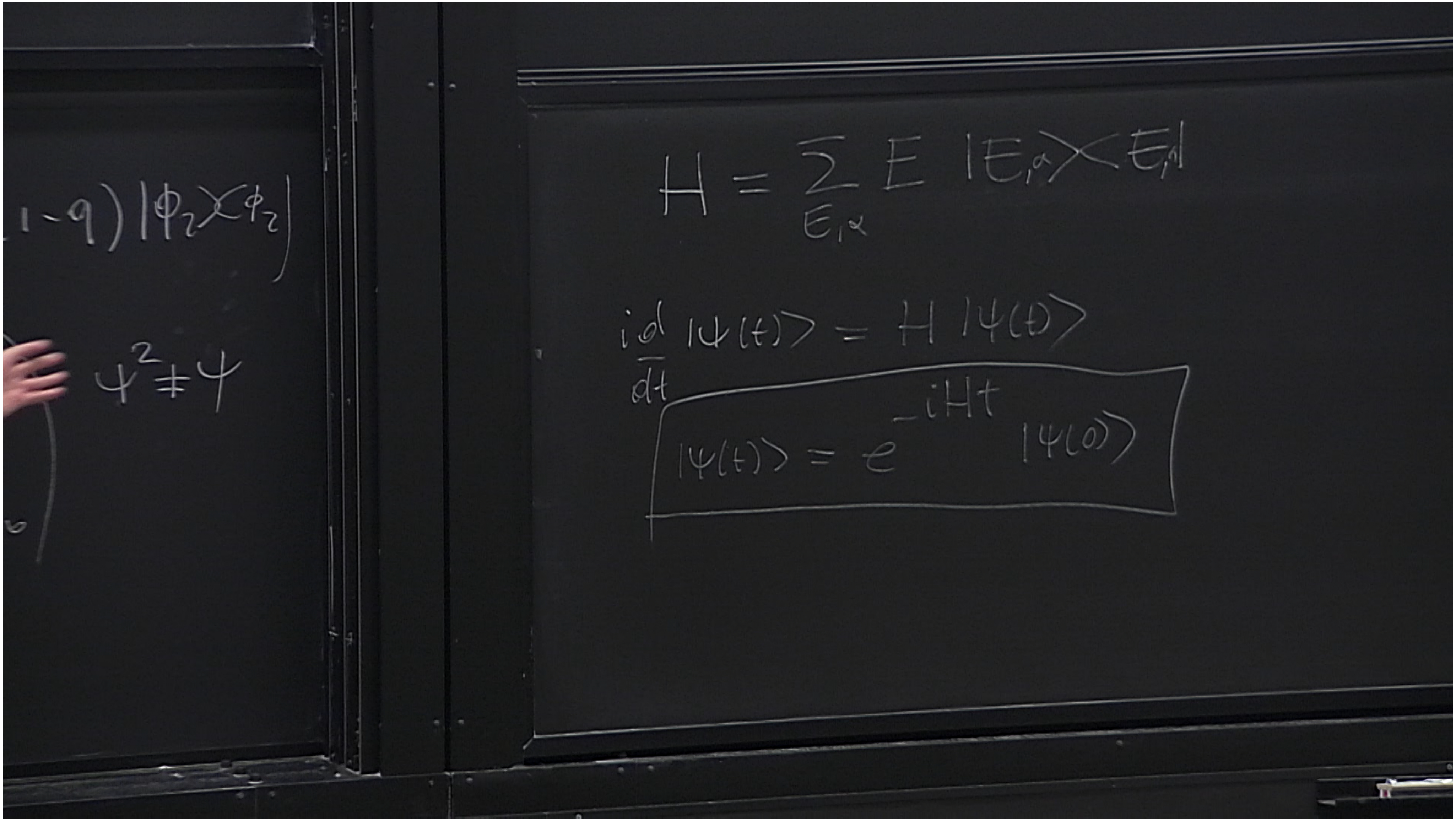
$$\langle \psi | A | \psi \rangle$$

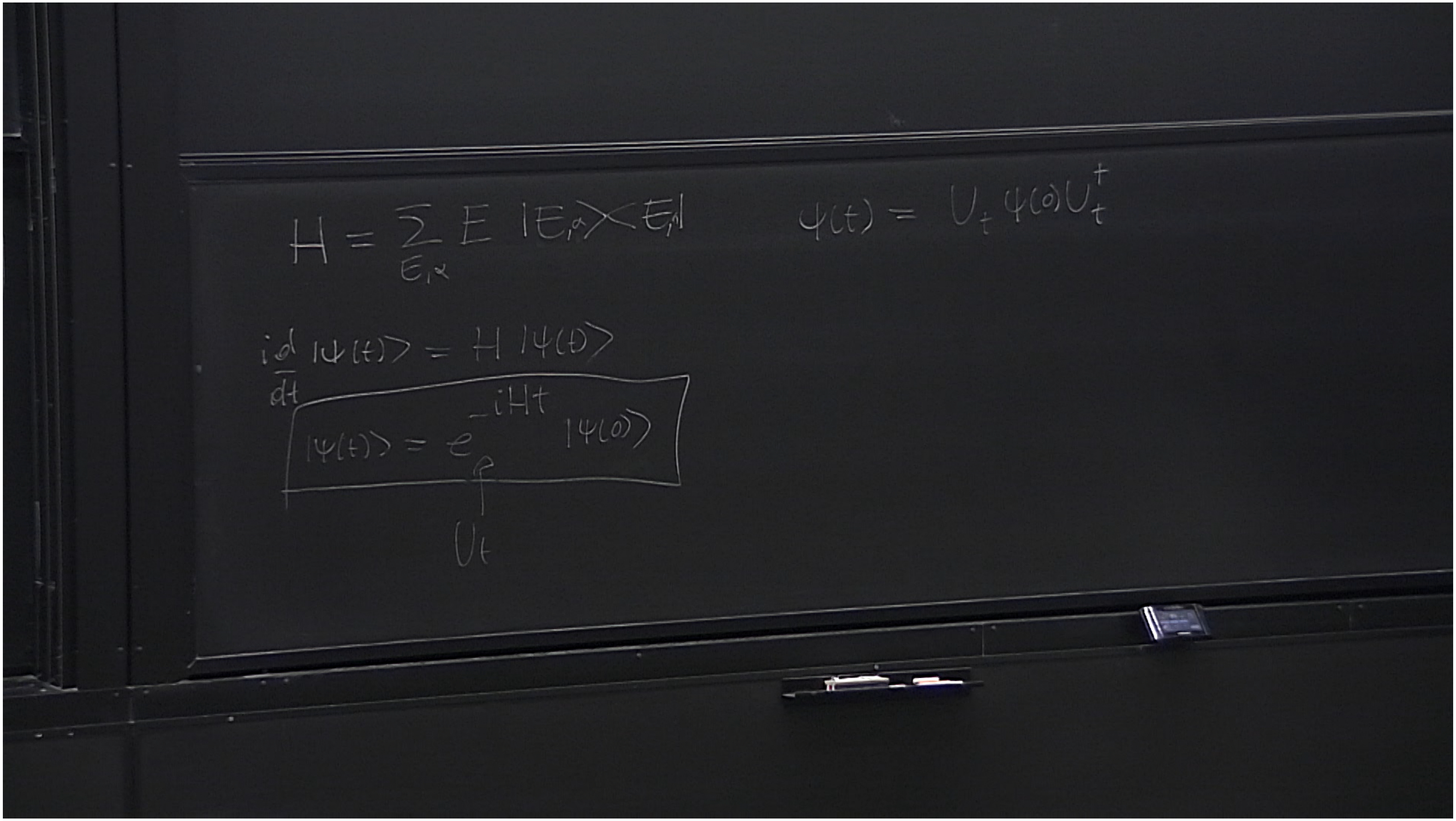
$$|\psi\rangle\langle\psi| \neq q\,|\phi_1\rangle\langle\phi_1| + (1-q)\,|\phi_2\rangle\langle\phi_2|$$

$$\underbrace{P_i\left(\textstyle\sum_j q_j \psi_j\right)\Big]}_{\psi} \qquad \mathrm{Tr}[A\psi]$$

$$\underbrace{|\psi\rangle\langle\psi|\psi\rangle}_{1}\langle\psi|$$

$$|\psi\rangle\langle\psi| \to \begin{pmatrix} 1 & & & & \\ & 0 & & & \\ & & 0 & & \\ & & & \ddots & \\ & & & & 0 \end{pmatrix} \qquad \begin{pmatrix} q & & & & \\ & 1-q & & & \\ & & 0 & & \\ & & & \ddots & \\ & & & & 0 \end{pmatrix} \qquad \psi^2 \neq \psi$$


$(1-q)|\phi_2\rangle\langle\phi_2|$
$\psi^2 \neq \psi$
$H = \sum_{E,\alpha} E\, |E,\alpha\rangle\langle E,\alpha|$
$i\frac{d}{dt}|\psi(t)\rangle = H|\psi(t)\rangle$
$\boxed{|\psi(t)\rangle = e^{-iHt}|\psi(0)\rangle}$


$H = \sum_{E,\alpha} E \, |E,\alpha\rangle\langle E,\alpha|$
$\psi(t) = U_t \, \psi(0) \, U_t^\dagger$
$i\frac{d}{dt}|\psi(t)\rangle = H|\psi(t)\rangle$
$\boxed{|\psi(t)\rangle = \underbrace{e^{-iHt}}_{U_t}|\psi(0)\rangle}$

$\rangle\langle E_n|$

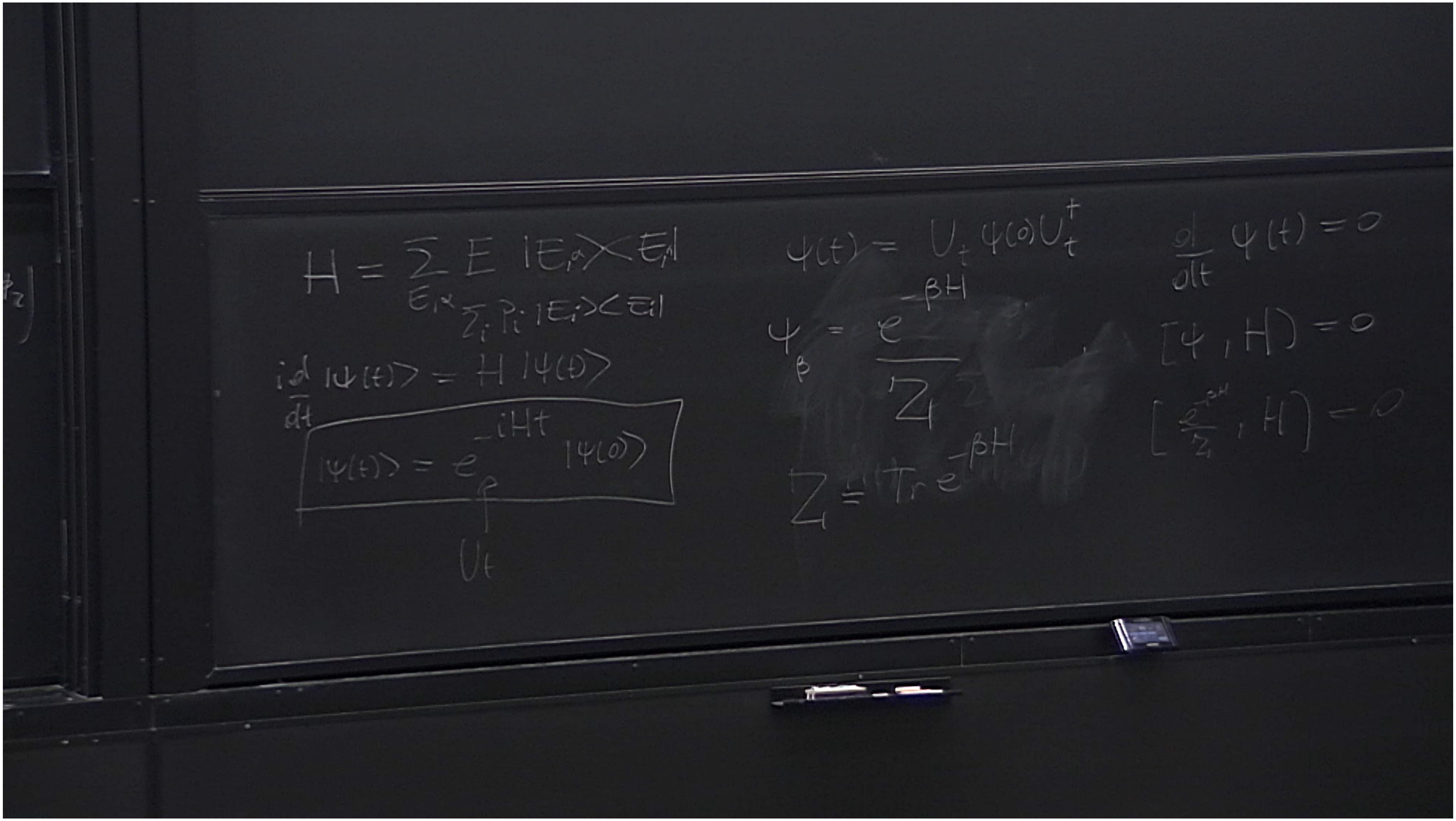
$$\psi(t) = U_t\,\psi(0)\,U_t^\dagger \qquad \frac{d}{dt}\psi(t) = 0$$

$$\psi(0) = \sum q_j \psi_j \qquad [\psi, H] = 0$$

$$\psi(t) = U_t \sum q_j \psi_j U_t^\dagger$$

$$\dot{\psi}(t) = [H, \psi(t)]$$

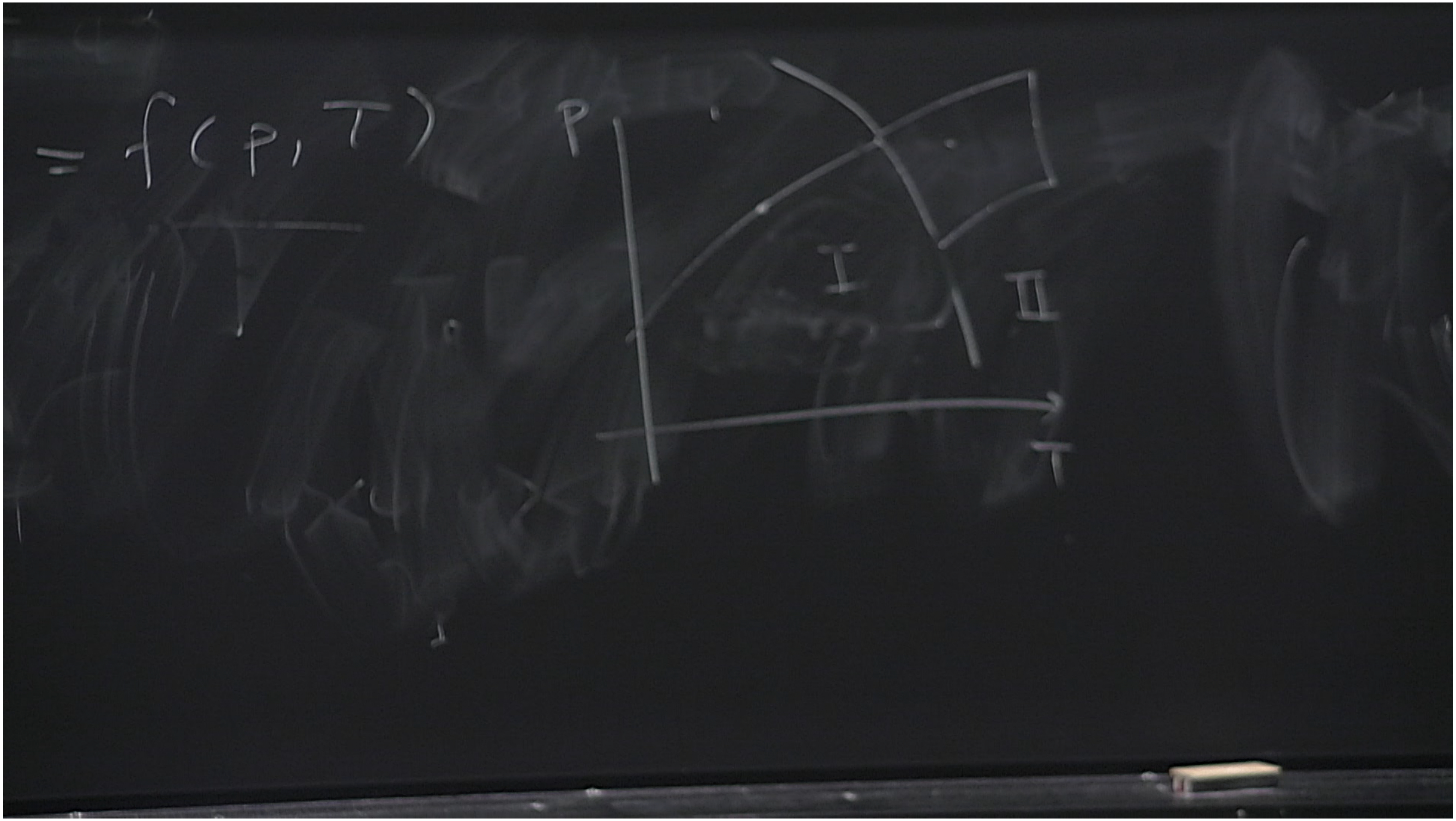
$\psi(0)\rangle$


$H = \sum_{E,\alpha} E \, |E,\alpha\rangle\langle E,\alpha|$
$\sum_i p_i \, |E_i\rangle\langle E_i|$
$i\frac{d}{dt} |\psi(t)\rangle = H |\psi(t)\rangle$
$|\psi(t)\rangle = e^{-iHt} |\psi(0)\rangle$
U_t
$\psi(t) = U_t \, \psi(0) \, U_t^\dagger$
$\psi_\beta = \frac{e^{-\beta H}}{Z}$
$Z = \mathrm{Tr} \, e^{-\beta H}$
$\frac{d}{dt} \psi(t) = 0$
$[\psi, H] = 0$
$[\frac{e^{-\beta H}}{Z}, H] = 0$

$$F = E - TS = k_B T \log Z$$

$$\beta = \frac{1}{k_B T} \qquad p(E) = \frac{e^{-\beta E}}{Z}$$

$$\lim_{N\to\infty} \frac{F}{N} = f(\rho, T)$$


= f(p, T)
P
I
II
T