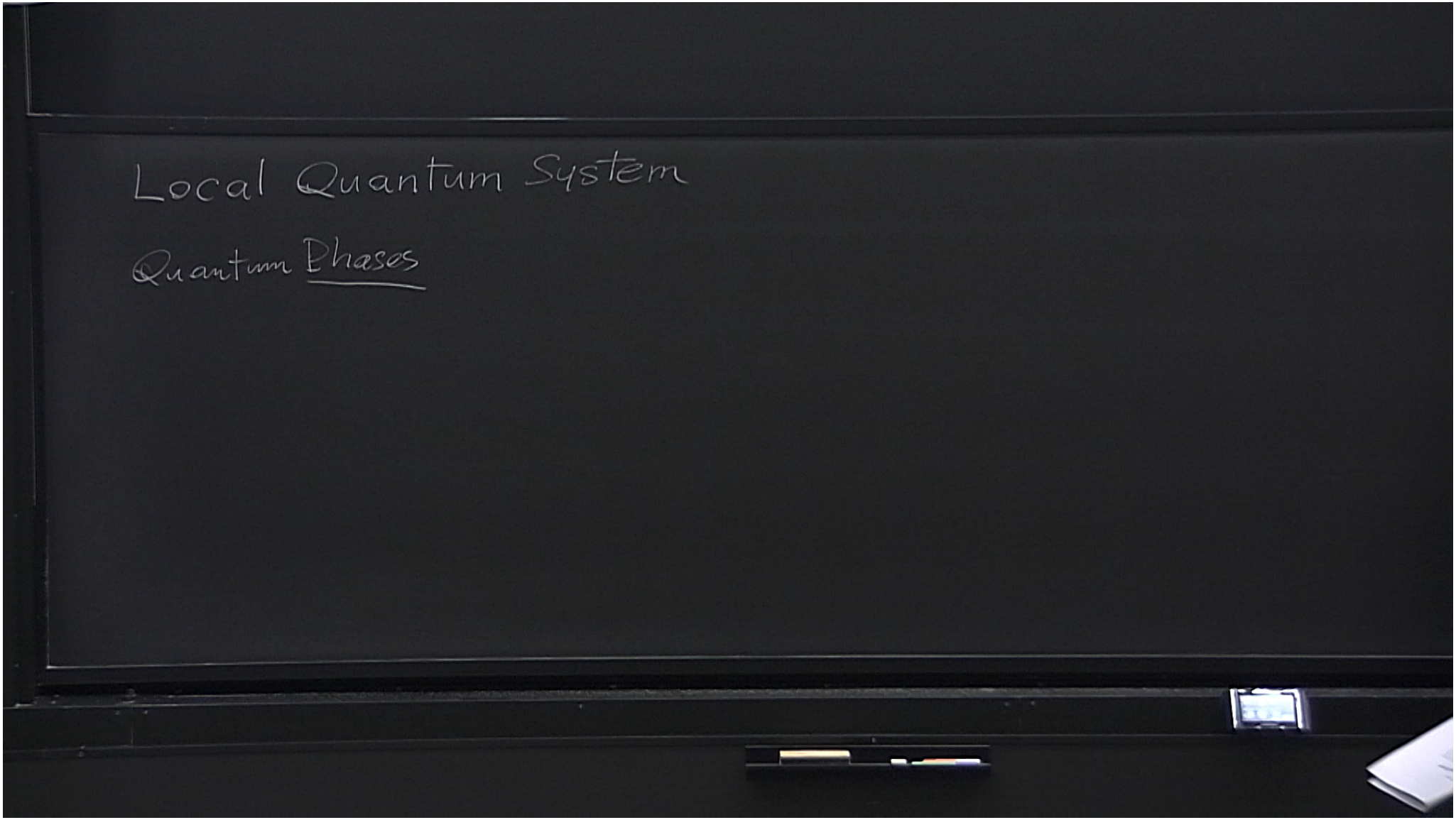
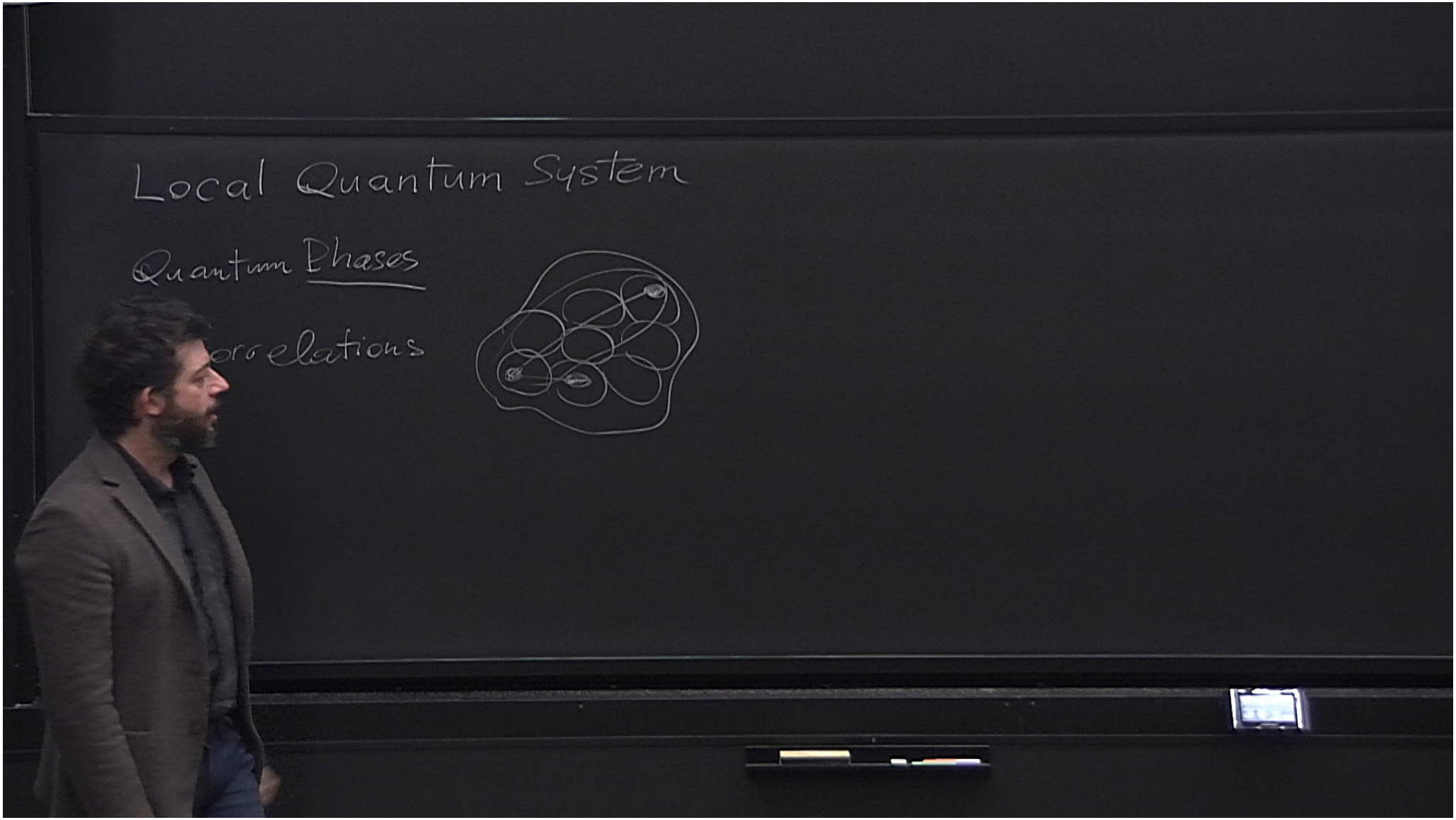
Title: PSI 17/18 - Condensed Matter - Lecture 1

Date: Jan 03, 2018 11:30 AM

URL: http://pirsa.org/18010034

Abstract:

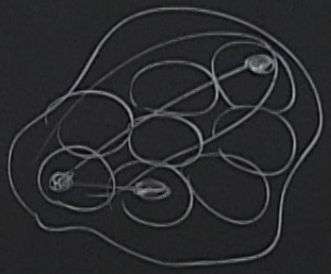

Local Quantum System
Quantum Phases


Local Quantum System
Quantum Phases
orrelations

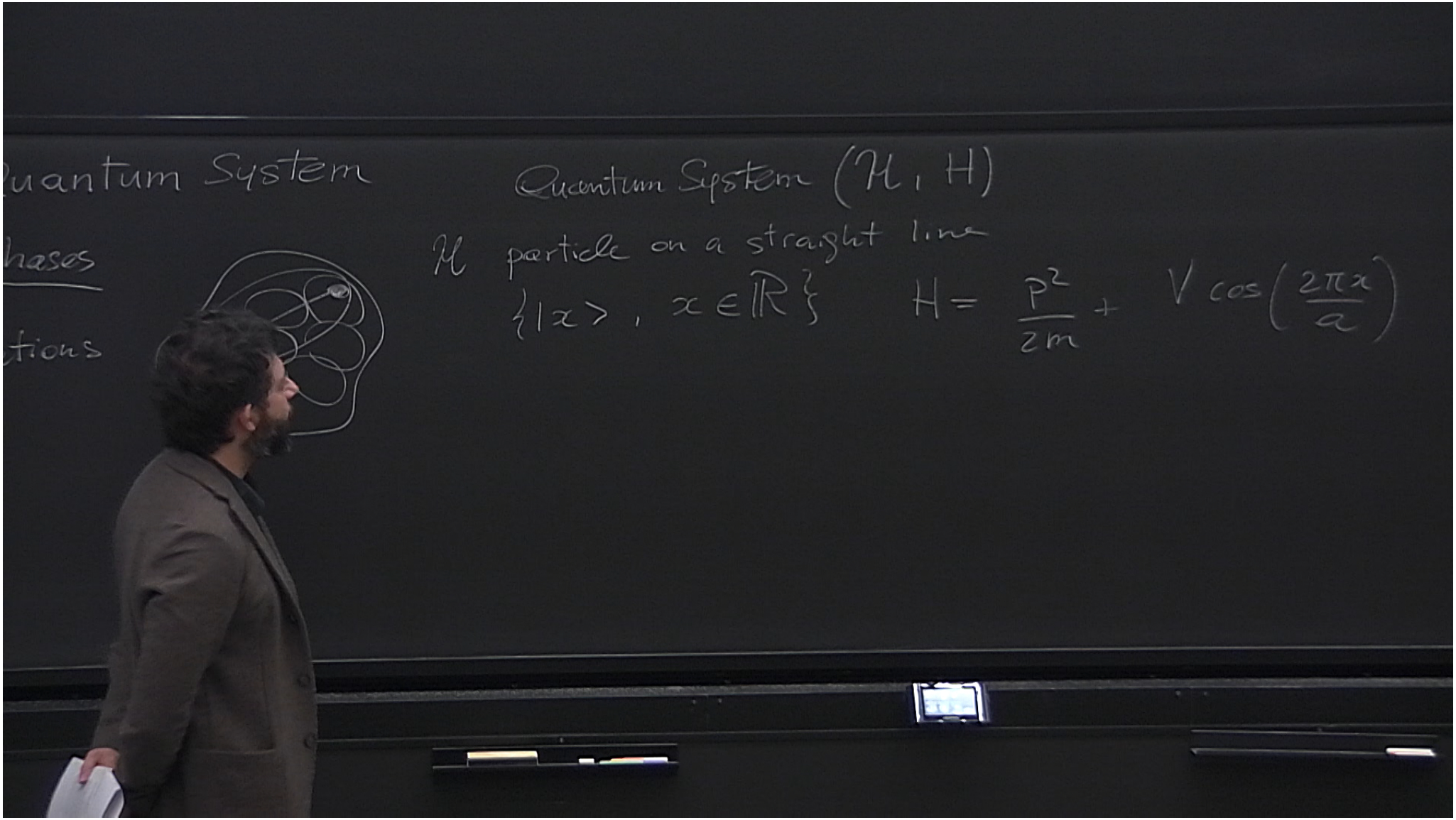
Local Quantum System

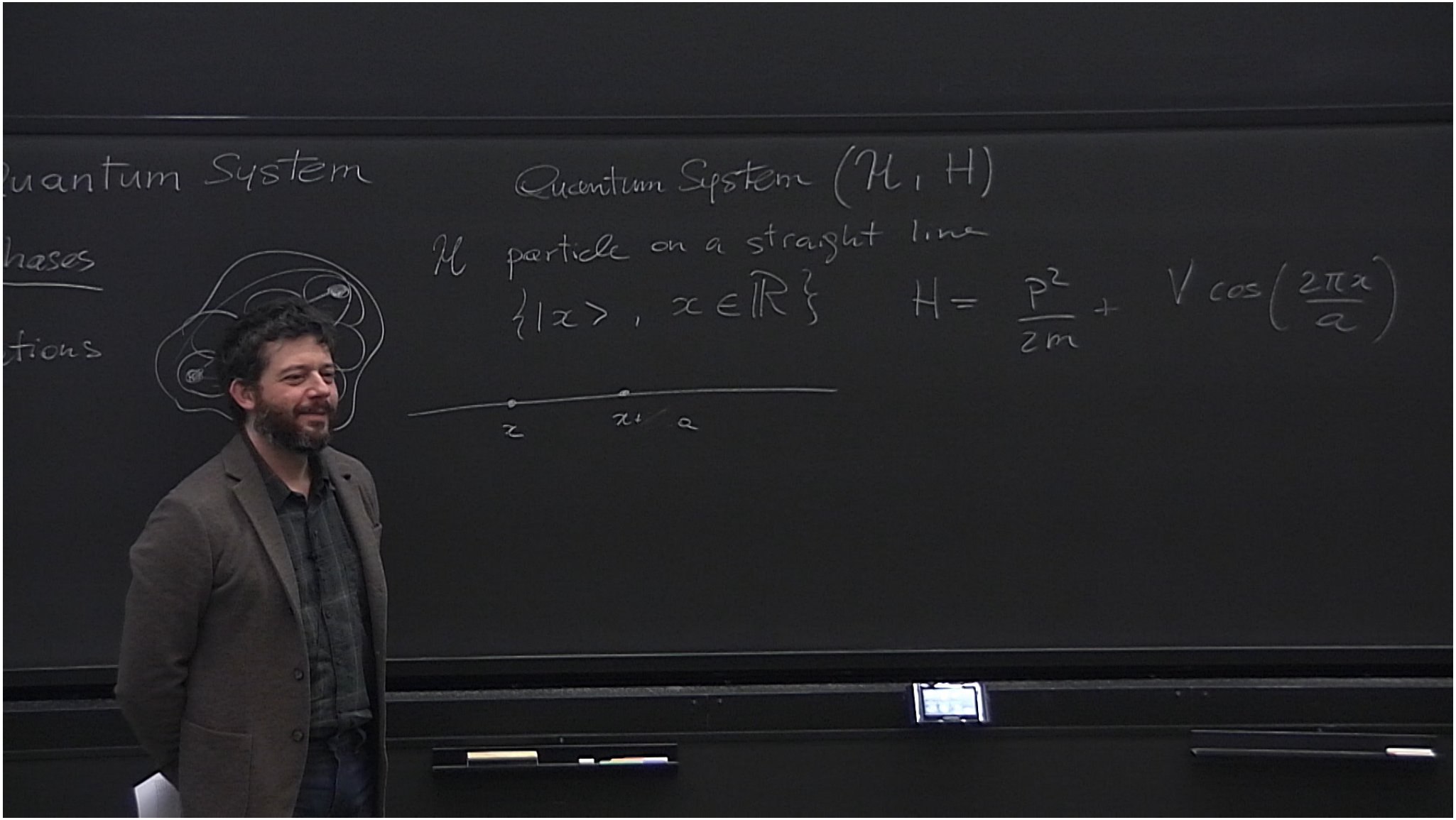
Quantum Phases

Correlations



Quantum System $(\mathcal{H}, H)$

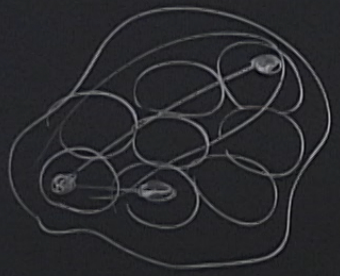

uantum System
hases
tions
Quantum System $(\mathcal{H}, H)$
$\mathcal{H}$ particle on a straight line
$\{|x\rangle, x \in \mathbb{R}\}$
$H = \frac{p^2}{2m} + V\cos\left(\frac{2\pi x}{a}\right)$


uantum System
hases
tions
Quantum System $(\mathcal{H}, H)$
$\mathcal{H}$ particle on a straight line
$\{|x\rangle, x \in \mathbb{R}\}$
$H = \frac{p^2}{2m} + V\cos\left(\frac{2\pi x}{a}\right)$
x
x+a

Quantum System

hases

tions

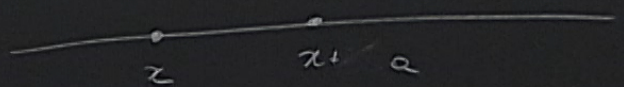


Quantum System $(\mathcal{H}, H)$

$\mathcal{H}$ particle on a straight line

$\{|x\rangle, x \in \mathbb{R}\}$

$$H = \frac{p^2}{2m} + V\cos\left(\frac{2\pi x}{a}\right)$$



$$T_y|x\rangle = |x+y\rangle$$

$$[T_a, H] = 0$$

$(\mathcal{H}, H)$

ht line

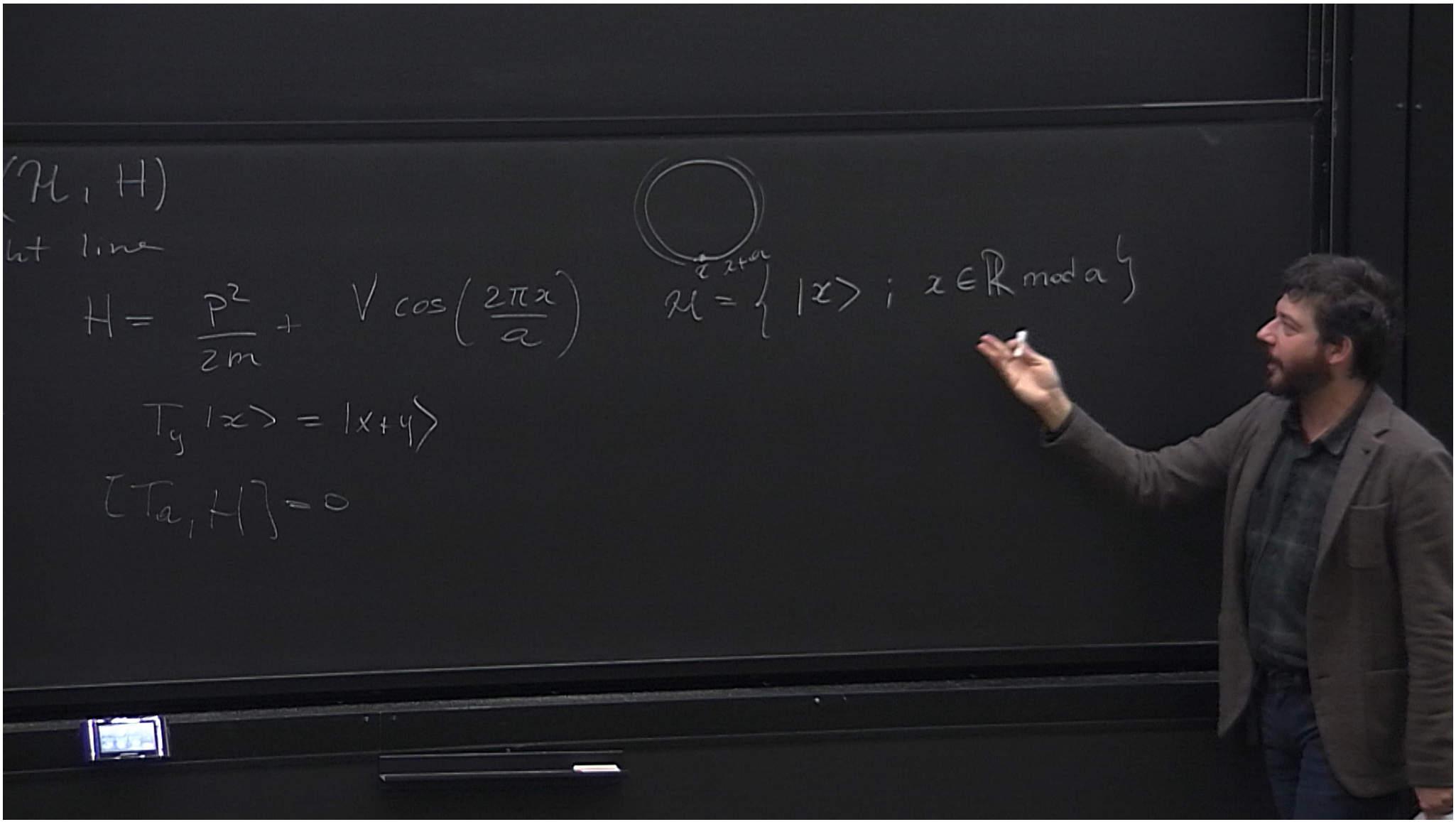
$$H = \frac{p^2}{2m} + V\cos\left(\frac{2\pi x}{a}\right)$$

$$\mathcal{H} = \{\, |x\rangle \;;\; x \in \mathbb{R} \bmod a \,\}$$

$x \sim x+a$

$$T_y |x\rangle = |x+y\rangle$$

$$[T_a, H] = 0$$



$(\mathcal{H}, H)$

ht line

$$H = \frac{p^2}{2m} + V\cos\left(\frac{2\pi x}{a}\right)$$

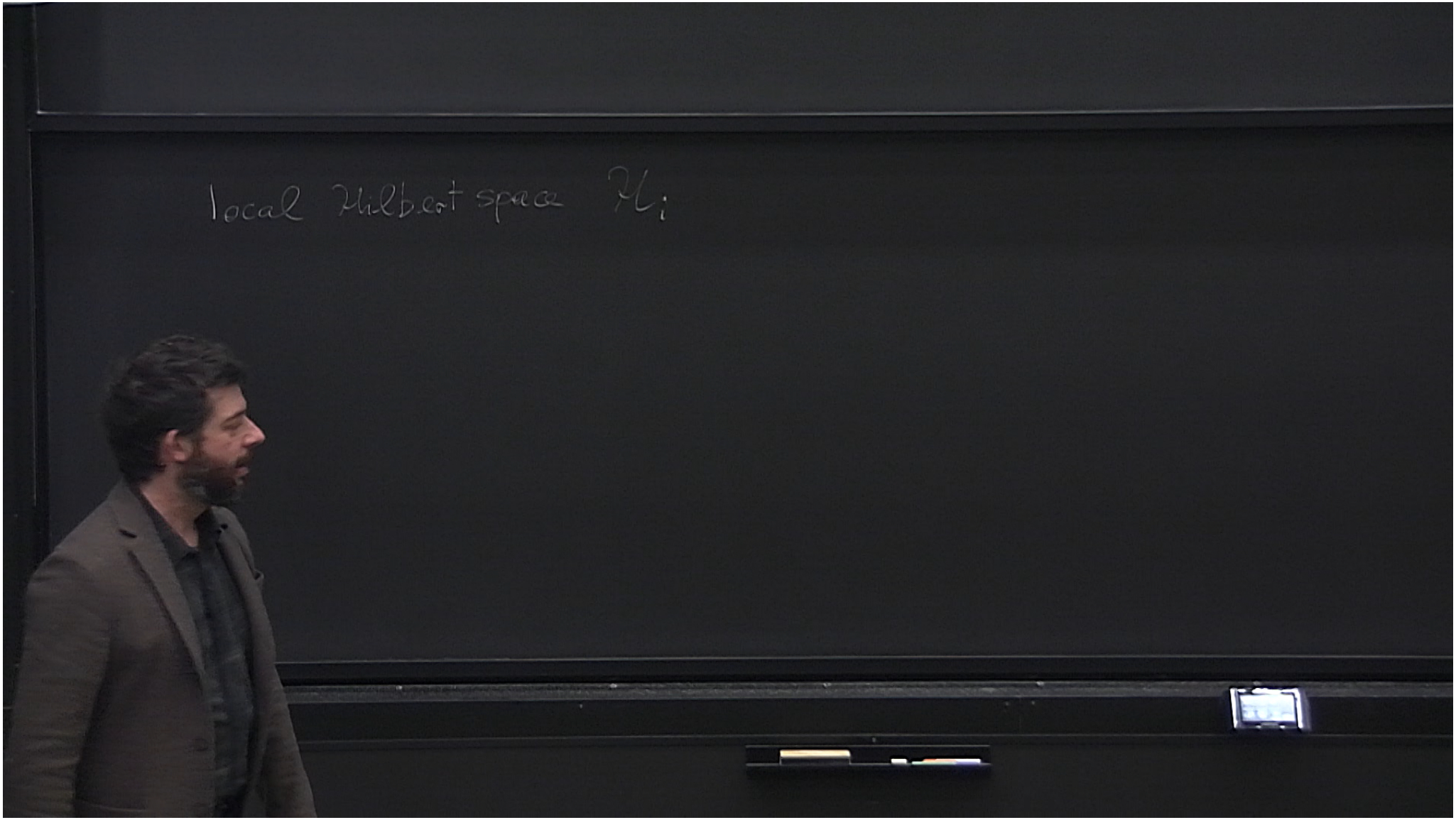
$$T_y|x\rangle = |x+y\rangle$$

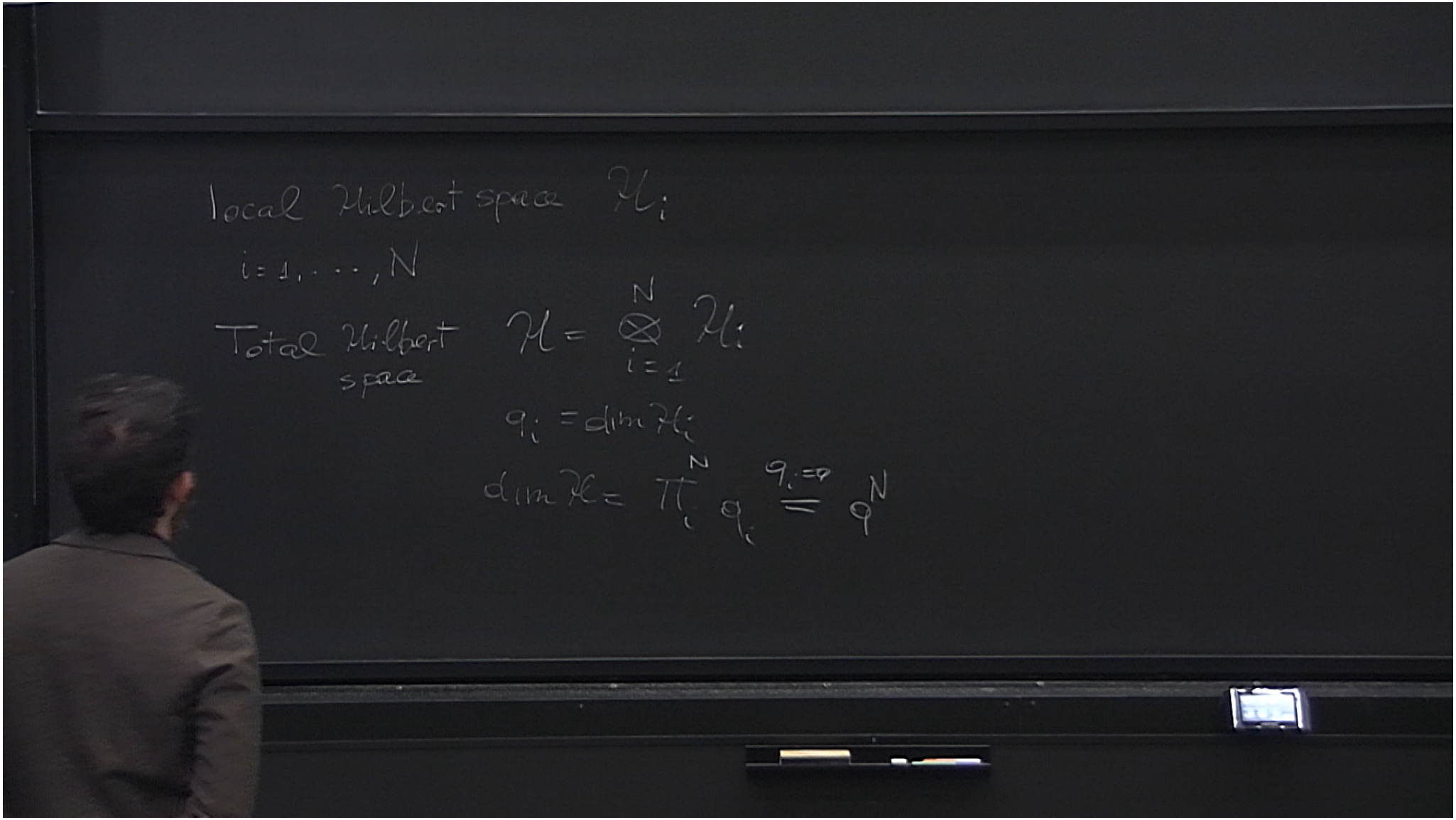
$$[T_a, H] = 0$$

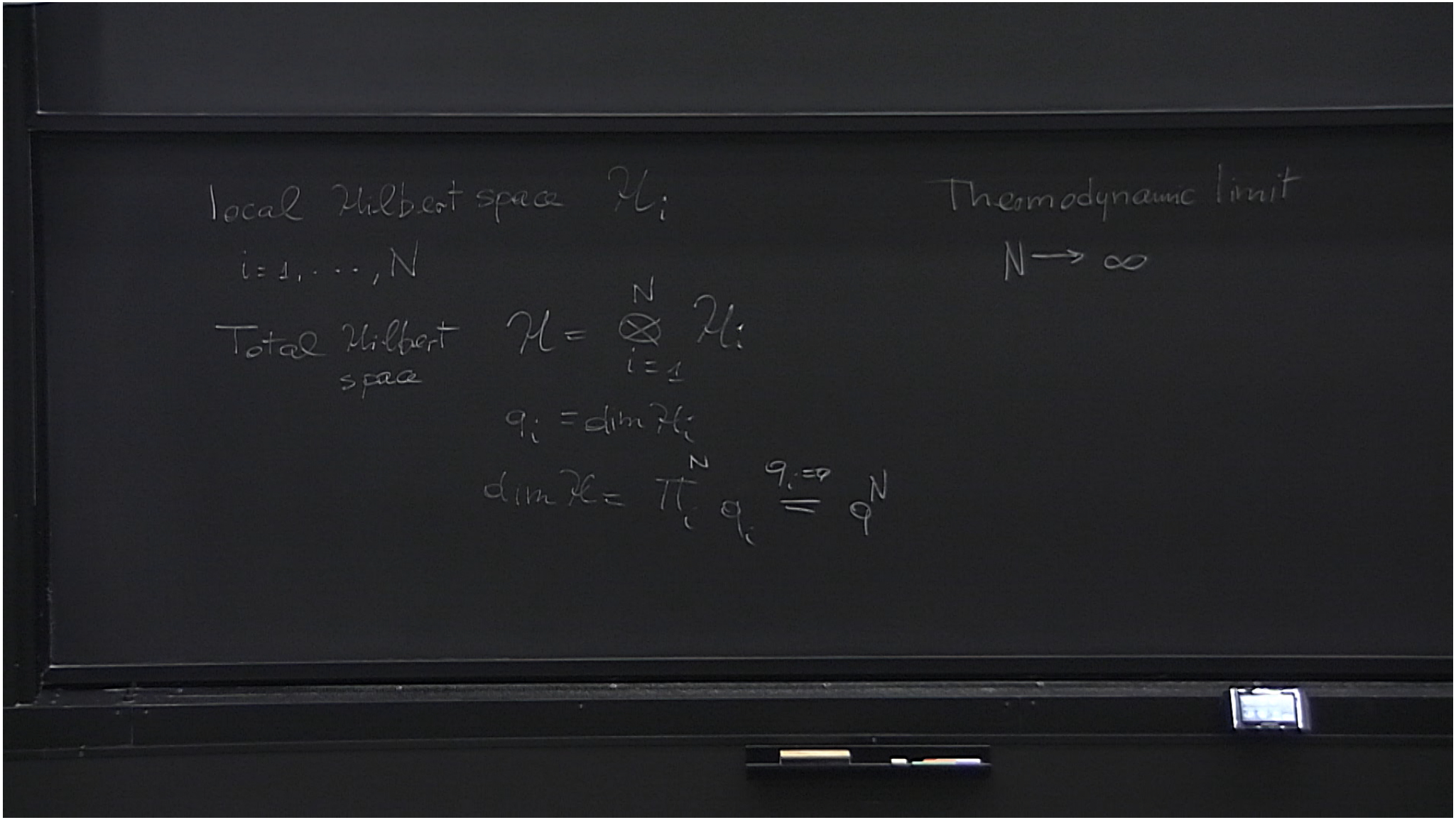


$$\mathcal{H} = \{\, |x\rangle \;;\; x \in \mathbb{R} \bmod a \,\}$$

$$|x\rangle \equiv |x+a\rangle$$


local Hilbert space $\mathcal{H}_i$


local Hilbert space $\mathcal{H}_i$
$i = 1, \dots, N$
Total Hilbert space $\mathcal{H} = \bigotimes_{i=1}^{N} \mathcal{H}_i$
$q_i = \dim \mathcal{H}_i$
$\dim \mathcal{H} = \prod_i^N q_i \overset{q_i = q}{=} q^N$


local Hilbert space $\mathcal{H}_i$
$i = 1, \dots, N$
Total Hilbert space $\mathcal{H} = \bigotimes_{i=1}^{N} \mathcal{H}_i$
$q_i = \dim \mathcal{H}_i$
$\dim \mathcal{H} = \prod_i^N q_i \overset{q_i = q}{=} q^N$
Thermodynamic limit
$N \to \infty$

local Hilbert space $\mathcal{H}_i$

$i = 1, \dots, N$

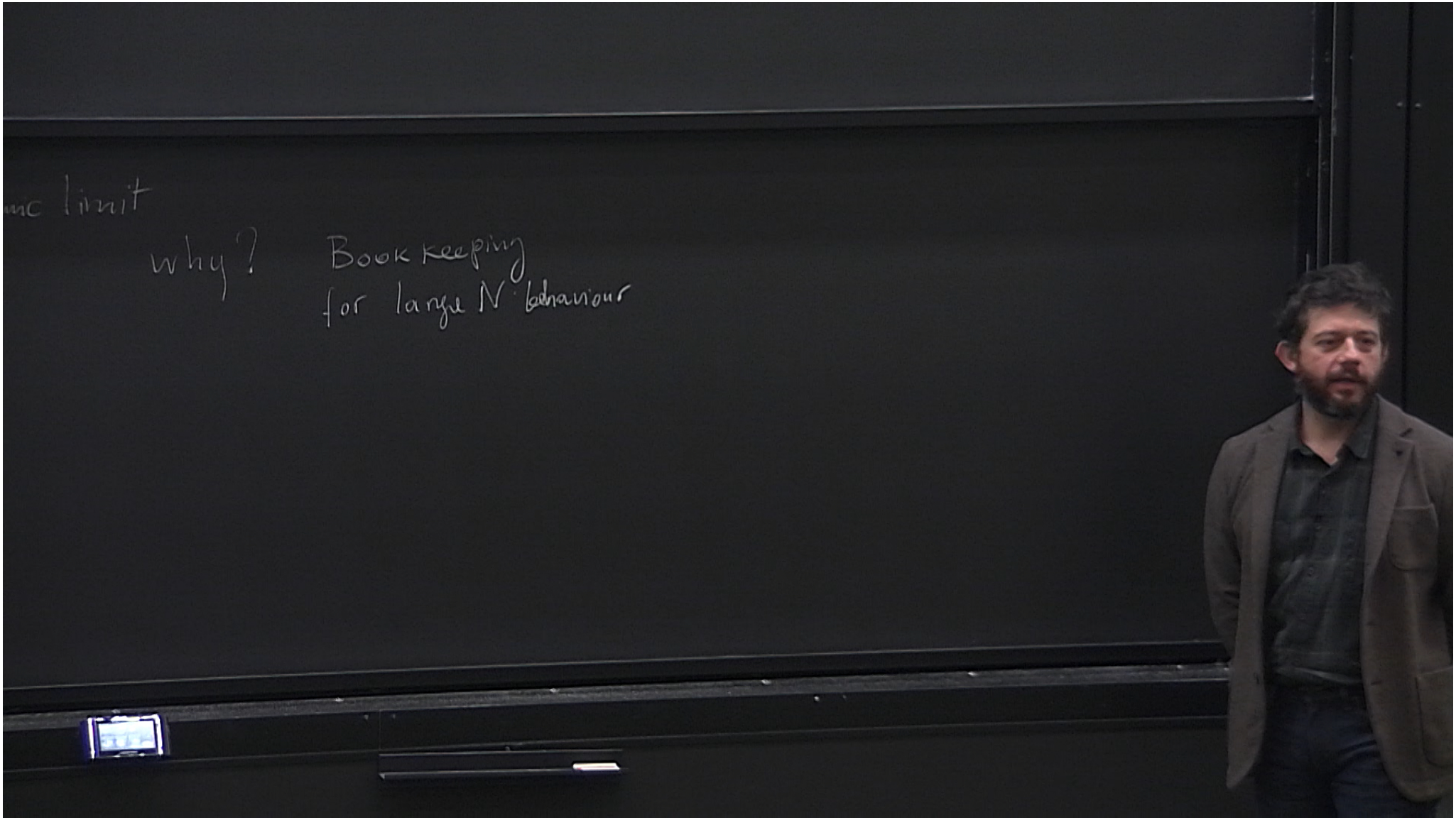
Total Hilbert space $\mathcal{H} = \bigotimes_{i=1}^{N} \mathcal{H}_i$

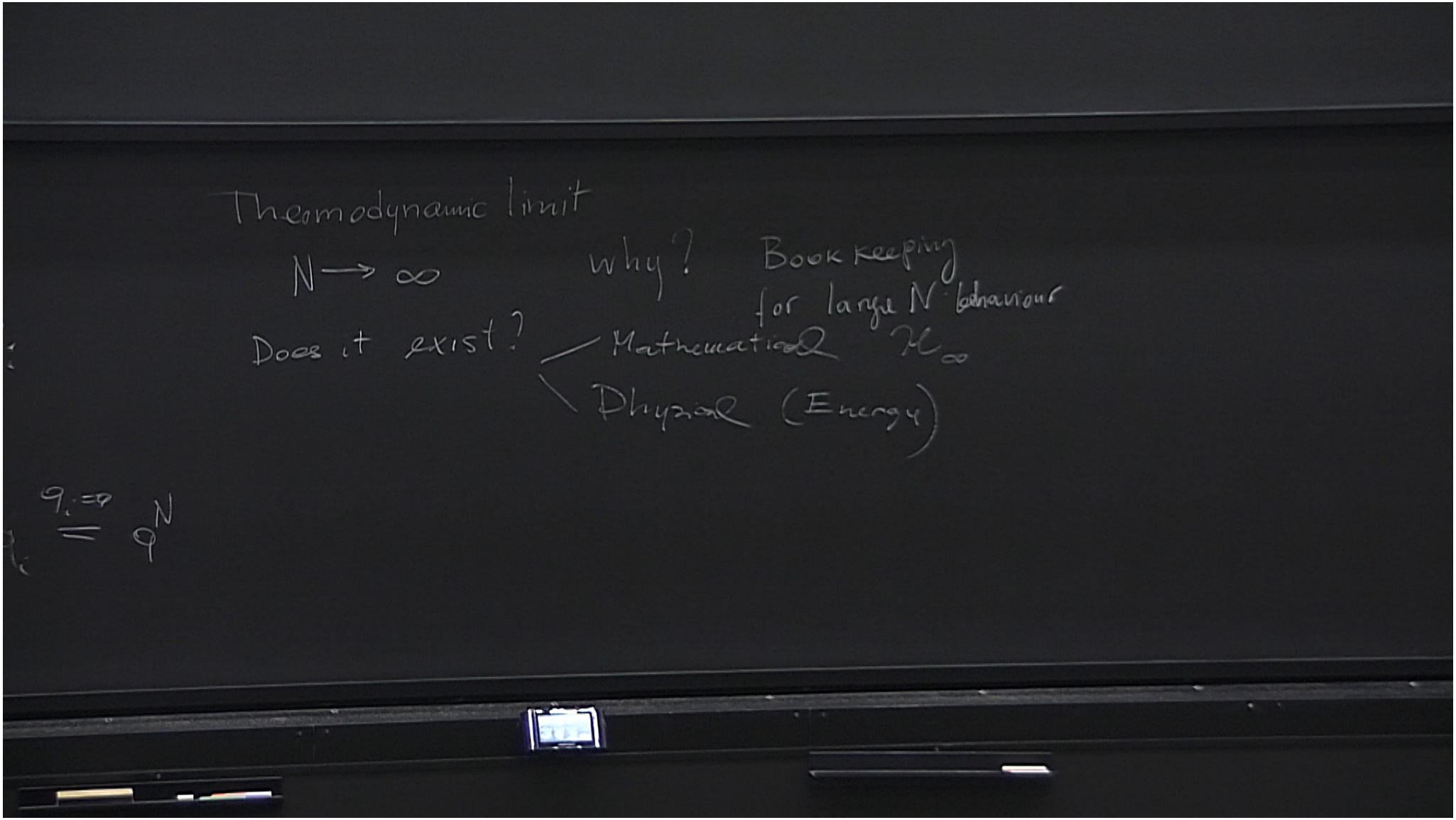
$q_i = \dim \mathcal{H}_i$

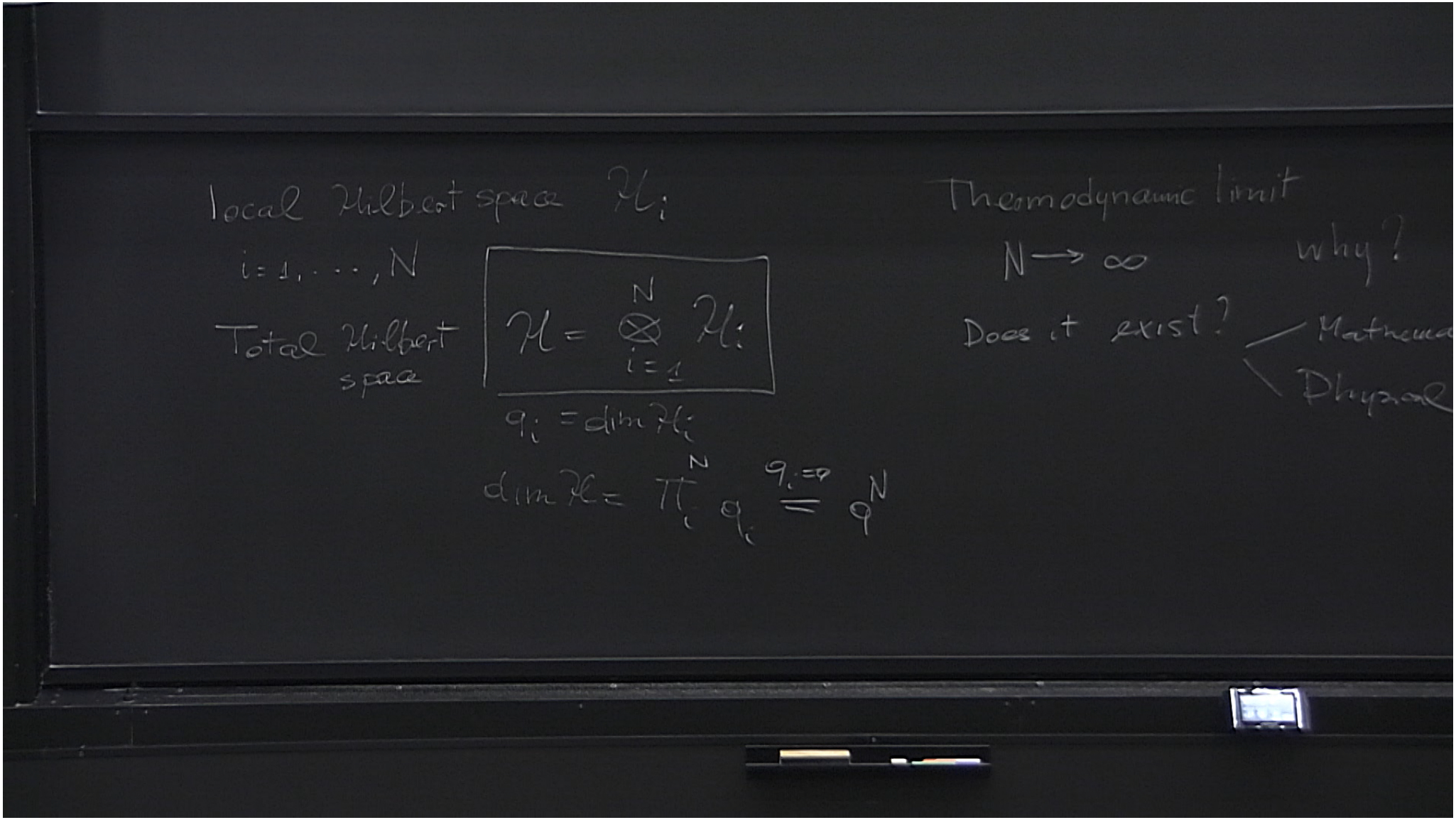
$\dim \mathcal{H} = \prod_i^N q_i \overset{q_i = q}{=} q^N$

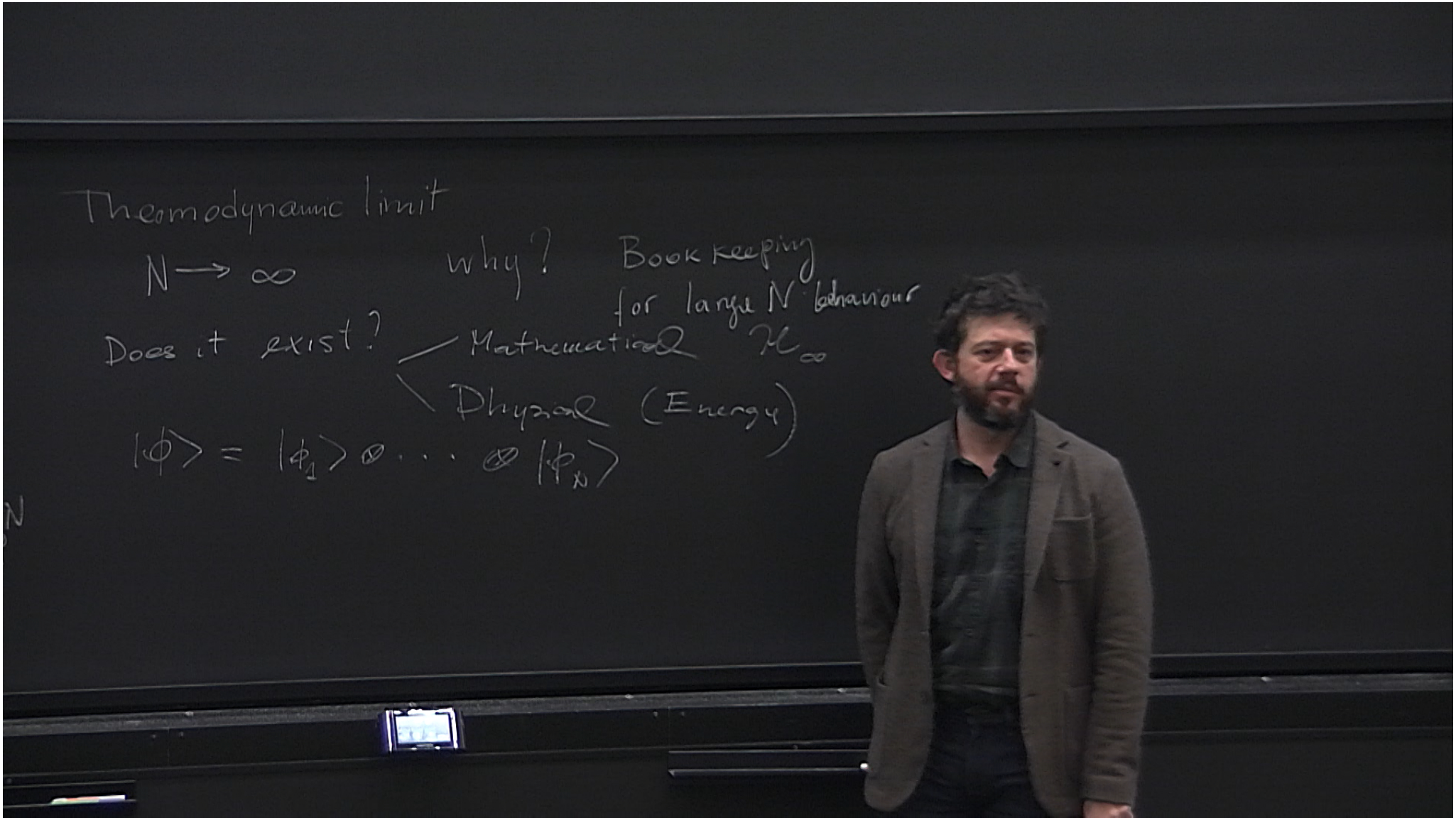
Thermodynamic limit

$N \to \infty$ why?


mc limit
why? Book keeping
for large N behaviour


Thermodynamic limit
$N \to \infty$
Why? Bookkeeping for large N behaviour
Does it exist?
Mathematical $\mathcal{H}_\infty$
Physical (Energy)
$q_i = q$
$= q^N$


local Hilbert space $\mathcal{H}_i$
$i = 1, \dots, N$
Total Hilbert space
$\mathcal{H} = \bigotimes_{i=1}^{N} \mathcal{H}_i$
$q_i = \dim \mathcal{H}_i$
$\dim \mathcal{H} = \prod_i^N q_i \overset{q_i = q}{=} q^N$
Thermodynamic limit
$N \to \infty$
why?
Does it exist?
Mathema
Physical


Thermodynamic limit
$N \to \infty$
why? Book keeping for large N behaviour
Does it exist? — Mathematical $\mathcal{H}_\infty$
\ Physical (Energy)
$|\phi\rangle = |\phi_1\rangle \otimes \cdots \otimes |\phi_N\rangle$

Thermodynamic limit

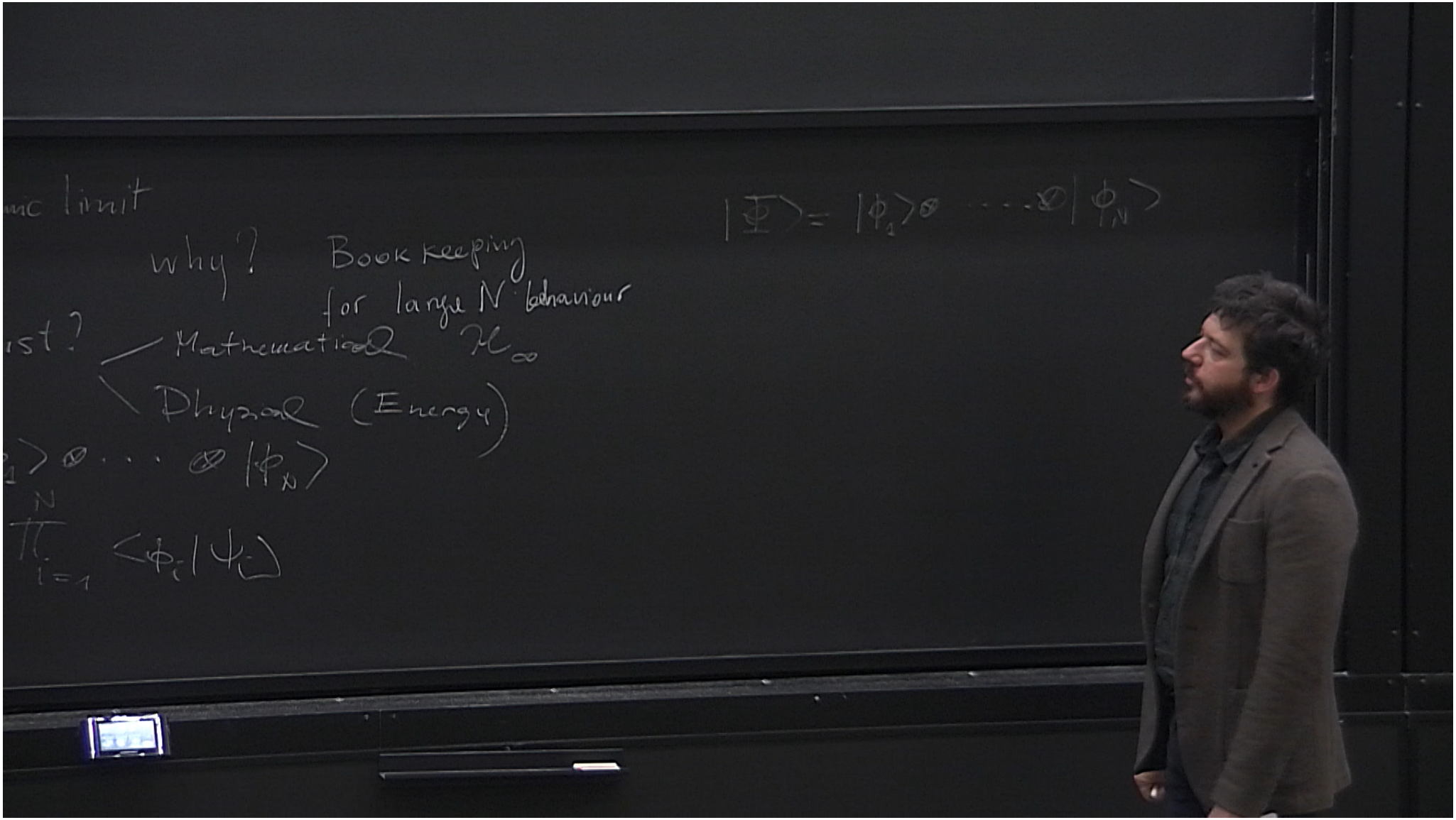
$N \to \infty$ why? Bookkeeping for large N behaviour

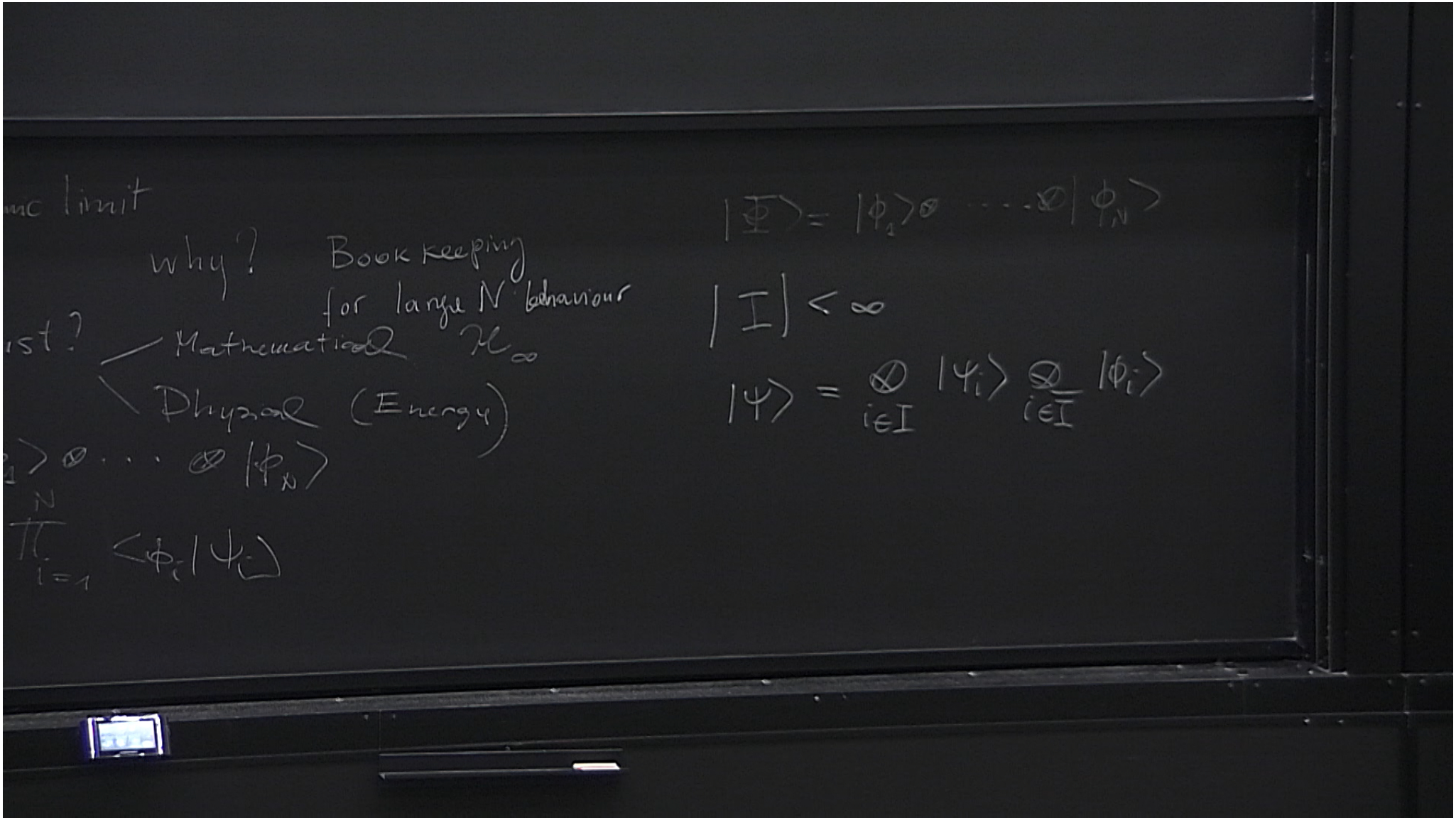
Does it exist? — Mathematical $\mathcal{H}_\infty$

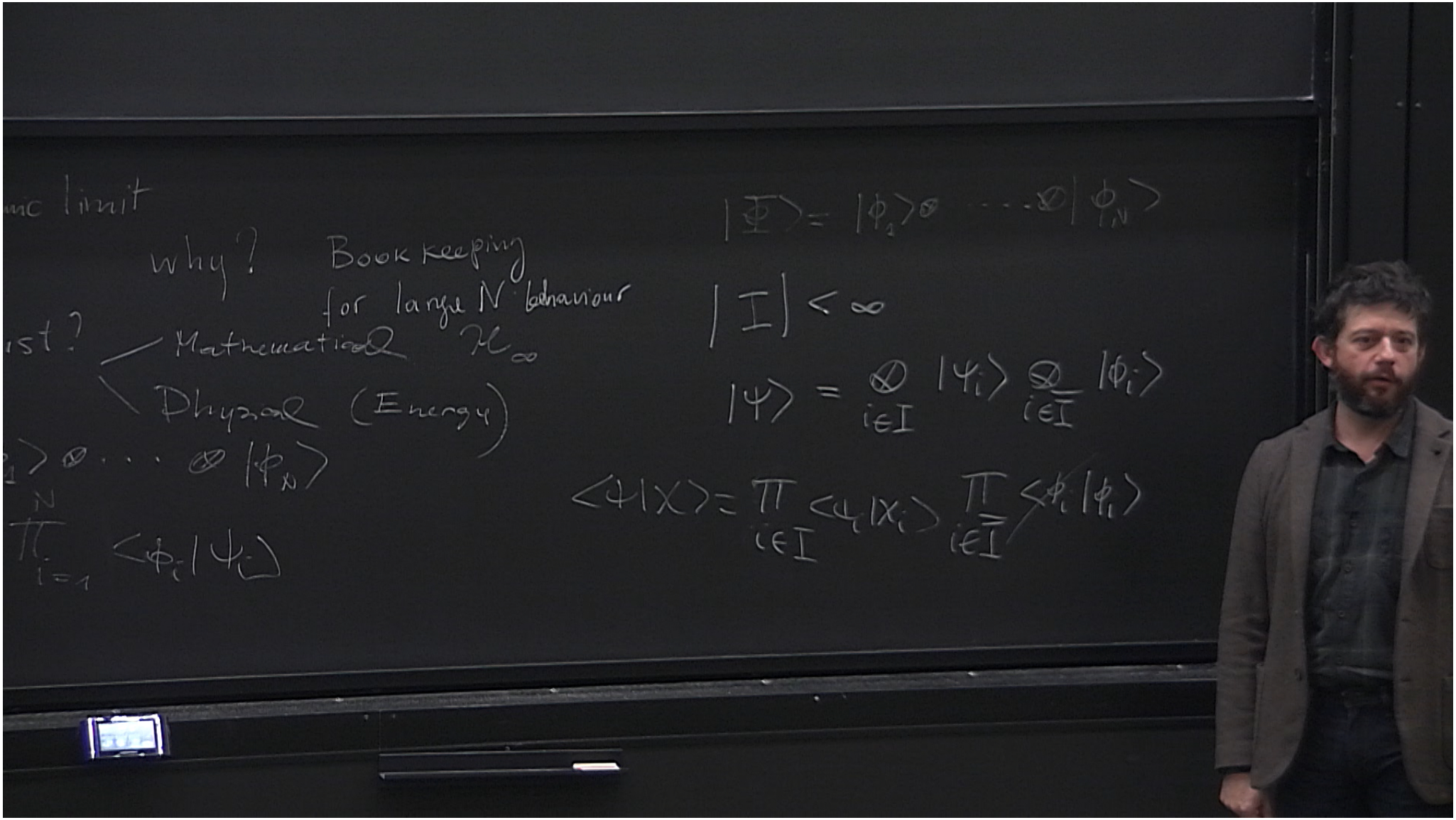
\ Physical (Energy)

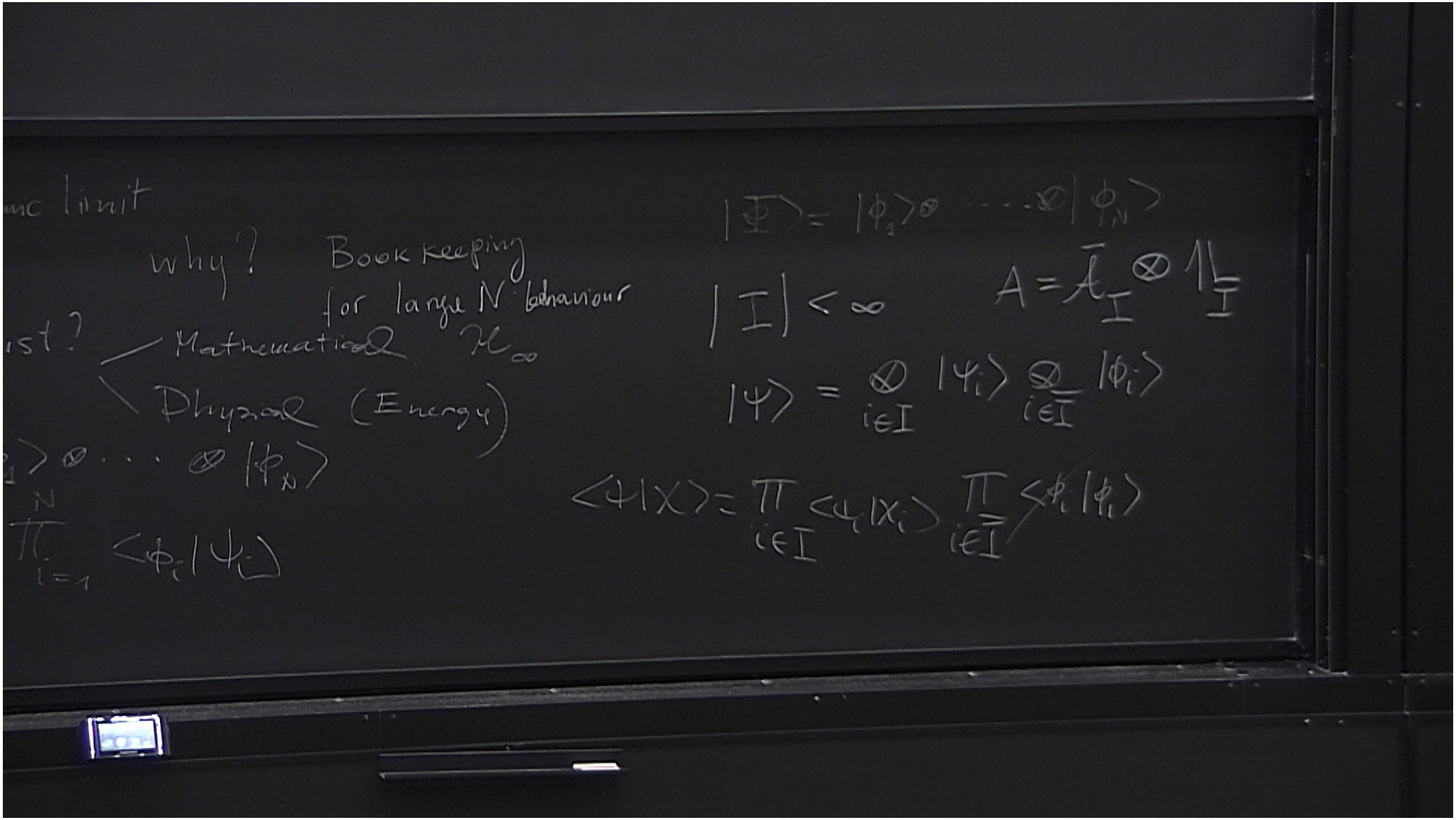
$|\phi\rangle = |\phi_1\rangle \otimes \cdots \otimes |\phi_N\rangle$

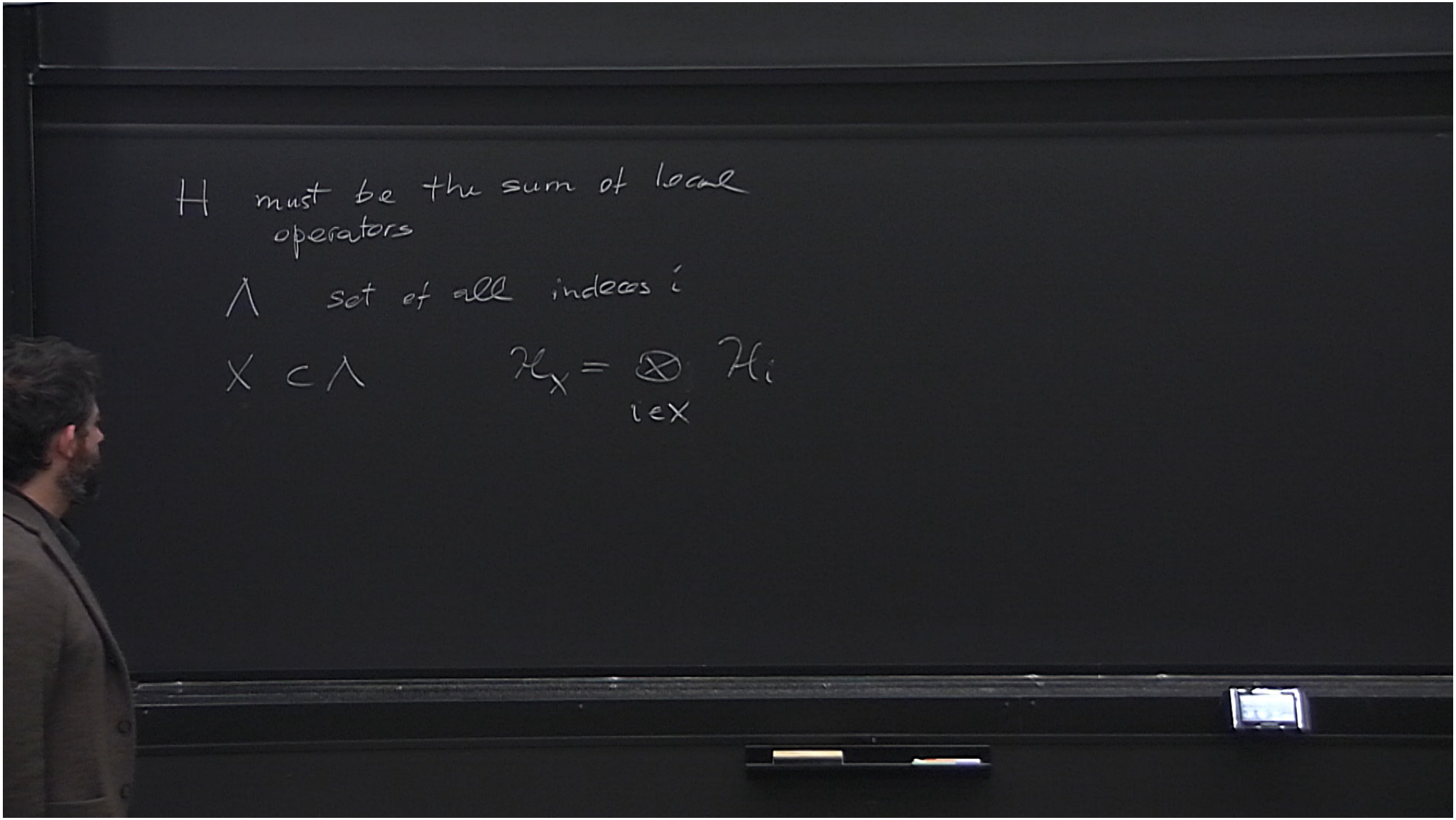
$\langle\phi|\psi\rangle = \prod_{i=1}^{N} \langle\phi_i|\psi_i\rangle$


mc limit
why? Bookkeeping
for large N behaviour
ist?
Mathematical
Physical (Energy)
$|\Psi\rangle = |\phi_1\rangle \otimes \cdots \otimes |\phi_N\rangle$
$|\phi_1\rangle \otimes \cdots \otimes |\phi_N\rangle$
$\prod_{i=1}^{N} \langle \phi_i | \psi_i \rangle$


mc limit
why? Bookkeeping
for large N behaviour
ist?
Mathematical $\mathcal{H}_\infty$
Physical (Energy)
$|\Phi\rangle = |\phi_1\rangle \otimes \cdots \otimes |\phi_N\rangle$
$|I| < \infty$
$|\Psi\rangle = \bigotimes_{i\in I} |\psi_i\rangle \bigotimes_{i\in \bar{I}} |\phi_i\rangle$
$\prod_{i=1}^{N} \langle\phi_i|\psi_i\rangle$


mc limit
why? Bookkeeping
for large N behaviour
ist?
Mathematical
Physical (Energy)
$|\Psi\rangle = |\phi_1\rangle \otimes \cdots \otimes |\phi_N\rangle$
$|I| < \infty$
$|\Psi\rangle = \bigotimes_{i\in I} |\psi_i\rangle \bigotimes_{i\in \bar{I}} |\phi_i\rangle$
$\langle\psi|\chi\rangle = \prod_{i\in I} \langle\psi_i|\chi_i\rangle \prod_{i\in \bar{I}} \langle\phi_i|\phi_i\rangle$
$\prod_{i=1}^{N} \langle\phi_i|\psi_i\rangle$


mc limit
why? Bookkeeping
for large N behaviour
ist?
Mathematical
Physical (Energy)
$|\Psi\rangle = |\phi_1\rangle \otimes \cdots \otimes |\phi_N\rangle$
$|I| < \infty$
$A = \bar{A}_I \otimes 1_{\bar{I}}$
$|\psi\rangle = \bigotimes_{i\in I} |\psi_i\rangle \bigotimes_{i\in\bar{I}} |\phi_i\rangle$
$\langle\psi|\chi\rangle = \prod_{i\in I} \langle\psi_i|\chi_i\rangle \prod_{i\in\bar{I}} \langle\phi_i|\phi_i\rangle$
$\prod_{i=1}^{N} \langle\phi_i|\psi_i\rangle$


H must be the sum of local operators
Λ set of all indices i
X ⊂ Λ
$\mathcal{H}_X = \bigotimes_{i \in X} \mathcal{H}_i$

H must be the sum of local operators

Λ set of all indices i

$X \subset \Lambda$ $\quad \mathcal{H}_X = \bigotimes_{i \in X} \mathcal{H}_i$

$|X| < R$ $\quad A = \tilde{A}_X \otimes \mathbb{1}_{\bar{X}}$

H must be the sum of local operators

Λ set of all indices i

$X \subset \Lambda$ $\quad \mathcal{H}_X = \bigotimes_{i \in X} \mathcal{H}_i$

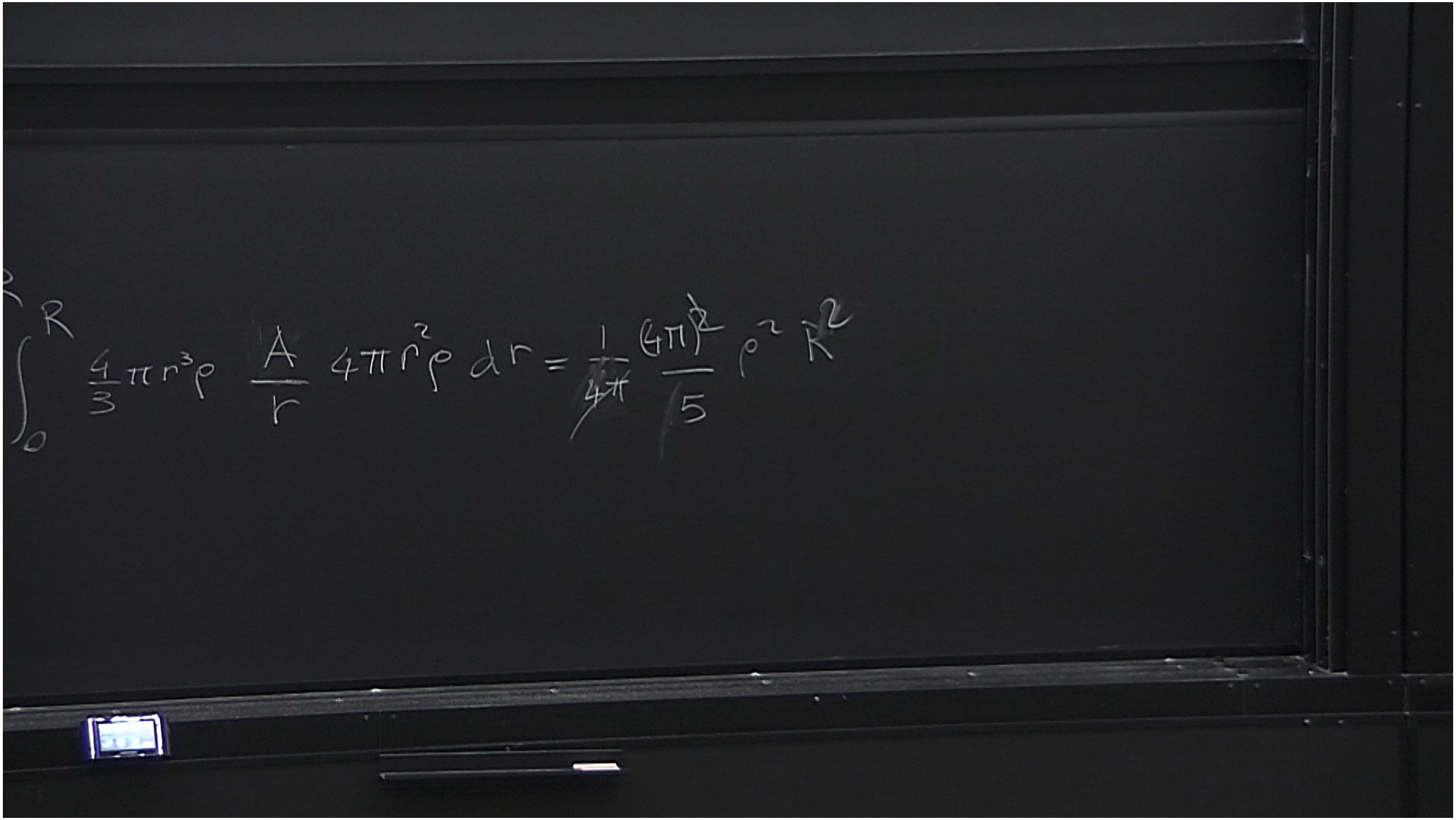
$|X| < R$ $\quad A = \tilde{A}_X \otimes \mathbb{1}_{\bar{X}}$

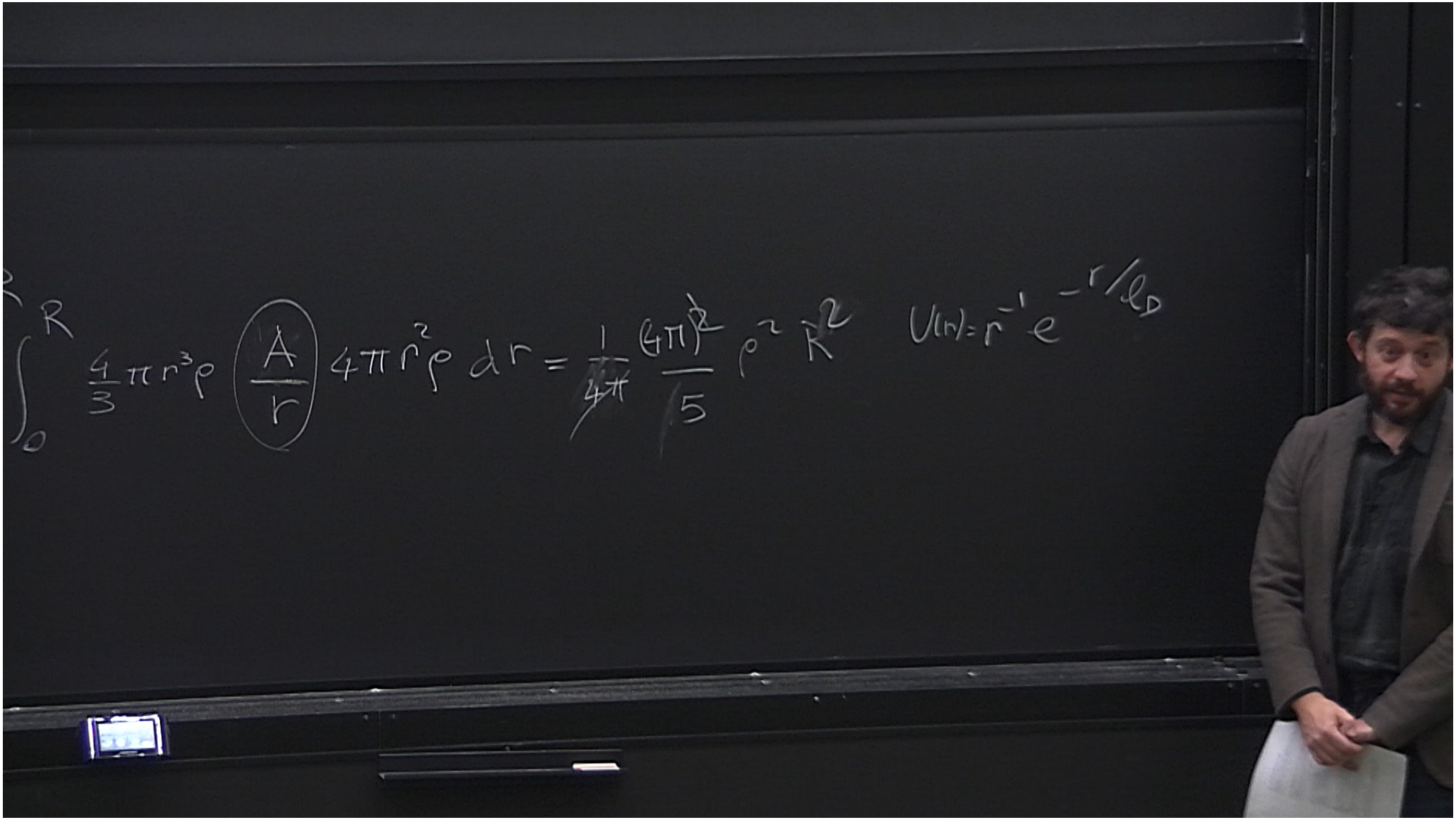
$U(r) = \frac{A}{r}$

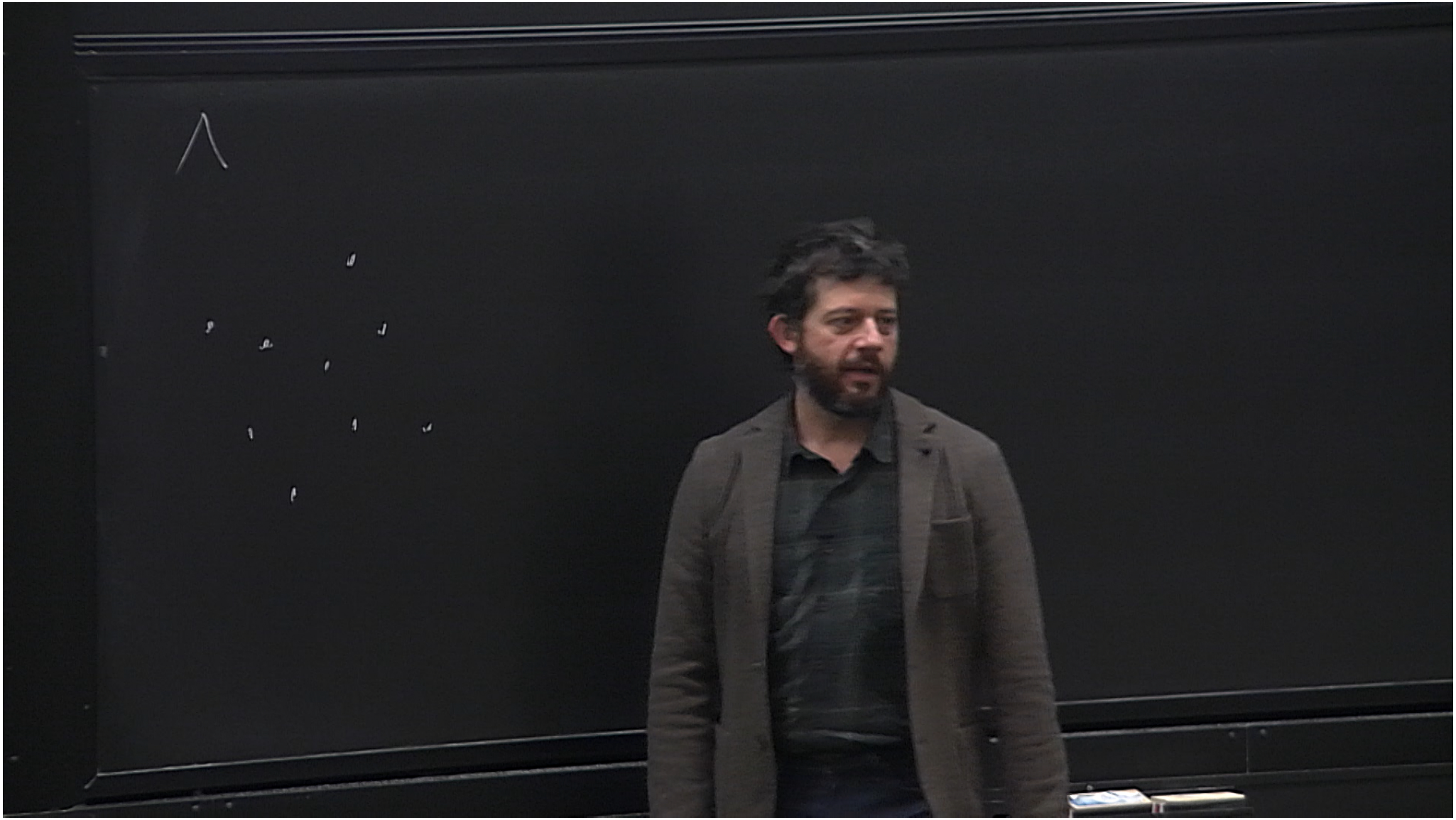
ρ radius R

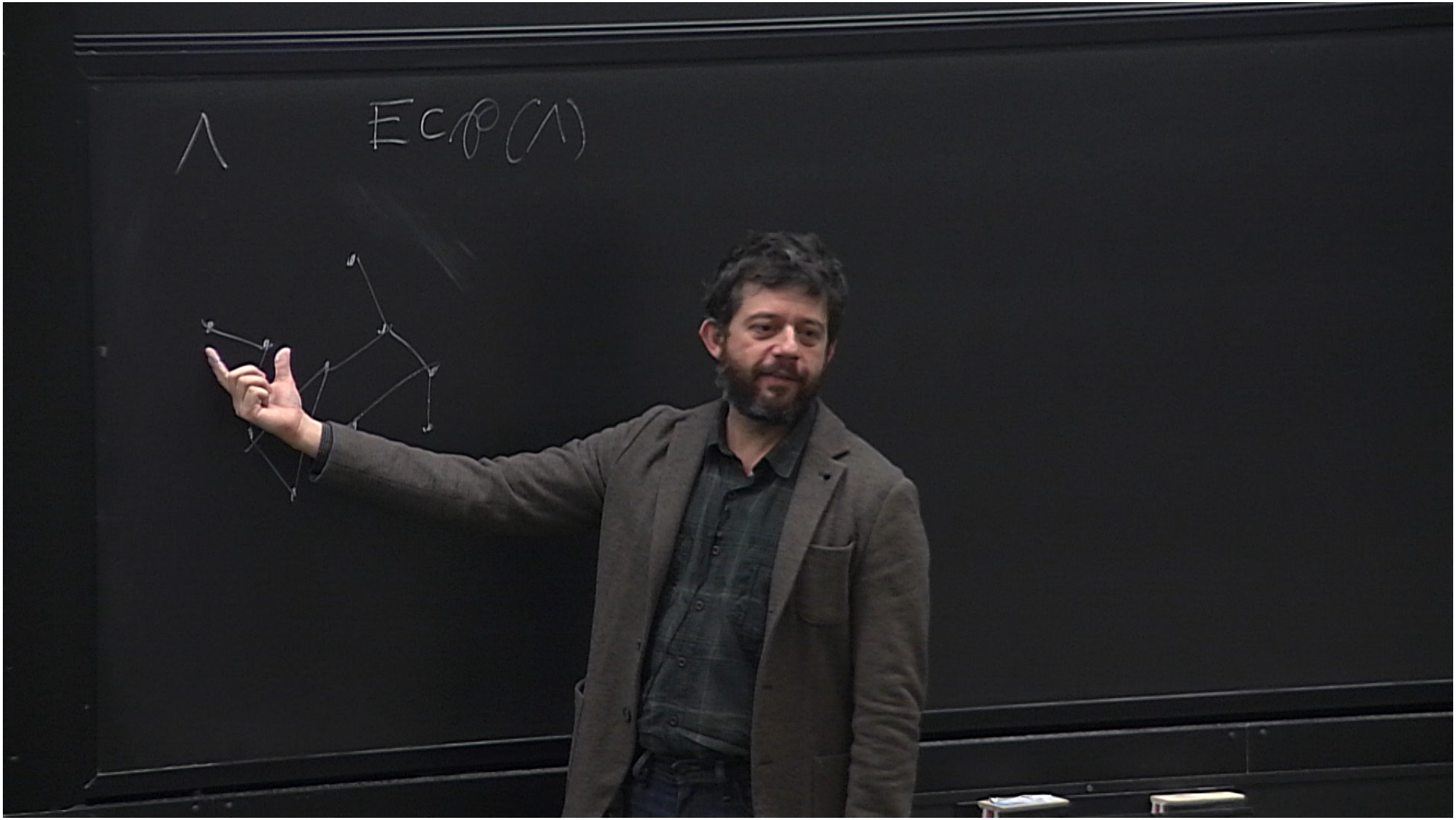
$\mathcal{E}(R) = \frac{E(R)}{V(R)} = \frac{1}{V} \int_0^R \frac{4}{3} \pi r^3 \rho \, \frac{A}{r}$

$\lim_{R \to \infty} \mathcal{E}(R)$

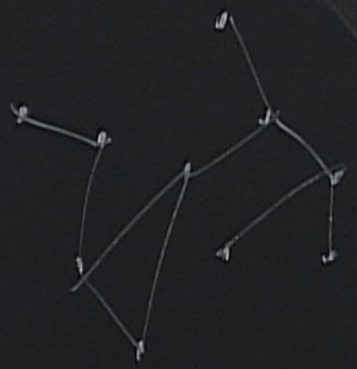

$\int_0^R \frac{4}{3}\pi r^3 \rho \, \frac{A}{r} \, 4\pi r^2 \rho \, dr = \frac{1}{4\pi} \frac{(4\pi)^2}{5} \rho^2 R^2$


$\int_0^R \frac{4}{3}\pi r^3 \rho \left(\frac{A}{r}\right) 4\pi r^2 \rho \, dr = \frac{1}{4\pi} \frac{(4\pi)^2}{5} \rho^2 R^2$
$U(r) = r^{-1} e^{-r/\ell_D}$



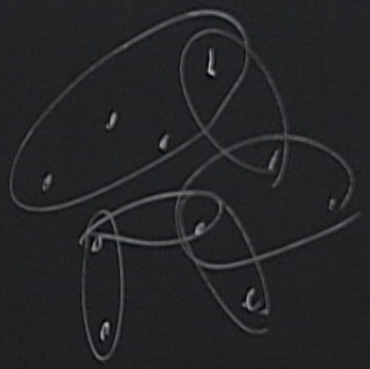

Λ
Ec(ρ(Λ)

Λ $E \subset \mathcal{P}(\Lambda)$ (Λ, E) is a graph



Λ $E \subset \mathcal{P}(\Lambda)$ (Λ, E) is a graph





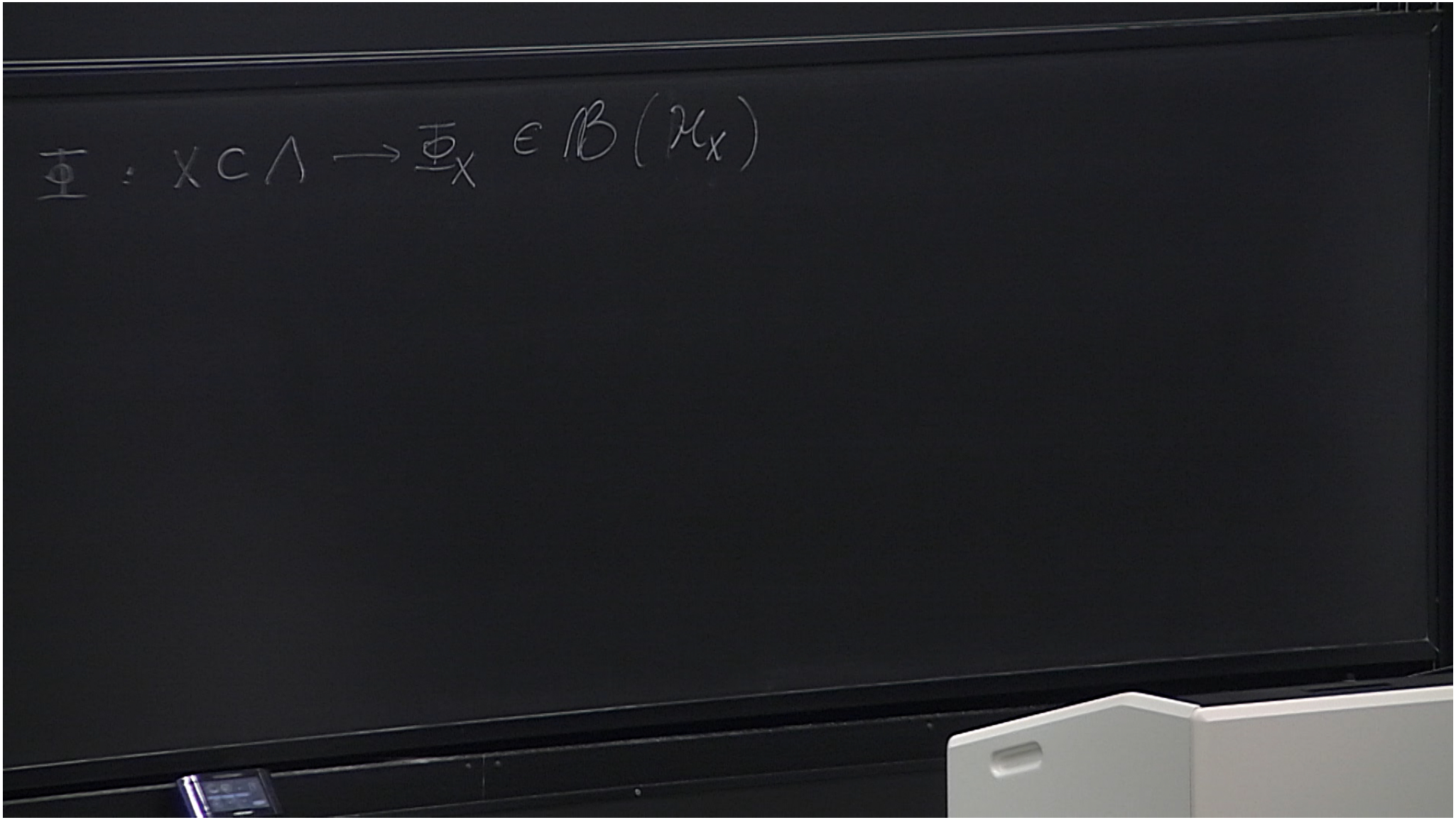
$\{E_i\}_{i=1,\dots,L}$

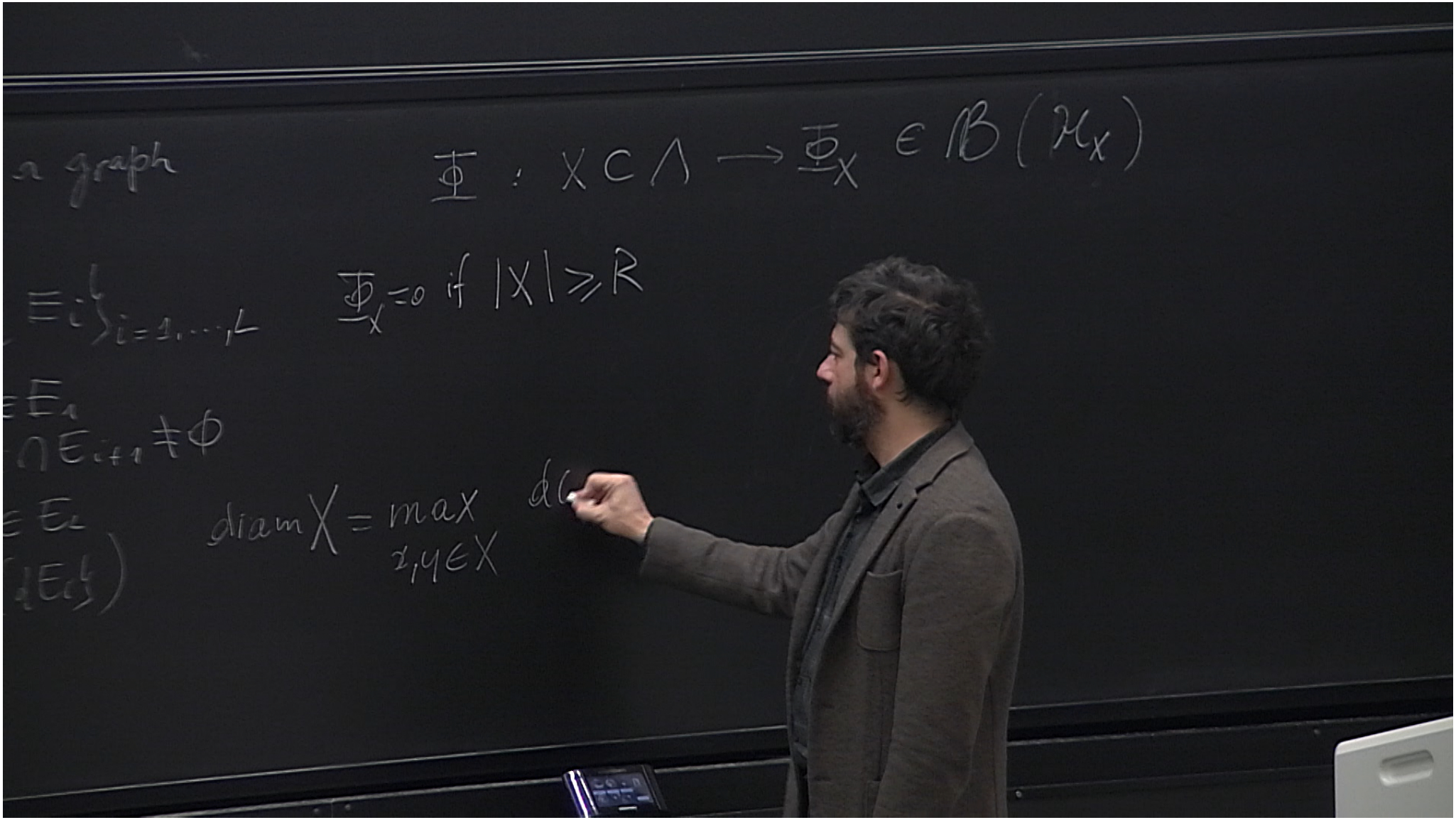
$A \in E_1$

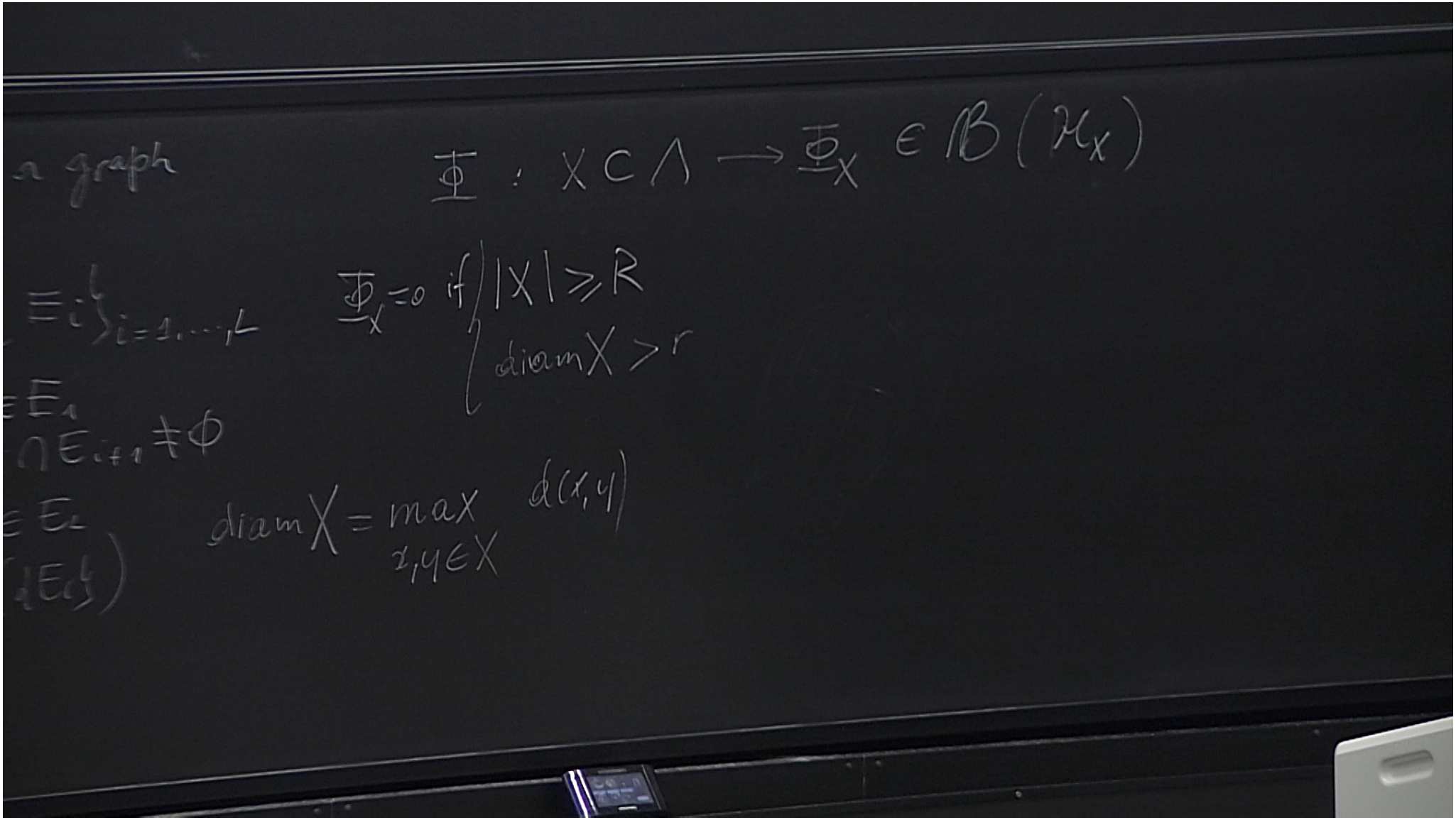
$E_i \cap E_{i+1} \neq \emptyset$

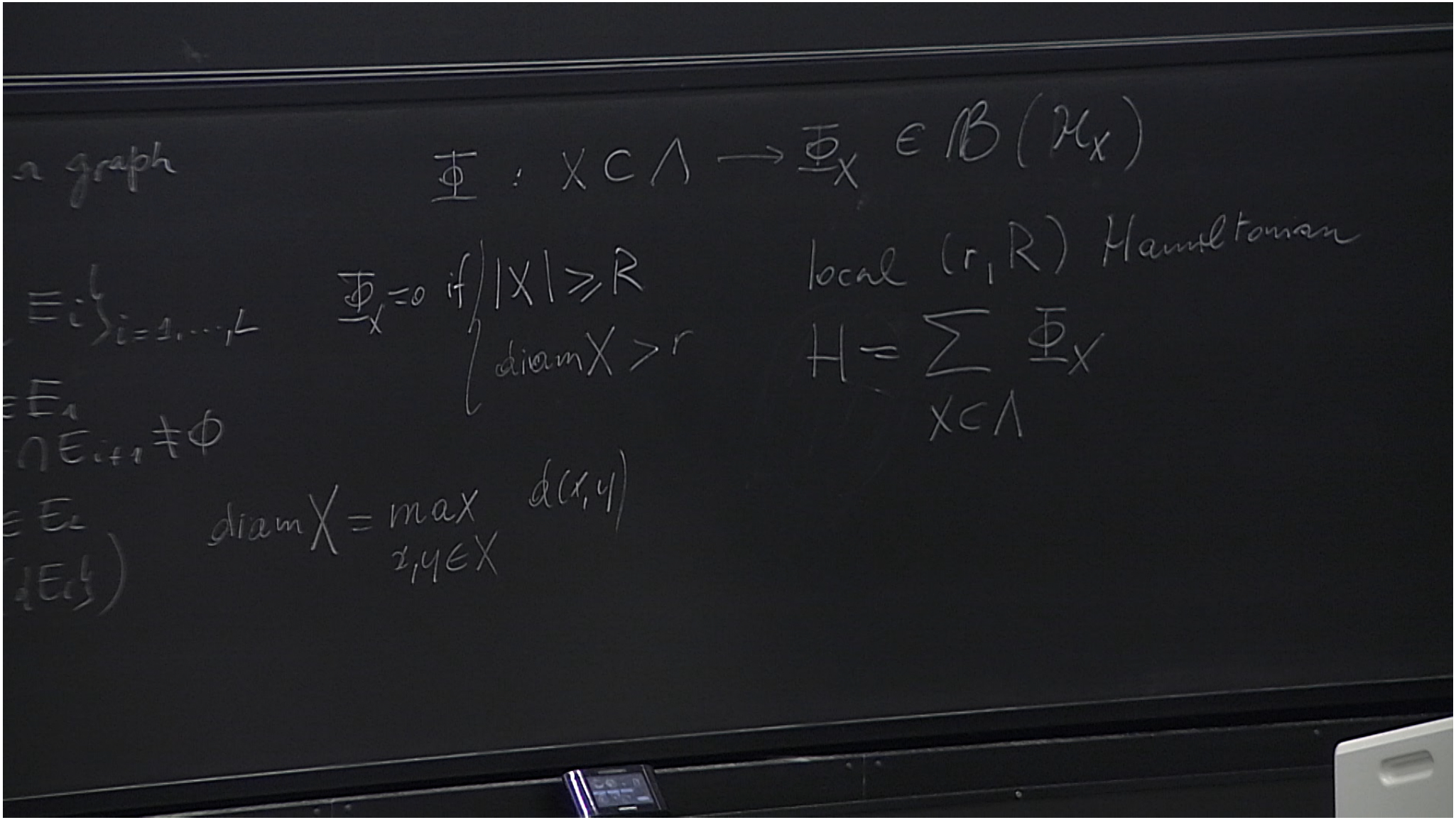
$B \in E_L$

$d(A,B) = \min_{\{E_i\}} L(\{E_i\})$

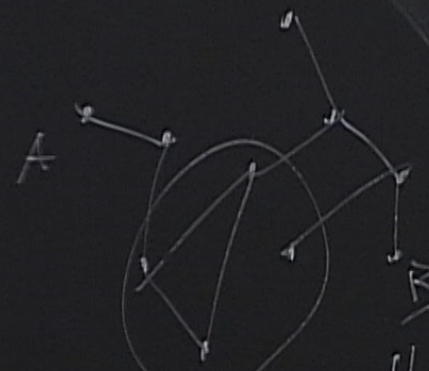

$\Phi : X \subset \Lambda \longrightarrow \Phi_X \in \mathcal{B}(\mathcal{H}_X)$

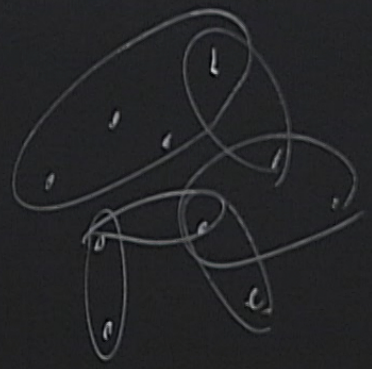

a graph
$\Phi : X \subset \Lambda \longrightarrow \Phi_X \in \mathcal{B}(\mathcal{H}_X)$
$\Phi_X = 0$ if $|X| \geq R$
$\{E_i\}_{i=1,\dots,L}$
E_1
$\cap E_{i+1} \neq \phi$
E_2
$\{E_i\}$)
$\operatorname{diam} X = \max_{x,y \in X}$


a graph
$\Phi : X \subset \Lambda \longrightarrow \Phi_X \in \mathcal{B}(\mathcal{H}_X)$
$\Phi_X = 0$ if $\begin{cases} |X| \geq R \\ \text{diam } X > r \end{cases}$
$\text{diam } X = \max_{x,y \in X} d(x,y)$


a graph
$\Phi : X \subset \Lambda \longrightarrow \Phi_X \in \mathcal{B}(\mathcal{H}_X)$
$\Phi_X = 0$ if $|X| \geq R$
$\mathrm{diam} X > r$
local (r, R) Hamiltonian
$H = \sum_{X \subset \Lambda} \Phi_X$
$\mathrm{diam} X = \max_{x,y \in X} d(x,y)$

Λ $E \subset \mathcal{P}(\Lambda)$ (Λ, E) is a graph Φ :





$\{E_i\}_{i=1,\dots,L}$ $\Phi_X = 0$ if $|X|$ diam

$A \in E_1$

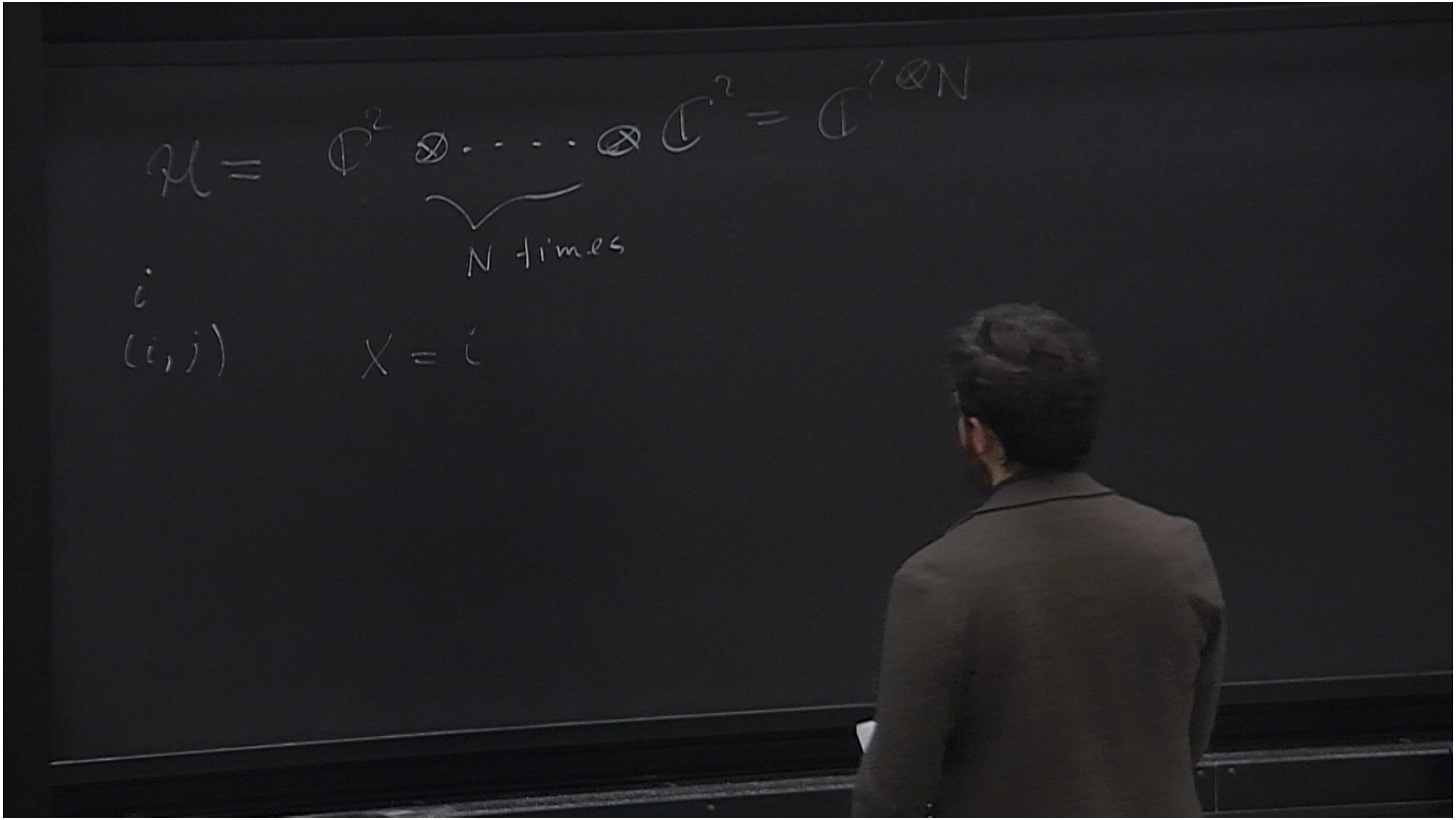
$E_i \cap E_{i+1} \neq \emptyset$

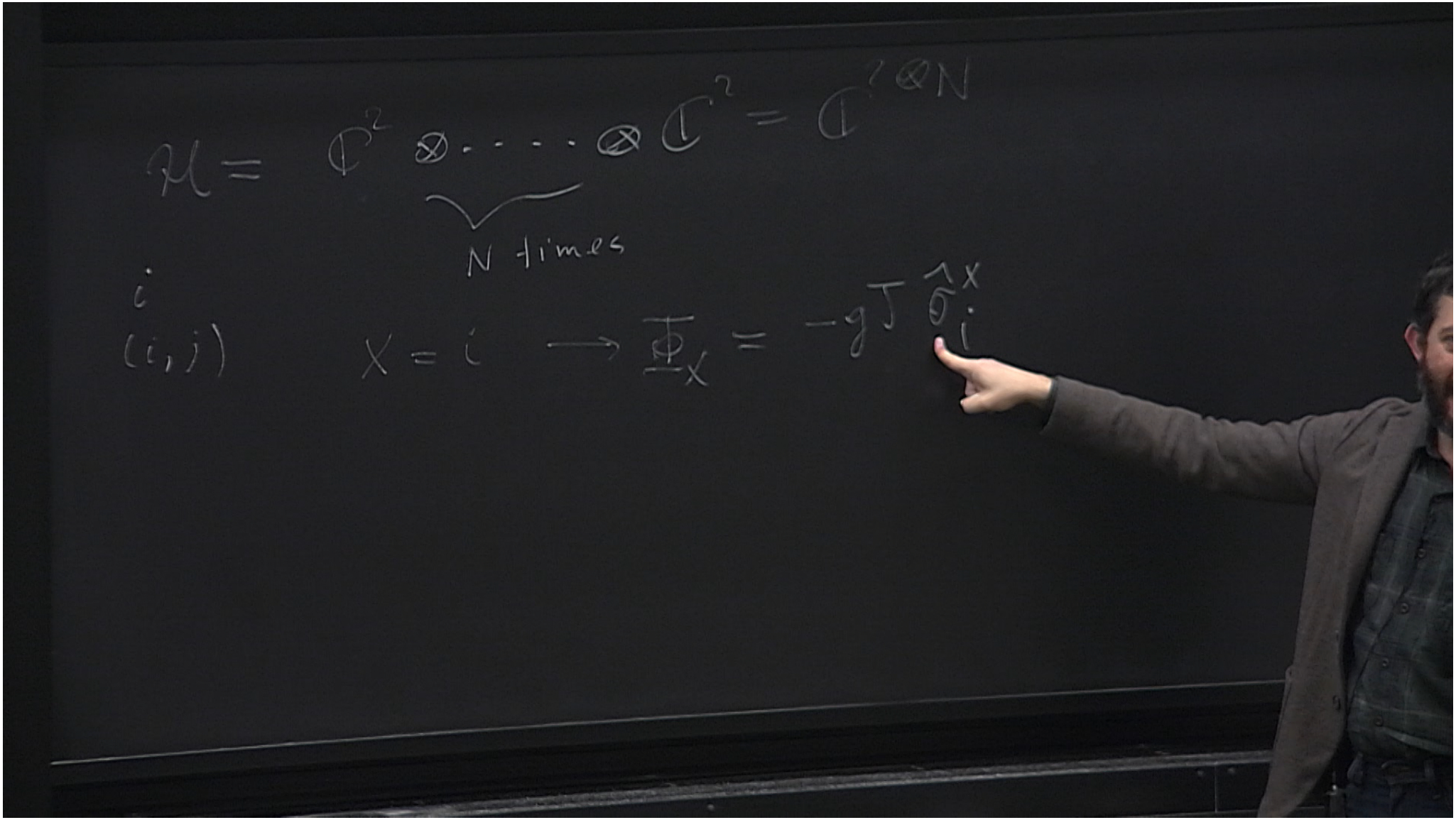
$B \in E_L$

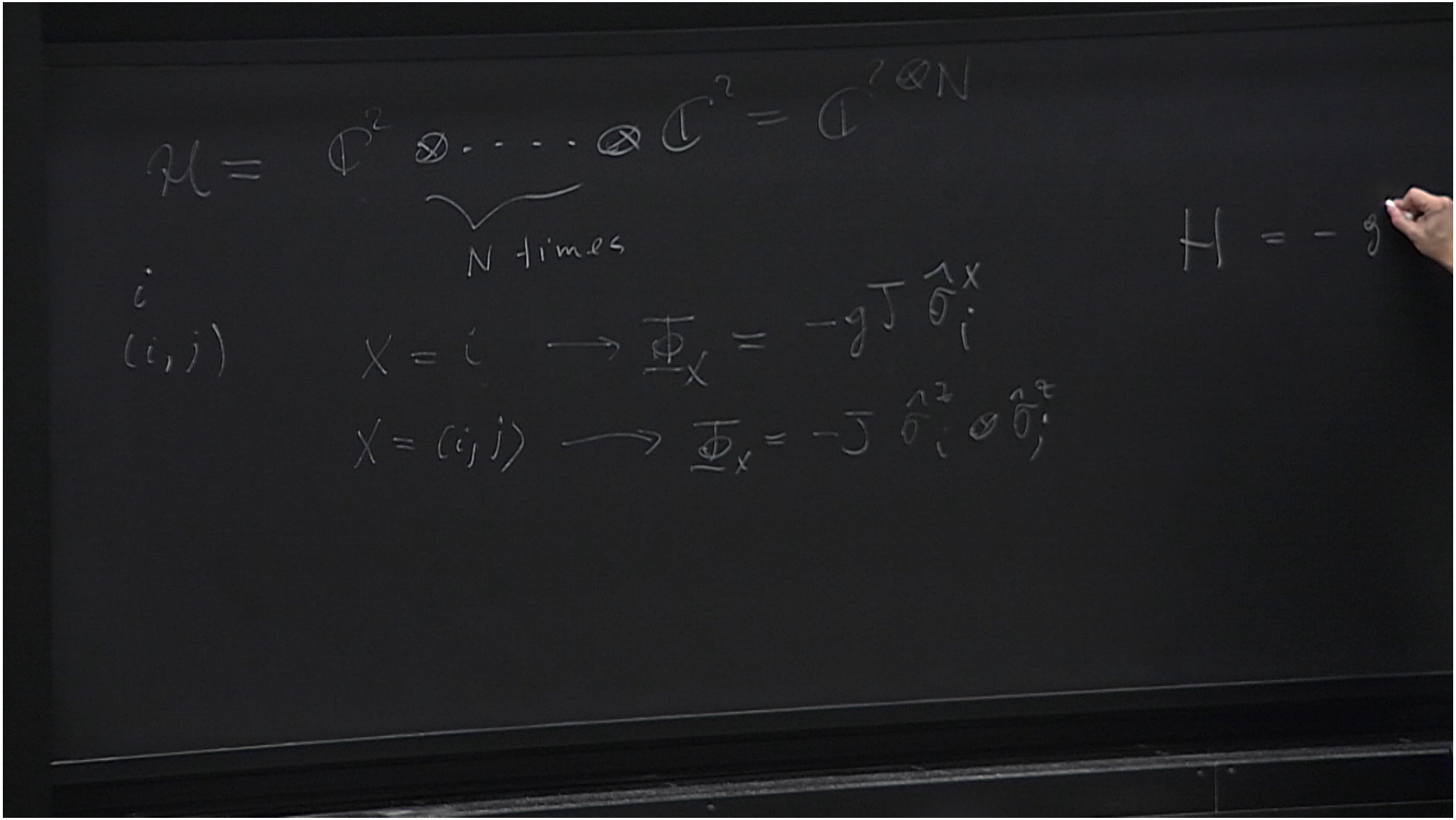
$\infty > \|A\| = \sup_{\||\psi\rangle\|=1} \|A|\psi\rangle\|$

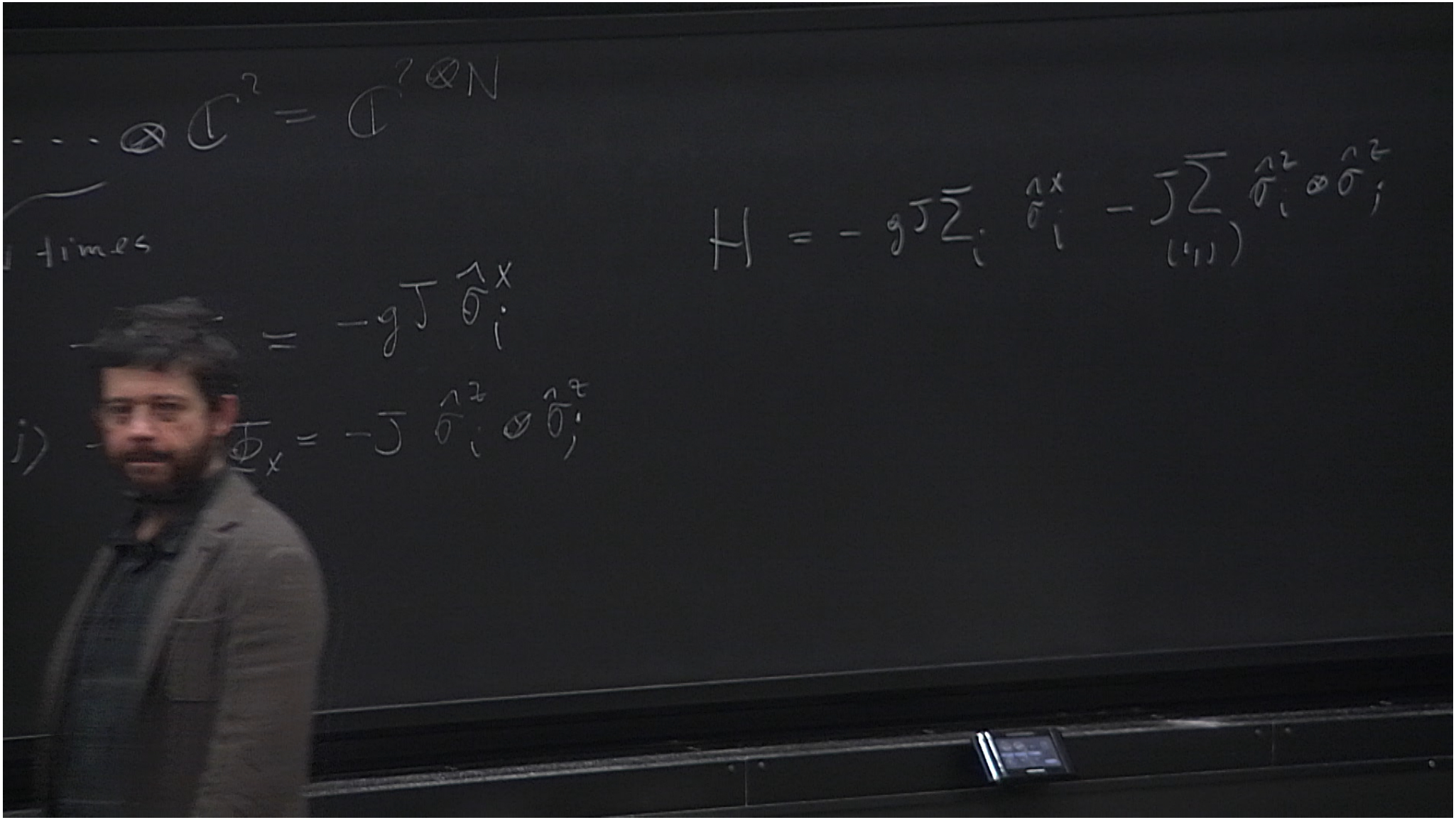
$d(A,B) = \min_{\{E_i\}} \left(\sum_{\{E_i\}} \right)$

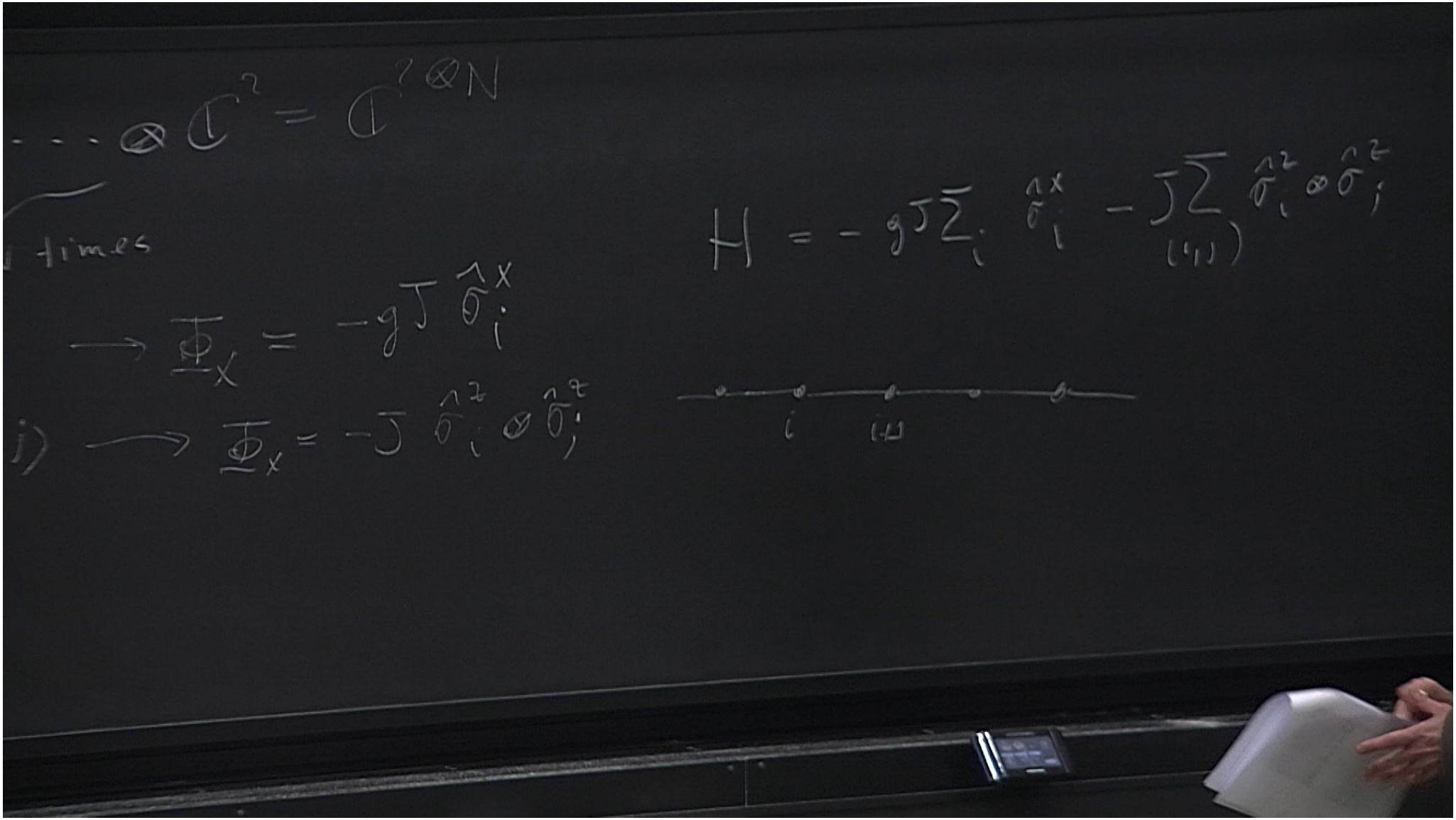
$\operatorname{diam} X = \max_{x,y \in X}$ d


$\mathcal{H} = \mathbb{C}^2 \otimes \cdots \otimes \mathbb{C}^2 = \mathbb{C}^{2^N}$
N times
i
(i, j)
X = i


$\mathcal{H} = \mathbb{C}^2 \otimes \cdots \otimes \mathbb{C}^2 = \mathbb{C}^{2^N}$
N times
i
(i,j)
$X = i \longrightarrow \Phi_X = -gJ \hat{\sigma}^x_i$


$\mathcal{H} = \mathbb{C}^2 \otimes \cdots \otimes \mathbb{C}^2 = \mathbb{C}^{2^N}$
N times
$H = -g$
i
(i,j)
$X = i \longrightarrow \Phi_X = -gJ\,\hat{\sigma}_i^x$
$X = (i,j) \longrightarrow \Phi_X = -J\,\hat{\sigma}_i^z \otimes \hat{\sigma}_j^z$


$\mathbb{C}^2 = \mathbb{C}^{2^N}$
times
$= -gJ\hat{\sigma}_i^x$
$H = -gJ\sum_i \hat{\sigma}_i^x - J\sum_{(i,j)} \hat{\sigma}_i^z \otimes \hat{\sigma}_j^z$


$\cdots \otimes \mathbb{C}^2 = \mathbb{C}^{2^N}$
times
$\rightarrow \Phi_x = -gJ\hat{\sigma}_i^x$
$\rightarrow \Phi_x = -J\hat{\sigma}_i^z \otimes \hat{\sigma}_j^z$
$H = -gJ\sum_i \hat{\sigma}_i^x - J\sum_{(ij)} \hat{\sigma}_i^z \otimes \hat{\sigma}_j^z$
i
i+1