Title: PSI 17/18 - Gravitational Physics - Lecture 13

Date: Jan 19, 2018 10:15 AM

URL: http://pirsa.org/18010033

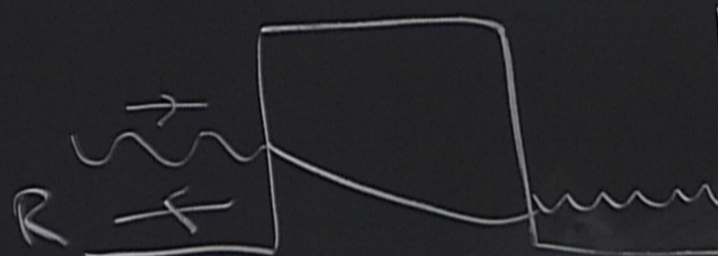
Abstract:

Coleman : PRD $\underline{15}$ 2929 (77)

" & de Luccia : PRD $\underline{21}$ 3305 (80)

Recall QM tunneling

$$\Omega^2 = \frac{2m(V_0 - E)}{\hbar^2}$$



$$|T|^2 = \frac{1}{1 + \frac{V_0^2 \sinh^2 \Omega d}{4E(V_0 - E)}} \propto e^{-2\Omega d}$$

$$\Omega d = \frac{1}{\hbar}\int_0^d \sqrt{2m(V_0 - E)}\,dx$$

"overall potential barrier"

Approximate wave fn by

$$\underline{\Psi} = e^{i\psi/\hbar}\, e^{-iEt/\hbar}$$

$$-\frac{\hbar^2}{2m}\underline{\Psi}'' + V\underline{\Psi} = \left[-\frac{i\hbar}{2m}\psi'' + \frac{\psi'^2}{2m} + V\right]\underline{\Psi} = E\underline{\Psi}$$

$$-\frac{\hbar^2}{2m}\underline{\Psi}'' + V\underline{\Psi} = \left[-\frac{i\hbar}{2m}\psi'' + \underbrace{\frac{\psi'^2}{2m} + V}\right]\underline{\Psi} = \underline{E\Psi}$$

$$V + \frac{\psi'^2}{2m} = E \qquad \psi \sim \pm\sqrt{2m(E-V)}\,x$$

$$-\frac{\hbar^2}{2m}\Psi'' + V\Psi = \left[-\frac{i\hbar}{2m}\varphi'' + \underbrace{\frac{\varphi'^2}{2m} + V}\right]\Psi = E\Psi$$

$$V + \frac{\varphi'^2}{2m} = E \qquad \varphi \sim \pm\sqrt{2m(E-V)}\;x$$

For $V=0$, $\quad \varphi \in \mathbb{R}$

$$-\frac{\hbar^2}{2m}\Psi'' + V\Psi = \left[-\frac{i\hbar}{2m}\varphi'' + \underbrace{\frac{\varphi'^2}{2m} + V}\right]\Psi = E\Psi$$

$$V + \frac{\varphi'^2}{2m} = E \qquad \varphi \sim \pm\sqrt{2m(E-V)}\,x$$

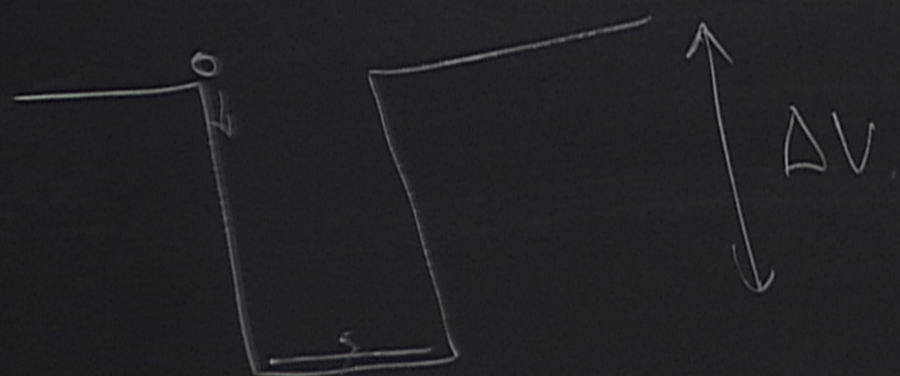
For $V=0$, $\varphi \in \mathbb{R}$, under barrier

$$\varphi \sim \pm i\Omega x$$

Now consider a (classical) particle in a potential well.

$$\int \sqrt{2\Delta V}\, dx = \int 2\Delta V\, d\tau$$

$$\frac{1}{2}\dot{x}^2 = \Delta V.$$



Now consider a (classical) particle in a potential well.

$$\int \sqrt{2\Delta V}\, dx = \int 2\Delta V \, d\tau$$

$$= \int \left(\Delta V + \tfrac{1}{2}\dot{x}^2\right) d\tau$$

$$\frac{1}{2}\dot{x}^2 = \Delta V.$$

Now consider a (classical) particle
in a potential well.

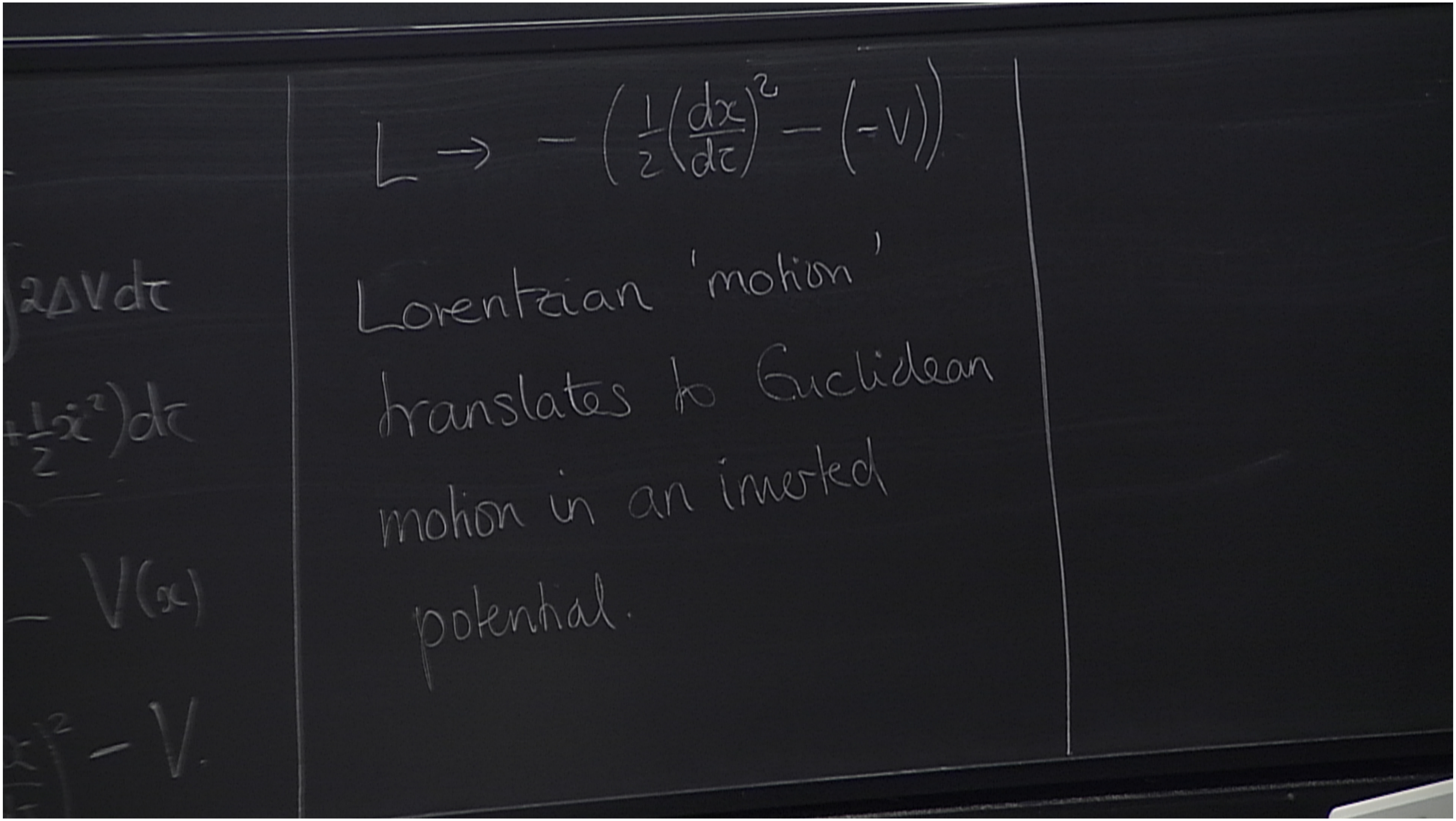
$$\frac{1}{2}\dot{x}^2 = \Delta V.$$

$$\int \sqrt{2\Delta V}\, dx = \int 2\Delta V\, d\tau$$

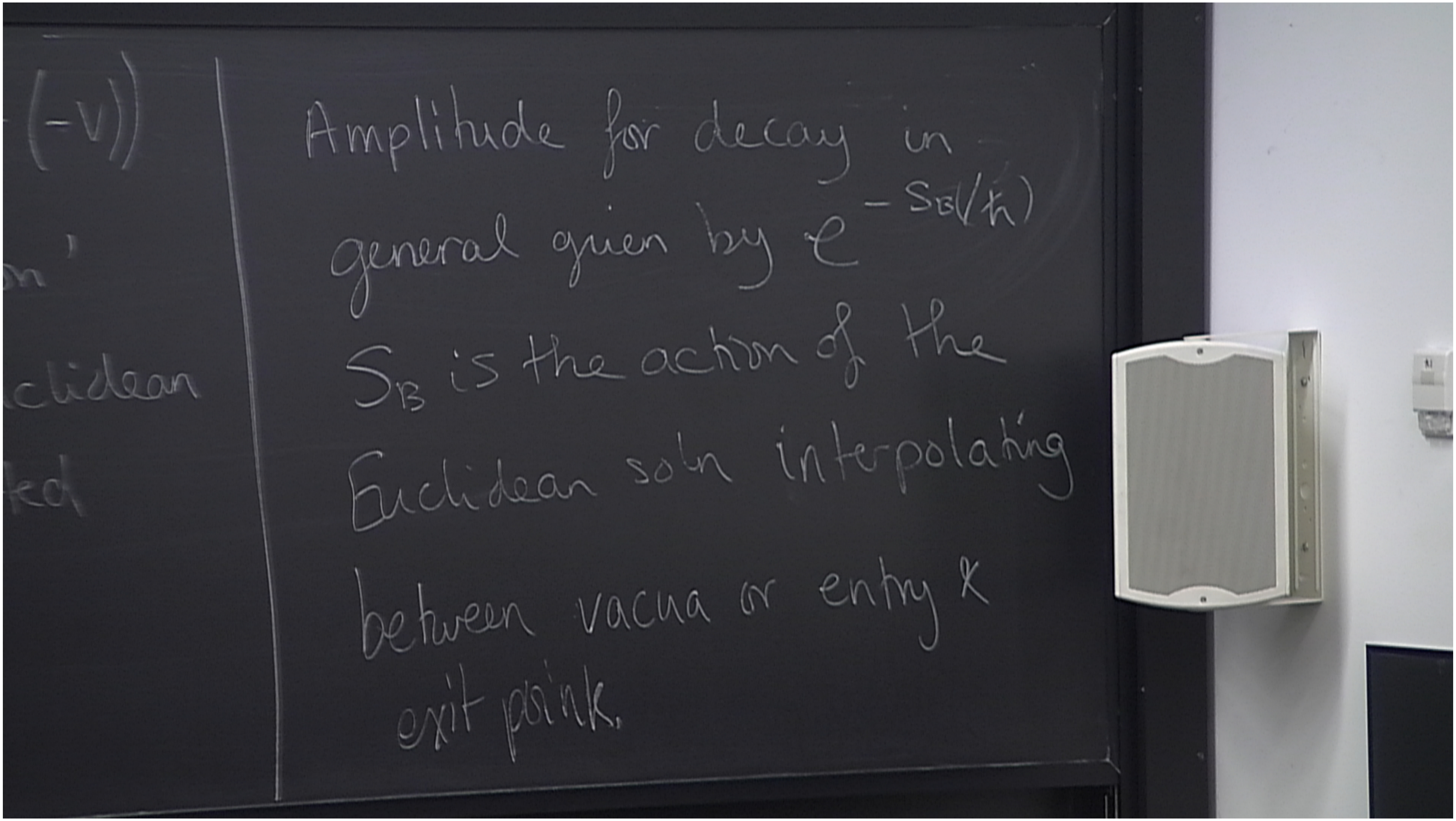
$$= \int \left(\underbrace{\Delta V + \tfrac{1}{2}\dot{x}^2}\right) d\tau$$

$$L = \frac{1}{2}\left(\frac{dx}{dt}\right)^2 - V(x)$$

$$\xrightarrow{t \to i\tau} -\frac{1}{2}\left(\frac{dx}{d\tau}\right)^2 - V.$$



$L \to -\left(\frac{1}{2}\left(\frac{dx}{d\tau}\right)^2 - (-V)\right)$
Lorentzian 'motion'
translates to Euclidean
motion in an inverted
potential.


Amplitude for decay in
general given by $e^{-S_B/\hbar}$
S_B is the action of the
Euclidean soln interpolating
between vacua or entry &
exit point.

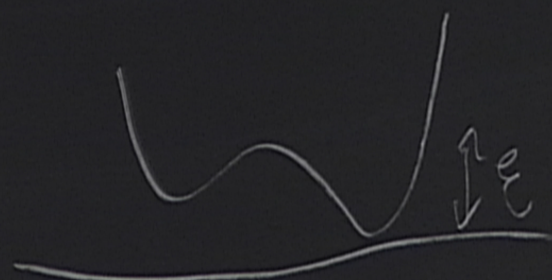
ptcle rolls to exit pt & back

"the bounce"

e.g. False vacuum decay

$$\mathcal{L}_\phi = \frac{1}{2}(\partial\phi)^2 - V(\phi)$$

$$V = \frac{\lambda}{2}(\phi^2 - \eta^2)^2 - \frac{\epsilon(\phi - \eta)}{2\eta}.$$



$\phi = -\eta \qquad V = \epsilon \qquad$ FALSE VAC

$\phi = \eta \qquad V = 0 \qquad$ TRUE VAC.

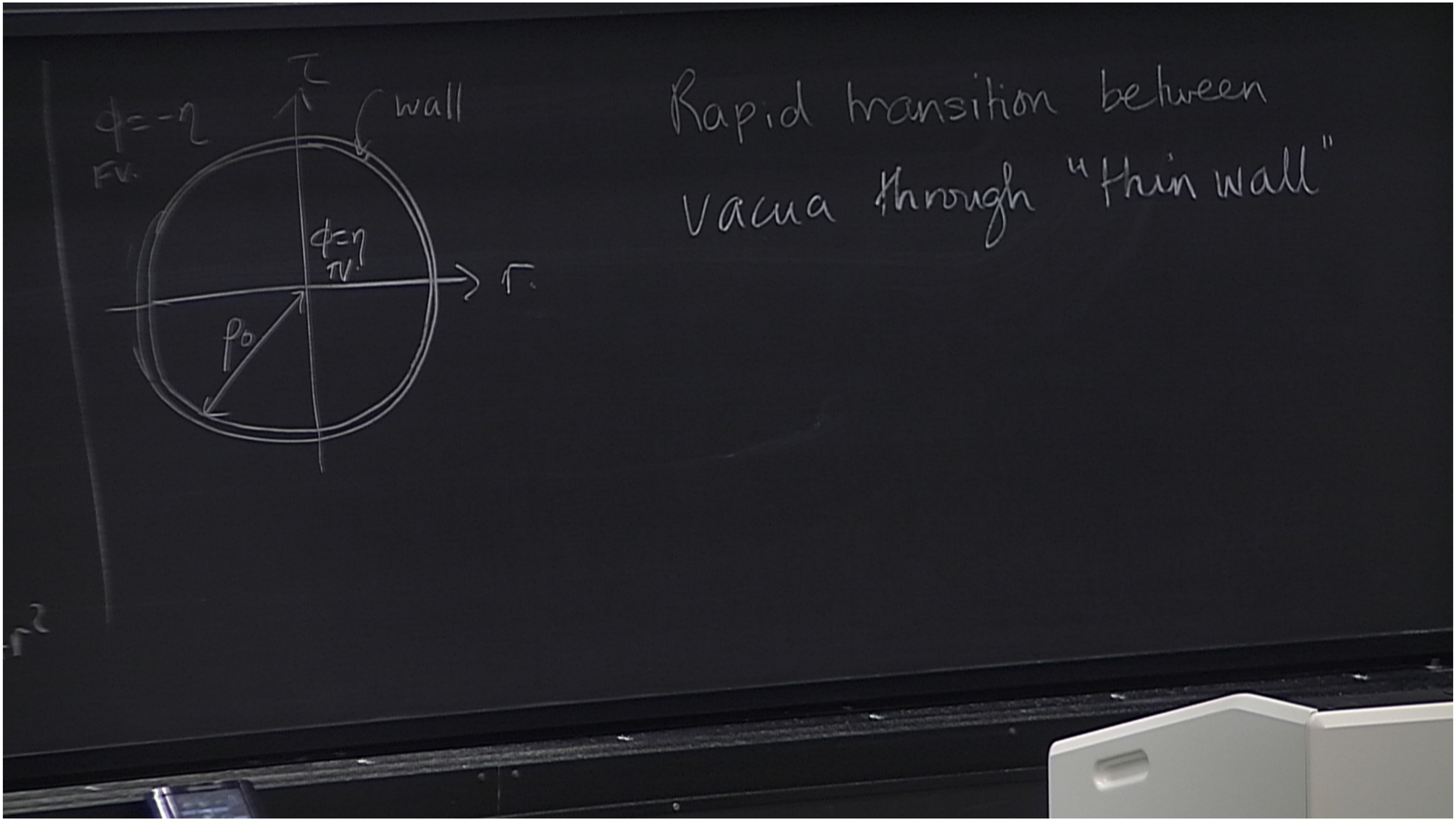
Expect F.V. to be metastable,
look for Euclidean soln to find
tunneling amplitude.

In $\tau = it$:

$$\frac{d^2\phi}{d\tau^2} + \nabla^2\phi = \frac{\partial V}{\partial \phi} = 2\lambda\phi(\phi^2-\eta^2) - \frac{\epsilon}{2\eta}$$

An approx soln is

$$\phi = -\eta \tanh\left[\sqrt{\lambda}\,\eta(\rho-\rho_0)\right]$$ where $\rho^2 = \tau^2 + r^2$


Rapid transition between
vacua through "thin wall"
τ
wall
φ = -η
F.V.
φ = η
T.V.
r
ρ₀

Euclidean action:

$$I_E = \int \frac{1}{2}\left[\left(\frac{d\phi}{d\tau}\right)^2 + (\nabla\phi)^2\right] + V$$

$$\simeq \int \frac{1}{2}\phi'^2 + V \sim \sigma \cdot 2\pi^2 \rho_0^3 \quad \text{WALL ENERGY}$$

$$+ \, \varepsilon \int_{\rho_0}^{\infty} 2\pi^2 \rho^3 \, d\rho$$

Euclidean action:

$$I_E = \int \frac{1}{2}\left[\left(\frac{d\phi}{d\tau}\right)^2 + (\underline{\nabla}\phi)^2\right] + V$$

$$\simeq \int \frac{1}{2}\phi'^2 + V \sim \sigma \cdot 2\pi^2 \rho_0^3$$ WALL ENERGY

$$+ \varepsilon \int_{\rho_0}^{\infty} 2\pi^2 \rho^3 \, d\rho$$ F.V. ENERGY

a description of decay by tunnelling [NON-PERTURBATIVE]

$$I_{FV} = \epsilon \int_0^{\infty} 2\pi^2 \rho^3 d\rho = \frac{\pi^2}{2} R^4 \Big|_{R \to \infty}$$

$$I_B = I_E - I_{FV} = \underbrace{2\pi^2 \rho_0^3 \sigma}_{\text{"COST FOR WALL"}} - \underbrace{\frac{\pi^2}{2} \rho_0^4 \epsilon}_{\text{GAIN FROM VACUUM}}$$

Euclidean soln is at a stationary pt in action $\frac{\partial I_B}{\partial \rho_0} = 0$

$$6\pi^2 \rho_0^2 \sigma = 2\pi^2 \rho_0^3 \varepsilon$$

$$\Rightarrow \quad \rho_0 = 3\sigma/\varepsilon \gg 1$$

tanh approxn required ρ_0 large.

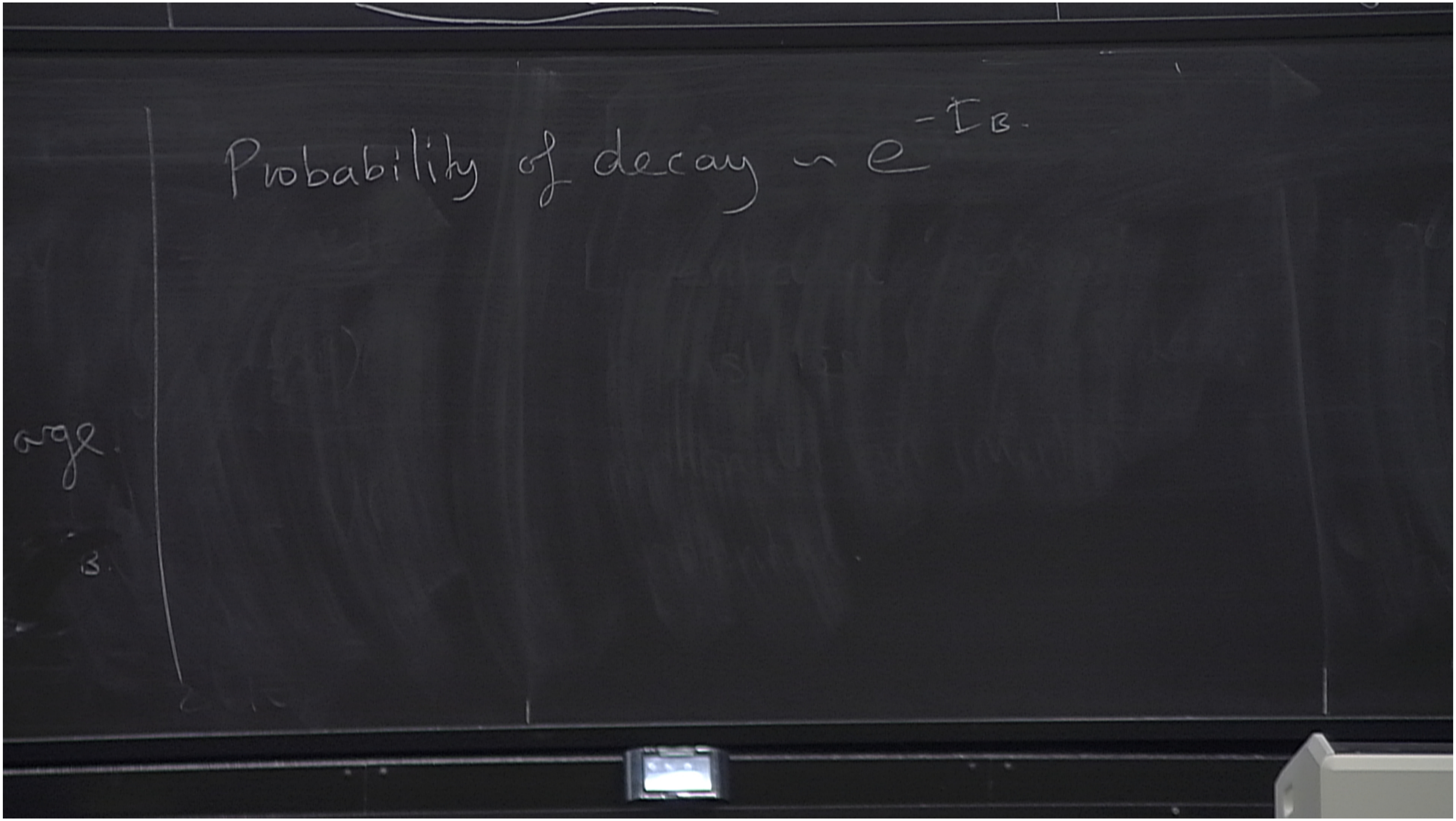
$$6\pi^2 \rho_0^2 \sigma = 2\pi^2 \rho_0^3 \varepsilon$$

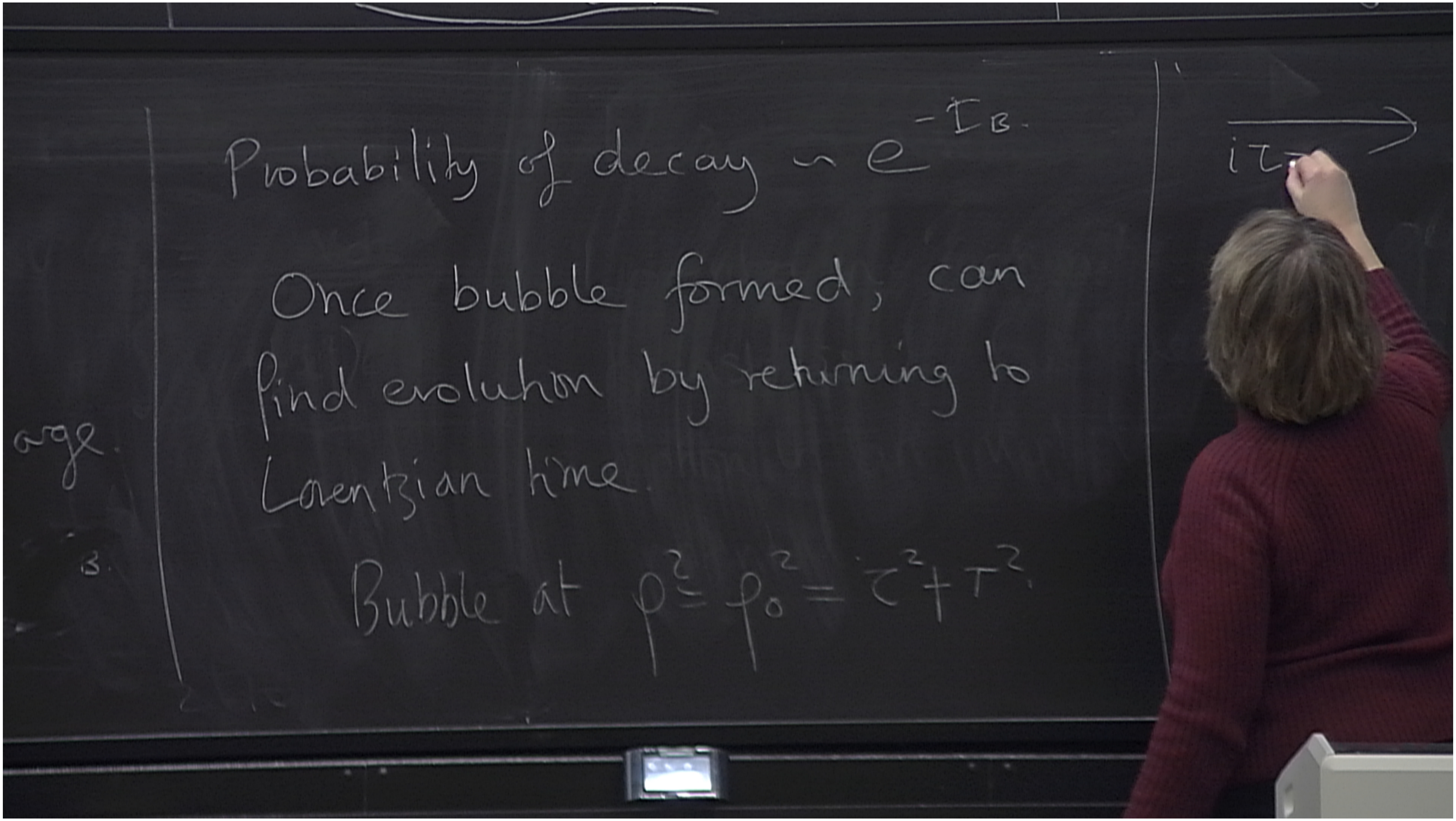
$$\Rightarrow \quad \rho_0 = 3\sigma/\varepsilon \gg 1$$

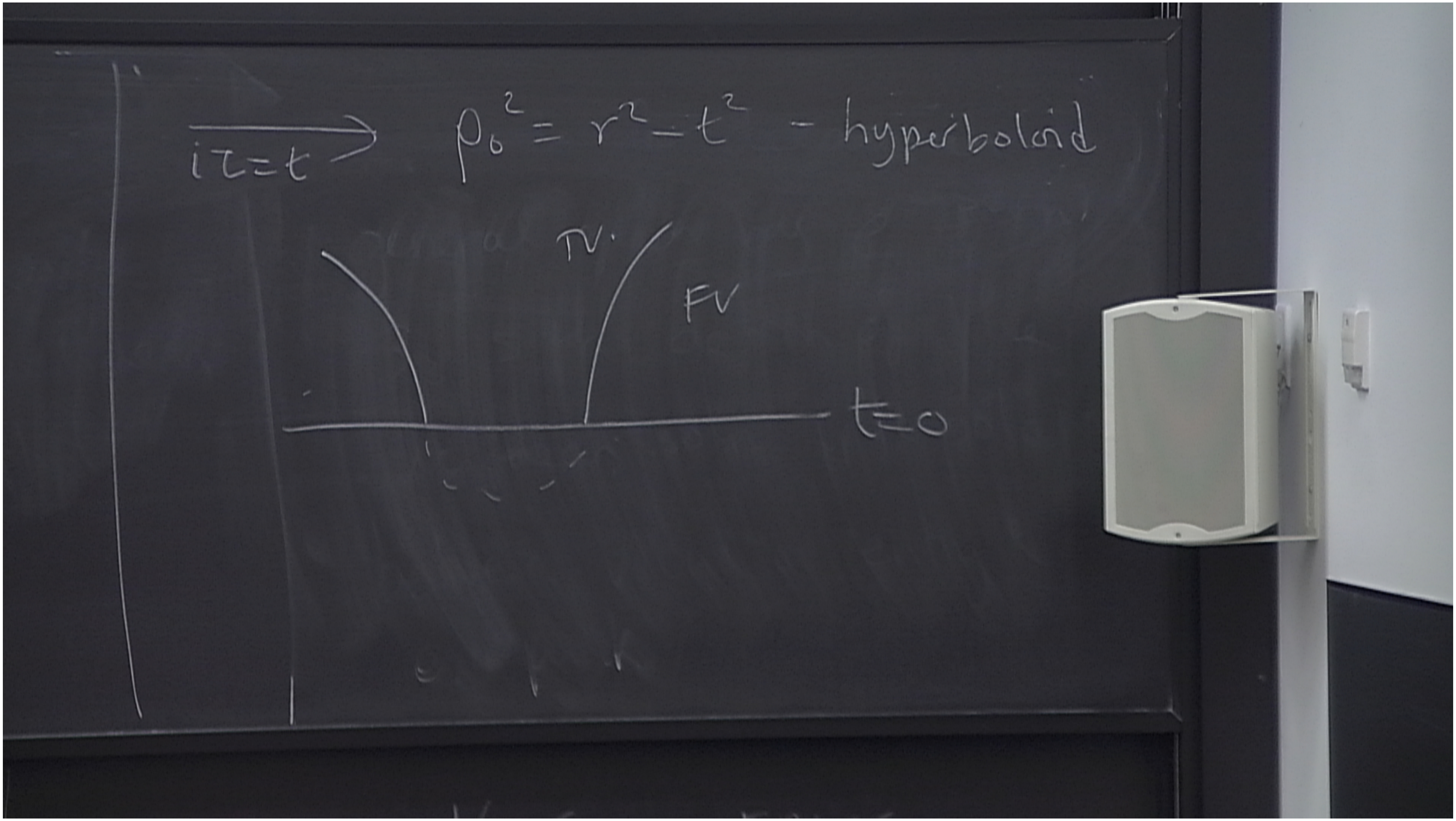
tanh approxn required ρ_0 large.

$$I_B = \frac{27\pi^2}{2} \frac{\sigma^4}{\varepsilon^3}$$

Probability of


Probability of decay ~ e^{-I_B}.


Probability of decay $\sim e^{-I_B}$.
Once bubble formed, can
find evolution by returning to
Lorentzian time.
Bubble at $\rho^2 = \rho_0^2 = \tau^2 + r^2$.
$i\tau$


iτ=t
ρ₀² = r² − t² − hyperboloid
TV.
FV
t=0

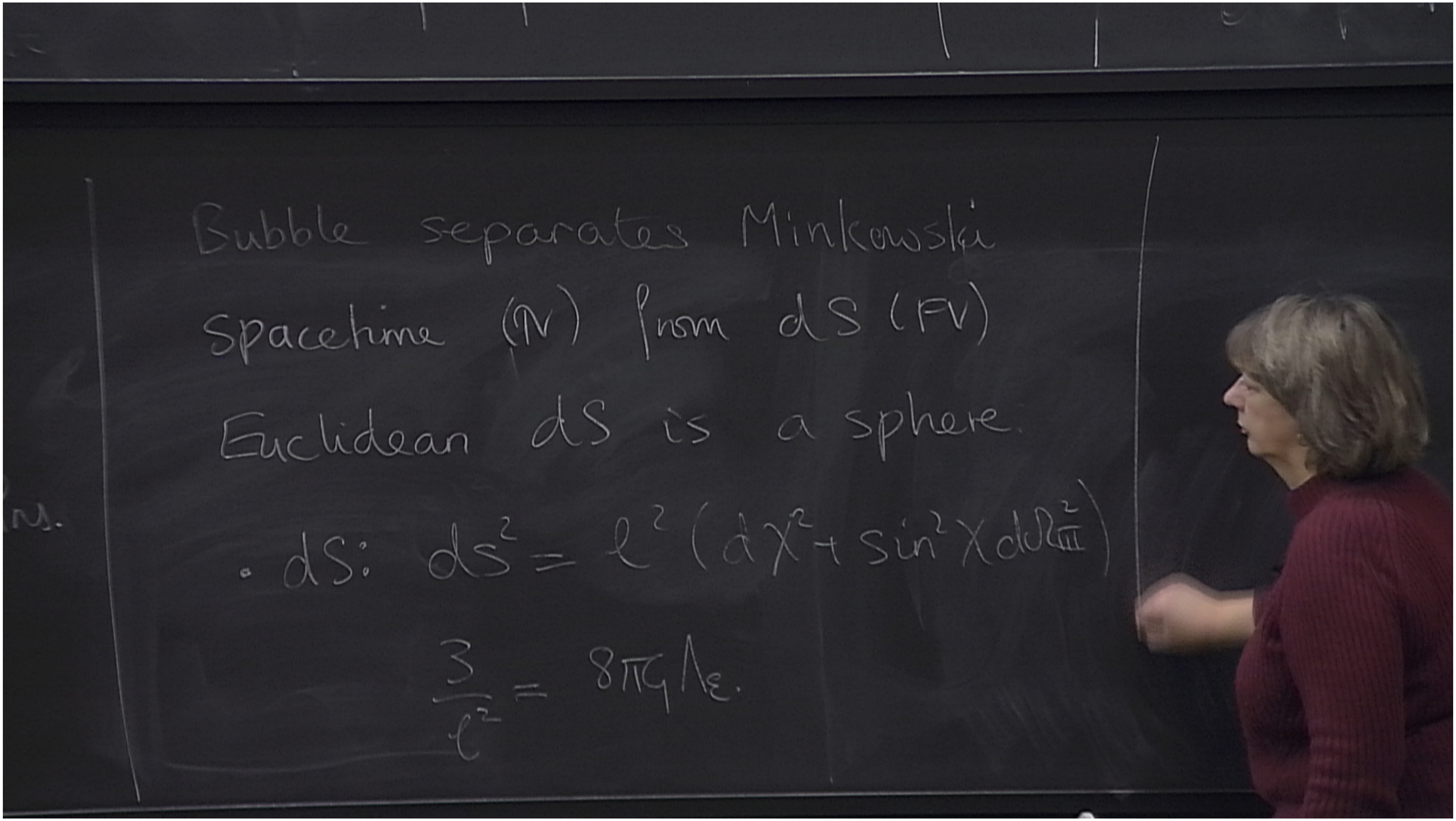
But vacuum energy gravitates

Include gravity in calculation

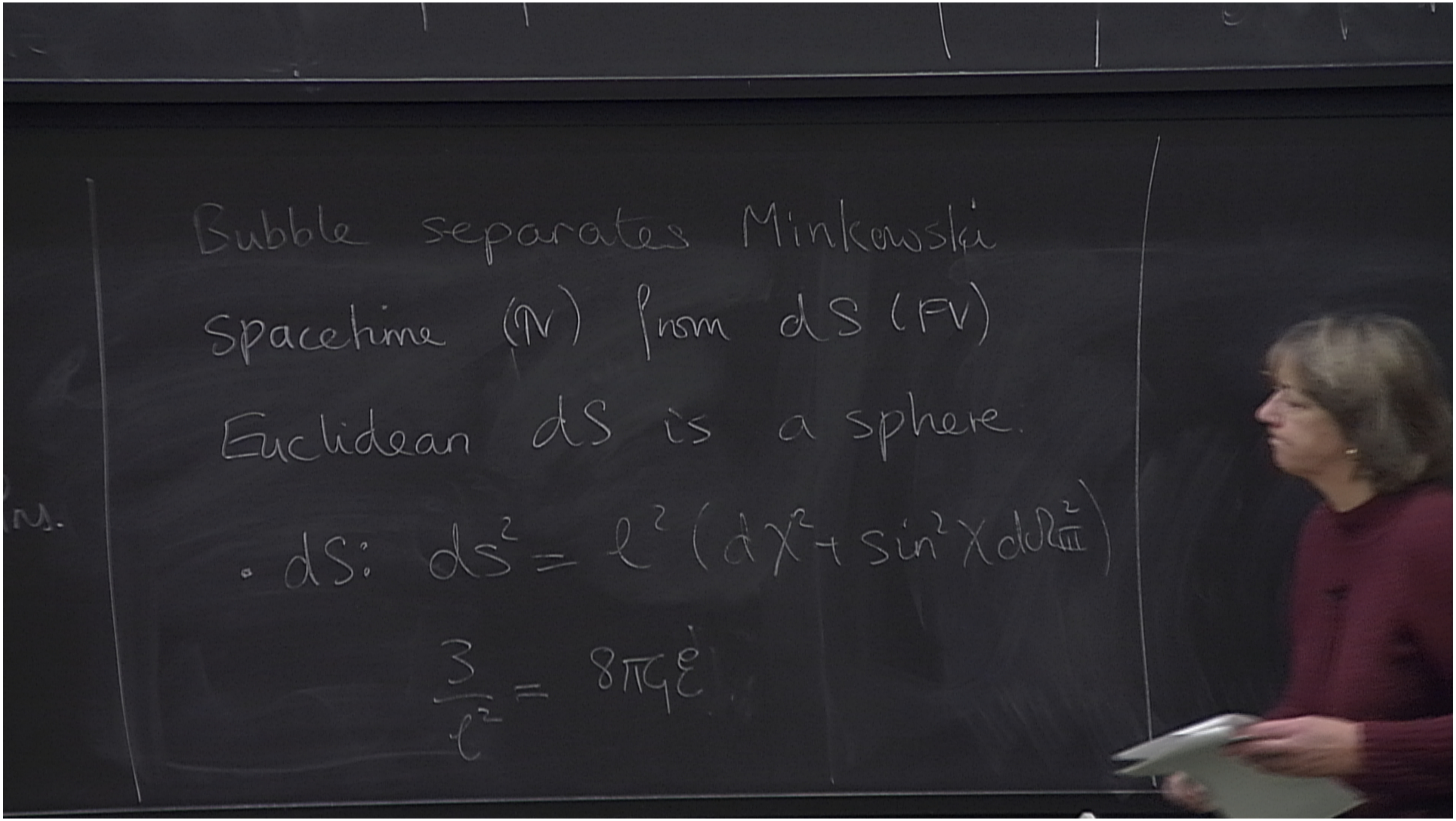
by finding Euclidean Einstein solns.

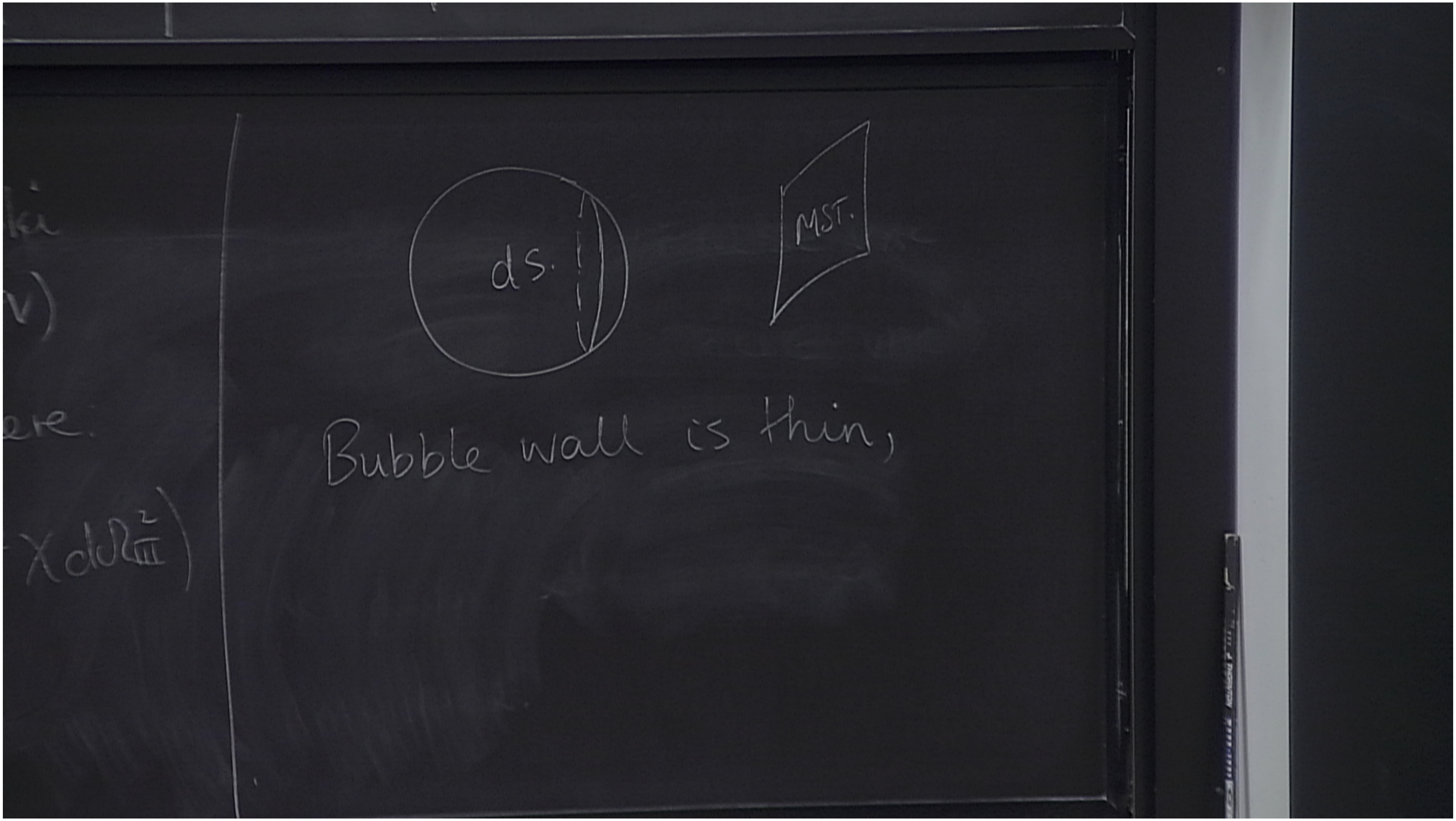
F.V. has energy $\mathcal{E} = \dfrac{\Lambda_F}{8\pi G}$

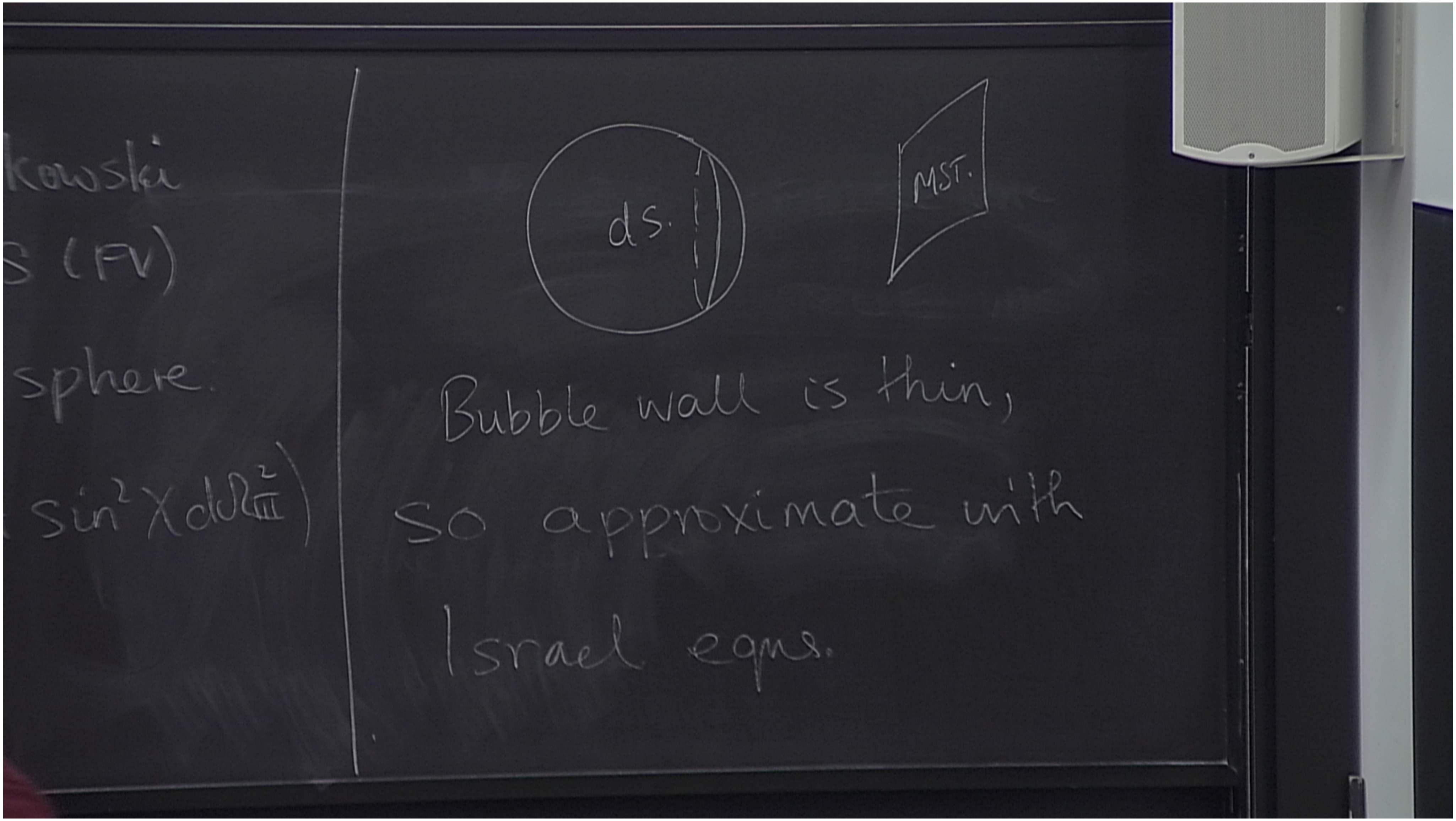
Bubble separates Minkowski
spacetime (TV) from dS (FV)

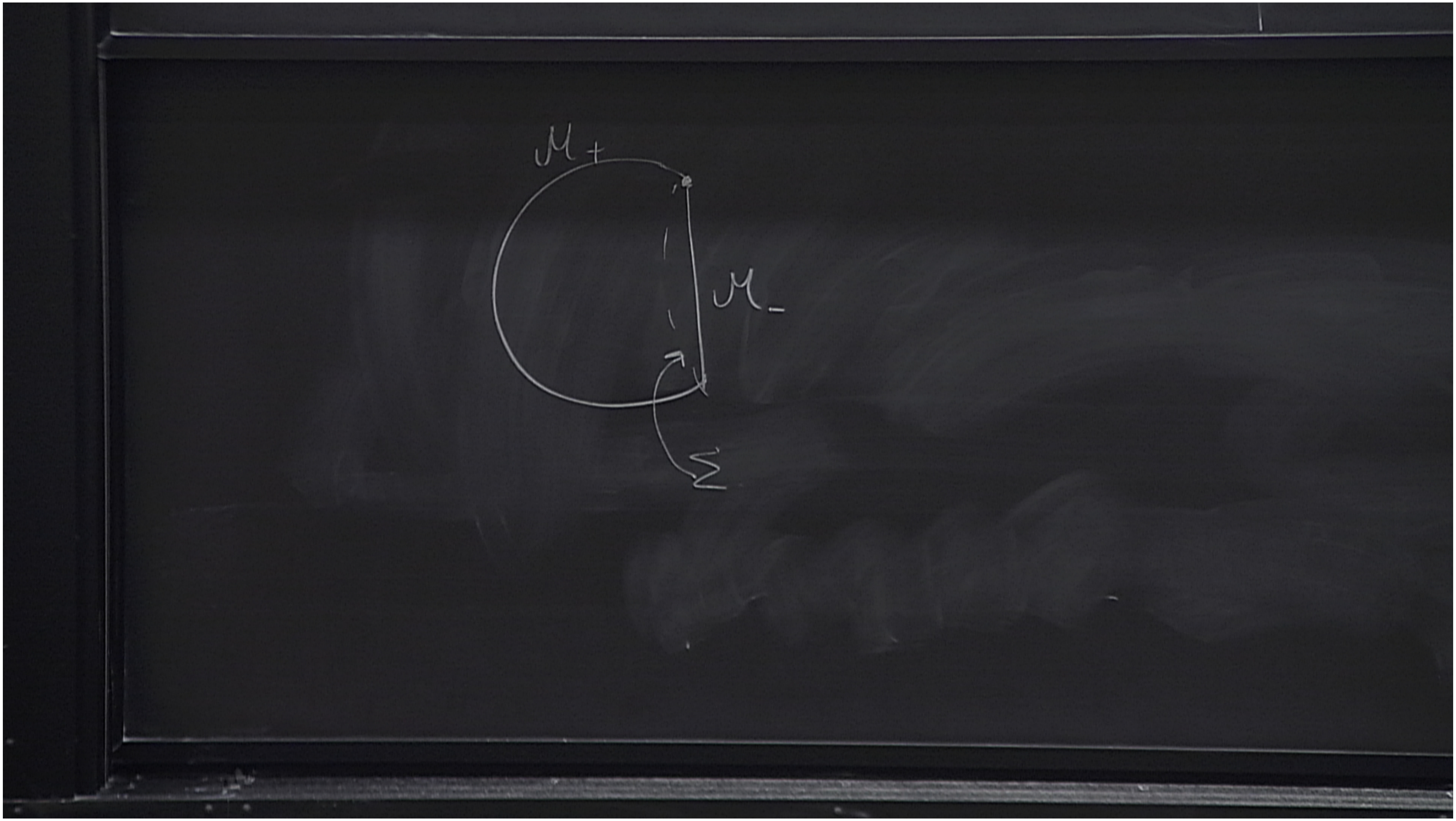


Bubble separates Minkowski
spacetime (TV) from dS (FV)
Euclidean dS is a sphere.
· dS: $ds^2 = \ell^2(d\chi^2 + \sin^2\chi \, d\Omega_{III}^2)$
$\frac{3}{\ell^2} = 8\pi G \Lambda_\varepsilon$.


Bubble separates Minkowski
spacetime (TV) from dS (FV)
Euclidean dS is a sphere.
• dS: $ds^2 = \ell^2 (d\chi^2 + \sin^2\chi \, d\Omega_{III}^2)$
$\frac{3}{\ell^2} = 8\pi G \varepsilon$


dS.
MST.
Bubble wall is thin,


kowski
S (FV)
sphere.
dS.
MST.
Bubble wall is thin,
so approximate with
Israel eqns.


$\mathcal{M}_+$
$\mathcal{M}_-$
Σ

Minkowski

dS (FV)

a sphere.

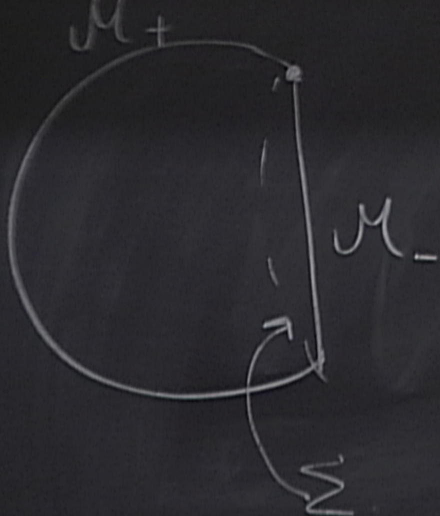
$\chi^2 + \sin^2\chi \, d\Omega_{III}^2)$

Bubble wall is thin,

so approximate with

Israel eqns

$$\Delta K_{ab} - \Delta K h_{ab} = 8\pi G \sigma h_{ab}.$$



For dS $\quad n_+ = \frac{1}{\ell}\frac{\partial}{\partial \chi}$

Wall at χ_0

$$K_{+\alpha\beta} = -\frac{1}{\ell}\Gamma^{\chi}_{\alpha\beta} = \frac{\cot \chi_0}{\ell}\, g_{\alpha\beta}$$

$\Delta K_{ab} - \Delta K h_{ab} =$

MST $\quad ds^2 = d\rho^2 + \rho^2 d\Omega_{III}^2$

Wall at $\rho_0 = l \sin\chi_0$

$n_- = \frac{\partial}{\partial\rho} \quad K_{-\alpha\beta} = -\Gamma^{\rho}_{\alpha\beta} = \frac{1}{\rho_0} g_{\alpha\beta}$

$$\Delta K_{\alpha\beta} - \Delta K\, g_{\alpha\beta} = -\frac{2}{\ell}\cot\chi_0 + \frac{2}{\rho_0}$$

$$= \frac{2}{\rho_0}\left[1 - \sqrt{1 - \rho_0^2/\ell^2}\right]$$

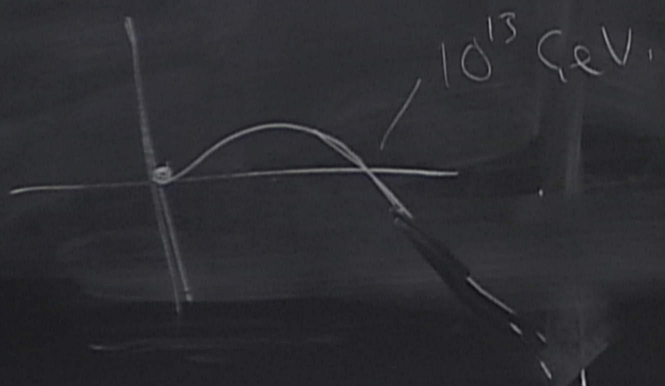
$$= 8\pi G\sigma.$$

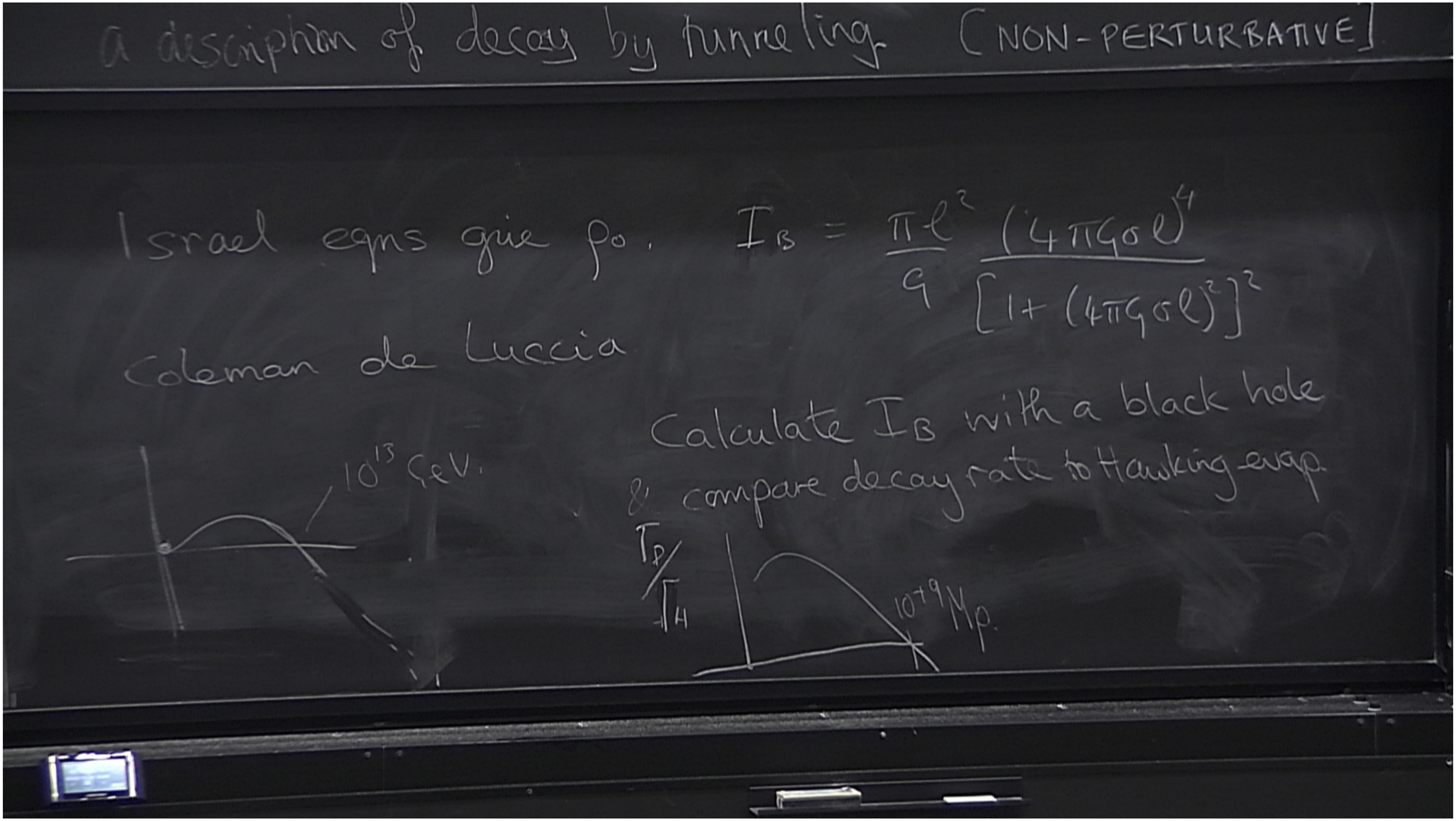
Israel eqns give ρ_0. $\quad I_B = \dfrac{\pi \ell^2}{G} \dfrac{(4\pi G \sigma \ell)^4}{[1 + (4\pi G\sigma\ell)^2]^2}$

Israel eqns give ρ_0.

$$I_B = \frac{\pi \ell^2}{G} \frac{(4\pi G \sigma \ell)^4}{\left[1+(4\pi G \sigma \ell)^2\right]^2}$$

Coleman de Luccia




a description of decay by tunneling [NON-PERTURBATIVE]
Israel eqns give ρ_0.
$I_B = \frac{\pi l^2}{9} \frac{(4\pi G\sigma l)^4}{[1+(4\pi G\sigma l)^2]^2}$
Coleman de Luccia
Calculate I_B with a black hole
& compare decay rate to Hawking evap.
10^{13} GeV.
Γ_D / Γ_H
$10^{+9} M_p$.