Title: PSI 17/18 - Gravitational Physics - Lecture 12

Date: Jan 18, 2018 10:15 AM

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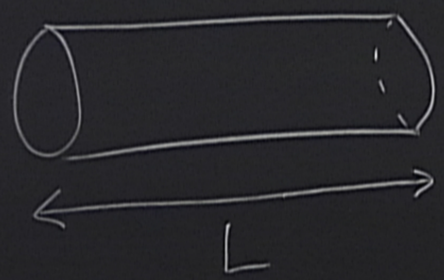
Abstract:

LECTURE 12 Gravitational Perturbation Theory

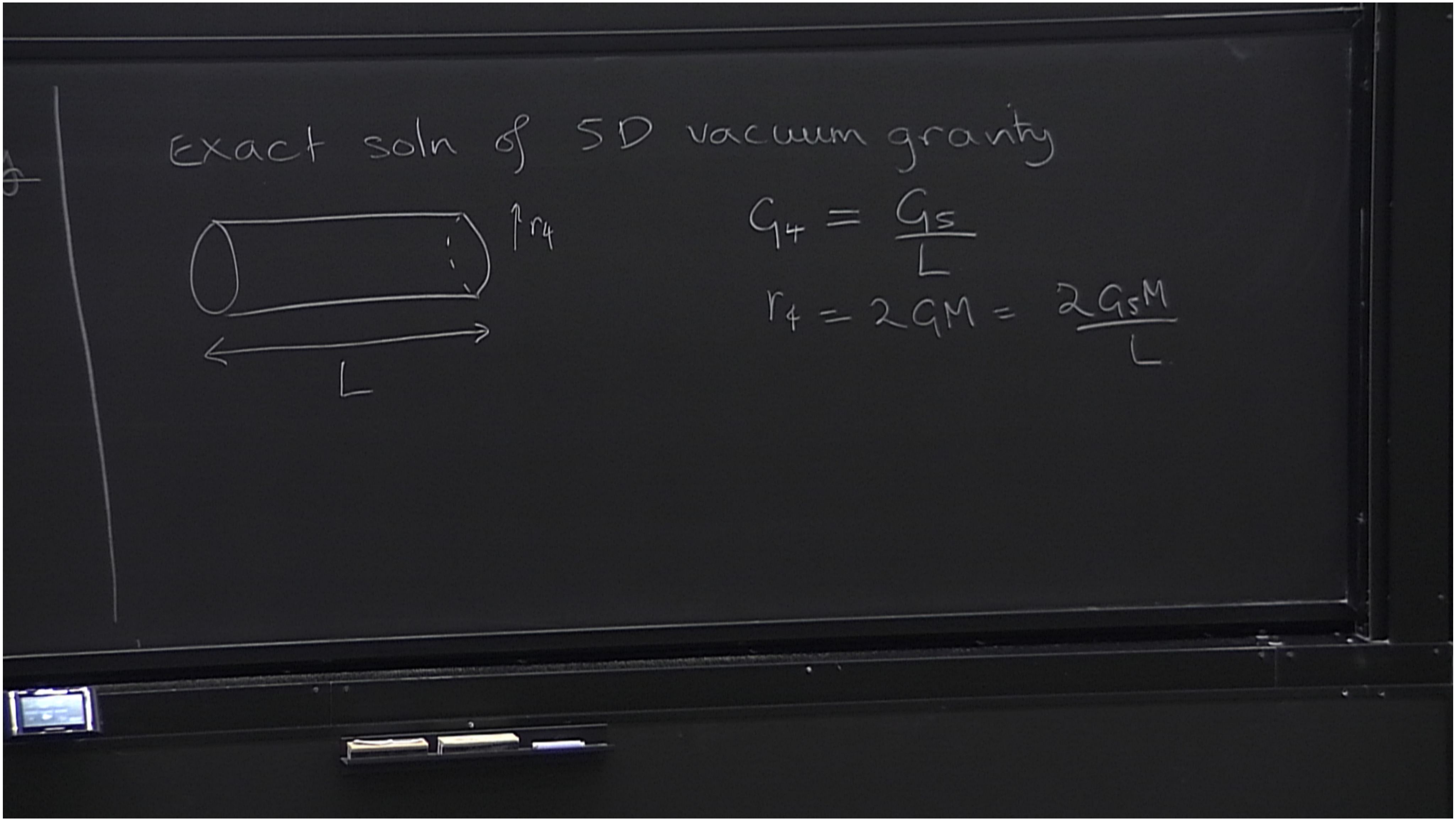
Recall black string:

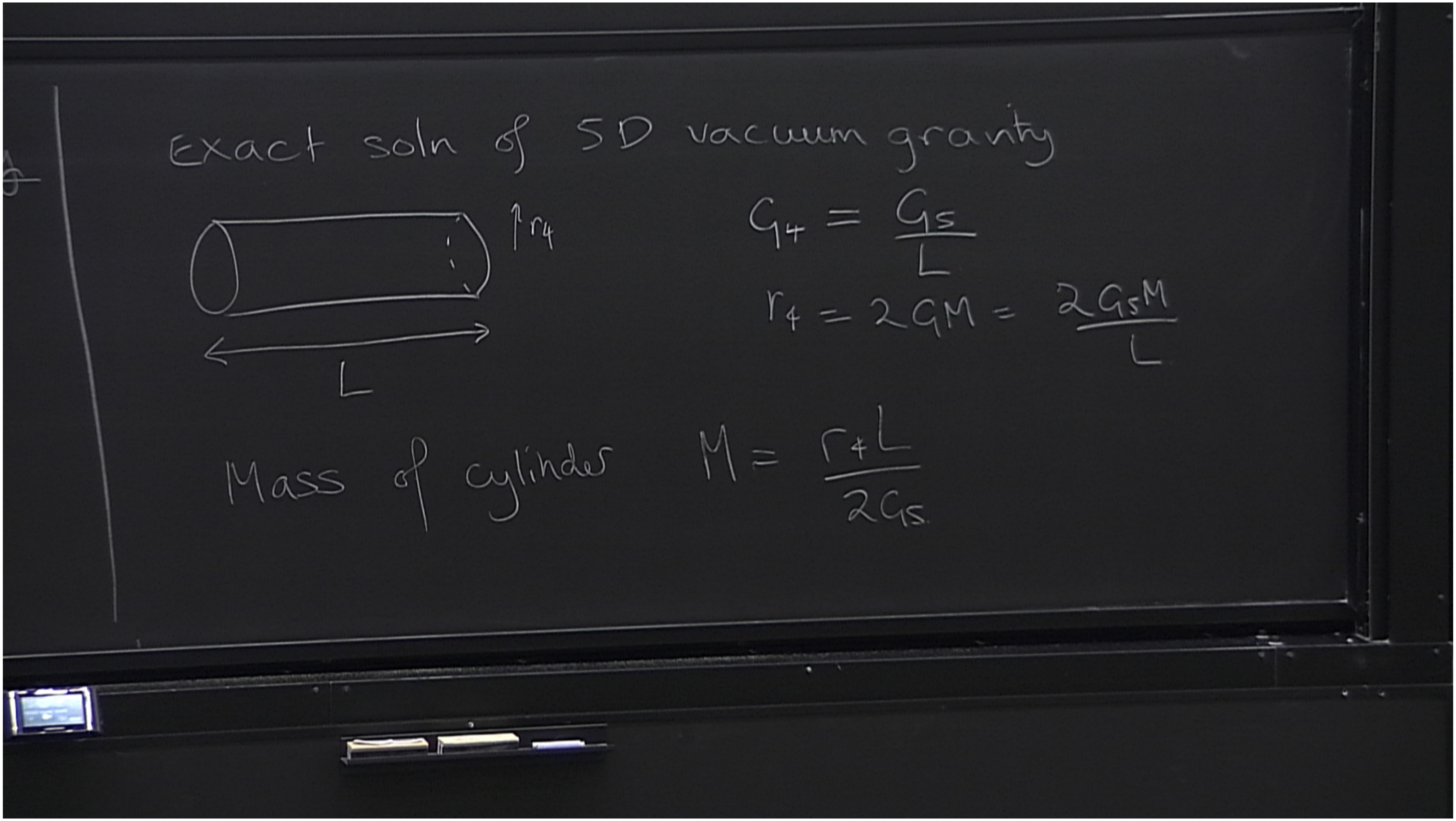
$$\underbrace{ds^2 = \left(1-\frac{2GM}{r}\right)dt^2 - \frac{dr^2}{\left(1-\frac{2GM}{r}\right)} - r^2 d\Omega_2^2}_{SCH_4} - dz^2 \quad \times \begin{Bmatrix} \mathbb{R} \\ S^1 \end{Bmatrix}$$

Exact soln of 5D vacuum gravity



$$G_4 = \frac{G_5}{L}$$


Exact soln of 5D vacuum gravity
r_4
L
$G_4 = \frac{G_5}{L}$
$r_4 = 2GM = \frac{2G_5 M}{L}$


Exact soln of 5D vacuum gravity
r4
L
G4 = G5 / L
r4 = 2GM = 2G5M / L
Mass of cylinder M = r4L / 2G5

Exact soln of 5D vacuum gravity

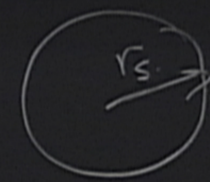
$$G_4 = \frac{G_5}{L}$$

$$r_4 = 2GM = \frac{2G_5M}{L}$$

Mass of cylinder $\quad M = \dfrac{r_4 L}{2G_5}$

Entropy $\quad S = \dfrac{4\pi r_4^2 L}{4G_5} = 4\pi G_4 M^2$

5D black hole:

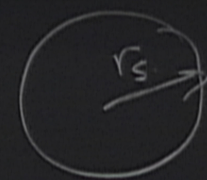


$$ds^2 = \left(1 - \frac{r_s^2}{\rho^2}\right) dt^2 - \frac{d\rho^2}{(1 - r_s^2/\rho^2)} - \rho^2 d\Omega_{III}^2$$

EX: $$\frac{16\pi G M}{(D-2) A_{D-2}} = r_s^2$$ (Show from $G^0_0 = 8\pi G T^0_0$)

IN DIM D

5D black hole:



$$ds^2 = \left(1 - \frac{r_s^2}{\rho^2}\right) dt^2 - \frac{d\rho^2}{(1 - r_s^2/\rho^2)} - \rho^2 \, d\Omega_{III}^2$$

EX: $$\frac{16\pi G_D M}{(D-2) A_{D-2}} = r_D^{D-3}$$ (Show from $G^0_0 = 8\pi G T^0_0$)

IN DIM D

$$M_{BH} = \frac{3 \times 2\pi^2 \times r_s^2}{16\pi G_5} = \frac{3\pi \; r_s^2}{8 G_5}$$

$$S_{BH} = \frac{1}{4G_5} \cdot 2\pi^2 r_s^3 = \frac{\pi^2}{2G_5}\left(\frac{8 G_5 M_{BH}}{3\pi}\right)^{3/2}$$

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Note $G_5 S_{BS} \propto \frac{(G_5 M_{BS})^2}{L}$; $G_5 S_{BH} \propto (G_5 M_{BH})^{3/2}$

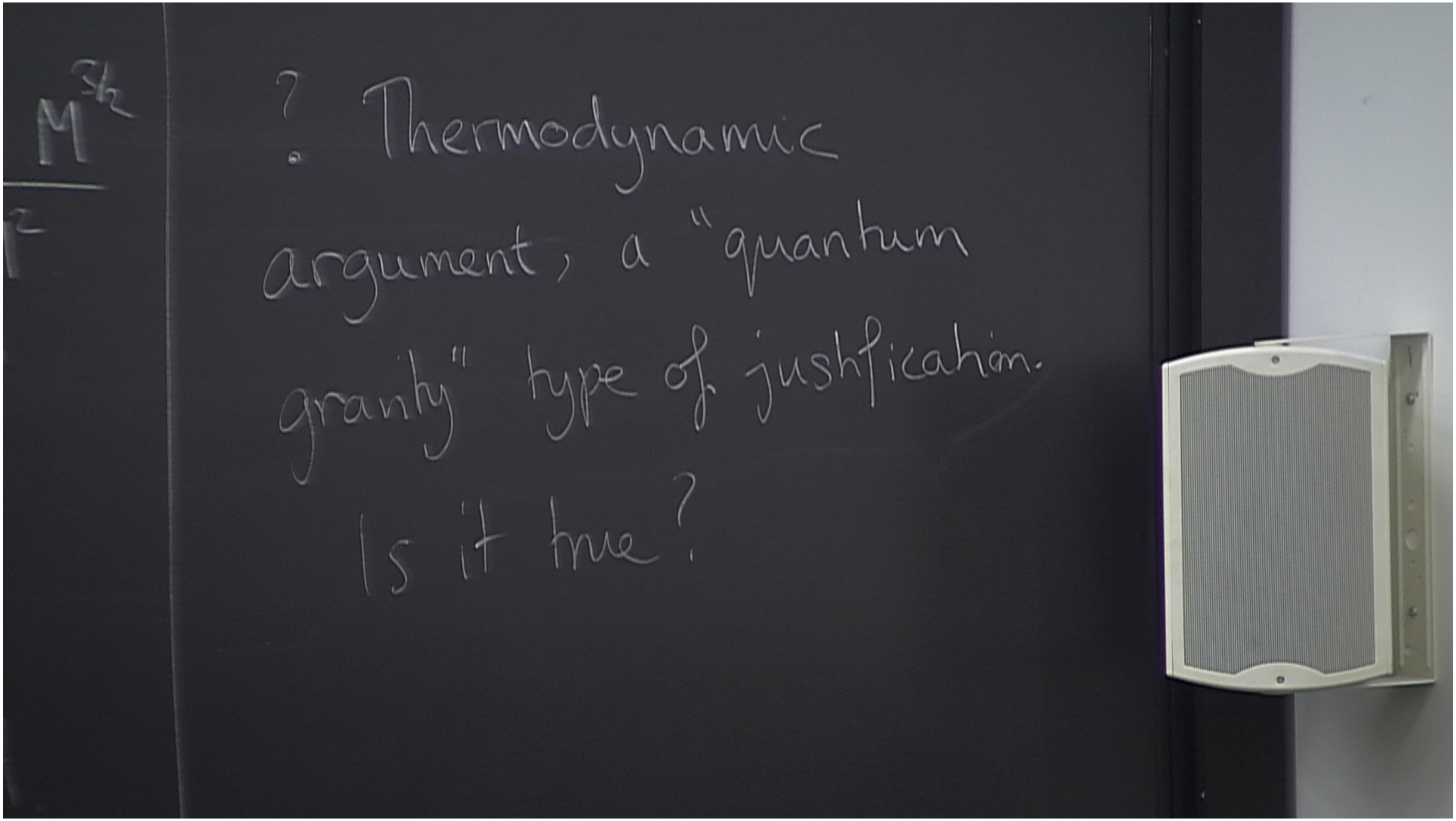
For a fixed mass, entropy of string is higher for smaller L, lower for larger L : skinny black strings should be unstable.

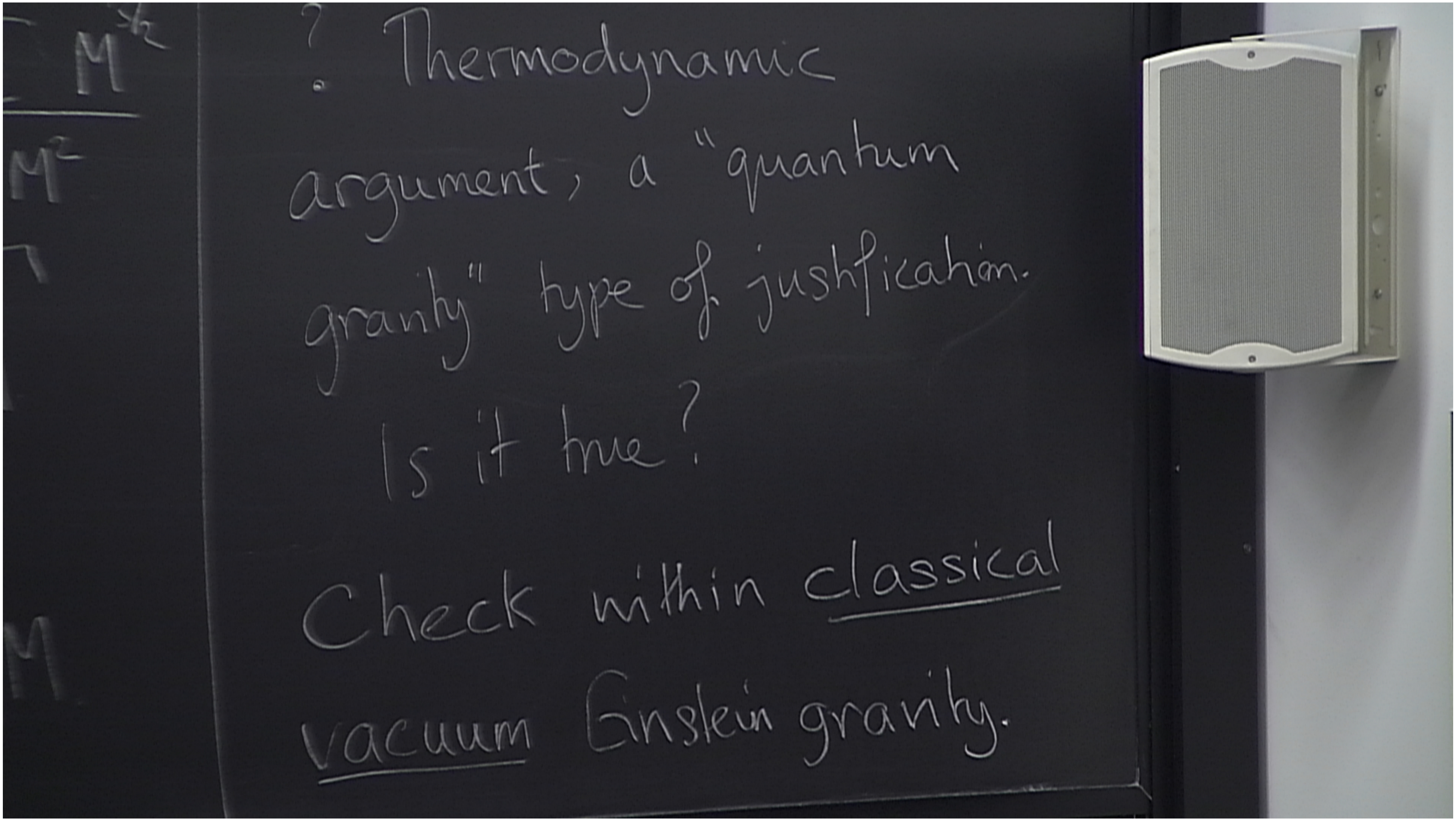
$$\frac{S_{BH}}{S_{BS}} = \sqrt{\frac{\pi}{27}} \, \frac{8\sqrt{2}\,\sqrt{G_4 L}\; M^{3/2}}{4\pi G_4 M^2}$$

$$= \sqrt{\frac{8}{27\pi}} \, \sqrt{\frac{L}{G_4 M}}$$

Instability emerges when

$$L > \frac{27\pi}{8} G_4 M$$


? Thermodynamic
argument, a "quantum
gravity" type of justification.
Is it true?


? Thermodynamic
argument, a "quantum
gravity" type of justification.
Is it true?
Check within classical
vacuum Einstein gravity.

Explore via perturbation theory.

- solar system tests.
- cosmology.
- gravitational radiation

Explore via perturbation theory.

- solar system tests.
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Background + perturbation

$g_{ab} \rightarrow g_{0ab} + h_{ab}$

theory.

Note h_{ab} is δg_{ab} not 1^{st} f.f.

and <u>not</u> δg^{-1} (like variation).

Assume h is in some sense small & ignore $O(h^2)$

$$\delta \Gamma^a_{bc} = \frac{1}{2}\left(g_0^{ad} - h^{ad}\right)\left(g_{0db,c} + h_{db,c} + \dots\right)$$

$$= \frac{1}{2}\left(\nabla_c h^a{}_b + \nabla_b h^a{}_c - \nabla^a h_{bc}\right)$$

(NC) $= \frac{1}{2}\Big($

$$\delta R_{ab} = \nabla_c \delta\Gamma^c{}_{ab} - \nabla_b \delta\Gamma^c{}_{ac}$$

$$= \frac{1}{2}\nabla_c \nabla_a h^c{}_b + \frac{1}{2}\nabla_c \nabla_b h^c{}_a$$

$$- \frac{1}{2}\Box h_{ab} - \frac{1}{2}\nabla_b \nabla_a h$$

Use Riemann identity

$$\nabla_c \nabla_a h^c{}_b = \nabla_a \nabla_c h^c{}_b + R^c{}_{dca} h^d{}_b$$

$$+ R_{bdca} h^{dc}$$

$$\Rightarrow \quad \delta R_{ab} = -\frac{1}{2}\Box h_{ab} - \frac{1}{2}\nabla_a\nabla_b h$$

$$+ \frac{1}{2}\nabla_a\nabla^c h_{cb} + \frac{1}{2}\nabla_b\nabla_c h^c{}_a$$

$$+ R_{ad}h^d{}_b + R_{bd}h^d{}_a$$

$$- 2R_{acbd}h^{cd}.$$

$$\Rightarrow \quad \delta R_{ab} = -\frac{1}{2}\Box h_{ab} - \frac{1}{2}\nabla_a\nabla_b h$$

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$$- 2R_{acbd}h^{cd}.$$

Define $\bar{h}_{ab} = h_{ab} - \frac{1}{2}h g_{ab}$.

Get the Lichnerowicz Operator:

$$\delta R_{ab} = -\frac{1}{2}\Delta_L h_{ab}$$

$$= -\frac{1}{2}\left[\Box h_{ab} + 2R_{acbd}h^{cd} - 2R_{(a}{}^{d}h_{b)d} - 2\nabla_{(a}\nabla^{d}h_{b)d}\right]$$

Gauge

$$X^\mu \to X^\mu + \xi^\mu \quad \leftarrow \text{"small"}$$

Under $X \to X + \xi$.

$$g_{ab} \to g_{ab} + \mathcal{L}_\xi g_{ab}$$

$$\equiv \Box \xi_b + R^a{}_b \xi_a = \text{source}$$

- well posed dif eq.

If $\nabla_a \bar{h}^a{}_b$ is not zero,

solve for ξ s.t. $\Box \xi^a + R^a{}_b \xi^b = \nabla_b \bar{h}^{ab}$

then take new coords $x - \xi$ in

which $\nabla_a \bar{h}^{ab} = 0$

ξ_a

$$= \Box \xi_b + R^a{}_b \xi_a = \text{source}$$

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which $\nabla_a \bar{h}^{ab} = 0$ <u>DE-DONDER</u>

ξ_a

small & ignore

$$\delta\Gamma^a_{bc} = \frac{1}{2}(g_0^{ad} - h^{ad})(g_{0db,c} + h_{db,c} + \dots)$$

$$(NC) = \frac{1}{2}\left(\nabla_c h^a{}_b + \nabla_b h^a{}_c - \nabla^a h_{bc}\right)$$

Use Riemann identity

$$\nabla_c \nabla_a h^c{}_b = \nabla_a \nabla_c h^c{}_b + R^c{}_{dca} h^d{}_b$$

$$+ R_{bdca} h^{dc}$$

Get the Lichnerowicz Operator:

$$\delta R_{ab} = -\frac{1}{2}\Delta_L h_{ab}$$

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Gauge

$$x^\mu \to x^\mu + \xi^\mu \leftarrow \text{"small"}$$

under $x \to x + \xi$:

$$g_{ab} \to g_{ab} + \mathcal{L}_\xi g_{ab}$$

= source

zero,

$+ R^a{}_b \xi^b = \nabla_b \bar{h}^{ab}$

$- \xi$ in

DE-DONDER

Remaining gauge freedom?

$$x^\mu \to x^\mu + \chi^\mu$$

$$\Box \chi^a + R^a{}_b \chi^b = 0$$

to preserve de Donder gauge

= source

zero,

$+R^a{}_b \xi^b = \nabla_b \bar{h}^{ab}$

ξ in

DE-DONDER

Remaining gauge freedom?

$$x^\mu \to x^\mu + \chi^\mu$$

$$\Box \chi^a + R^a{}_b \chi^b = 0$$

to preserve de Donder gauge

D solns.

Count degrees of freedom (d.o.f.)

h_{ab} sym $\longleftrightarrow$ $\frac{D(D+1)}{2}$ cpts.

$\nabla_a \bar{h}^{ab} = 0$ $\longleftrightarrow$ D constraints.

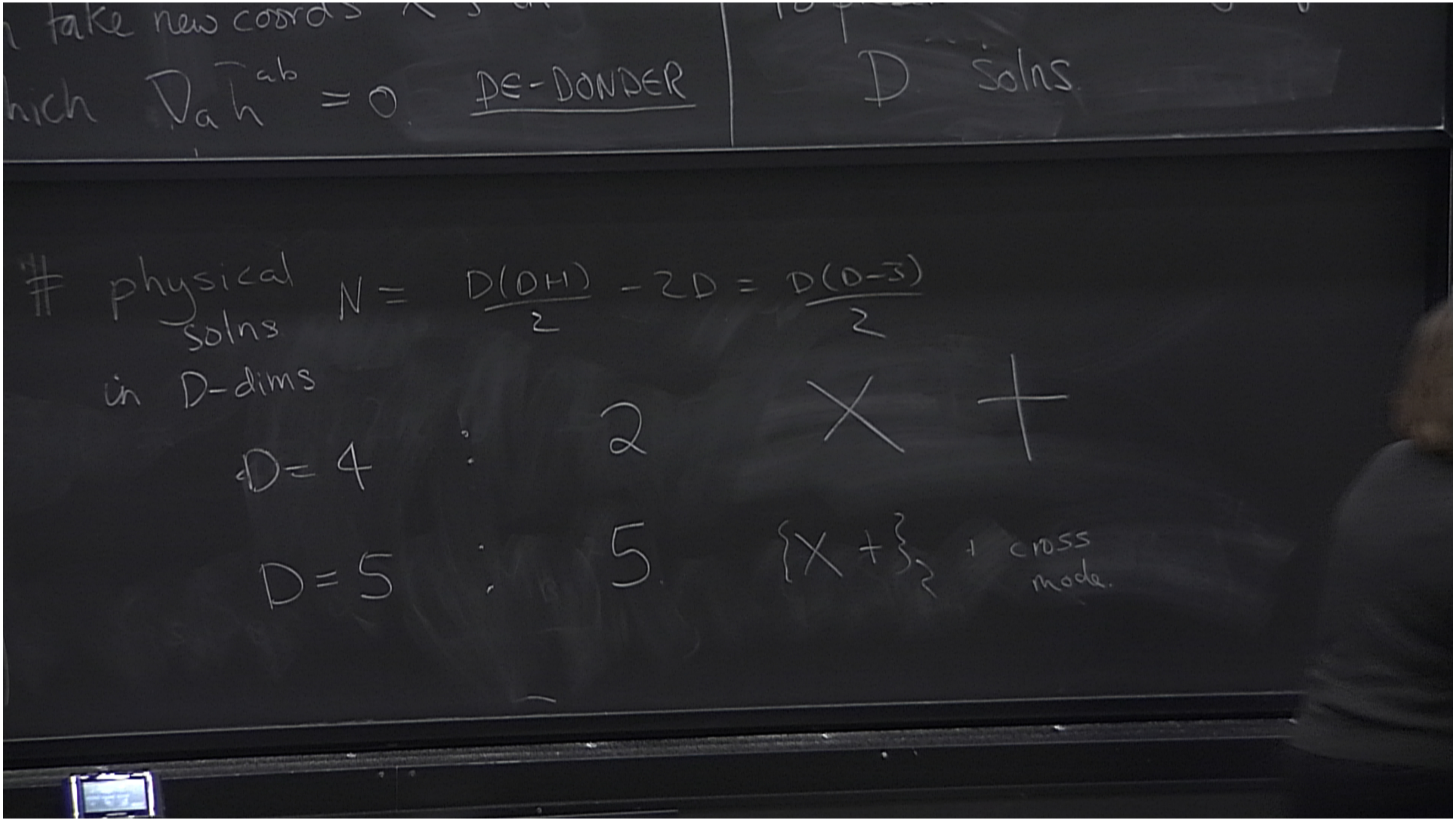
X^a $\longleftrightarrow$ D remaining gauge d.o.f.

which $\nabla_a \bar{h}^{ab} = 0$ DE-DONDER | D solns.

\# physical solns in D-dims $N = \frac{D(D+1)}{2} - 2D = \frac{D(D-3)}{2}$

$D = 4$: 2 $\quad \times \quad +$

$D = 5$: 5 $\quad \{\times\ +\}_2$ + cross mode.


take new coords
which $\nabla_a \bar{h}^{ab} = 0$ DE-DONDER
D solns
# physical solns in D-dims $N = \frac{D(D+1)}{2} - 2D = \frac{D(D-3)}{2}$
D = 4 : 2 × +
D = 5 : 5 {× +}₂ + cross mode.

$g_{ab} \rightarrow g_{0ab} + h_{ab}$

$(NC) = \frac{1}{2}$

For the black string, have ∂_t, ∂_z & $SO(3)$ Killing vectors. Decompose h_{ab} w.r.t. symmetries.

$\partial_t \longleftrightarrow e^{i\omega t}$ OR $e^{\Omega t}$ for instability.

$\partial_z \longleftrightarrow e^{i\mu z}$

$SO(3) \longleftrightarrow h_{ab}$ has specific angular momentum, $e^{im\phi}$ & l. $\leftrightarrow Y_{lm}(\theta, \phi)$ for scalars

Simplest mode has $l=0$

(s-wave). Cylinder $\rightarrow$ sphere,

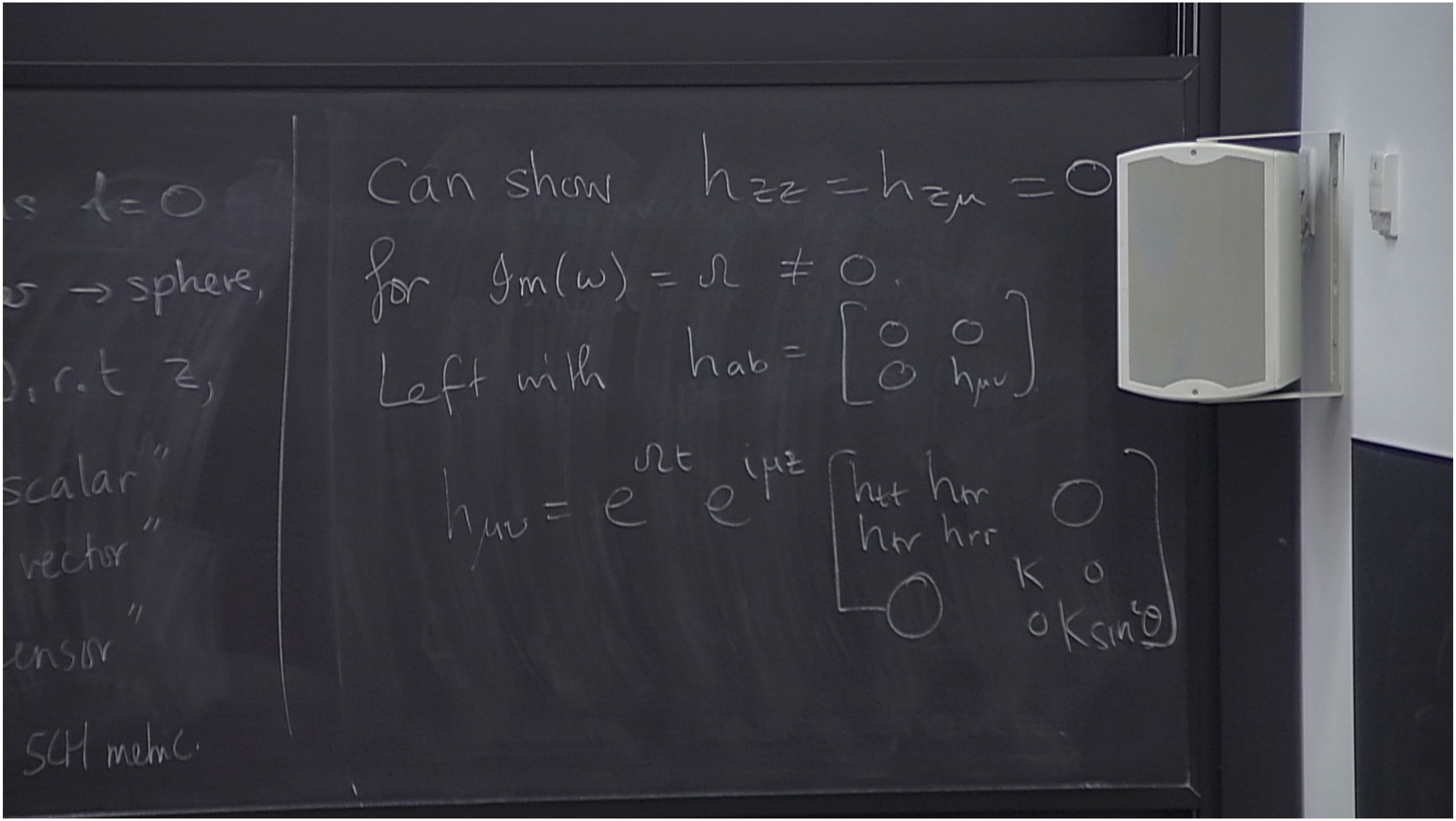
so expect $e^{i\mu z}$. W.r.t z,

have h_{zz} — "scalar"

$h_{z\mu}$ — "vector"

$h_{\mu\nu}$ — "tensor"

w.r.t 4D SCH metric.


$\ell = 0$
→ sphere,
r.t z,
scalar"
vector"
ensor"
SCH metric.
Can show $h_{zz} = h_{z\mu} = 0$
for $\Im(\omega) = \Omega \neq 0$.
Left with $h_{ab} = \begin{bmatrix} 0 & 0 \\ 0 & h_{\mu\nu} \end{bmatrix}$
$h_{\mu\nu} = e^{\Omega t} e^{i\mu z} \begin{bmatrix} h_{tt} & h_{tr} & 0 & 0 \\ h_{tr} & h_{rr} & 0 & 0 \\ 0 & 0 & K & 0 \\ 0 & 0 & 0 & K\sin^2\theta \end{bmatrix}$

$SO(3) \longleftrightarrow h_{ab}$ has specific angular momentum, $e^{im\phi}$ & l. $\longleftrightarrow Y_{lm}(\theta,\phi)$ for scalars

$h_{z\mu}$

$h_{\mu\nu}$ — "ten

w.r.t 4D SO

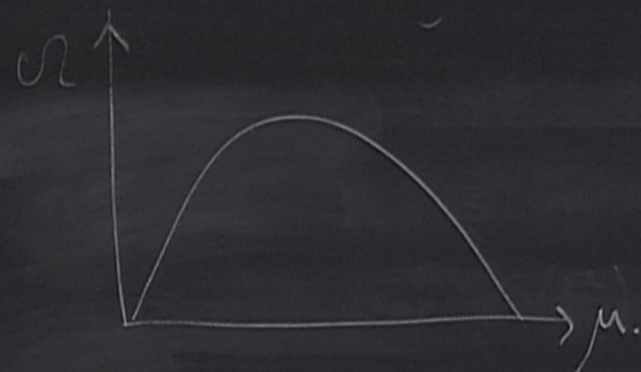
Because $h_{\mu\nu}$ depends on z,

cannot gauge away.

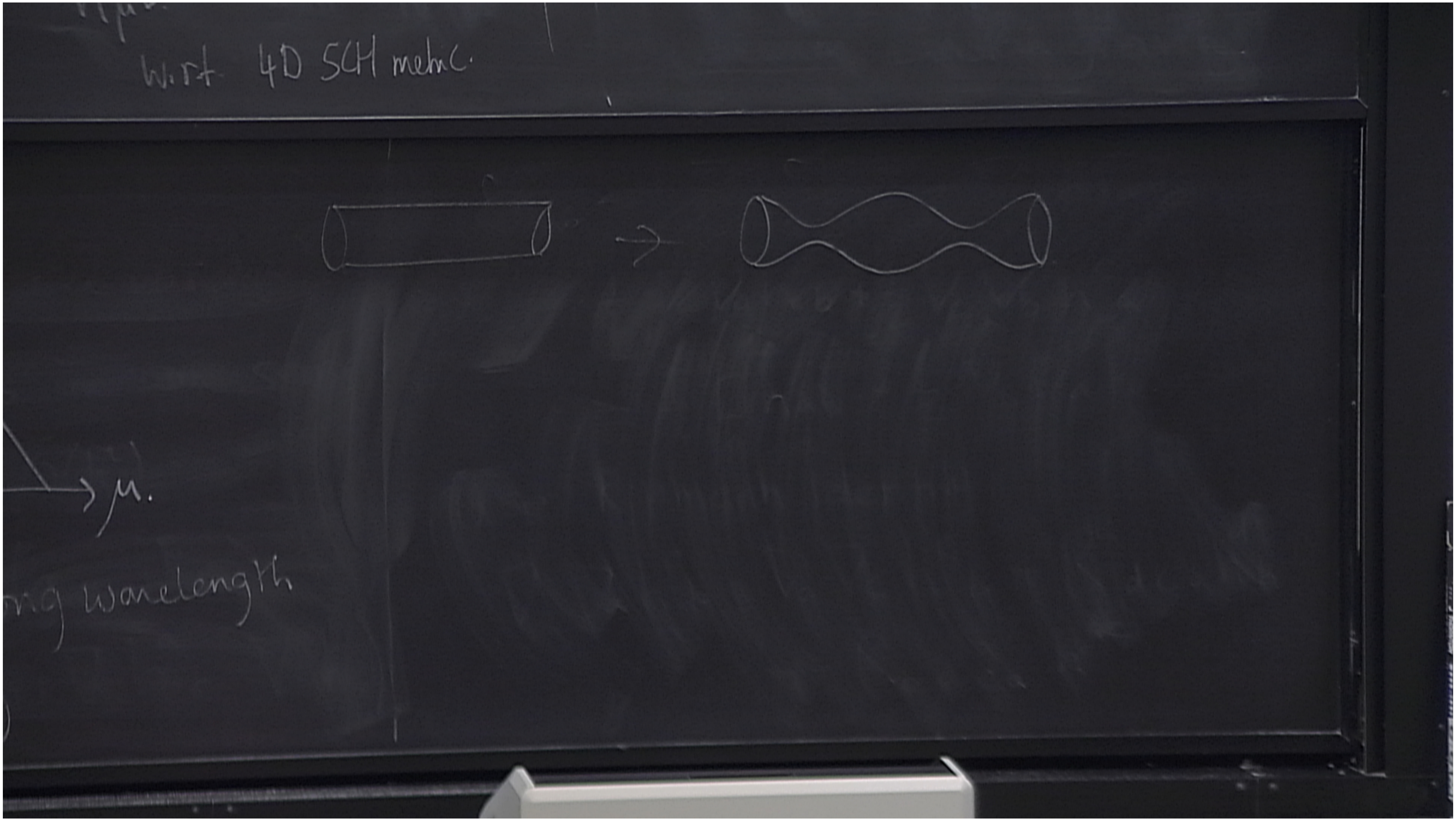
Lichnerowicz + gauge constraints

give 2nd order DE. in h_{tr}.

$$h''_{tr} + f(r, h'_{tr}, h_{tr}) = \mu^2 h_{tr}$$



Small μ. $\longleftrightarrow$ long wavelength instability


w.r.t 4D SCH metric.
μ.
wavelength