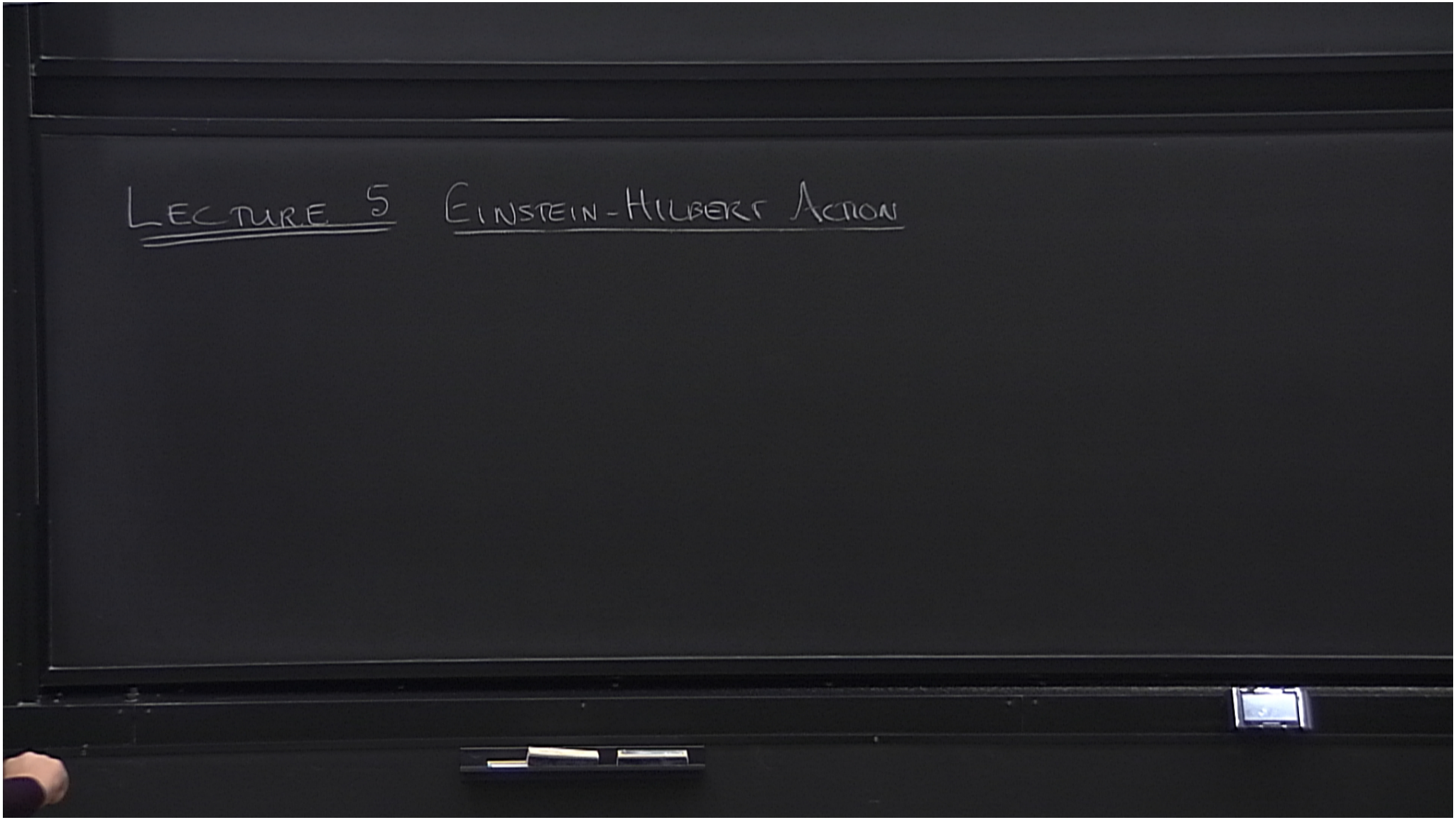
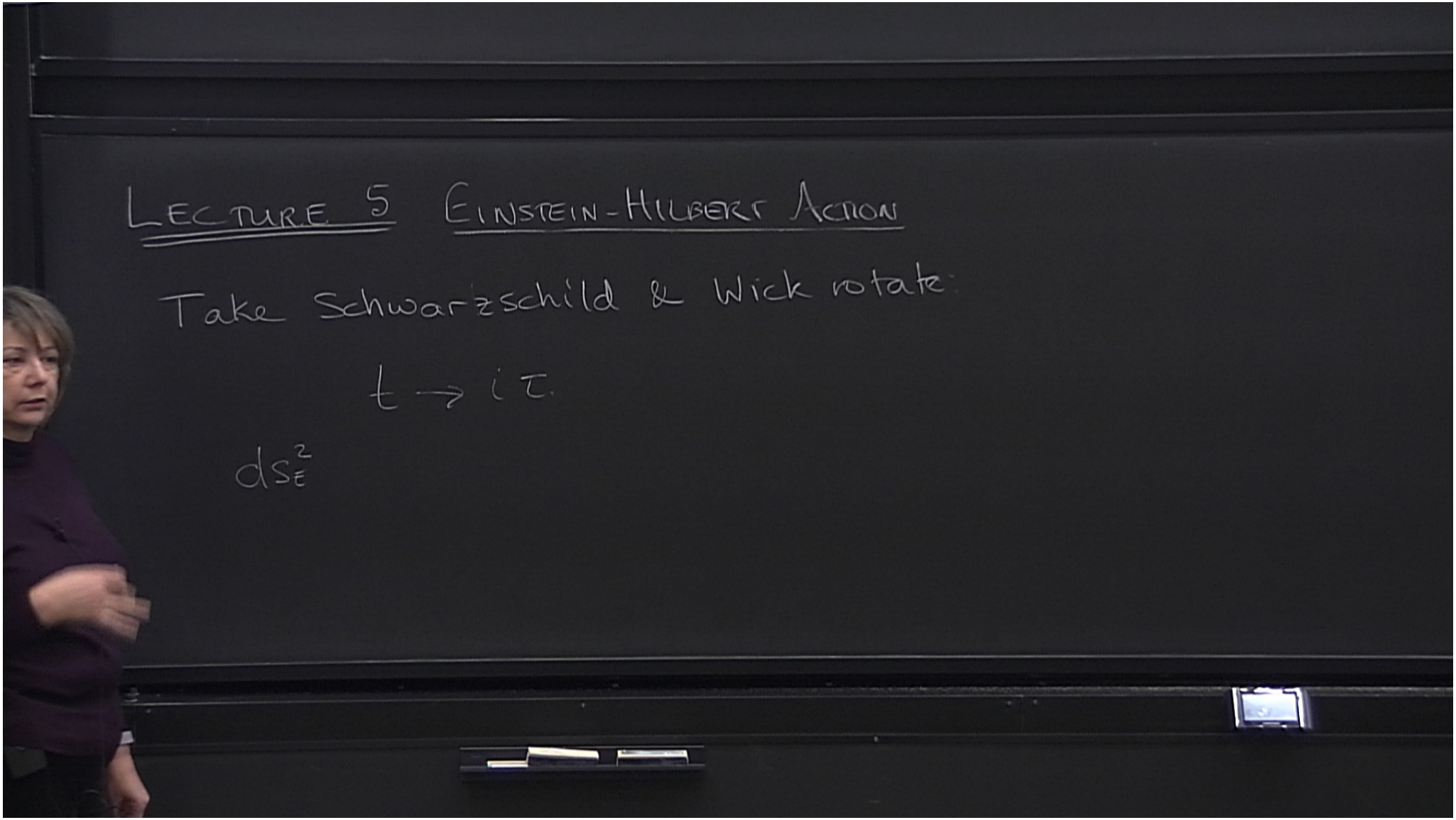
Title: PSI 17/18 - Gravitational Physics - Lecture 5

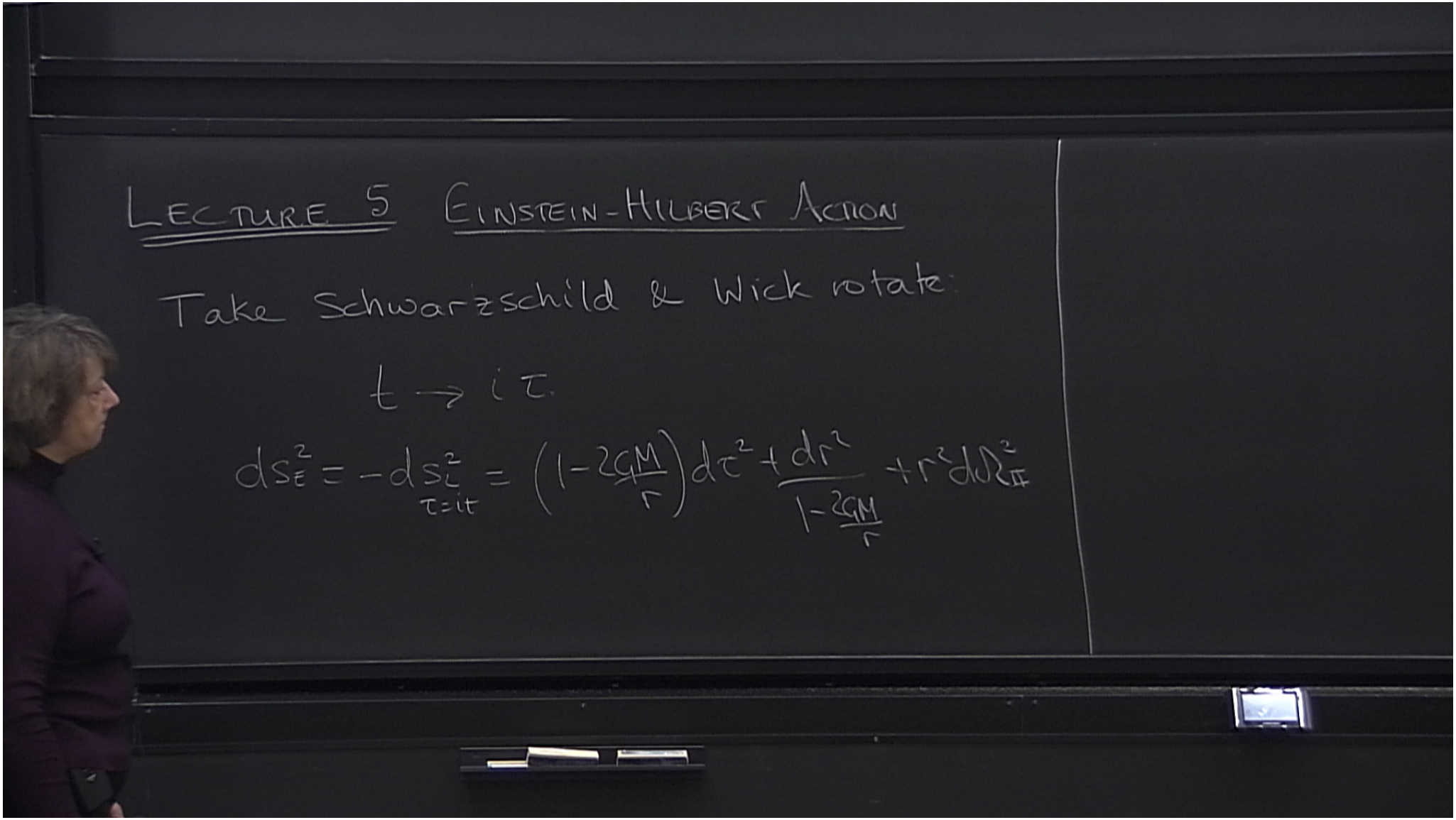
Date: Jan 09, 2018 10:15 AM

URL: http://pirsa.org/18010025

Abstract:


LECTURE 5 EINSTEIN-HILBERT ACTION


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Take Schwarzschild & Wick rotate:
$t \to i\tau$
ds_E^2


LECTURE 5 EINSTEIN-HILBERT ACTION
Take Schwarzschild & Wick rotate:
$t \to i\tau$
$ds_E^2 = -ds_L^2 \Big|_{\tau=it} = \left(1-\frac{2GM}{r}\right)d\tau^2 + \frac{dr^2}{1-\frac{2GM}{r}} + r^2 d\Omega_{II}^2$

Lecture 5 Einstein-Hilbert Action

Take Schwarzschild & Wick rotate:

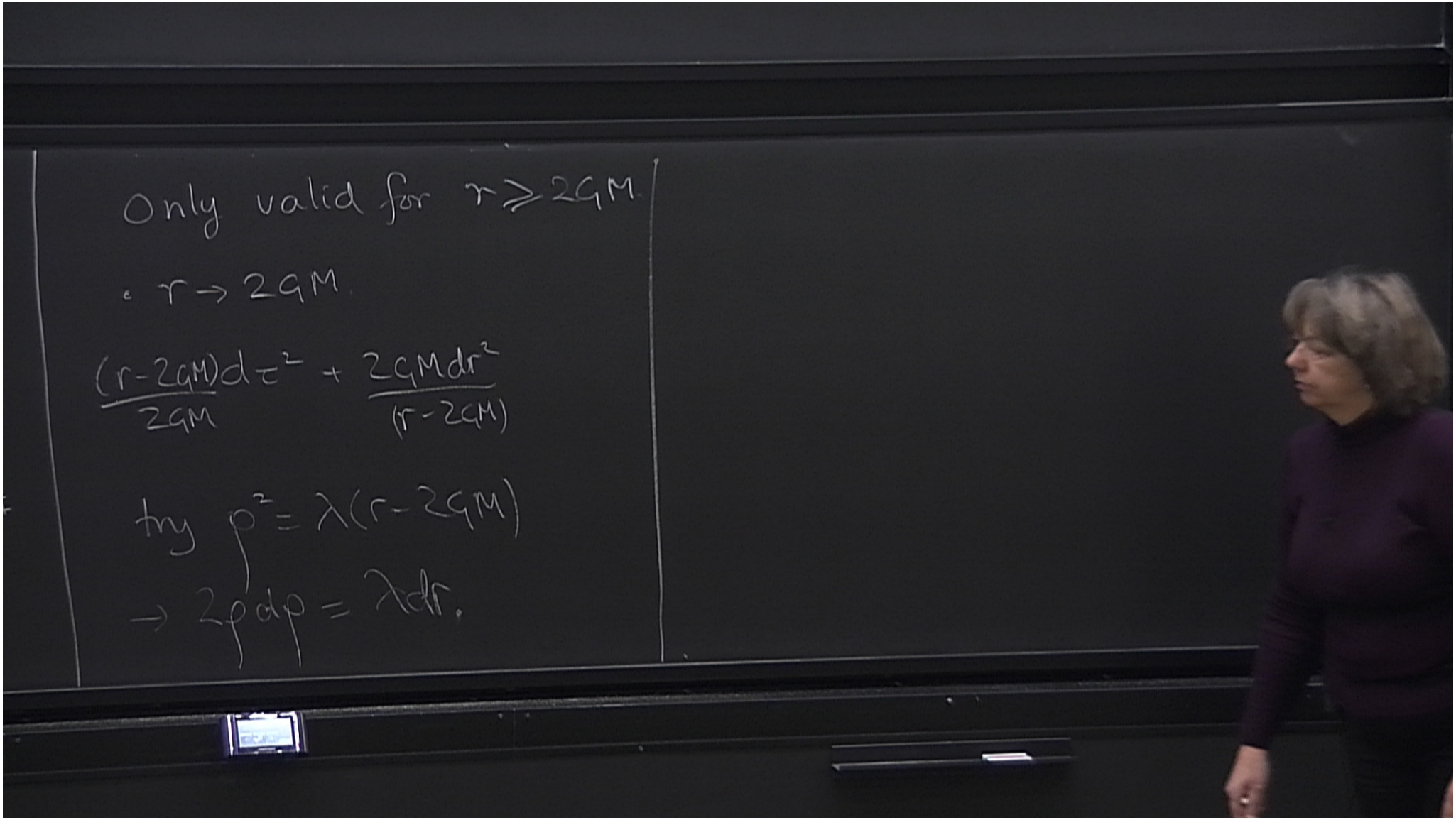
$$t \to i\tau$$

$$ds_E^2 = -ds_L^2\Big|_{\tau=it} = \left(1-\frac{2GM}{r}\right)d\tau^2 + \frac{dr^2}{1-\frac{2GM}{r}} + r^2 d\Omega_{II}^2$$

Only valid for $r \geq 2GM$

- $r \to 2GM$

$$\frac{(r-2GM)}{2GM}d\tau^2 + \frac{2GM\,dr^2}{(r-2GM)}$$


Only valid for $r \geq 2GM$
$r \to 2GM$
$\frac{(r-2GM)}{2GM} dt^2 + \frac{2GM dr^2}{(r-2GM)}$
try $\rho^2 = \lambda(r-2GM)$
$\to 2\rho d\rho = \lambda dr.$

only valid for $r \geq 2GM$.

$r \to 2GM$.

$\frac{-2GM}{2GM} dt^2 + \frac{2GM dr^2}{(r-2GM)}$

thy $\rho^2 = \lambda(r-2GM)$

$\to 2\rho d\rho = \lambda dr.$

$4\rho^2 d\rho^2 = 4\lambda(r-2GM)d\rho^2 = \lambda^2 dr^2$

$\Rightarrow \quad \frac{2GM\,dr^2}{r-2GM} = \frac{4}{\lambda} \cdot 2GM \cdot d\rho^2$

Set $\lambda = 8GM$

only valid for $r \geq 2GM$

$r \to 2GM$

$\frac{2GM)}{2GM} d\tau^2 + \frac{2GM dr^2}{(r-2GM)}$

$\rho^2 = \lambda(r-2GM)$

$2\rho d\rho = \lambda dr.$

$4\rho^2 d\rho^2 = 4\lambda(r-2GM)d\rho^2 = \lambda^2 dr^2$

$\Rightarrow \quad \frac{2GM\,dr^2}{r-2GM} = \frac{4}{\lambda} \cdot 2GM \cdot d\rho^2$

Set $\lambda = 8GM$, gives

$$d\rho^2 + \frac{\rho^2}{16G^2M^2} d\tau^2$$

Now take $\chi = \frac{r}{4GM}$, then

have $d\rho^2 + \rho^2 d\chi^2$

Now take $\chi = \frac{\tau}{4GM}$, then

have $d\rho^2 + \rho^2 d\chi^2$

– metric near origin in polars,

<u>provided</u> $\Delta\chi = 2\pi$

$\rightarrow \Delta\tau = \beta = 8\pi GM$

$\tau = it$ $\left(\quad\quad\right)$ $1 - \frac{2GM}{r}$

$\rightarrow 2\rho\, d\rho = \lambda\, dr.$

Now take $\chi = \frac{\tau}{4GM}$, then

have $d\rho^2 + \rho^2 d\chi^2$

– metric near origin in polars,

provided $\Delta\chi = 2\pi$

$\rightarrow \Delta\tau = \beta = 8\pi GM$

Periodic Euclidean time is indicative

$\rightarrow 2\rho d\rho = \lambda dr.$

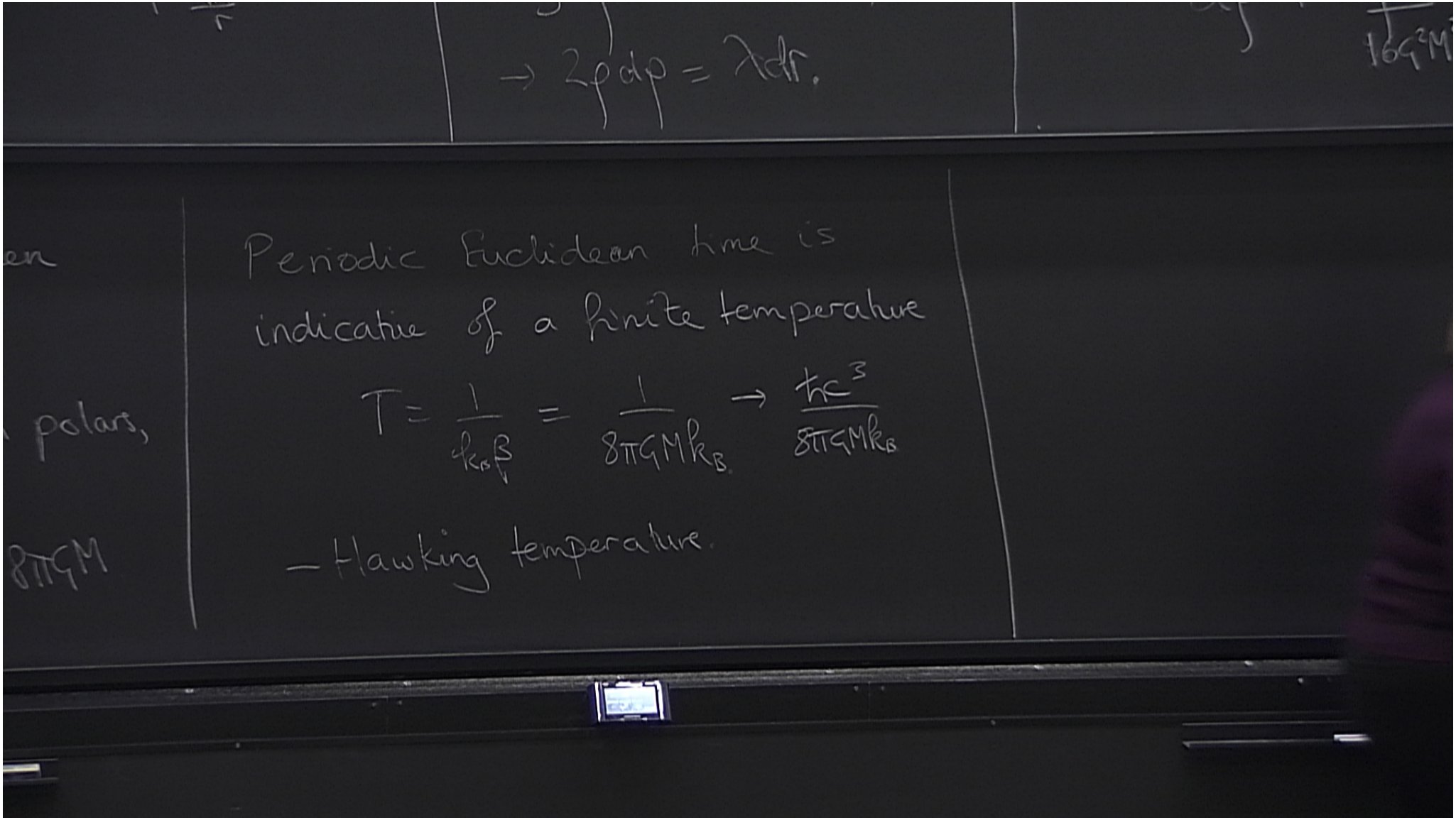
Periodic Euclidean time is indicative of a finite temperature

$$T = \frac{1}{k_B \beta} = \frac{1}{8\pi G M k_B}$$

$\rightarrow 2\rho d\rho = \lambda dr.$

Periodic Euclidean time is indicative of a finite temperature

$$T = \frac{1}{k_B \beta} = \frac{1}{8\pi G M k_B} \rightarrow \frac{\hbar c^3}{8\pi G M k_B}$$


$\rightarrow 2\rho d\rho = \lambda dr.$
Periodic Euclidean time is indicative of a finite temperature
$T = \frac{1}{k_B \beta} = \frac{1}{8\pi G M k_B} \rightarrow \frac{\hbar c^3}{8\pi G M k_B}$
– Hawking temperature.

$2\rho\, d\rho = \lambda\, dr.$

$$d\rho + \frac{\rho}{16G^2M^2}\, dr$$

Euclidean time is

a finite temperature

$$= \frac{1}{8\pi G M k_B} \rightarrow \frac{\hbar c^3}{8\pi G M k_B}$$

temperature.

For SCH

$$M = \frac{r_+}{2G}$$

$2\rho d\rho = \lambda dr.$

$$d\rho + \frac{\rho}{16G^2M^2}\, dr$$

...lidean time is

...nite temperature

$$\frac{1}{8\pi GMk_B} \rightarrow \frac{\hbar c^3}{8\pi GMk_B}$$

...ature.

For SCH

$$M = \frac{r_+}{2G}$$

in a dynamical process.

$$\delta M = \frac{\delta r_+}{2G}$$

$2\rho d\rho = \lambda dr.$

$d\rho + \frac{\rho}{16G^2M^2} dr$

clidean time is

a finite temperature

$= \frac{1}{8\pi GMk_B} \rightarrow \frac{\hbar c^3}{8\pi GMk_B}$

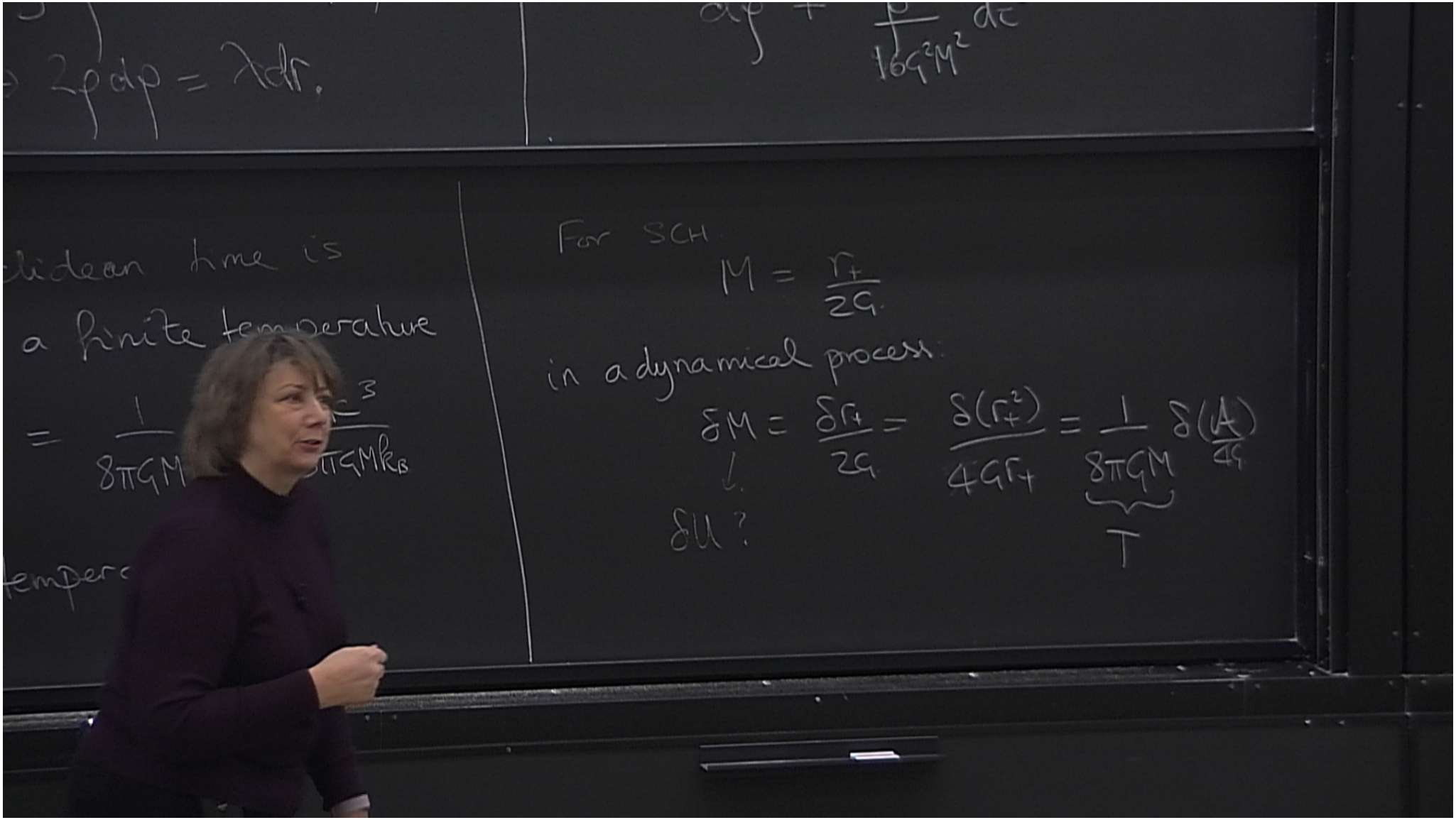
perature.

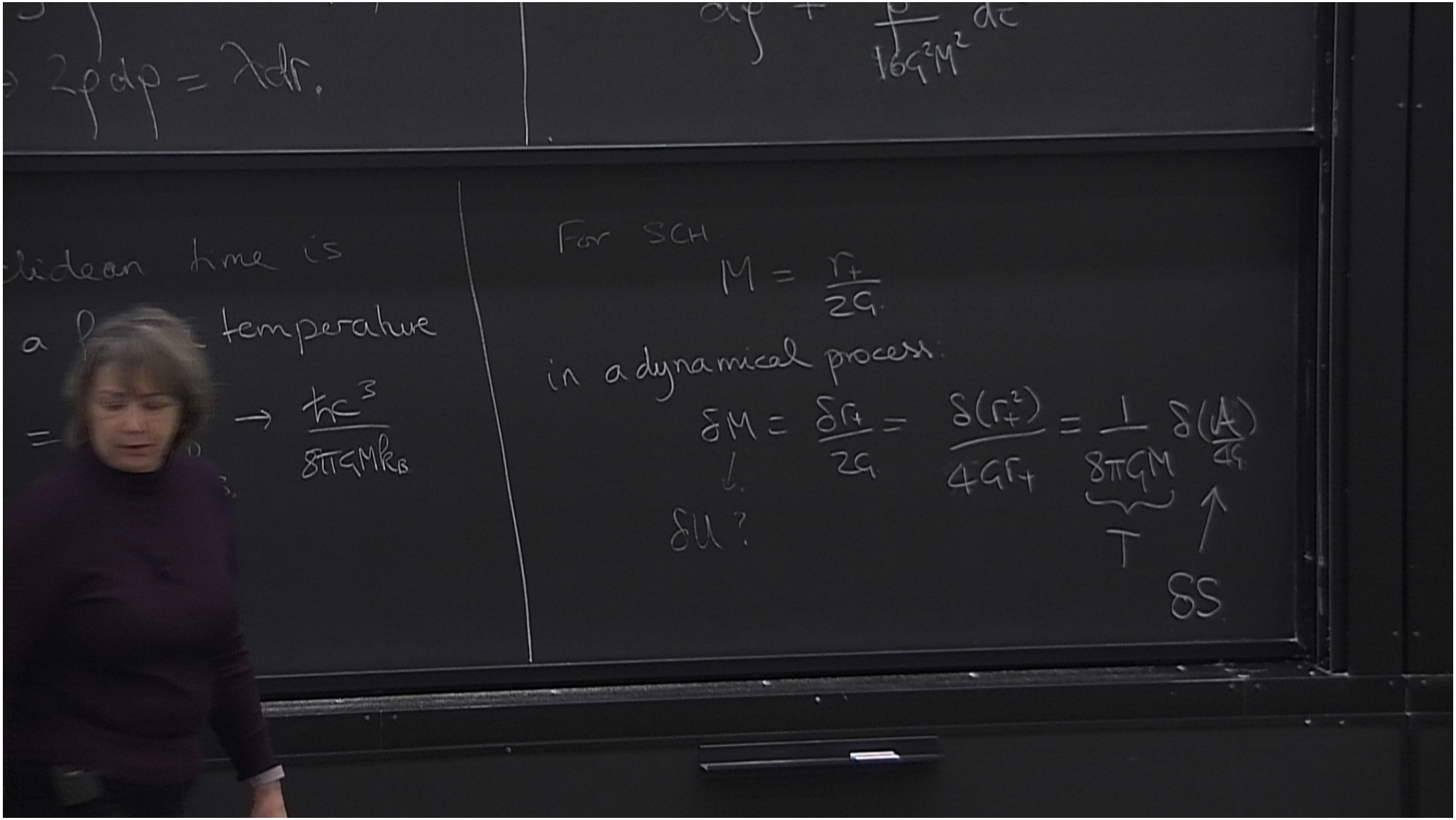
For SCH

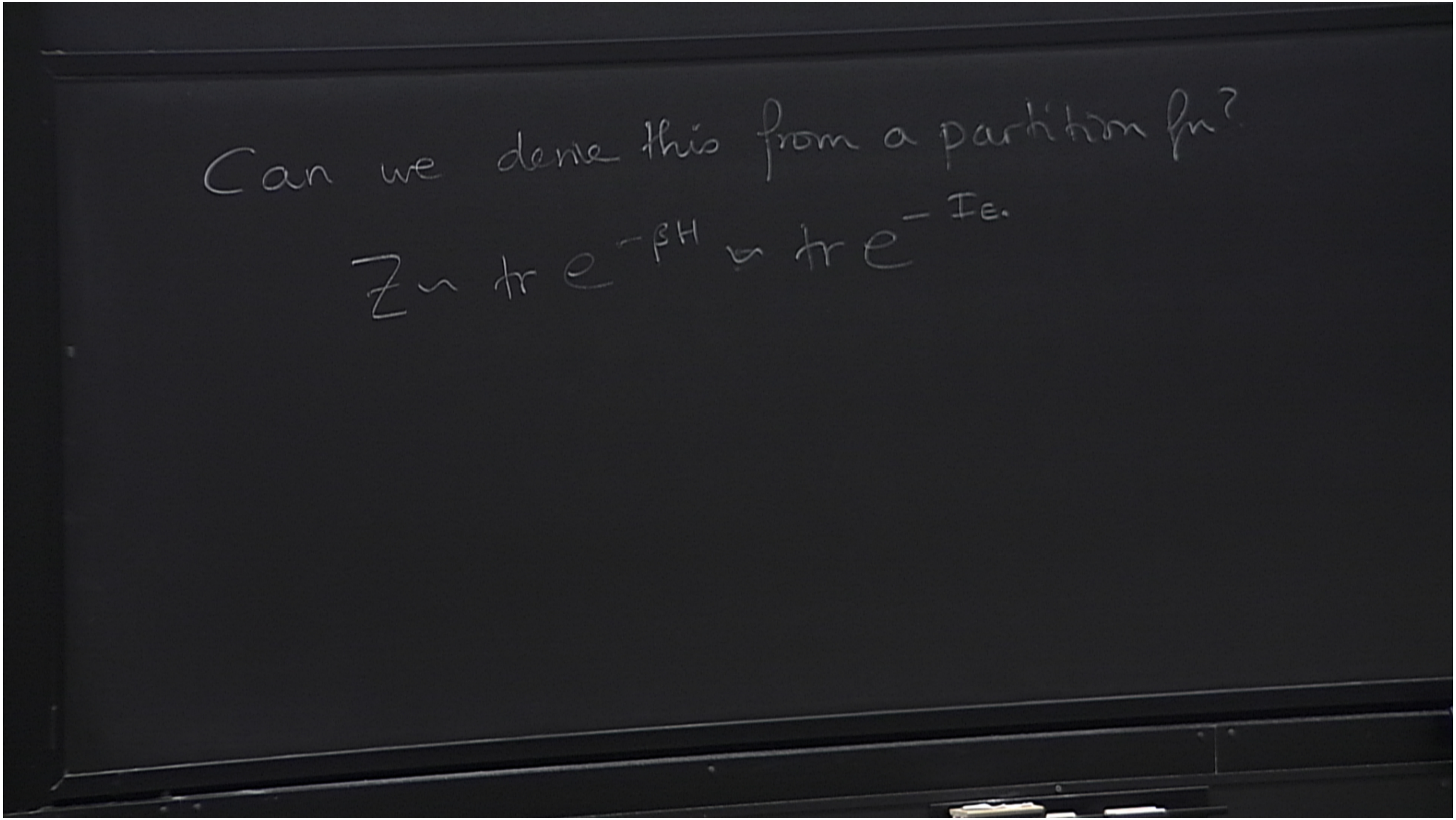
$M = \frac{r_+}{2G}$

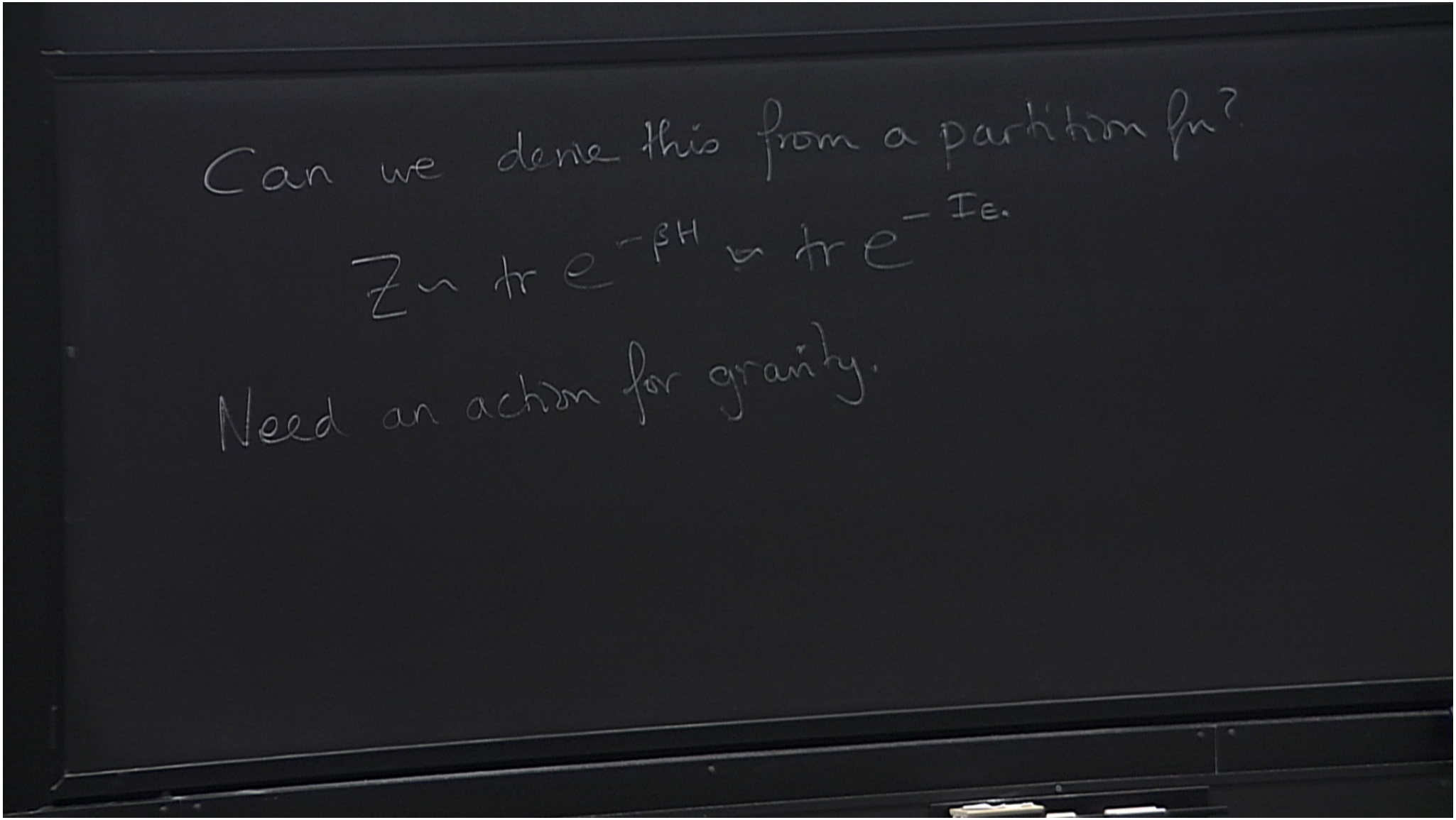
in a dynamical process

$\delta M = \frac{\delta r_+}{2G} = \frac{\delta(r_+^2)}{4Gr_+} = \frac{1}{8\pi GM} \delta\left(\frac{A}{4G}\right)$


$2\rho d\rho = \lambda dr.$
$\frac{1}{16G^2M^2} dt$
Euclidean time is
a finite temperature
$= \frac{1}{8\pi GM}$
$8\pi GMk_B$
For SCH
$M = \frac{r_+}{2G}$
in a dynamical process:
$\delta M = \frac{\delta r_+}{2G} = \frac{\delta(r_+^2)}{4Gr_+} = \frac{1}{8\pi GM} \delta\left(\frac{A}{4G}\right)$
δU ?
T


$2\rho d\rho = \lambda dr.$
$\frac{1}{16G^2M^2} d\tau$
Euclidean time is
a
temperature
$\rightarrow \frac{\hbar c^3}{8\pi G M k_B}$
For SCH
$M = \frac{r_+}{2G}$
in a dynamical process
$\delta M = \frac{\delta r_+}{2G} = \frac{\delta(r_+^2)}{4Gr_+} = \frac{1}{8\pi GM} \delta\left(\frac{A}{4G}\right)$
δU ?
T
δS


Can we derive this from a partition fn?
$Z \sim \mathrm{tr}\, e^{-\beta H} \sim \mathrm{tr}\, e^{-I_E}$

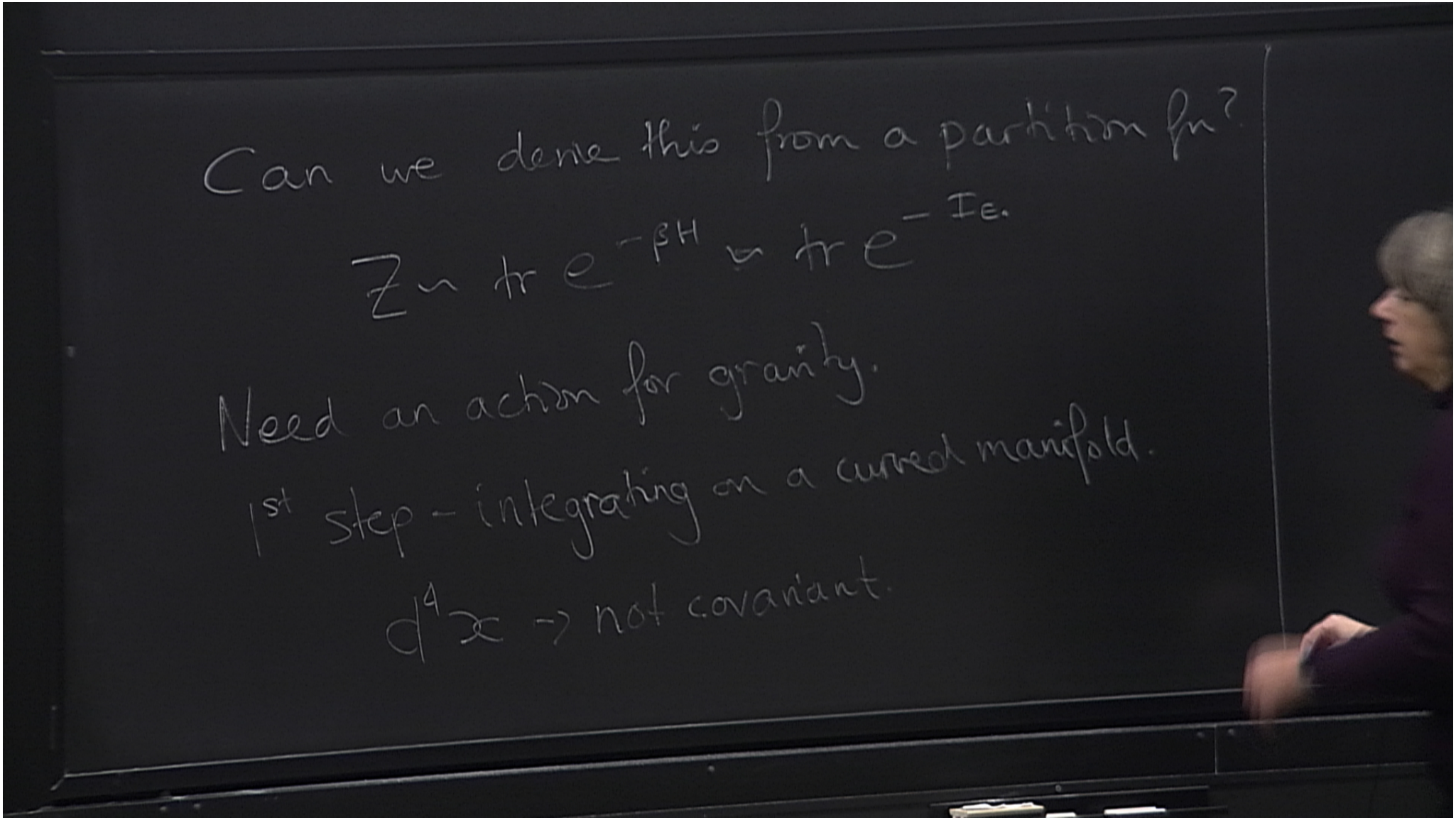

Can we derive this from a partition fn?
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Need an action for gravity.

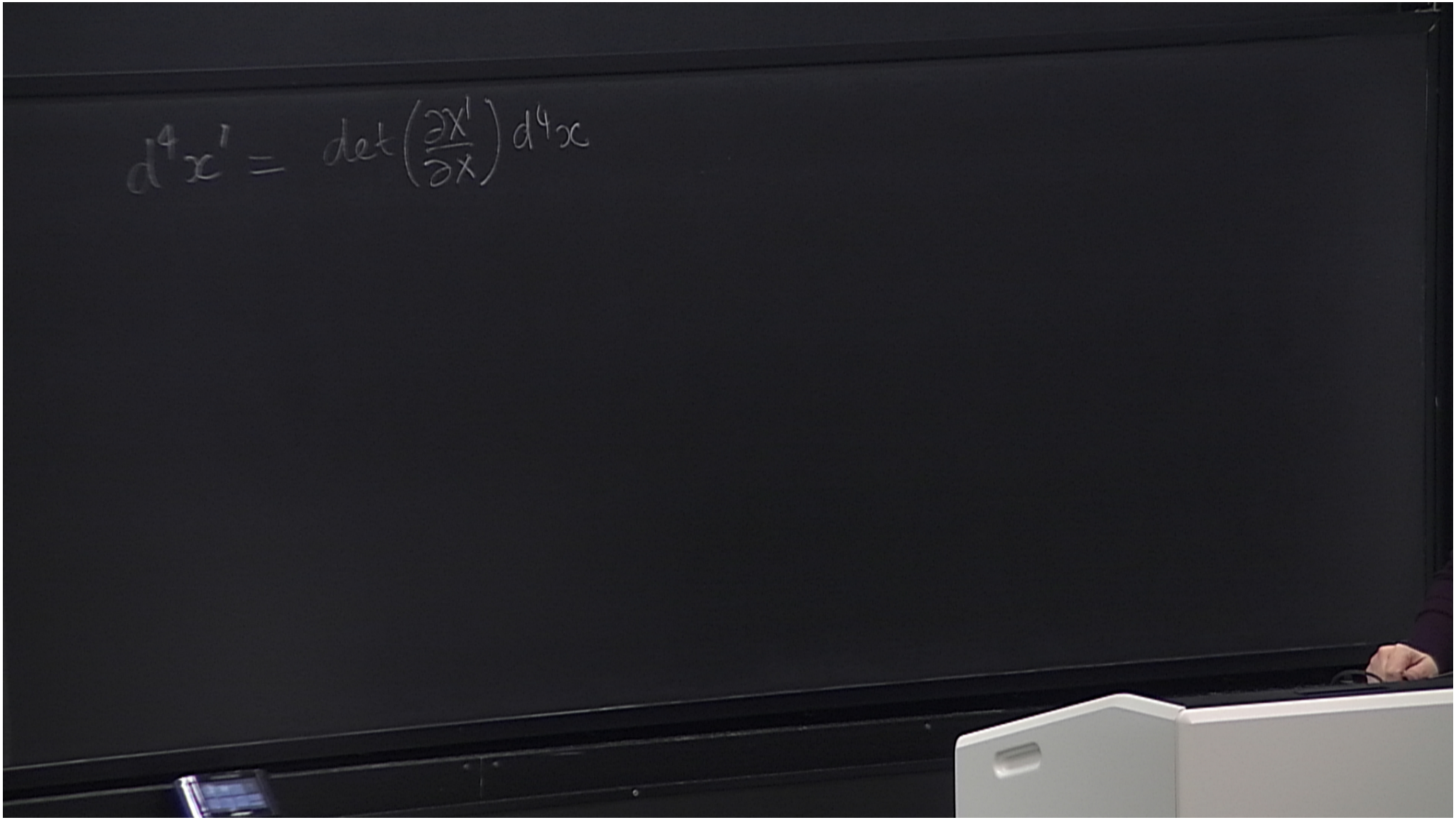
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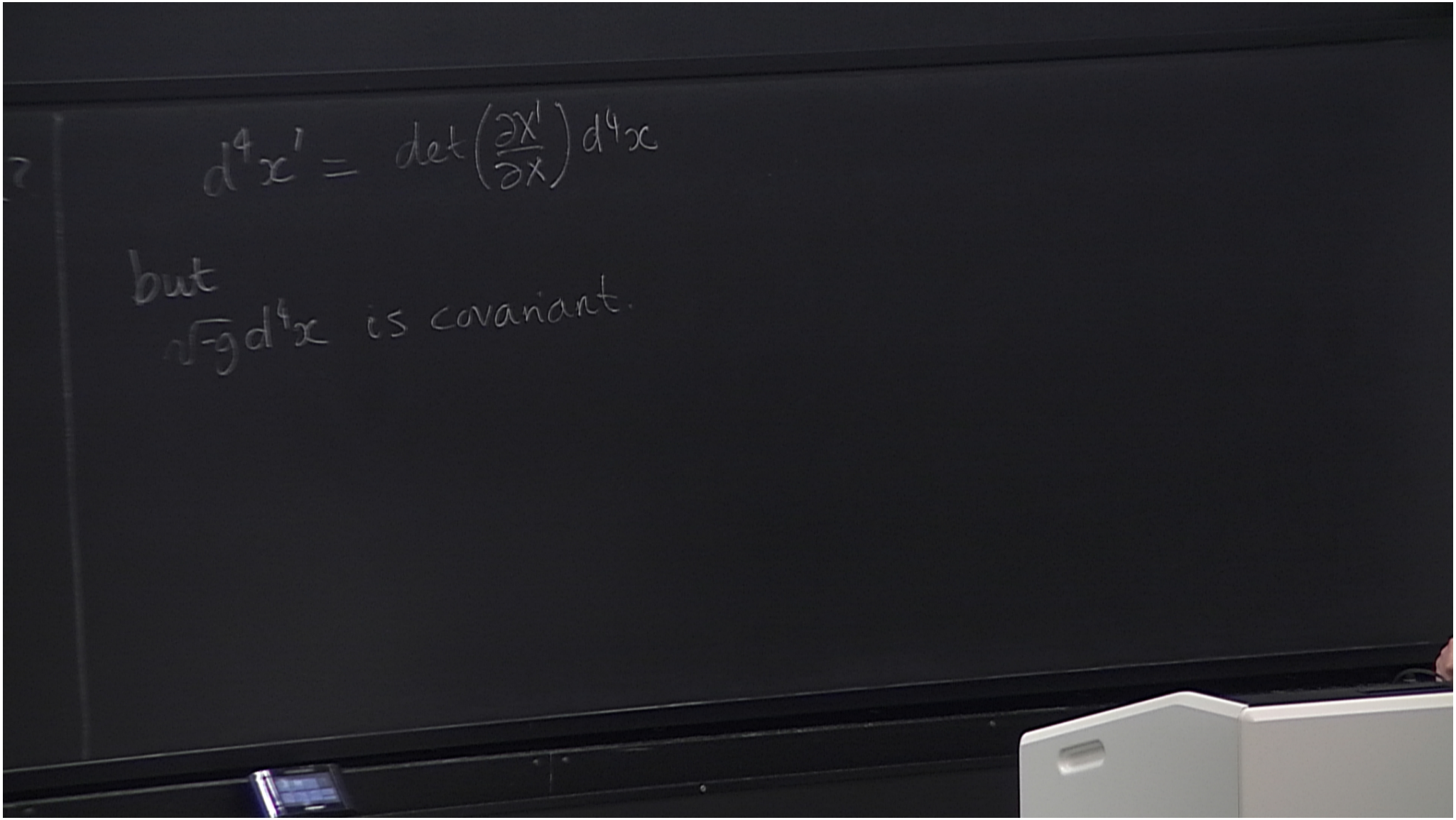
$$Z \sim \mathrm{tr}\, e^{-\beta H} \sim \mathrm{tr}\, e^{-I_E}$$

Need an action for gravity.

1st step - integrating on a curved manifold.


Can we derive this from a partition fn?
$Z \sim \mathrm{tr}\, e^{-\beta H} \sim \mathrm{tr}\, e^{-I_E}$
Need an action for gravity.
1st step - integrating on a curved manifold.
d^4x -> not covariant.


$d^4x' = \det\left(\frac{\partial x'}{\partial x}\right) d^4x$


$d^4x' = \det\left(\frac{\partial x'}{\partial x}\right) d^4x$
but
$\sqrt{-g}\, d^4x$ is covariant.

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Physical volume in coord

box dt, dx, dy, dz



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Physical volume in coord box dt, dx, dy, dz



In GR, the metric is our physical field, so will vary any action w.r.t. g.

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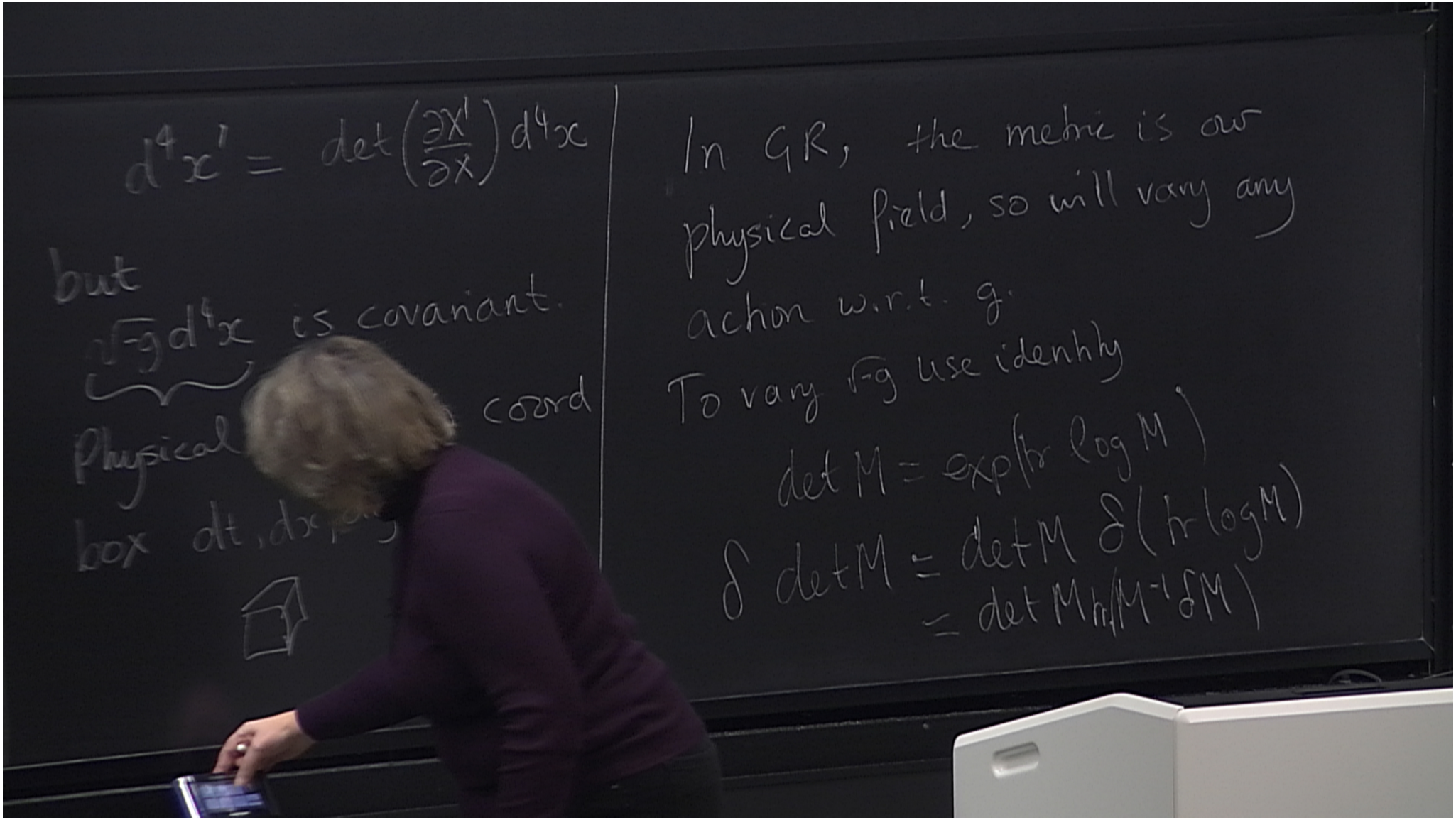
Physical volume in coord box dt, dx, dy, dz



In GR, the metric is our physical field, so will vary any action w.r.t. g.

To vary $\sqrt{-g}$ use identity

$$\det M = \exp(\mathrm{tr}\log M)$$


$d^4x' = \det\left(\frac{\partial x'}{\partial x}\right) d^4x$
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Physical
coord
box dt, dx,
In GR, the metric is our physical field, so will vary any action w.r.t. g.
To vary $\sqrt{-g}$ use identity
$\det M = \exp(\mathrm{tr}\log M)$
$\delta \det M = \det M\, \delta(\mathrm{tr}\log M)$
$= \det M \,\mathrm{tr}(M^{-1}\delta M)$

$$\delta\sqrt{-g} = \frac{1}{2}\sqrt{-g}\, g^{ab}\delta g_{ab}$$

$$= -\frac{1}{2}\sqrt{-g}\, g_{ab}\delta g^{ab}$$

$$\delta\sqrt{-g} = \frac{1}{2}\sqrt{-g}\, g^{ab}\delta g_{ab}$$

$$= -\frac{1}{2}\sqrt{-g}\, g_{ab}\delta g^{ab}$$

e.g. massless scalar field

$$\mathcal{L}_\phi = \frac{1}{2}(\partial\phi)^2 = \frac{1}{2} g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

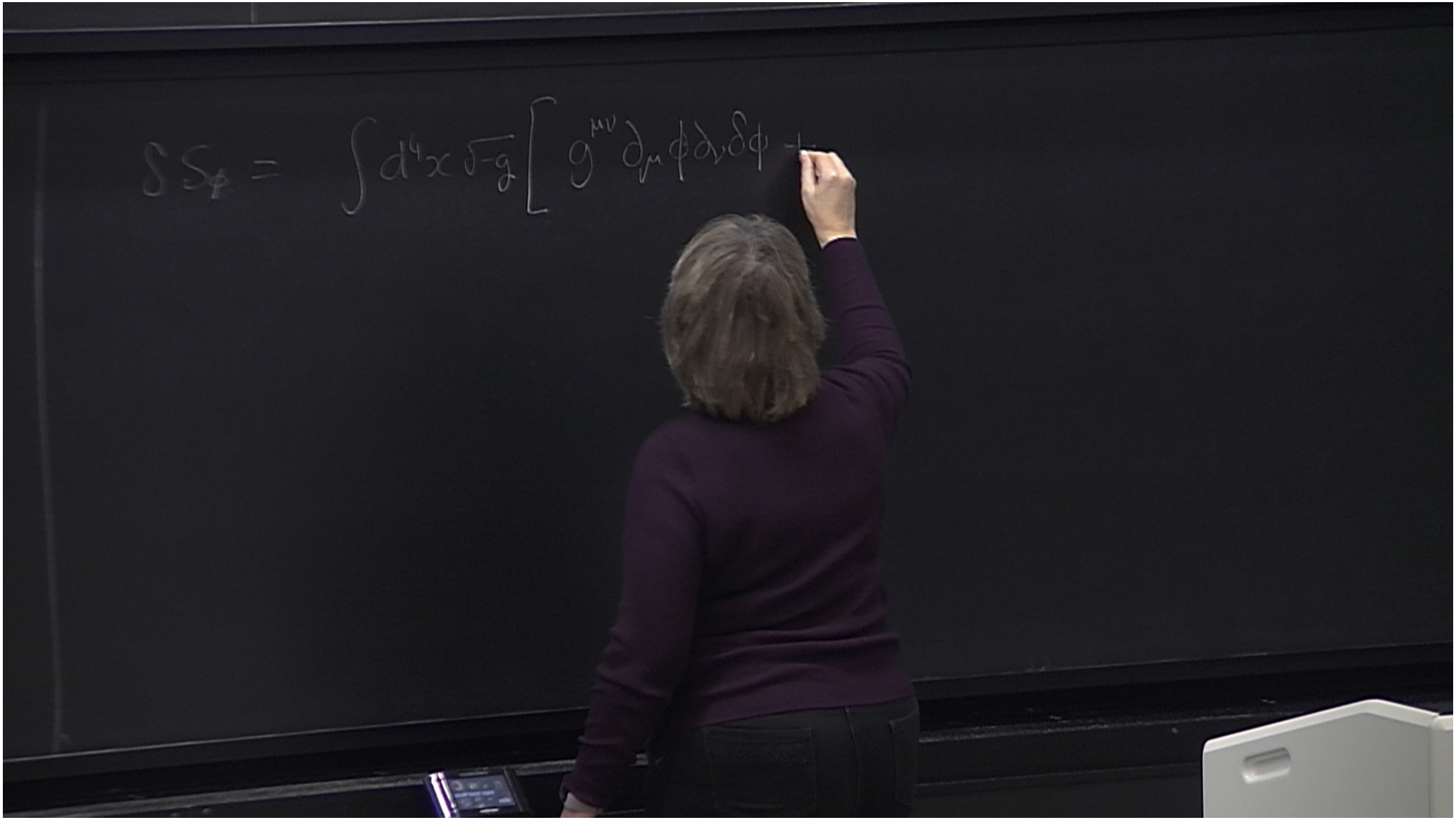
$$\delta\sqrt{-g} = \frac{1}{2}\sqrt{-g}\, g^{ab}\delta g_{ab}$$

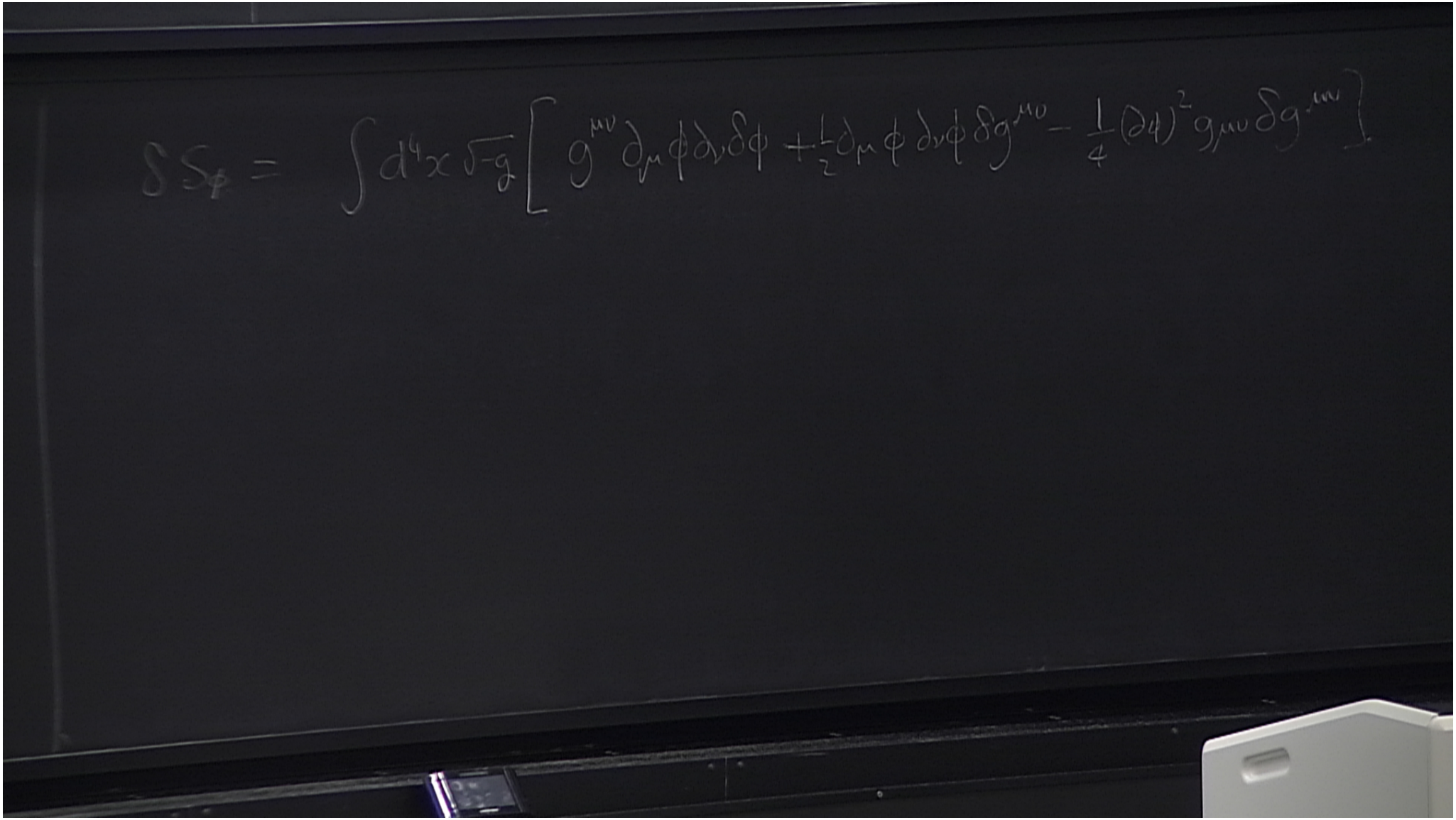
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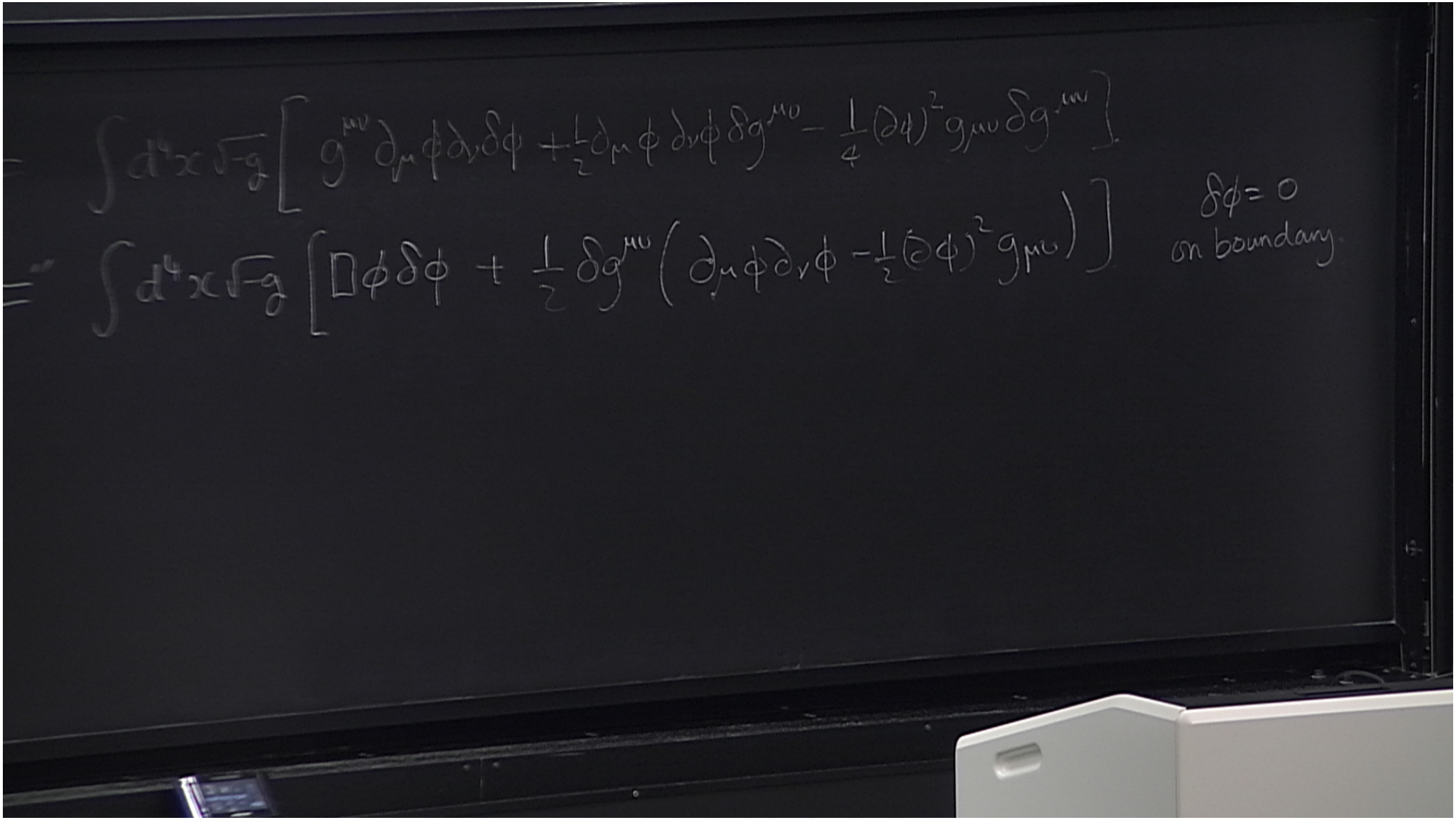
$$S_\phi = \int d^4x \sqrt{g}\, \mathcal{L}_\phi$$


$\delta S_\phi = \int d^4x \sqrt{-g} \left[g^{\mu\nu} \partial_\mu \phi \partial_\nu \delta\phi + \right.$


$\delta S_\phi = \int d^4x \sqrt{-g} \left[g^{\mu\nu} \partial_\mu \phi \partial_\nu \delta\phi + \frac{1}{2} \partial_\mu \phi \partial_\nu \phi \delta g^{\mu\nu} - \frac{1}{4} (\partial\phi)^2 g_{\mu\nu} \delta g^{\mu\nu} \right]$

$$\delta S_\phi = \int d^4x \sqrt{-g} \left[g^{\mu\nu} \partial_\mu \phi \partial_\nu \delta\phi + \frac{1}{2} \partial_\mu \phi \partial_\nu \phi \delta g^{\mu\nu} - \frac{1}{4} (\partial\phi)^2 g_{\mu\nu} \delta g^{\mu\nu} \right]$$

$$= \int d^4x \sqrt{-g} \left[\Box\phi \delta\phi + \frac{1}{2} \delta g^{\mu\nu} \left(\partial_\mu \phi \partial_\nu \phi - \frac{1}{2} (\partial\phi)^2 g_{\mu\nu} \right) \right]$$


$= \int d^4x \sqrt{-g} \left[g^{\mu\nu} \partial_\mu \phi \partial_\nu \delta\phi + \frac{1}{2} \partial_\mu \phi \partial_\nu \phi \delta g^{\mu\nu} - \frac{1}{4} (\partial\phi)^2 g_{\mu\nu} \delta g^{\mu\nu} \right]$
$= \int d^4x \sqrt{-g} \left[\Box\phi \delta\phi + \frac{1}{2} \delta g^{\mu\nu} \left(\partial_\mu \phi \partial_\nu \phi - \frac{1}{2} (\partial\phi)^2 g_{\mu\nu} \right) \right]$
$\delta\phi = 0$
on boundary.

$$\delta S_\phi = \int d^4x \sqrt{-g} \left[g^{\mu\nu} \partial_\mu \phi \partial_\nu \delta\phi + \frac{1}{2} \partial_\mu \phi \partial_\nu \phi \delta g^{\mu\nu} - \frac{1}{4} (\partial\phi)^2 g_{\mu\nu} \delta g^{\mu\nu} \right]$$

$$\text{"}=\text{"} \int d^4x \sqrt{-g} \left[\Box\phi \delta\phi + \frac{1}{2} \delta g^{\mu\nu} \left(\partial_\mu \phi \partial_\nu \phi - \frac{1}{2} (\partial\phi)^2 g_{\mu\nu} \right) \right]$$

$\delta\phi = 0$ on boundary.

$$= \int d^4x \sqrt{-g} \left[\frac{\delta S}{\delta \phi} \delta\phi + \frac{\delta S}{\delta g^{\mu\nu}} \delta g^{\mu\nu} \right]$$

$$\delta S_\phi = \int d^4x \sqrt{-g} \left[g^{\mu\nu} \partial_\mu \phi \partial_\nu \delta\phi + \frac{1}{2} \partial_\mu \phi \partial_\nu \phi \delta g^{\mu\nu} - \frac{1}{4} (\partial\phi)^2 g_{\mu\nu} \delta g^{\mu\nu} \right]$$

$$\text{"}=\text{"} \int d^4x \sqrt{-g} \left[\Box\phi \delta\phi + \frac{1}{2} \delta g^{\mu\nu} \left(\partial_\mu \phi \partial_\nu \phi - \frac{1}{2} (\partial\phi)^2 g_{\mu\nu} \right) \right]$$

$\delta\phi = 0$ on boundary.

$$= \int d^4x \sqrt{-g} \left[\frac{\delta S}{\delta \phi} \delta\phi + \frac{\delta S}{\delta g^{\mu\nu}} \delta g^{\mu\nu} \right]$$

$= \det M_{ij} M^{-1} \delta M$

$$\delta S_\phi = \int d^4x \sqrt{-g} \left[g^{\mu\nu} \partial_\mu \phi \partial_\nu \delta\phi + \tfrac{1}{2} \partial_\mu \phi \, \partial_\nu \phi \, \delta g^{\mu\nu} - \tfrac{1}{4} (\partial\phi)^2 g_{\mu\nu} \delta g^{\mu\nu} \right]$$

$$\text{"}=\text{"} \int d^4x \sqrt{-g} \left[\Box\phi \delta\phi + \frac{1}{2} \delta g^{\mu\nu} \left(\partial_\mu \phi \partial_\nu \phi - \frac{1}{2} (\partial\phi)^2 g_{\mu\nu} \right) \right] \qquad \delta\phi = 0 \text{ on boundary}$$

$$= \int d^4x \sqrt{-g} \left[\frac{\delta S}{\delta \phi} \delta\phi + \frac{\delta S}{\delta g^{\mu\nu}} \delta g^{\mu\nu} \right]$$

$$\frac{\delta S}{\delta \phi} = 0 \Rightarrow \Box\phi = 0.$$

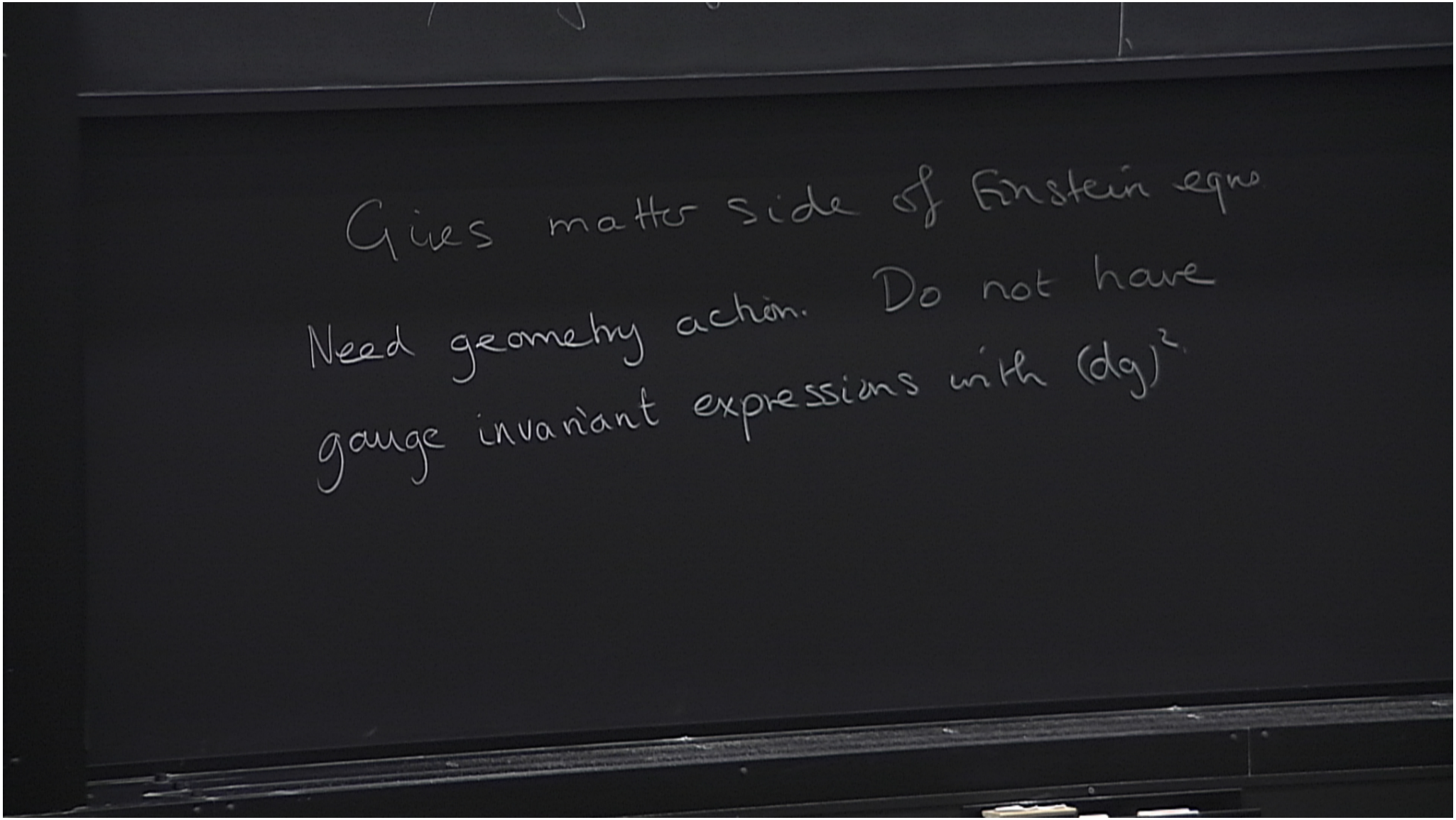
$$\frac{2\delta S}{\delta g^{\mu\nu}} = \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} (\partial\phi)^2 g_{\mu\nu}$$

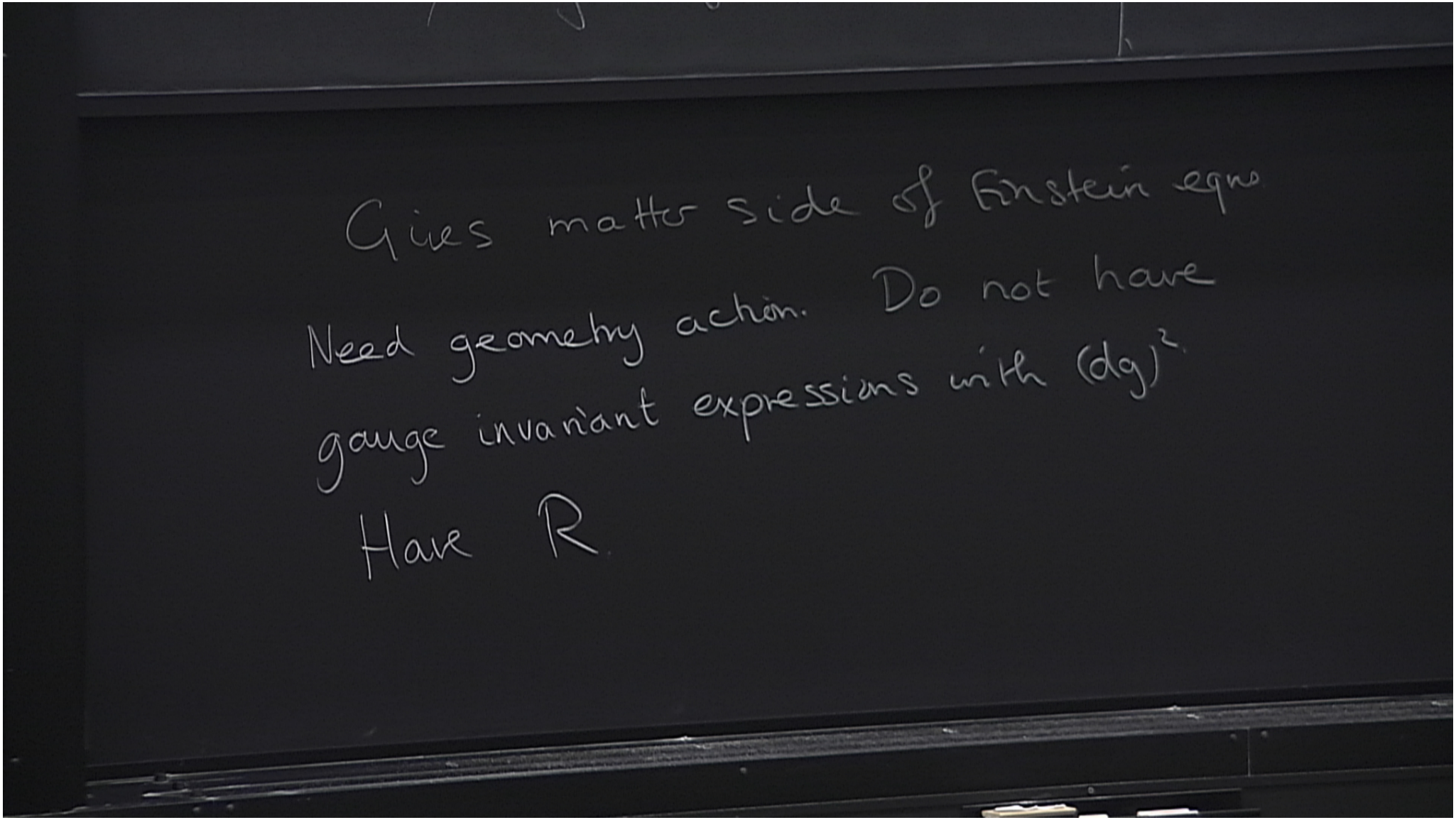
$$= T_{\phi \mu\nu}$$

$S_\phi = \int d^4x \sqrt{-g}\, \mathcal{L}_\phi$

Gives matter side of Einstein eqns.

Need geometry action.

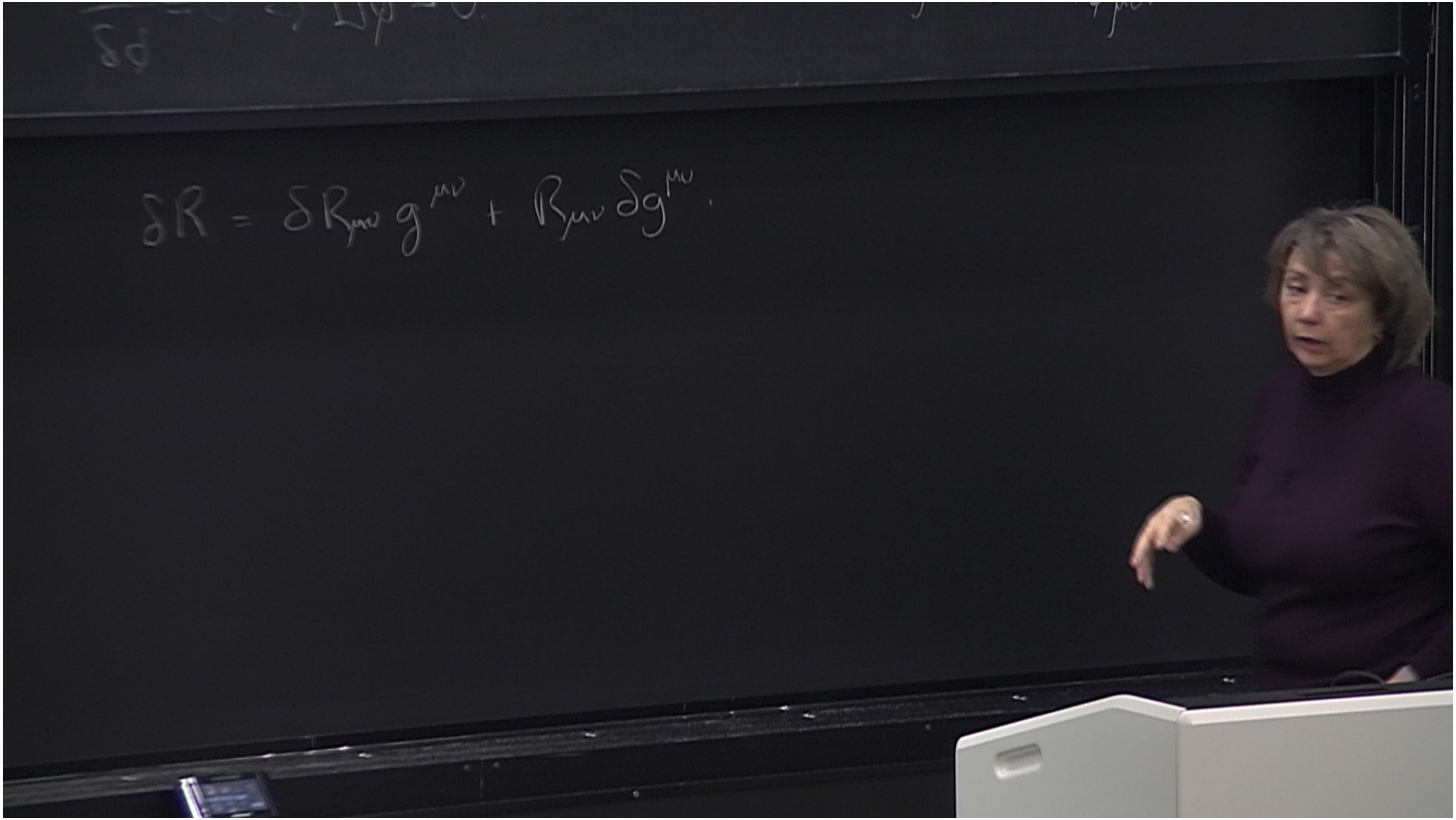

Gives matter side of Einstein eqns
Need geometry action. Do not have
gauge invariant expressions with $(dg)^2$.

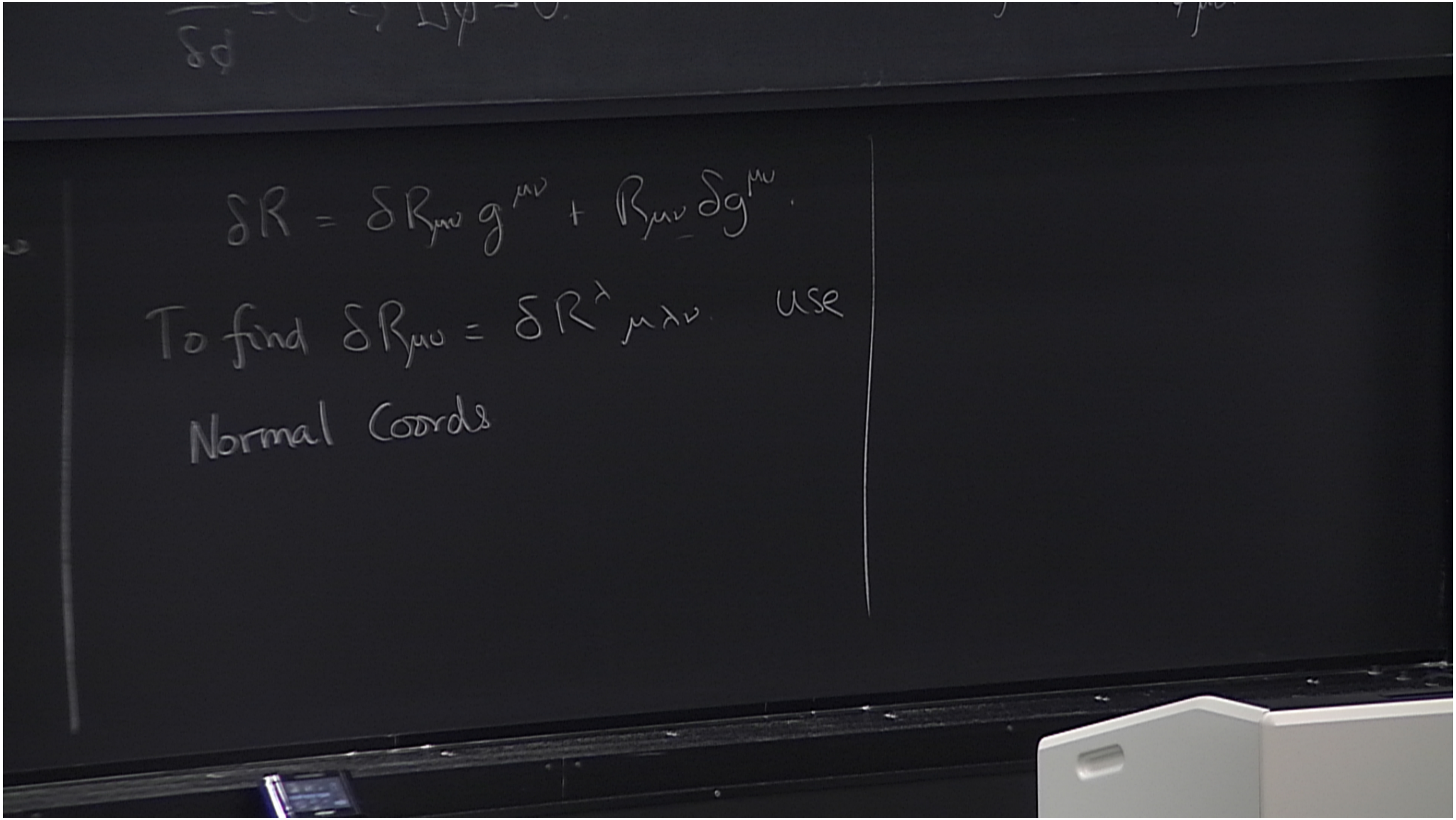

Gives matter side of Einstein eqn
Need geometry action. Do not have
gauge invariant expressions with $(dg)^2$.
Have R

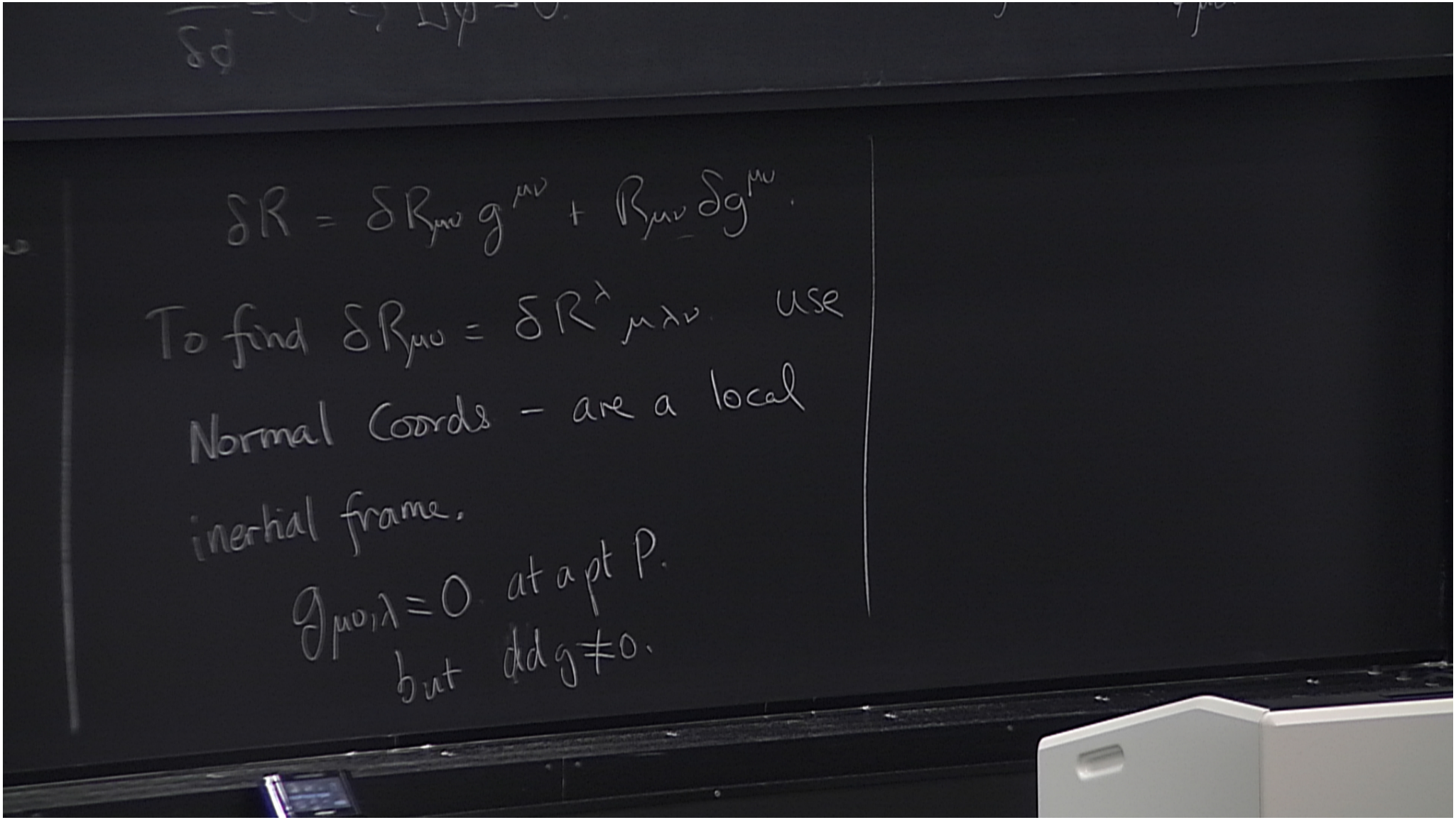
Gives matter side of Einstein eqns.

Need geometry action. Do not have gauge invariant expressions with $(dg)^2$.

Have $R = R_{\mu\nu} g^{\mu\nu}$.


$\delta R = \delta R_{\mu\nu} g^{\mu\nu} + R_{\mu\nu} \delta g^{\mu\nu}$.


$\delta R = \delta R_{\mu\nu} g^{\mu\nu} + R_{\mu\nu} \delta g^{\mu\nu}$.
To find $\delta R_{\mu\nu} = \delta R^{\lambda}{}_{\mu\lambda\nu}$. use
Normal Coords


$\delta R = \delta R_{\mu\nu} g^{\mu\nu} + R_{\mu\nu} \delta g^{\mu\nu}$.
To find $\delta R_{\mu\nu} = \delta R^{\lambda}{}_{\mu\lambda\nu}$ use
Normal Coords - are a local
inertial frame.
$g_{\mu\nu,\lambda} = 0$ at a pt P.
but $ddg \neq 0$.

$\delta R = \delta R_{\mu\nu} g^{\mu\nu} + R_{\mu\nu} \delta g^{\mu\nu}$.

To find $\delta R_{\mu\nu} = \delta R^{\lambda}{}_{\mu\lambda\nu}$ use Normal Coords - are a local inertial frame.

$g_{\mu\nu,\lambda} = 0$ at a pt P.

but $d d g \neq 0$.

In N.C.

$R^{\lambda}{}_{\mu\lambda\nu} = \Gamma^{\lambda}_{\mu\nu,\lambda} - \Gamma^{\lambda}_{\mu\nu,\lambda}$

$\delta R = \delta R_{\mu\nu} g^{\mu\nu} + R_{\mu\nu} \delta g^{\mu\nu}$.

To find $\delta R_{\mu\nu} = \delta R^{\lambda}{}_{\mu\lambda\nu}$ use Normal Coords - are a local inertial frame.

$g_{\mu\nu,\lambda} = 0$ at a pt P.

but $ddg \neq 0$.

In N.C. $\nabla = \partial$

$\delta R^{\lambda}{}_{\mu\lambda\nu} = \delta\Gamma^{\lambda}_{\mu\nu,\lambda} - \delta\Gamma^{\lambda}_{\mu\lambda,\nu}$

$\delta R = \delta R_{\mu\nu} g^{\mu\nu} + R_{\mu\nu} \delta g^{\mu\nu}$.

To find $\delta R_{\mu\nu} = \delta R^{\lambda}{}_{\mu\lambda\nu}$ use Normal Coords - are a local inertial frame.

$g_{\mu\nu;\lambda} = 0$ at a pt P.

but $\partial\partial g \neq 0$.

In N.C. $\nabla = \partial$.

$\delta R^{\lambda}{}_{\mu\lambda\nu} = \delta\Gamma^{\lambda}_{\mu\nu,\lambda} - \delta\Gamma^{\lambda}_{\mu\nu,\lambda}$

$= \delta\Gamma^{\lambda}_{\mu\nu;\lambda} - \delta\Gamma^{\lambda}_{\mu\nu;\lambda}$.

$\delta R = \delta R_{\mu\nu} g^{\mu\nu} + R_{\mu\nu} \delta g^{\mu\nu}$.

To find $\delta R_{\mu\nu} = \delta R^{\lambda}{}_{\mu\lambda\nu}$ use Normal Coords - are a local inertial frame.

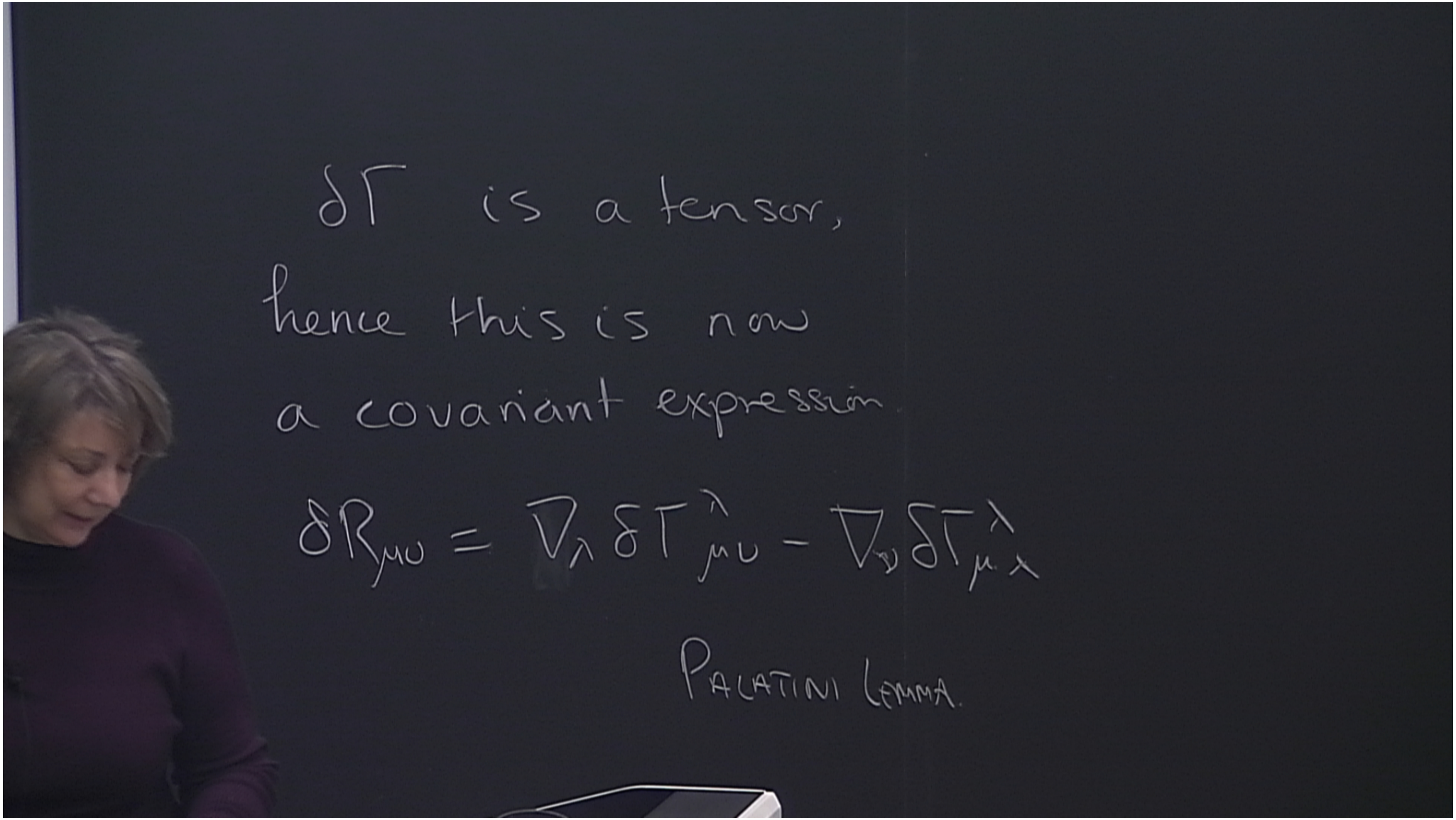
$g_{\mu\nu;\lambda} = 0$ at a pt P.

but $\partial\partial g \neq 0$.

In N.C. $\nabla = \partial$

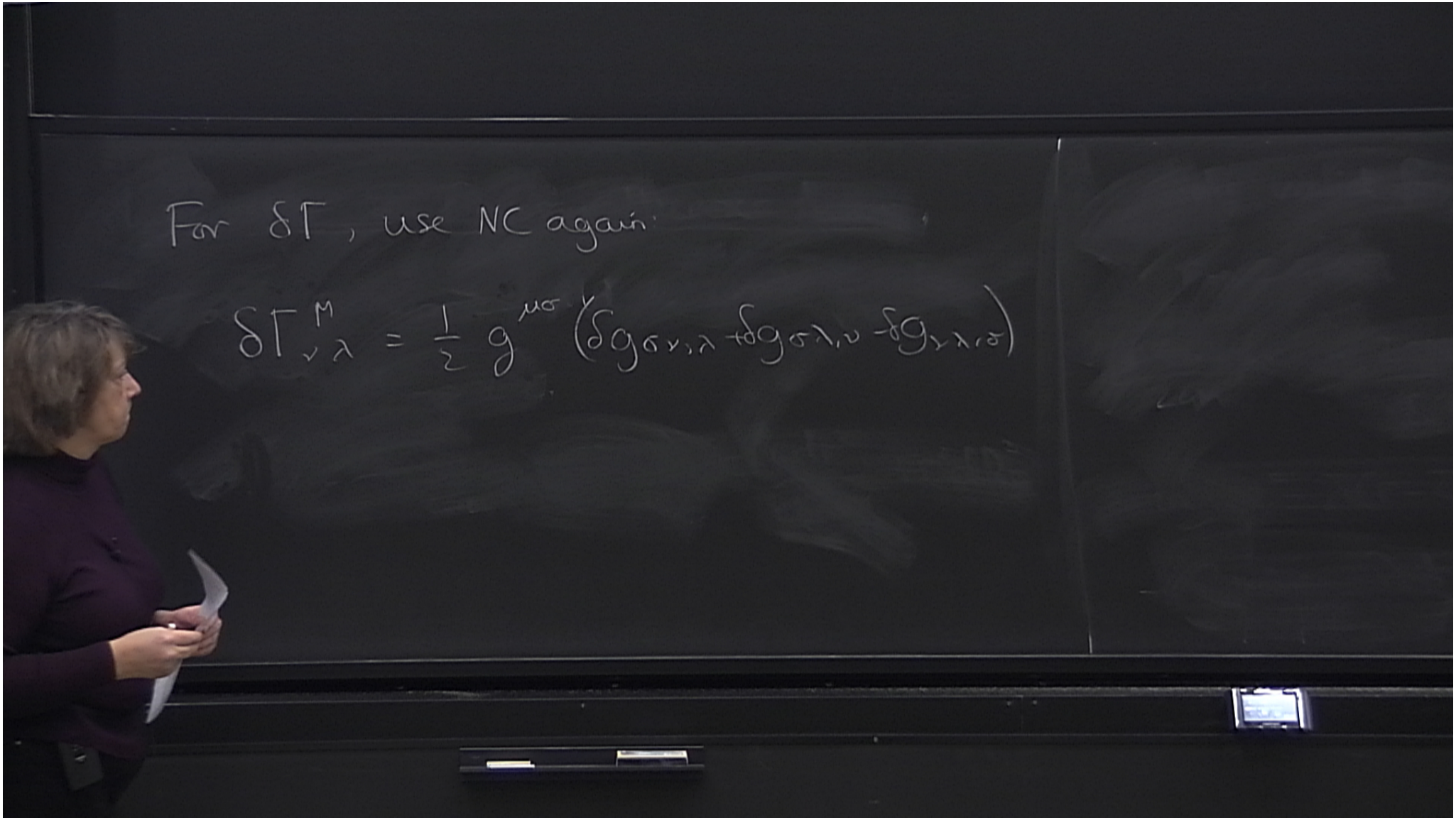
$\delta R^{\lambda}{}_{\mu\lambda\nu} = \delta\Gamma^{\lambda}_{\mu\nu,\lambda} - \delta\Gamma^{\lambda}_{\mu\lambda,\nu}$

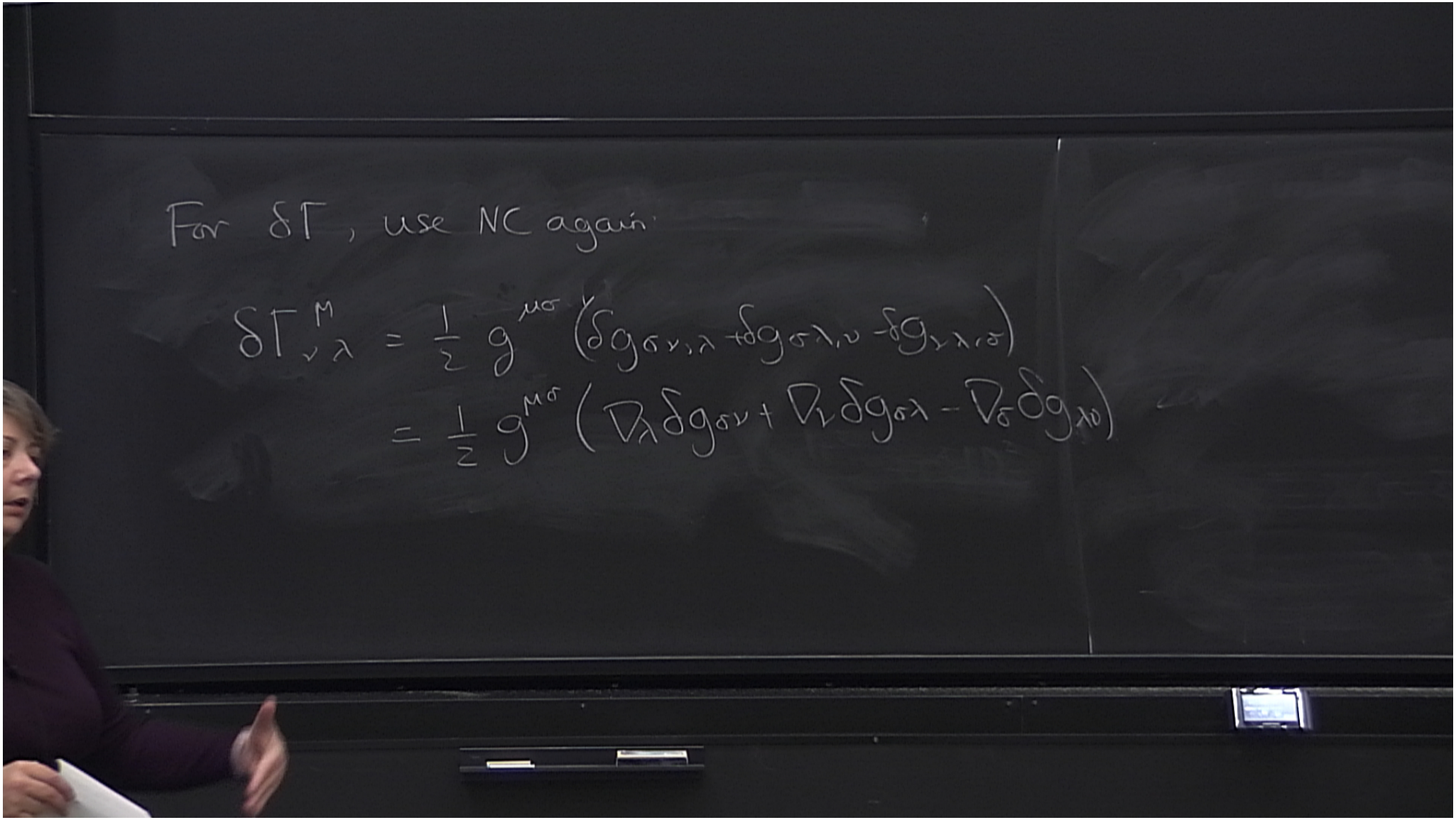
$= \delta\Gamma^{\lambda}_{\mu\nu;\lambda} - \delta\Gamma^{\lambda}_{\mu\lambda;\nu}$.

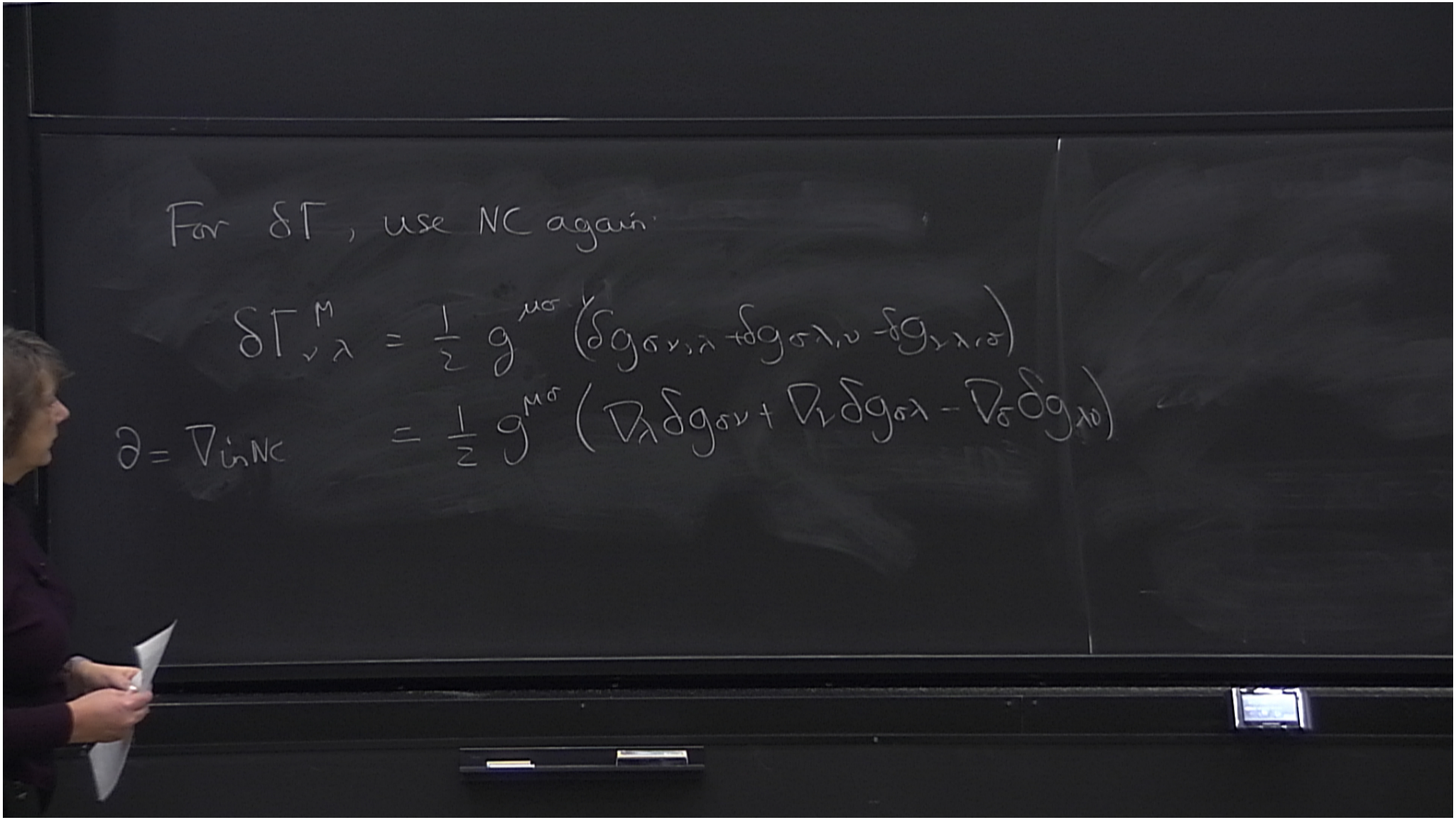

δΓ is a tensor,
hence this is now
a covariant expression
$\delta R_{\mu\nu} = \nabla_\lambda \delta\Gamma^\lambda_{\mu\nu} - \nabla_\nu \delta\Gamma^\lambda_{\mu\lambda}$
PALATINI LEMMA

For $\delta\Gamma$, use NC again:

$$\Gamma^{M}_{\nu\lambda} = \frac{1}{2} g^{\mu\sigma} \left(g_{\sigma\nu,\lambda} + g_{\sigma\lambda,\nu} - g_{\nu\lambda,\sigma} \right)$$


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$= \frac{1}{2} g^{\mu\sigma} (\nabla_\lambda \delta g_{\sigma\nu} + \nabla_\nu \delta g_{\sigma\lambda} - \nabla_\sigma \delta g_{\nu\lambda})$


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$\delta\Gamma^{\mu}_{\nu\lambda} = \frac{1}{2} g^{\mu\sigma} (\delta g_{\sigma\nu,\lambda} + \delta g_{\sigma\lambda,\nu} - \delta g_{\nu\lambda,\sigma})$
$\partial = \nabla_{\text{in NC}}$ $= \frac{1}{2} g^{\mu\sigma} (\nabla_\lambda \delta g_{\sigma\nu} + \nabla_\nu \delta g_{\sigma\lambda} - \nabla_\sigma \delta g_{\nu\lambda})$

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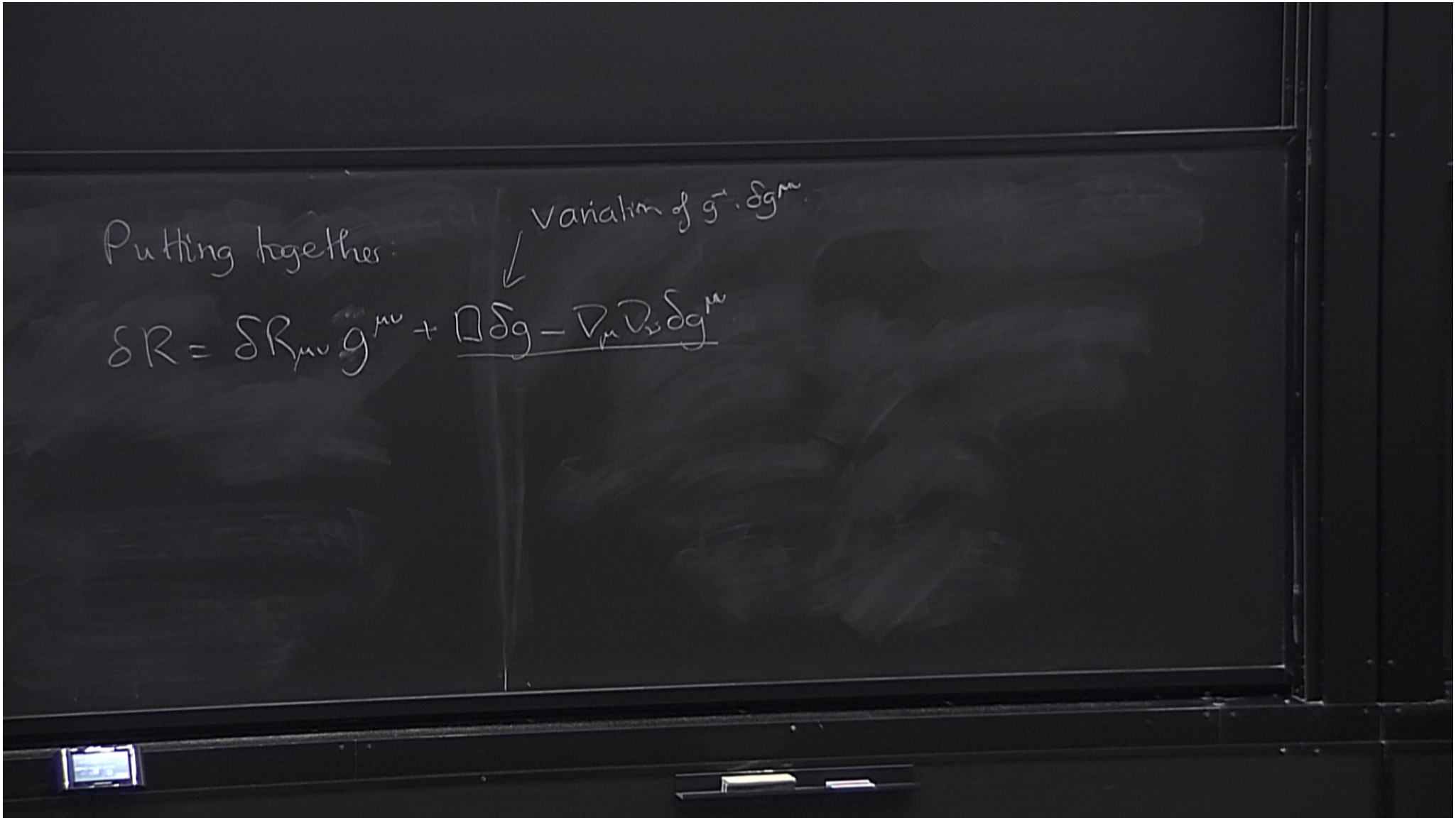
$$\delta\Gamma^{\mu}_{\nu\lambda} = \frac{1}{2} g^{\mu\sigma} \left(\delta g_{\sigma\nu;\lambda} + \delta g_{\sigma\lambda;\nu} - \delta g_{\nu\lambda;\sigma} \right)$$

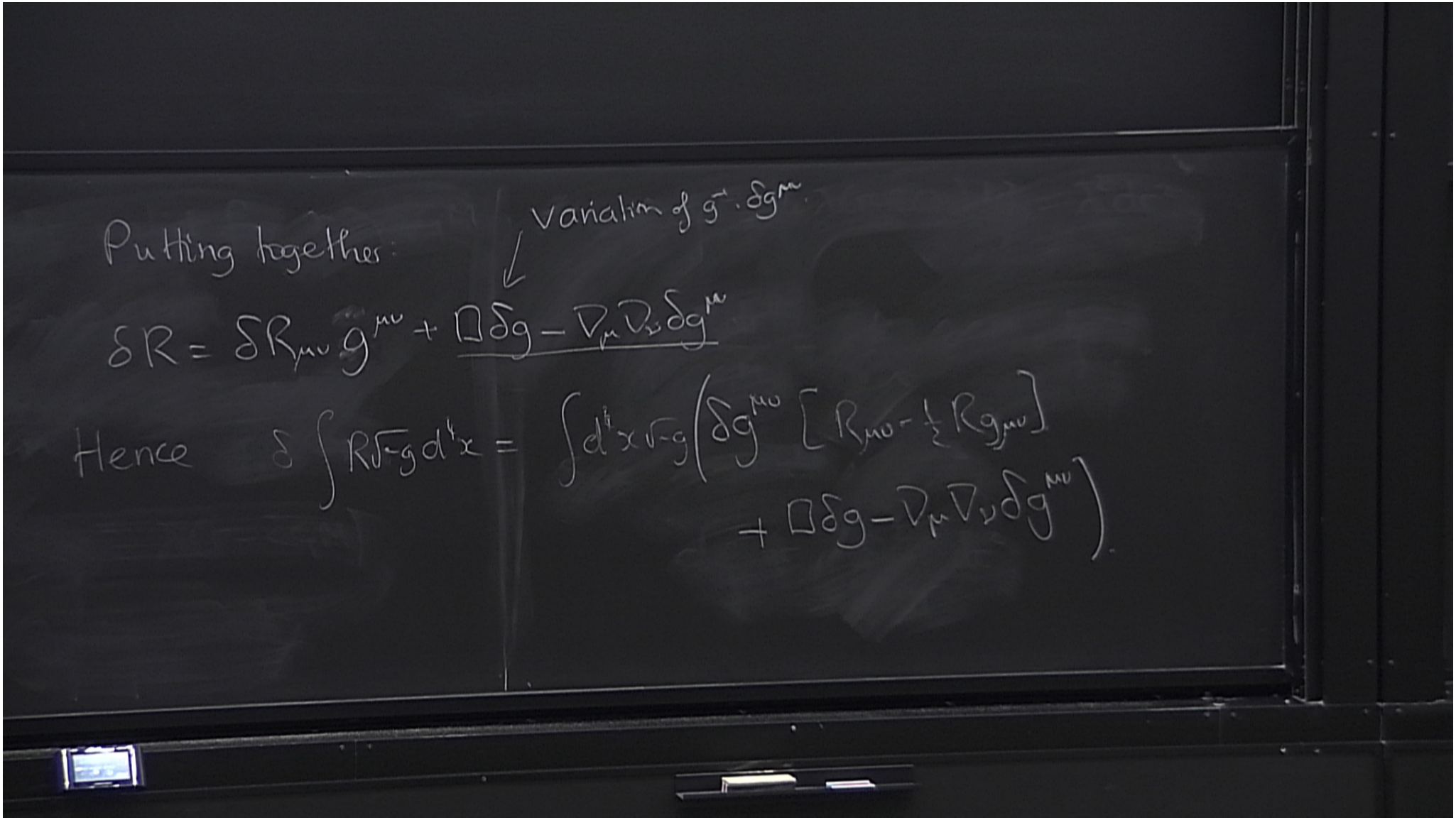
$$\partial = \nabla_{\text{in NC}} \quad = \frac{1}{2} g^{\mu\sigma} \left(\nabla_\lambda \delta g_{\sigma\nu} + \nabla_\nu \delta g_{\sigma\lambda} - \nabla_\sigma \delta g_{\lambda\nu} \right)$$

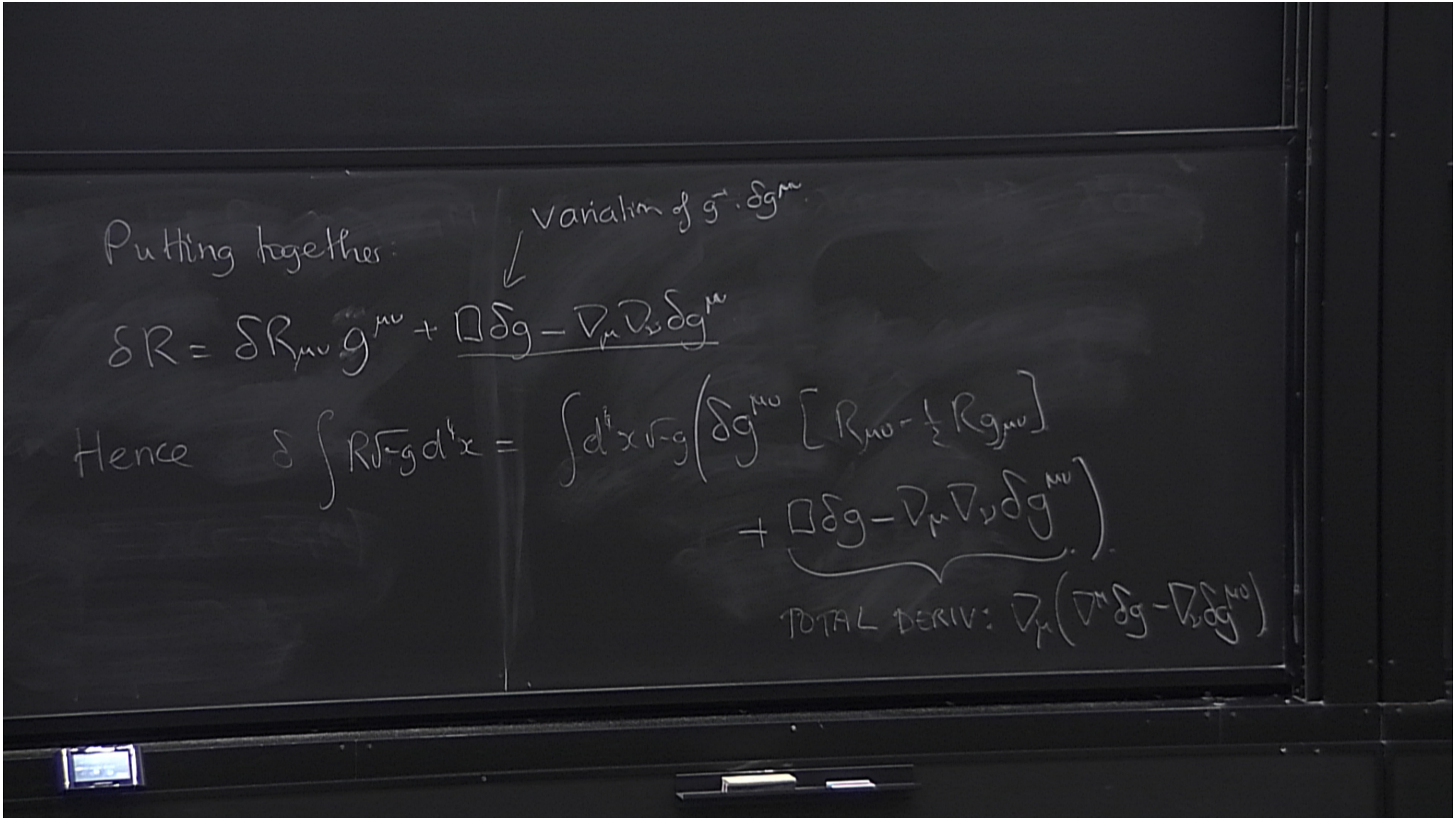
$$\Rightarrow \delta R_{\mu\nu} = \frac{1}{2} \Box \delta g^{-1}{}_{\mu\nu} - \frac{1}{2} \nabla_\lambda \nabla_\mu \delta g^{-1}{}^{\lambda}{}_{\nu}$$
$$+ \frac{1}{2} \nabla_\mu \nabla_\nu \delta g^{-1}{}^{\lambda\mu} - \frac{1}{2} \nabla_\lambda \nabla_\nu \delta g^{-1}{}^{\lambda}{}_{\mu}$$

Putting together:

$$\delta R = \delta R_{\mu\nu} g^{\mu\nu} + \Box \delta g - D_\mu D_\nu \delta g^{\mu\nu}$$


Putting together:
Variation of $g^{\mu\nu}\delta g_{\mu\nu}$
$\delta R = \delta R_{\mu\nu} g^{\mu\nu} + \Box \delta g - \nabla_\mu \nabla_\nu \delta g^{\mu\nu}$


Putting together:
Variation of $g^{\mu\nu}$: $\delta g^{\mu\nu}$
$\delta R = \delta R_{\mu\nu} g^{\mu\nu} + \Box\delta g - \nabla_\mu\nabla_\nu\delta g^{\mu\nu}$
Hence $\delta\int R\sqrt{-g}\,d^4x = \int d^4x\sqrt{-g}\left(\delta g^{\mu\nu}\left[R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}\right] + \Box\delta g - \nabla_\mu\nabla_\nu\delta g^{\mu\nu}\right)$


Putting together:
Variation of $g^{\mu\nu}$: $\delta g^{\mu\nu}$
$\delta R = \delta R_{\mu\nu} g^{\mu\nu} + \Box\delta g - \nabla_\mu\nabla_\nu\delta g^{\mu\nu}$
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$\left. + \Box\delta g - \nabla_\mu\nabla_\nu\delta g^{\mu\nu}\right)$
TOTAL DERIV: $\nabla_\mu\left(\nabla^\mu\delta g - \nabla_\nu\delta g^{\mu\nu}\right)$

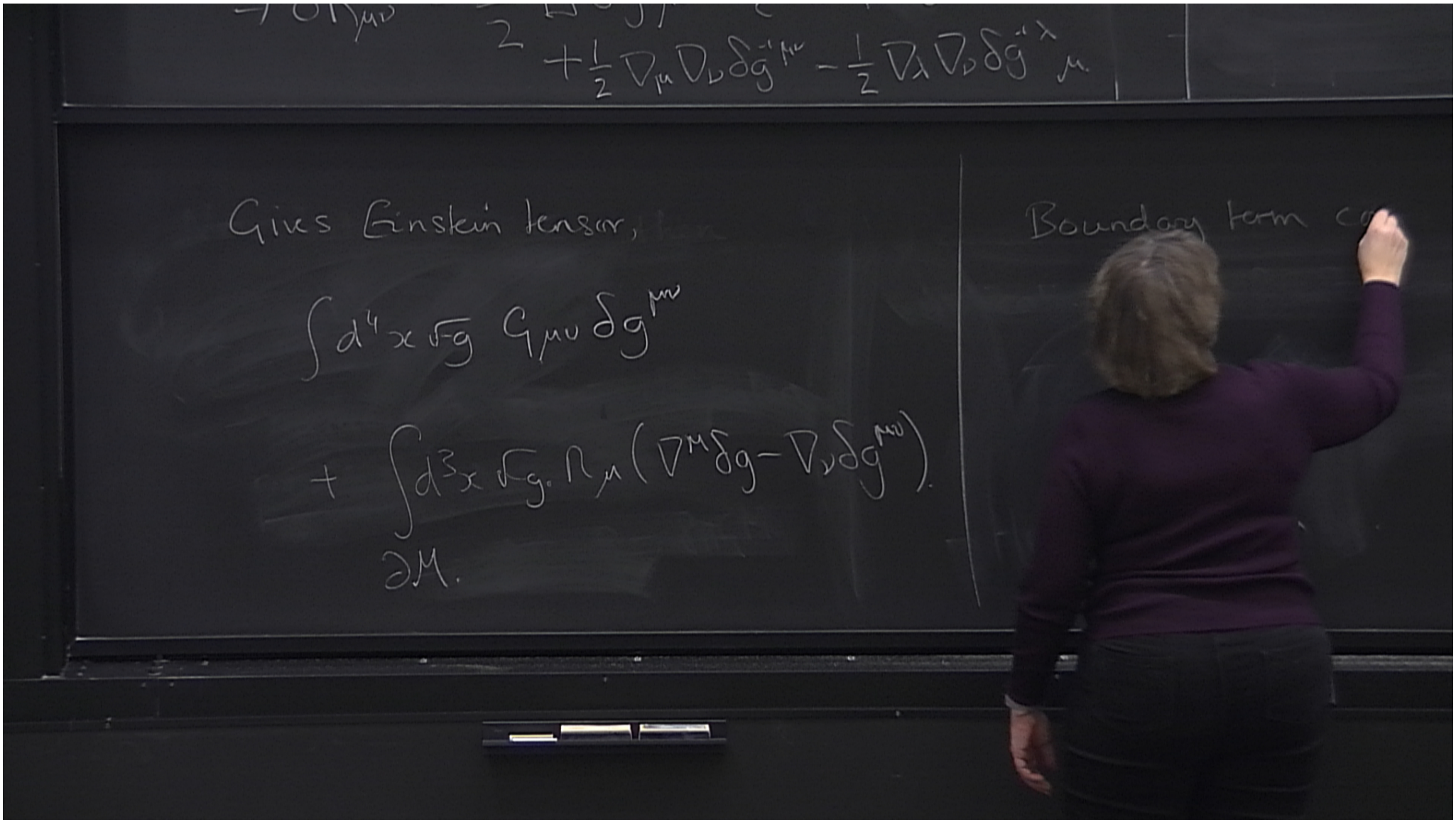
$+\frac{1}{2} D_\mu D_\nu \delta g^{\mu\nu} - \frac{1}{2} D_\lambda D_\nu \delta g^{\nu\lambda}{}_{\mu}$

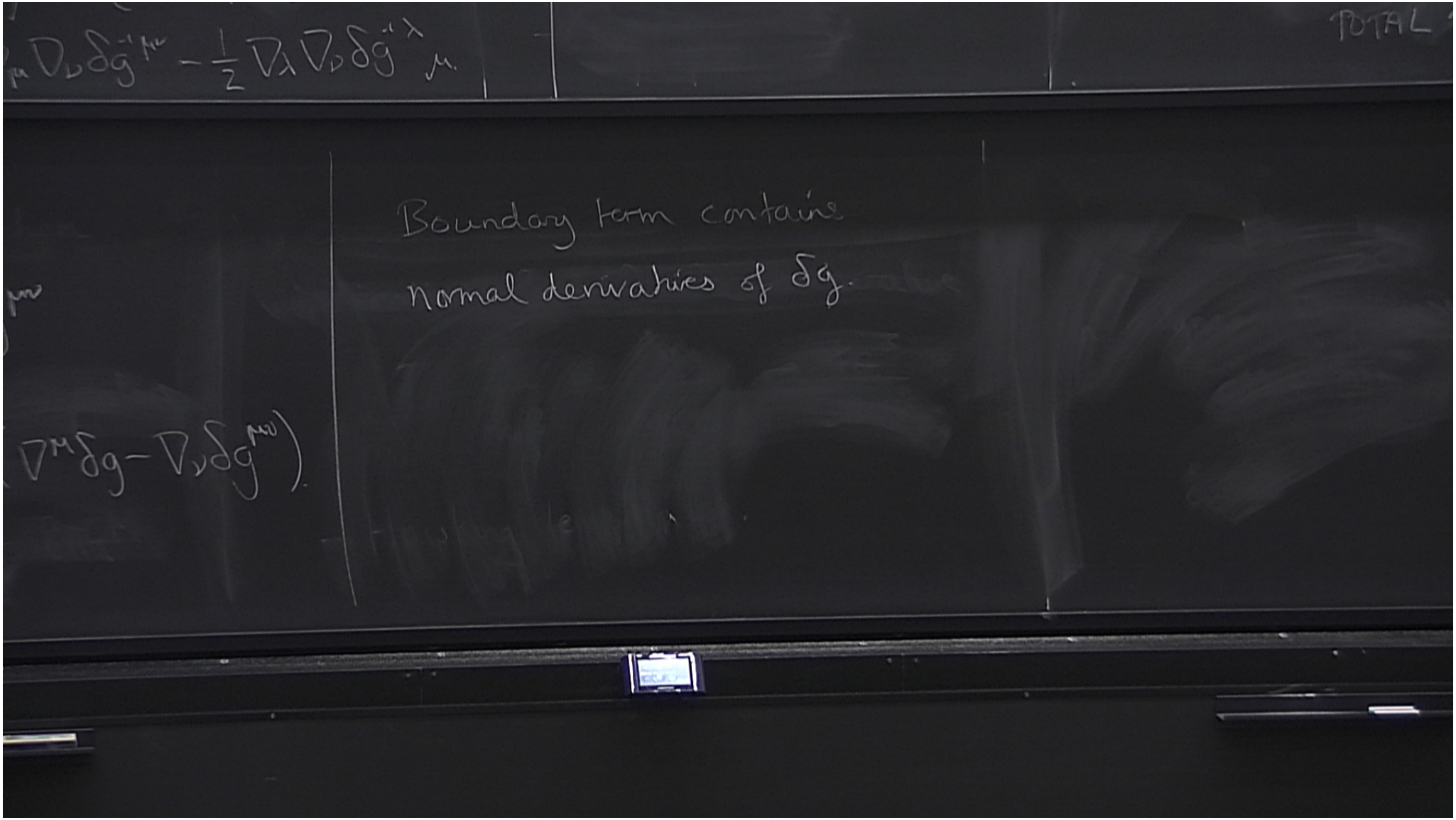
Gives Einstein tensor,

$$\int d^4x \sqrt{-g}\, G_{\mu\nu} \delta g^{\mu\nu}$$

$$+ \int_{\partial M} d^3x \sqrt{-g}\, n_\mu \left(D^\mu \delta g - D_\nu \delta g^{\mu\nu} \right).$$

Boundary term co




TOTAL
Boundary term contains
normal derivatives of δg.

$D_\nu \delta g^{\mu\nu} - \frac{1}{2} D_\lambda D_\nu \delta g^{\nu\lambda}{}_{\mu}$

TOTAL

$\mu\nu$

$D^\mu \delta g - D_\nu \delta g^{\mu\nu})$

Boundary term contains normal derivatives of δg.

So $\delta g = 0$ on boundary does not guarantee that boundary term vanishes.

TOTAL DERIV: $\nabla_\mu(\nabla^\mu \delta g - \nabla_\nu \delta g^{\mu\nu})$

y term contains
derivatives of δg.
-0 on boundary
guarantee that
y term vanishes.

For eqns of motion, this is not
nec. a problem, unless ∂M is
dynamical.

$$S_{EH} = \frac{1}{16\pi G}\int R\sqrt{g}\, d^4x$$

For $\delta\Gamma$, use NC again:

$$\delta\Gamma^{M}_{\nu\lambda} = \frac{1}{2} g^{\mu\sigma} \left(\delta g_{\sigma\nu;\lambda} + \delta g_{\sigma\lambda;\nu} - \delta g_{\nu\lambda;\sigma} \right)$$

$$\partial = \nabla_{\text{in NC}} = \frac{1}{2} g^{\mu\sigma} \left(\nabla_\lambda \delta g_{\sigma\nu} + \nabla_\nu \delta g_{\sigma\lambda} - \nabla_\sigma \delta g_{\nu\lambda} \right)$$

$$\Rightarrow \delta R_{\mu\nu} = \frac{1}{2} \Box \delta g^{\lambda}_{\ \mu\nu} - \frac{1}{2} \nabla_\lambda \nabla_\mu \delta g^{\sigma\lambda}_{\ \ \ \nu} + \frac{1}{2} \nabla_\mu \nabla_\nu \delta g^{\sigma\lambda}_{\ \ \ \sigma} - \frac{1}{2} \nabla_\lambda \nabla_\nu \delta g^{\sigma\lambda}_{\ \ \ \mu}$$

Putting t

$\delta R = \delta$

Hence

Gives Einstein tensor,

$$\int d^4x \sqrt{-g}\, G_{\mu\nu} \delta g^{\mu\nu}$$

Boundary term contain

normal derivatives of δg