Title: PSI 17/18 - Standard Model - Lecture 12

Date: Jan 18, 2018 09:00 AM

URL: http://pirsa.org/18010019

Abstract:

Higgs low-energy theorem

Return to $h \to gg$ Trick to compute $h \to gg$

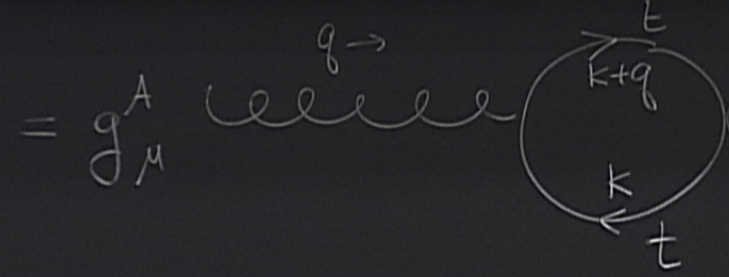
Valid in limit that $m_h << m_t$.

Compute contribution to QCD running from t quark.

to compute $h \to gg$ amplitude from β-fn for QCD.

m_t.

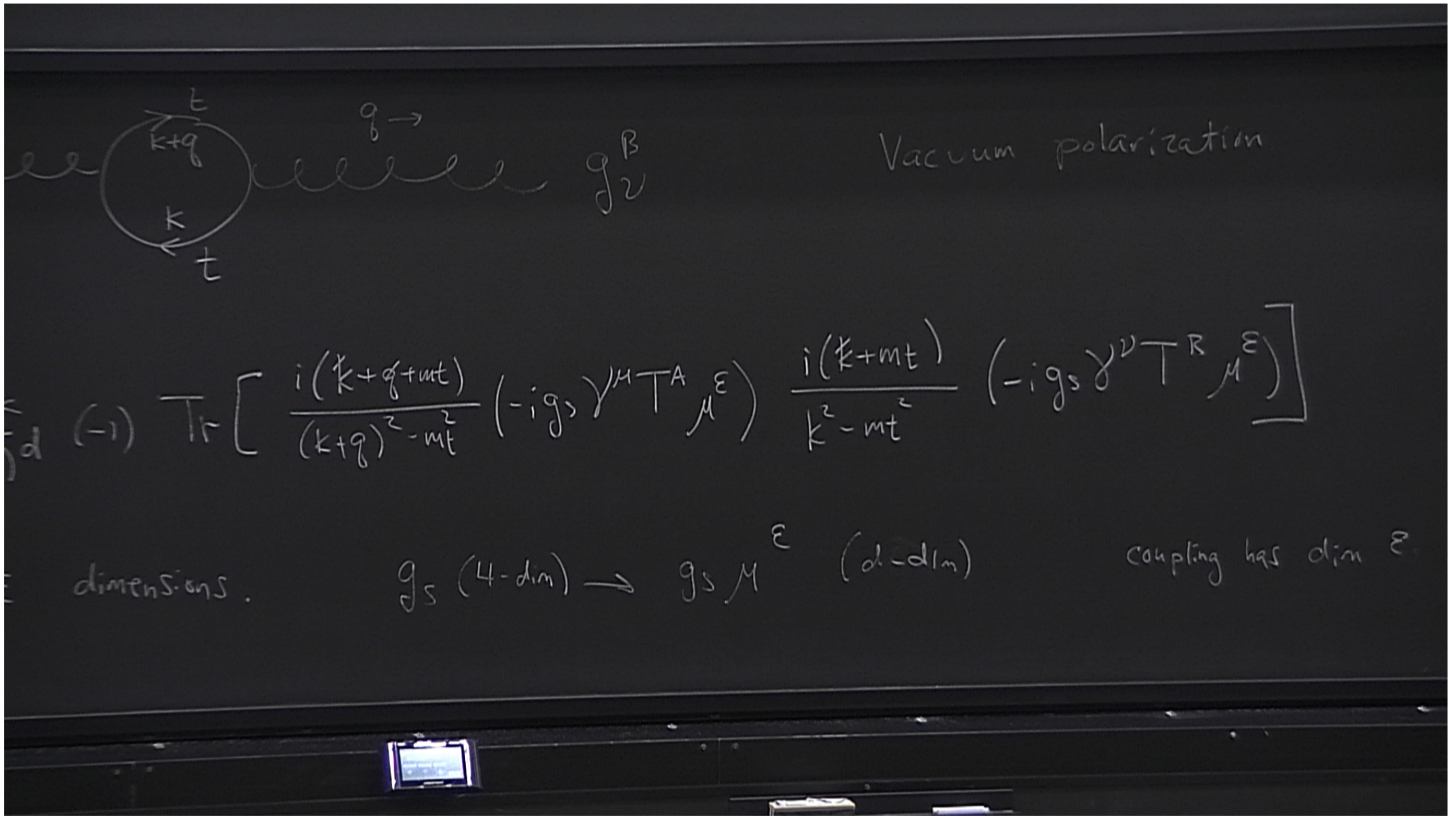
running from t quark.

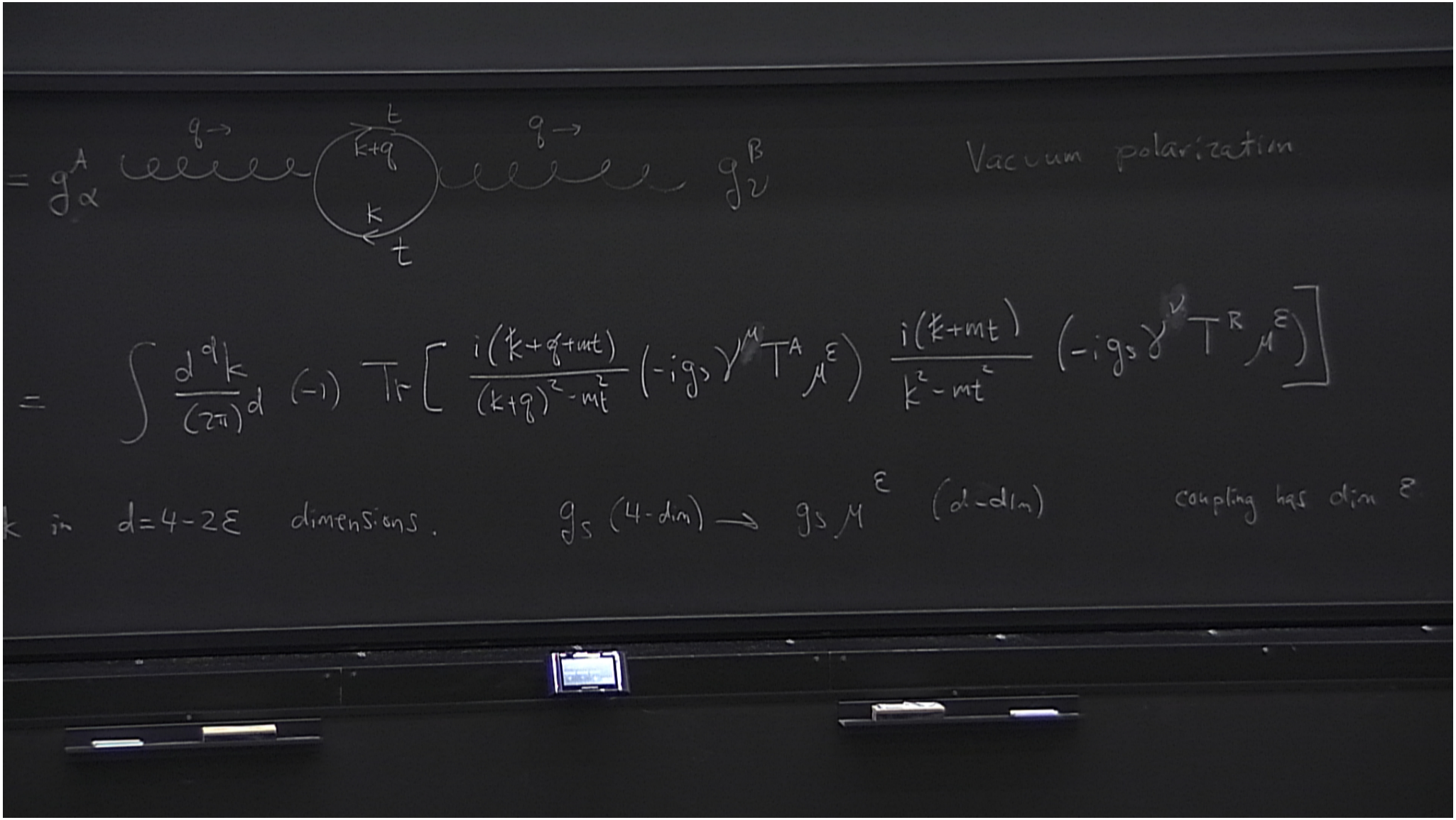
$i\mathcal{M} = g_\mu^A$ 

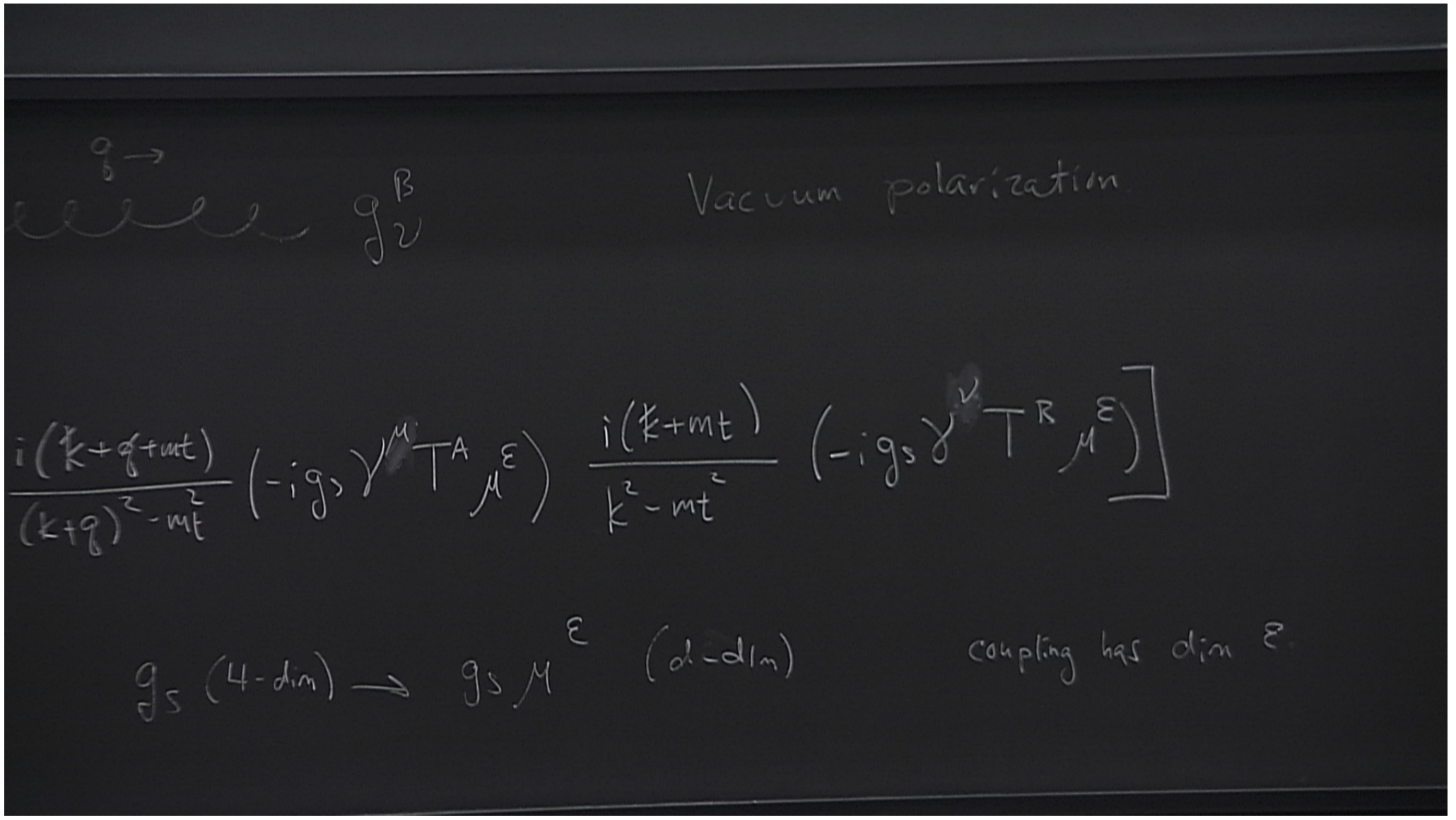
g_ν^B Vacu

$$= \int \frac{d^d k}{(2\pi)^d} (-1)\, \mathrm{Tr}\left[\frac{i(\not{k}+\not{q}+m_t)}{(k+q)^2 - m_t^2} (-i g_s \gamma^\mu T^A \mu^\varepsilon) \frac{i(\not{k}+m_t)}{k^2 - m_t^2} \right.$$

Work in $d = 4 - 2\varepsilon$ dimensions. $g_s\,(4\text{-dim}) \longrightarrow g_s \mu^\varepsilon$ $(d_i - d_{\ln})$


t
k+q
k
t
q →
Vacuum polarization
$\int d$ (-1) Tr [$\frac{i(\not{k}+\not{q}+m_t)}{(k+q)^2-m_t^2}(-ig_s\gamma^\mu T^A{}_\mu{}^\varepsilon)\frac{i(\not{k}+m_t)}{k^2-m_t^2}(-ig_s\gamma^\nu T^R{}_\mu{}^\varepsilon)$]
dimensions.
$g_s(4-dim) \rightarrow g_s \mu^\varepsilon \quad (d-dim)$
coupling has dim ε


q→
t
k+q
k
t
q→
Vacuum polarization
$= g_\alpha^A$
g_2^B
$= \int \frac{d^d k}{(2\pi)^d} (-1)\, \mathrm{Tr}\left[\frac{i(\not{k}+\not{q}+m_t)}{(k+q)^2 - m_t^2} (-i g_s \gamma^\mu T^A \mu^\varepsilon) \frac{i(\not{k}+m_t)}{k^2 - m_t^2} (-i g_s \gamma^\nu T^R \mu^\varepsilon) \right]$
k in $d = 4 - 2\varepsilon$ dimensions.
g_s (4-dim) → $g_s \mu^\varepsilon$ (d-dim)
coupling has dim ε


Vacuum polarization
$\frac{i(\not{k}+\not{q}+m_t)}{(k+q)^2-m_t^2}(-ig_s\gamma^\mu T^A{}_\mu{}^\varepsilon)\frac{i(\not{k}+m_t)}{k^2-m_t^2}(-ig_s\gamma^\nu T^R{}_\mu{}^\varepsilon)]$
$g_s(4-dim) \rightarrow g_s\mu^{\varepsilon}\ (d-d_{lm})$
coupling has dim ε

Result:

$$i\mathcal{M} = \frac{i\alpha_s}{4\pi} \mathrm{Tr}[T^A T^B] (q^\mu q^\nu - q^2 \eta^{\mu\nu}) \int_0^1 dx \; 8x(1-x)$$

Same as in QED, $\alpha_{em} \to \alpha_s \mathrm{Tr}[T^A T^B]$

Introduce counter term to cancel divergence:

$$\mathcal{L}_{CT} = -(Z_3^t - 1) \frac{1}{2} \mathrm{Tr}[G_{\mu\nu} G^{\mu\nu}]$$

$$-q^2\eta^{\mu\nu}\Big)\int_0^1 dx\; 8x(1-x)\left(\frac{1}{\varepsilon}-\gamma_E+\log 4\pi+\log\frac{\mu^2}{m_t^2-q^2x(1-x)}\right)$$

$$\alpha_s\,\mathrm{Tr}\left[T^A T^B\right]$$

divergence:

$$\left[G_{\mu\nu}G^{\mu\nu}\right]$$

$$Z_3^t - 1 = \frac{1}{2}\,\frac{\alpha_s}{4\pi}\int_0^1 dx\; 8x(1-x)\left(\frac{1}{\epsilon} - \gamma_E + \log\right.$$

$\overline{\text{MS}}$ ("modified" minimal subtraction)

Z_i = renormalization co

Bare vs. renormalized coupling.

$$g_s^{(B)} = \frac{Z_1}{Z_2\sqrt{Z_3}}\,\mu^\epsilon g_s = \frac{1}{\sqrt{Z_3^T}}\,\mu^\epsilon g_s + \ldots$$

$$\delta z(1-z)\left(\frac{1}{\epsilon} - \gamma_E + \log 4\pi\right) = -\frac{\alpha_s}{6\pi}\left(\frac{1}{\epsilon} - \gamma_E + \log 4\pi\right)$$

subtraction)

Z_i = renormalization constants

$$\frac{1}{\frac{\epsilon}{\epsilon_3}} A^{\epsilon} g_s + \dots$$

$$\alpha_s = Z_3^t \, \mu^{-2\varepsilon} \, \alpha_s^{(B)}$$

$$= \left(1 - \frac{\alpha_s}{6\pi}\left(\frac{1}{\varepsilon} - \gamma_E + \log 4\pi\right)\right) \mu^{-2\varepsilon} \, \alpha_s^{(B)}$$

Determine t contribution to β-fn.

$$\beta_s^t = \frac{d\alpha_s}{d\log\mu} = \mu \frac{d\alpha_s}{d\mu} = \lim_{\varepsilon \to 0} \frac{d}{d\log\mu}\left(Z_3^t \, \mu^{-2\varepsilon} \, \alpha_s^{(B)}\right)$$

$$\alpha_s^{(B)}$$

$$\left(\frac{1}{\epsilon} - \gamma_E + \log 4\pi\right)\Big) \mu^{-2\epsilon} \alpha_s^{(B)}$$

ation to β-fn.

$$\mu \frac{d\alpha_s}{d\mu} = \lim_{\epsilon \to 0} \frac{d}{d\log\mu}\left(Z_3^t \mu^{-2\epsilon} \alpha_s^{(B)}\right) = -2\epsilon Z_3^t \mu^{-2\epsilon} \alpha_s^{(B)}$$

$$\beta_s^{(1)} = \lim_{\varepsilon \to 0} \frac{\alpha_s}{3\pi} \underbrace{\mu^{-2\varepsilon} \alpha_s^{(B)}}_{\alpha_s + \mathcal{O}(\alpha_s^2)} = \frac{\alpha_s^2}{3\pi} = b_t \alpha_s^2$$

Full QCD β-fn. $\beta_s = \frac{d\alpha_s}{d\log\mu} = -\left(11 - \frac{2}{3} N_F\right) \frac{\alpha_s^2}{2\pi}$

$$= \cdots + \underbrace{\frac{2}{3} \cdot \frac{\alpha_s^2}{2\pi}}_{b_t \alpha_s^2}$$

$$= \frac{\alpha_s^2}{3\pi} \equiv b_t \alpha_s^2 \qquad b_t = \frac{1}{3\pi}.$$

$O(\alpha_s^2)$

$$\beta_s = \frac{d\alpha_s}{d\log\mu} = -\left(11 - \frac{2}{3}N_F\right)\frac{\alpha_s^2}{2\pi} \qquad N_F = \#\ \text{fermions}.$$

$$\equiv \cdots + \underbrace{\frac{2}{3}\cdot\frac{\alpha_s^2}{2\pi}}_{b_t \alpha_s^2}$$

Recall: $i\mathcal{M}(h \to gg) = -\frac{i\alpha_s}{3\pi v}\,\varepsilon_\mu(p)\,\varepsilon_\nu(p')\,(p^\nu p'^$

Represent by effective hgg interaction:

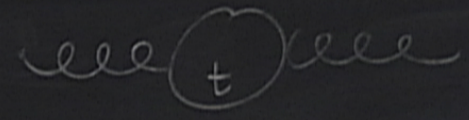
$$\mathcal{L}_{hgg} = \frac{\alpha_s}{6\pi v}\, h\, \mathrm{Tr}[G_{\mu\nu} G^{\mu\nu}]$$

$$= -\frac{i\alpha_s}{3\pi v}\,\varepsilon_\mu(p)\,\varepsilon_\nu(p')\left(p^\nu p'^\mu - p\cdot p'\,\eta^{\mu\nu}\right)\delta^{AB}$$

ive hgg interaction:

$$\mathcal{L}_{hgg} = \frac{\alpha_s}{6\pi v}\,h\,\mathrm{Tr}\left[G_{\mu\nu}G^{\mu\nu}\right]$$

Vacuum polarization:



$$i\mathcal{M} = i\, b_t\, \alpha_s \left(q^\mu q^\nu - q^2 \eta^{\mu\nu}\right) \mathrm{Tr}\left[T^A T^B\right] \left(\frac{1}{\varepsilon} -$$

Higgs field couples to $t\bar{t}$ via interaction: $m_t \bar{t} t \left(1+\right.$

Generate hgg vertex from gg amplitude via shift m_t

$$i\mathcal{M} = i\, b_t\, \alpha_s \left(q^\mu q^\nu - q^2 \eta^{\mu\nu}\right) \mathrm{Tr}\left[T^A T^B\right] \left(\frac{1}{\varepsilon} - \gamma_E + \log 4\pi + \log \right.$$

$$T^A T^B \left(\frac{1}{\varepsilon} - \gamma_E + \log 4\pi + \log \frac{M^2}{m_t^2} \right) \qquad m_t^2 \gg q^2$$

tion: $m_t \to m_t \left(1 + \frac{h}{v}\right)$

ude via shift $m_t \to m_t \left(1 + \frac{h}{v}\right)$

$$\left(-\gamma_E + \log 4\pi + \log \frac{M^2}{m_t^2} - 2 \log \left(1 + \frac{h}{v}\right) \right)$$

$$i\mathcal{M} = \ldots - i\,2\,b_t\,\alpha_s\,(q^\mu q^\nu - q^2\eta^{\mu\nu})\,\mathrm{Tr}[T^A T^B]\left(\frac{h}{v}\right) + \ldots$$

Represent hgg by interaction:

$$\mathcal{L}_{hgg} = \frac{b_t\,\alpha_s}{2v}\,h\,\mathrm{Tr}[G_{\mu\nu}G^{\mu\nu}]$$

$$\frac{\alpha_s}{6\pi v}, \quad b_t = \frac{1}{3\pi}$$

$\eta^{\mu\nu}) \; Tr[T^A T^B] \left(\frac{h}{v}\right) + \ldots$ = kept linear term only.

$\frac{1}{h} \; Tr[G_{\mu\nu} G^{\mu\nu}]$

$b_2 = \frac{1}{3\pi}$

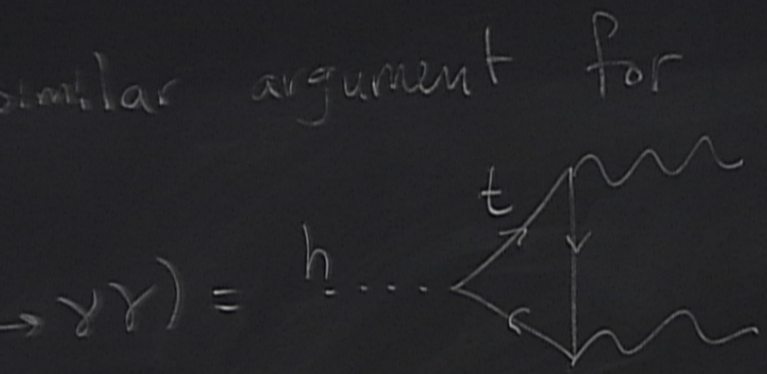
+ ...

kept linear term only.

Use β-fns to compute $h \to$ gauge bosons couplings at loop order.

Valid if $m_h \ll m_t$

Use similar argument for $h \to \gamma\gamma$.

$i\mathcal{M}(h \to \gamma\gamma) =$ 

Similar to $h \to gg$

Very complicated.

Use low-energy theorem: QED β function contrib

Compute vacuum polarizations (only divergent part)

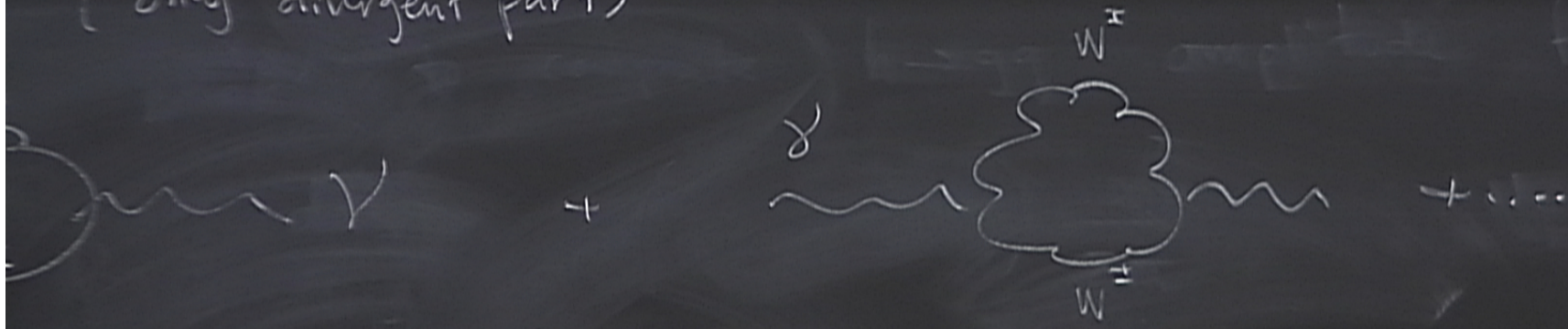
e.g. 

+ ...

$$\beta_{em} = \frac{d\alpha_{em}}{d\log\mu} = b_t \alpha_{em}^2 + b_W \alpha_{em}^2 + \ldots$$

QED β function contributions from t & W

(only divergent part)



$b_t \alpha_{em}^2 + b_W q_{em}^2 + \ldots$

Shift in masses $m_t \to m_t\left(1+\frac{h}{v}\right)$, $m_W \to m_W\left(1+\frac{h}{v}\right)$,

Then generate $h\gamma\gamma$ coupling:

$$\mathcal{L}_{h\gamma\gamma} = \frac{(b_t + b_W)\,\alpha_{em}}{4v}\, h F^{\mu\nu} F_{\mu\nu}$$

$$b_t = \frac{8}{9\pi}, \quad b_W = -\frac{7}{2\pi}$$

Only valid for $m_h \ll m_t, m_W$.

Relevant parameter:

$$\left(\frac{m_h}{2m_t}\right)^2 \approx 0.1$$

$$\left(\frac{m_h}{2m_W}\right)^2 \approx 0.6$$

Problems with SM

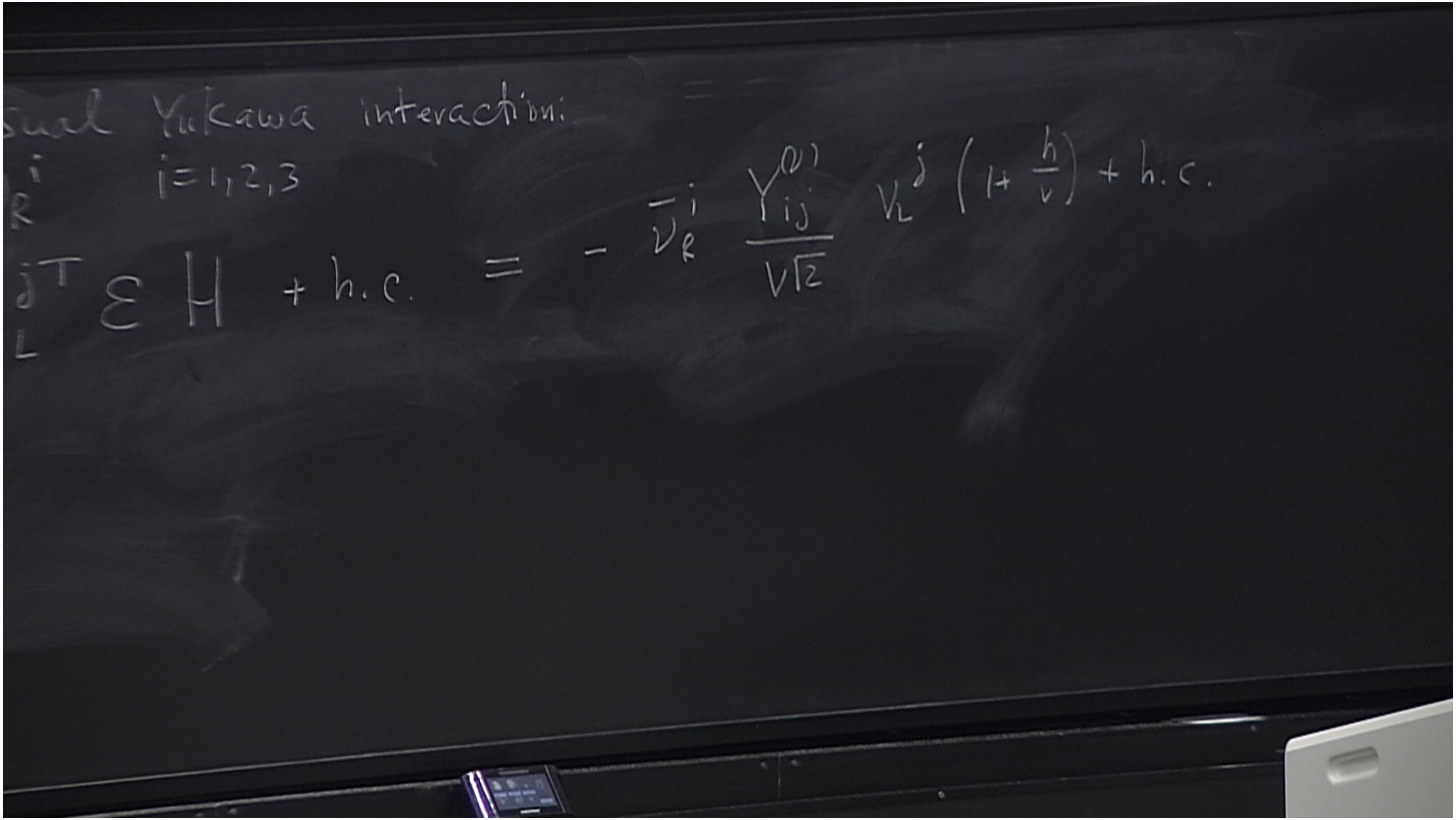
(1) Neutrino masses: Small ($<$ eV)

Why so small but nonzero?

Include ν masses via usual Yukawa interaction:

Introduce RH ν field: ν_R^i $\quad i = 1,2,3$

$$\mathcal{L}_{\text{Yukawa}} = - Y^{(\nu)}_{ij} \bar{\nu}^i_R L^{jT}_L \varepsilon H + \text{h.c.} =$$


Yukawa interaction:
$i = 1,2,3$
$-\bar{\nu}_R^i \frac{Y_{ij}^{(\nu)}}{v\sqrt{2}} \nu_L^j \left(1 + \frac{h}{v}\right) + h.c.$
$+ h.c. =$

Include ν masses via usual Yukawa interaction:

Introduce RH ν field: ν_R^i $\quad i=1,2,3$

$$\mathcal{L}_{\text{Yukawa}} = - Y^{(\nu)}_{ij} \bar{\nu}^i_R L^{j\,T}_L \varepsilon H + \text{h.c.} =$$

Diagonalize: $\nu^i_{L,R} = (U_{\nu_{L,R}})_{ij} \nu'^j_{L,R}$

Yukawa interaction:

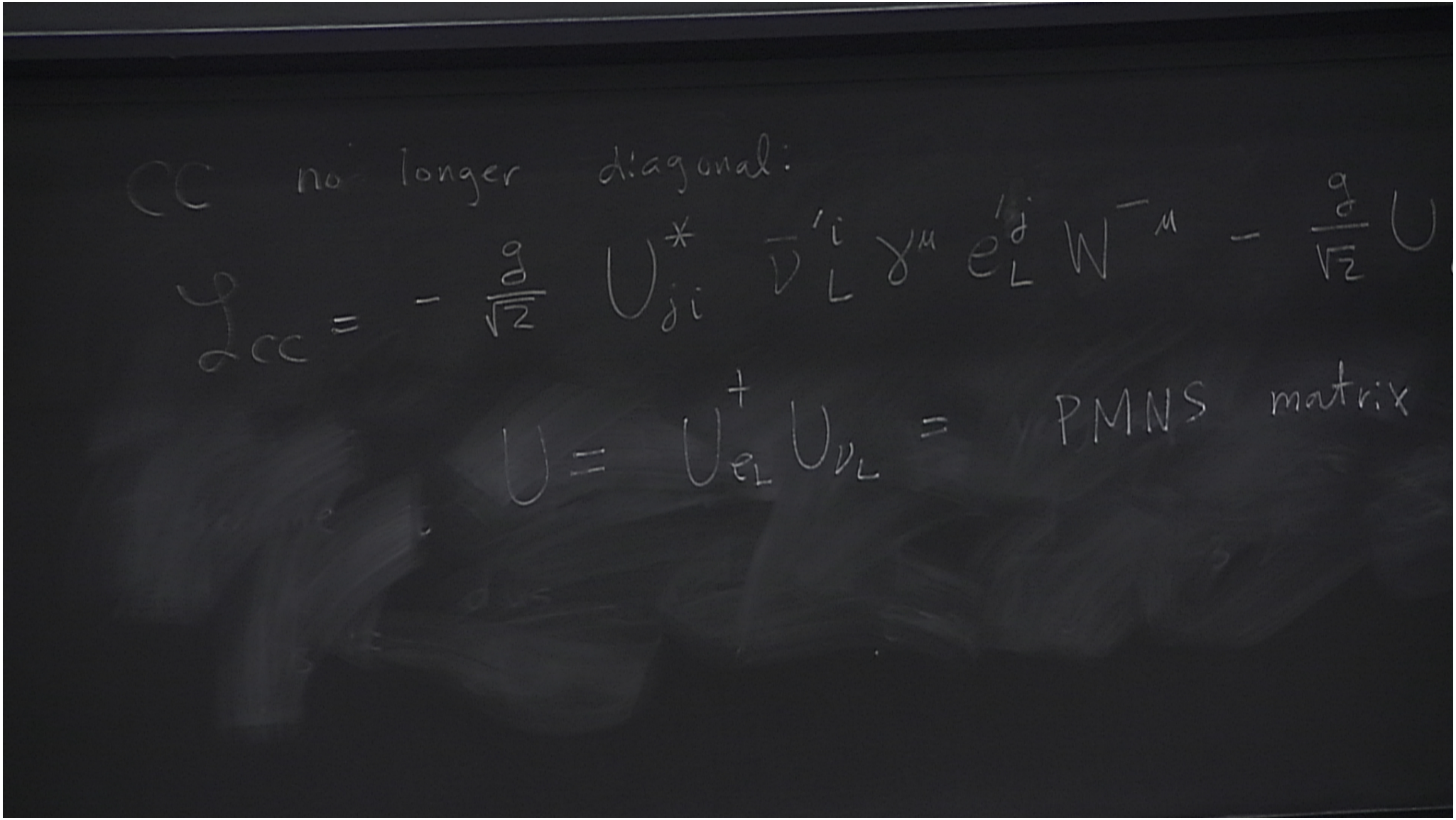
$i = 1,2,3$

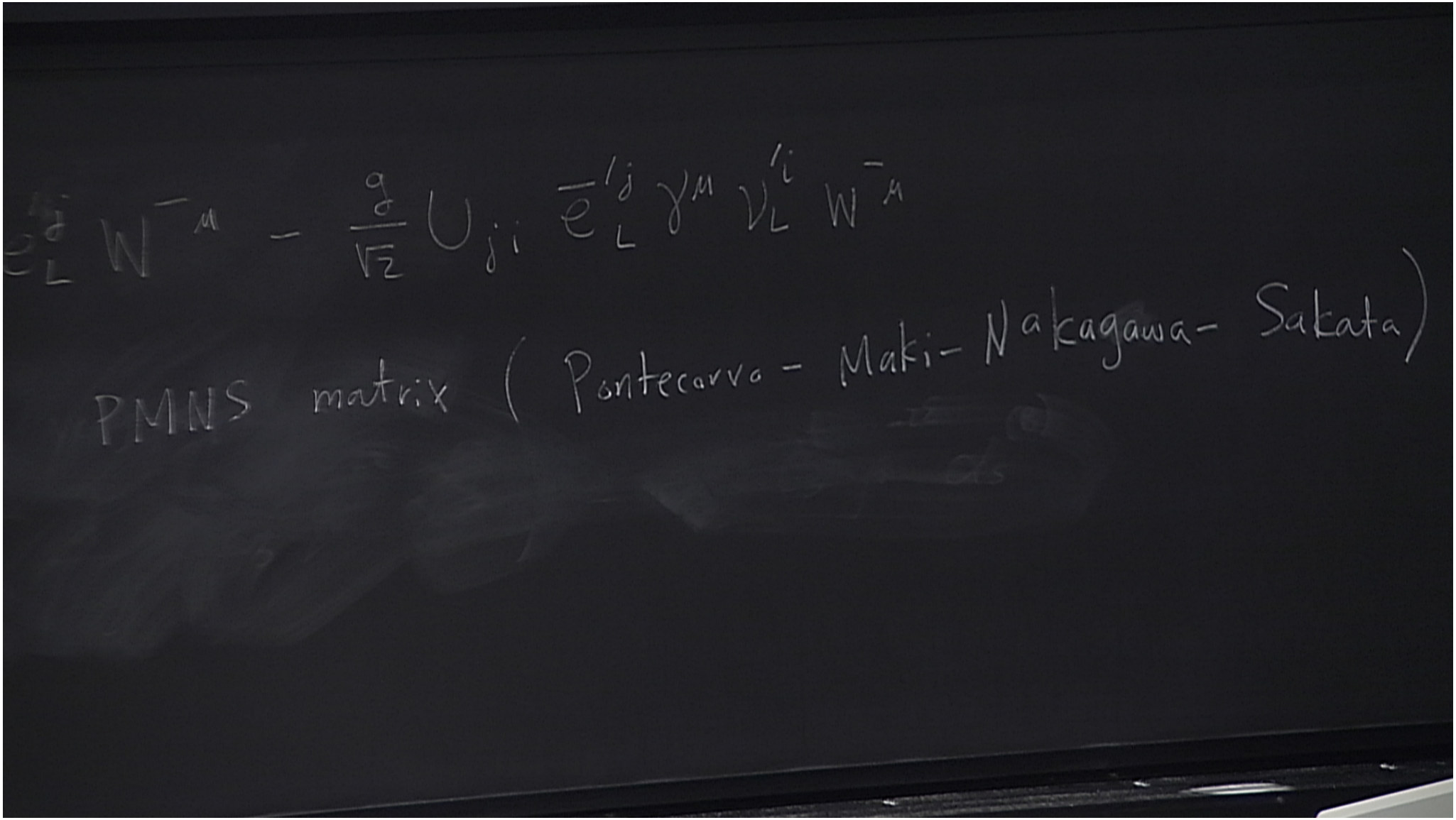
$$H + h.c. = -\bar{\nu}_R^i \frac{Y^{(2)}_{ij}}{v\sqrt{2}} \nu_L^j \left(1 + \frac{h}{v}\right) + h.c.$$

$$(\nu_{L,R})_{ij}\, \nu'^{j}_{L,R}$$

$\nu'^{i}_{L,R}$ = mass eigenstates

$\nu^{i}_{L,R}$ = gauge eigenstates.


CC no longer diagonal:
$\mathcal{L}_{CC} = -\frac{g}{\sqrt{2}} U^{*}_{ji} \bar{\nu}'^{i}_{L} \gamma^{\mu} e'^{j}_{L} W^{-}_{\mu} - \frac{g}{\sqrt{2}} U$
$U = U^{\dagger}_{e_L} U_{\nu_L}$ = PMNS matrix


$\bar{e}^{j}_{L} W^{-\mu} - \frac{g}{\sqrt{2}} U_{ji} \bar{e}'^{j}_{L} \gamma^{\mu} \nu'^{i}_{L} W^{-\mu}$
PMNS matrix (Pontecarvo - Maki - Nakagawa - Sakata)