Title: PSI 17/18 - Standard Model - Lecture 9

Date: Jan 15, 2018 09:00 AM

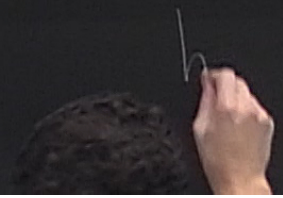
URL: http://pirsa.org/18010016

Abstract:

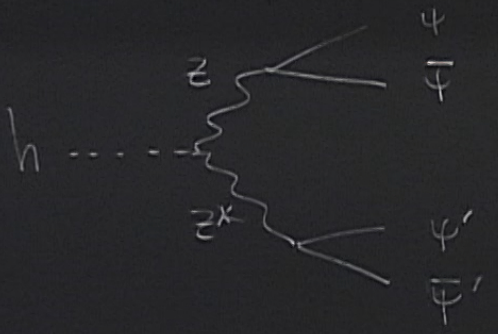
# Higgs decays & production

Tree-level: $h \to$ fermions

| | |
|---|---|
| $h \to b\bar{b}$ | 58% |
| $h \to c\bar{c}$ | 2.9% |
| $h \to \tau\tau$ | 6.3% |



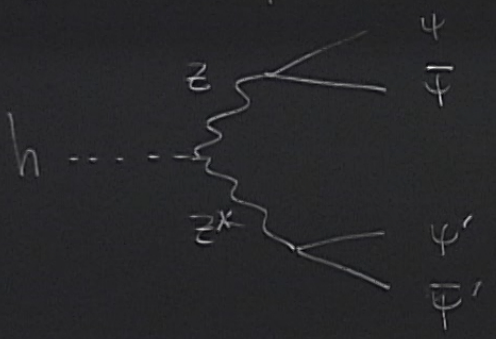
$h \to W^+W^-, ZZ$




forbidden for both gauge boson on-shell
One (or both) must be virtual

"golden mode" $\psi, \psi' = e, \mu$

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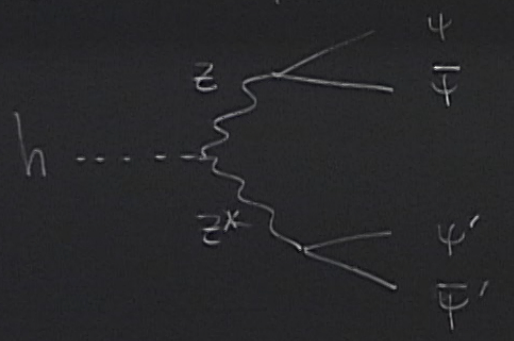



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$h \to ZZ^* \to 4$ leptons. ($e^+e^-$ or $\mu^+\mu^-$)

$h \to W^+W^-, ZZ$




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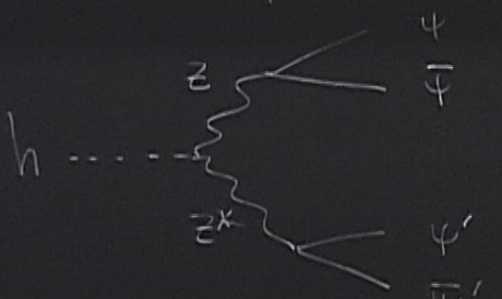
"golden mode" $\psi, \psi' = e, \mu$

$h \to ZZ^* \to 4$ leptons. ($e^+e^-$ or $\mu^+\mu^-$)

$\Gamma(h \to ZZ^*) \approx 0.11$ MeV (2.6 %)

$\Gamma(h \to WW^*) \approx 0.88$ MeV (22%)

$h \to W^+W^-, ZZ$




$\sim \frac{1}{p^2 - m_Z^2 + i\varepsilon}$

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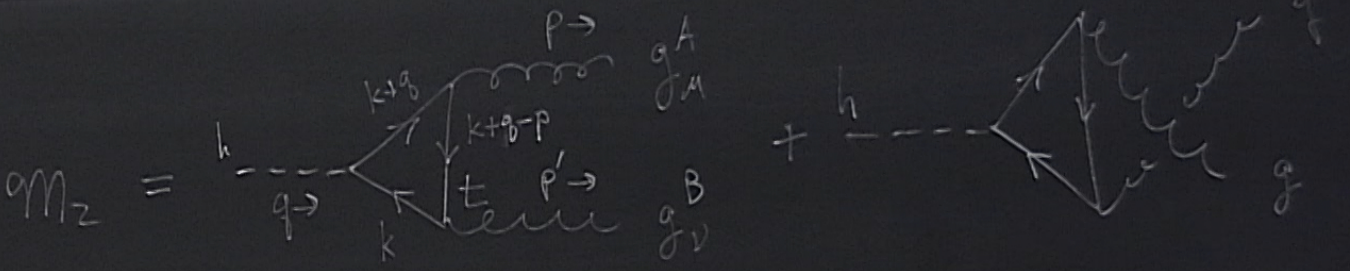
Loop decays:

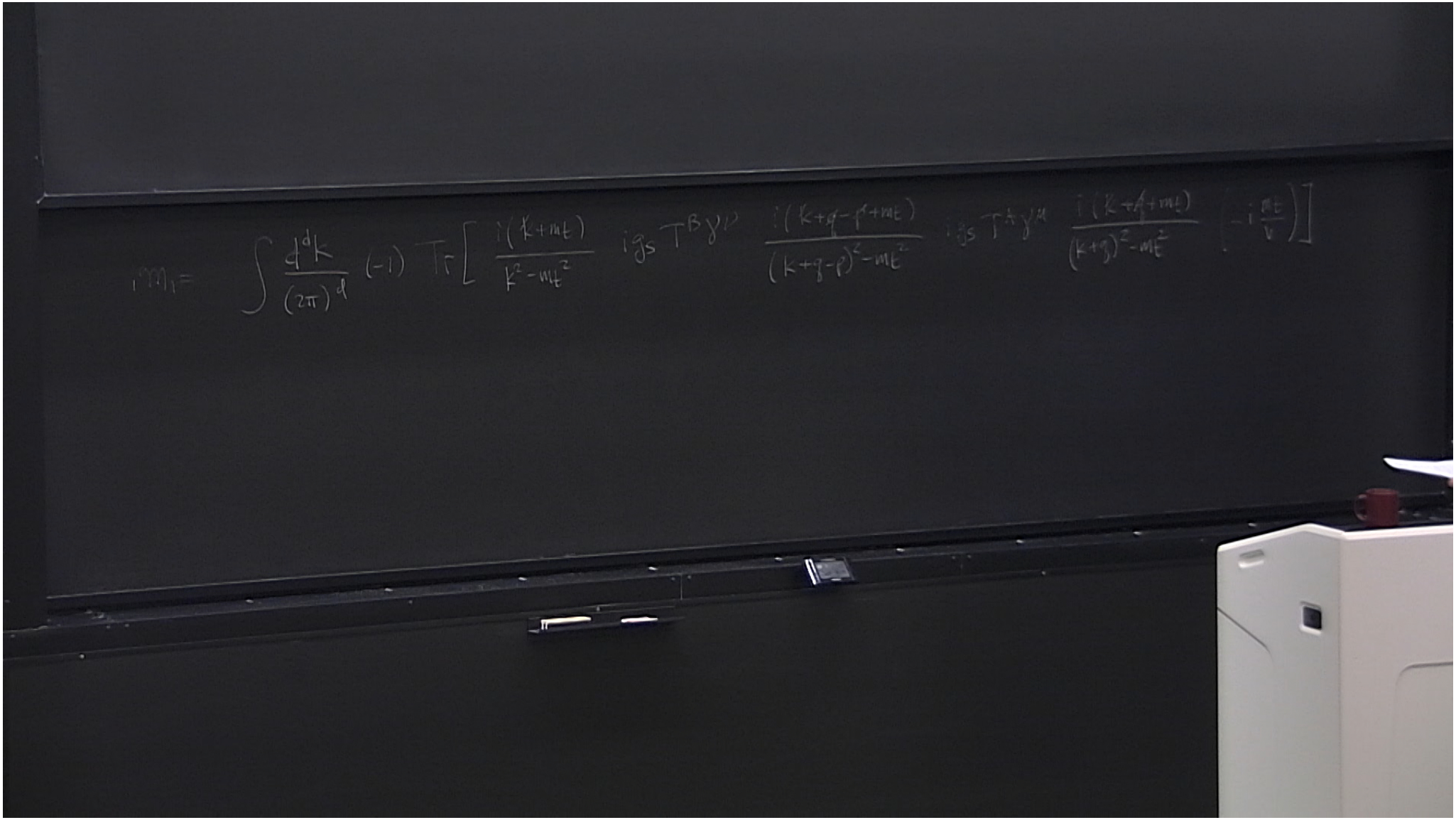
$\Gamma(h \to gg) \simeq 0.35$ MeV (8.6 %)

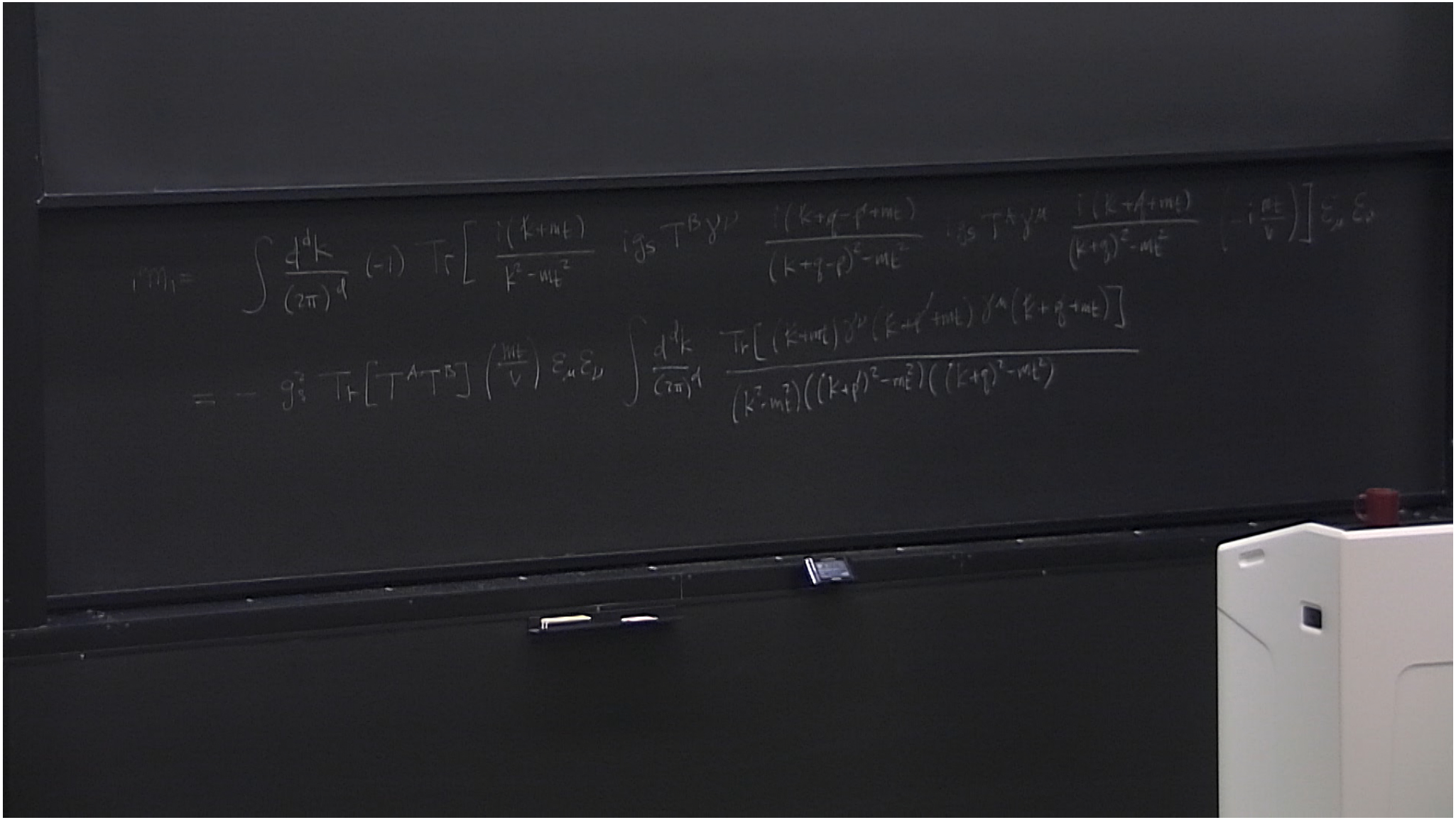
$\Gamma(h \to \gamma\gamma) \simeq 6 \times 10^{-3}$ MeV (0.15 %)

Decay $h \to gg$

$$i\mathcal{M} = i\mathcal{M}_1 + i\mathcal{M}_2 =$$




$$i\mathcal{M}_1 = \int \frac{d^4k}{(2\pi)^4} (-1)\, \mathrm{Tr}\left[ \frac{i(\not{k}+m_t)}{k^2-m_t^2}\, ig_s T^B \gamma^\nu\, \frac{i(\not{k}+\not{q}-\not{p}+m_t)}{(k+q-p)^2-m_t^2}\, ig_s T^A \gamma^\mu\, \frac{i(\not{k}+\not{A}+m_t)}{(k+q)^2-m_t^2}\, \left(-i\frac{m_t}{v}\right)\right]$$


$$i\mathcal{M}_1 = \int \frac{d^4k}{(2\pi)^4}(-1)\,\mathrm{Tr}\left[\frac{i(\not{k}+m_t)}{k^2-m_t^2}\,ig_s T^B \gamma^\nu \frac{i(\not{k}+\not{q}-\not{p}'+m_t)}{(k+q-p')^2-m_t^2}\,ig_s T^A \gamma^\mu \frac{i(\not{k}+\not{q}+m_t)}{(k+q)^2-m_t^2}\left(-i\frac{m_t}{v}\right)\right]\varepsilon_\mu \varepsilon_\nu$$
$$= -g_s^2\,\mathrm{Tr}\left[T^A T^B\right]\left(\frac{m_t}{v}\right)\varepsilon_\mu \varepsilon_\nu \int \frac{d^4k}{(2\pi)^4} \frac{\mathrm{Tr}\left[(\not{k}+m_t)\gamma^\nu(\not{k}+\not{p}'+m_t)\gamma^\mu(\not{k}+\not{q}+m_t)\right]}{(k^2-m_t^2)((k+p')^2-m_t^2)((k+q)^2-m_t^2)}$$

Feynman parameters:

$$i\mathcal{M}_1 = -g_s^2 \, \mathrm{Tr}[T^A T^B] \, \frac{m_t}{v} \, \varepsilon_\mu \varepsilon_\nu \times \int dx\, dy\, dz \; 2\,\delta(1-x-y-z) \, \frac{\mathrm{Tr}[\dots]}{\left[(k^2 - m_t^2)x + ((k+p')^2 - m_t^2)y + ((k+q)^2 - m_t^2)z\right]^3}$$

Feynman parameters:

$$\frac{1}{ABC} = \int dx\,dy\,dz\; \frac{2!}{(Ax+By+Cz)^3}\,\delta(1-x-y-z)$$

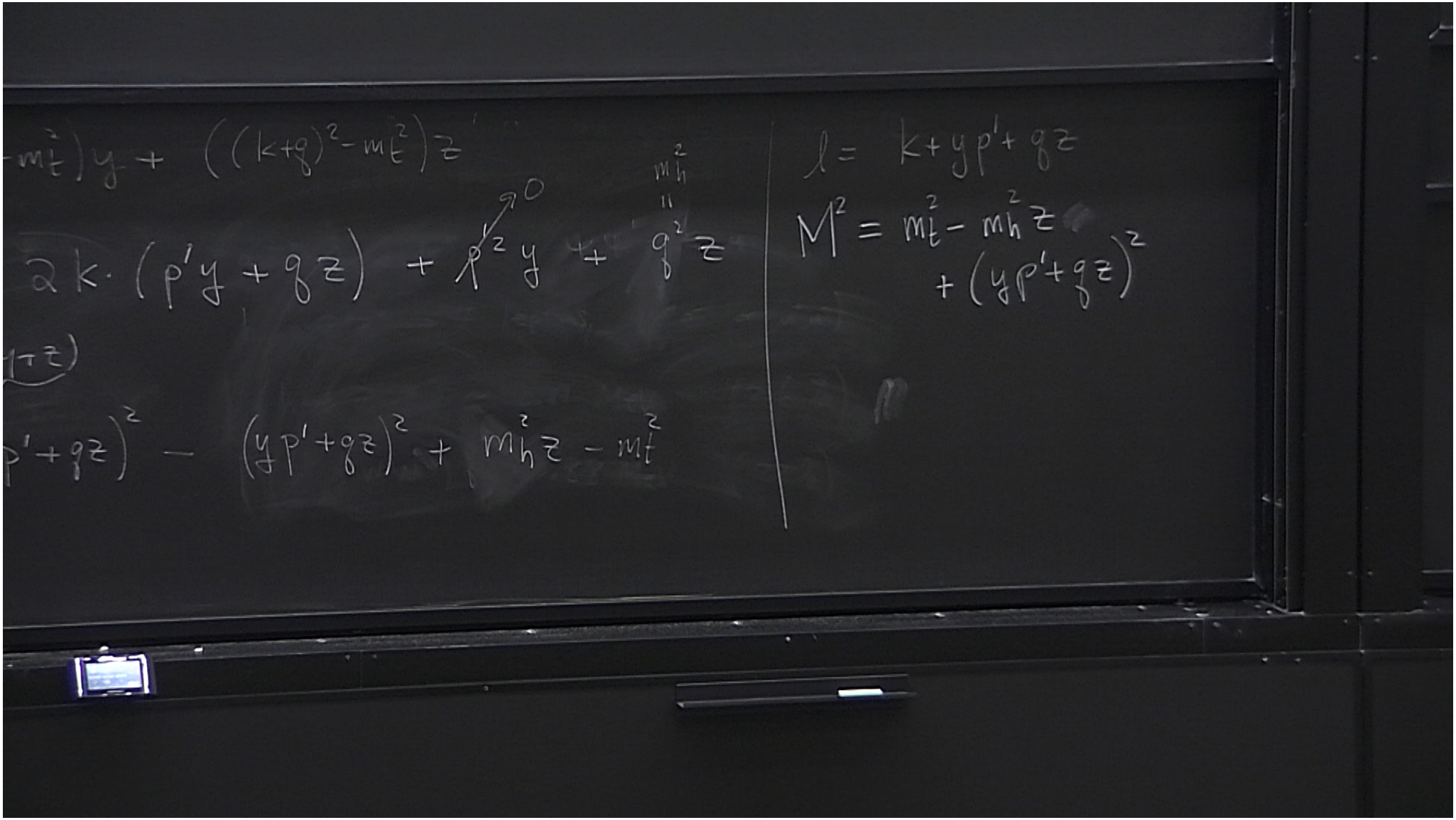
$$i\mathcal{M}_1 = -g_s^2\,\mathrm{Tr}[T^A T^B]\,\frac{m_t}{v}\,\varepsilon_\mu \varepsilon_\nu \int \frac{d^4k}{(\pi)^4} \int dx\,dy\,dz\; 2\,\delta(1-x-y-z)\,\frac{\mathrm{Tr}[\dots]}{\left[(k^2-m_t^2)x + ((k+p')^2-m_t^2)y + ((k+q)^2-m_t^2)z\right]^3}$$

$$(k^2 - m_t^2)x + ((k+p')^2 - m_t^2)y + ((k+q)^2 - m_t^2)z$$

$$k^2 \underbrace{(x+y+z)}_{1} + 2k\cdot(p'y + qz) + \cancel{p'^2}^{\,0}\, y + m_h^2 z$$

$$- m_t^2 \underbrace{(x+y+z)}_{1}$$

$$= (k + yp' + qz)^2 - (yp' + qz)^2 + m_h^2 z - m_t^2$$


$-m_t^2)y + ((k+q)^2 - m_t^2)z$
$2k\cdot(p'y+qz) + p'^2 y + q^2 z$
$m_h^2$
$p'^2 = 0$
$(1-z)$
$p'+qz)^2 - (yp'+qz)^2 + m_h^2 z - m_t^2$
$l = k + yp' + qz$
$M^2 = m_t^2 - m_h^2 z + (yp'+qz)^2$

$$M^2 = m_t^2 - m_h^2 z + m_h^2 z^2 + 2yz\, p' \cdot q$$

$$= m_t^2 - m_h^2 z \underbrace{(1 - z - y)}_{x}$$

$$= m_t^2 - xz m_h^2$$

$$p' \cdot q = p' \cdot (p + p') = p \cdot p' = \frac{1}{2}(p+p')^2 = \frac{1}{2} q^2 = \frac{1}{2} m_h^2$$

$$i\mathcal{M} = -g_s^2 \, \mathrm{Tr}[T^A T^B] \, \frac{m_t}{V} \, \varepsilon_\mu \varepsilon_\nu \int \frac{d^4\ell}{(2\pi)^4} \int dx\, dy\, dz \; 2\delta(1-x-y-z) \; \frac{\mathrm{Tr}[\dots]}{(\ell^2 - M^2)^3}$$

$$\mathrm{Tr}[\dots] = 16\, m_t\, \ell^\mu \ell^\nu - 4\ell^2 m_t \eta^{\mu\nu} + 4m_t^3 \eta^{\mu\nu} - 2m_t m_h^2 (1-2xz)\eta^{\mu\nu} + 4(1-4xz) p^\mu p'^\nu m_t$$

Neglected terms linear in $\ell$ and terms with $p^\mu, p'^\nu$

$$\int \frac{d^4k}{(2\pi)^4} (-1) \, \mathrm{Tr}\left[ \frac{i(\not{k}+m_t)}{k^2 - m_t^2} \, ig_s T^B \gamma^\nu \, \frac{i(\not{k}+\not{q}-\not{p}'+m_t)}{(k+q-p')^2 - m_t^2} \, ig_s T^A \gamma^\mu \right.$$

d- dimensions: $l^\mu l^\nu \longrightarrow \frac{1}{d} l^2 \eta^{\mu\nu}$ under the integral.

$$C = \frac{1}{d} \int \frac{d^d l}{(2\pi)^d} \frac{l^2}{(l^2 - M^2)^3}$$

if $\mu \neq \nu$ integral $\int \frac{d^d l}{(2\pi)^d} \frac{l^\mu l^\nu}{(l^2 - M^2)^3}$ vanishes.

$$\int \frac{d^d l}{(2\pi)^d} \frac{l^\mu l^\nu}{(l^2 - M^2)^3} = \int \frac{d^d l}{(2\pi)^d} \frac{l^2/d}{(l^2 - M^2)^3}$$

$$\eta_{\mu\nu} \int \frac{d^d l}{(2\pi)^d} \frac{l^\mu l^\nu}{(l^2 - M^2)^3} = C \, \eta^{\mu\nu} \eta_{\mu\nu} = C d$$

$$= \int \frac{d^d l}{(2\pi)^d} \frac{l^2}{(l^2 - M^2)^3}$$

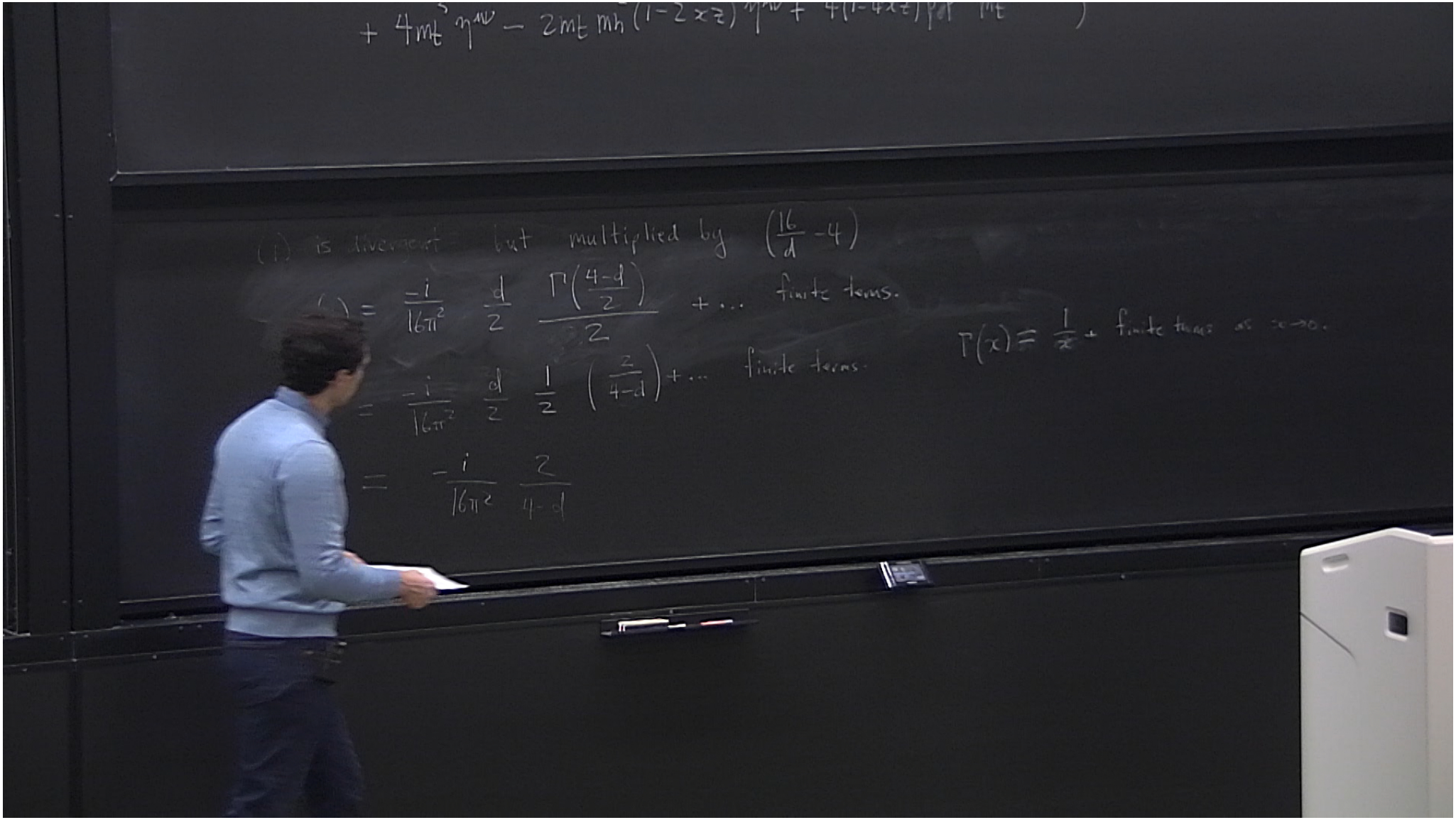
$$i\mathcal{M}_1 = -g_s^2\,\mathrm{Tr}[T^A T^B]\,\frac{m_t^2}{v}\,\varepsilon_\mu\varepsilon_\nu \int \frac{d^d\ell}{(2\pi)^d}$$

$$\times \int dx\,dy\,dz\; 2\,\delta(1-x-y-z)\cdot \frac{1}{(\ell^2-M^2)^3}$$

$$\times \left( \left(\frac{16}{d}-4\right)\ell^2\,\eta^{\mu\nu} + 2\left(2m_t^2 - m_h^2(1-2xz)\right)\eta^{\mu\nu} + 4(1-4xz)\,p^\nu p'^\mu \right)$$

$$(1) = \int \frac{d^d\ell}{(2\pi)^d} \frac{\ell^2}{(\ell^2 - M^2)^3} = \frac{-i}{(4\pi)^{d/2}} \frac{d}{2} \frac{\Gamma(3 - \frac{d}{2} - 1)}{\Gamma(3)} \left(\frac{1}{M^2}\right)^{3 - \frac{d}{2} - 1}$$

$$(2) = \int \frac{d^d\ell}{(2\pi)^d} \frac{1}{(\ell^2 - M^2)^3} = \frac{i}{(4\pi)^{d/2}} \frac{\Gamma(3 - d/2)}{\Gamma(3)} \left(\frac{1}{M^2}\right)^{3 - d/2}$$


$+ 4m_t^2 \eta^{\mu\nu} - 2m_t m_h$
(1) is divergent but multiplied by $\left(\frac{16}{d} - 4\right)$
$= \frac{-i}{16\pi^2} \frac{d}{2} \frac{\Gamma\left(\frac{4-d}{2}\right)}{2} + \ldots$ finite terms.
$= \frac{-i}{16\pi^2} \frac{d}{2} \frac{1}{2} \left(\frac{2}{4-d}\right) + \ldots$ finite terms.
$= \frac{-i}{16\pi^2} \frac{2}{4-d}$
$\Gamma(x) = \frac{1}{x} +$ finite terms as $x \to 0$.

(1) is divergent but multiplied by $\left(\frac{16}{d}-4\right)$

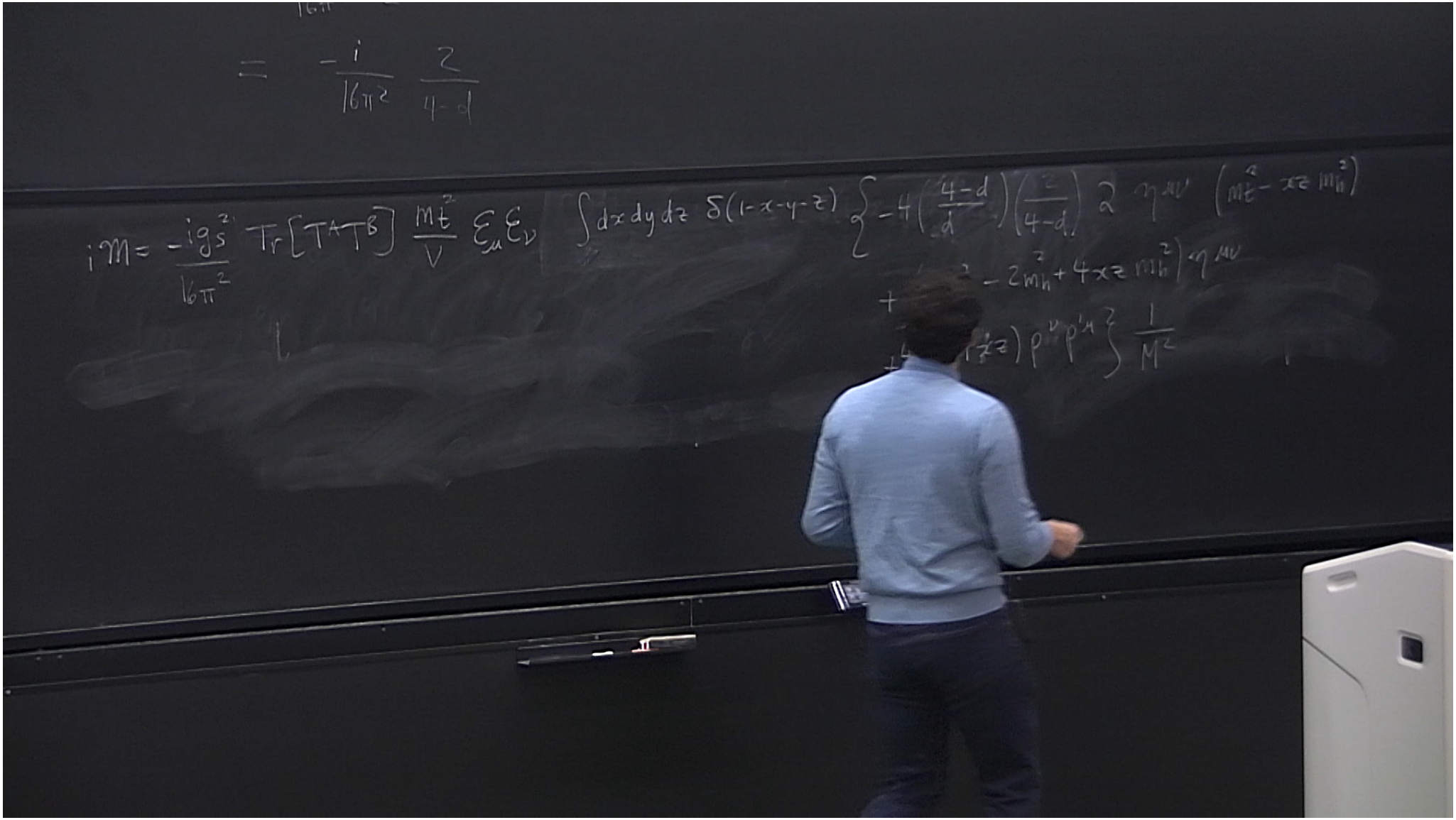
$$(1) = \frac{-i}{16\pi^2}\,\frac{d}{2}\,\frac{\Gamma\left(\frac{4-d}{2}\right)}{2} + \ldots \text{ finite terms.}$$

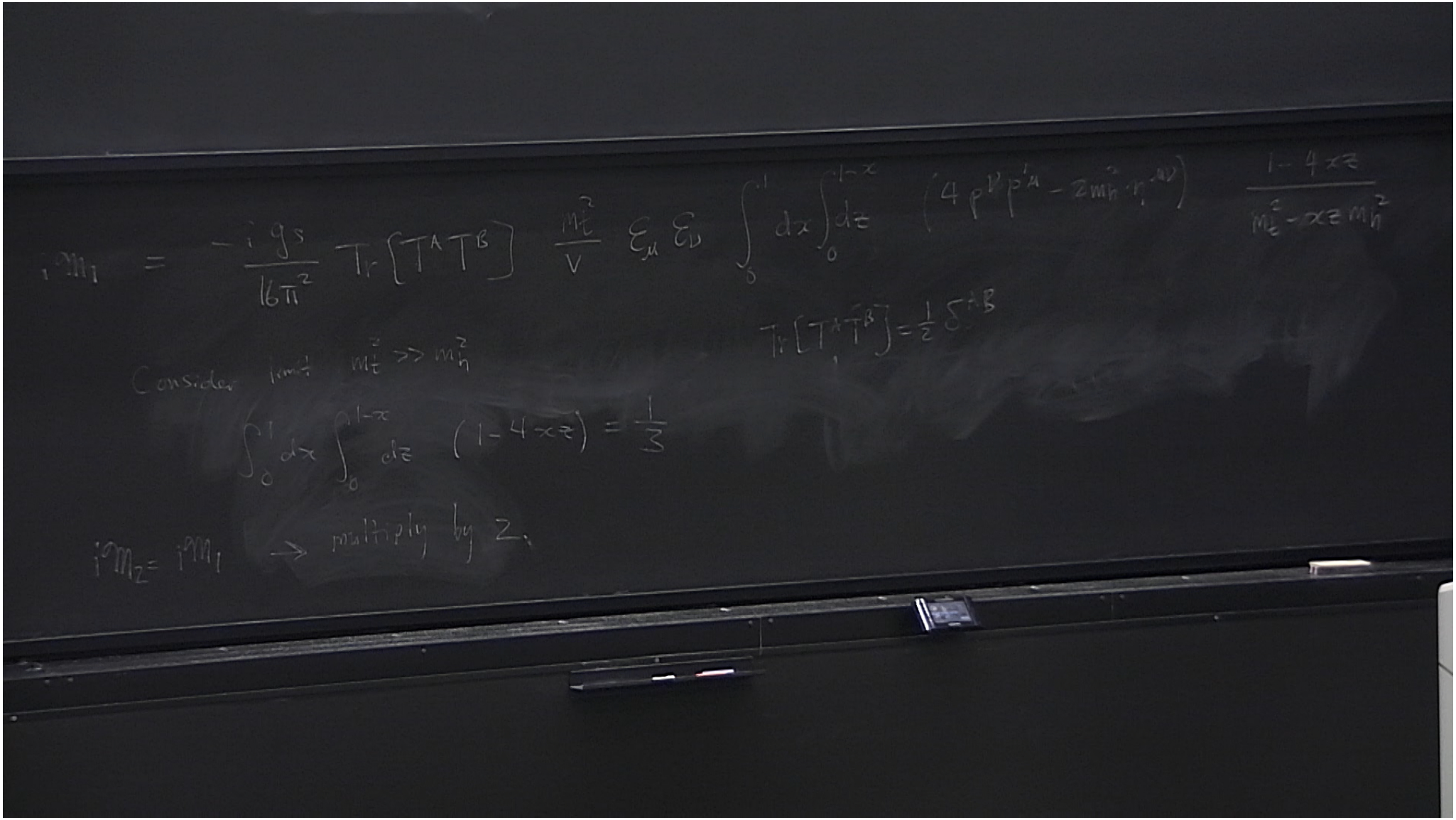
$$= \frac{-i}{16\pi^2}\,\frac{d}{2}\,\frac{1}{2}\left(\frac{2}{4-d}\right) + \ldots \text{ finite terms.}$$

$$= -\frac{i}{16\pi^2}\,\frac{2}{4-d}$$

$\Gamma(x) = \frac{1}{x} +$ finite terms as $x \to 0$.

$$M = -g_s^2\,\mathrm{Tr}[T^A T^B]\,\frac{mt}{V}\,\varepsilon_\mu \varepsilon_\nu \int \frac{d^d\ell}{(2\pi)^d} \int dx\,dy\,dz\; 2\,\delta(1-x-y-z)\,\frac{\mathrm{Tr}[\ldots]}{(\ell^2 - M^2)^3}$$

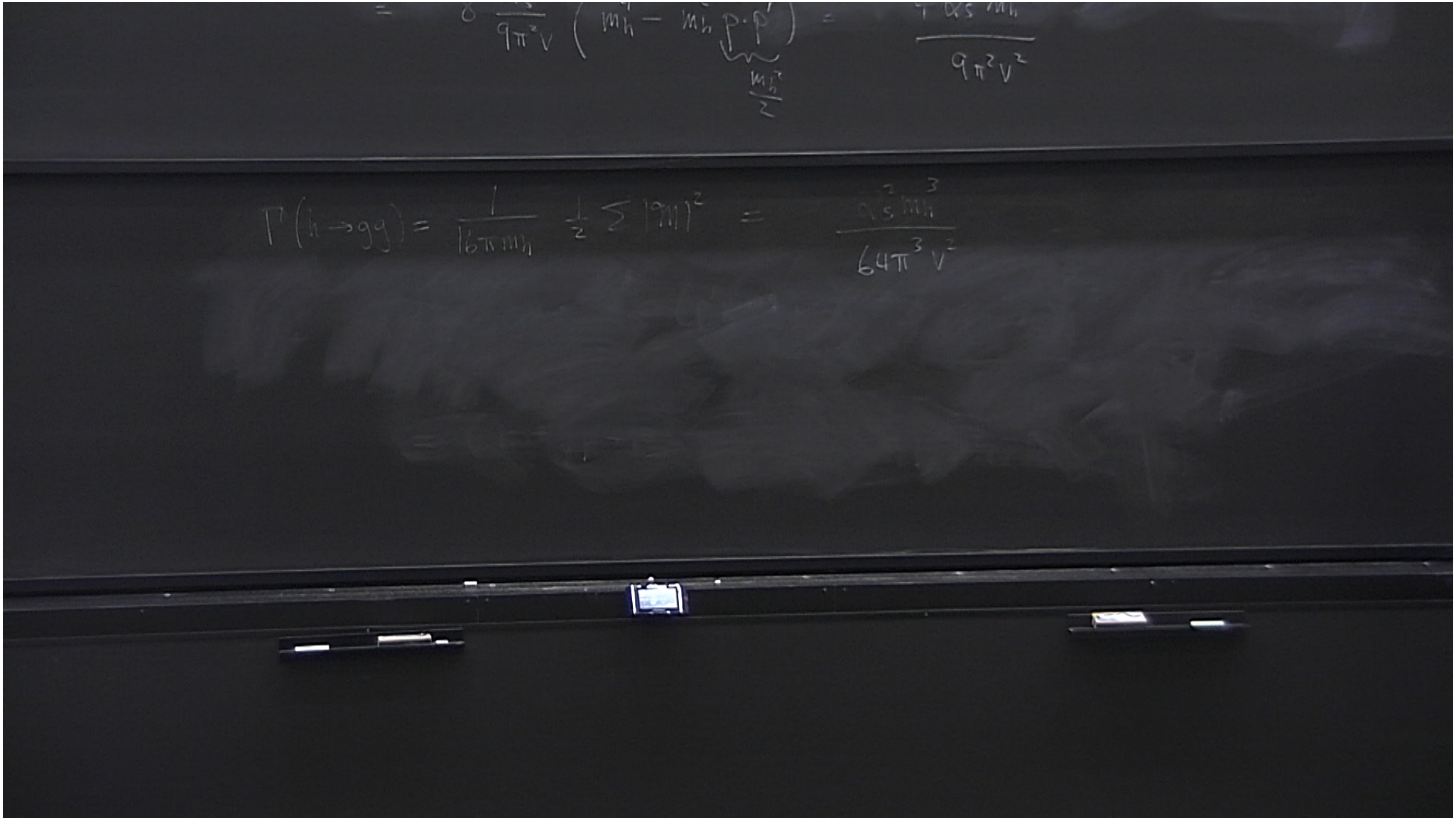

$= \frac{-i}{16\pi^2} \frac{2}{4-d}$
$i\mathcal{M} = \frac{-ig_s^2}{16\pi^2} \mathrm{Tr}[T^A T^B] \frac{m_t^2}{v} \varepsilon_\mu \varepsilon_\nu \int dx\,dy\,dz\, \delta(1-x-y-z) \left\{ -4\left(\frac{4-d}{d}\right)\left(\frac{2}{4-d}\right) 2\,\eta^{\mu\nu} \left(m_t^2 - xz\,m_h^2\right) \right.$
$- 2m_h^2 + 4xz\,m_h^2) \cdot \eta^{\mu\nu}$
$+$
$\left. xz)\, p^\nu p'^\mu \right\} \frac{1}{M^2}$


$i\mathcal{M}_1 = \frac{-ig_s}{16\pi^2} \mathrm{Tr}[T^A T^B] \frac{m_t^2}{V} \varepsilon_\mu \varepsilon_\nu \int_0^1 dx \int_0^{1-x} dz \left(4 p^\nu p'^\mu - 2 m_h^2 \eta^{\mu\nu}\right) \frac{1-4xz}{m_t^2 - xz m_h^2}$
$\mathrm{Tr}[T^A T^B] = \frac{1}{2}\delta^{AB}$
Consider $\lim \frac{m_t^2}{} \gg m_h^2$
$\int_0^1 dx \int_0^{1-x} dz \; (1-4xz) = \frac{1}{3}$
$i\mathcal{M}_2 = i\mathcal{M}_1 \rightarrow$ multiply by 2.

$$i\mathcal{M} = -\frac{i\alpha_s}{3\pi V}\,\varepsilon_\mu \varepsilon_\nu \left(p^\nu p'^\mu - \frac{m_h^2}{2}\eta^{\mu\nu}\right)\delta^{AB}$$

$$\sum |\mathcal{M}|^2 = \left(\frac{\alpha_s}{3\pi V}\right)^2 \underbrace{\delta^{AB}\delta_{AB}}_{8}\left(p^\nu p'^\mu - \frac{m_h^2}{2}\eta^{\mu\nu}\right)\left(p_\nu p'_\mu - \frac{m_h^2}{2}\eta_{\mu\nu}\right)$$

$$= 8\,\frac{\alpha_s^2}{9\pi^2 V}\left(m_h^4 - m_h^2 \underbrace{p\cdot p'}_{\frac{m_h^2}{2}}\right) = \frac{4\alpha_s^2 m_h^4}{9\pi^2 V^2}$$


$\Gamma(h \to gg) = \frac{1}{16\pi m_h} \frac{1}{2} \sum |\mathcal{M}|^2 = \frac{\alpha_s^2 m_h^3}{64\pi^3 v^2}$