

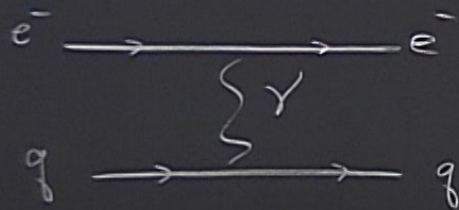
Title: PSI 17/18 - Standard Model - Lecture 8

Date: Jan 12, 2018 09:00 AM

URL: <http://pirsa.org/18010015>

Abstract:

parton level cross section.

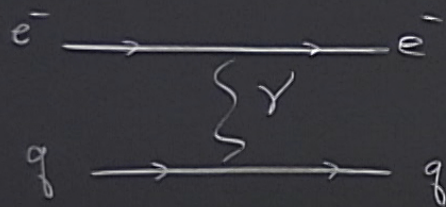


$$\frac{1}{4} \sum |\mathcal{M}|^2 = \frac{2e^4 Q_q^2}{t^2} (\hat{s}^2 + \hat{u}^2)$$

$$\hat{\sigma}(e^- q \rightarrow e^- q) = \frac{1}{2E_q 2E_p |v_{rel}|} \int \frac{d^3 \ell'}{(2\pi)^3} \frac{1}{2E_{\ell'}} \int \frac{d^3 p'}{(2\pi)^3} \frac{1}{2E_{p'}} (2\pi)^4 \delta^4(\ell + p - \ell' - p') \cdot \frac{1}{4} \sum |\mathcal{M}|^2$$

=

Parton level cross section:



$$\frac{1}{4} \sum |\mathcal{M}|^2 = \frac{2e^4 Q_q^2}{\hat{s}^2} (\hat{s}^2 + \hat{u}^2)$$

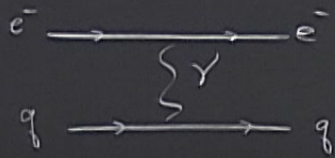
$$\hat{\sigma}(e^- q \rightarrow e^- q) = \frac{1}{2E_l 2E_p |v_{rel}|} \int \frac{d^3 l'}{(2\pi)^3} \frac{1}{2E_{l'}} \int \frac{d^3 p'}{(2\pi)^3} \frac{1}{2E_{p'}} (2\pi)^4 \delta^4(l+p-l'-p') \cdot \frac{1}{4} \sum |\mathcal{M}|^2$$

Evaluate in CM frame.

$$= \frac{1}{2\hat{s}} \int \frac{d^3 l'}{(2\pi)^3} (2\pi) \delta(\sqrt{\hat{s}} - 2E_{l'}) \cdot \frac{1}{4} \sum |\mathcal{M}|^2$$

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Parton level cross section



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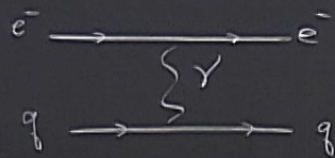
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parton level cross section



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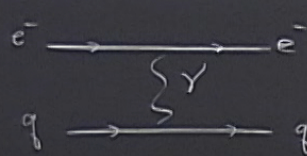
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$$\begin{aligned} \hat{s} &= (p+l)^2 \\ \hat{t} &= (l-l')^2 = q^2 \\ \hat{u} &= (p-l')^2 \end{aligned}$$

→ q

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$$\hat{t} = -2\ell \cdot \ell' = -2E_{\ell} E_{\ell'} (1 - \cos\theta) = -\frac{1}{2} \hat{S} (1 - \cos\theta)$$

$$d\hat{t} = \frac{1}{2} \hat{S} d\cos\theta$$

$$\frac{d\hat{\sigma}}{d\hat{t}} = \frac{d\hat{\sigma}}{d\cos\theta} \left(\frac{d\cos\theta}{d\hat{t}} \right) = \frac{2\pi \alpha^2 Q_g^2}{\hat{s}^2 \hat{t}^2} (\hat{s}^2 + \hat{u}^2)$$

$$\frac{2}{\hat{s}}$$

parton level $\rightarrow$ hadron level

$$\hat{t} = q^2 = -Q^2$$

p = 4-momentum of initial quark

P = 4-momentum of initial proton.

The "struck" quark

Parton level $\rightarrow$ hadron level

$$\frac{1}{t} = q^2 = -Q^2$$

p = 4-momentum of initial quark

P = 4-momentum of initial proton.

The "struck" quark that interacts with γ carries a fraction x of total proton momentum.

$$p = xP, \quad 0 < x < 1$$

Solve for x : $m_g = 0$. $p^2 = p'^2 = 0$.

$$p'^2 = (p + l - l')^2 = p^2 + (l - l')^2 + 2p \cdot (l - l') = p^2 + q^2 + 2p \cdot q$$

$$0 = p \cdot q + q^2$$

Solve for x : $m_g = 0$. $p^z = p'^z = 0$.

$$p'^z = (p + l - l')^2 = p^2 + (l - l')^2 + 2p \cdot (l - l') = p^2 + q^2 + 2p \cdot q$$

$$0 = 2p \cdot q + q^2 = 2x P \cdot q + q^2$$

$$x = \frac{-q^2}{2P \cdot q} = \frac{Q^2}{2P \cdot q}$$

Same x as before.

$$\hat{S} = (l+p)^2 = 2l \cdot p = x 2l \cdot p = x s$$

$$y = \frac{p \cdot q}{p \cdot l} = \frac{x \frac{p \cdot q}{x p \cdot l}}{x p \cdot l} = \frac{p \cdot q}{p \cdot l} = \frac{p \cdot (p' - p)}{p \cdot l} = \frac{2p \cdot p'}{2p \cdot l} = \frac{-\hat{t}}{\hat{S}}$$

$$\hat{t} = -y\hat{S} = -xys$$

$$\hat{u} = -\hat{S} - \hat{t} = -s - x(1-y)$$

$$\frac{d\hat{t}}{dy} = \frac{d\hat{t}}{d\hat{t}}$$

$$\hat{u} = -\hat{s} - \frac{\hat{t}}{t} = -s x(1-y)$$

$$\frac{d\hat{\sigma}}{dy} = \frac{d\hat{\sigma}}{d\hat{t}} \left| \frac{d\hat{t}}{dy} \right| = \frac{2\pi\alpha^2 Q_f^2}{Q^4} (y^2 + 2(1-y))$$

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Probability of finding quark q inside proton with a given momentum fraction x :

$$f_q(x) dx = \text{prob. for } q \text{ to have momentum between } x \text{ and } x+dx.$$

$$f_q(x) = \text{parton distribution function (pdf)}$$

$$\sigma(\bar{e}p \rightarrow e^-X) = \sum_g \int_0^1 dx f_g(x) \hat{\sigma}(\bar{e}g \rightarrow e^-g)$$

$$\frac{d\sigma}{dx} = \sum_g f_g(x) \hat{\sigma}(\bar{e}g \rightarrow e^-g)$$

$$\frac{d^2\sigma}{dx dy} = \sum_g f_g(x) \frac{d\hat{\sigma}}{dy}$$

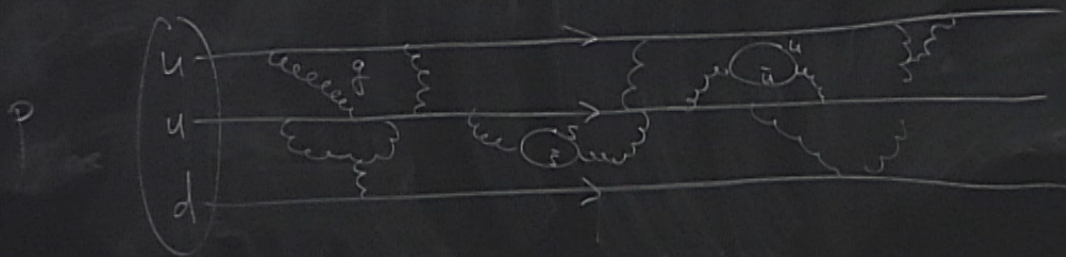
$dx dy$

Parton distribution functions

proton = uud

These are just valence quarks.

But also have $q\bar{q}$ pairs that fluctuate from vacuum (sea quarks)



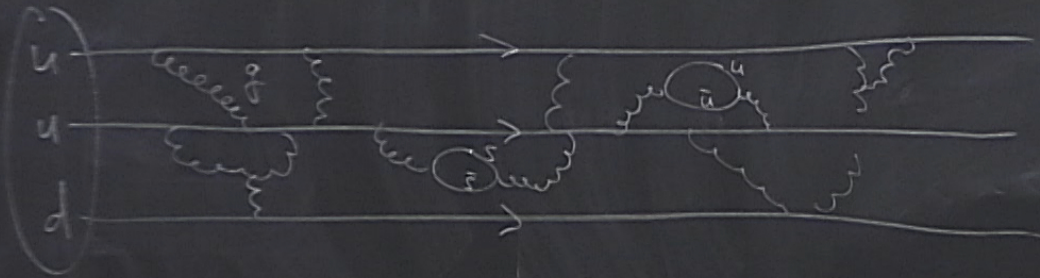
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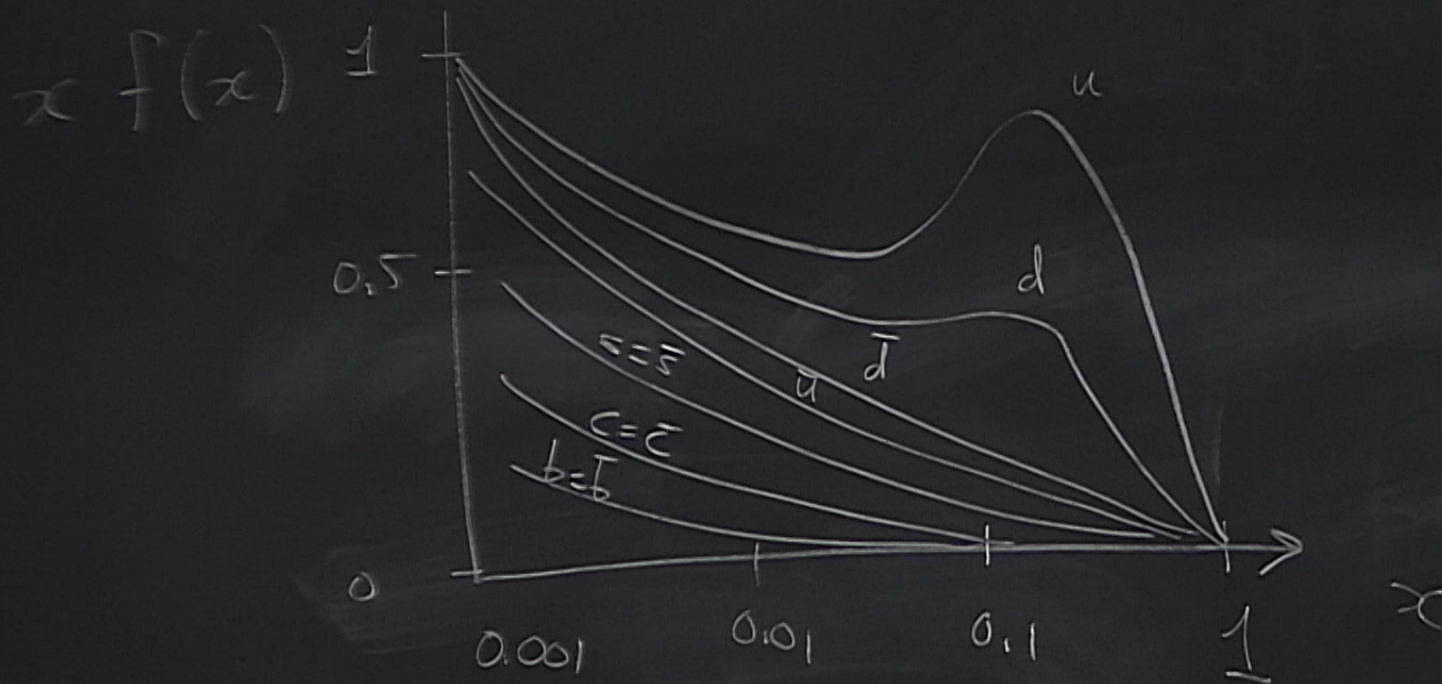
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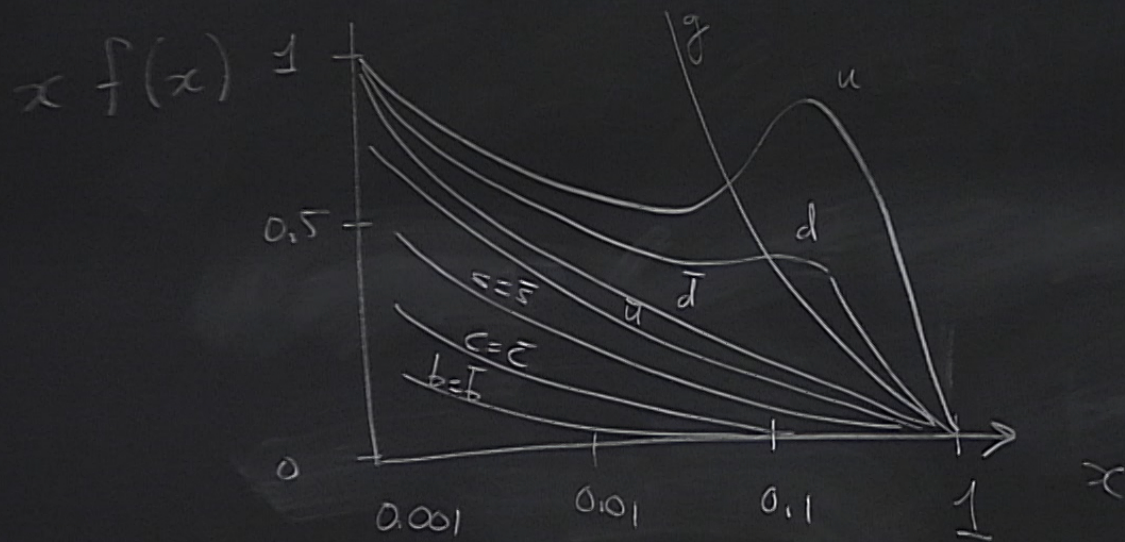
Photon can scatter off
of valence or sea quarks



$$m_q = 0. \quad p^2 = p'^2 = 0.$$

$$p^2 + q^2 + 2p \cdot q$$





$$\begin{aligned}
 \frac{d^2\sigma(\bar{e}p \rightarrow \bar{e}x)}{dx dy} &= \sum_g f_g(x) \frac{d\hat{\sigma}}{dy} \\
 &= \sum_g x f_g(x) \frac{2\pi\alpha^2 Q_g^2 S}{Q^4} \left(y^2 + 2(1-y) \right)
 \end{aligned}$$

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 &= \sum_g x f_g(x) \frac{2\pi\alpha^2 Q_g^2 S}{Q^4} (y^2 + 2(1-y)) \\
 &= \frac{2\pi\alpha^2}{Q^4} \left[F_1^e(x, Q^2) x y^2 + 2 F_2^e(x, Q^2) (1-y) \right]
 \end{aligned}$$

$$\begin{aligned}
 x F_1^e(x, Q^2) &= F_2^e(x, Q^2) = \sum_q x Q_q^2 f_q(x) \\
 &= x \left(\frac{2}{3}\right)^2 (f_u(x) + f_{\bar{u}}(x)) \\
 &\quad + x \left(\frac{1}{3}\right)^2 (f_d(x) + f_{\bar{d}}(x)) \\
 &\quad + x \left(\frac{1}{3}\right)^2 (f_s(x) + f_{\bar{s}}(x)) + \dots
 \end{aligned}$$

General calculation: $F_{1,2}^e(x, Q^2)$

parton level calculation: $F_{1,2}^e(x, Q^2)$ } Bjorken scaling.
(1). $F_{1,2}^e(x, Q^2)$ } only functions of x
not of Q^2 }
proton is made from spin- $\frac{1}{2}$ quarks

General calculation: $F_{1,2}^e(x, Q^2)$

parton level calculation: proton is made from spin- $\frac{1}{2}$ quarks

(1). $F_{1,2}^e(x, Q^2)$ only functions of x } Bjorken scaling.
not of Q^2

Underlying scattering process is elastic $\rightarrow$ only one independent variable.

(2) $x F_1^e(x) = F_2^e(x)$ Callan-Gross relation

(2) $x F_1^e(x) = F_2^e(x)$ Callan-Gross relation

Scaling violations: $O(\alpha_s)$ QCD corrections lead to violation of Bjorken scaling.
In agreement with expt. data.

Higgs decays & production

Higgs boson has largest couplings to fields with largest mass i.e. $W^\pm, Z, t$
Mass $m_h \approx 125 \text{ GeV}$ $h \rightarrow W^+ W^-, Z Z, t \bar{t}$ not allowed

This is a good thing: explore also subleading coupling to lighter SM particles ($h \rightarrow b \bar{b}, h \rightarrow \tau \bar{\tau}$)

$$\left(\frac{2}{3}\right)^2 \begin{pmatrix} f_u(x) + f_{\bar{u}}(x) \\ f_d(x) + f_{\bar{d}}(x) \end{pmatrix}$$

Tree-level decays

$$\bullet h \rightarrow \psi \bar{\psi}$$

$$\mathcal{L}_{\text{int}} = -\frac{m_{\psi}}{v} h \bar{\psi} \psi$$



$$i\mathcal{M}(h \rightarrow \psi \bar{\psi}) = -i \frac{m_{\psi}}{v} \bar{u}(p) v(p')$$

$$\sum |\mathcal{M}|^2 = \frac{m_{\psi}^2}{v^2} 4 p \cdot p' = \frac{2 m_{\psi}^2 m_h^2}{v^2}$$

$$q^2 = (p + p')^2 = p^2 + p'^2 + 2 p \cdot p'$$

$$\Gamma(h \rightarrow \psi \bar{\psi}) = \frac{1}{16\pi m_h} \sum |\mathcal{M}|^2 = \frac{m_\psi^2 m_h}{8\pi v^2}$$

$$\Gamma(h \rightarrow \tau \bar{\tau}) = \frac{m_\tau^2 m_h}{8\pi v^2} \approx 0.26 \text{ MeV} \quad (2.6\% \text{ Branching ratio})$$

$$\Gamma(h \rightarrow b \bar{b}) = \frac{m_b^2 m_h}{8\pi v^2}$$

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RG running of m_b from $m_b \rightarrow m_h$
 Suppresses m_b by factor of ~ 2

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RG running of m_b from $m_b \rightarrow m_h$
 suppresses m_b by factor of α_s
 smaller.

$$\Gamma(h \rightarrow c \bar{c}) = \frac{3m_c^2}{8\pi V^2} \approx 0.4 \text{ MeV} \quad (2.9\%)$$

RG running suppresses m_c by α_s .