

Title: PSI 17/18 - Standard Model - Lecture 7

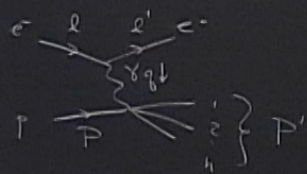
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Abstract:

Inclusive e^-p scattering

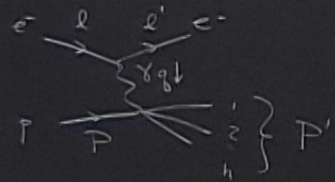
$$\sigma(e^-p \rightarrow e^-X) = \sum_H \sigma(e^-p \rightarrow e^-H)$$



$$= \sum_H \frac{1}{2E_q 2E_P |V_{rel}|} \int \frac{d^3q'}{(2\pi)^3} \frac{1}{2E_{q'}} \prod_{i=1}^n \int \frac{d^3P'_i}{(2\pi)^3} \frac{1}{2E_{P'_i}} (2\pi)^4 \delta^4(q + P - q' - \sum_{i=1}^n P'_i) \frac{1}{4} \sum |M(e^-p \rightarrow e^-H)|^2$$

Inclusive e^-p scattering

$$\sigma(e^-p \rightarrow e^-X) = \sum_H v(e^-p \rightarrow e^-H)$$



$$= \sum_H \frac{1}{2E_l 2E_p |v_{rel}|} \int \frac{d^3 l'}{(2\pi)^3} \frac{1}{2E_{l'}} \frac{1}{2E_{q'}} \int \frac{d^3 P'}{(2\pi)^3} \frac{1}{2E_{P'}} (2\pi)^4 \delta^4(l+P-l'-\sum_{i=1}^n P'_i) \frac{1}{4} \sum |M(e^-p \rightarrow e^-H)|^2$$

$$iM(e^-p \rightarrow e^-H) = \bar{u}(l') i\epsilon \gamma_\mu u(l) \frac{-i}{q^2} \langle H | e j_{em}^\mu | p \rangle$$

$$e j_{em}^\mu = \text{EM current that couples to } A_\mu \text{ in QED}$$

$$= e \sum_f Q_f \bar{\psi}_f \gamma^\mu \psi_f$$

$$P \rightarrow \left\{ \begin{array}{l} P' \\ h \end{array} \right.$$

$$i\mathcal{M}(e^-p \rightarrow e^-H) = \bar{u}(l') i e \gamma_\mu u(l) \frac{1}{q^2} \langle H | e j_{em}^\mu | P \rangle$$

$$\begin{aligned} \frac{1}{4} \sum_{\text{spins}} |\mathcal{M}|^2 &= \frac{e^4}{q^4} \frac{1}{4} \text{Tr}[\not{l}' \gamma^\mu \not{l} \gamma^\nu] \langle H | e j_{em}^\mu | P \rangle \langle P | e j_{em}^\nu | H \rangle \\ \sigma(e^-p \rightarrow e^-X) &= \frac{1}{2E_l |V_{rel}|} (2\pi) \int \frac{d^3 l'}{(2\pi)^3} \frac{1}{2E_{l'}} \frac{e^4}{q^4} \frac{1}{2} \text{Tr}[\not{l}' \gamma^\mu \not{l} \gamma^\nu] \\ &\times \frac{1}{2E_P} \sum_H \prod_{i=1}^n \int \frac{d^3 P_i'}{(2\pi)^3} \frac{1}{2E_{P_i'}} (2\pi)^4 \delta^4(l+P-l'-\sum_{i=1}^n P_i') \\ &\times \frac{1}{2} \sum_{\text{spins}} \langle H | j_{em}^\mu | P \rangle \langle P | j_{em}^\nu | H \rangle \end{aligned}$$

$$= \sum_H \frac{1}{2E_Q 2E_P |V_{rel}|} \int \frac{d^3 \ell}{(2\pi)^3} \frac{1}{2E_{Q'}} \frac{1}{2E_{P'}} \int \frac{d^3 P'}{(2\pi)^3 2E_{P'}}$$

$$i\mathcal{M}(e^- p \rightarrow e^- H) = \bar{u}(\ell') i\epsilon \gamma_\mu u(\ell) \frac{-i}{q^2} \langle H | e j_{em}^\mu | p \rangle$$

$e j_{em}^\mu$ = EM current that couples to A_μ in $\mathcal{L}$.

$$= e \sum_q Q_q \bar{q} \gamma^\mu q$$

$$\frac{1}{4} \sum_{\text{spins}} |\mathcal{M}|^2 = \frac{e^4}{q^4} \frac{1}{4} \text{Tr}[\not{\ell}' \gamma^\mu \not{\ell} \gamma^\nu] \langle H | e j_{em}^\mu | p \rangle \langle p | e j_{em}^\nu | H \rangle$$

$$\sigma(e^- p \rightarrow e^- X) = \frac{1}{2E_Q |V_{rel}|} (2\pi) \int \frac{d^3 \ell'}{(2\pi)^3} \frac{1}{2E_{Q'}} \frac{e^4}{q^4} \frac{1}{2} \text{Tr}[\not{\ell}' \gamma^\mu \not{\ell} \gamma^\nu]$$

$$\times \frac{1}{2E_P} \sum_H \prod_{i=1}^n \int \frac{d^3 P'_i}{(2\pi)^3} \frac{1}{2E_{P'_i}} (2\pi)^4 \delta^4(\ell + P - \ell' - \sum_{i=1}^n P'_i)$$

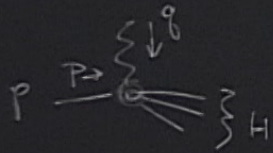
$$\times \frac{1}{2} \sum_{\text{spins}} \langle H | j_{em}^\mu | p \rangle \langle p | j_{em}^\nu | H \rangle$$

$$\left. \begin{array}{l} \\ \\ \end{array} \right\} W_{\mu\nu}$$

Unknown hadronic physics in $W_{\mu\nu}$

Leptonic tensor $L_{\mu\nu} = \frac{1}{2} \text{Tr}[\not{\epsilon}' \gamma_\mu \not{\epsilon} \gamma_\nu] = 2(l_\mu l'_\nu + l_\nu l'_\mu - l \cdot l' \eta_{\mu\nu})$

Use Lorentz invariance to restrict form of $W_{\mu\nu}$.
 $q, P =$ only 4-momenta $W_{\mu\nu}$ can depend on

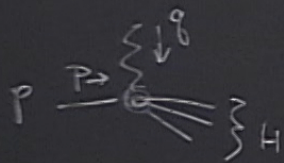


Unknown hadronic physics in $W_{\mu\nu}$

Leptonic tensor $L_{\mu\nu} = \frac{1}{2} \text{Tr}[\not{\epsilon}' \gamma_\mu \not{\epsilon} \gamma_\nu] = 2(l_\mu l'_\nu + l_\nu l'_\mu - l \cdot l' \eta_{\mu\nu})$

Use Lorentz invariance to restrict form of $W_{\mu\nu}$.

q, P = only 4-momenta $W_{\mu\nu}$ can depend on



$$W_{\mu\nu} \supset \eta_{\mu\nu}, P_\mu P_\nu, q_\mu q_\nu, P_\mu q_\nu, P_\nu q_\mu, \epsilon_{\alpha\beta\mu\nu} P^\alpha q^\beta$$

$g_\mu g_\nu$, $g_\mu P_\nu$, $g_\nu P_\mu$ all vanish when contracted with $L_{\mu\nu}$.
 Ex: $g^{\alpha\beta} P^\alpha P^\beta$ not allowed by Parity conservation. (EM is not chiral)
 $\eta_{\mu\nu} \rightarrow P_\mu P_\nu$

$g^{\mu}g_{\nu}$, $g_{\mu}P_{\nu}$, $g_{\nu}P_{\mu}$ all vanish when contracted with $L_{\mu\nu}$.
 $\epsilon_{\alpha\beta\mu\nu} g^{\alpha}P^{\beta}$ not allowed by Parity conservation. (EM is not chiral)

$$W_{\mu\nu} = -W_1 \eta_{\mu\nu} + W_2 \frac{P_{\mu}P_{\nu}}{M_p^2}$$

$W_{1,2}$ are Lorentz scalars

They depend on Lorentz scalar quantities e.g. $W_{1,2}(x, Q^2)$

$$\sigma(e^- p \rightarrow e^- X) = \frac{1}{2E_l |v_{rel}|} (2\pi) \int \frac{d^3 l'}{(2\pi)^3} \frac{1}{2E_{l'}} \frac{e^4}{g^4} L^{\mu\nu} W_{\mu\nu}$$

Evaluate in lab frame: p initially at rest $P^\mu = (m_p, 0, 0, 0)$

$$|v_{rel}| = 1$$

Also assume: $E_l, E_{l'}, \nu \gg m_p$

CM energy sq. $s = (l + p)^2 = m_p^2 + 2P \cdot l = m_p^2 + 2m_p E_l \approx 2m_p E_l$

$$L^{\mu\nu} W_{\mu\nu} = 2 (l^\mu l'^\nu + l^\nu l'^\mu - l \cdot l' \eta^{\mu\nu}) \left(-W_1 \eta_{\mu\nu} + W_2 \frac{p^\mu p^\nu}{m_p^2} \right)$$

$$= 4 l \cdot l' W_1 + W_2 \left[\frac{4(p \cdot l)(p \cdot l')}{m_p^2} - \dots \right]$$

$$= 2 Q^2 W_1 + W_2 \left[4 E_l E_{l'} - \dots \right]$$

$$L^{\mu\nu} W_{\mu\nu} = 2 (l^\mu l'^\nu + l^\nu l'^\mu - l \cdot l' \eta^{\mu\nu}) \left(-W_1 \eta_{\mu\nu} + W_2 \frac{P^\mu P^\nu}{m_P^2} \right)$$

$$= 4 l \cdot l' W_1 + W_2 \left[\frac{4(P \cdot l)(P \cdot l')}{m_P^2} - 2 l \cdot l' \right]$$

$$= 2 Q^2 W_1 + W_2 [4 E_l E_{l'} - Q^2]$$

$$u - l \cdot l' \eta^{\mu\nu}) \left(-W_1 \eta_{\mu\nu} + W_2 \frac{p_\mu p_\nu}{m_p^2} \right)$$

$$N_2 \left[\frac{4(p \cdot l)(p \cdot l')}{m_p^2} - 2 l \cdot l' \right]$$

$$N_2 \left[4E_l E_{l'} - Q^2 \right] = 2E_l E_{l'} \left[2W_1(1 - \cos\theta) + W_2(1 + \cos\theta) \right]$$

$$q^2 = -Q^2 = (l - l')^2 = -2l \cdot l'$$

$$Q^2 = 2l \cdot l' = 2E_l E_{l'}(1 - \cos\theta)$$

$$= 4m_p \nu x W_1 + (2E_e E_{e'} - \underbrace{2m_p \nu x}_{m_p \ll E_e, E_{e'}, \nu}) W_2$$

$$= 4m_p E_e y x W_1 + 4E_e^2 (1-y) W_2$$

$$\sigma(e^- p \rightarrow e^- X) = \frac{1}{2E_e} \frac{1}{4\pi^2} \int \frac{E_{e'}^2 dE_{e'} d\cos\theta}{2E_{e'}} (2\pi) \frac{e^4}{q^4} L_{\mu\nu} W^{\mu\nu}$$

$$\begin{aligned}
&= 4m_p \gamma x W_1 + \underbrace{(2E_e E_{e'} - 2m_p \gamma x)}_{m_p \ll E_e, E_{e'}, D} W_2 \\
&= 4m_p E_e y x W_1 + 4E_e^2 (1-y) W_2
\end{aligned}$$

$$\sigma(e^- p \rightarrow e^- X) = \frac{1}{2E_e} \frac{1}{4\pi^2} \int \frac{E_{e'}^2 dE_{e'} d\cos\theta}{2E_{e'}} (2\pi) \frac{e^4}{q^4} L_{\mu\nu} W^{\mu\nu}$$

Convert to integral over $\int dx dy$.

$$y = 1 - \frac{E_{e'}}{E_e} \rightarrow dy = - \frac{dE_{e'}}{E_e} \rightarrow dE_{e'} = - E_e dy$$

$$x = \frac{Q^2}{2m_p V} = \frac{2E_{e'}E_e(1 - \cos\theta)}{2m_p V} \rightarrow dx = - \frac{E_e E_{e'}}{m_p V} d\cos\theta$$

$$d\cos\theta = \frac{m_p V}{E_e E_{e'}} dx$$

$$\begin{aligned}
 \sigma(e^- p \rightarrow e^- X) &= \frac{1}{8\pi E_e} \int dx dy \frac{e^4}{Q^4} \left[4m_p^2 E_e y x \nu W_1 + 4E_e^2 (1-y) W_2 m_p \nu \right] \\
 &= \int dx dy \frac{2\pi\alpha^2}{Q^4} \left[S xy (2m_p W_1) + 2 S (1-y) (\nu W_2) \right]
 \end{aligned}$$

$$\begin{aligned}
 \sigma(e^- p \rightarrow e^- X) &= \frac{1}{8\pi E_e} \int dx dy \frac{e^4}{Q^4} \left[4m_p^2 E_e y x \nu W_1 + 4E_e^2 (1-y) W_2 m_p \nu \right] \\
 &= \int dx dy \frac{2\pi\alpha^2}{Q^4} \left[s xy \underbrace{(2m_p W_1)} + 2s(1-y)(\nu W_2) \right]
 \end{aligned}$$

$$\begin{aligned}
 \sigma(e^- p \rightarrow e^- X) &= \frac{1}{8\pi E_\ell} \int dx dy \frac{e^4}{Q^4} \left[4m_p^2 E_\ell y x \nu W_1 + 4\vec{E}_\ell^2 (1-y) W_2 m_p \nu \right] \\
 &= \int dx dy \frac{2\pi\alpha^2}{Q^4} \left[s xy \underbrace{(2m_p W_1)}_{F_1^e} + 2s(1-y) \underbrace{(\nu W_2)}_{F_2^e} \right]
 \end{aligned}$$

$$= 2\pi F_1(x, Q^2)$$

$$\sigma(e^- p \rightarrow e^- X) = \int_0^1 dx dy \frac{2\pi\alpha^2 s}{Q^4} \left[F_1^e(x, Q^2) xy^2 + 2F_2^e(x, Q^2)(1-y) \right]$$

$$\frac{d^2\sigma(e^- p \rightarrow e^- X)}{dx dy} = \frac{2\pi\alpha^2 s}{Q^4} \left[F_1^e xy^2 + 2F_2^e(1-y) \right]$$

Re-do calculation in SM. Use factorization:

$$\sigma(e^- p \rightarrow e^- H) = \sum_{\substack{\text{quark } q \\ \text{mom } p}} \text{Prob}(q, p) \times \hat{\sigma}(e^- q \rightarrow e^- q) \times \text{Prob}(H)$$

prob for finding
quark q w/ mom p
in proton
scattering cross section
for a given quark q
with momentum p .
prob for producing
hadronic state H

$$\begin{aligned}
 &= 4m_p \gamma x W_1 + (2E_e E_{e'} - \underbrace{2m_p \gamma x}_{m_p \ll E_e, E_{e'}, \gamma}) W_2 \\
 &= 4m_p E_e y x W_1 + 4E_e^2 (1-y) W_2
 \end{aligned}$$

$$\sigma(e^- p \rightarrow e^- X) = \frac{1}{2E_e} \frac{1}{4\pi^2} \int \frac{E_{e'}^2 dE_{e'} d\cos\theta}{2E_{e'}} (2\pi) \frac{e^4}{q^4} L_{\mu\nu} W^{\mu\nu}$$

Convert to integral over $\int dx dy$.

$\text{Prob}(H)$ is unknown. But don't need it for inclusive cross section

$$\sigma(e^-p \rightarrow e^-X) = \sum_H \sigma(e^-p \rightarrow e^-H) = \sum_{q,p} \text{Prob}(q,p) \hat{\sigma}(\bar{e}q \rightarrow \bar{e}q) \underbrace{\sum_H \text{Prob}(H)}_{=1}$$

$\text{Prob}(H)$ is unknown. But don't need it for inclusive cross section

$$\begin{aligned}\sigma(e^-p \rightarrow e^-X) &= \sum_H \sigma(e^-p \rightarrow e^-H) = \sum_{q,p} \text{Prob}(q,p) \hat{\sigma}(\bar{e}q \rightarrow \bar{e}q) \underbrace{\sum_H \text{Prob}(H)}_{=1 \text{ by confinement.}} \\ &= \sum_{q,p} \text{Prob}(q,p) \hat{\sigma}(e^-q \rightarrow e^-q)\end{aligned}$$

Parton model

$\text{Prob}(H)$ is unknown. But don't need it for inclusive cross section

$$\sigma(e^-p \rightarrow e^-X) = \sum_H \sigma(e^-p \rightarrow e^-H) = \sum_{q,p} \text{Prob}(q,p) \hat{\sigma}(\bar{e}q \rightarrow \bar{e}q) \underbrace{\sum_H \text{Prob}(H)}_{=1 \text{ by confinement.}}$$

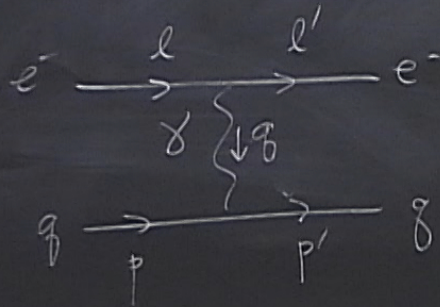
$$= \sum_{q,p} \text{Prob}(q,p) \hat{\sigma}(e^-q \rightarrow e^-q) \quad \text{Parton model}$$

Partons are constituents inside hadron. (e.g. quarks, antiquarks, gluons)

Parton quantities given hats (e.g. $\hat{\sigma}$)

Here EM scattering $\rightarrow$ only quarks & antiquarks relevant.

Parton-level cross section.



$$i\mathcal{M}(\bar{e}q \rightarrow \bar{e}q) = \bar{u}(l') i e \gamma^\mu u(l) \frac{-i}{q^2} \bar{u}(p') (-e Q_q \gamma_\mu) u(p)$$

$$\sigma(\bar{e}p \rightarrow \bar{e}X) = \frac{1}{8\pi E_\ell} \int dx dy \frac{e^4}{Q^4} \left[4m_p^2 E_\ell y x \nu W_1 + 4 \right]$$

$$\begin{aligned}
 \frac{1}{4} \sum |\mathcal{M}|^2 &= \frac{1}{4} \frac{e^4}{g^4} Q_g^2 \text{Tr}[\not{x}' \gamma^\mu \not{x} \gamma^\nu] \text{Tr}[\not{p}' \gamma_\mu \not{p} \gamma_\nu] \\
 &= \frac{4e^4}{g^4} Q_g^2 (l \cdot p \, l' \cdot p' + l \cdot p' \, l' \cdot p)
 \end{aligned}$$

$$\begin{aligned}\frac{1}{4} \sum |M|^2 &= \frac{1}{4} \frac{e^4}{g^4} Q_g^2 \text{Tr}[\not{x}' \gamma^\mu \not{x} \gamma^\nu] \text{Tr}[\not{p}' \gamma_\mu \not{p} \gamma_\nu] \\ &= \frac{4e^4}{g^4} Q_g^2 (l \cdot p \, l' \cdot p' + l \cdot p' \, l' \cdot p)\end{aligned}$$

Mandelstam variables:

$$\begin{aligned}\frac{1}{4} \sum |M|^2 &= \frac{1}{4} \frac{e^4}{q^4} Q_g^2 \text{Tr}[x' \gamma_\mu x \gamma_\nu] \text{Tr}[p' \gamma_\mu p \gamma_\nu] \\ &= \frac{4e^4}{q^4} Q_g^2 (l \cdot p \, l' \cdot p' + l \cdot p' \, l' \cdot p) = \frac{2e^4 Q_g^2}{t} (\hat{s}^2 + \hat{u}^2)\end{aligned}$$

Mandelstam variables:

$$\hat{s} = (p + l)^2 = 2p \cdot l = 2p' \cdot l'$$

$$\hat{t} = (l - l')^2 = q^2$$

$$\hat{u} = (l - p)^2 = -2l \cdot p' = -2l' \cdot p$$

$$\hat{s} + \hat{t} + \hat{u} = 0$$