Title: PSI 17/18 - Standard Model - Lecture 5

Date: Jan 09, 2018 09:00 AM

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Abstract:

Fermion mass terms

$\bar{\psi}\psi$ is forbidden by gauge symmetry.

SU(2) group theory:

Two 2-component vectors that are fundamentals

Two ways to write SU(2)-invariant contractions

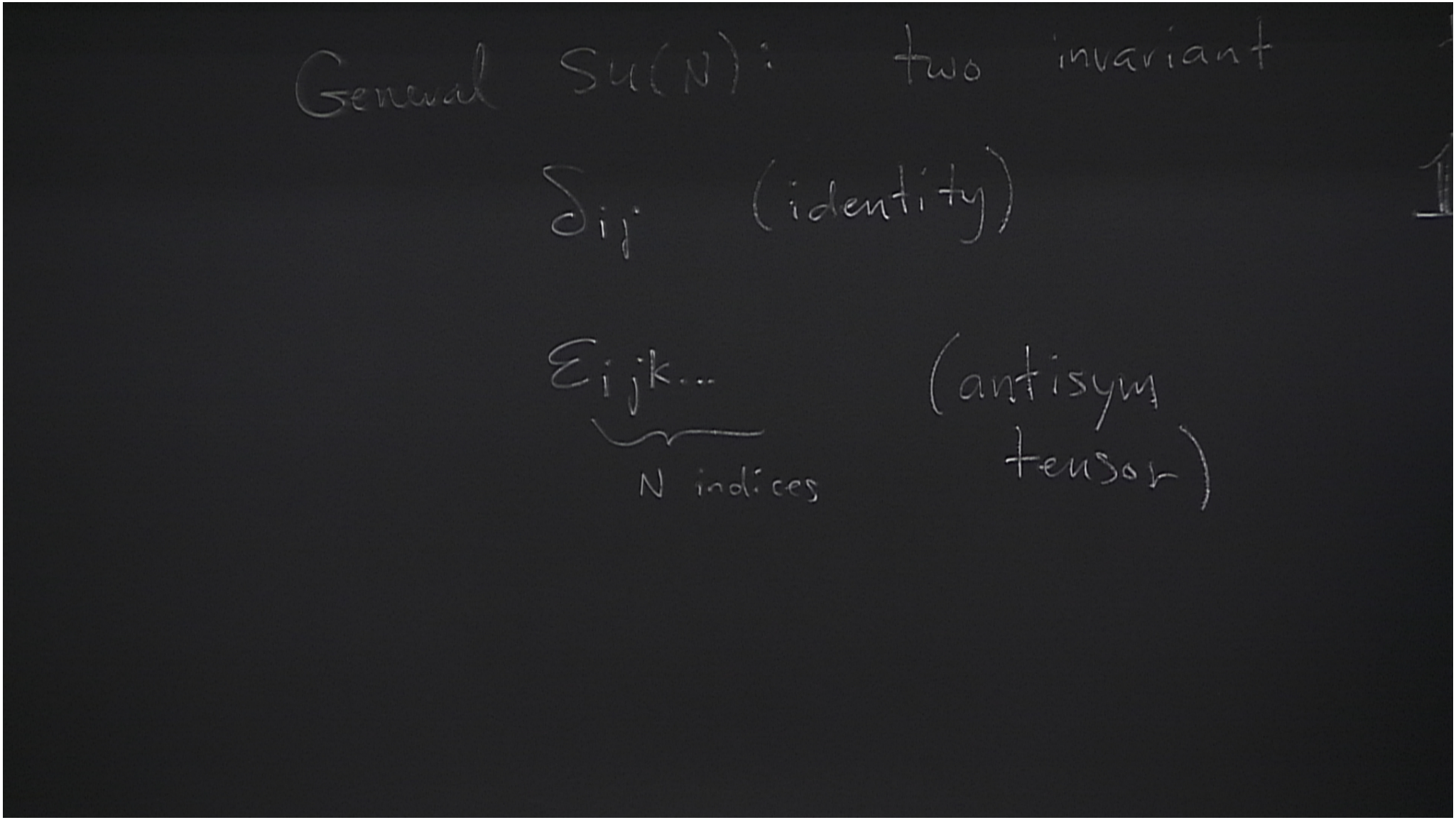
$\eta^\dagger \xi$ or $\eta^T \varepsilon \xi$ $\varepsilon =$

...metry.

... are fundamentals under $SU(2)$: η, ξ

... contractions

ξ $\qquad \varepsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ antisym tensor acting on $SU(2)_L$ indices.


General SU(N): two invariant
δ_{ij} (identity)
$\varepsilon_{ijk\ldots}$ (antisym tensor)
N indices

ant tensors:

$$\mathbb{1} \rightarrow U^\dagger \mathbb{1} U = \mathbb{1} U^\dagger U = \mathbb{1}$$

$$\varepsilon_{ijk\ldots} \rightarrow U_{ii'} U_{jj'} \cdots \varepsilon_{i'j'\ldots}$$

$$= \underbrace{\det(U)}_{1} \varepsilon_{ij\ldots} = \varepsilon_{ij\ldots}$$

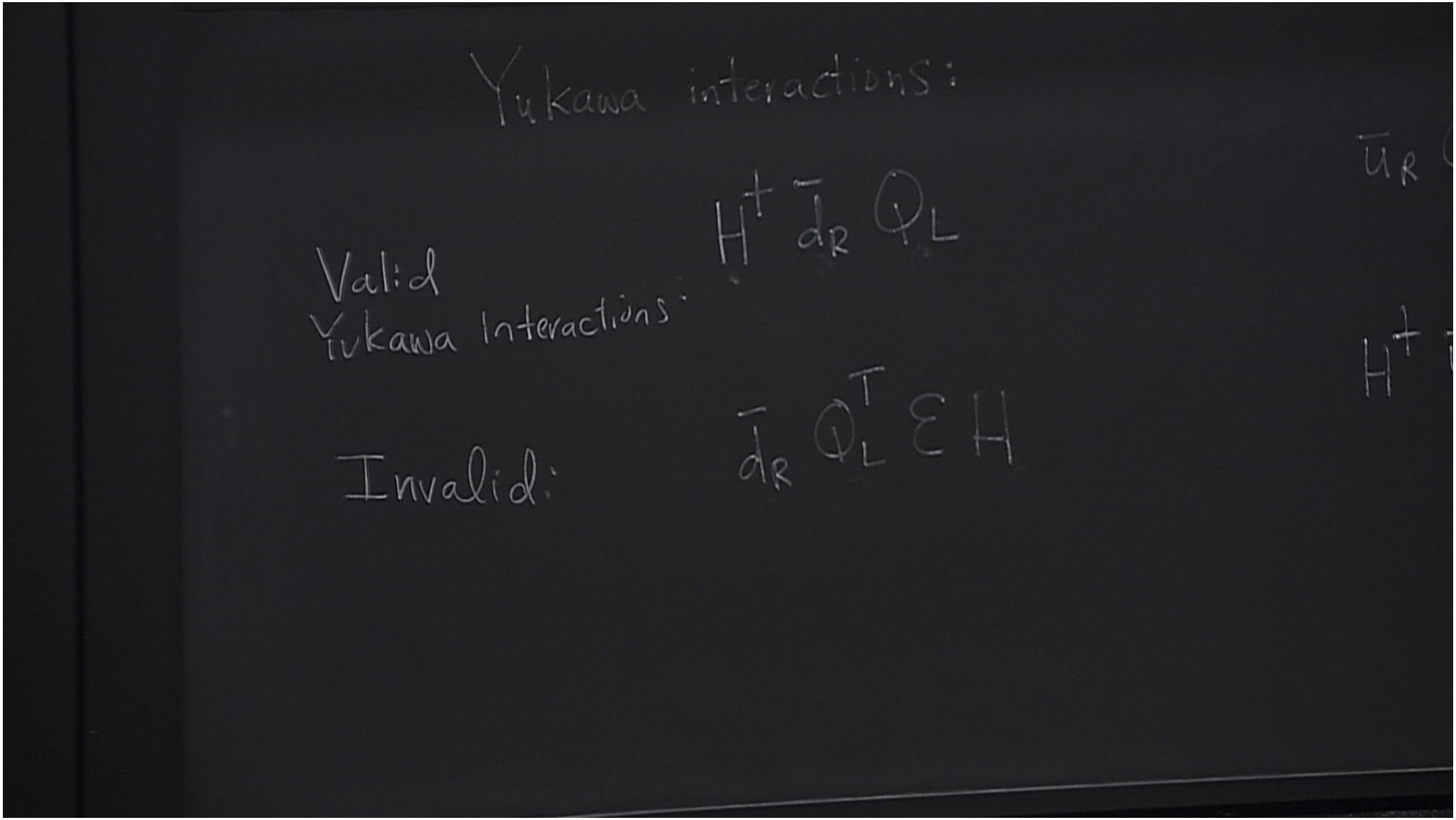
Yukawa interactions:

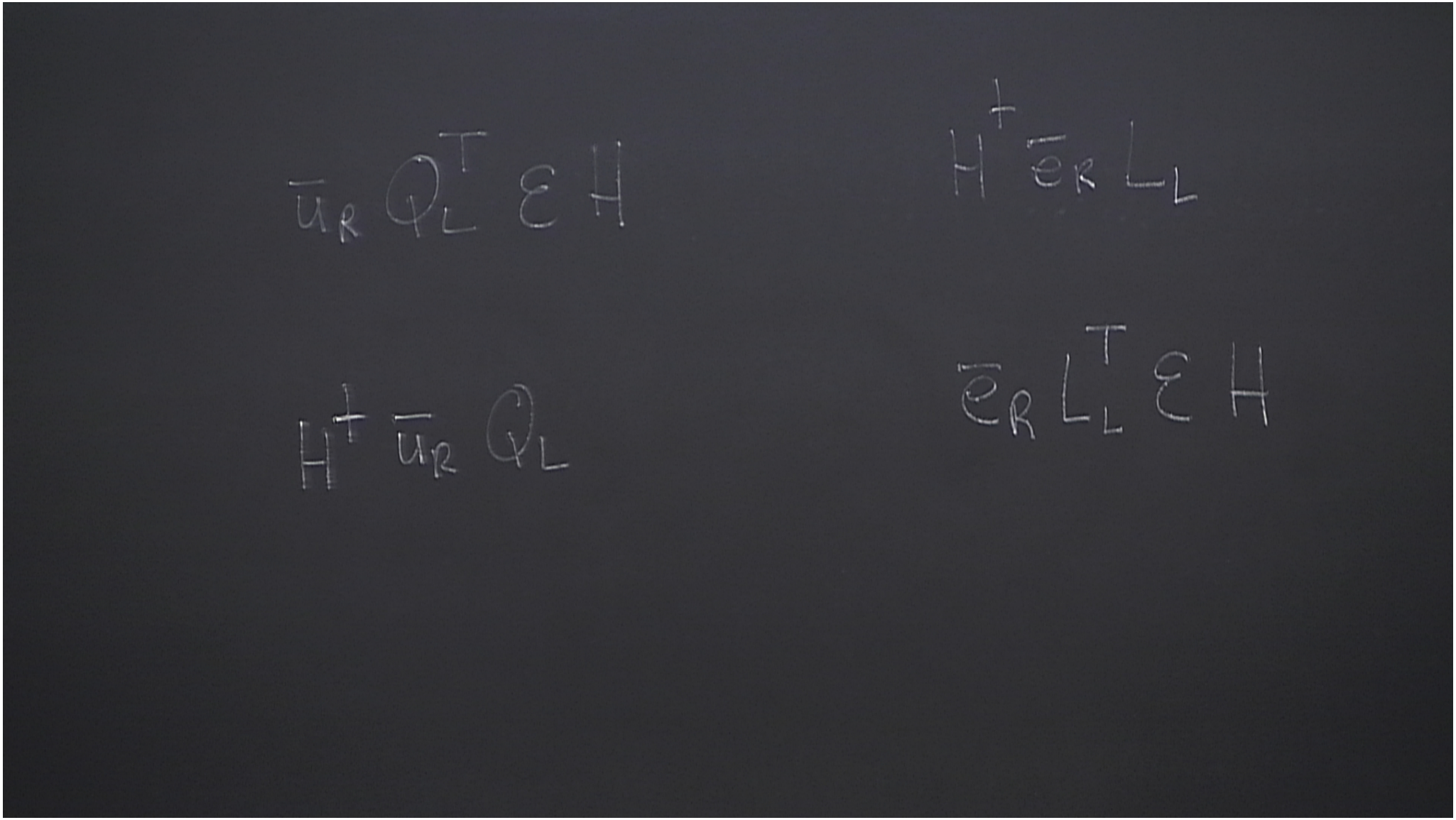
$H^\dagger \bar{d}_R Q_L$

$\bar{d}_R Q_L^T \varepsilon H$

$\bar{u}_R Q_L^T \varepsilon H$

$H^\dagger \bar{u}_R Q_L$


Yukawa interactions:
Valid Yukawa Interactions:
$H^\dagger \bar{d}_R Q_L$
Invalid:
$\bar{d}_R Q_L^T \varepsilon H$


$\bar{u}_R Q_L^T \varepsilon H$
$H^\dagger \bar{e}_R L_L$
$H^\dagger \bar{u}_R Q_L$
$\bar{e}_R L_L^T \varepsilon H$

Any generations i, j can be coupled via Yuka

$$\mathcal{L}_{\text{Yukawa}} = - Y^{(u)}_{ij} \, \bar{u}^{i}_{R} Q^{j\,T}_{L} \varepsilon H - Y^{(d)}_{ij} H^{\dagger}$$

$$= - Y^{(u)}_{ij} \frac{v}{\sqrt{2}} \bar{u}^{i}_{R} u^{j}_{L} \left(1 + \frac{h}{v}\right) - Y^{(d)}_{ij} \frac{v}{\sqrt{2}}$$

ed via Yukawa interaction:

$$-\mathcal{L} \supset Y^{(d)}_{ij}\, H^\dagger\, \bar{d}^i_R\, Q^j_L - Y^{(e)}_{ij}\, H^\dagger\, \bar{e}^i_R\, L^j_L + \text{h.c.}$$

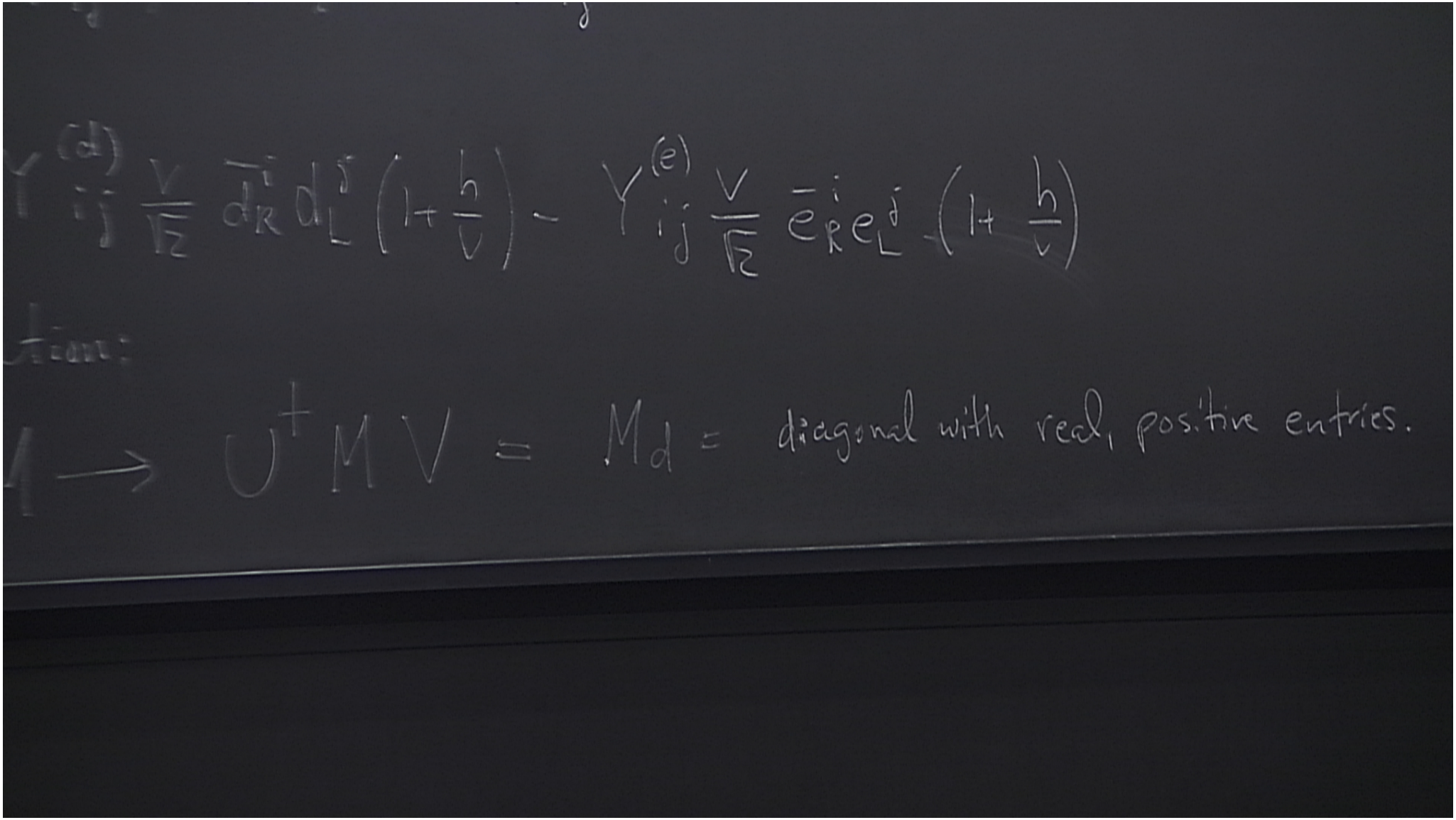
$$\supset -Y^{(d)}_{ij}\,\frac{v}{\sqrt{2}}\,\bar{d}^i_R\, d^j_L\left(1+\frac{h}{v}\right) - Y^{(e)}_{ij}\,\frac{v}{\sqrt{2}}\,\bar{e}^i_R\, e^j_L\left(1+\frac{h}{v}\right)$$

$\mathcal{L}_{\text{Yukawa}} = -$

$= -\, Y^{(u)}_{ij} \frac{v}{\sqrt{2}} \bar{u}^i_R u^j_L \left(1 + \frac{h}{v}\right) - Y^{(d)}_{ij} \frac{v}{\sqrt{2}}$

Diagonalize using biunitary transformation:

Square matrix M: diagonalize $M \rightarrow$


$Y^{(d)}_{ij} \frac{v}{\sqrt{2}} \bar{d}^i_R d^j_L \left(1+\frac{h}{v}\right) - Y^{(e)}_{ij} \frac{v}{\sqrt{2}} \bar{e}^i_R e^j_L \left(1+\frac{h}{v}\right)$
$\rightarrow U^\dagger M V = M_d =$ diagonal with real, positive entries.

Proof: Square matrix M.

$MM^\dagger$ is Hermitian with real, positive eigenvalue

Diagonalize: $U^\dagger M M^\dagger U = M_d^2 =$ diagon

$U^\dagger M \; M^\dagger U$

Square matrix M.

$MM^\dagger$ is Hermitian with real, positive eigenvalues.

Diagonalize: $U^\dagger M M^\dagger U = M_d^2 =$ diagonal matrix

$$U^\dagger M \underbrace{\left(M^\dagger U M_d^{-1}\right)}_{V} = M_d$$

$$V V^{\dagger} = (M_d^{-1} U^{\dagger} M)(M^{\dagger} U M_d^{-1})$$

$$= M_d^{-1} \underbrace{U^{\dagger} M M^{\dagger} U}_{M_d^2} M_d^{-1} = \underline{1}$$

$$\mathcal{L}_{\text{Yukawa}} = -\bar{u}_R M^{(u)} u_L \left(1+\frac{h}{v}\right) - \bar{d}_R M^{(d)} d_L \left(1+\frac{h}{v}\right)$$

e.g. $$M^{(u)}_{ij} = Y^{(u)}_{ij} \frac{v}{\sqrt{2}}$$

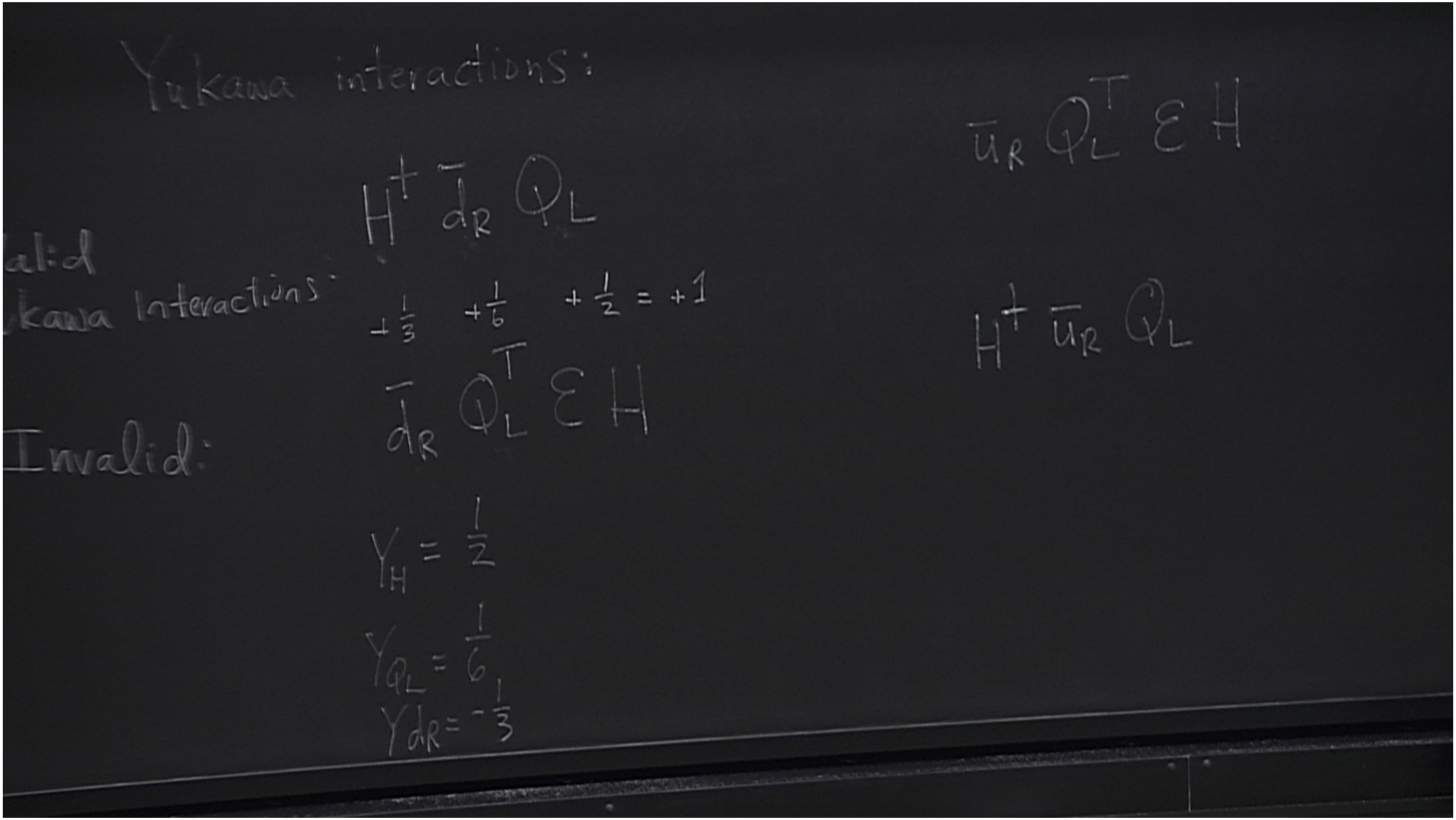
$$u_L = \begin{pmatrix} u_L^1 \\ u_L^2 \\ u_L^3 \end{pmatrix}$$ vector over generations.

$$U^{\dagger}_{u_R} M^{(u)} U_{u_L} = m^{(u)} = \text{diag}\left(m_1^{(u)}, m_2^{(u)}, m_3^{(u)}\right) =$$

$$\left(1+\frac{h}{v}\right) - \bar{d}_R M^{(d)} d_L \left(1+\frac{h}{v}\right) - \bar{e}_R M^{(e)} e_L \left(1+\frac{h}{v}\right) + h.c.$$

over generations:

$$= \text{diag}\left(m_1^{(u)}, m_2^{(u)}, m_3^{(u)}\right) = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix}$$

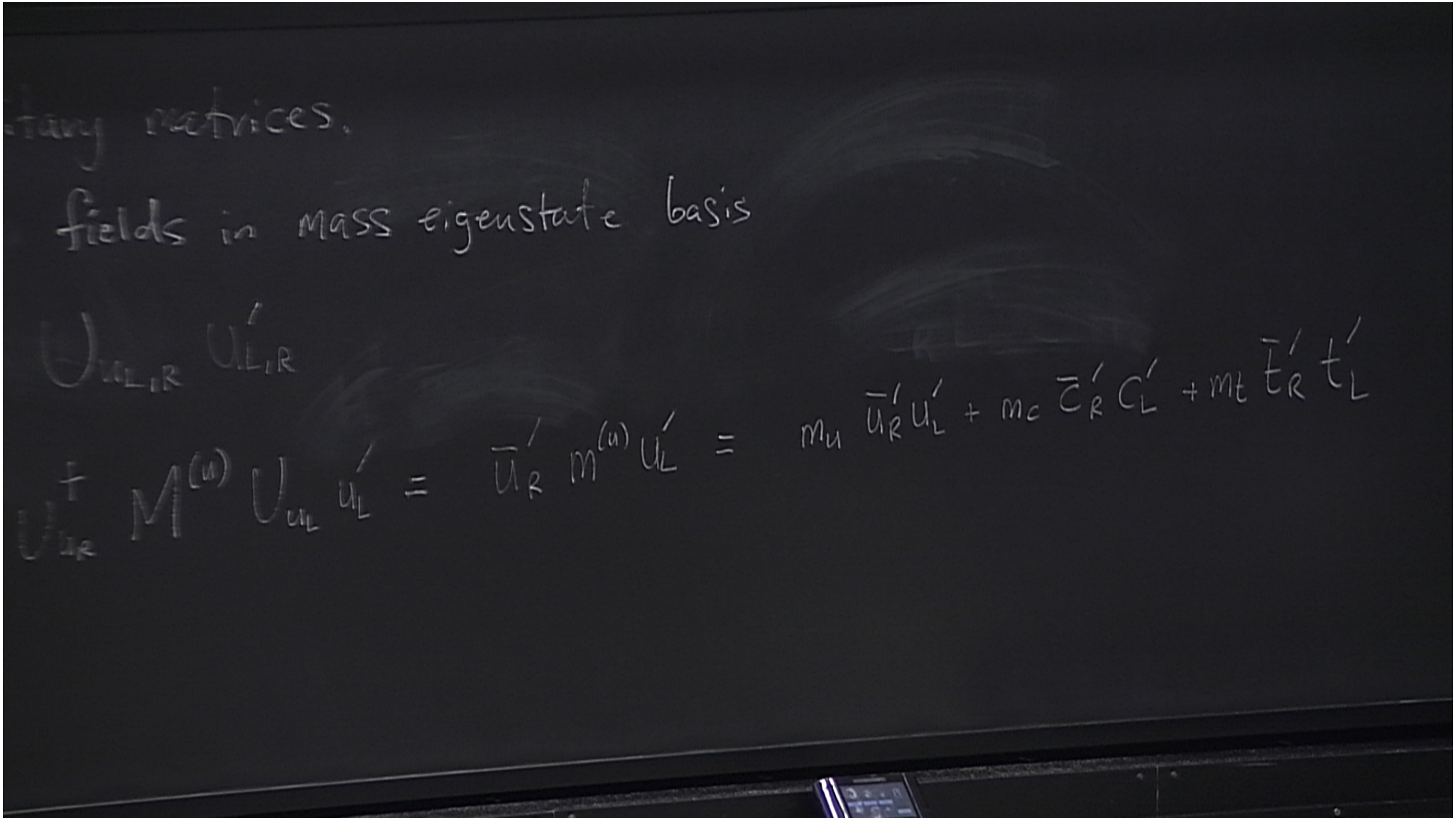

Yukawa interactions:
$H^\dagger \bar{d}_R Q_L$
$+\frac{1}{3} \quad +\frac{1}{6} \quad +\frac{1}{2} = +1$
$\bar{d}_R Q_L^T \varepsilon H$
$\bar{u}_R Q_L^T \varepsilon H$
$H^\dagger \bar{u}_R Q_L$
Invalid:
$Y_H = \frac{1}{2}$
$Y_{Q_L} = \frac{1}{6}$
$Y_{d_R} = -\frac{1}{3}$

U_{u_L}, U_{u_R} = unitary matrices.

Define $u'_{L,R}$ fields in mass eigenstate basis

$$u_{L,R} = U_{u_{L,R}} u'_{L,R}$$

$$\bar{u}_R M^{(u)} u_L = \bar{u}'_R U^{\dagger}_{u_R} M^{(u)} U_{u_L} u'_L = \bar{u}'_R m^{(u)} u'_L$$


tary matrices.
fields in mass eigenstate basis
$U_{u_{L,R}}\ u'_{L,R}$
$U^{\dagger}_{u_R} M^{(u)} U_{u_L} u'_L = \bar{u}'_R m^{(u)} u'_L = m_u \bar{u}'_R u'_L + m_c \bar{c}'_R c'_L + m_t \bar{t}'_R t'_L$

original basis

mass eigenstate basis.

$$d_{L,R} = U_{d_{L,R}} d'_{L,R}$$

$$e_{L,R} = U_{e_{L,R}} e'_{L,R}$$

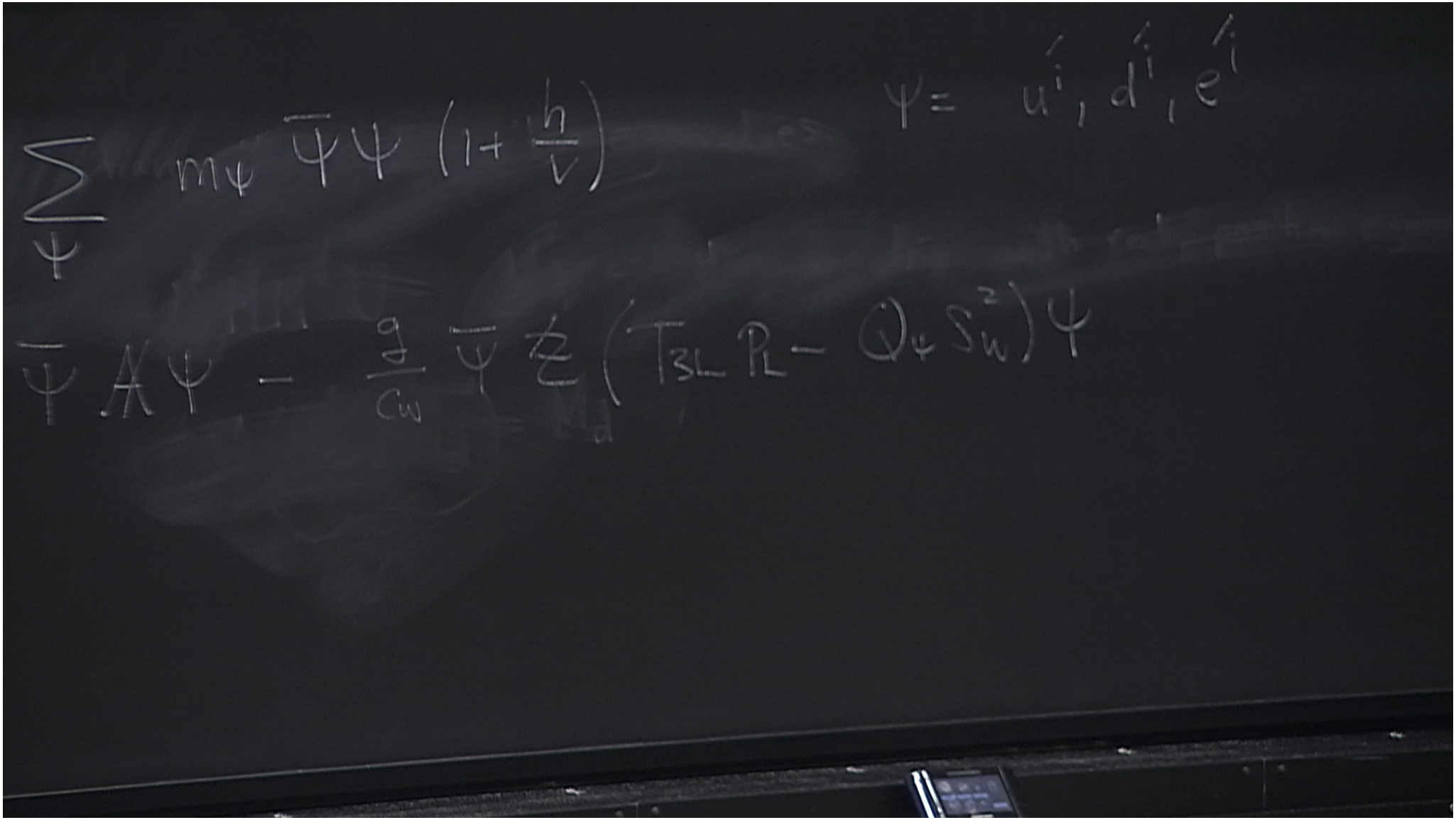
original basis

mass eigenstate basis.

$$d_{L,R} = U_{d_{L,R}} d'_{L,R}$$

$$e_{L,R} = U_{e_{L,R}} e'_{L,R}$$

$$\nu_L = U_{e_L} \nu'_L$$


$\sum_\psi m_\psi \bar{\psi}\psi \left(1 + \frac{h}{v}\right)$
$\psi = u^i, d^i, e^i$
$\bar{\psi} A \psi - \frac{g}{c_w} \bar{\psi} Z \left(T_{3L} P_L - Q_\psi s_w^2\right) \psi$

In this basis:

$$\mathcal{L}_{Yukawa} = -\sum_{\psi} m_\psi \bar{\psi}\psi \left(1 + \frac{h}{v}\right)$$

$$\mathcal{L}_{NC} = -e Q_\psi \bar{\psi} \not{A} \psi - \frac{g}{c_W} \bar{\psi} \not{Z} \left(T_{3L} P_L -\right.$$

Consider $\psi = u^i$

$m_\psi \bar{\psi}\psi \left(1 + \frac{h}{v}\right)$ $\psi = u^i, d^i, e^i$

$\psi - \frac{g}{c_W} \bar{\psi} \not{Z} \left(T_{3L} P_L - Q_\psi s_W^2\right) \psi$

$$\mathcal{L}_{NC} = -e\, Q_u \left(\bar{u}_L^i \not{A} u_L^i + \bar{u}_R^i \not{A} u_R^i \right)$$

$$- \frac{g}{c_w} \left(\bar{u}_L^i \not{Z} \left(T_{3L} - Q_u s_w^2 \right) u_L^i - \bar{u}_R^i \not{Z} Q_u s_w^2 u_R^i \right)$$

Transform to mass eigenstate basis:

$$\bar{u}_L^i \gamma^\mu u_L^i = \bar{u}_L^{j\prime} \left(U_{u_L}^\dagger \right)_{ji} \gamma^\mu \left(U_{u_L} \right)_{ik} u_L^{k\prime} =$$

$$\left. - Q_u s_W^2 \right) u_L^i - \bar{u}_R^i \not{Z} Q_u s_W^2 u_R^i \Big)$$

basis:

$$(U_u^\dagger)_{ji} \gamma^\mu (U_{uL})_{ik} u_L^{k'} = \bar{u}_L'^i \gamma^\mu u_L^{i'}$$

$$-\frac{g}{c_w}\left(\bar{u}_L^i \not{Z} \left(T_{3L} - Q_u s_W^2\right) u_L^i - \bar{u}_R^i \not{Z} Q\right.$$

Transform to mass eigenstate basis:

$$\bar{u}_L^i \gamma^\mu u_L^i = \bar{u}_L^{j\prime} \left(U_{u_L}^\dagger\right)_{ji} \gamma^\mu \left(U_{u_L}\right)_{ik}$$

NC is diagonal in mass eigenstate basis.

$$\mathcal{L}_{Yuk} = -\bar{u}_R M^{(u)} u_L \left(1 + \frac{h}{v}\right) - \bar{d}_R M^{(d)}$$

rrent interactions do not change fermion flavor (g

$$\frac{g}{\sqrt{2}}\left(\bar{u}_L^i \not{W}^+ d_L^i + \bar{\nu}_L^i \not{W}^+ e_L^i\right) + \text{h.c.}$$

$$\frac{g}{\sqrt{2}}\Big(\bar{u}_L^{j'} (U_{u_L}^\dagger)_{ji} \not{W}^+ (U_{d_L})_{ik} d_L^{k'}$$

$$+ \bar{\nu}_L^{j'} (U_{e_L}^\dagger)_{ji} \not{W}^+ (U_{e_L})_{ik} e_L^{k'}\Big) + \text{h.c.}$$

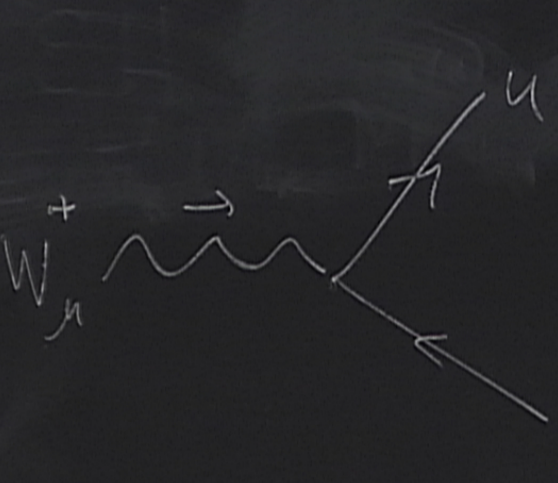
Define $V' = U_{u_L}^{\dagger} U_{d_L} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$

$$\mathcal{L}_{CC} = -\frac{g}{\sqrt{2}} \left(\bar{u}_L^{\prime i} V_{ij} \not{W}^{+} d_L^{j\prime} + \bar{\nu}_L^{i\prime} \not{W}^{+} e_L^{i} \right)$$

Define $V' = U_{u_L}^{\dagger} U_{d_L} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$

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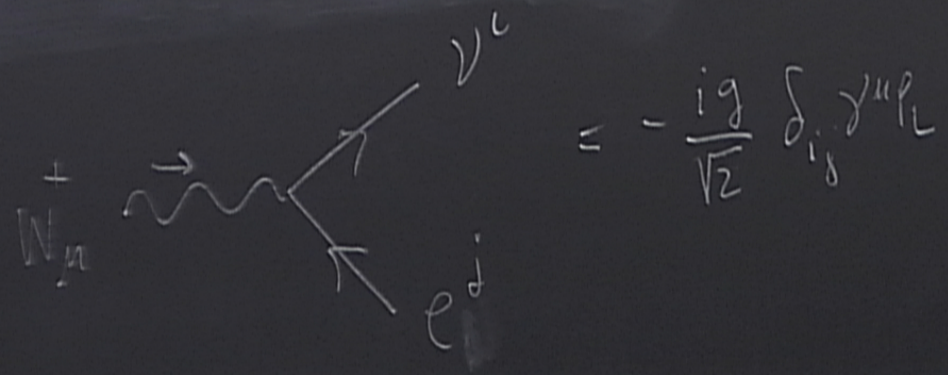
Feynman rules.



$$= -\frac{ig}{\sqrt{2}} V_{ij} \gamma^{\mu} P_L$$

Cabibbo - Kobayashi - Maskawa matrix (CKM)

$+ h.c.$



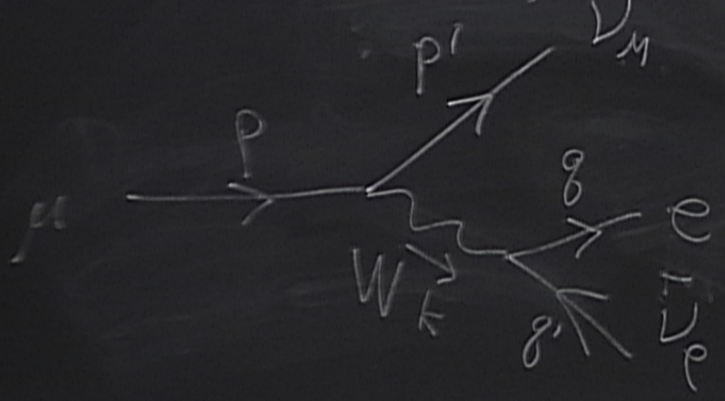
$$= -\frac{ig}{\sqrt{2}} \delta_{ij} \gamma^\mu P_L$$

Effective field theory for weak interaction

Integrate out W, Z and express weak interactions

Valid provided $E \ll m_W, m_Z$

ex. μ-decay

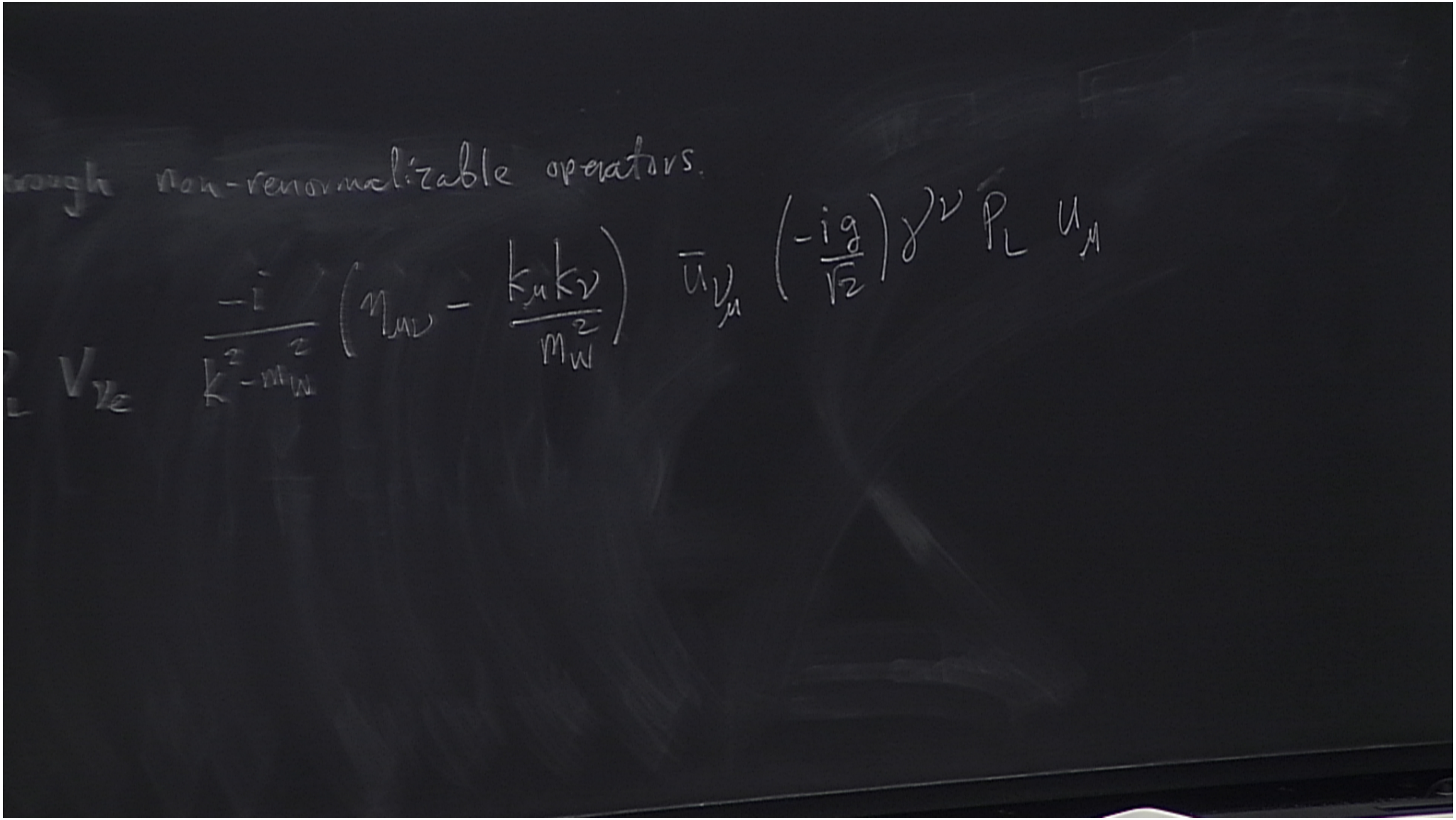


weak interaction

and express weak interactions through non-renormalizable operators.

m_W, M_Z

$$i\mathcal{M}(\mu \to e \nu_\mu \bar{\nu}_e) = \bar{u}_e \left(\frac{-ig}{\sqrt{2}}\right) \gamma^\mu P_L V_{\nu_e} \frac{-i}{k^2 - m_W^2} \left(\eta_{\mu\nu} - \frac{k_\mu k_\nu}{m_W^2}\right)$$


rough non-renormalizable operators.
$\frac{-i}{k^2 - m_W^2}\left(\eta_{\mu\nu} - \frac{k_\mu k_\nu}{m_W^2}\right)$
$\bar{u}_{\nu_\mu}\left(\frac{-ig}{\sqrt{2}}\right)\gamma^\nu P_L u_\mu$
$V_{\nu e}$

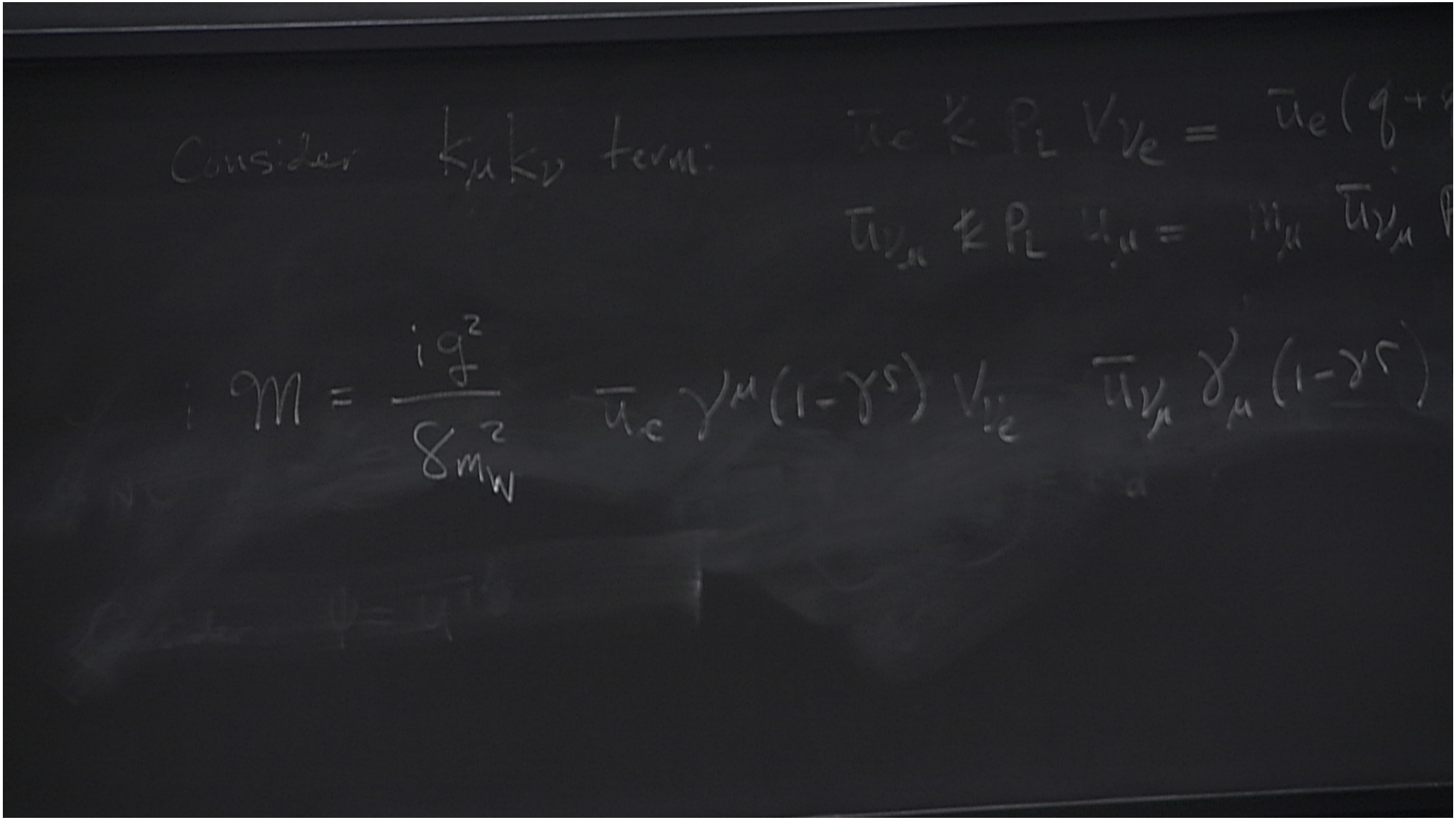
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m_W, m_Z

$$i\mathcal{M}(\mu \to e \nu_\mu \bar{\nu}_e) = \bar{u}_e \left(\frac{-ig}{\sqrt{2}}\right) \gamma^\mu P_L V_{\nu e} \frac{-i}{k^2 - m_W^2} \left(\eta_{\mu\nu} - \frac{k_\mu k_\nu}{m_W^2}\right)$$

$$k^2 \ll m_W^2$$

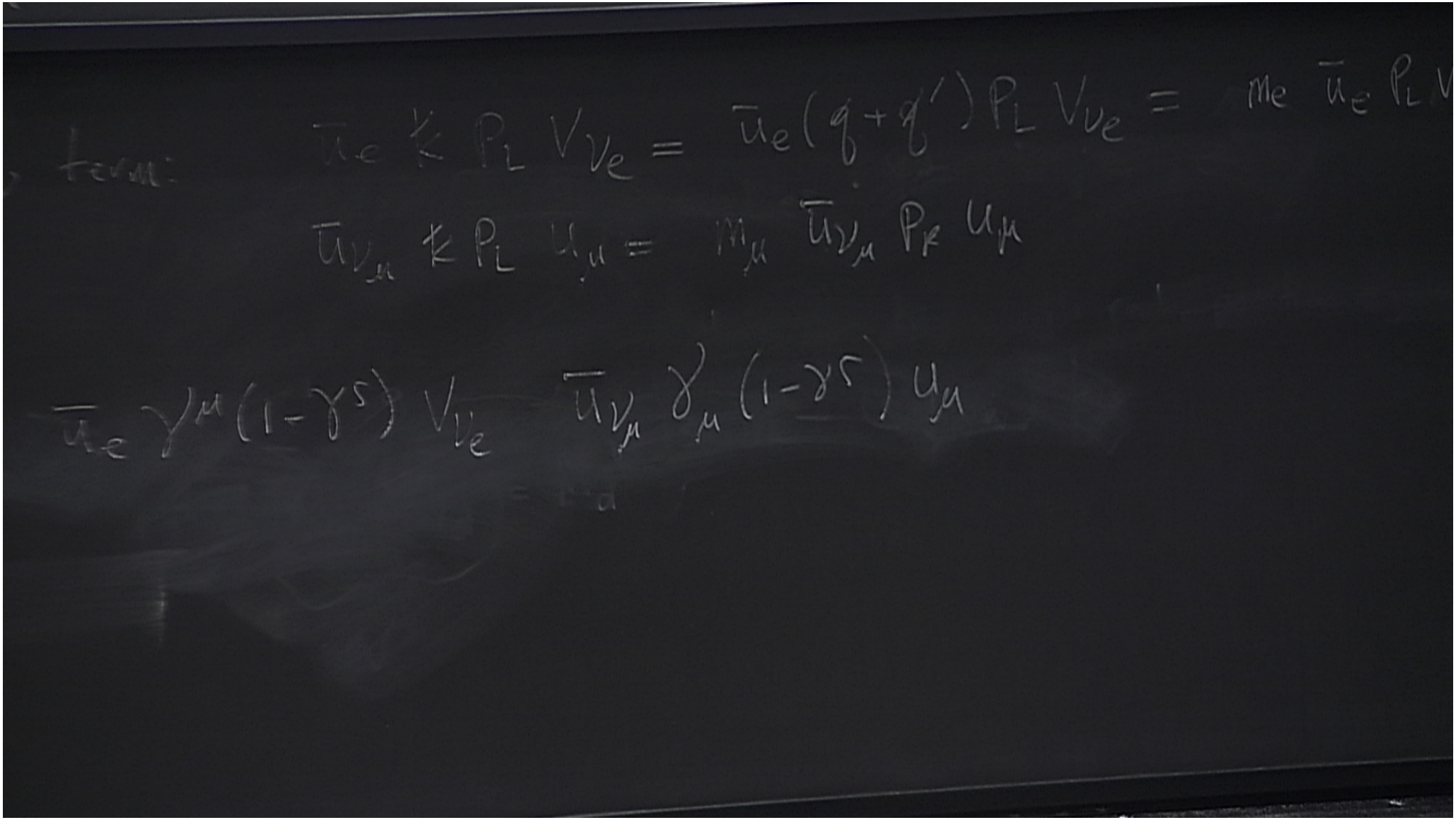

Consider $k_\mu k_\nu$ term:
$\bar{u}_e \not{k} P_L v_{\nu_e} = \bar{u}_e (q +$
$\bar{u}_{\nu_\mu} \not{k} P_L u_\mu = \bar{u}_\mu \bar{u}_{\nu_\mu}$
$\mathcal{M} = \frac{i g^2}{8 m_W^2} \bar{u}_e \gamma^\mu (1-\gamma^5) v_{\nu_e} \bar{u}_{\nu_\mu} \gamma_\mu (1-\gamma^5)$

Consider $k_\mu k_\nu$ term:

$$\bar{u}_e \not{k} P_L v_{\nu_e} = \bar{u}_e (\not{q} + \not{$$

$$\bar{u}_{\nu_\mu} \not{k} P_L v_\mu = \not{i}_\mu \bar{u}_{\nu_\mu} P$$

$$\mathcal{M} = \frac{i g^2}{8 m_W^2} \bar{u}_e \gamma^\mu (1-\gamma^5) v_{\nu_e} \bar{u}_{\nu_\mu} \gamma_\mu (1-\gamma^5)$$


term:
$\bar{u}_e \not{k} P_L \nu_e = \bar{u}_e (q + q') P_L \nu_e = m_e \bar{u}_e P_L \nu$
$\bar{u}_{\nu_\mu} \not{k} P_L u_\mu = m_\mu \bar{u}_{\nu_\mu} P_R u_\mu$
$\bar{u}_e \gamma^\mu (1-\gamma^5) \nu_e \quad \bar{u}_{\nu_\mu} \gamma_\mu (1-\gamma^5) u_\mu$

Consider $k_\mu k_\nu$ term: $\bar{u}_e \not{k} P_L v_{\nu_e} = \bar{u}_e (\not{q} + \not{q}') P$

$\bar{u}_{\nu_\mu} \not{k} P_L u_\mu = m_\mu \bar{u}_{\nu_\mu} P_R u$

$$\mathcal{M} = \frac{i g^2}{8 m_W^2} \bar{u}_e \gamma^\mu (1-\gamma^5) v_{\nu_e} \; \bar{u}_{\nu_\mu} \gamma_\mu (1-\gamma^5) u_\mu$$

$$\mathcal{L}_{eff} = \frac{G_F}{\sqrt{2}} \bar{e} \gamma^\mu (1-\gamma^5) \nu_e \; \bar{\nu}_\mu \gamma_\mu (1-\gamma^5) \mu + h.c.$$

$= \bar{u}_e (q + q') P_L v_{\nu_e} = m_e \bar{u}_e P_L v_{\nu_e}$ } Suppressed by $\frac{m_e m_\mu}{m_W^2} \sim 10^{-8}$.

$= \bar{u}_{\nu_\mu} P_L u_\mu$

G_F = Fermi's const.

$\gamma_\mu (1-\gamma^5) u_\mu$

$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8 m_W^2}$

$(1-\gamma^5) \mu + h.c.$

$\nu^i \quad \bar{D}^i \gamma_\mu (1-\gamma^5) e^j + h.c.$ leptonic.

$\gamma^\mu (1-\gamma^5) d^j \quad \bar{e}^k \gamma_\mu (1-\gamma^5) \nu^k + h.c.$ semileptonic

$\bar{u}^i \gamma^\mu (1-\gamma^5) d^j \quad \bar{d}^l \gamma_\mu (1-\gamma^5) u^k + h.c.$ hadronic.

$$\mathcal{L}_{eff}^{CC} = \frac{G_F}{\sqrt{2}} \sum_{i,j} \bar{e}^i \gamma^\mu (1-\gamma^5) \nu^i \; \bar{\nu}^j \gamma_\mu (1-\gamma^5) e^j + h.c.$$

$$+ \frac{G_F}{\sqrt{2}} \sum_{i,j,k} V_{ij} \; \bar{u}^i \gamma^\mu (1-\gamma^5) d^j \; \bar{e}^k \gamma_\mu (1-\gamma^5) \nu$$

$$+ \frac{G_F}{\sqrt{2}} \sum_{ijkl} V_{ij} V_{kl}^* \; \bar{u}^i \gamma^\mu (1-\gamma^5) d^j \; \bar{d}^l \gamma_\mu (1-\gamma^5)$$

$SU(2)_L \times U(1)_Y$ gauge theory: $(V-A) \times (V-A)$ universal structure.