Title: PSI 17/18 - Standard Model - Lecture 3

Date: Jan 05, 2018 09:00 AM

URL: http://pirsa.org/18010010

Abstract:

Abelian Higgs model

Complex scalar ϕ + local $U(1)$

$$\mathcal{L} = (D_\mu \phi)^\dagger (D^\mu \phi) - V(\phi) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$D_\mu = \partial_\mu + i g A_\mu$$

$$V(\phi) = -\mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2$$

vev: $v = \sqrt{\frac{\mu^2}{\lambda}}$

plug in shifted fields: $\phi = \frac{v + \phi_1' + i\phi_2'}{\sqrt{2}}$

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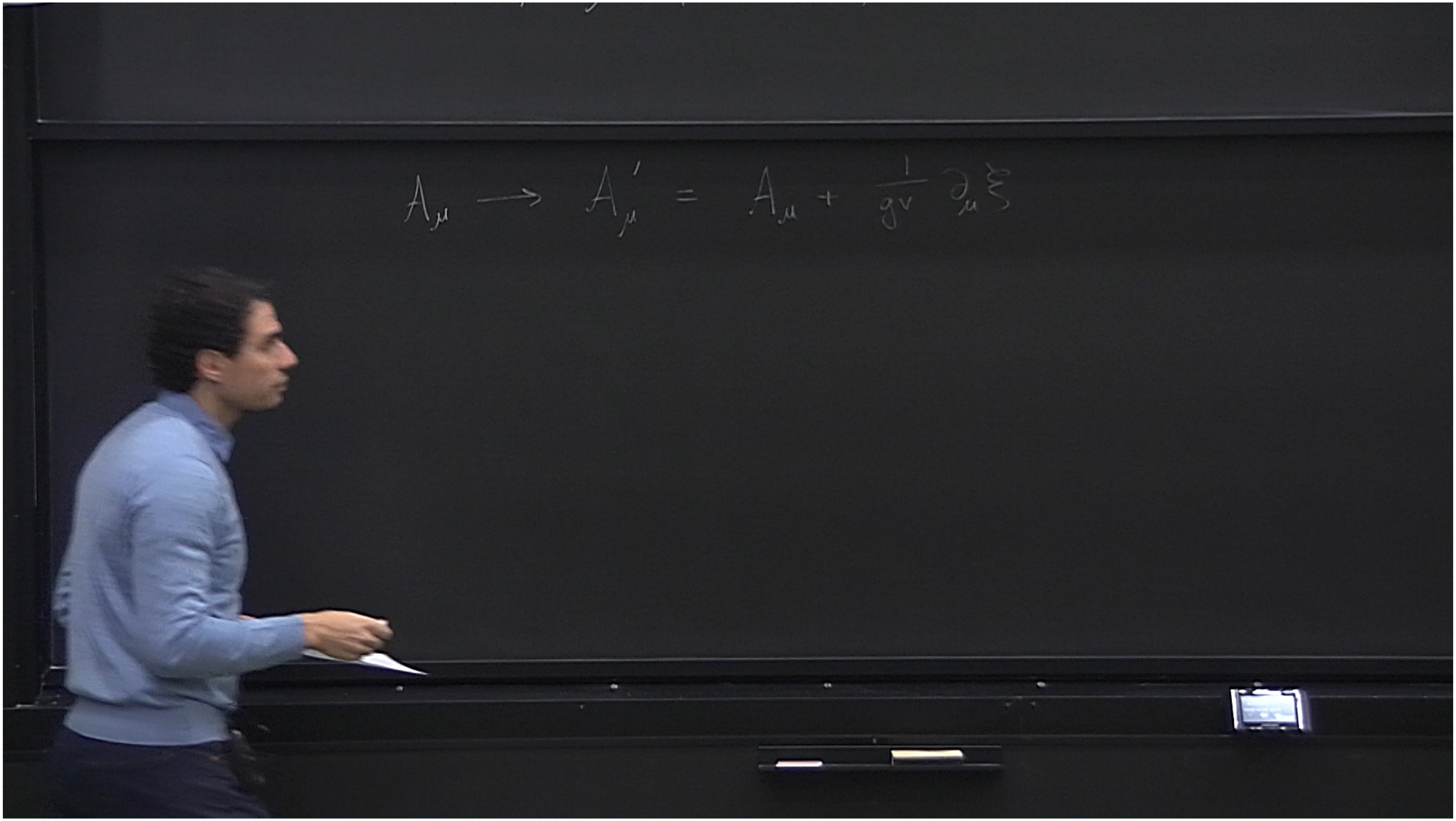
Unitary gauge.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$$\mathcal{L} = \frac{1}{2}\left|\left(\partial_\mu + i g A'_\mu\right)(v+h)\right|^2 + \frac{\mu^2}{2}(v+h)^2 - \frac{\lambda}{4}(v+h)^4 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

Unitary gauge.

$$ig A'_\mu)\,(v+h)$$

$$(v+h)^2 - \frac{\lambda}{4}(v+h)^4 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

drop prime.

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$$= \frac{1}{2}(\partial_\mu h)^2 - \mu^2 h^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}g^2 v^2 A_\mu A^\nu + \mathcal{L}_{int}$$

Higgs mass $m_h^2 = 2\mu^2 \longrightarrow m_h = \sqrt{2}\mu$

Vector boson mass: $m_A^2 = g^2 v^2 \longrightarrow m_A = gv$

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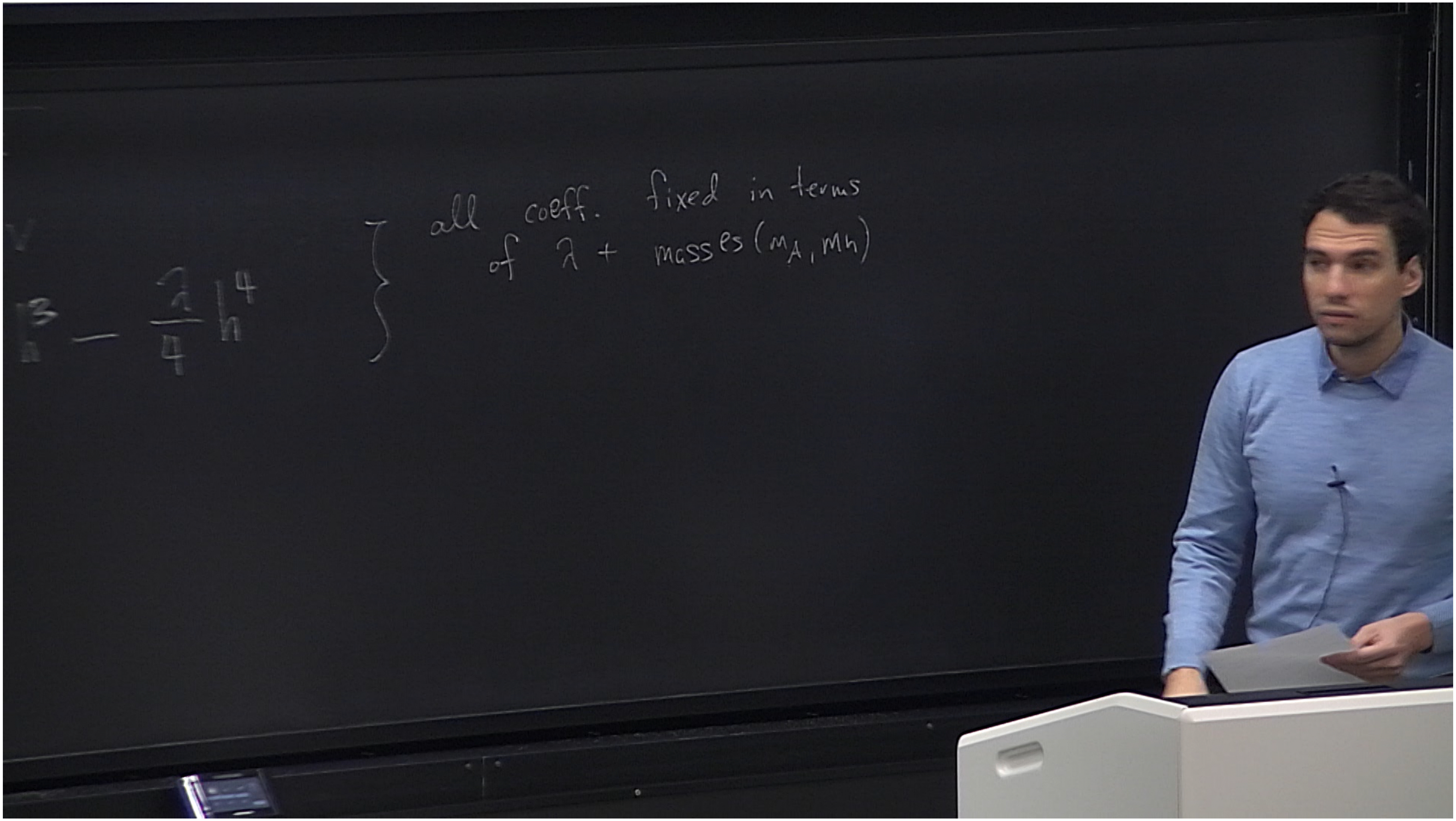
$h^3 - \frac{\lambda}{4} h^4$
all coeff. fixed in terms
of λ + masses (m_A, m_h)

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Would-be Goldstone boson ξ:

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massless A_μ

2 d.o.f.

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massless A_μ + complex scalar field

2 d.o.f. $\qquad$ 2 dof

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2 d.o.f.   2 dof

$\xi \swarrow \quad \searrow h$

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$$= g^2 v\, A_\mu A^\mu h + \frac{1}{2} g^2 h^2 A_\mu A^\mu - \lambda v h^3 - \frac{\lambda}{4} h^4$$

} all coeff. fi... of λ + m...

...-be Goldstone boson ξ is eaten by A_μ

$\underbrace{\text{massless } A_\mu}_{2 \text{ d.o.f.}}$ + complex scalar field $\longrightarrow$ $\underbrace{\text{massive } A_\mu}_{3 \text{ d.o.f.}}$ + $\underbrace{\text{real scalar } h}_{1 \text{ d.o.f.}}$

2 dof

$\xi \swarrow \quad \searrow h$

Fermion masses

Add fermion ψ with chiral gauge couplings

$\psi_{L,R}$ transform differently.

$\bar{\psi}\psi$

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Gauge coupling g.

Charges of fields Q:

ϕ

ψ_L

ψ_R

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ψ_L : Q_L

ψ_R Q_R

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ψ_L : Q_L $\quad g_L = Q_L g$

ψ_R $\quad Q_R$ $\quad g_R = Q_R g$

Gauge transform: $\phi \to e^{-igQ_\phi \alpha} \phi$

$\psi_{L,R} \to e^{-igQ_{L,R}\alpha} \psi_{L,R}$

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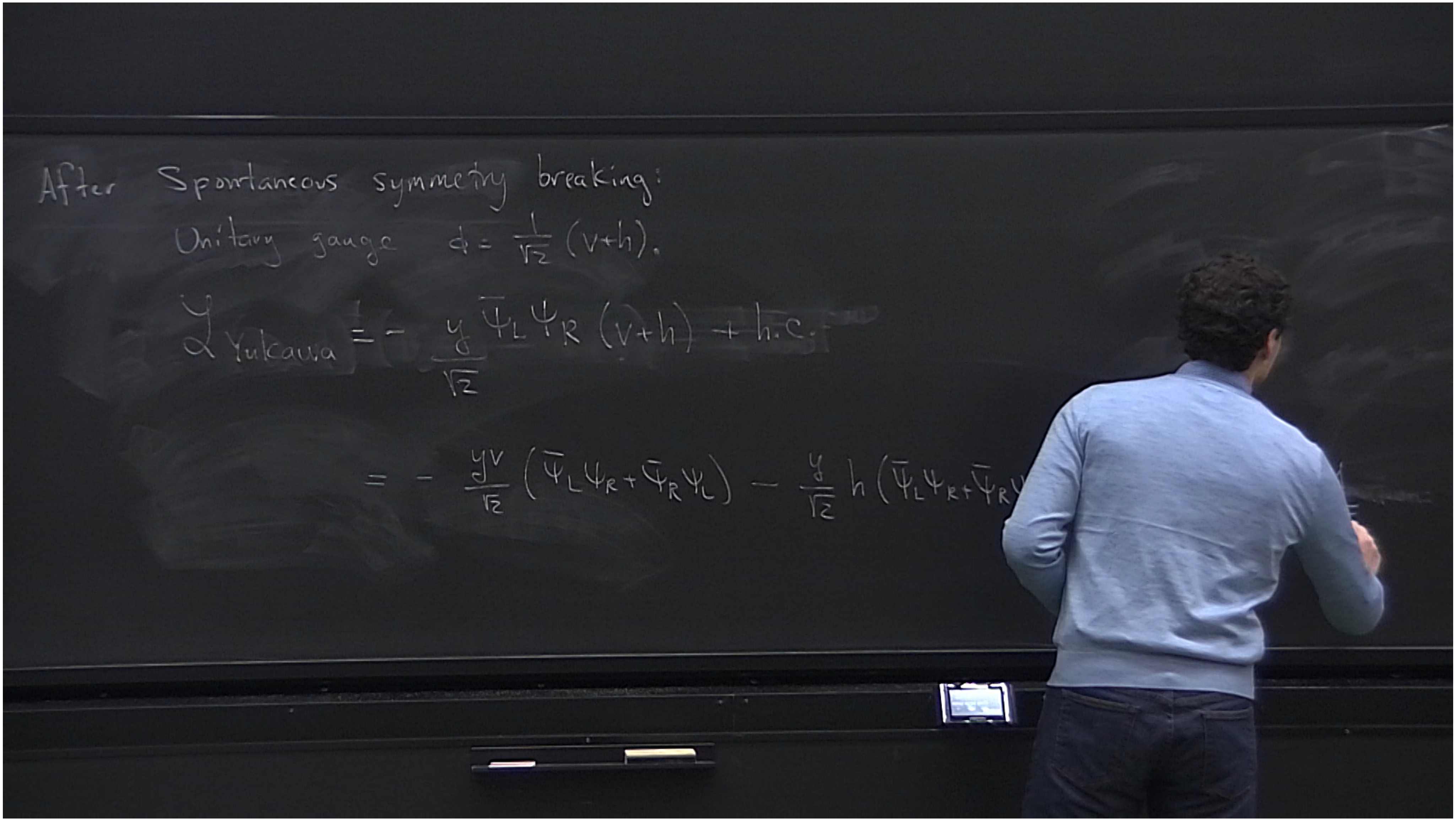
Assume $Q_L = Q_R + Q_\phi$; then we can write:

$$\mathcal{L}_{\text{Yukawa}} = -y \bar{\psi}_L \psi_R \phi + \text{h.c.}$$


After Spontaneous symmetry breaking:
Unitary gauge $\phi = \frac{1}{\sqrt{2}}(v+h)$.
$\mathcal{L}_{Yukawa} = -\frac{y}{\sqrt{2}}\bar{\psi}_L\psi_R(v+h) + h.c.$
$= -\frac{yv}{\sqrt{2}}(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L) - \frac{y}{\sqrt{2}}h(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi$

s symmetry breaking:

auge $\phi = \frac{1}{\sqrt{2}}(v+h)$.

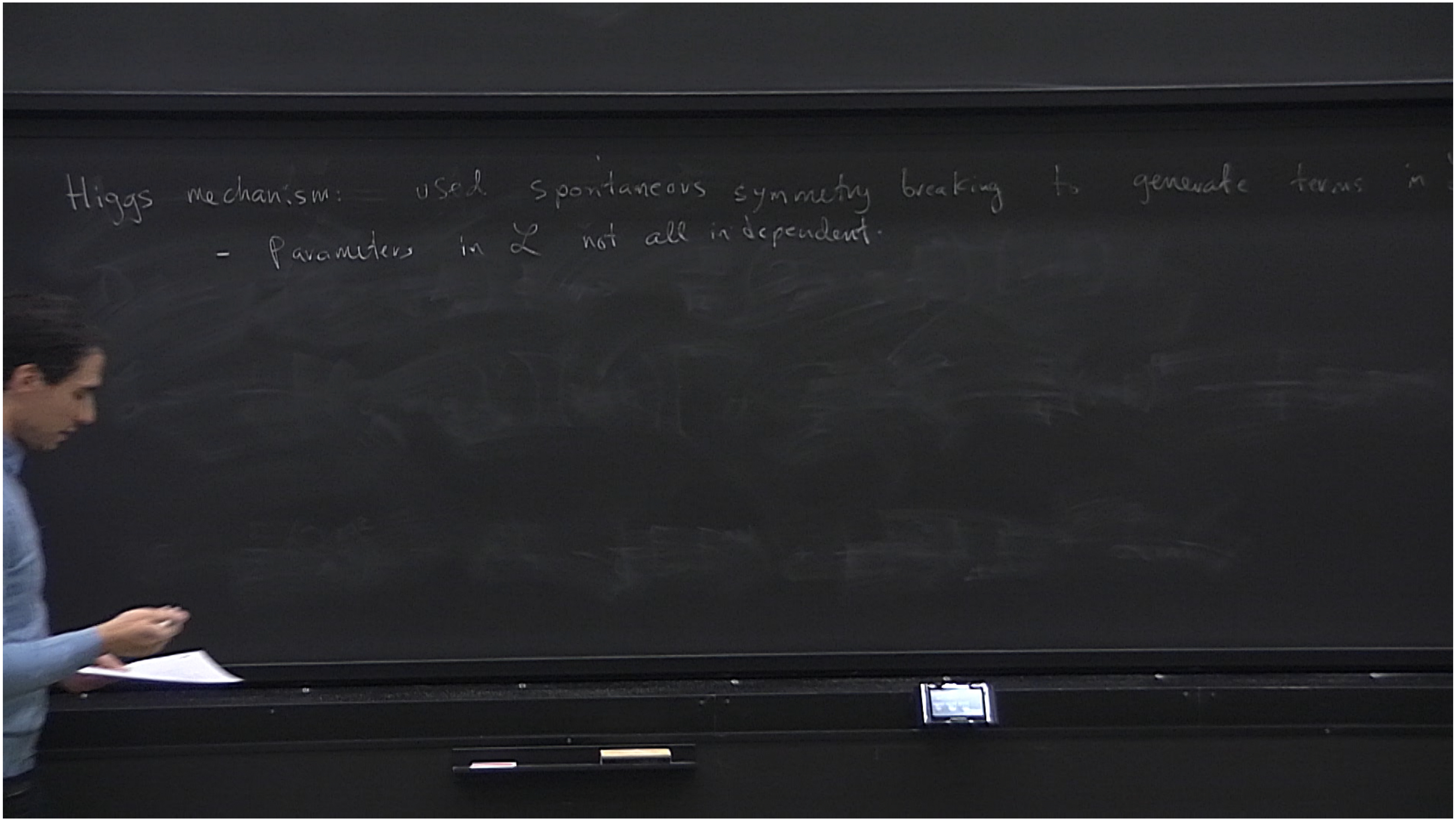
$= - \frac{y}{\sqrt{2}} \bar{\Psi}_L \Psi_R (v+h) + h.c.$

$= - \frac{yv}{\sqrt{2}} (\bar{\Psi}_L \Psi_R + \bar{\Psi}_R \Psi_L) - \frac{y}{\sqrt{2}} h (\bar{\Psi}_L \Psi_R + \bar{\Psi}_R \Psi_L) = - m_\Psi \bar{\Psi}\Psi - \frac{y}{\sqrt{2}} h \bar{\Psi}\Psi \qquad m_\Psi = \frac{yv}{\sqrt{2}}$

$$(\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L) = -m_\psi \bar{\psi}\psi - \frac{y}{\sqrt{2}} h \bar{\psi}\psi \qquad m_\psi = \frac{yv}{\sqrt{2}}$$

$$= -\frac{yv}{\sqrt{2}}\left(\bar\psi_L\psi_R+\bar\psi_R\psi_L\right)-\frac{y}{\sqrt{2}}h\left(\bar\psi_L\psi_R+\bar\psi_R\psi_L\right) = -m_\psi\bar\psi\psi-\frac{y}{\sqrt{2}}h\bar\psi\psi \qquad m_\psi=\frac{yv}{\sqrt{2}}$$

Higgs mechanism: used spontaneous symmetry breaking to generate terms in $\mathcal{L}$ that aren't gauge invariant.


Higgs mechanism: used spontaneous symmetry breaking to generate terms in
- Parameters in $\mathcal{L}$ not all independent.

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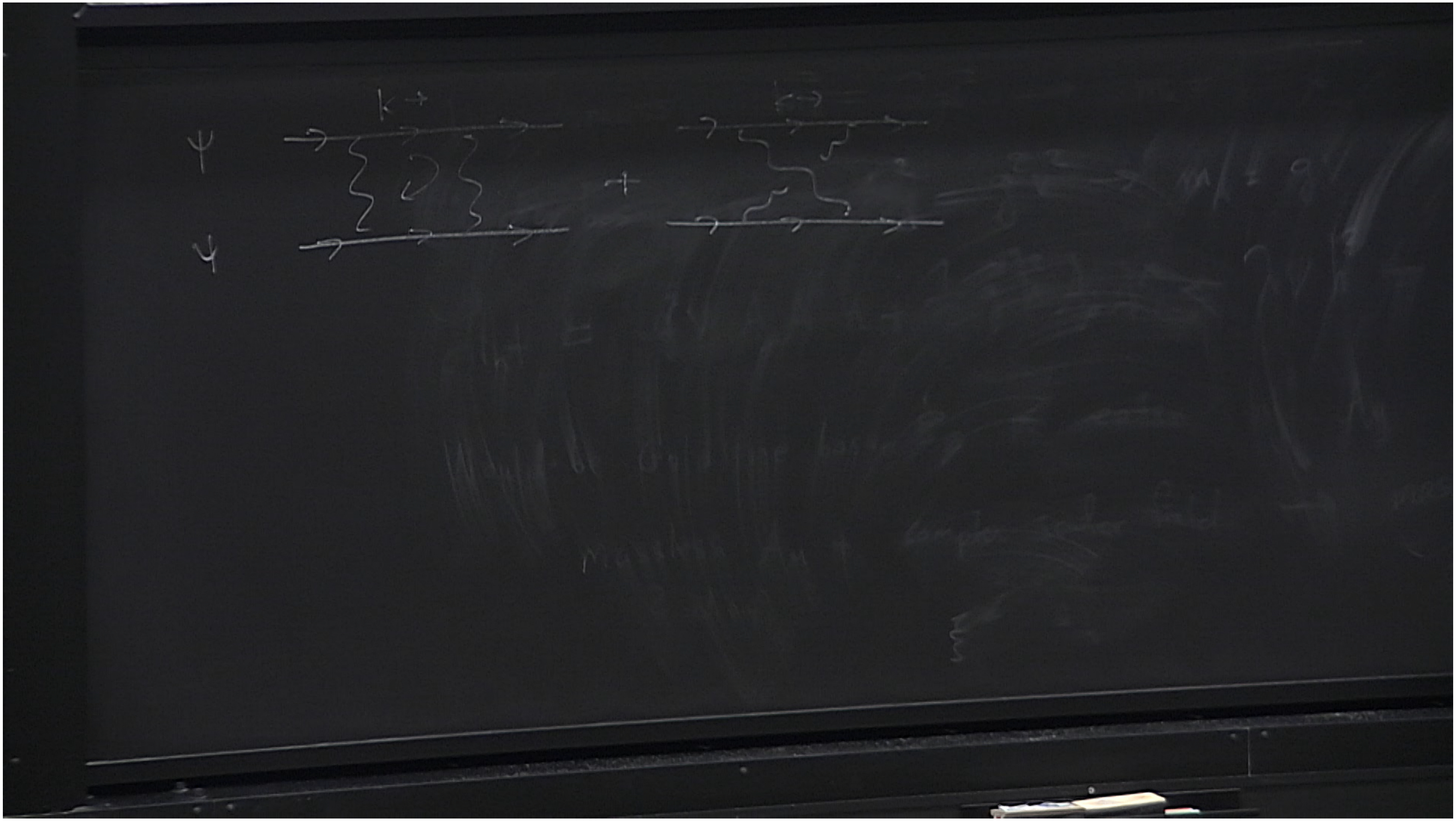
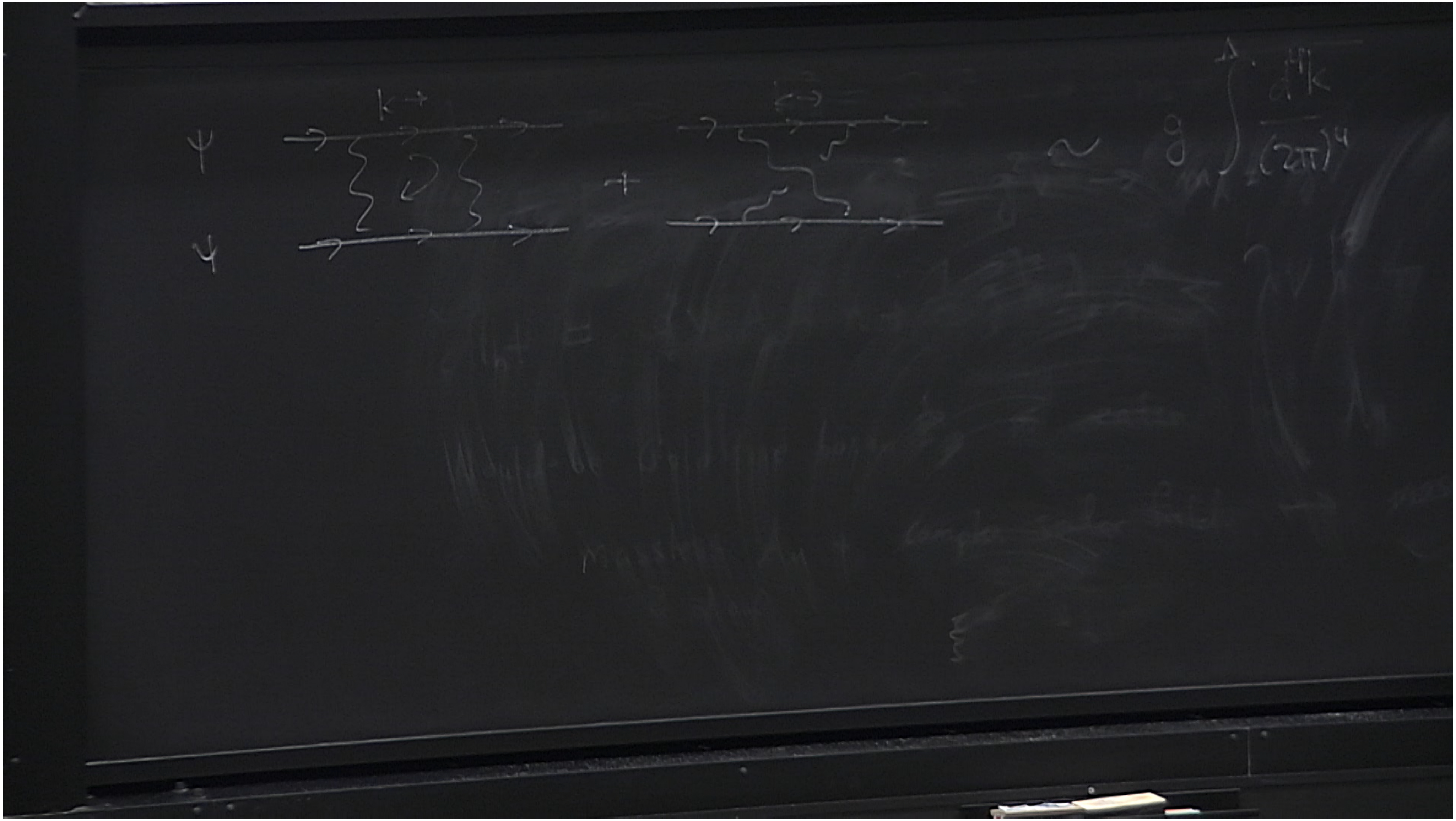
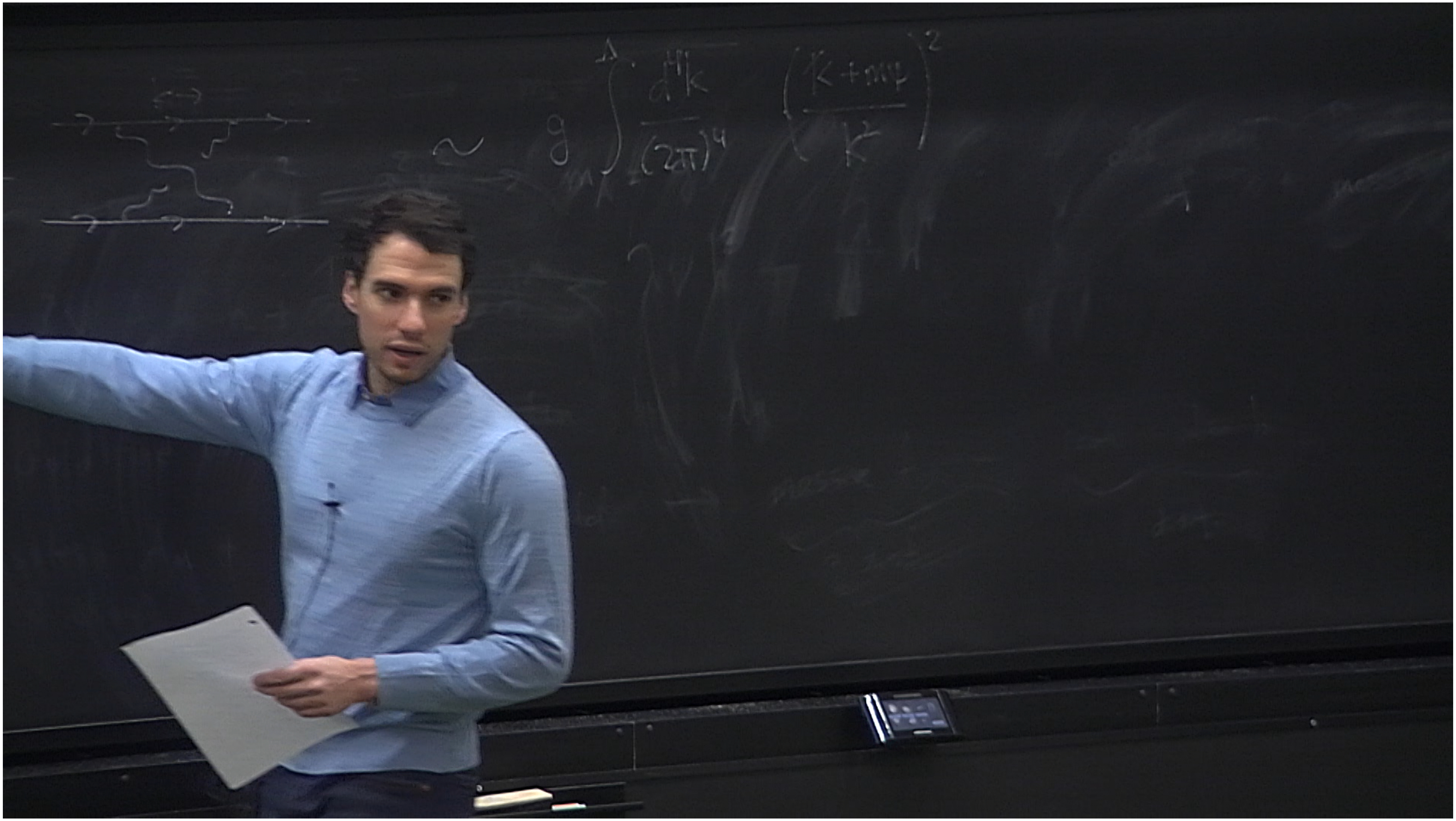
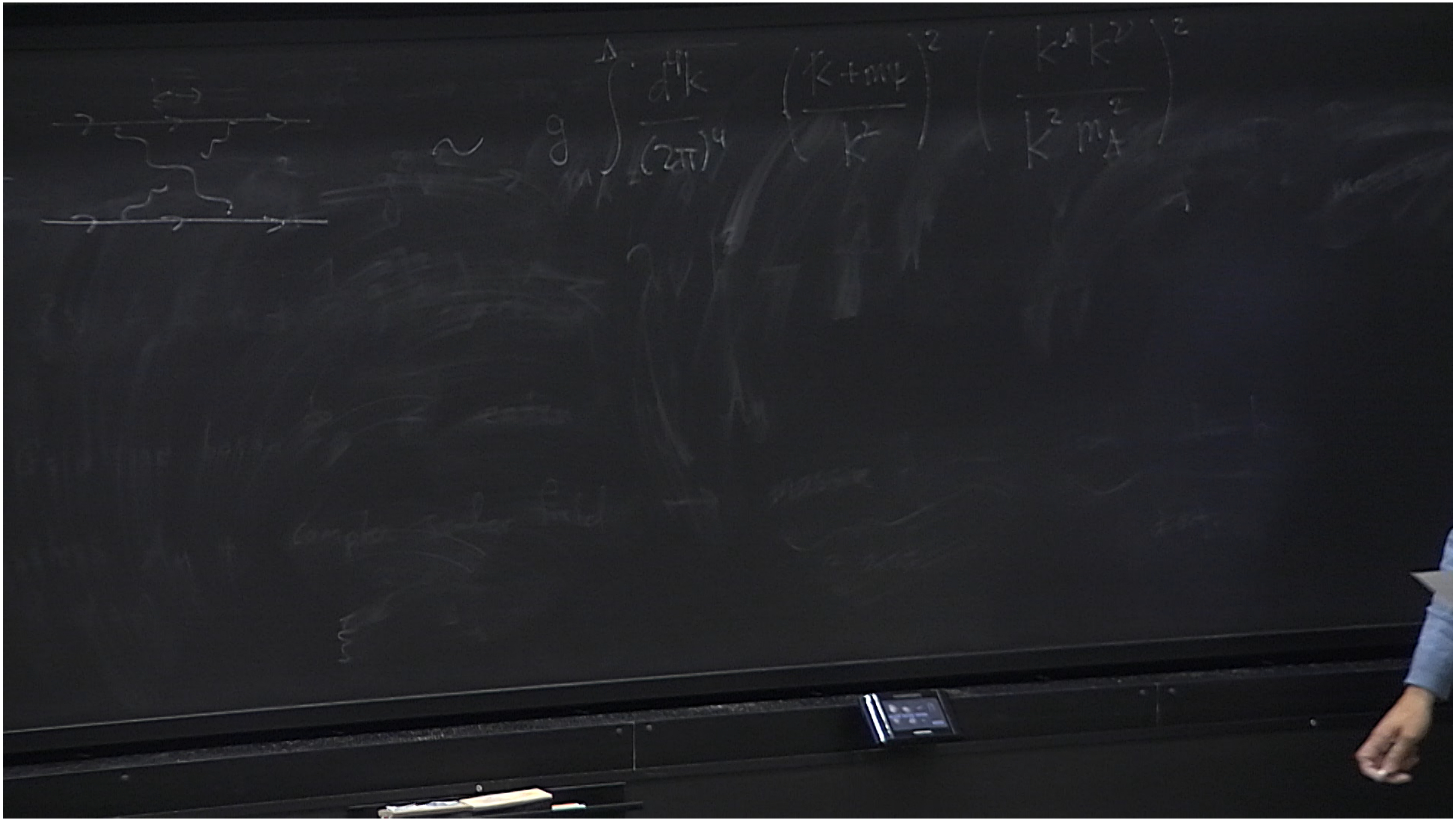
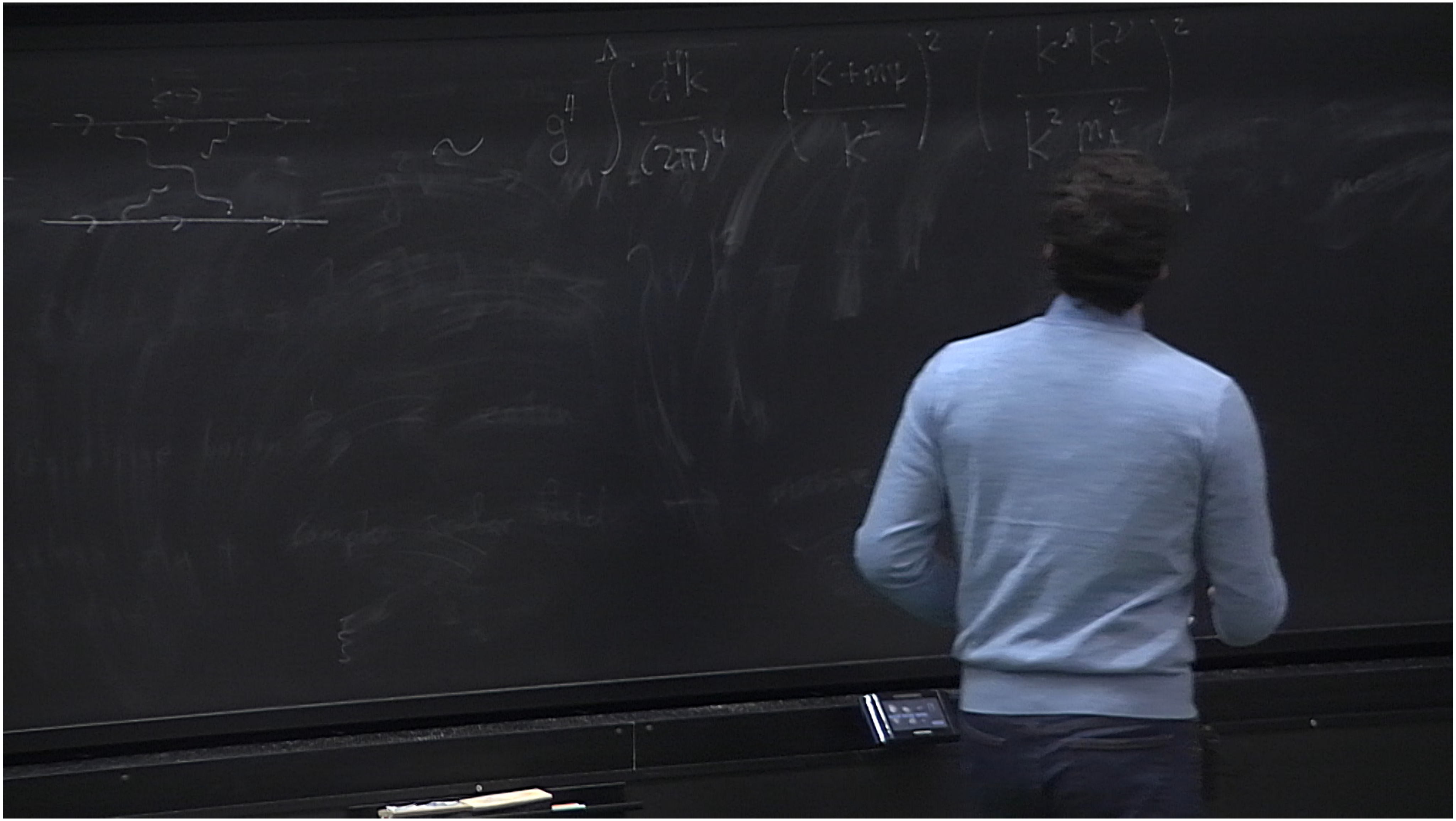
- Parameters in $\mathcal{L}$ not all independent.
- Left over d.o.f. $\rightarrow$ Higgs field h.

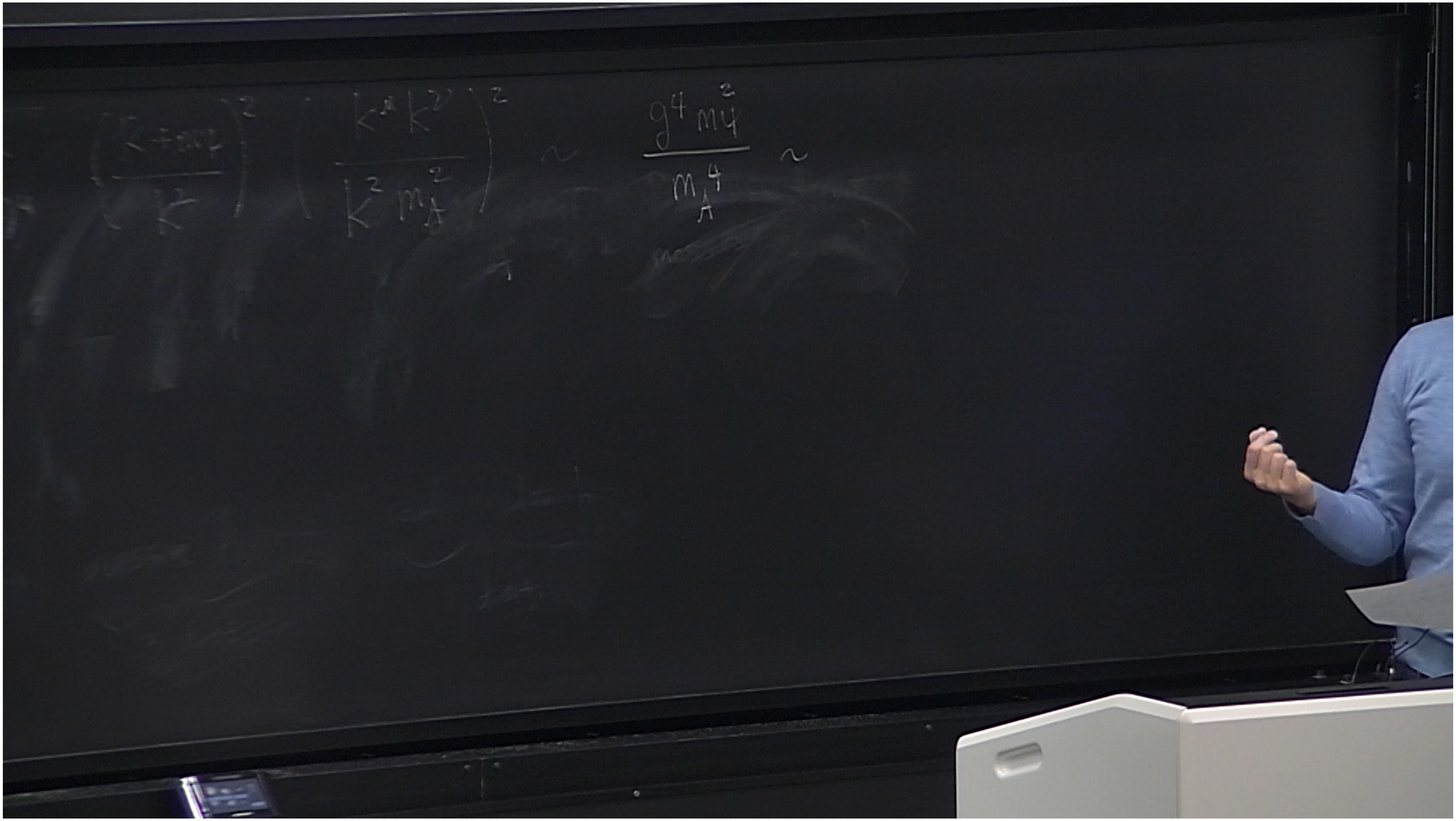


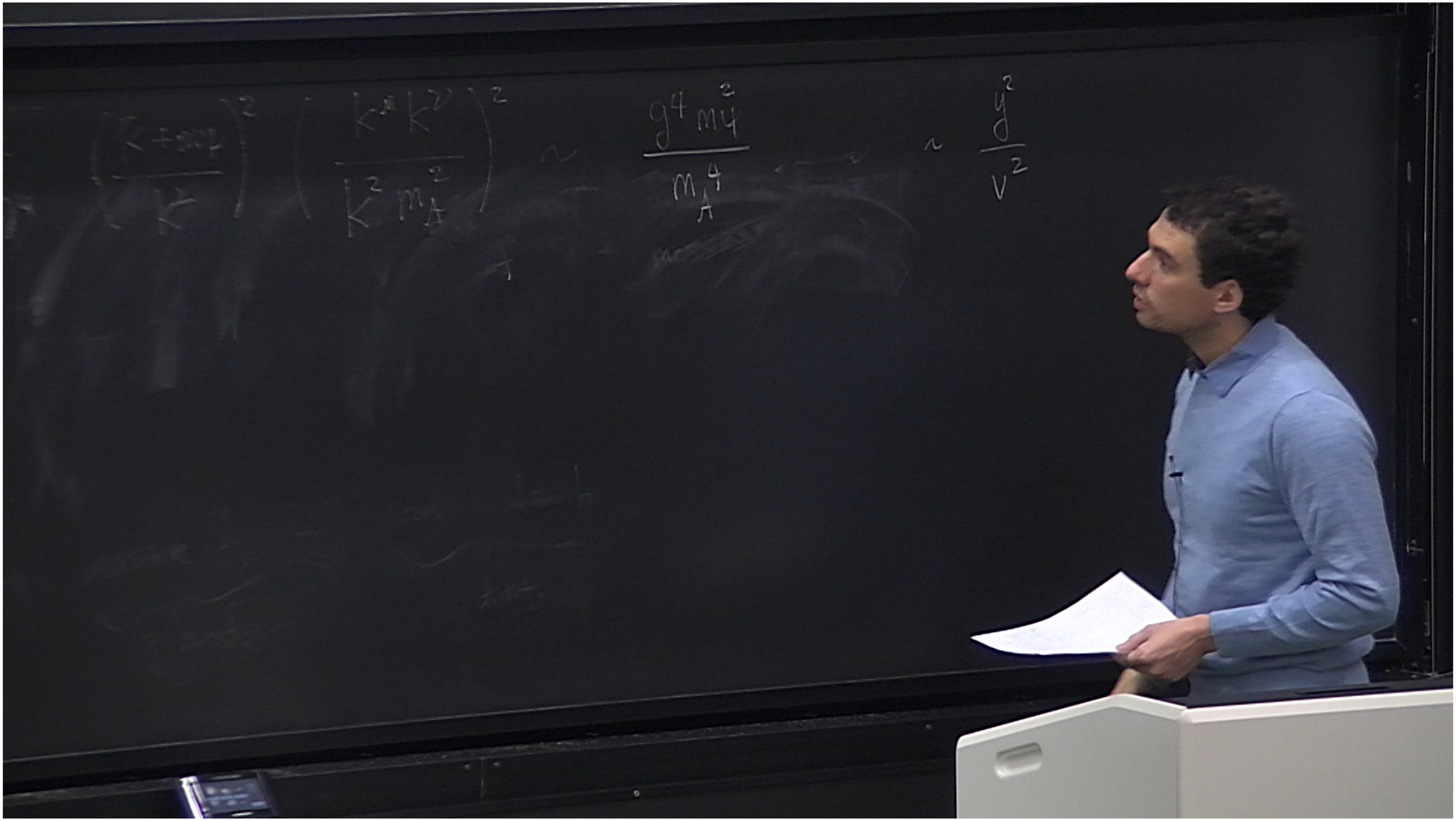

ψ
ψ
k →
+
∼ g ∫^Λ d^4k / (2π)^4

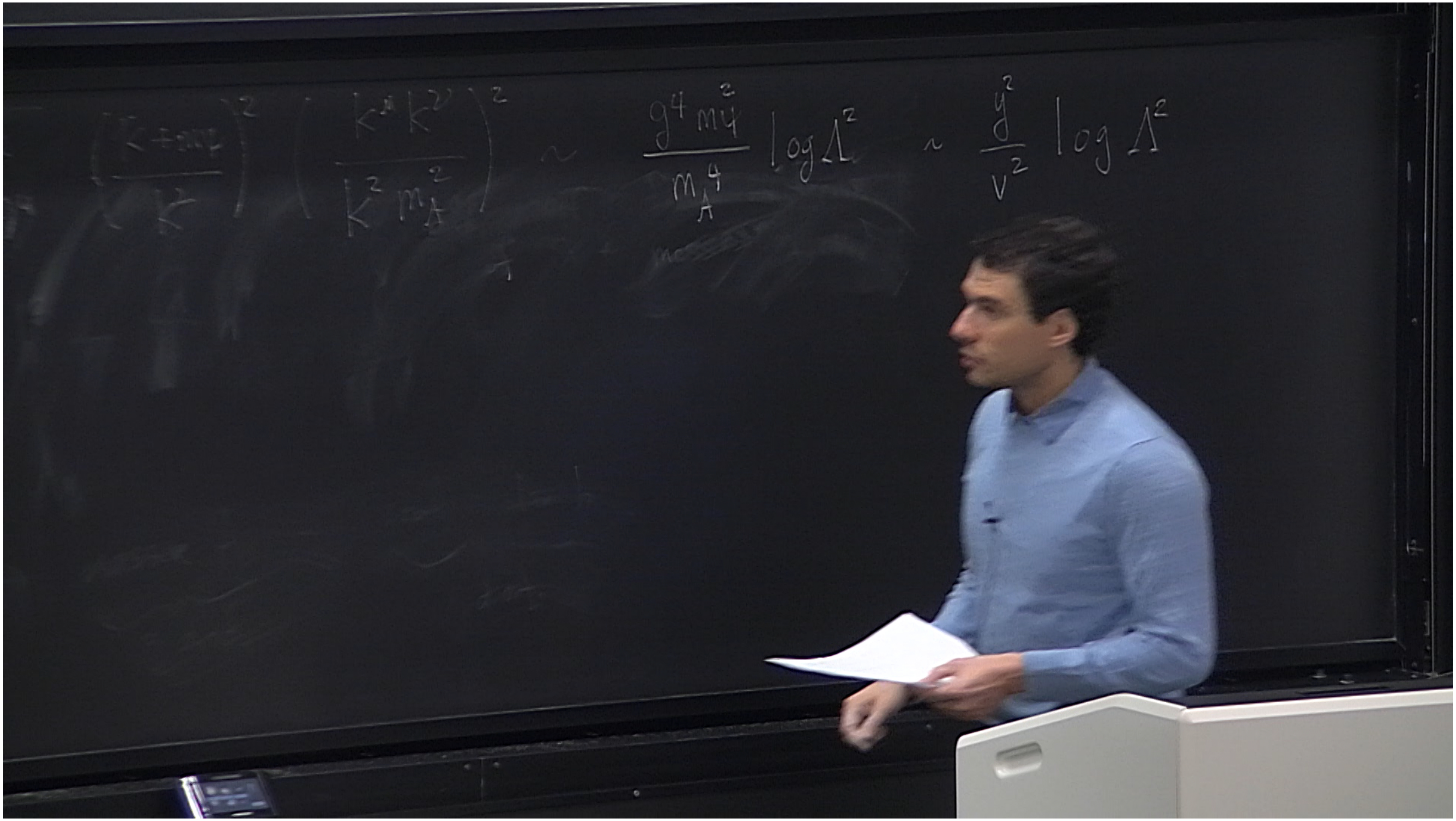

$\sim g \int^{\Lambda} \frac{d^4k}{(2\pi)^4} \left(\frac{k + m\psi}{k^2} \right)^2$

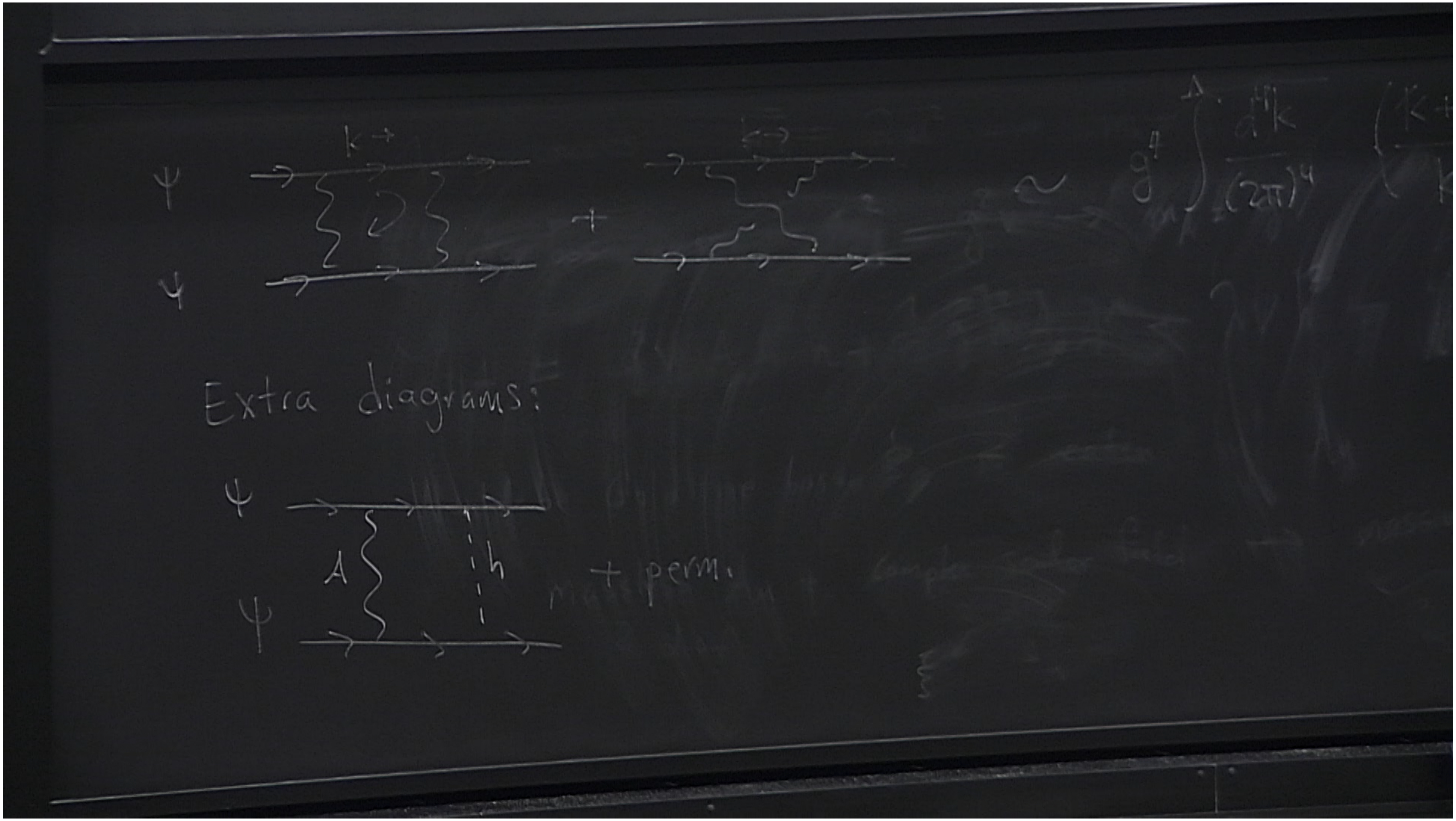

$\sim g \int^{\Lambda} \frac{d^4k}{(2\pi)^4} \left(\frac{k + m_\psi}{k^2}\right)^2 \left(\frac{k^\mu k^\nu}{k^2 m_A^2}\right)^2$

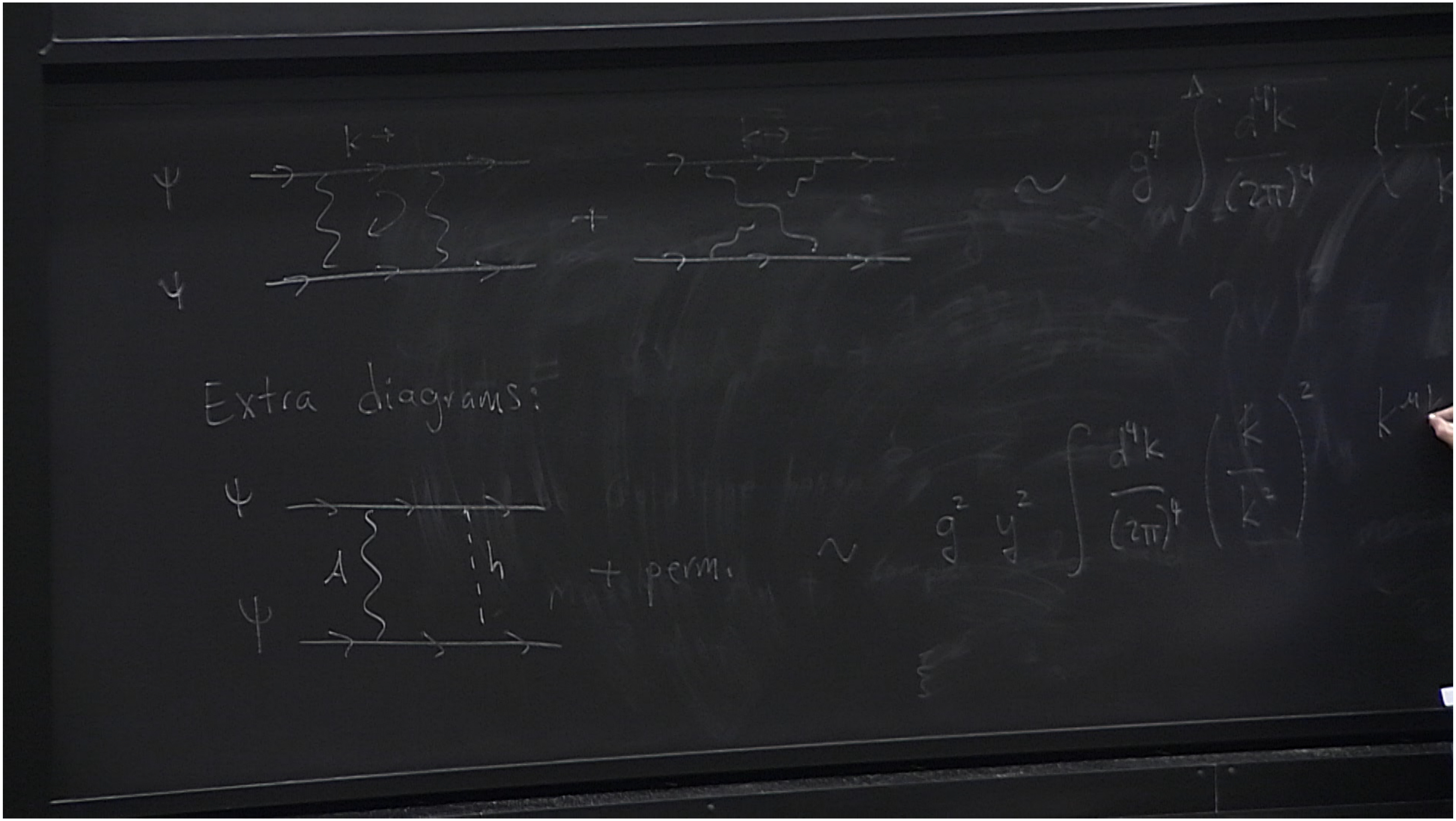

$\sim g^4 \int^{\Lambda} \frac{d^4k}{(2\pi)^4} \left(\frac{k + m_\psi}{k^2}\right)^2 \left(\frac{k^\mu k^\nu}{k^2 m_1^2}\right)^2$


$\left(\frac{k^\mu k^\nu}{k^2 m_A^2}\right)^2 \sim \frac{g^4 m_\psi^2}{m_A^4} \sim$


$\left(\frac{k^\mu k^\nu}{k^2 m_A^2}\right)^2 \sim \frac{g^4 m_\psi^2}{m_A^4} \sim \frac{y^2}{v^2}$


$\frac{g^4 m_\psi^2}{m_A^4} \log \Lambda^2 \sim \frac{y^2}{v^2} \log \Lambda^2$


ψ
ψ
k →
+
~ g⁴ ∫^Λ d⁴k/(2π)⁴
Extra diagrams:
ψ
A
h
ψ
+ perm.


ψ
ψ
+
$\sim g^4 \int^{\Lambda} \frac{d^4k}{(2\pi)^4}$
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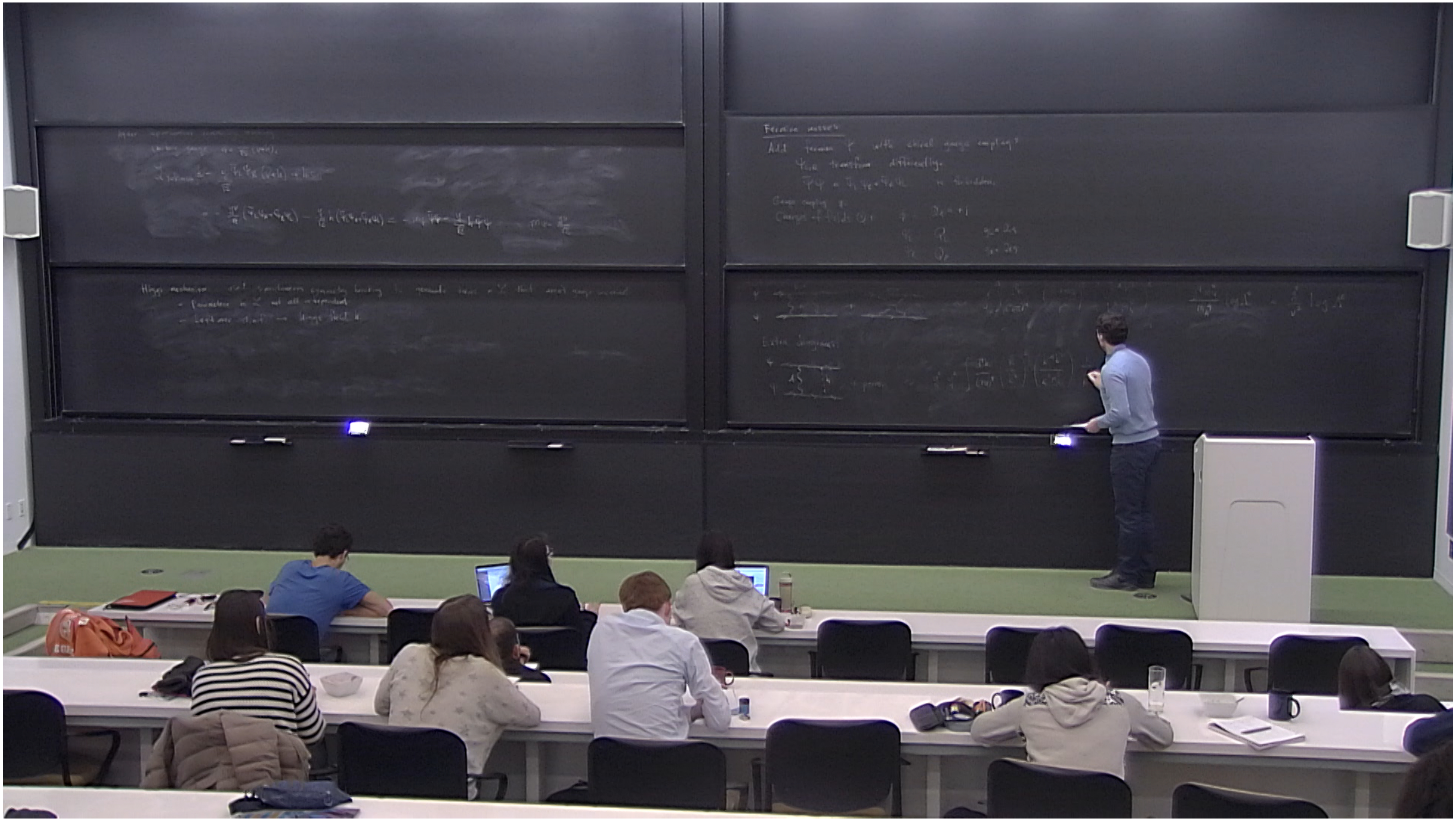
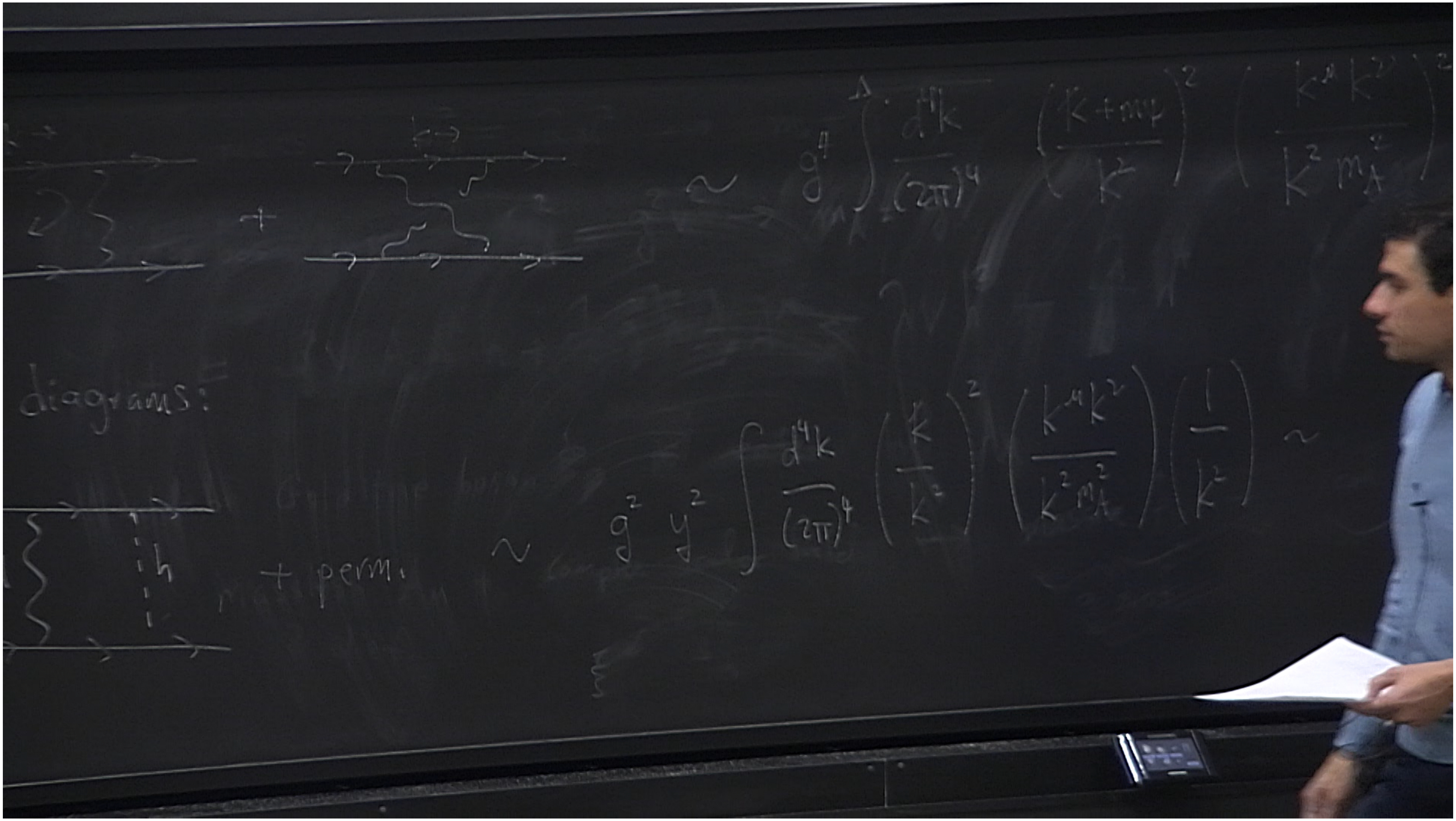
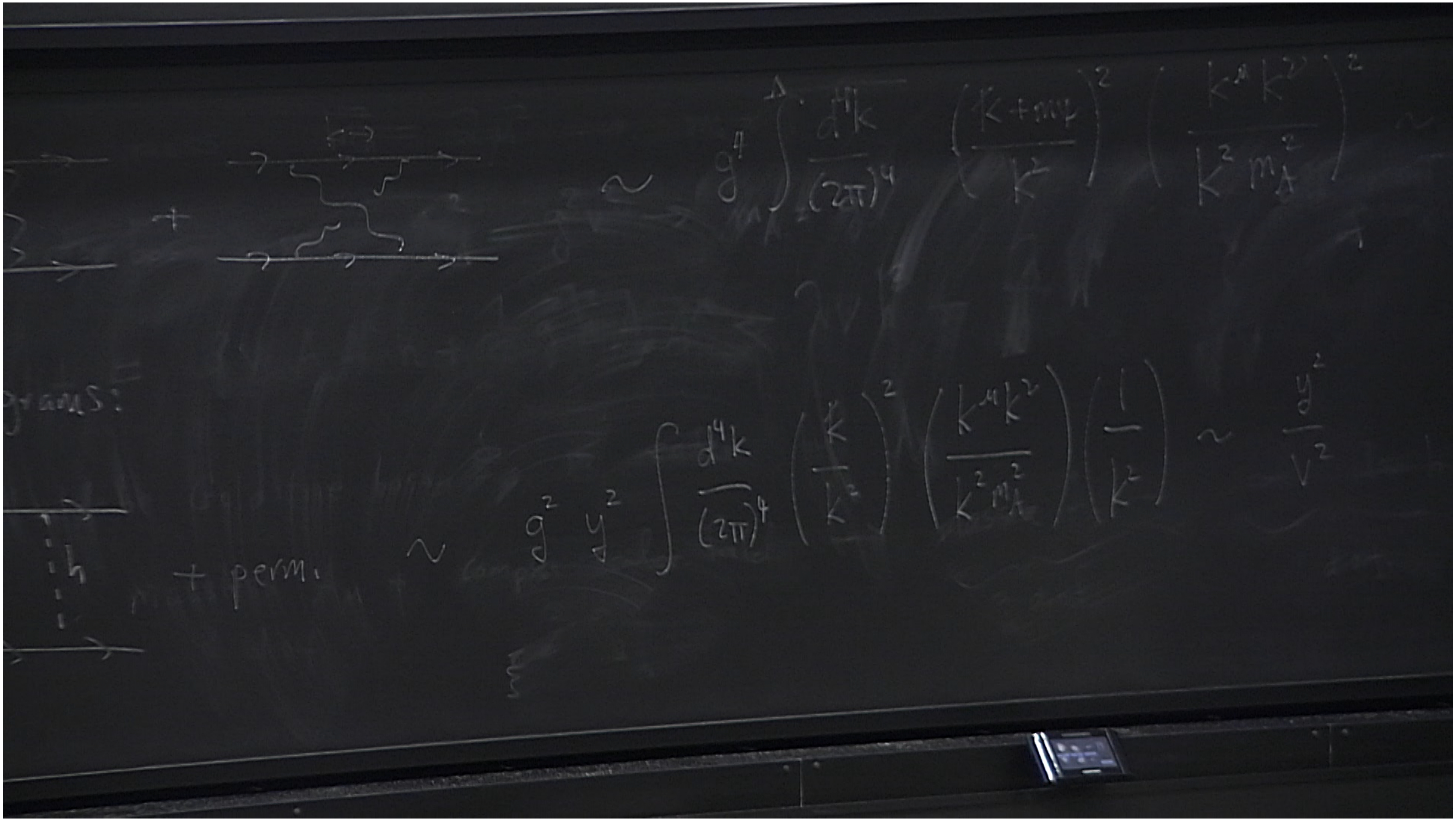
$\sim g^2 y^2 \int \frac{d^4k}{(2\pi)^4} \left(\frac{k}{k^2}\right)^2$

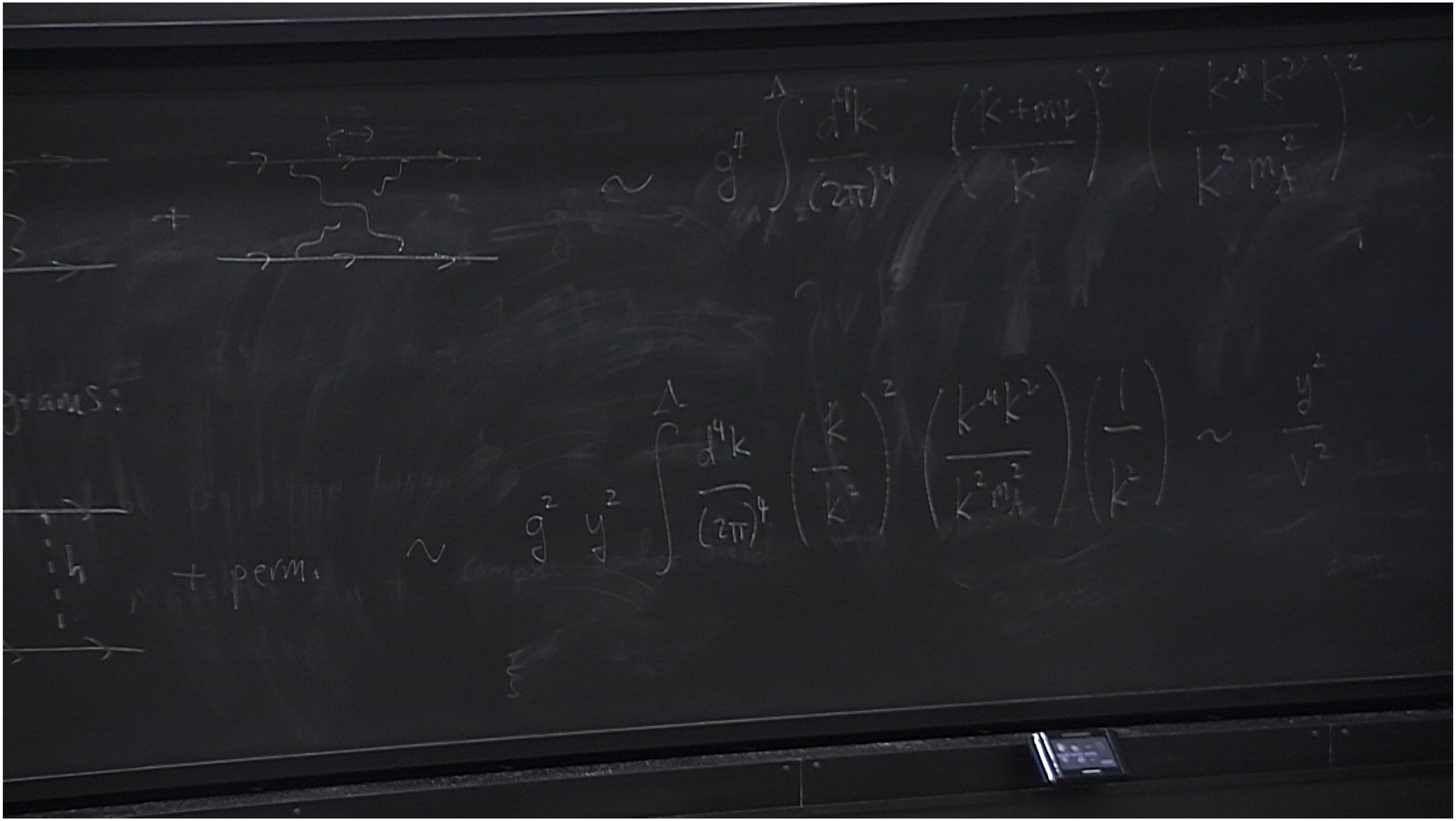




diagrams:
+ perm.
~ g^4 ∫^Λ d^4k/(2π)^4 ((k + mψ)/k^2)^2 (k^μ k^ν/(k^2 m_A^2))^2
~ g^2 y^2 ∫ d^4k/(2π)^4 (k/k^2)^2 (k^μ k^ν/(k^2 m_A^2)) (1/k^2) ~



$\sim \quad g^4 \int^{\Lambda} \frac{d^4k}{(2\pi)^4} \left(\frac{\not{k}+m_\psi}{k^2}\right)^2 \left(\frac{k^\mu k^\nu}{k^2 m_A^2}\right)^2$
grams:
h
+ perm.
$\sim \quad g^2\, y^2 \int \frac{d^4k}{(2\pi)^4} \left(\frac{\not{k}}{k^2}\right)^2 \left(\frac{k^\mu k^\nu}{k^2 m_A^2}\right)\left(\frac{1}{k^2}\right) \sim \frac{y^2}{v^2}$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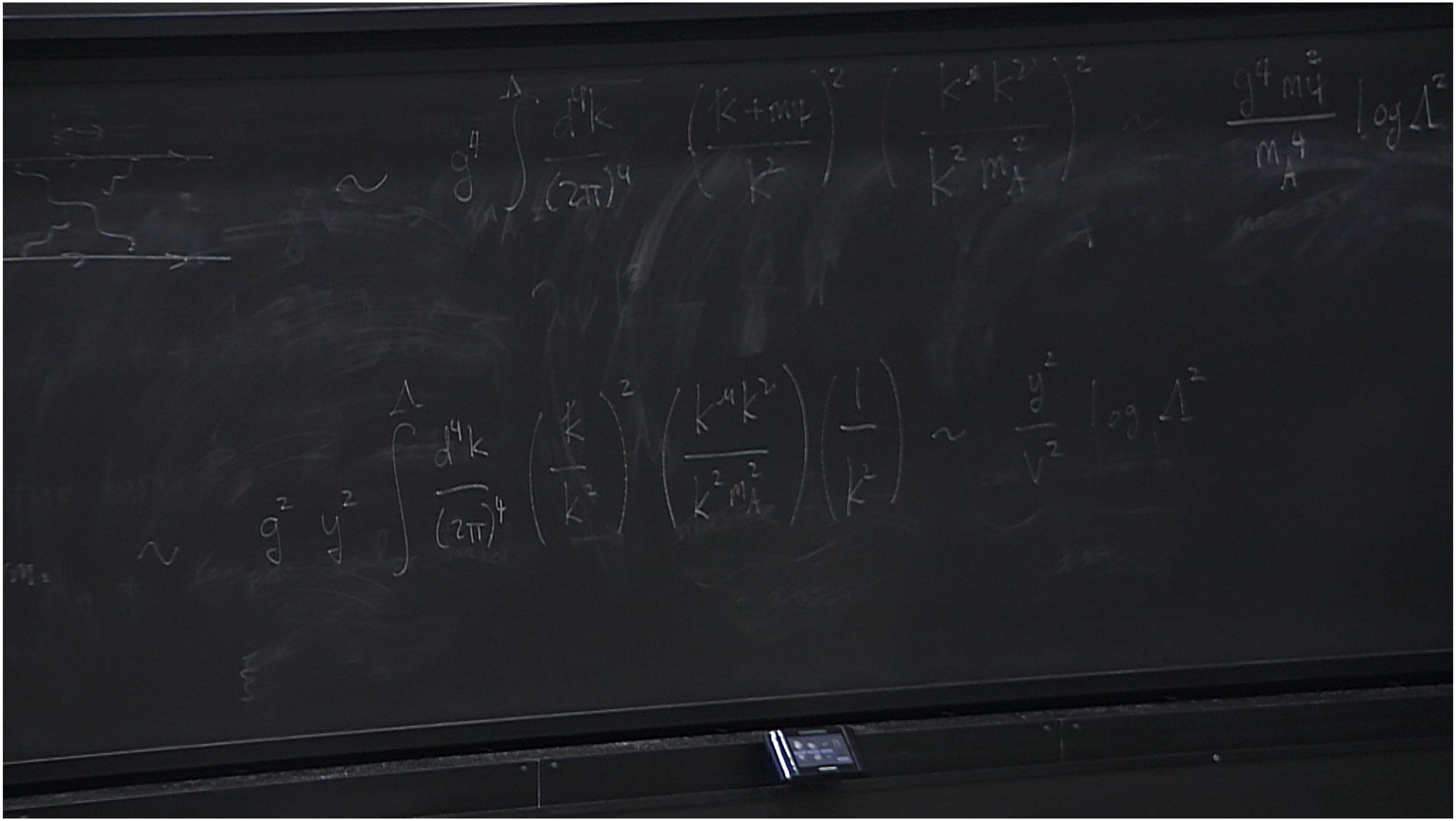
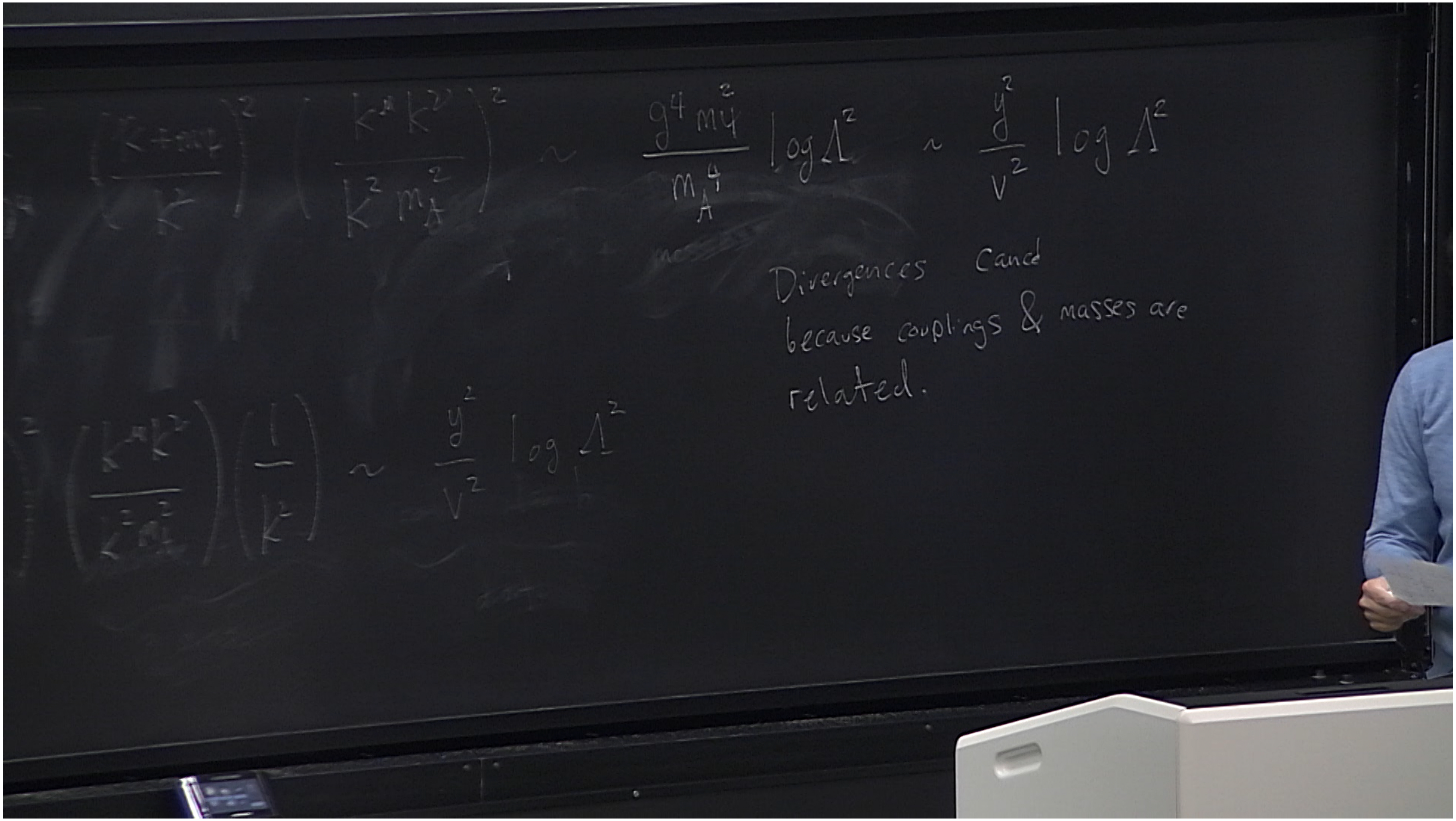
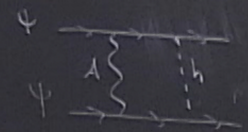

$\sim g^4 \int^\Lambda \frac{d^4k}{(2\pi)^4} \left(\frac{k+m_\psi}{k^2}\right)^2 \left(\frac{k^\mu k^\nu}{k^2 m_A^2}\right)^2 \sim \frac{g^4 m_\psi^2}{m_A^4} \log \Lambda^2$
$\sim g^2 y^2 \int^\Lambda \frac{d^4k}{(2\pi)^4} \left(\frac{k}{k^2}\right)^2 \left(\frac{k^\mu k^\nu}{k^2 m_A^2}\right) \left(\frac{1}{k^2}\right) \sim \frac{y^2}{v^2} \log \Lambda^2$


$\left(\frac{k^\mu k^\nu}{k^2 m_A^2}\right)^2 \sim \frac{g^4 m_\psi^2}{m_A^4} \log \Lambda^2 \sim \frac{y^2}{v^2} \log \Lambda^2$
Divergences Cancel
because couplings & masses are related.
$\left(\frac{k^\mu k^\nu}{k^2 m_A^2}\right)\left(\frac{1}{k^2}\right) \sim \frac{y^2}{v^2} \log \Lambda^2$

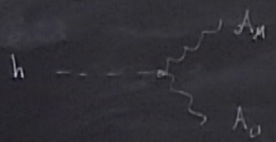
Extra diagrams:



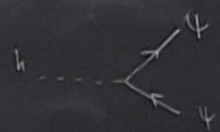
$$+\text{ perm.} \sim g^2 y^2 \int^{\Lambda} \frac{d^4k}{(2\pi)^4} \left(\frac{k}{k^2}\right)^2 \left(\frac{k^{-4}k^2}{k^2 m_A^2}\right)\left(\frac{1}{k^2}\right) \sim \frac{y^2}{v^2} \log \Lambda^2$$

Divergences cancel because couplings & masses are related.

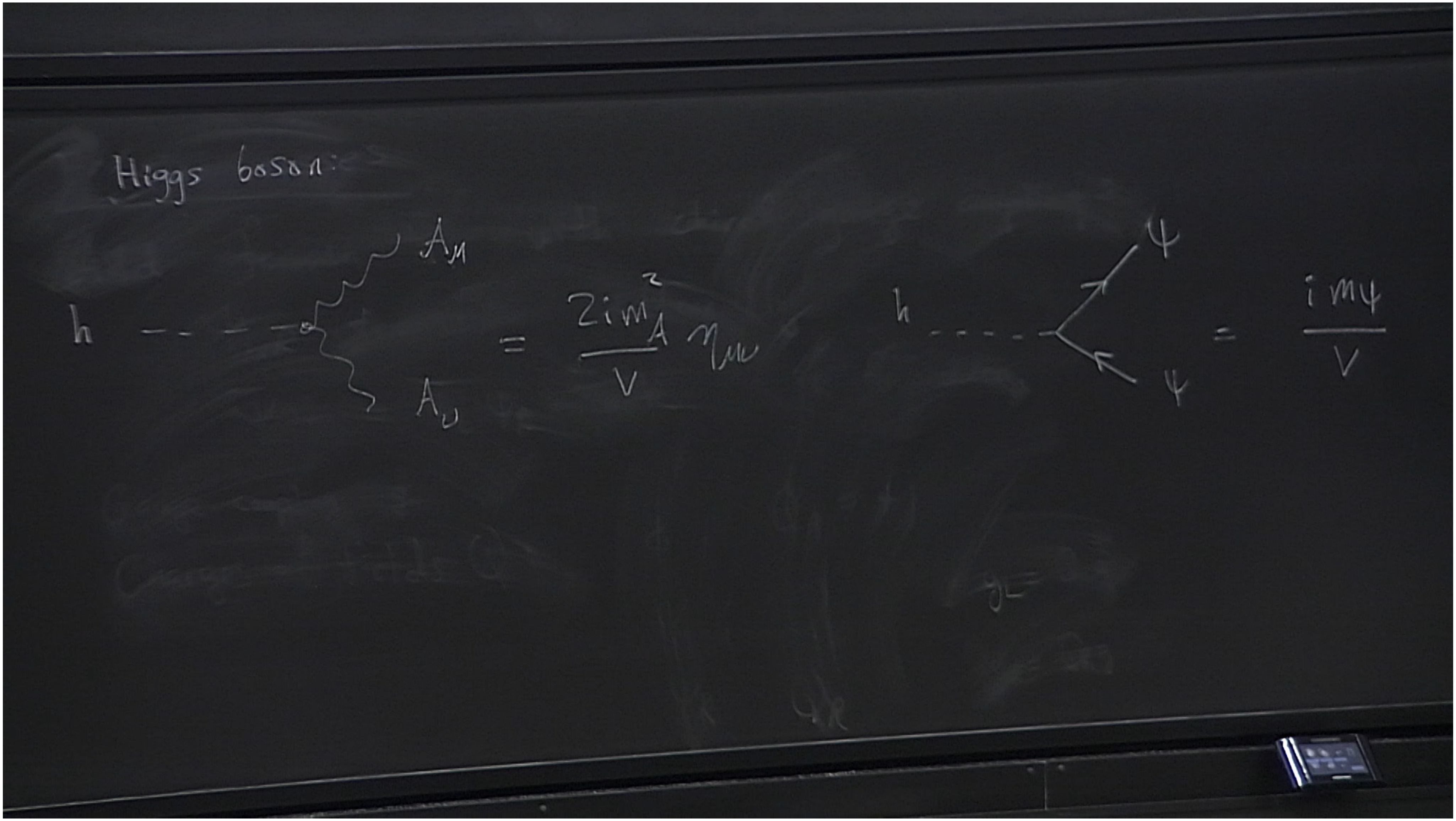
Higgs boson



$$= \frac{2 i m_A^2}{v} \eta_{\mu\nu}$$



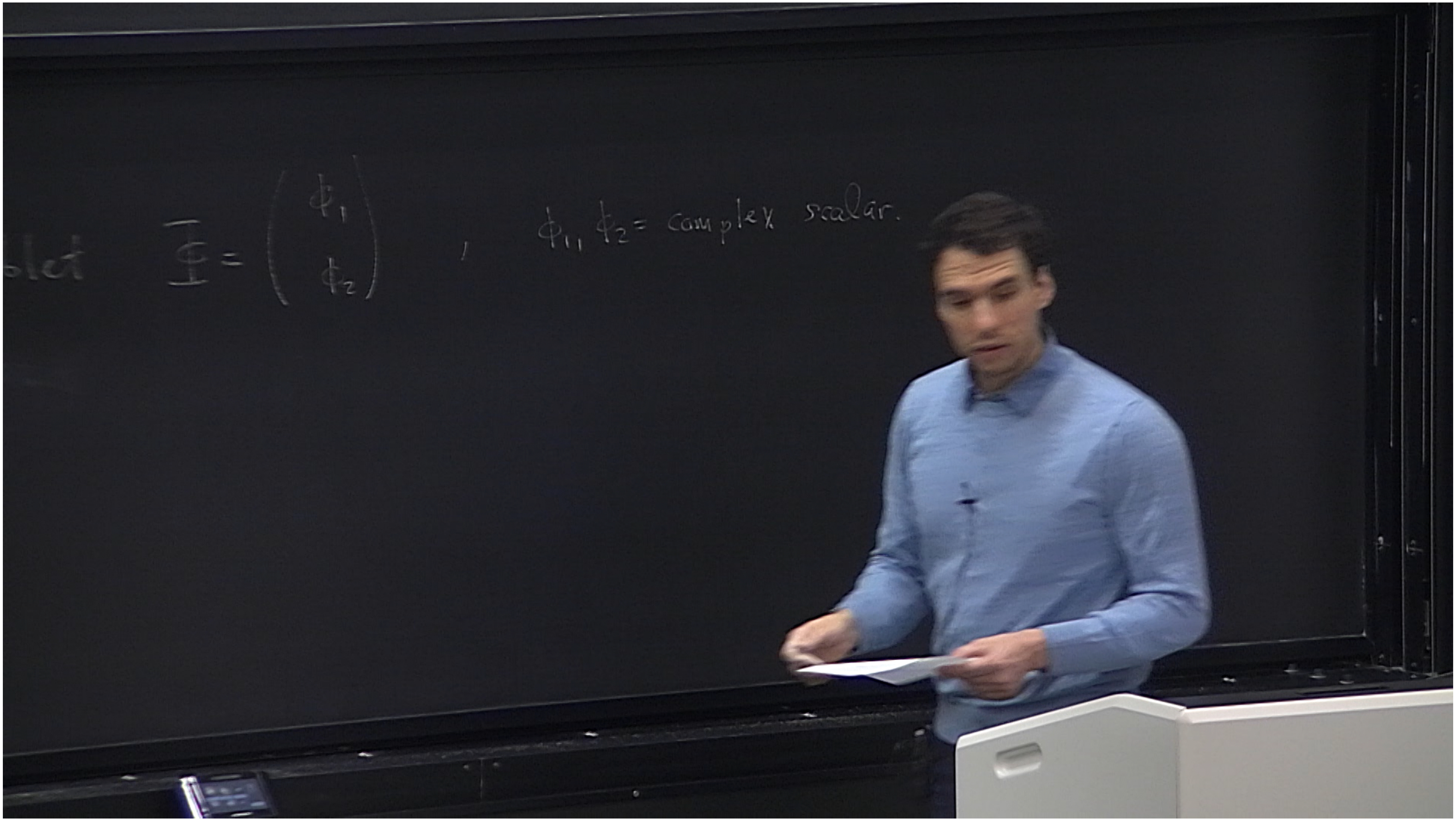
$$= \frac{i m_\psi}{v}$$


Higgs boson:
h
A_μ
A_ν
$= \frac{2im_A^2}{v}\eta_{\mu\nu}$
h
ψ
ψ
$= \frac{im_\psi}{v}$

Non abelian Higgs model

SU(2) gauge theory + complex scalar doublet

$\mathcal{L}_{Yukawa}$


$\Phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$,
ϕ_1, ϕ_2 = complex scalar.

Non abelian Higgs model

SU(2) gauge theory + complex scalar doublet $\Phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$

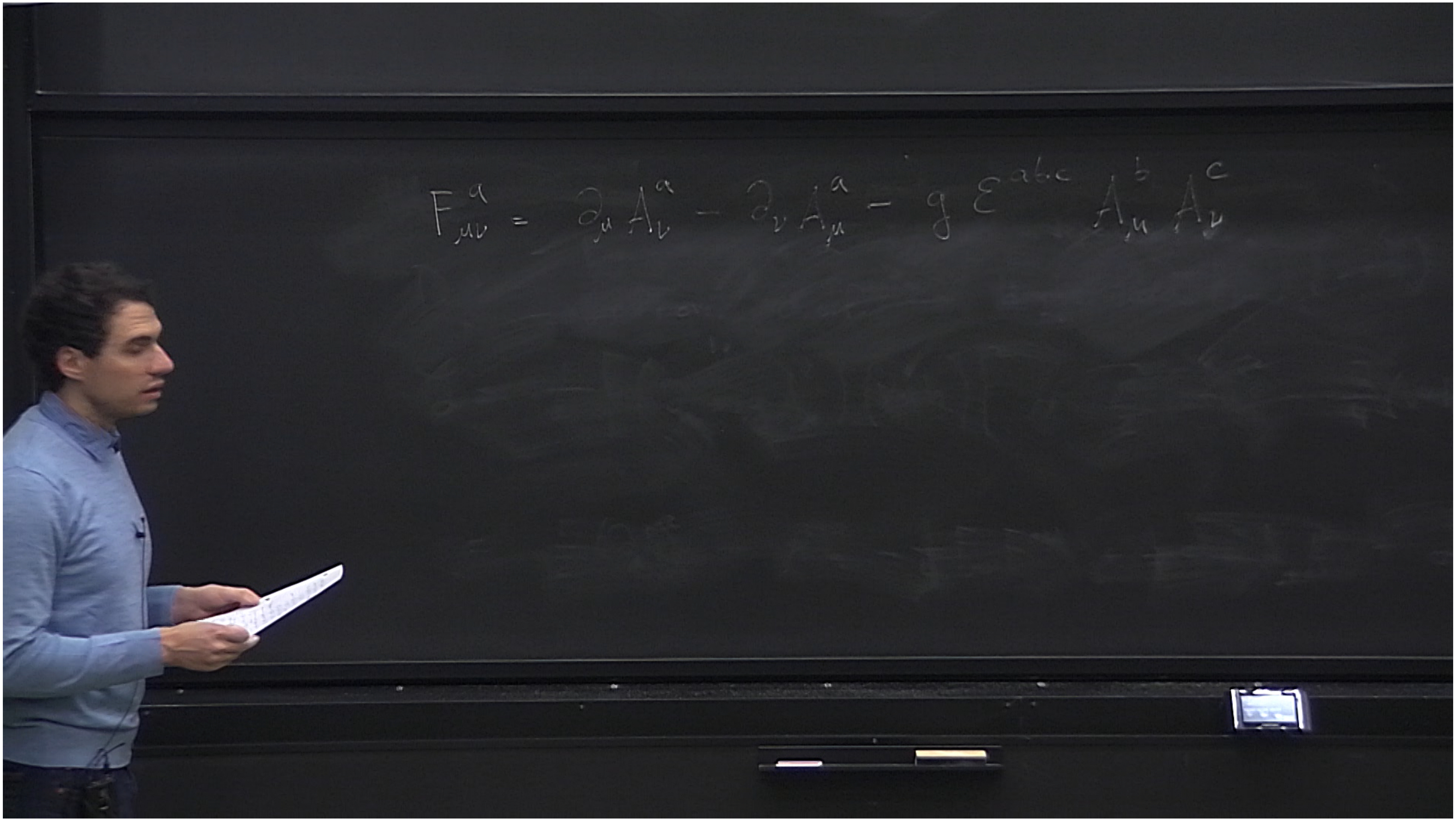
$$\mathcal{L} = (D_\mu \Phi)^\dagger (D^\mu \Phi) - V(\Phi) - \frac{1}{2} \mathrm{Tr}\left[F_{\mu\nu} F^{\mu\nu} \right]$$

$$D_\mu \Phi = \left(\partial_\mu + i g T^a A^a_\mu \right) \Phi$$

complex scalar doublet $\Phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$, ϕ_1, ϕ_2 = complex scalar.

$$\Phi) - V(\Phi) - \frac{1}{2} \mathrm{Tr}\left[F_{\mu\nu} F^{\mu\nu} \right]$$

$$ig T^a A^a_\mu) \Phi \, , \qquad T^a = \frac{\sigma^a}{2}$$ generator of $su(2)$


$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g\,\varepsilon^{abc} A^b_\mu A^c_\nu$

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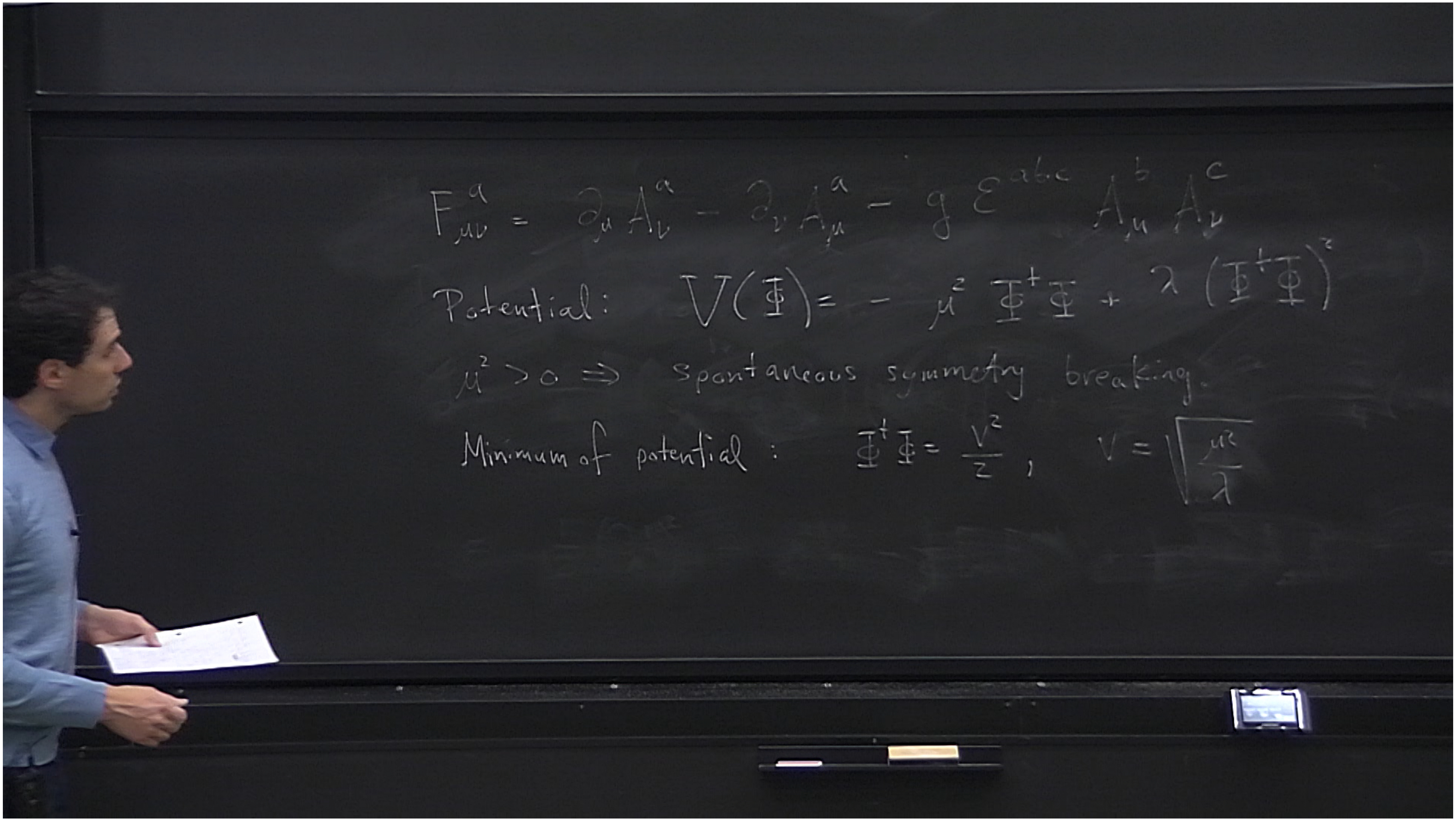
Potential: $V(\Phi) = -\mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2$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Potential: $V(\Phi) = -\mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2$
$\mu^2 > 0 \Rightarrow$ spontaneous symmetry breaking.
Minimum of potential: $\Phi^\dagger \Phi = \frac{v^2}{2}, \quad v = \sqrt{\frac{\mu^2}{\lambda}}$

Work in unitary gauge.

$$\underline{\Phi} = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$$

...ork in unitary gauge.

$$\underline{\Phi} = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} = \begin{pmatrix} \phi_{1r} + i\phi_{1i} \\ \phi_{2r} + i\phi_{2i} \end{pmatrix} = \exp\left(i\, T^a \xi^a(x)/v \right) \begin{pmatrix} 0 \\ \frac{v+h(x)}{\sqrt{2}} \end{pmatrix}$$

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Real fields $\xi_1 - \xi_3$ & h.

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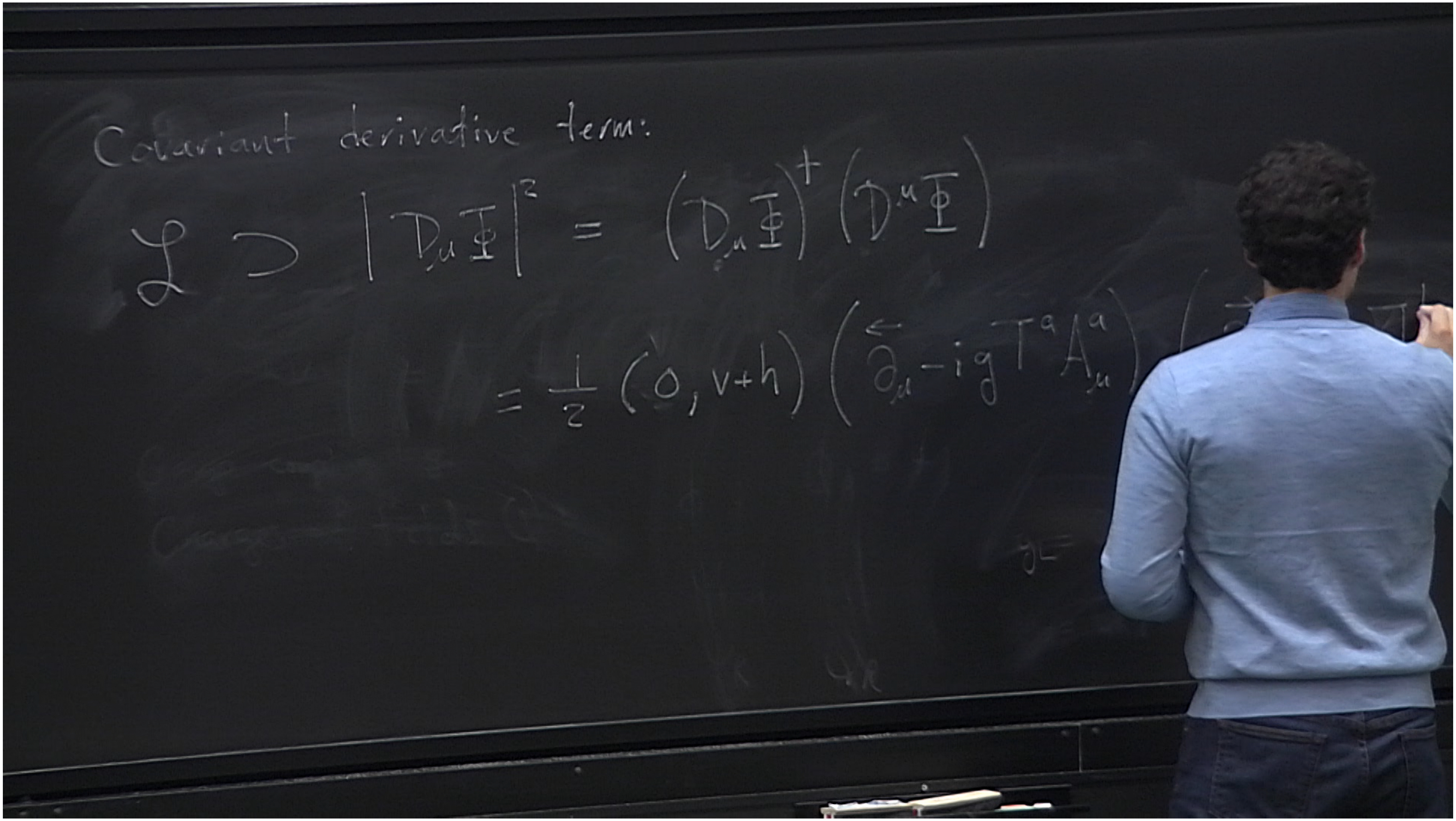
Covariant derivative term:
$\mathcal{L} \supset |D_\mu \Phi|^2 = (D_\mu \Phi)^\dagger (D^\mu \Phi)$
$= \frac{1}{2}(0, v+h)\left(\overleftarrow{\partial}_\mu - i g T^a A^a_\mu\right)$

tive term:

$$\Phi|^2 = (D_\mu \Phi)^\dagger (D^\mu \Phi)$$

$$= \frac{1}{2}(0, v+h)\left(\overleftarrow{\partial}_\mu - i g T^a A^a_\mu\right)\left(\overrightarrow{\partial}^\mu + i g T^b A^b_\mu\right)\begin{pmatrix} 0 \\ v+h \end{pmatrix}$$

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$$= \frac{1}{2}(\partial_\mu h)^2 + \frac{g^2}{8}$$

...ive term:

$$|\ldots\Phi|^2 = (D_\mu \Phi)^\dagger (D^\mu \Phi)$$

$$= \frac{1}{2}(0, v+h)\left(\overleftarrow{\partial}_\mu - igT^a A^a_\mu\right)\left(\overrightarrow{\partial}^\mu + igT^b A^b_\mu\right)\begin{pmatrix}0\\ v+h\end{pmatrix}$$

$$= \frac{1}{2}(\partial_\mu h)^2 + \frac{g^2}{8}(0, v+h)\begin{pmatrix}A^3_\mu & A^1_\mu - iA^2_\mu \\ A^1_\mu + iA^2_\mu & -A^3_\mu\end{pmatrix}^2\begin{pmatrix}0\\ v+h\end{pmatrix}$$

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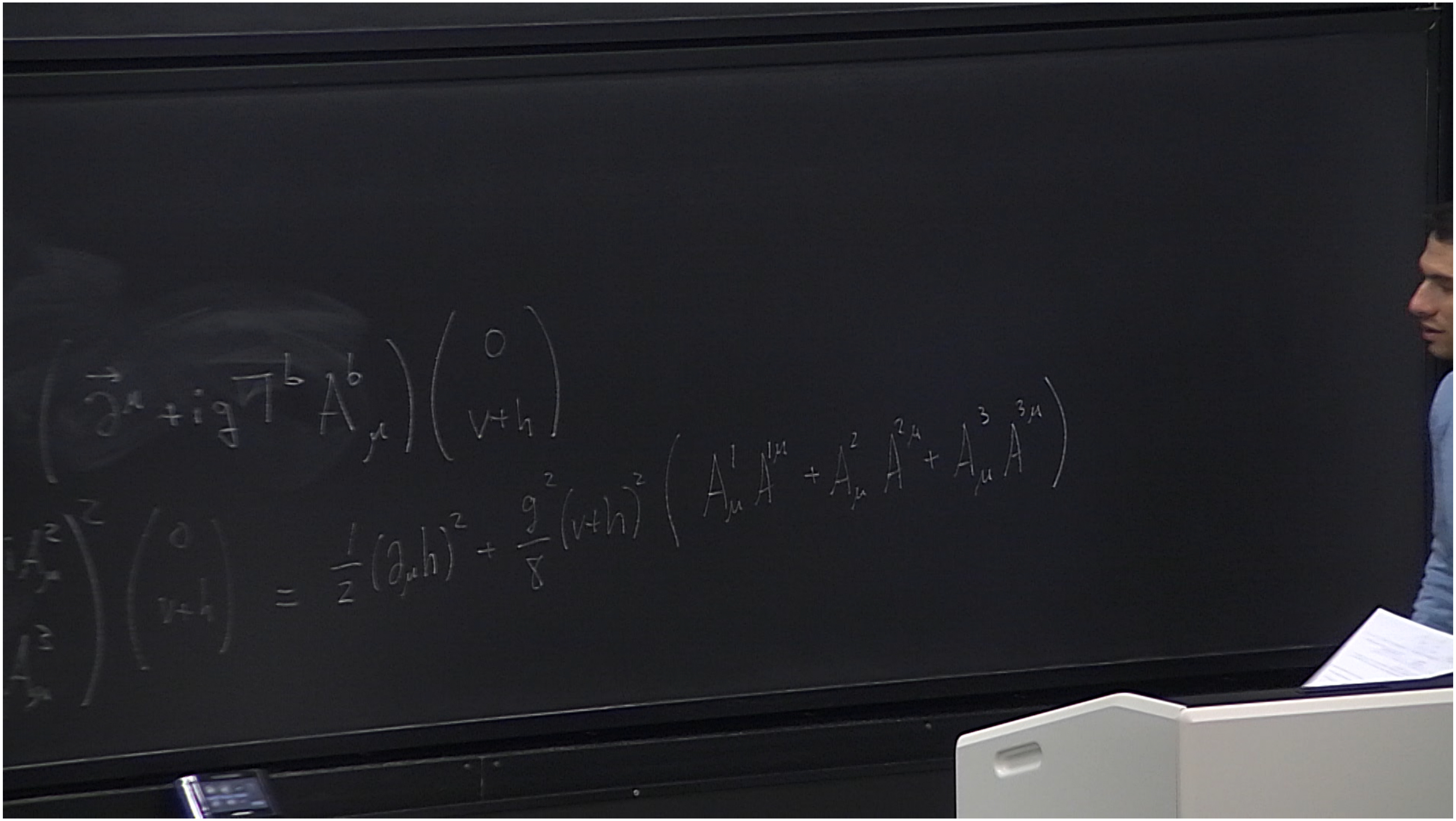
$= \frac{1}{2}(\partial_\mu h)^2 + \frac{g^2}{8}(v+h)^2\left(A^1_\mu A^{1\mu} + A^2_\mu A^{2\mu} + A^3_\mu A^{3\mu}\right)$

$$= \frac{1}{2}(\partial_\mu h)^2 + \frac{1}{2}\left(\frac{gv}{2}\right)^2 A^a_\mu A^{a\mu}$$

$$= \frac{1}{2}(\partial_\mu h)^2 + \frac{1}{2}\left(\frac{gv}{2}\right)^2 A^a_\mu A^{a\mu}\left(1+\frac{h}{v}\right)^2$$

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Three gauge bosons: $A^1_\mu, A^2_\mu, A^3_\mu$ all have same mass $m_A = \frac{gv}{2}$

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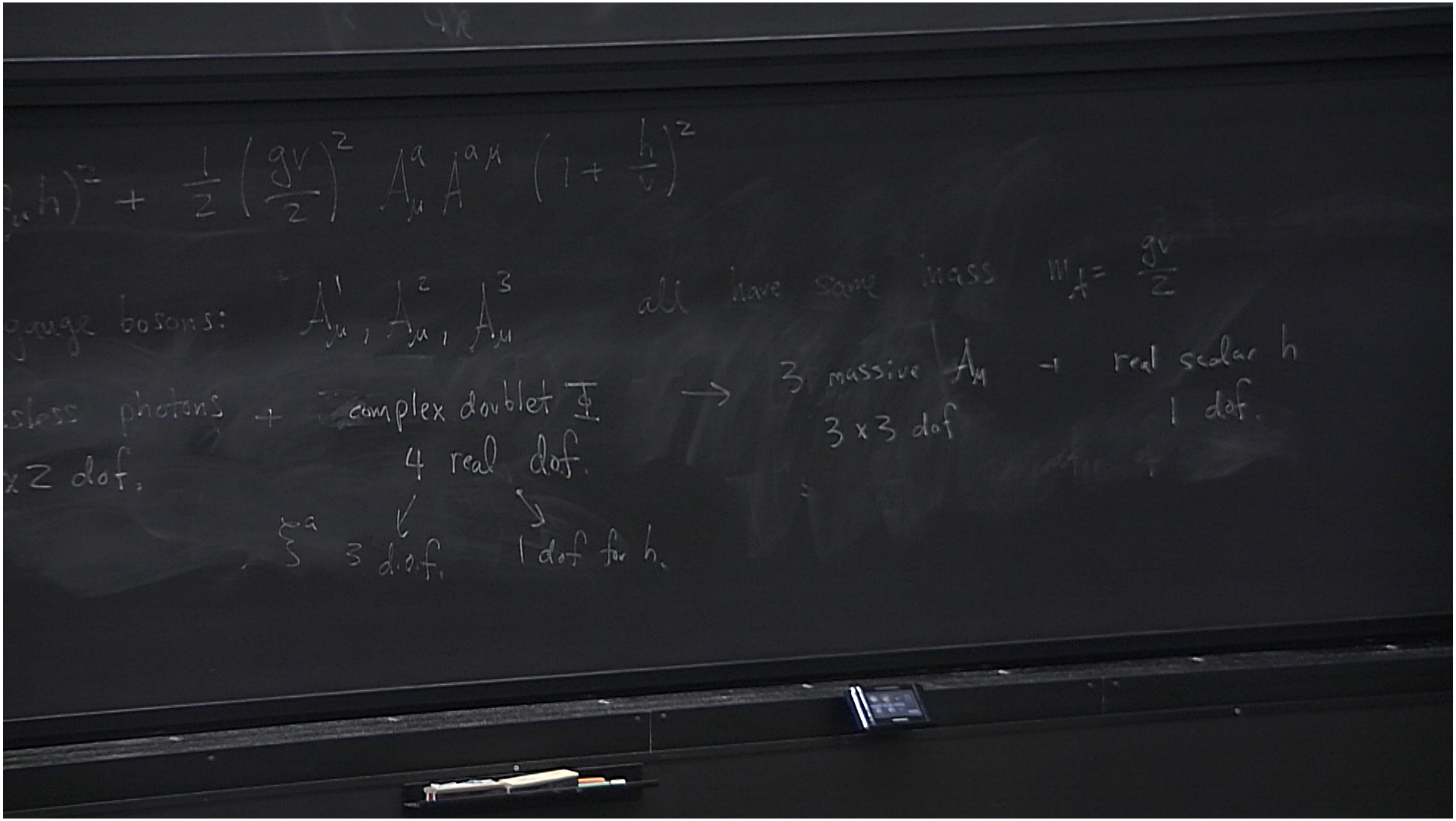
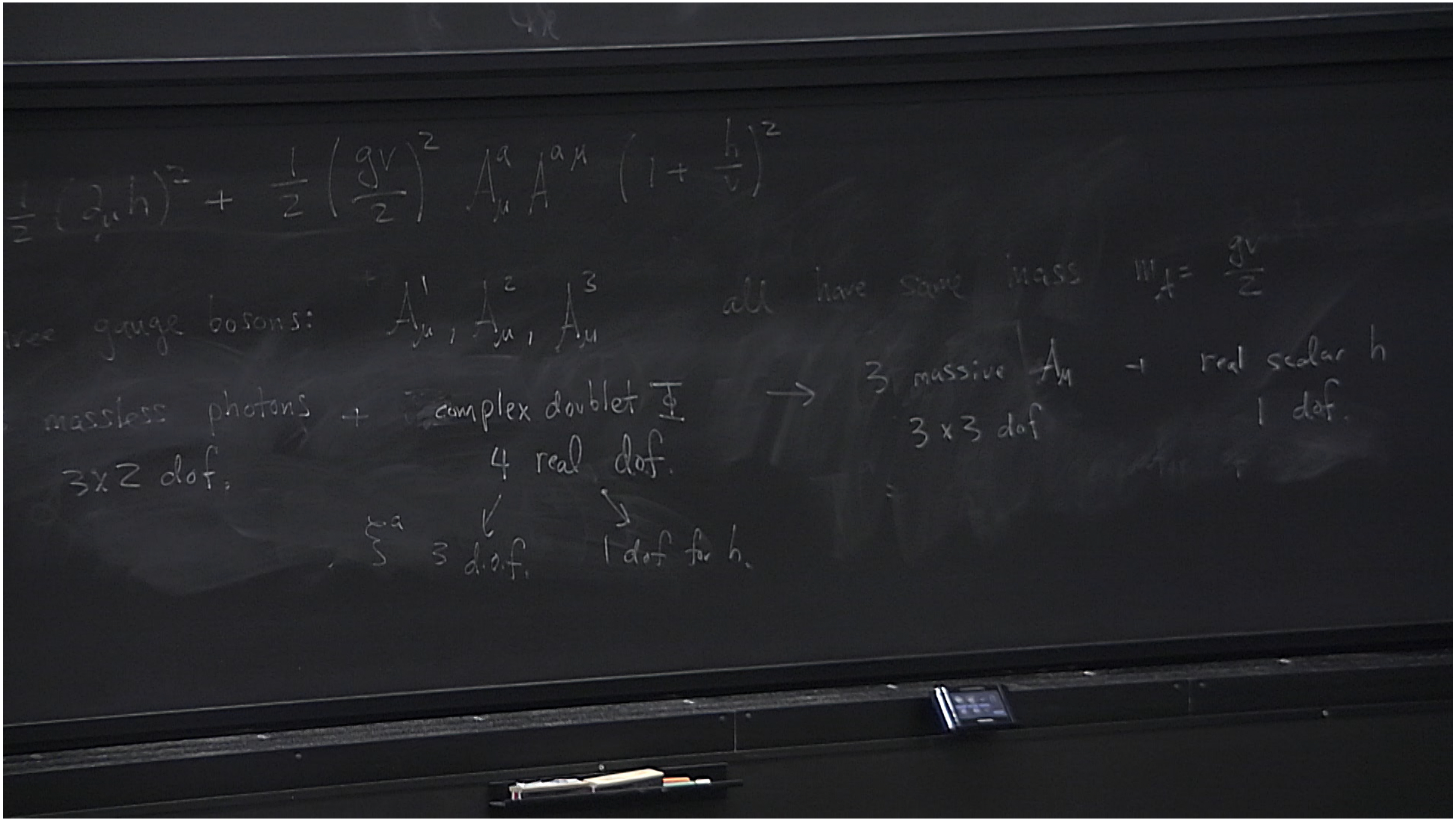
3 massless photons + complex doublet Φ

3×2 dof. 4 real dof.

ξ^a 3 d.o.f. 1 dof for h.



$(\partial_\mu h)^2 + \frac{1}{2}\left(\frac{gv}{2}\right)^2 A^a_\mu A^{a\mu}\left(1+\frac{h}{v}\right)^2$
gauge bosons: $A^1_\mu, A^2_\mu, A^3_\mu$
all have same mass $m_A = \frac{gv}{2}$
massless photons + complex doublet Φ
$\times 2$ dof.
4 real dof.
ξ^a 3 d.o.f.
1 dof for h.
→ 3 massive A_μ + real scalar h
3 x 3 dof
1 dof.


$\frac{1}{2}(\partial_\mu h)^2 + \frac{1}{2}\left(\frac{gv}{2}\right)^2 A^a_\mu A^{a\mu}\left(1+\frac{h}{v}\right)^2$
gauge bosons: $A^1_\mu, A^2_\mu, A^3_\mu$
all have same mass $m_A = \frac{gv}{2}$
massless photons + complex doublet Φ $\longrightarrow$ 3 massive A_μ + real scalar h
3x2 dof.
4 real dof.
3 x 3 dof
1 dof.
ξ^a 3 d.o.f.
1 dof for h.