Title: Applications of convex-split technique in Quantum Resource theory and Quantum Communication.

Date: Dec 20, 2017 10:00 AM

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Abstract: <p>We discuss some applications of a result on the convex combination of the quantum states (that we refer to as convex-split technique) and its variants. In the framework of Quantum Resource theory, we provide an operational way of characterizing the amount of resource in a given quantum state, for a large class of resource theories. Our results use the convex-split technique in the achievability proof, with a matching converse in the one-shot setting. Building upon the ideas involved in this achievability proof, we show how the technique (and its close counterpart of position-based decoding) can lead to a large family of entanglement-assisted protocols for quantum communication. We sketch close connections between these protocols and the port-based teleportation scheme of Ishizaka and Hiroshima [Phys. Rev. Lett. 101, 240501]. We exploit these connections to obtain a new protocol for entanglement assisted point to point quantum channel coding in the asymptotic and i.i.d. setting, with the rate of entanglement cost and achievable communication matching that of Bennett et. al. [IEEE Trans. Inf. Theory, 48, 2002]. Based on the works arXiv:1410.3031, arXiv:1708.00381,   arXiv:1702.01940.</p>


Talk overview
Resource destruction in Quantum resource theory
Protocols for quantum communication
A new protocol for entanglement assisted communication over qua
Applications of convex-split technique to quantum resource theory and quantum communication
Anurag Anshu
Centre for Quantum Technologies, NUS, Singapore
December 20, 2017
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Outline for section 1

1. Talk overview
2. Resource destruction in Quantum resource theory
   - Background
   - The problem of destroying correlation
   - The problem of destroying entanglement
   - Generalization to other resource theories
3. Protocols for quantum communication
   - Quantum state splitting
   - Connection to decoupling theorems
   - Connection to port-based teleportation
4. A new protocol for entanglement assisted communication over quantum channel
   - Position-based decoding
   - Achievin

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Part 2

- Convex-split lemma with an external reference
- Protocols for quantum communication.
- Connections with the decoupling theorem.

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Part 2

- Convex-split lemma with an external reference
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Outline for section 2

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3. Protocols for quantum communication
   - Quantum state splitting
   - Connection to decoupling theorems
   - Connection to port-based teleportation
4. A new protocol for entanglement assisted communication over quantum channel
   - Position-based decoding
   - Achieving optimal rate of entanglement

Notations

- σ_B

Notations

- Ψ_B


Notations

- $\mathrm{D}_{\max}(\Psi_B \| \sigma_B) = \inf\{\lambda : \Psi_B \preceq 2^{\lambda}\sigma_B\}$.
- $\mathrm{D}(\Psi_B \| \sigma_B) = \mathrm{Tr}\left(\Psi_B \log \Psi_B - \Psi_B \log \sigma_B\right)$.

Notations

$\frac{1}{2}$  $+$ $\frac{1}{2}$ 

$$\frac{1}{2}\Psi_{B_1} \otimes \sigma_{B_2} + \frac{1}{2}\sigma_{B_1} \otimes \Psi_{B_2}$$

Convex-split Lemma: simplest version

$B_1, B_2, \ldots, B_N$

$$\overset{\varepsilon}{\approx} \frac{1}{N} + \frac{1}{N} \cdots\cdots + \frac{1}{N}$$

$$\text{If } \log N \geq \mathrm{D}_{\max}(\Psi_B \| \sigma_B) + \log \frac{1}{\varepsilon}.$$

A., Devabathini, Jain [Phys. Rev. Lett. 2017, arXiv 2014].

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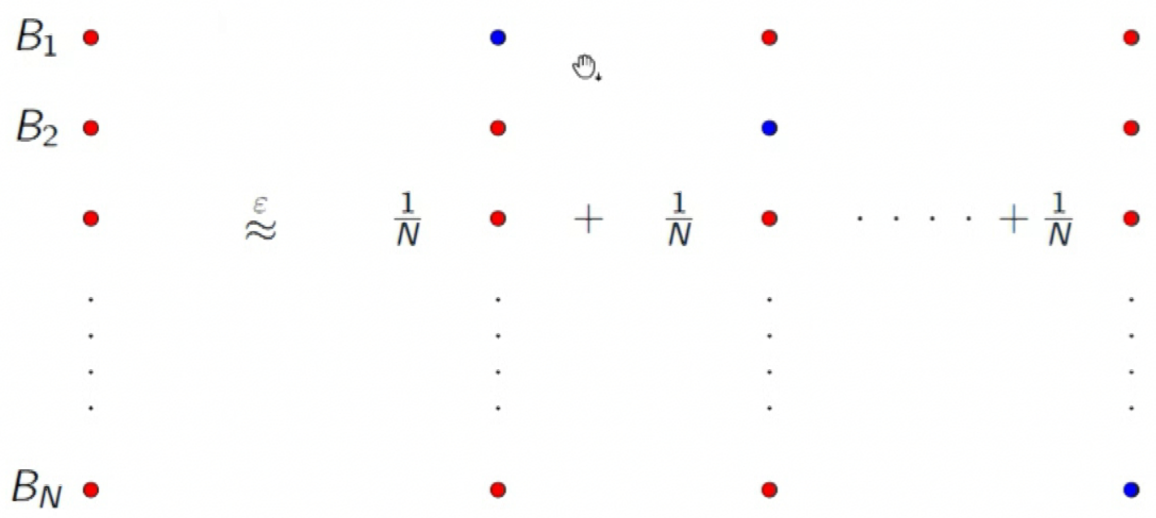
Convex-split Lemma: simplest version



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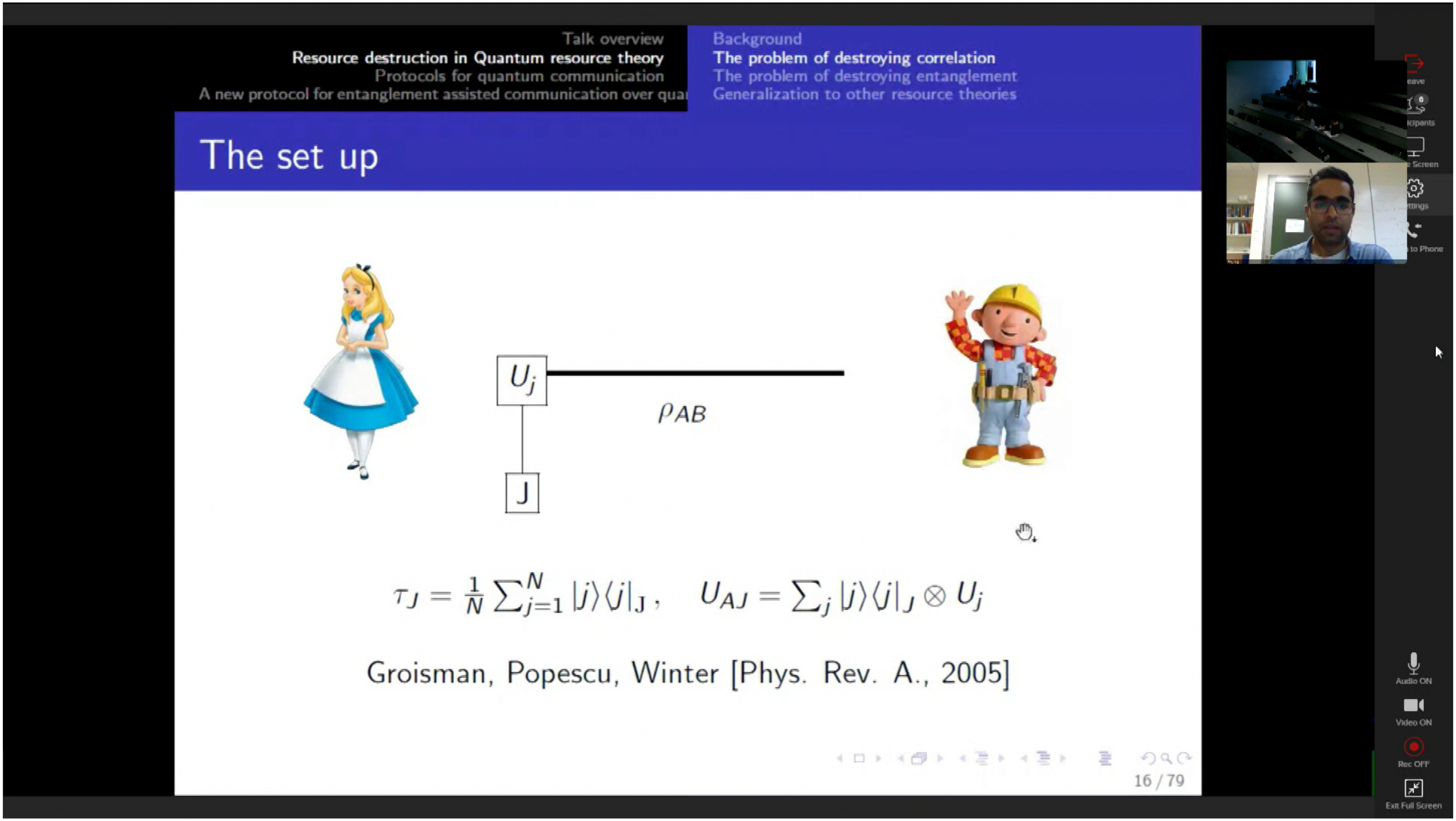
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The set up
U_j
ρ_{AB}
J
$\tau_J = \frac{1}{N}\sum_{j=1}^{N} |j\rangle\langle j|_J\,, \quad U_{AJ} = \sum_j |j\rangle\langle j|_J \otimes U_j$
Groisman, Popescu, Winter [Phys. Rev. A., 2005]
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The set up

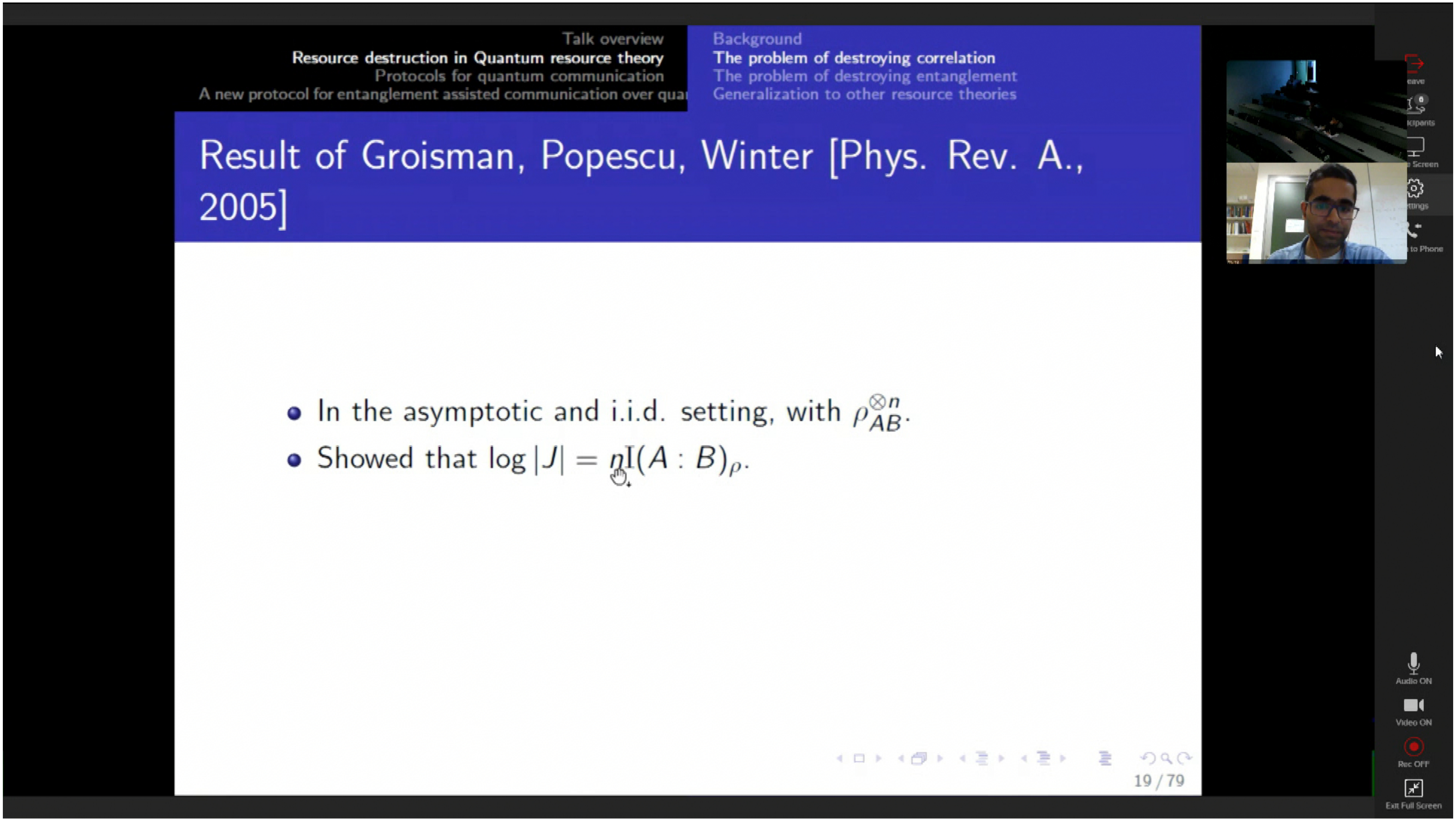
ρ'_{AB}

$$\rho'_{AB} = \mathrm{Tr}_J(U_{AJ}\rho_{AB} \otimes \tau_J U^\dagger_{AJ})$$

Groisman, Popescu, Winter [Phys. Rev. A., 2005]

Result of Groisman, Popescu, Winter [Phys. Rev. A., 2005]

- In the asymptotic and i.i.d. setting, with $\rho_{AB}^{\otimes n}$.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Result of Groisman, Popescu, Winter [Phys. Rev. A., 2005]
In the asymptotic and i.i.d. setting, with $\rho_{AB}^{\otimes n}$.
Showed that $\log |J| = n\mathrm{I}(A:B)_\rho$.
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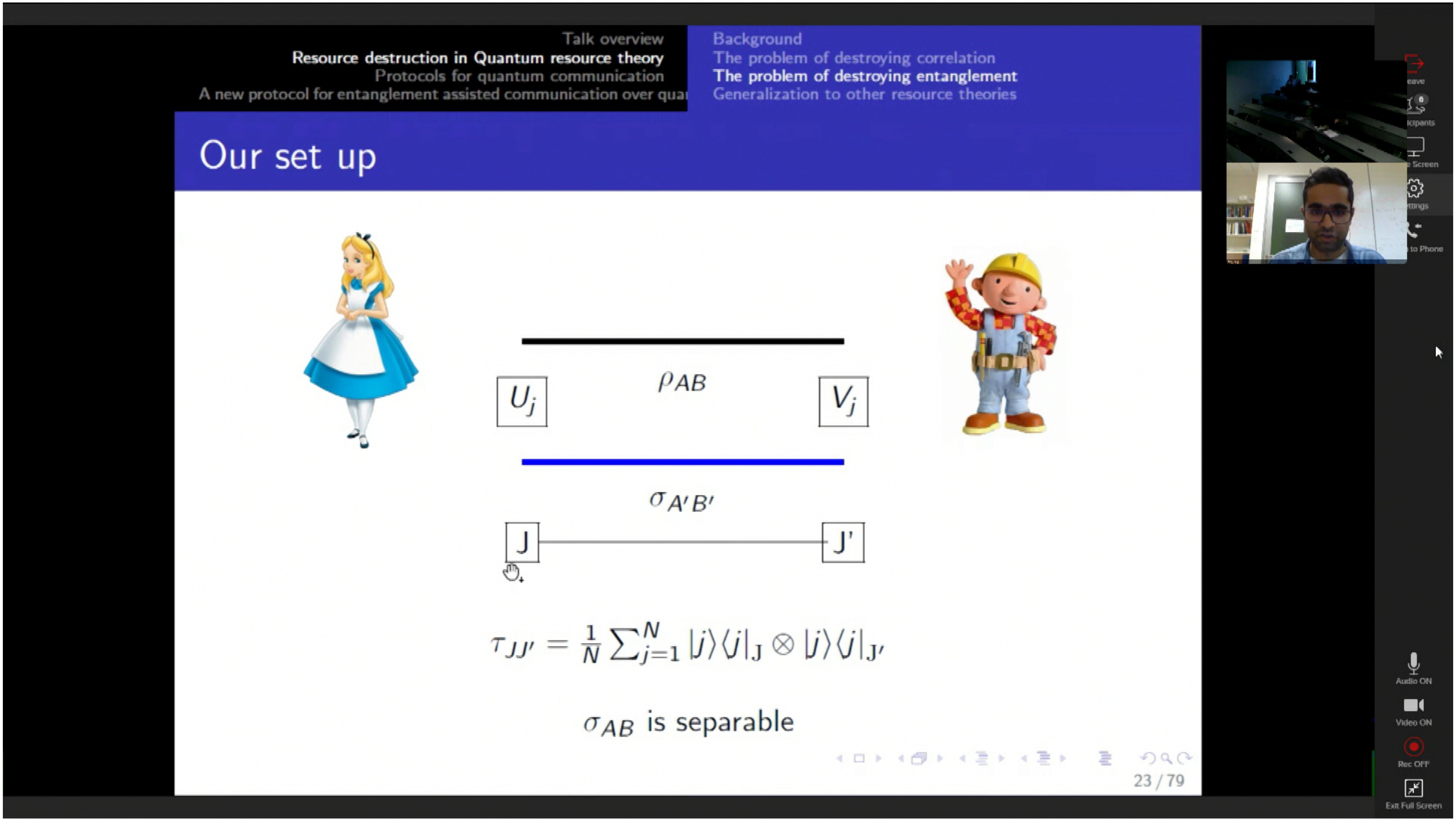
Result of Groisman, Popescu, Winter [Phys. Rev. A., 2005]

- In the asymptotic and i.i.d. setting, with $\rho_{AB}^{\otimes n}$.
- Showed that $\log |J| = n\mathrm{I}(A:B)_\rho$.
- $\mathrm{I}(A:B)_\rho = S(\rho_A) + S(\rho_B) - S(\rho_{AB})$.
- $\mathrm{I}(A:B)_\rho = \mathrm{D}(\rho_{AB} \| \rho_A \otimes \rho_B) = \inf_{\sigma_A} \mathrm{D}(\rho_{AB} \| \sigma_A \otimes \rho_B)$.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Our set up
U_j
ρ_{AB}
V_j
$\sigma_{A'B'}$
J
J'
$\tau_{JJ'} = \frac{1}{N}\sum_{j=1}^{N} |j\rangle\langle j|_{J} \otimes |j\rangle\langle j|_{J'}$
σ_{AB} is separable
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Our set up

$$\omega_{AA'BB'} \approx \theta_{AB} \otimes \sigma_{A'B'}$$

θ_{AB} is separable

Our results

- Definitions:

$$E(\rho_{AB}) = \inf_{\sigma_{AB} \in SEP(A:B)} \mathrm{D}(\rho_{AB} \| \sigma_{AB})$$

$$E_{\varepsilon}^{one-shot}(\rho_{AB}) = \inf_{\sigma_{AB} \in SEP(A:B)} \mathrm{D}^{\varepsilon}_{\max}(\rho_{AB} \| \sigma_{AB})$$

Our results

- Definitions:

$$E(\rho_{AB}) = \inf_{\sigma_{AB} \in SEP(A:B)} \mathrm{D}(\rho_{AB} \| \sigma_{AB})$$

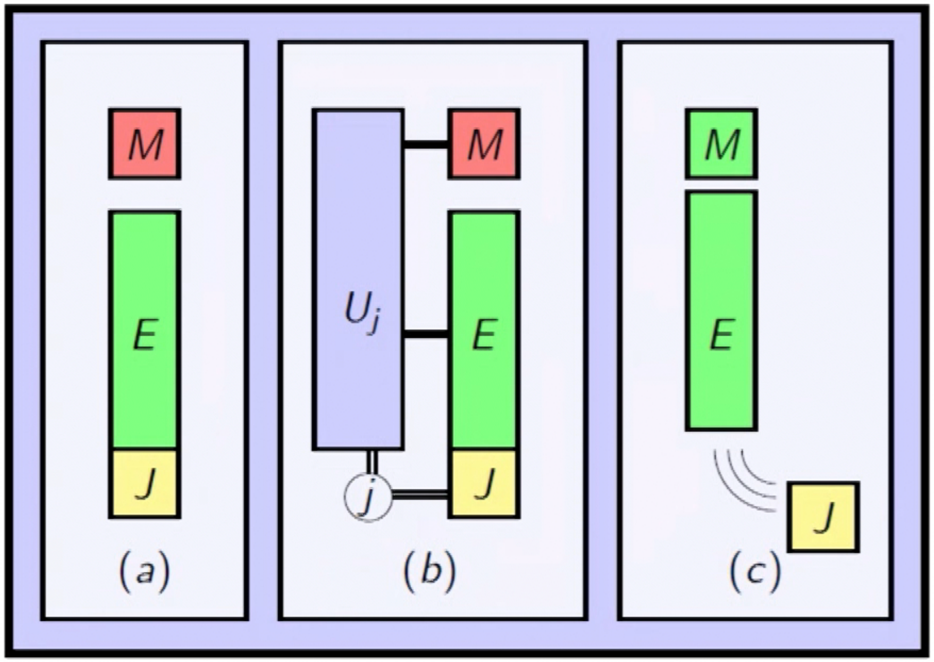
$$E_{\varepsilon}^{one-shot}(\rho_{AB}) = \inf_{\sigma_{AB} \in SEP(A:B)} \mathrm{D}_{\max}^{\varepsilon}(\rho_{AB} \| \sigma_{AB})$$

- $\mathrm{D}_{\max}(\rho \| \sigma) = \inf\{k : \rho \preceq 2^k \sigma\}$.
- $\sigma = \frac{\rho + (2^k - 1)\rho'}{1 + (2^k - 1)}$ for some quantum state ρ'. "Robustness of ρ".

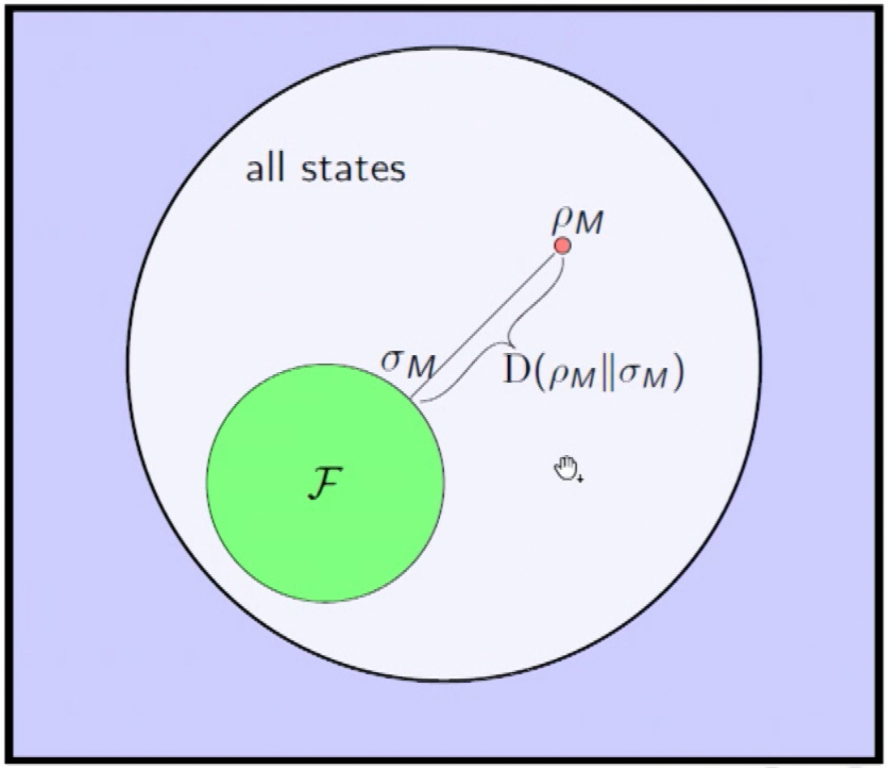
Resource theories basics

- Free states given by $\mathcal{F}$. We assume:
  - Convex set.
  - Closed under tensor product.
  - Closed under partial trace.

Our task



Relative entropy of resource: geometric view



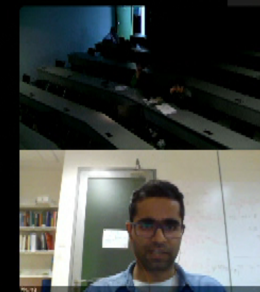
Our results

- A., Hsieh, Jain [arXiv 2017]: In the one shot setting, the number of bits discarded is characterized by $E_{\varepsilon}^{one-shot}(\rho_M)$.
    - Achievable protocol with error $\varepsilon + \delta$ requires discarding $E_{\varepsilon}^{one-shot}(\rho_M) + 2\log\frac{1}{\delta}$ bits.
    - Any protocol with error ε must discard $E_{\varepsilon}^{one-shot}(\rho_M)$.
- In the asymptotic and i.i.d. setting, this becomes $\lim_{n\to\infty}\frac{1}{n}E(\rho_M^{\otimes n})$.

The role of relative entropy of resource

- Brandao, Gour [Phys. Rev. Lett., 2015] show that in the asymptotic and i.i.d. setting, the rate of transformation of ρ to σ is

$$\frac{E(\rho)}{E(\sigma)}.$$



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The role of relative entropy of resource

- Brandao, Gour [Phys. Rev. Lett., 2015] show that in the asymptotic and i.i.d. setting, the rate of transformation of ρ to σ is

$$\frac{E(\rho)}{E(\sigma)}.$$

- Suggests that $E(\rho)$ is a natural measure for quantifying the amount of resource in a state.
- Interesting question to extend their result in the one-shot setting.

Protocol

ρ_M ●

σ_{M_1} ●

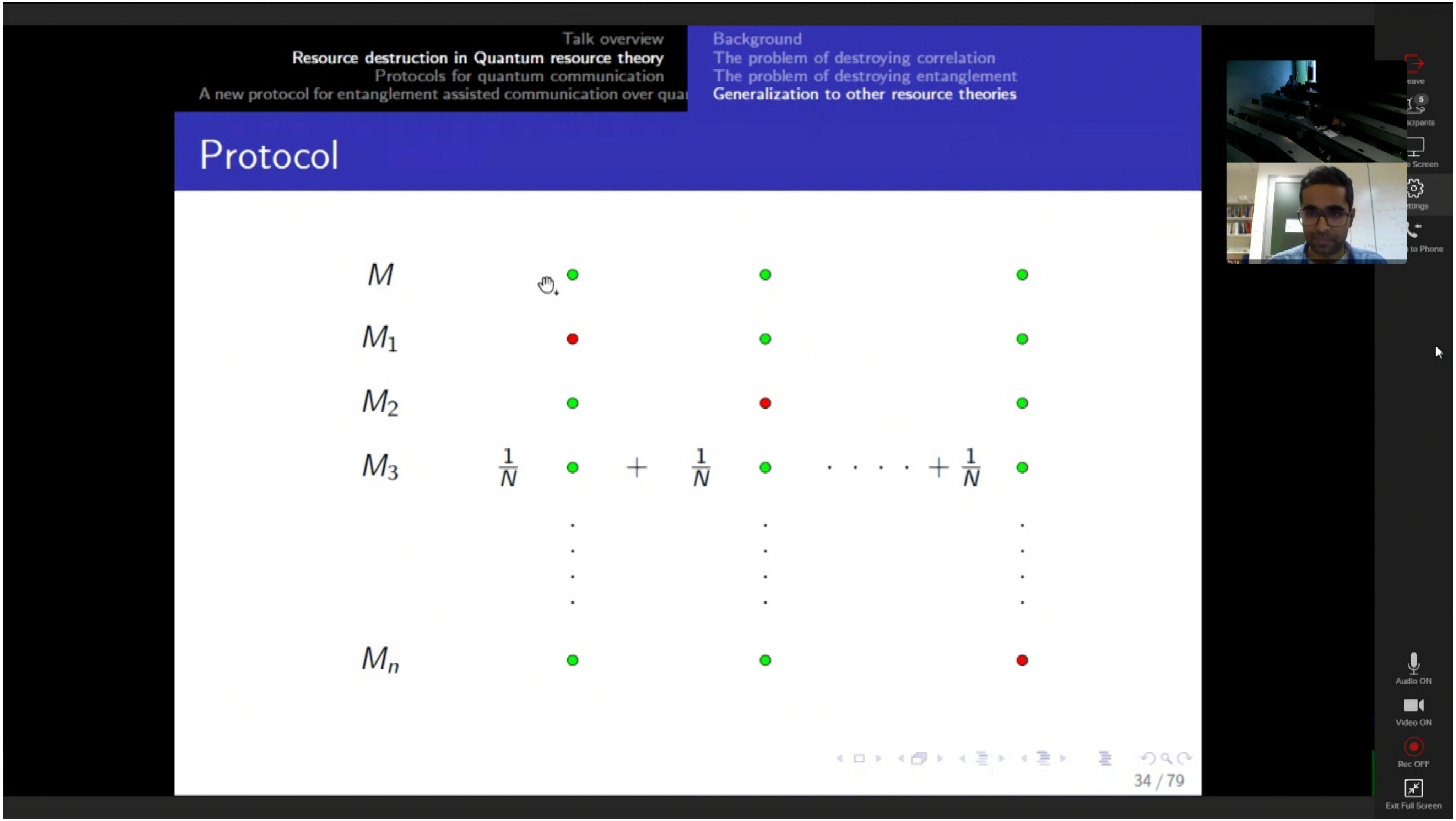
σ_{M_2} ●

⋮

σ_{M_n} ●

σ is the free state that minimizes

$$E_{\varepsilon}^{one-shot}(\rho) = \inf_{\sigma \in \mathcal{F}} \mathrm{D}^{\varepsilon}_{\max}(\rho \| \sigma).$$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Protocol
M
M1
M2
M3
Mn
1/N + 1/N . . . . + 1/N
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Resource theories in our framework

- Controlled Swap operation belongs to $\mathcal{U}$ for a large class of resource theories.
- Coherence.
  - Braumgatz, Cramer, Plenio [Phys. Rev. Lett. 2014]; Winter, Yang [Phys. Rev. Lett., 2015]; Streltsov, Adesso, Plenio [Rev. Mod. Phys., 2017].
- Asymmetry.
  - Wakakuwa [Phys. Rev. A., 2017], where relative entropy of frameness is non-zero. Does not apply to the formulation in Gour, Marvian, Spekkens [Phys. Rev. A., 2009].

Overview of applications of convex-split technique

- Quantum source compression settings, with the aid of a technique of position-based decoding (discussed later).
- Quantum channel coding settings, with the aid of a technique of position-based decoding.
- Miscellaneous applications.

Quantum state redistribution

- Special subcases are Quantum state merging/splitting
  - Horodecki, Oppenheim, Winter [Comm. Math. Phys. 2007, Nature 2005]; Abeyesinghe, Devetak, Hayden, Winter [Proc. Roy. Soc., 2009]; Berta [Master's thesis, 2009]; Berta, Christandl, Renner [Comm. Math. Phys. 2011]; A., Devabathini, Jain [Phys. Rev. Lett., 2017].
- Found applications in quantum communication complexity (Touchette [STOC, 2015]).

Other quantum source compression settings

- Distributed quantum source compression: A., Jain, Warsi [IEEE TIT, to appear]
  - Earlier works: Abeyesinghe, Devetak, Hayden, Winter [Proc. Roy. Soc., 2009]; Hsieh, Watanabe [ITW, 2015]; Dutil, Hayden [arXiv, 2010].
- Quantum measurement compression: A., Jain, Warsi [arXiv, 2017].
  - Earlier works: Winter [Comm. Math. Phys., 2004]; Wilde, Hayden, Buscemi, Hsieh [Jour. Phys. A., 2012]).
- To be presented at QIP 2018.

Quantum channel coding settings

- Point to point quantum channel, Gelf'and-Pinsker quantum channel, quantum broadcast channel, compound quantum channel: A., Jain, Warsi [arXiv 2017, QIP 2018].
- Earlier works:
  - Point to point channel: Schumacher, Westmoreland [1997]; Holevo [1998]; Hayashi-Nagaoka [2002]; Renner, Wang[2012]; Llyod [1997]; Shor [2000]; Devetak [2003]; Hayden, Horodecki, Winter, Yard [2007]; Datta, Hsieh [2011].
  - Quantum broadcast channel: Allahverdyan-Saakian [1998], Yard-Hayden-Devetak [2006], Dupuis' thesis [2010].
  - Gelf'and-Pinsker quantum channel: Dupuis' thesis [2010].
  - Compound quantum channel: Boche et. al. [2009-2017], Hayashi [2009], Berta-Gharibiyan-Walter [2016].

Miscellaneous applications

- Catalytic decoupling (Majenz et. al. [Phys. Rev. Lett., 2017]; discussed later).
- A weak version of Operator Chernoff bound (Ahlswede, Winter [IEEE TIT, 2003]) in one-shot setting (A., Jain, Warsi [arXiv, 2017]).
  - A 'proper' one-shot analogue of Operator Chernoff bound in Radhakrishnan, Sen, Warsi [Qcrypt 2017].
- Some port-based teleportation schemes (Ishizaka, Hiroshima [Phys. Rev. Lett, 2008]; discussed later) in A., Devabathini, Jain [Phys. Rev. Lett., 2017].
- Private capacity of quantum channel (Cai, Winter, Yeung [Prob. Inf. Trans., 2004], Devetak [IEEE TIT, 2005]) in one-shot setting (Wilde [Quant. Inf. Proc. 2017]).

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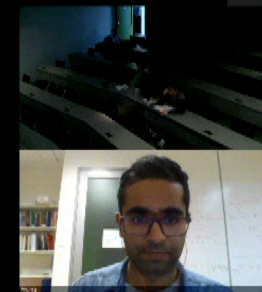

Quantum channel coding settings

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- Earlier works:
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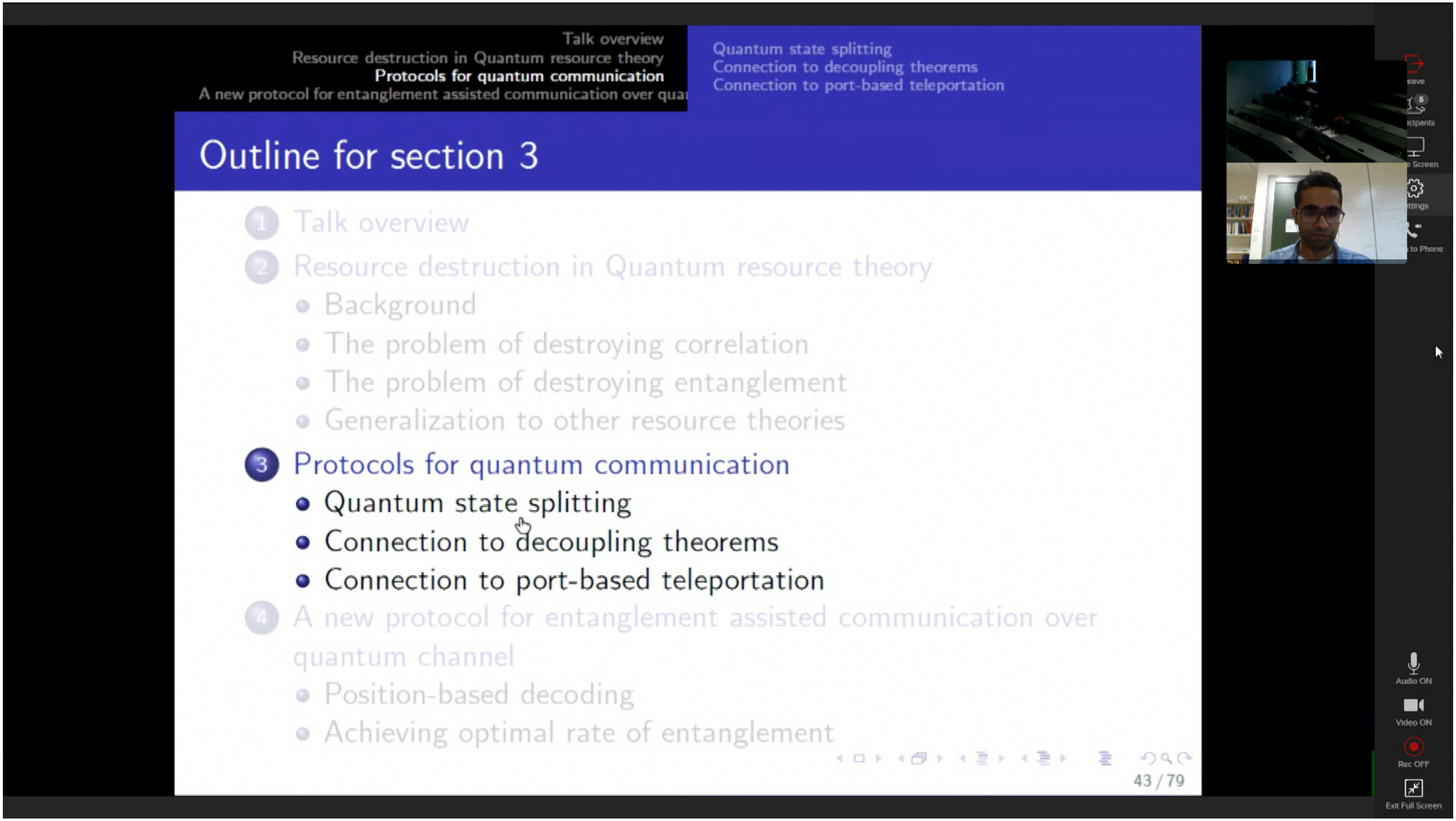
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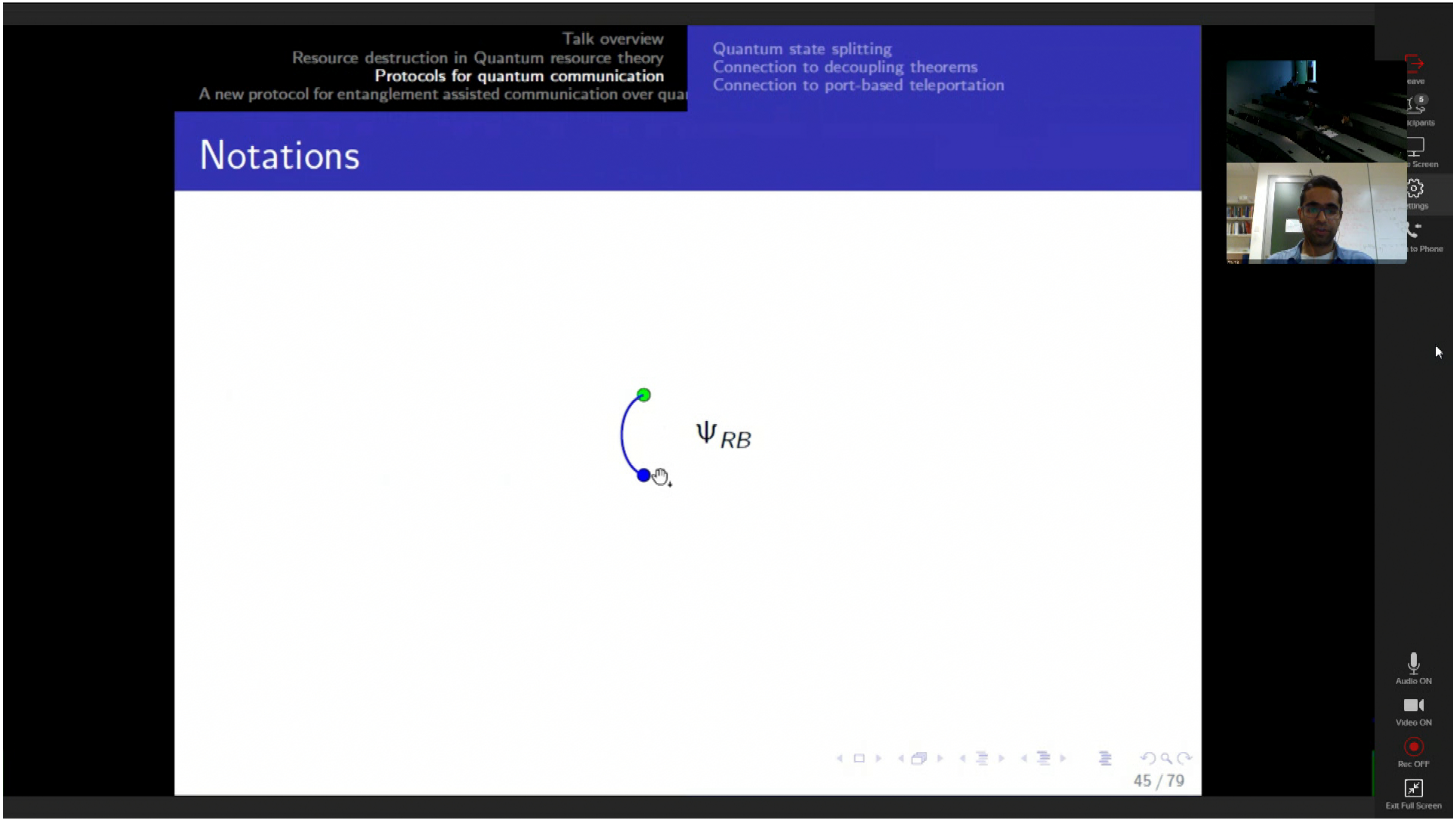
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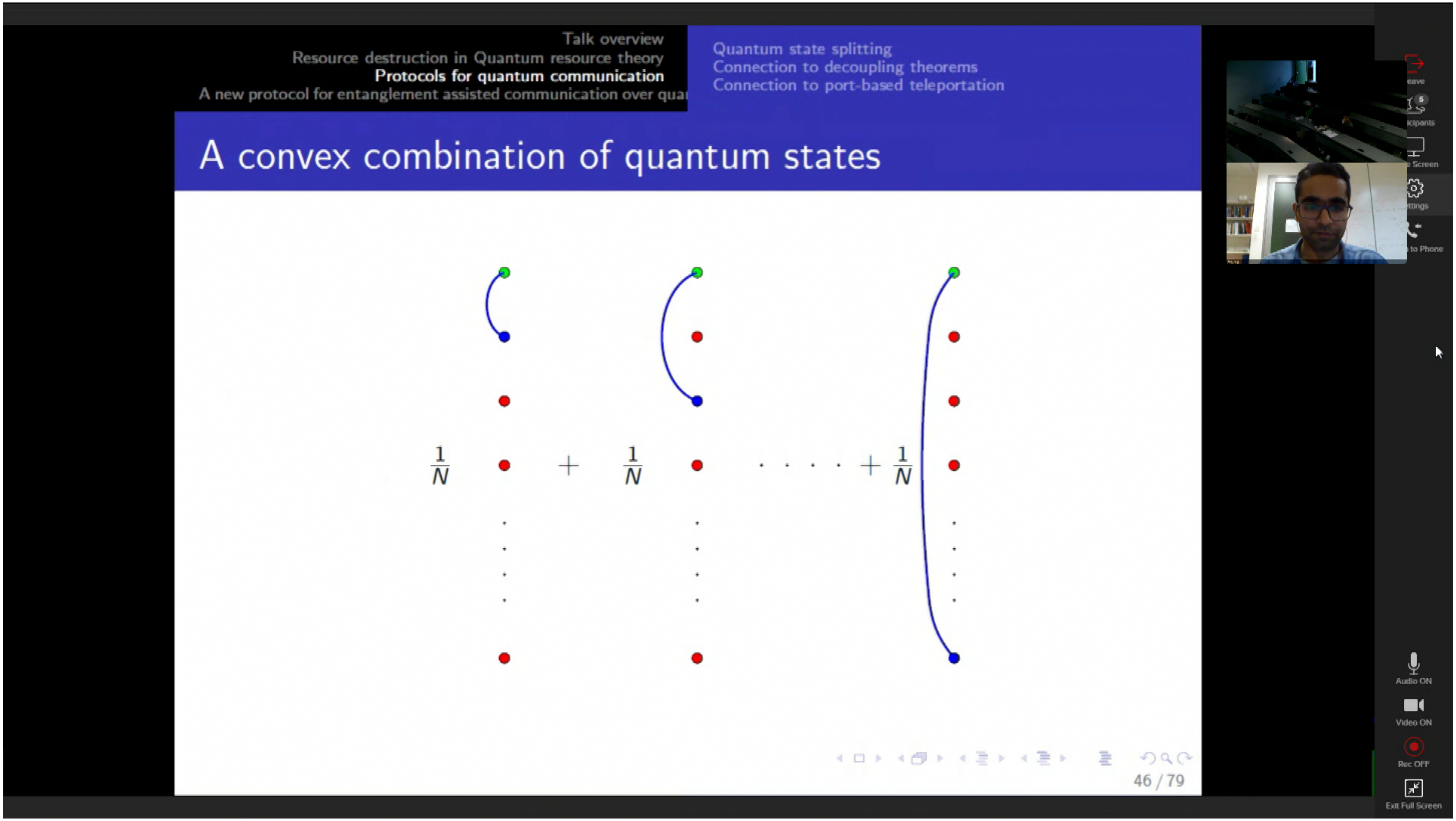
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$\frac{1}{N}$ + $\frac{1}{N}$ . . . . + $\frac{1}{N}$
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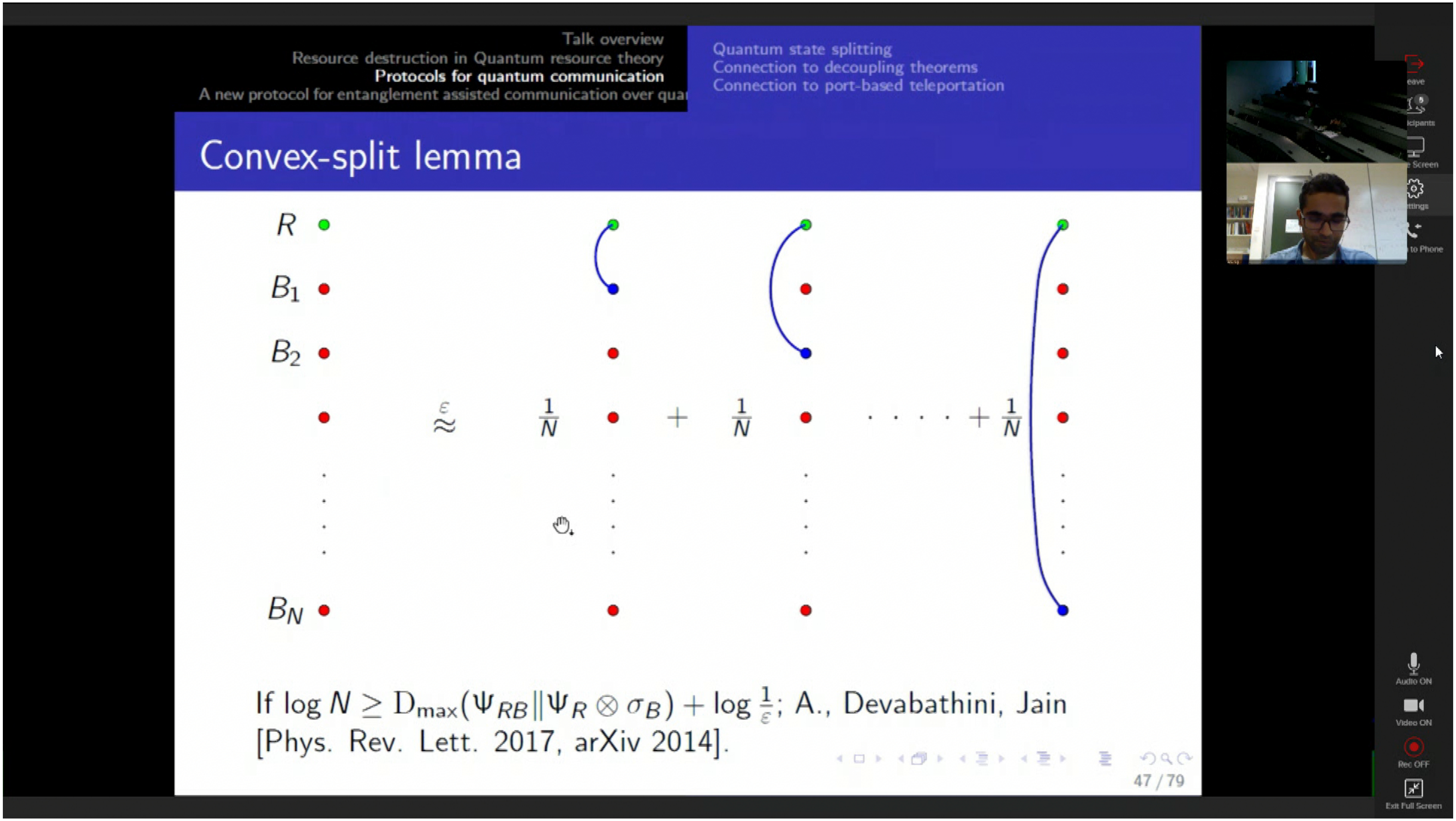
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B_1
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$\overset{\varepsilon}{\approx}$ $\frac{1}{N}$ + $\frac{1}{N}$ $\cdots\cdots$ + $\frac{1}{N}$
If $\log N \geq \mathrm{D}_{\max}(\Psi_{RB} \| \Psi_R \otimes \sigma_B) + \log \frac{1}{\varepsilon}$; A., Devabathini, Jain [Phys. Rev. Lett. 2017, arXiv 2014].
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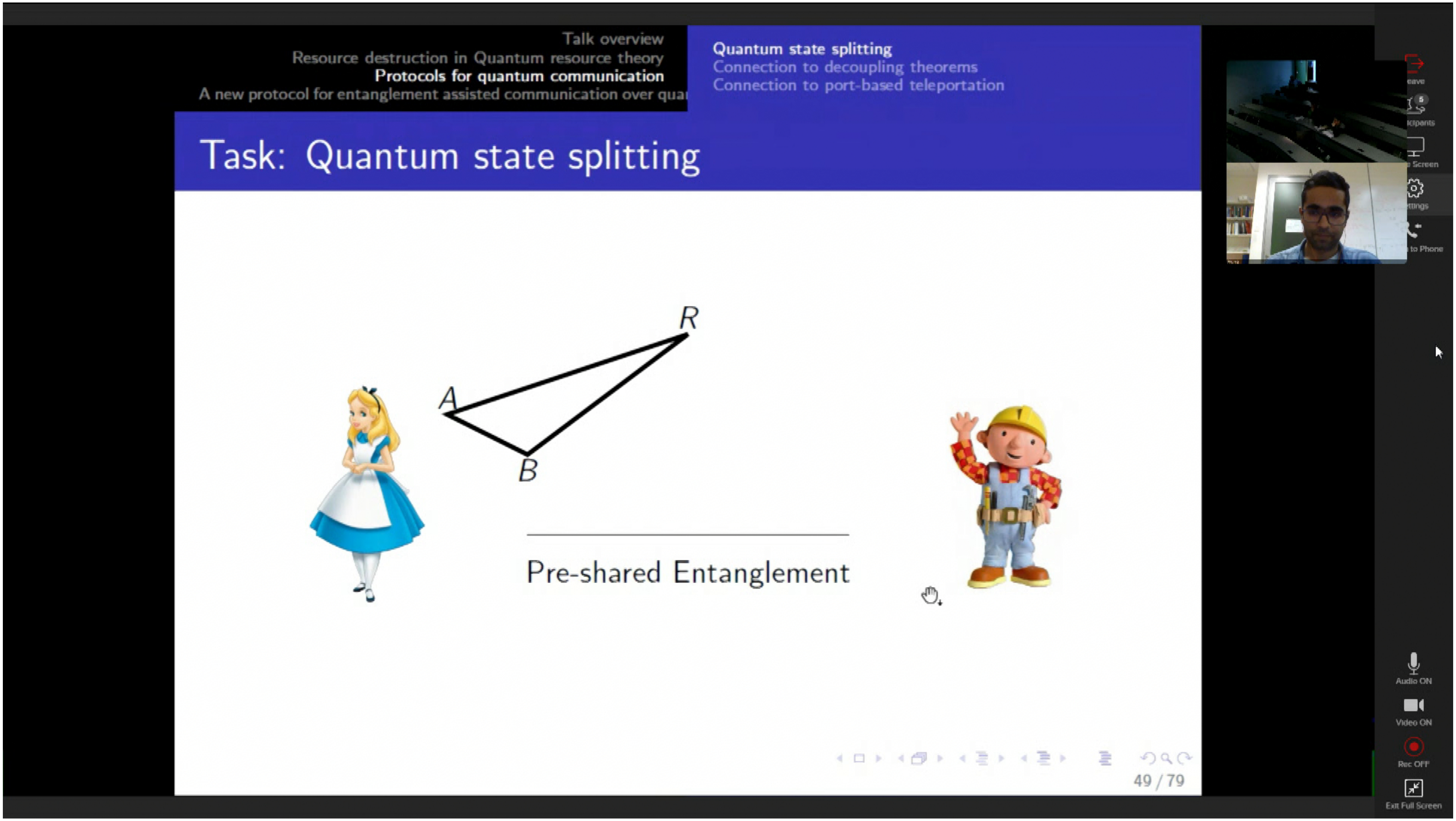
Task: Quantum state splitting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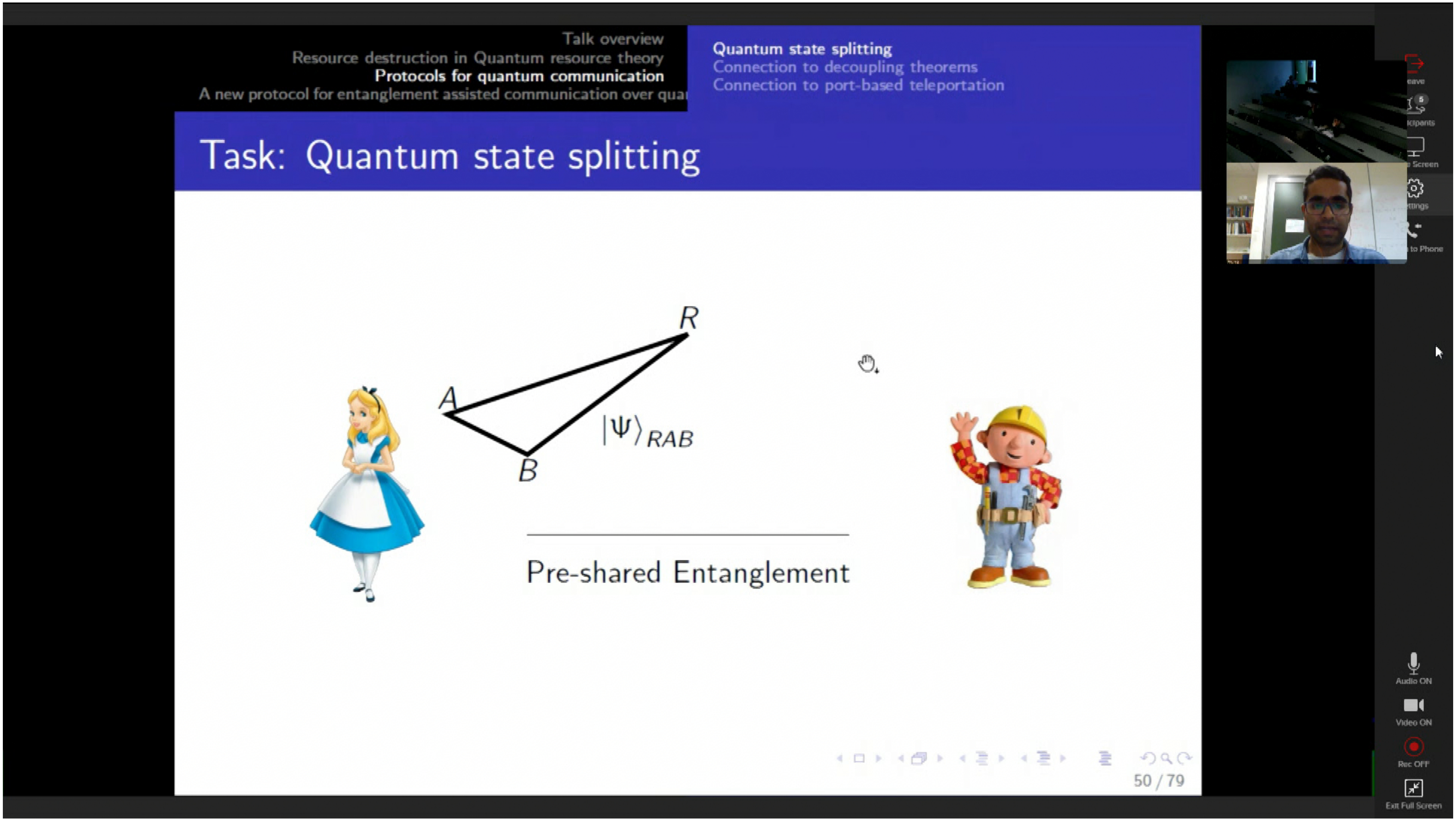
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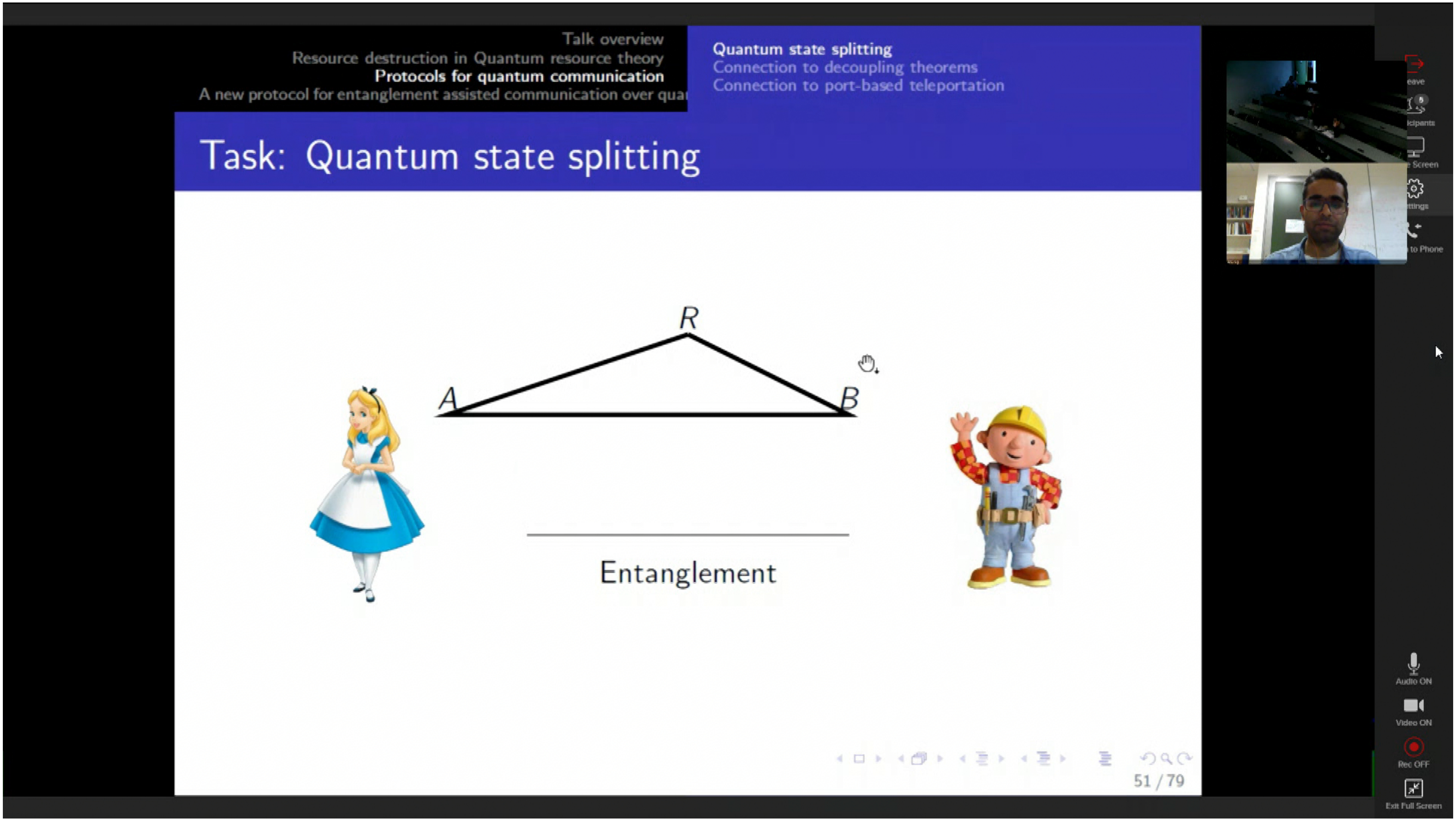
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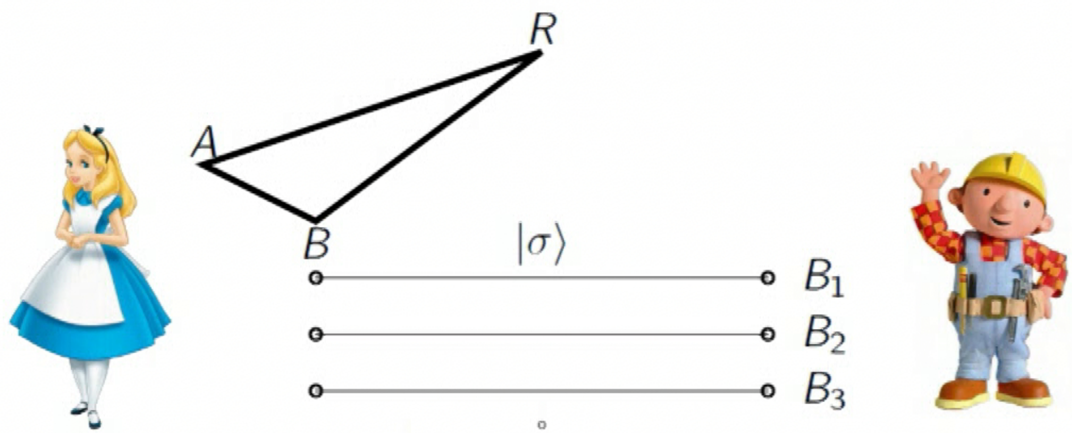
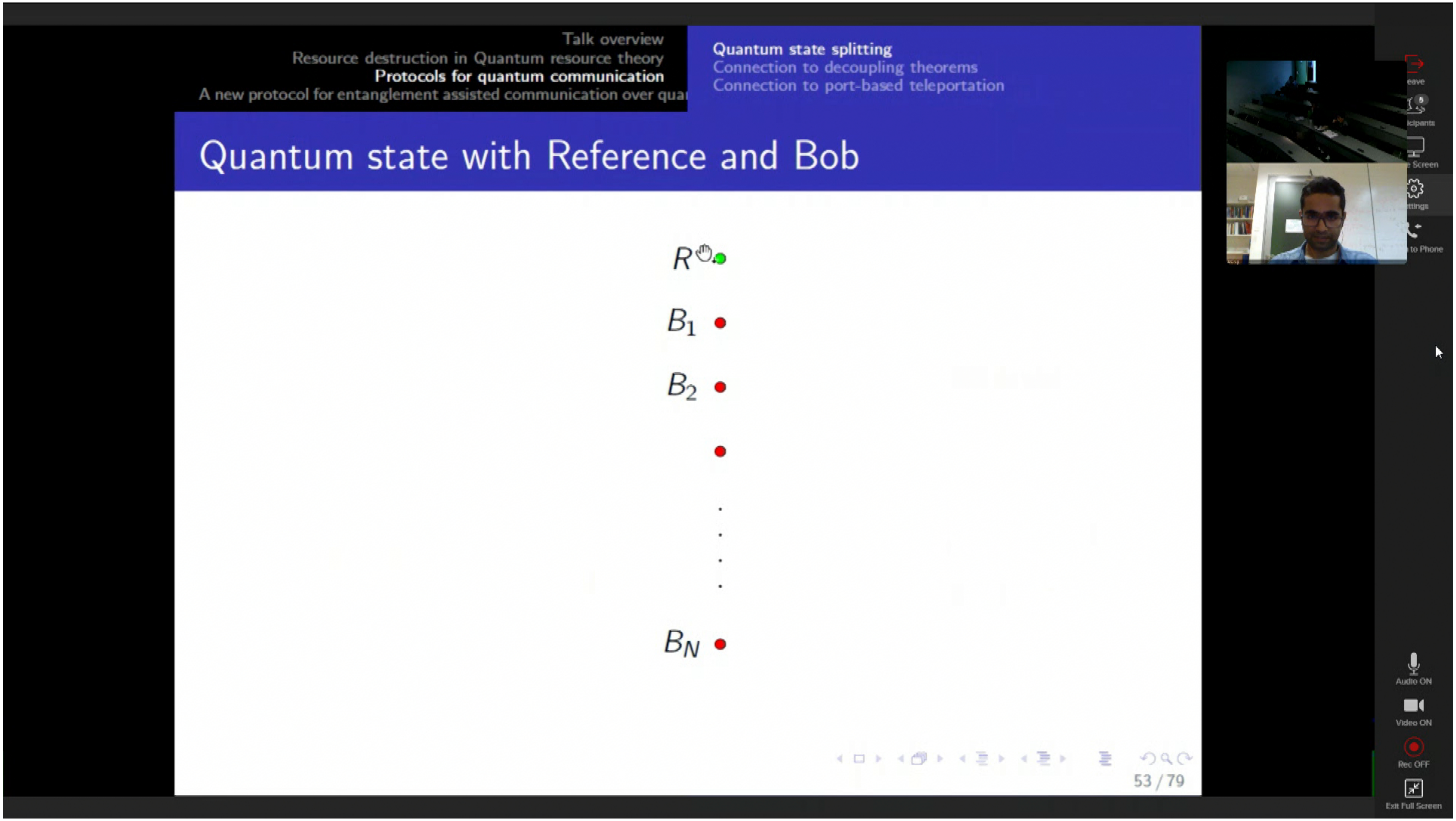
Our protocol: form of pre-shared entanglement

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Quantum state with Reference and Bob
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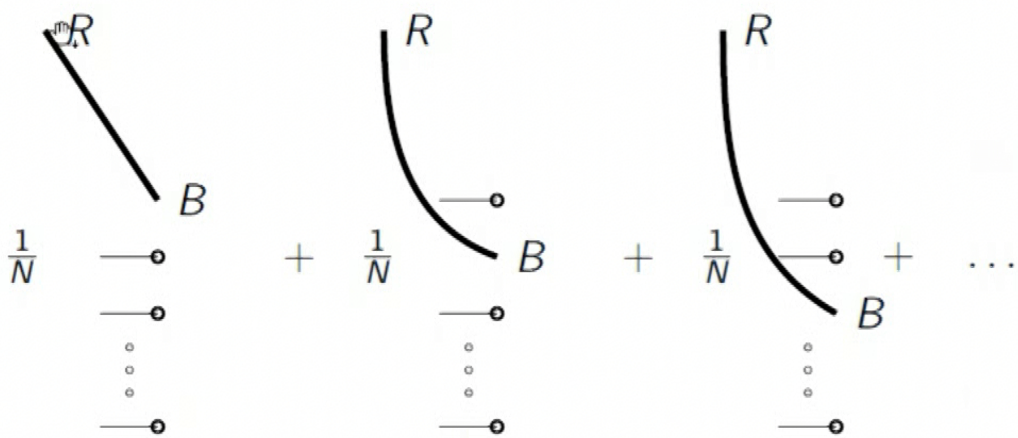
Alice sees the following state



$$\log N = \mathrm{D}_{\max}(\Psi_{RB} \| \Psi_R \otimes \sigma_B) + \log \tfrac{1}{\varepsilon}.$$

Resulting protocol

- If two quantum states are close, there exist equally close purifications. (Uhlmann [Rep. Math. Phys., 1976])
- Alice uses this fact. Measures and communicates $\log N = \mathrm{D}_{\max}(\Psi_{RB} \| \Psi_R \otimes \sigma_B) + \log \frac{1}{\varepsilon}$.
- Optimize over σ_B to achieve:

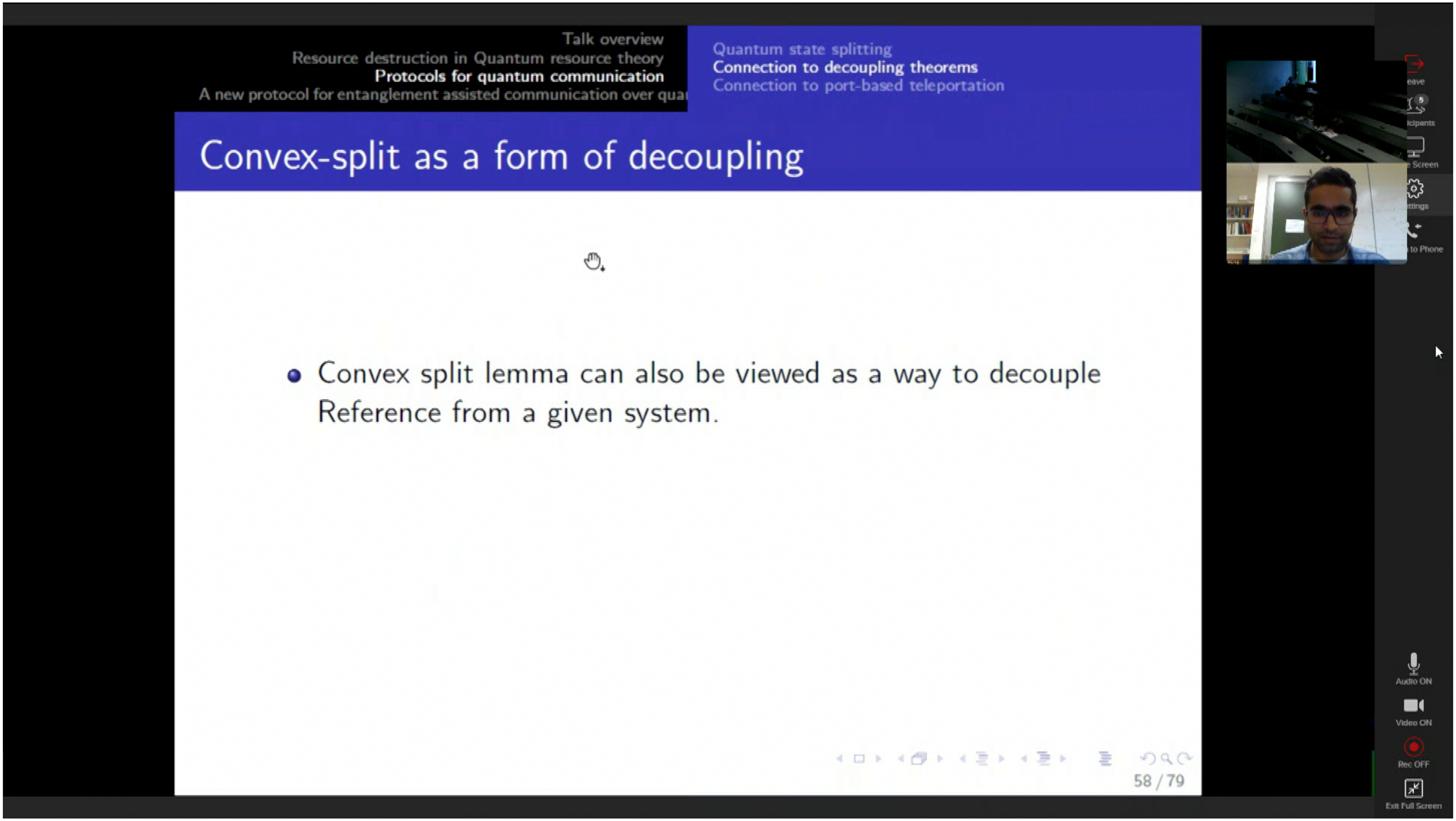
$$\mathrm{I}_{\max}(R:B) = \inf_{\sigma_B} \mathrm{D}_{\max}(\Psi_{RB} \| \Psi_R \otimes \sigma_B).$$

Resulting protocol

- We achieve $\mathrm{I}^{\varepsilon}_{\max}(R : B) + \log \frac{1}{\varepsilon}$ for error 2ε.
- Lower bound $\mathrm{I}^{\varepsilon}_{\max}(R : B)$ for error ε.
- Best earlier work (Berta, Christandl, Renner [Comm. Math. Phys., 2011]) achieved $\mathrm{I}^{\varepsilon}_{\max}(R : B) + \log\log|B| + \log \frac{1}{\varepsilon}$.
- Quantum state merging $\overset{timereverse}{=}$ Quantum state spitting.

Decoupling via random unitary

- Alice applies a random unitary on her local system.
- If she communicates enough qubits to Bob, her local system gets decoupled with the Reference.
- Key proof technique in many previous works on quantum source compression.
  - See for example, Horodecki, Oppenheim, Winter [Comm. Math. Phys. 2007, Nature 2005]; Hayden, Horodecki, Yard, Winter [Open Sys. Inf. Dyn., 2008]; Abeyeshinghe, Devetak, Hayden, Winter [Proc. Roy Soc., 2009]; Devetak, Yard [Phys. Rev. Lett., 2008]; Dupuis [PhD thesis, 2010]; Szehr [Master's thesis, 2012]; and references therein.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Convex-split as a form of decoupling
Convex split lemma can also be viewed as a way to decouple Reference from a given system.
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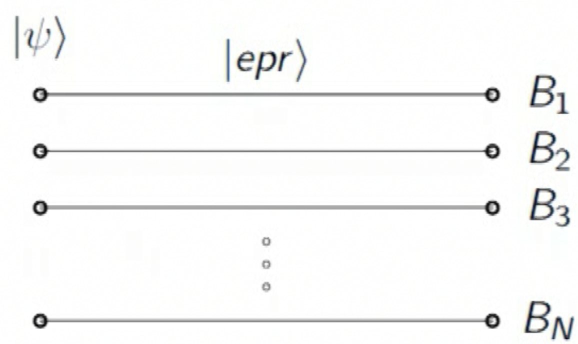
Convex-split as a form of decoupling

- Convex split lemma can also be viewed as a way to decouple Reference from a given system.
- As shown in Majenz et. al. [Phys. Rev. Lett. 2017], it gives near-optimal amount of discarded qubits under the framework of catalytic decoupling.

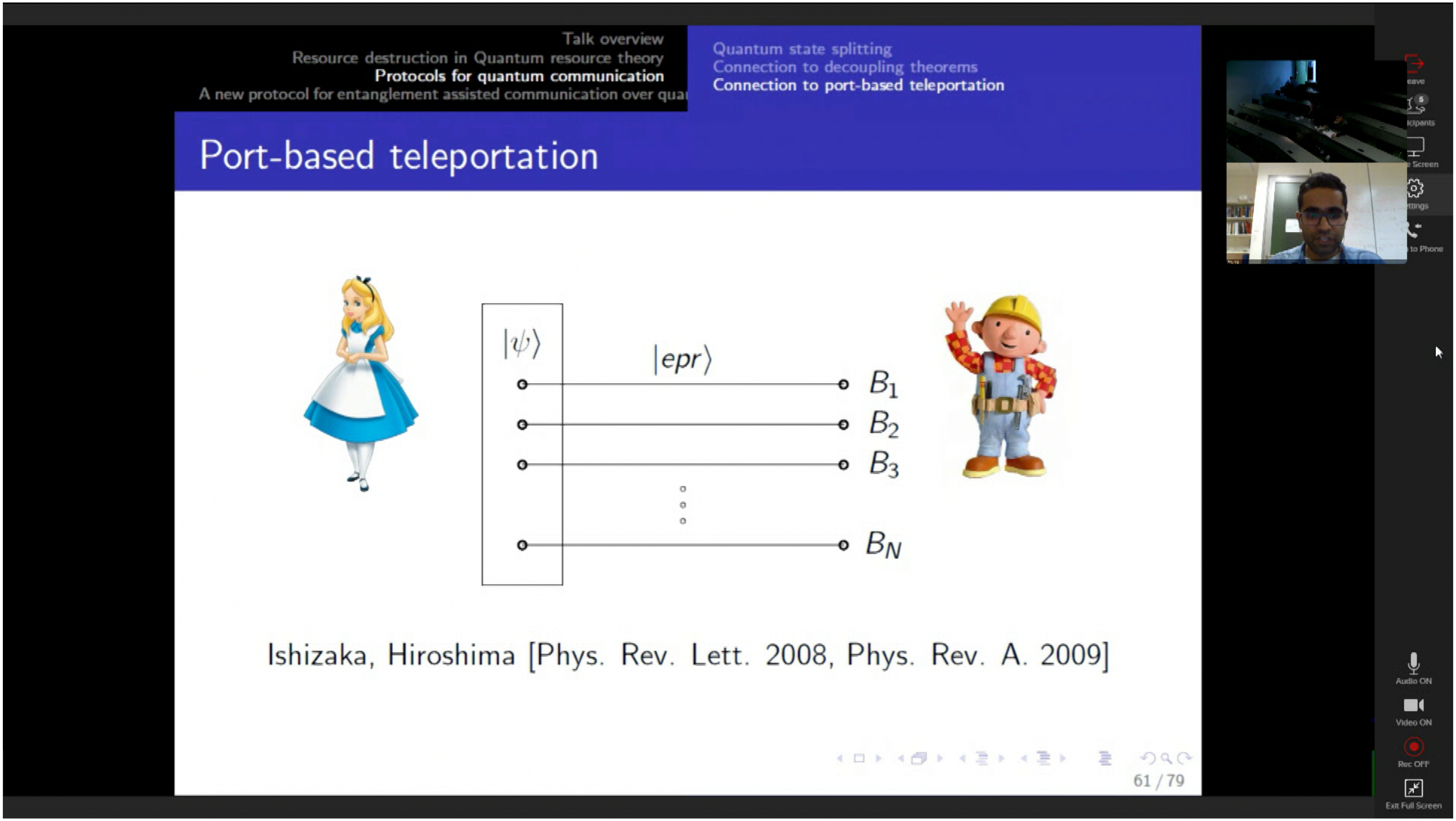
Further context

- Decoupling via random unitary can be viewed as a quantum analogue of the well known 'random binning argument'. (Slepian, Wolf [IEEE TIT, 1973]).
- Convex-split technique is a quantum analogue of the rejection sampling method in communication complexity.
  - Rejection sampling approach formally given by Von-Neumann [Nat. Bur. Stan., 1951].
  - Used in classical communication complexity in Jain, Radhakrishnan, Sen [ICALP 2003]; Harsha, Jain, McAllester, Radhakrishnan [IEEE TIT, 2010]; Braverman, Rao [FOCS, 2011].
  - For classical-quantum context, see Jain, Radhakrishnan, Sen [FOCS 2002, CCC 2005]; A., Mukhopadhyay, Jain, Shayeghi, Yao [IEEE TIT, 2016]; Shayeghi [Master's thesis, UWaterloo 2015].

Port-based teleportation



Ishizaka, Hiroshima [Phys. Rev. Lett. 2008, Phys. Rev. A. 2009]


Talk overview
Resource destruction in Quantum resource theory
Protocols for quantum communication
A new protocol for entanglement assisted communication over qua
Quantum state splitting
Connection to decoupling theorems
Connection to port-based teleportation
Port-based teleportation
$|\psi\rangle$
$|epr\rangle$
B_1
B_2
B_3
B_N
Ishizaka, Hiroshima [Phys. Rev. Lett. 2008, Phys. Rev. A. 2009]
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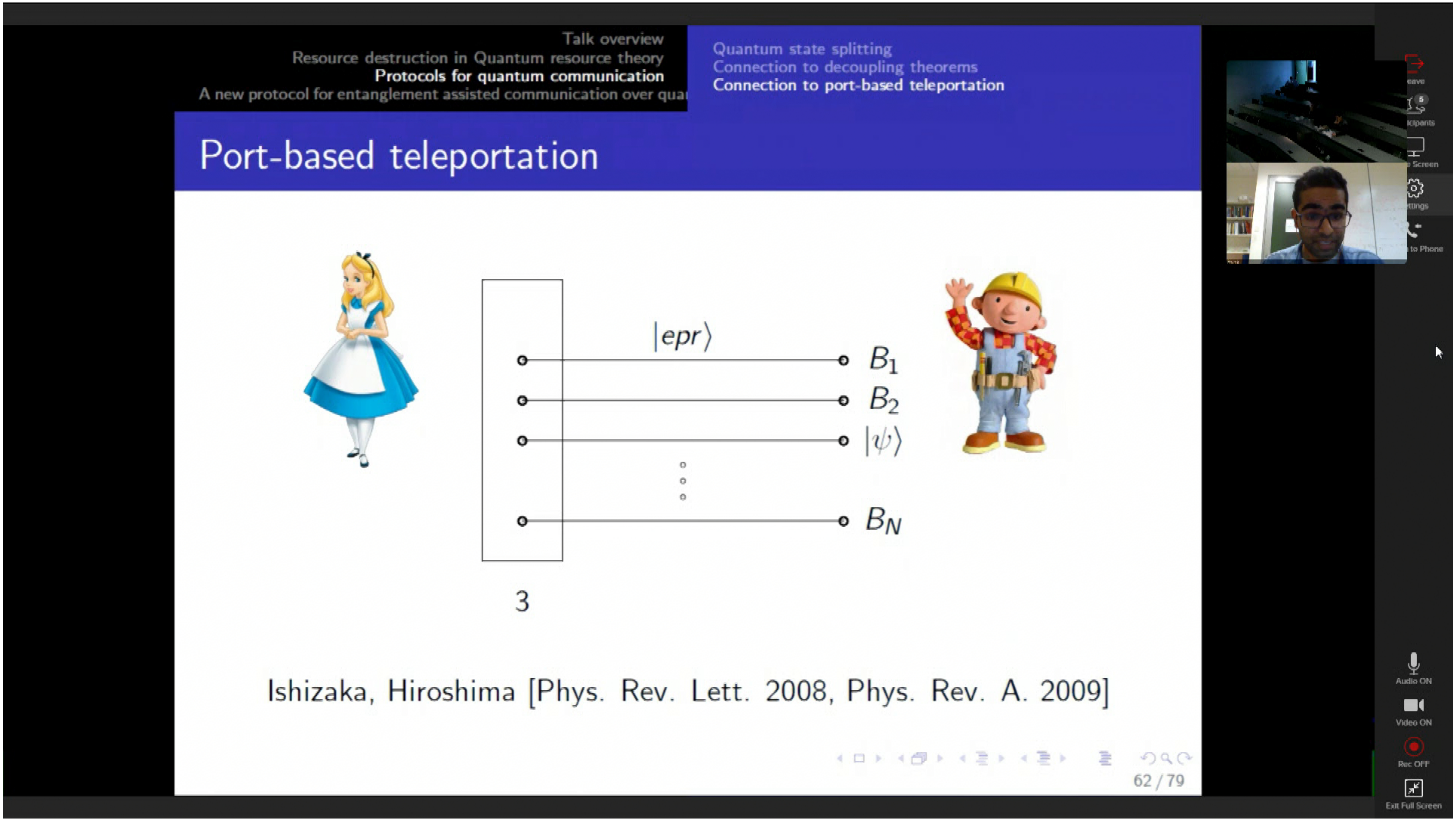

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Quantum state splitting
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Port-based teleportation
|epr⟩
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BN
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Ishizaka, Hiroshima [Phys. Rev. Lett. 2008, Phys. Rev. A. 2009]
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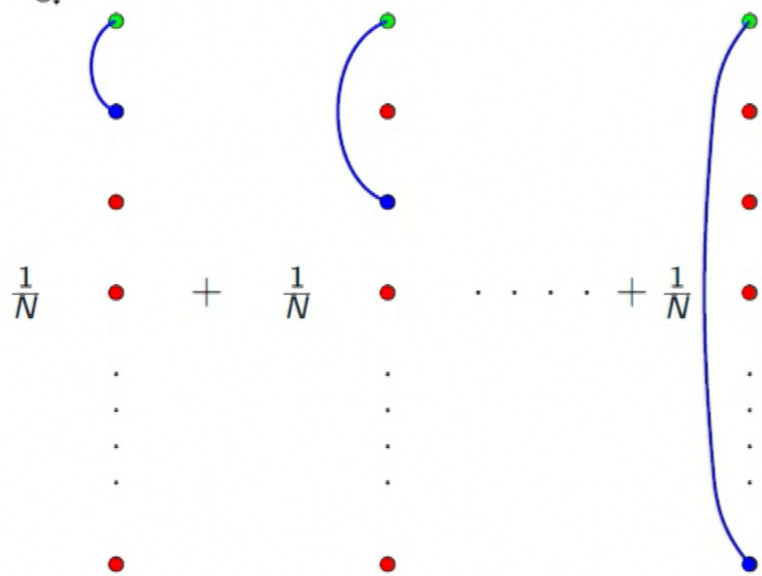
Connection to convex-split technique

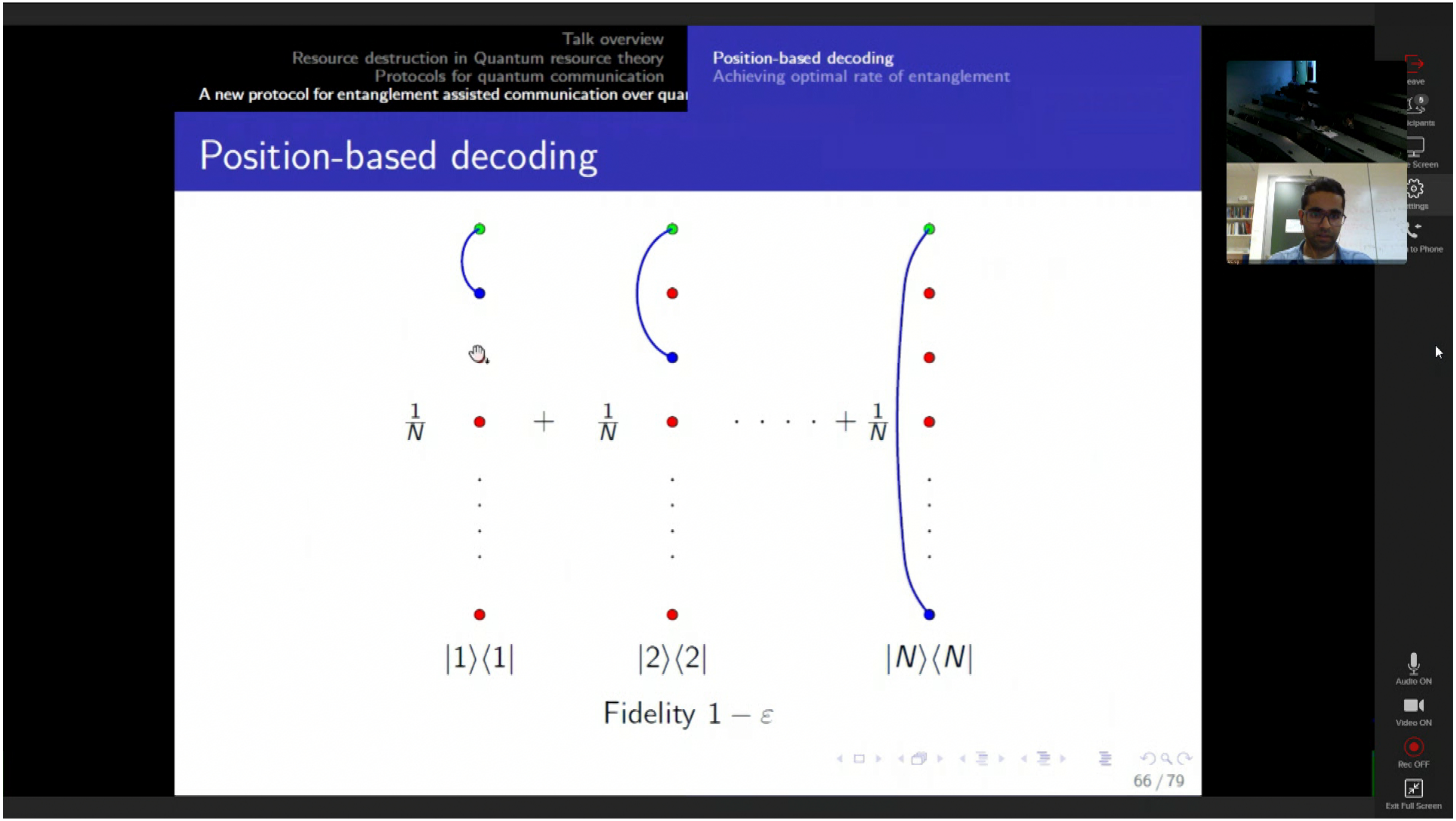
- Average fidelity of teleportation is related to the fidelity of transferring one of the EPR pair (Horodecki, Horodecki, Horodecki [Phys. Rev. A., 1999]).
- Further extension of port-based scheme: if parties know that the state $|\psi\rangle$ comes from a known ensemble then the number of ports can be further saved. (A., Devabathini, Jain [Phys. Rev. Lett. 2017]).

Outline for section 4

Position-based decoding



A., Jain, Warsi [arXiv 2017]


Talk overview
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A new protocol for entanglement assisted communication over qua
Position-based decoding
Achieving optimal rate of entanglement
Position-based decoding
$\frac{1}{N}$ + $\frac{1}{N}$ · · · · + $\frac{1}{N}$
$|1\rangle\langle 1|$
$|2\rangle\langle 2|$
$|N\rangle\langle N|$
Fidelity $1-\varepsilon$
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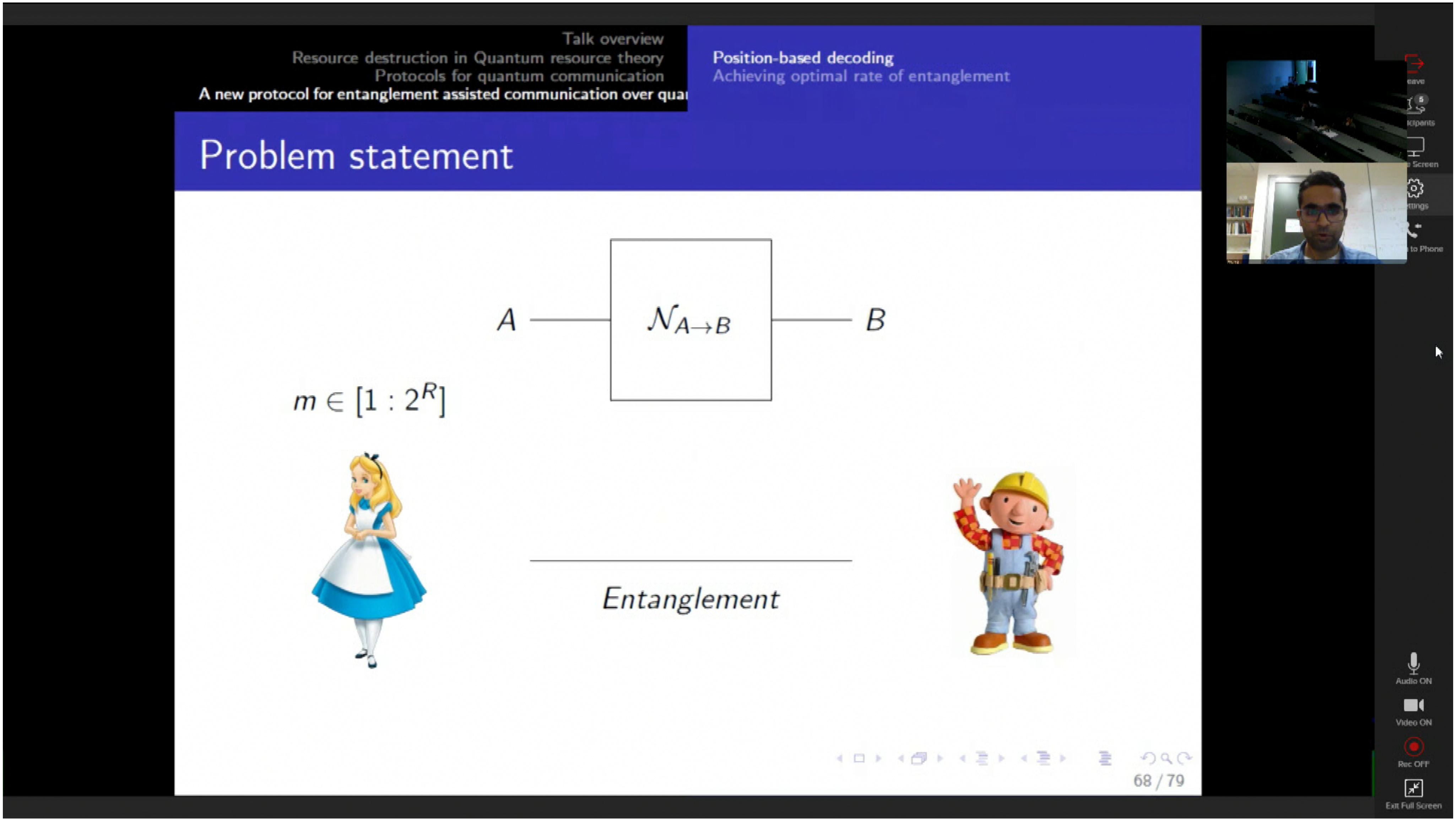
Position-based decoding

- Distinguishing possible if $N \leq \varepsilon \cdot 2^{\mathrm{D}^{\varepsilon}_{\mathrm{H}}(\Psi_{RB} \| \Psi_R \otimes \sigma_B)}$ (A., Jain, Warsi [arXiv 2017]).
- $\mathrm{D}^{\varepsilon}_{\mathrm{H}}(\rho \| \sigma) = -\inf_{\Pi} \log \mathrm{Tr}(\Pi\sigma)$, where Π is such that $\mathrm{Tr}(\Pi\rho) \geq 1 - \varepsilon$.
- Either use quantum hypothesis testing or pretty-good measurement.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Position-based decoding
Achieving optimal rate of entanglement
Problem statement
A
$\mathcal{N}_{A\to B}$
B
$m \in [1 : 2^R]$
Entanglement
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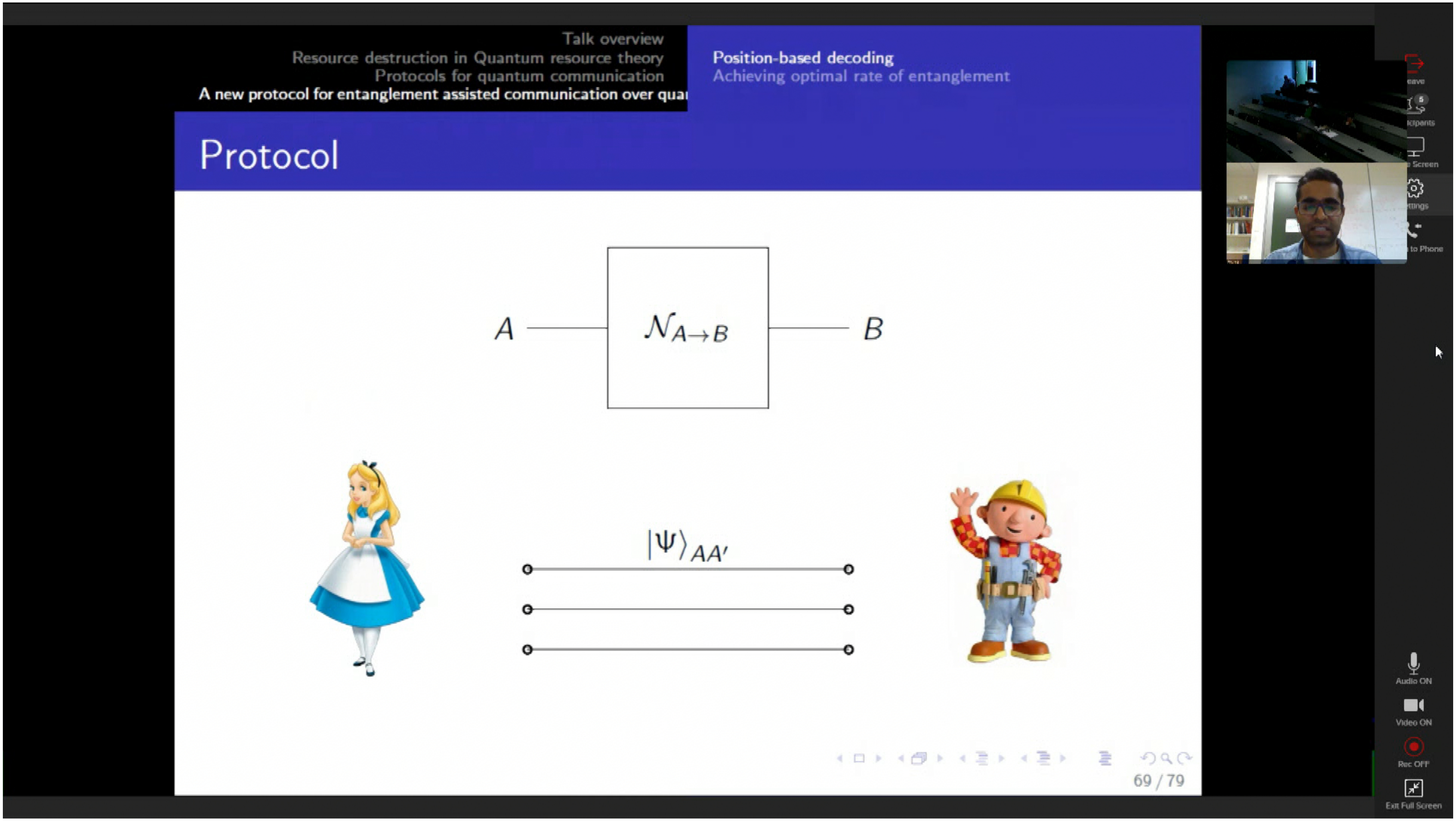
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Protocol
A
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$|\Psi\rangle_{AA'}$
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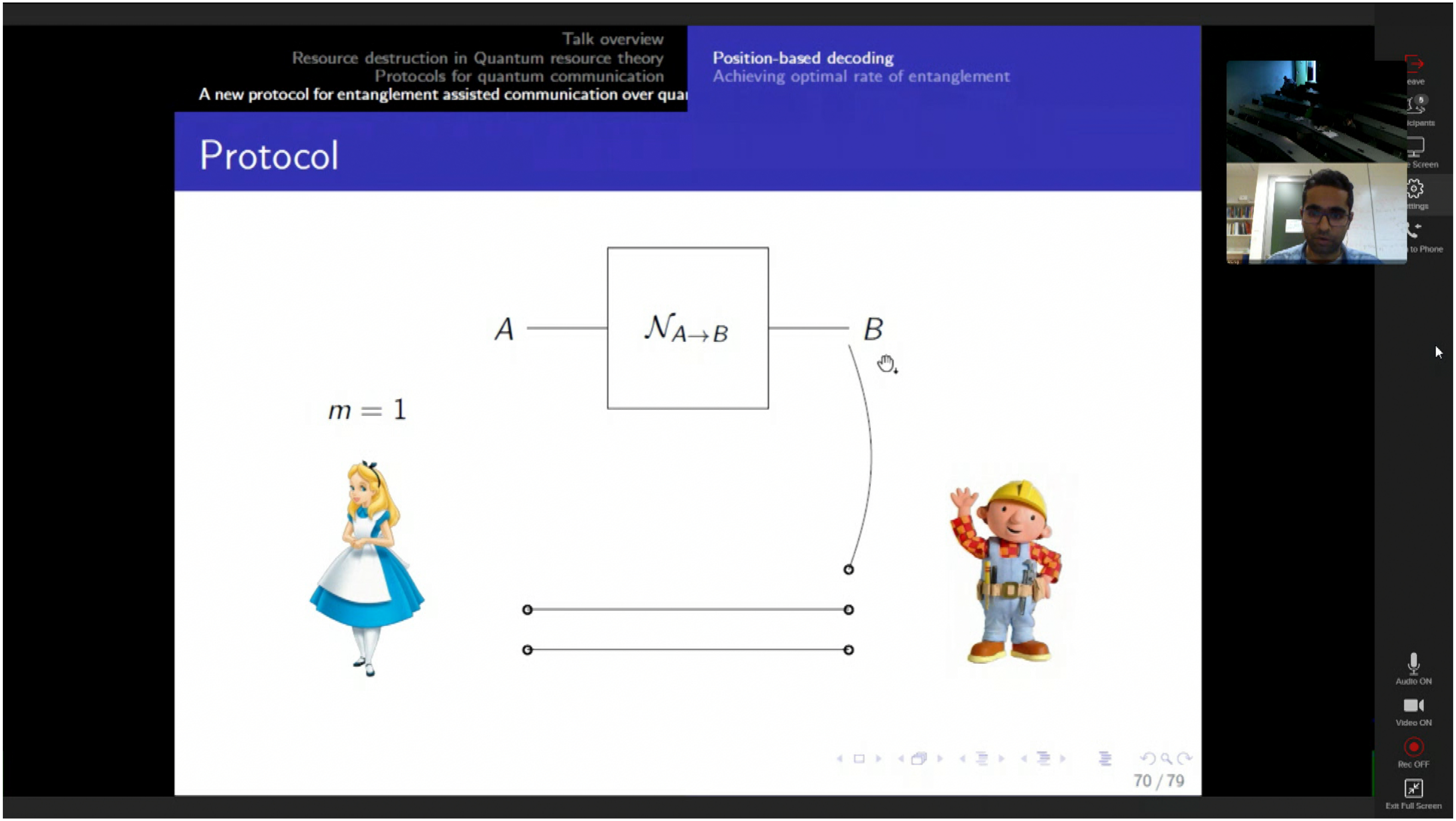
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Protocol
A
$\mathcal{N}_{A\to B}$
B
$m = 1$
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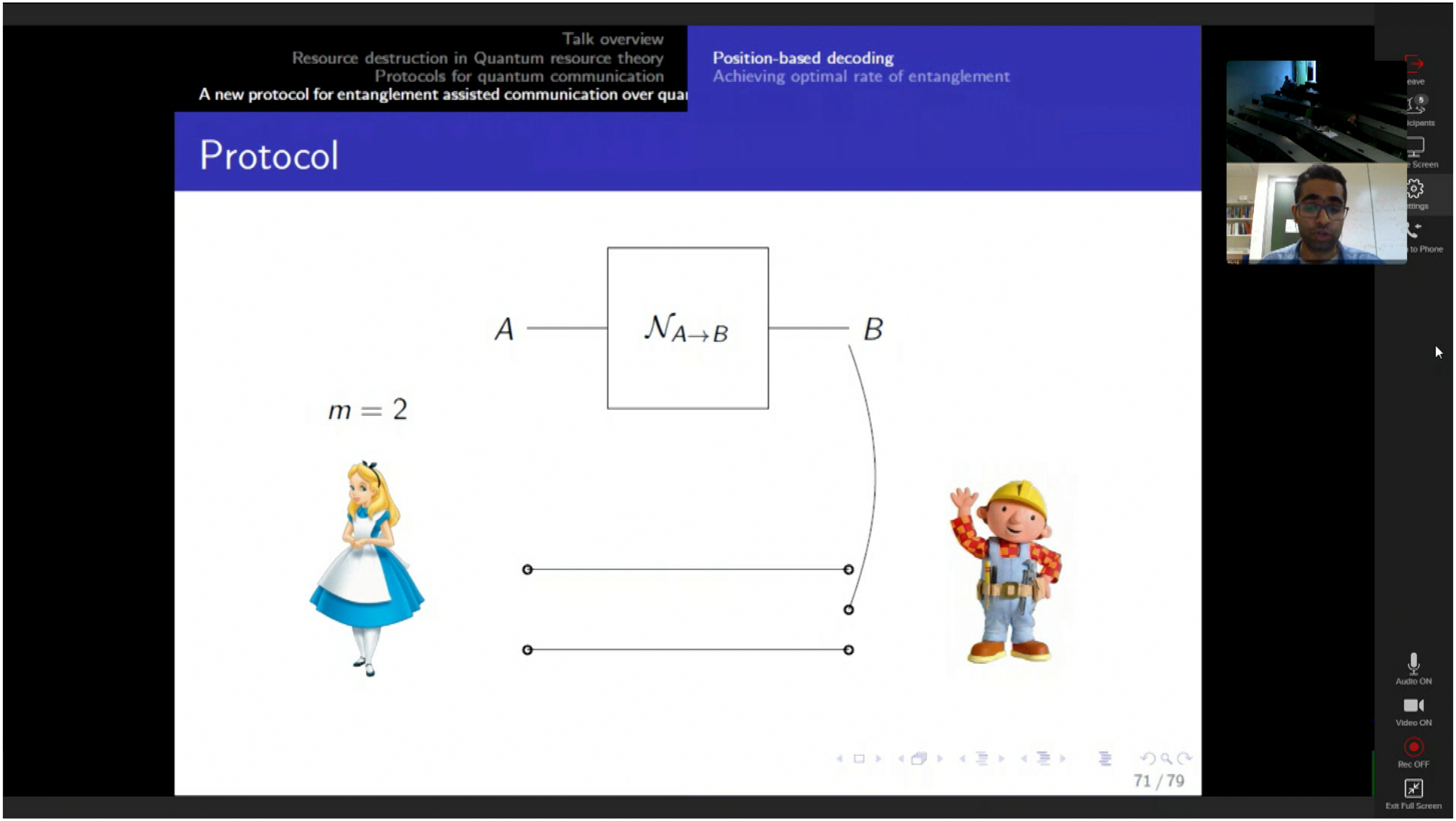
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Protocol
A
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B
$m = 2$
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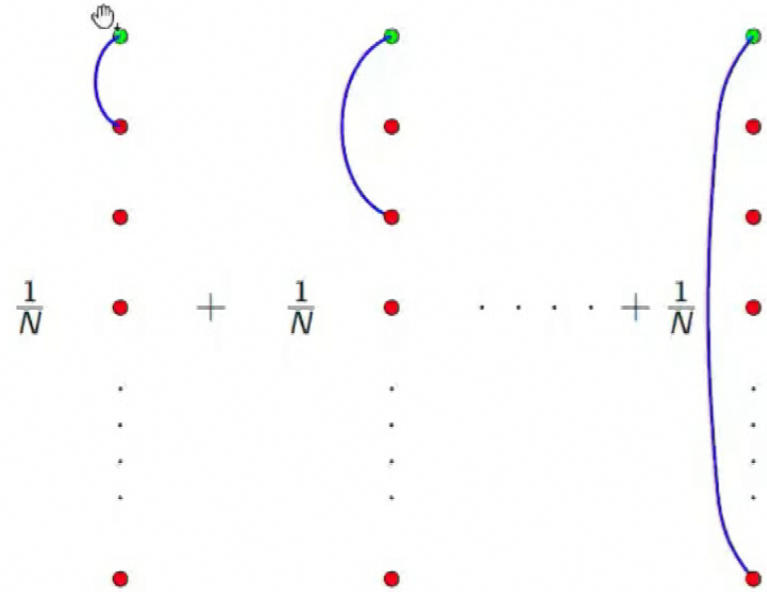
Quantum state with Bob for uniform input



Achievable rate

- Reliable communication possible if $R \leq \mathrm{D}^{\varepsilon}_{\mathrm{H}}(\Psi_{A'B} \| \Psi_{A'} \otimes \sigma_B)$, $\Psi_{A'B} = \mathcal{N}_{A\to B}(\Psi_{AA'})$.
- Recovers the result of Bennett, Shor, Smolin, Thapliyal [IEEE TIT, 2002] for entanglement assisted quantum capacity:

$$\max_{\Psi_{AA'}} \mathrm{I}(A' : B)_{\mathcal{N}_{A\to B}(\Psi_{AA'})}.$$

- Drawback: uses lots of entanglement.

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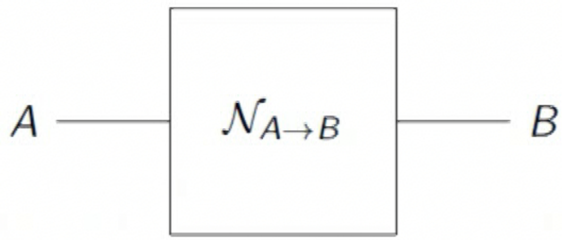
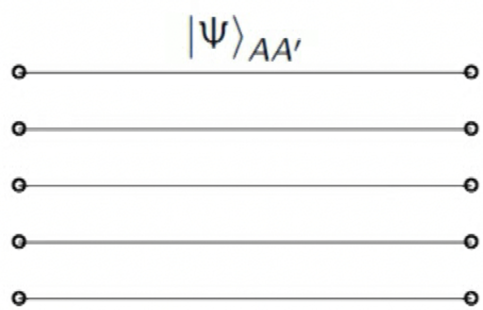
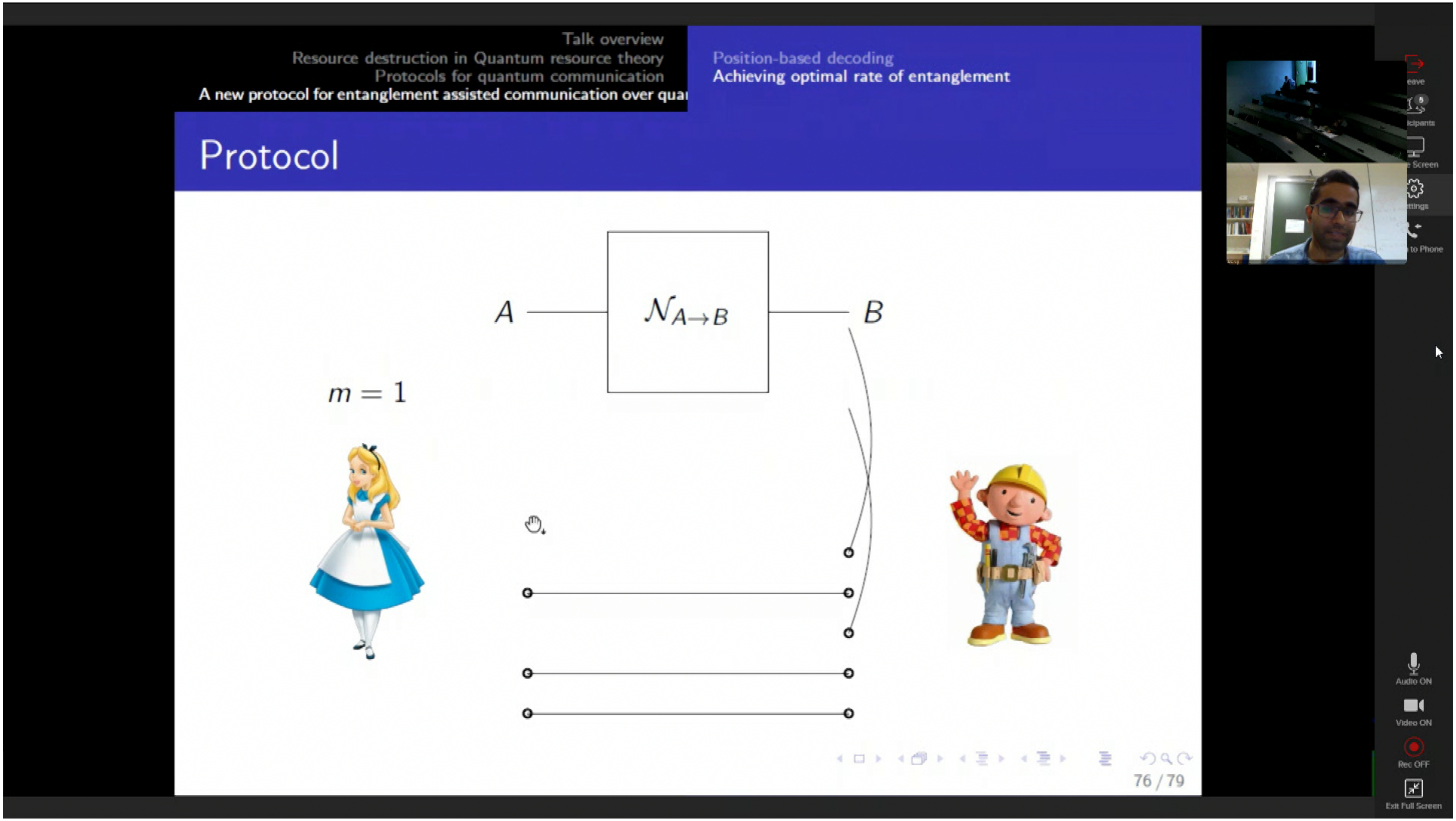
Entanglement recycling in port-based teleportation

- Introduced in Strelchuk, Horodecki, Oppenheim [Phys. Rev. Lett. 2014].
- Use multiple ports for teleportation.
- Leads to significant reduction in the number of ports: one can communicate $o(N)$ qubits through N ports.

Position-based decoding over subsets



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$\mathcal{N}_{A\to B}$
B
$m = 1$
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Achievable entanglement and communication rate

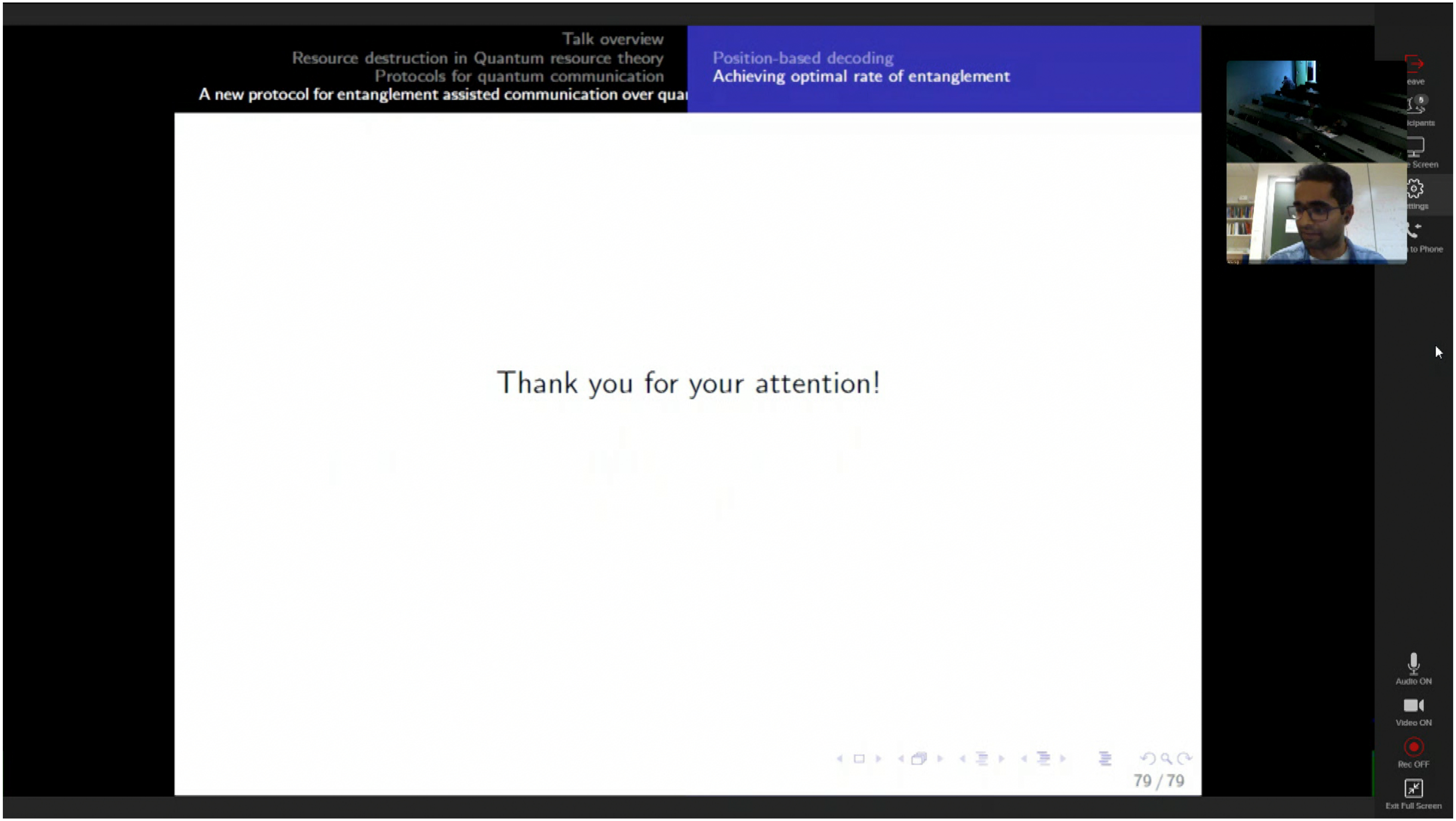
- For any pure quantum state $\Psi_{AA'}$, the number of bits communicated Q and the number of qubits of entanglement required E is

$$E \geq H(\Psi_{A'}), \quad Q \leq \mathrm{I}(A' : B)_{\mathcal{N}_{A\to B}(\Psi_{AA'})}.$$

- Matches both the entanglement rate and communication cost in the protocol of Bennett, Shor, Smolin, Thapliyal [IEEE TIT, 2002].

Outlook

- Port-based teleportation or protocols using convex-split, position based decoding form a family of protocols having similar encoding and decoding operations.
- They differ from earlier schemes of teleportation or quantum channel coding (that use decoupling via random unitary).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Thank you for your attention!
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