Title: The dynamics of entanglement in smooth quenches

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Abstract: <p>Many researchers have been studying the time evolution of entanglement entropy in the sudden quenches where a characteristic mass scale suddenly changes. It is well-know that in these quenches, the change of entanglement entropy become thermal entropy which is proportional to a subsystem size in the late time. However, we do not know which quenches thermalize a subsystem. In our works, we have been studied the time evolution of quantum entanglement in the global quenches with finite quench rate (smooth quenches). Thus, we found that diabaticity plays an important role, so that quenches thermalize the subsystem.</p>

Seminar @ PI 2017/12/12

The dynamics of entanglement in smooth quenches

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Collaborate with
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Introduction:

- EE is a candidate of **an entropy in Non-equilibrium physics**.
- In AdS/CFT correspondences, ***Entanglement*** in CFT living on the boundary is expected to be significantly related to ***Gravity*** in the bulk.

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The dynamics of entanglement ⟺ The dynamics of gravity

Thermalization ⟺ ***Black Hole Physics***

Introduction:

- EE is a candidate of **an entropy in Non-equilibrium physics**.

It is important to study the ***dynamical properties*** of Entanglement.

The dynamics of entanglement ⟺ The dynamics of gravity

Thermalization ***Black Hole Physics***

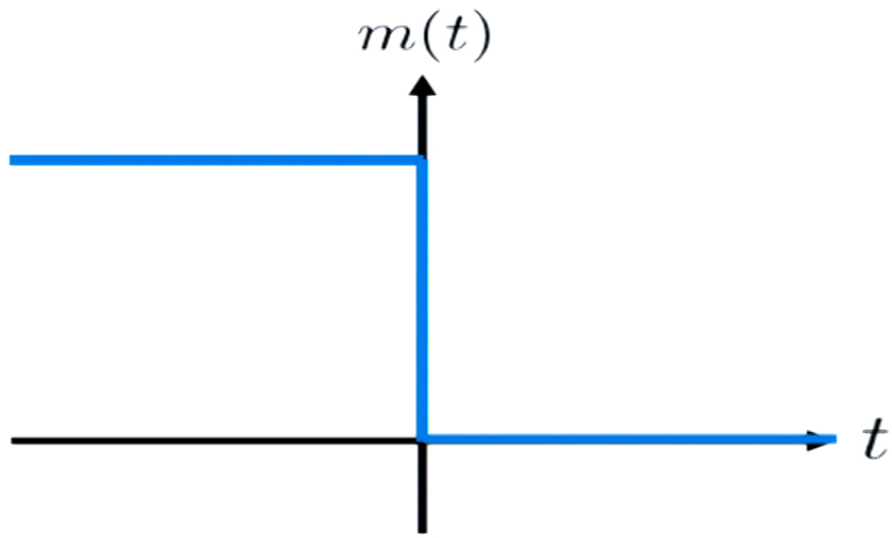
The Contents of Talk

- Introduction
- Motivation
- Profile
- Results
- Setup
- Method
- ECP
- CCP
- Summary and Future directions

Motivation

Our Motivation

In the sudden global quenches where Hamiltonian suddenly change, in the late time, the change of entanglement entropy ($\Delta S_A(t) = S_A(t) - S_A(t_{initial})$)is:



[Calabrese-Cardy, 06] [Hartman-Maldacena, 13] [Liu-Suh, 13]

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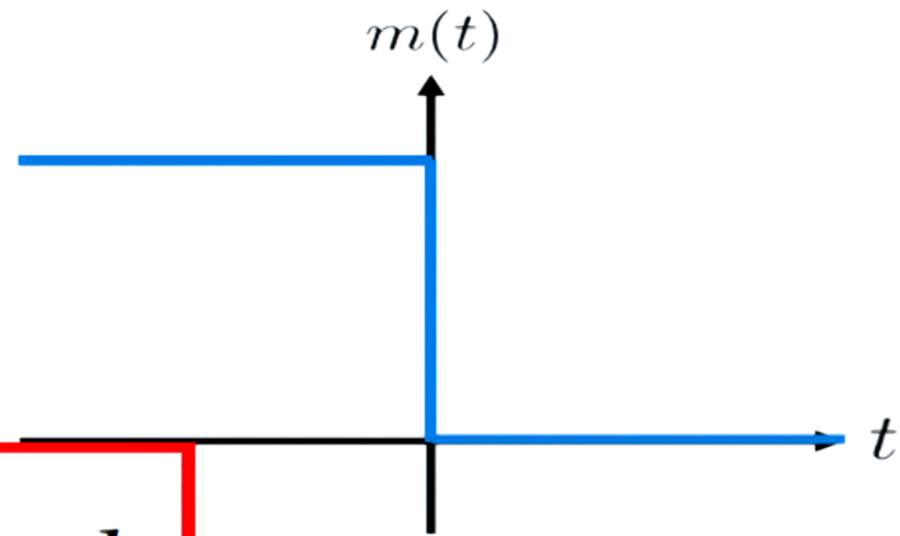


$$\Delta S_A \rightarrow S_{thermal}$$

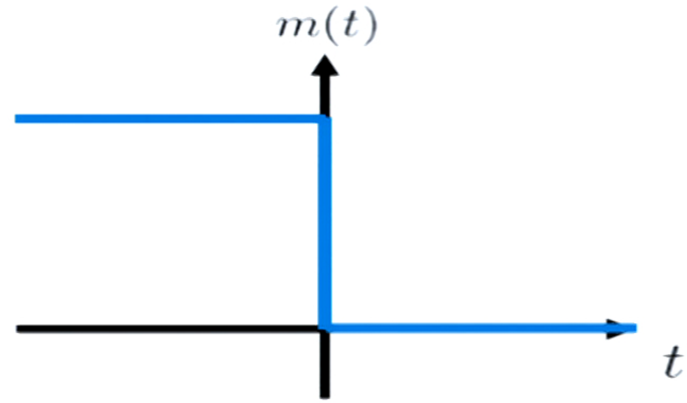
[Calabrese-Cardy, 06] [Hartman-Maldacena, 13] [Liu-Suh, 13]

Our Motivation

Sudden Quenches

Thermalized

$\Delta S_A \sim$ ***Volume of Subsystem***



Our Motivation

Sudden Quenches

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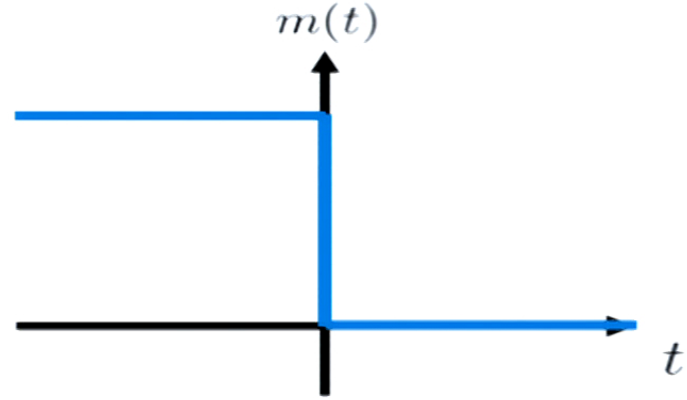


Is this unique behavior for sudden quenches?

Our Motivation

What creates

entangled particles (thermalizition) ?

or

What is important

when a subsystem is thermalized ?

Our Motivation

What creates

n) ?

Diadiabaticity is important!

What is important

when a subsystem is thermalized ?

Protocol

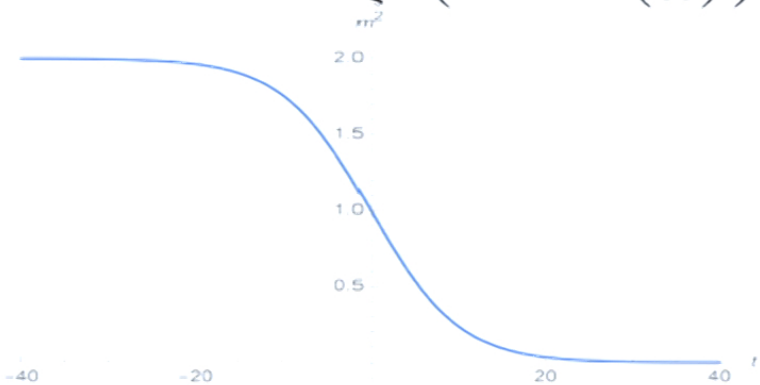
Our protocol (Smooth Quenches)

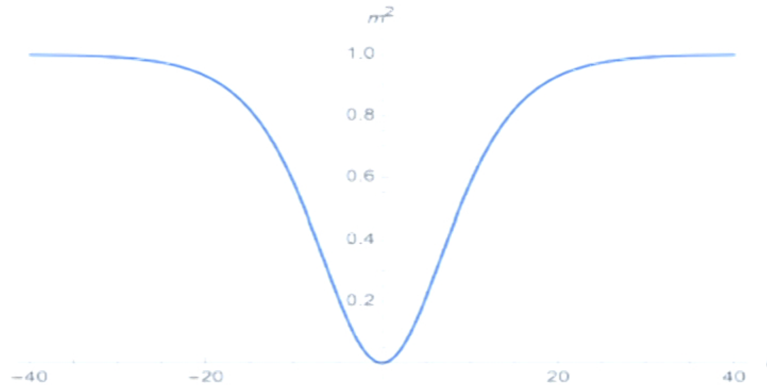
[Das-Galante-Myers, 14]

2d –Time-dependent Hamiltonian

$$H(t) = \frac{1}{2}\int dx\,\left[\Pi^2(x) + \partial_x\phi^2(x) + m^2(t)\phi^2(x)\right]$$

- ECP: $m^2(t) = \frac{1}{\xi^2}\left(1 - \tanh\left(\frac{t}{\delta t}\right)\right)$
- CCP: $m^2(t) = \frac{1}{\xi^2}\tanh^2\left(\frac{t}{\delta t}\right)$





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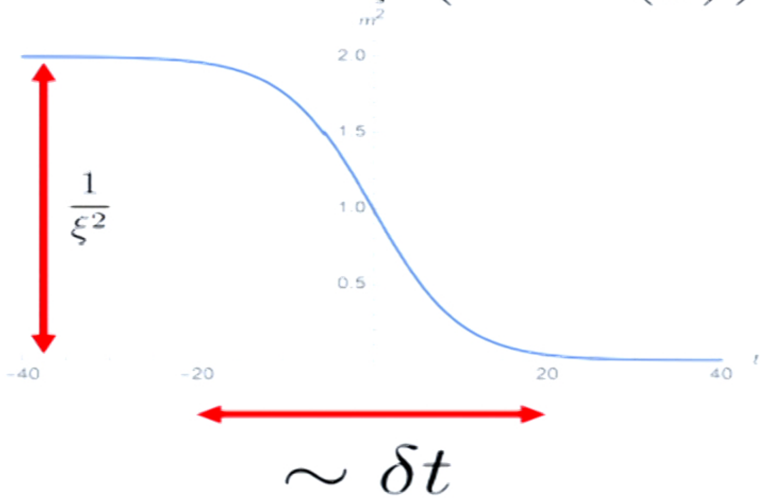

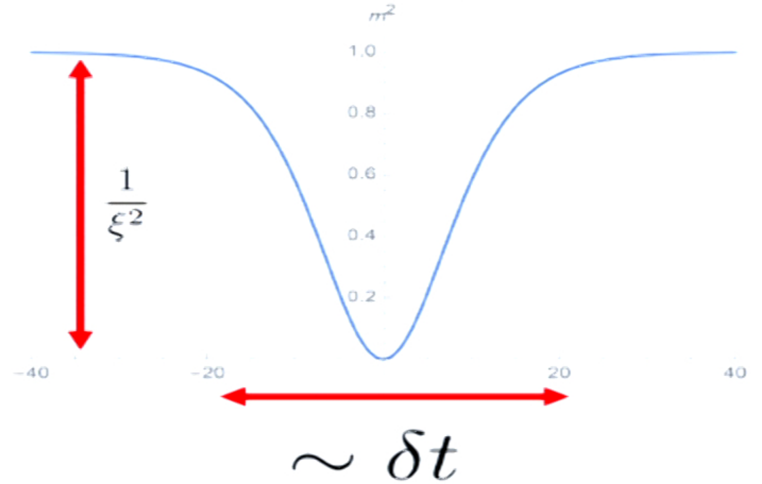

Fast Quench limit: $\omega \ll 1 \quad (\delta t \ll \xi)$

Slow Quench limit: $\omega \gg 1 \quad (\delta t \gg \xi)$

- ECP: $m^2(t) = \frac{1}{\xi^2}\left(1 - \tanh\left(\frac{t}{\delta t}\right)\right)$
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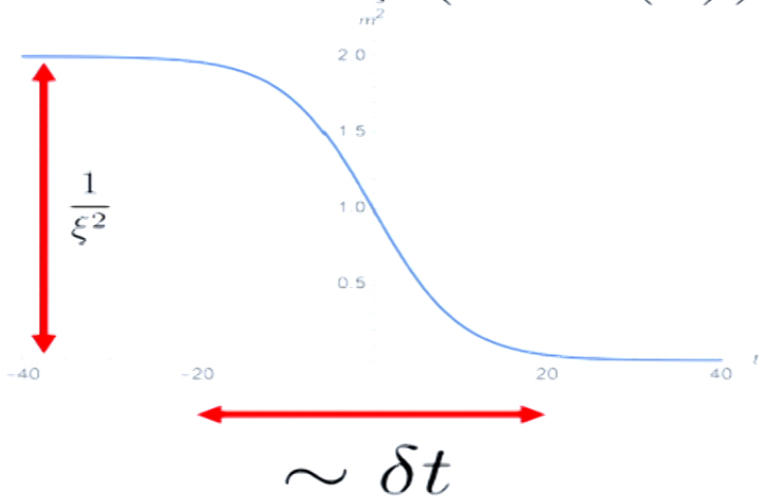

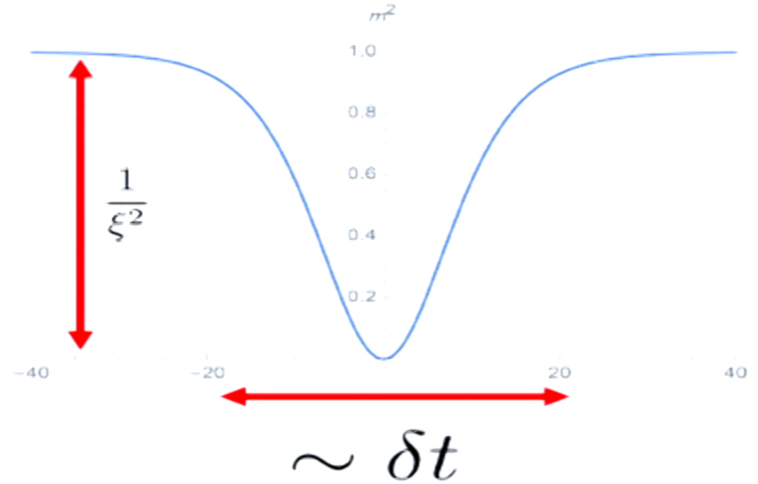

Fast Quench limit: $\omega \ll 1 \quad (\delta t \ll \xi)$

Slow Quench limit: $\omega \gg 1 \quad (\delta t \gg \xi)$

Results

Result1

- ECP: $m^2(t) = \frac{1}{\xi^2}\left(1 - \tanh\left(\frac{t}{\delta t}\right)\right)$
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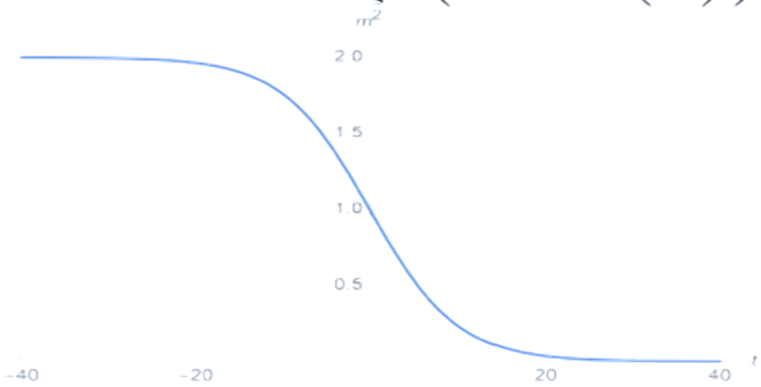
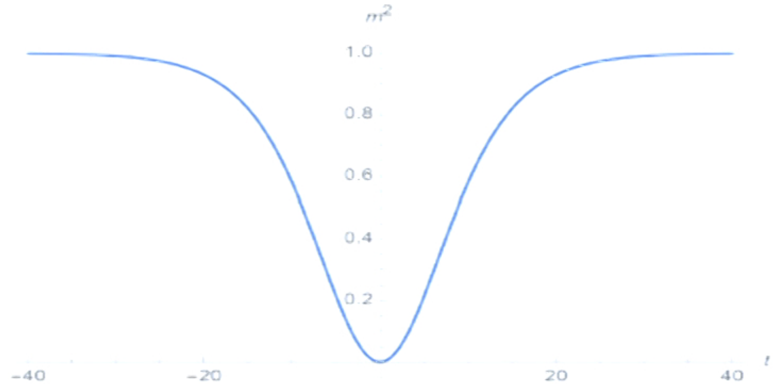


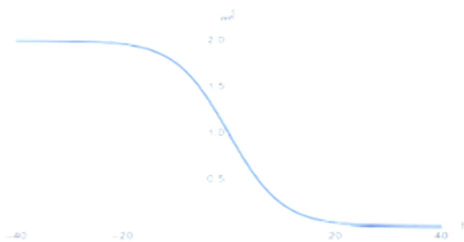


At late time,

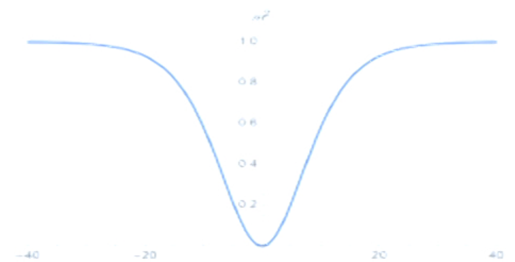
ΔS_A is proportional to a subsystem size ***l***.

Result1

- ECP:



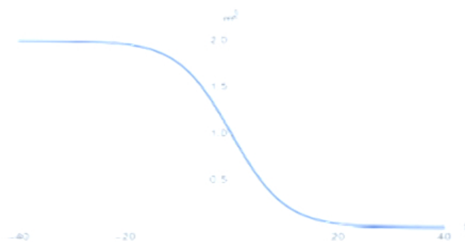
- CCP:



$$\Delta S_A \sim c \times l$$

Result1

• ECP:



$$m^2(t) = \frac{1}{\xi^2}\left(1 - \tanh\left(\frac{t}{\delta t}\right)\right)$$

$$\Delta S_A \sim c \times l$$

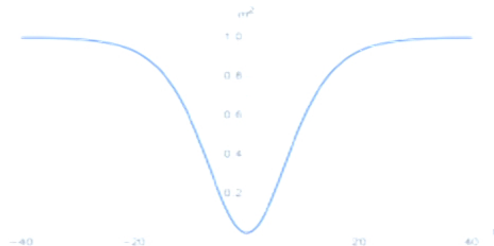
is set by

- ***an initial correlation length*** in a fast limit $\omega = \delta t \cdot m = \frac{\delta t}{\varepsilon} \ll 1$.
- ***an effective correlation length*** in a slow limit $\omega = \delta t \cdot m = \frac{\delta t}{\xi} \gg 1$.

A length scale when adiabaticity breaks down.

Result1

- CCP:



$$m^2(t) = \frac{1}{\xi^2}\tanh^2\left(\frac{t}{\delta t}\right)$$

$$\Delta S_A \sim c \times l$$

Depends on ω ***in both limits.***

Result1

Assumptions: $\frac{1}{m \cdot a} = \frac{\xi}{a} \gg 1$, a: is a lattice spacing.

- **ECP:**

Fast limit: $\omega = \delta t \cdot m = \frac{\delta t}{\xi} \ll 1$

$$\Delta S_A \sim C_1 \frac{l}{\xi}$$

slow limit: $\omega = \delta t \cdot m = \frac{\delta t}{\xi} \gg 1$, $E_{kz} = \frac{1}{\delta t} \ll 1$

$$\Delta S_A \sim C_2 E_{kz} \cdot l$$

- **CCP:**

Fast limit: $\omega = \delta t \cdot m = \frac{\delta t}{\xi} \ll 1$

slow limit: $\omega = \delta t \cdot m = \frac{\delta t}{\xi} \gg 1$, $\xi_{kz} = \sqrt{\frac{\delta t}{m}} = \sqrt{\xi \delta t} \gg 1$

$$\Delta S_A \sim C_3(\omega) \frac{l}{\xi}$$

Result1

Assumpt … g.

Constant.

- **ECP:**

Fast limit: $\omega = \delta t \cdot m = \frac{\delta t}{\xi} \ll 1$

$$\Delta S_A \sim C_1 \frac{l}{\xi}$$

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$$\Delta S_A \sim C_3(\omega) \frac{l}{\xi}$$

$C_3(\omega)$

In fast limit, keeping ξ constant and, ω decreases

⇒ $C_3(\omega)$ ***decreases***.

In slow limit, keeping ξ constant and, ω increases

⇒ $C_3(\omega)$ ***decreases***.

$C_3(\omega)$

In fast limit, keeping ξ constant and, ω decreases

Consistent with a number operator in late time.

In fast limit, keeping ξ constant and, ω increases

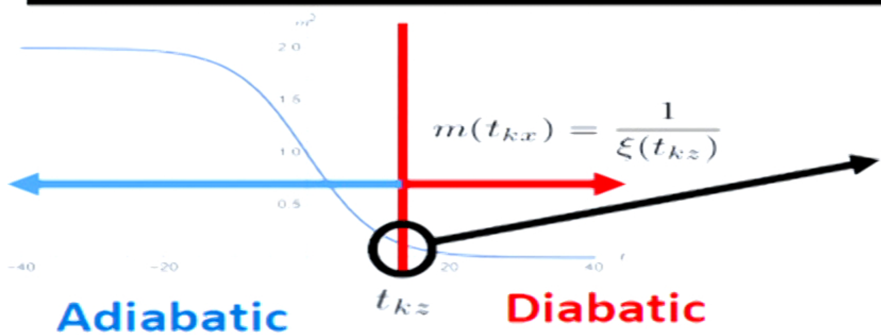
→ $C_3(\omega)$ ***decreases***.

Interpretation in slow ECP

- **How is the subsystem thermalized?**
  (Entangled particles are created?)

Interpretation in slow ECP

Entangled particles are created when the adiabaticity breaks down!
(Subsystems are thermalized!)



Entangled particles are created at t=t_{kz} and carry quantum entanglement.

Thermalized around $t = t_{kz} + \frac{l}{2}$

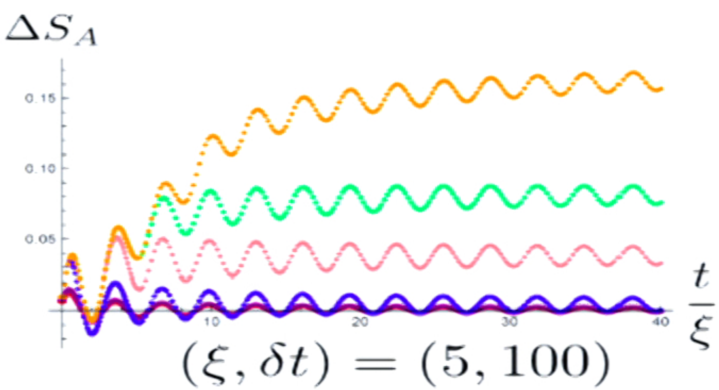
Interpretation In fast ECP and CCP

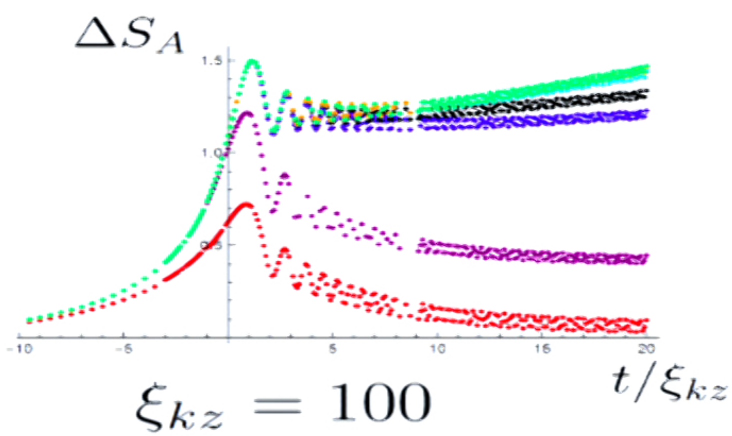
- Time Evolution of $\Delta S_A(t)$

Propagation of Entangled particles created @t=0.

Result2

Entanglement Oscillation





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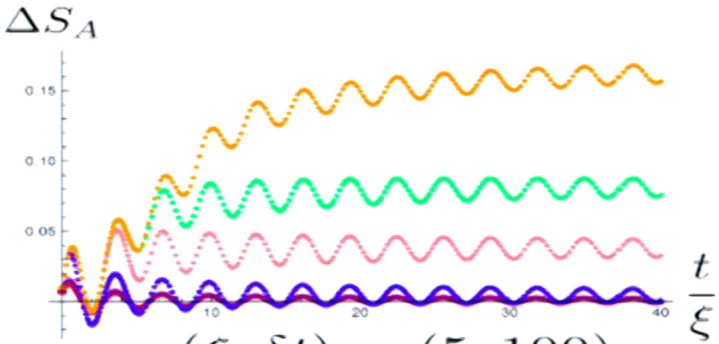
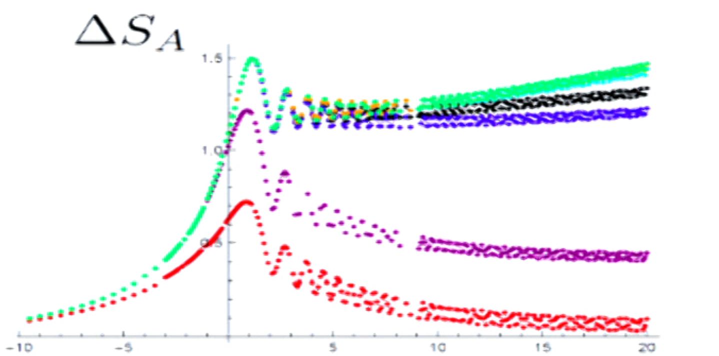


$(\xi, \delta t) = (5, 100)$



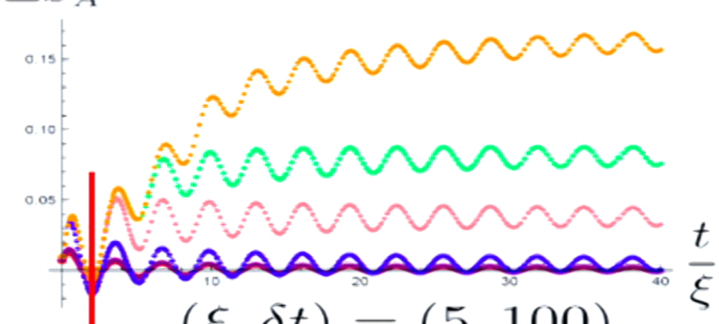
$\xi_{kz} = 100$

The period of oscillation @ late time.

↔ ***The periodicity of zero mode*** $\simeq \pi\xi$

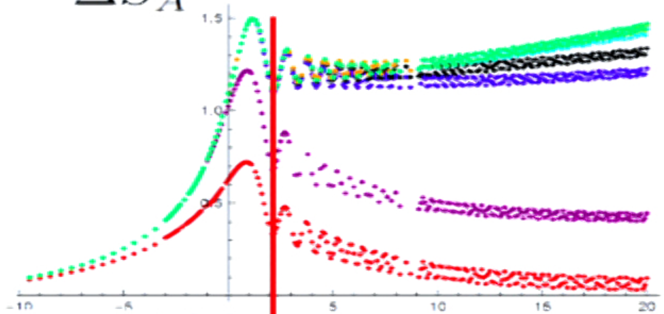
Result3

Time evolution is characterized by $\underline{t \simeq 2\xi_{kz}, 2\xi}$.



$(\xi, \delta t) = (5, 100)$

$t \simeq 2\xi_{kz}$



$\xi_{kz} = 100$ t/ξ_{kz}

$t \simeq 2\xi$

After $t = 2\xi, 2\xi_{kz}$, ΔS_A oscillates.


Setup

Smooth Quenches

- These quenches are ***more realistic***.
- Hamiltonian is not changed suddenly but is changed smoothly.
- We can excite the state ***slowly or fast***.
- This is a kind of generalization of sudden quenches.

Our protocol (Smooth Quenches)

[Das-Galante-Myers, 14]

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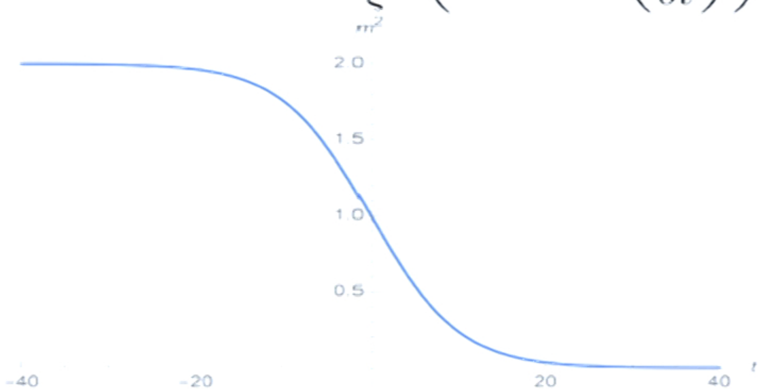

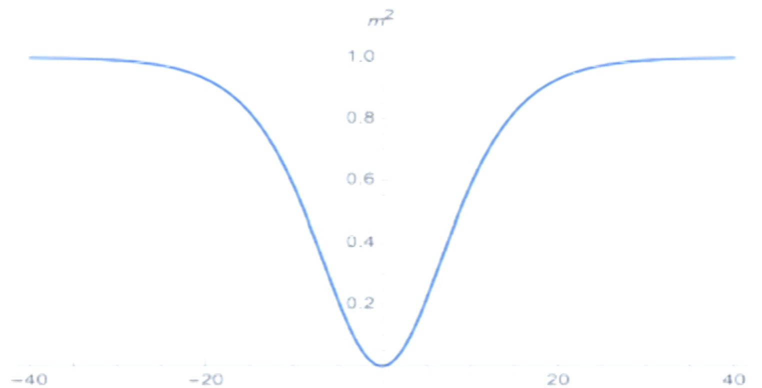

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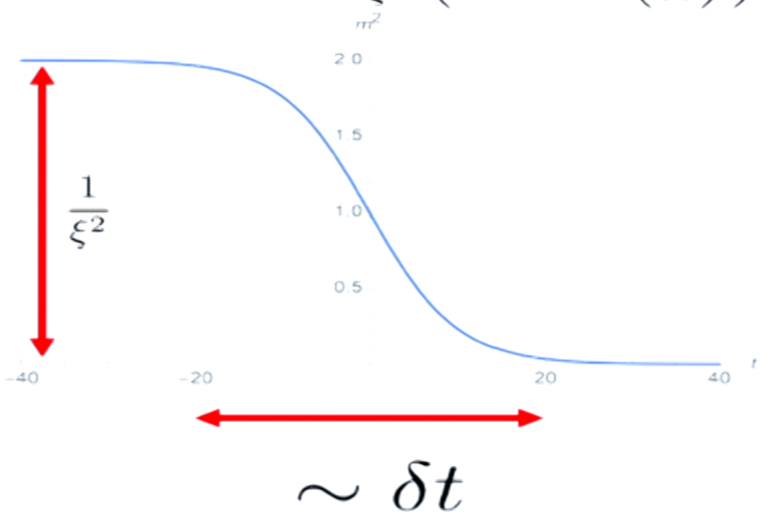

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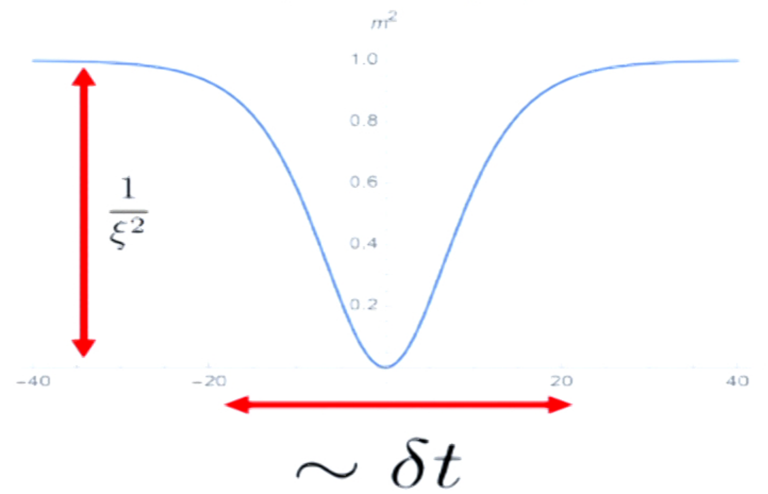
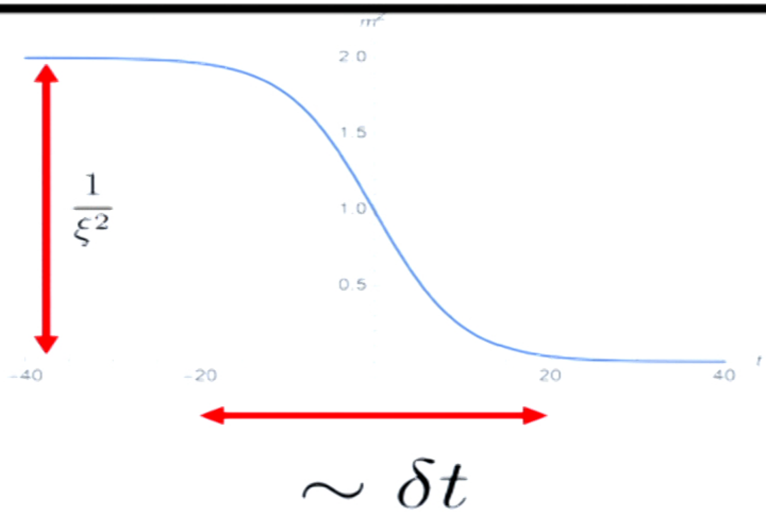
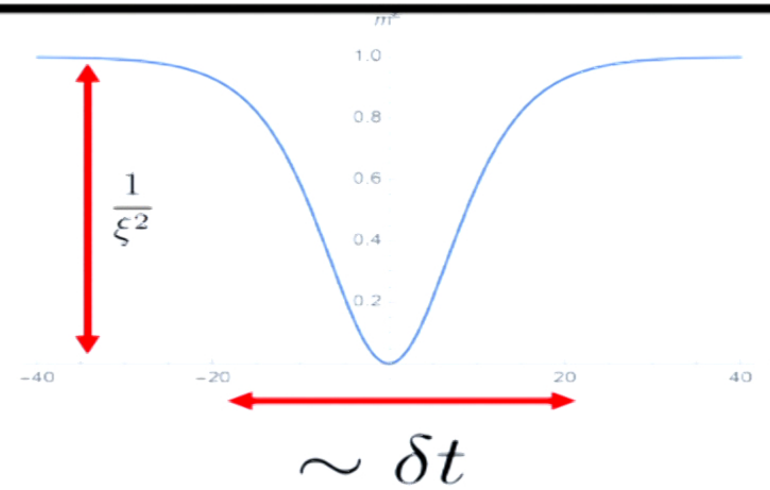


We have
two tunable parameters.

→ States are excited **slowly** and **rapidly.**



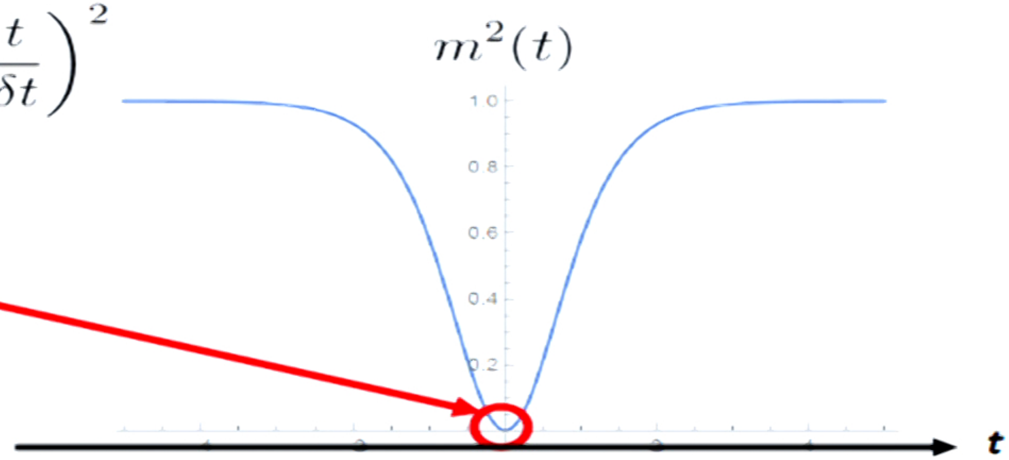


Our setup

- Theory 2d Free scalar with time dependent mass m(t).
- Put it on the lattice but take the ***thermodynamic limit***.

Mass profile: $m^2(t) = m^2 \tanh\left(\frac{t}{\delta t}\right)^2$

At ***t =0***, the theory is at critical point.



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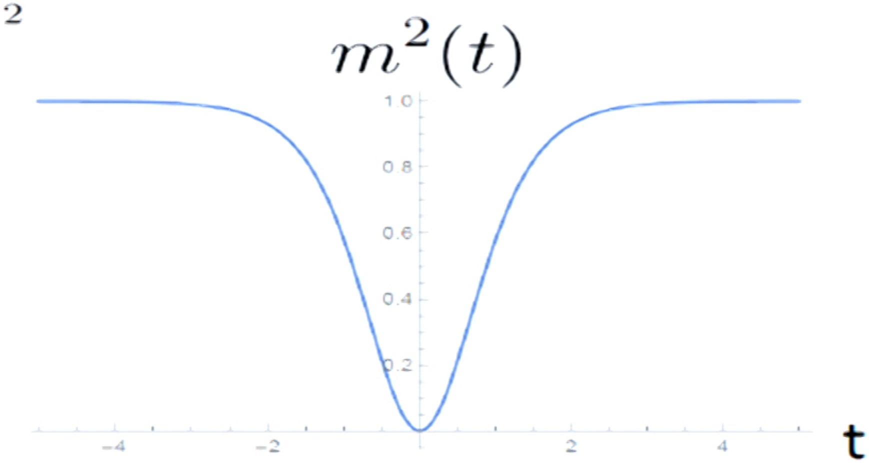
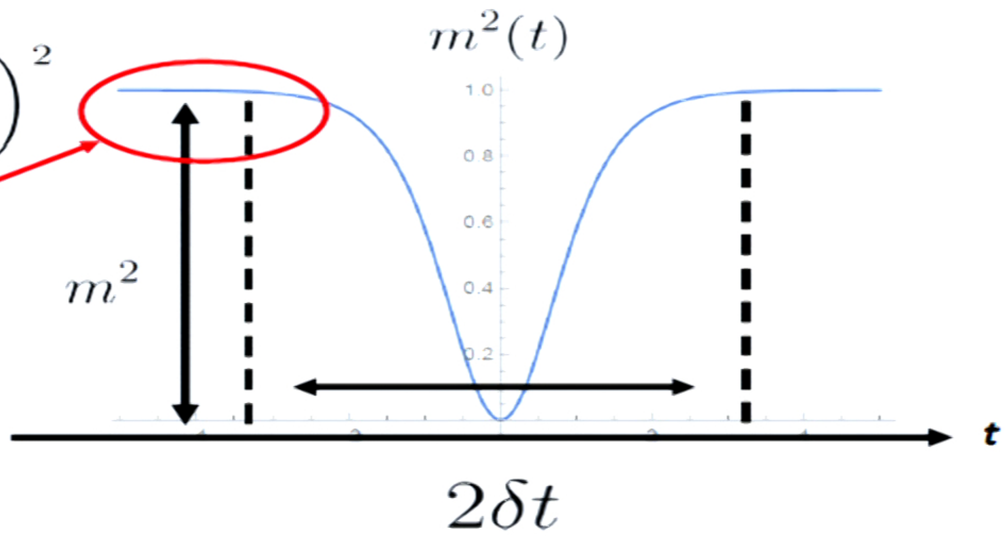


Initial state: The Ground state
for massive free scalar with mass m^2.

Slow Quenches

Mass profile: $m^2(t) = m^2 \tanh\left(\frac{t}{\delta t}\right)^2$

Very Early time:
Observables ***can***
be computed adiabatically .



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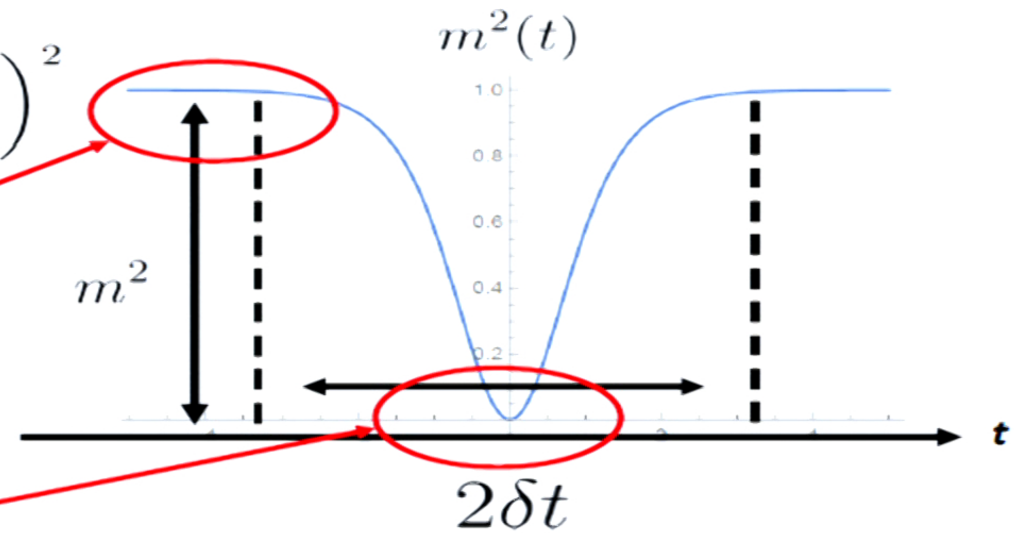

Very Early time:
Observables ***can***
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Around critical point:

Observables ***can not***
be computed adiabatically .

Slow Quenches

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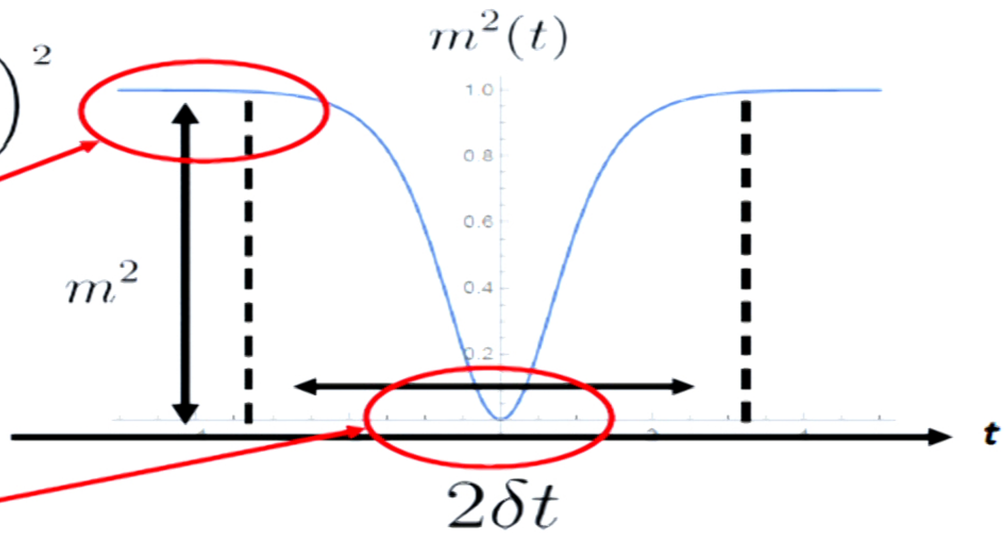

Very Early time:
Observables ***can*** be computed adiabatically .

Around critical point:

We assume that at $t = -t_{kz}$, adiabaticity breaks down.

(Around $t = t_{kz}$, time process becomes adiabatic again.)

Adiabatic Expansion

$$X_{ij} = X_{ij}^{(0)} + X_{ij}^{(1)} + \cdots$$

$$P_{ij} = P_{ij}^{(0)} + P_{ij}^{(1)} + \cdots$$

$$D_{ij} = D_{ij}^{(0)} + D_{ij}^{(1)} + \cdots$$

$$\langle \phi_i \phi_j \rangle = X_{ij}$$

$$\left\langle \dot{\phi}_i \dot{\phi}_j \right\rangle = P_{ij}$$

$$\frac{1}{2} \left\langle \{ \phi_i , \dot{\phi}_j \} \right\rangle = D_{ij}$$

Higher orders has higher derivative with respect to t.

Adiabatic Expansion

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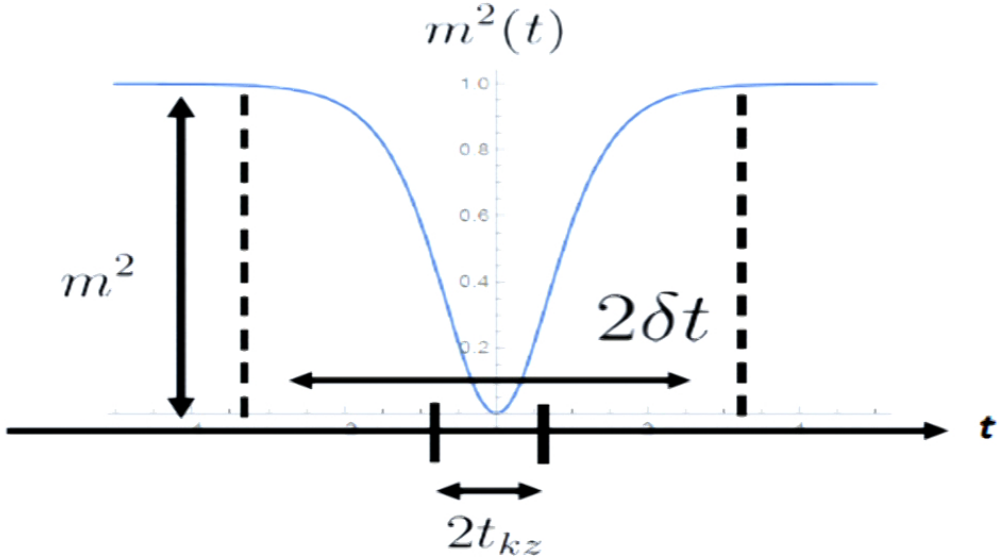
$$\frac{1}{2} \left\langle \{ \phi_i, \dot{\phi}_j \} \right\rangle = D_{ij}$$

Higher orders has higher derivative with respect to t.

Landau Criteria $\frac{1}{m^2(t)} \frac{dm(t)}{dt} \ll 1$ ⇒ ***Adiabaticity holds***

If t_{kz} *is so small,* ***the most of whole time evolution*** *is adiabatic.*

More precisely, $\frac{t_{kz}}{\delta t} \ll 1$



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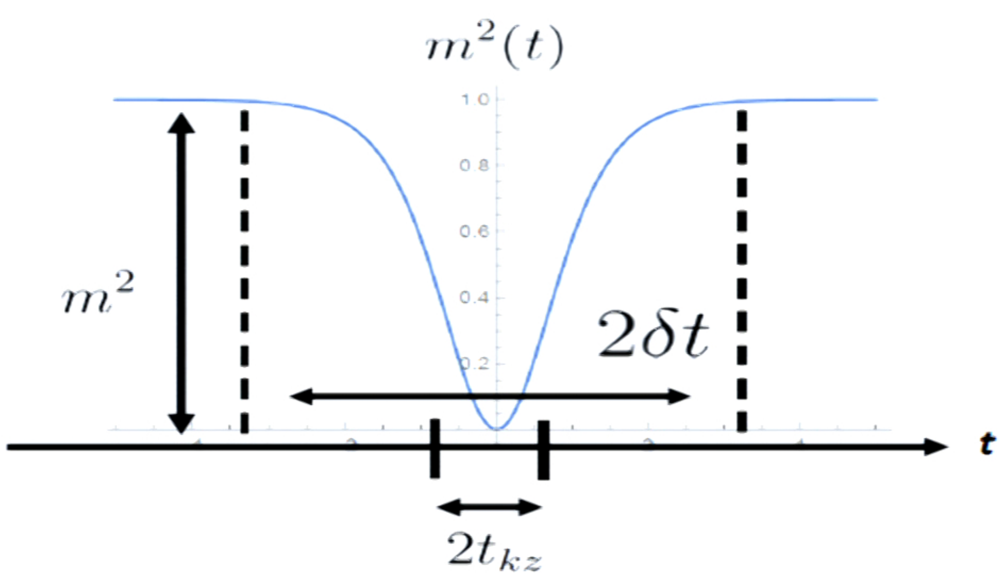

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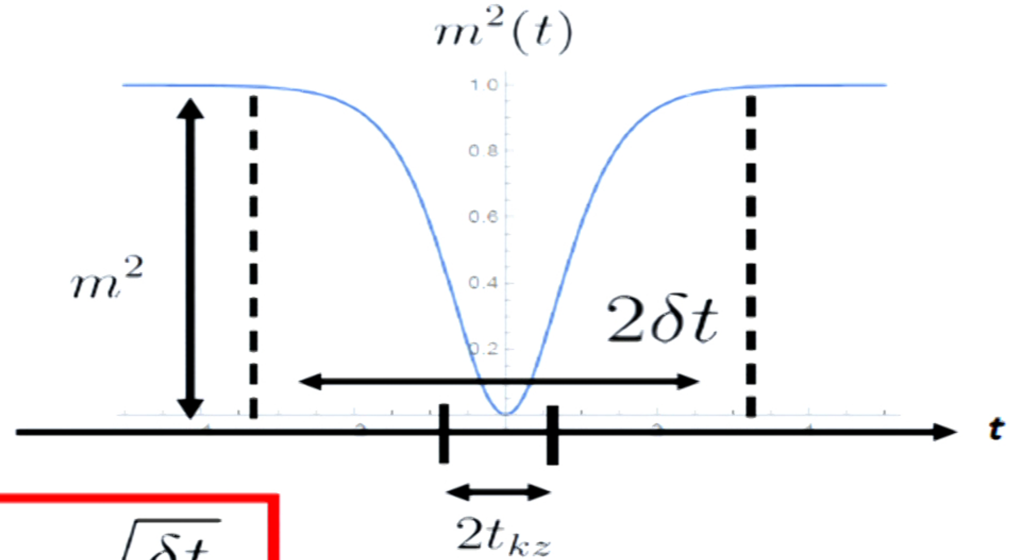

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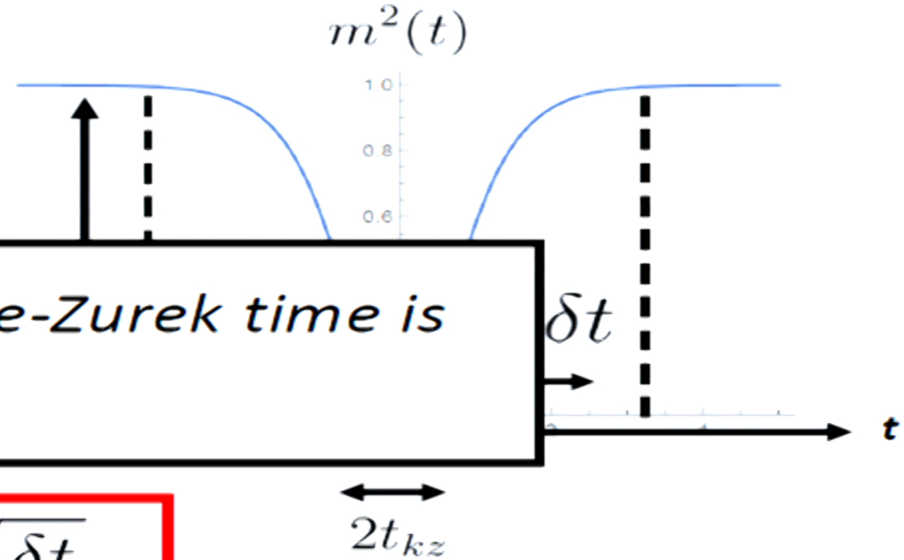
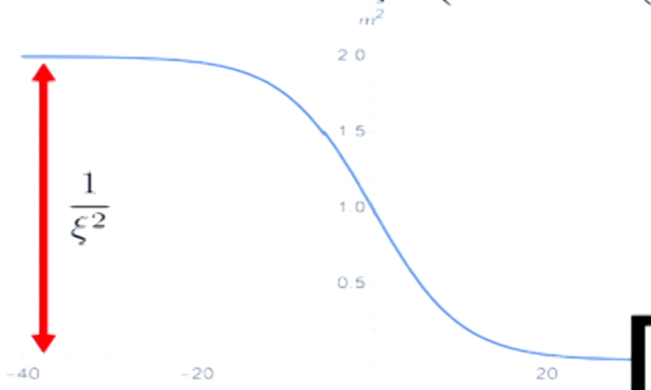
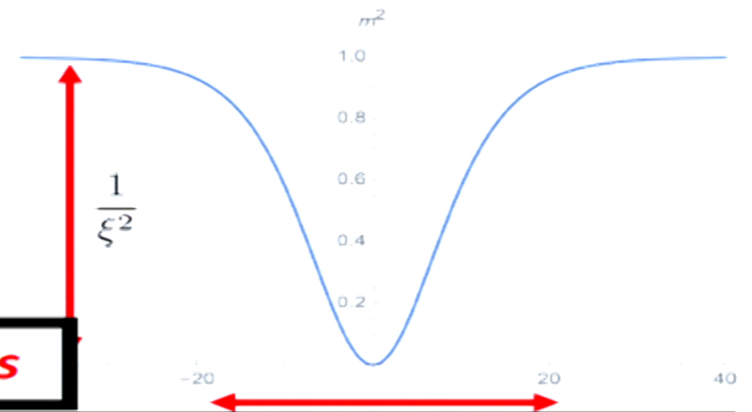


*In **slow quenches**, Kibble-Zurek time is small.*

$$t_{kz} = m(-t_{kz}) = \xi_{kz} = \sqrt{\frac{\delta t}{m}}$$

- ECP : $m^2(t) = \frac{1}{\xi^2}\left(1 - \tanh\left(\frac{t}{\delta t}\right)\right)$
- CCP: $m^2(t) = \frac{1}{\xi^2}\tanh^2\left(\frac{t}{\delta t}\right)$





Parameters

• ECP :	• CCP:
$\xi_{kz} = 1/E_{kz} = \delta t$ $t_{kz} = \delta t \log \omega$	$\xi_{kz} = t_{kz} = \sqrt{\delta t \cdot \xi}$

Method

Discretize

- We put our theory on ***the lattice*** so that we compute ΔS_A by the correlator method.

Correlator method

- This is a method to compute ΔS_A by using the correlation functions.

Conditions: 1. State is ***a Gaussian state***.
2. Local observables can be computed ***by Wick theorem***.

Correlator Method

- If an initial state $|\Psi\rangle$ is given by a gaussian state:

For example, $|\Psi\rangle$ ($a_k\,|\Psi\rangle = 0$)

Correlator Method

- If an initial state $|\Psi\rangle$ is given by a gaussian state:

For example, $|\Psi\rangle$ ($a_k |\Psi\rangle = 0$)

We assume that a reduced density matrix is given by

$$\rho_A = tr_B \rho \sim e^{-\sum \gamma_k b_k^\dagger b_k}$$

$$a_k = \alpha_k b_k + \beta_{-k} b^\dagger_{-k}$$

If ϕ_i, ϕ_j are included in A,

$$\langle \phi_i \phi_j \rangle = tr\left(\rho \phi_i \phi_j\right) = tr_A\left(\rho_A \phi_i \phi_j\right) = \langle \phi_i \phi_j \rangle_A$$

Correlator Method

$$\langle \phi_i \phi_j \rangle = tr\left(\rho \phi_i \phi_j\right) = tr_A\left(\rho_A \phi_i \phi_j\right) = \langle \phi_i \phi_j \rangle_A$$

Determined by E.O.M and so on.

$$f\left(\gamma_k\right)$$

Correlator Method

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$f\left(\gamma_k\right)$

Two point functions ⟷ γ_k

Correlator Method

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$f(\gamma_k)$

Two point functions ⟷ γ_k ➡ $\rho_A \sim e^{-\sum \gamma_k b_k^\dagger b_k}$

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$$\langle \phi_i \phi_j \rangle = tr\left(\rho \phi_i \phi_j\right) = tr_A\left(\rho_A \phi_i \phi_j\right) = \langle \phi_i \phi_j \rangle_A$$

Two point functions ⟷ γ_k ⟹ $\rho_A \sim e^{-\sum \gamma_k b_k^\dagger b_k}$

⟹ S_A ***is determined by two point functions.***

Correlator method

Entanglement Entropy:

$$S_A = \sum_{k=1}^{l} s_A(\gamma_k)$$

$$s_A(\gamma_k) = \left(\frac{1}{2} + \gamma_k\right) \log\left(\frac{1}{2} + \gamma_k\right) - \left(-\frac{1}{2} + \gamma_k\right) \log\left(-\frac{1}{2} + \gamma_k\right)$$

$$\Gamma = \begin{pmatrix} X_{ij} & \frac{1}{2} D_{ij} \\ \frac{1}{2} D_{ji} & P_{ij} \end{pmatrix} \quad J = \begin{pmatrix} 0 & \mathbf{1} \\ -\mathbf{1} & 0 \end{pmatrix}$$

$$X_{ij} = \langle \phi_i \phi_j \rangle \qquad P_{ij} = \langle \pi_i \pi_j \rangle \qquad D_{ij} = \langle \{\phi, \pi_j\} \rangle$$

- $M = iJ\Gamma$ has eigenvalues $\pm\gamma_k$.

Correlator method

Entanglement Entropy:

The subsystem size = *l*

$$S_A = \sum_{k=1}^{l} s_A(\gamma_k)$$

2*l* ×2*l* matrix

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By evaluating M, we can compute S_A.

$X_{ij} = \langle \phi_i \phi_j \rangle$ $\quad P_{ij} = \langle \pi_i \pi_j \rangle$ $\quad D_{ij} = \langle \{\phi_i, \pi_j\} \rangle$

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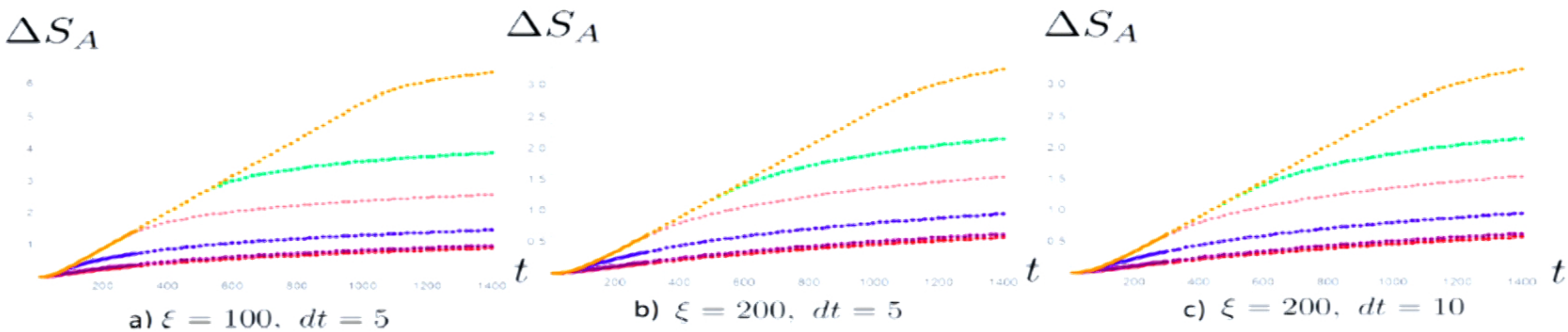
EE in ECP

Plot of EE in Fast ECP



Orange Curve: l=2000, Green Curve: l=1000, Pink Curve: l= 500,
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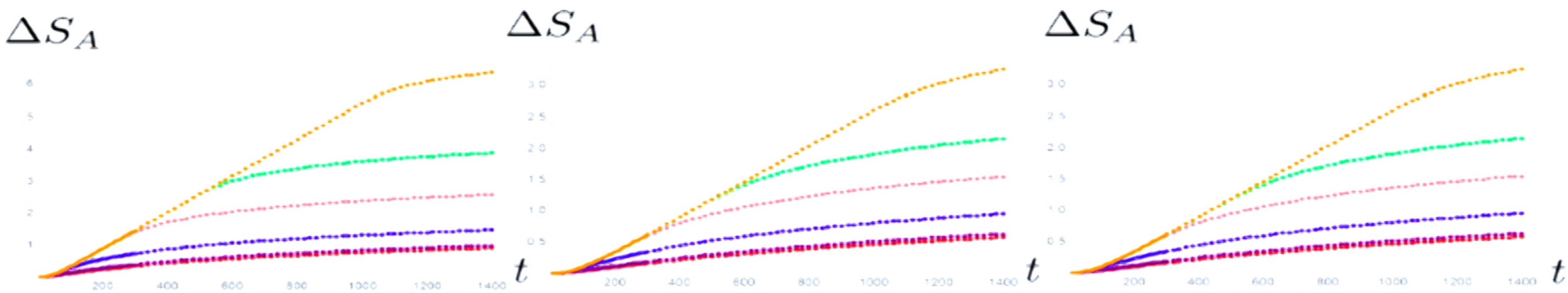
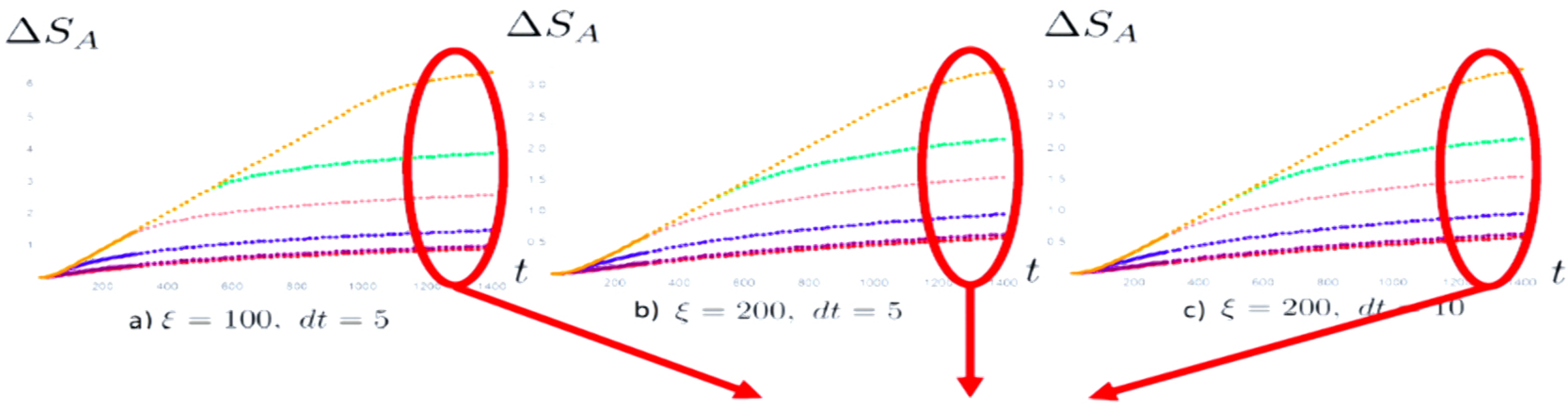


a) $\xi = 100,\ dt = 5$ b) $\xi = 200,\ dt = 5$ c) $\xi = 200,\ dt = 10$

If l is sufficiently larger than ξ and $\xi \ll t \leq l/2$,

$$\Delta S_A(t) \sim 0.57 \times \frac{t}{\xi}$$

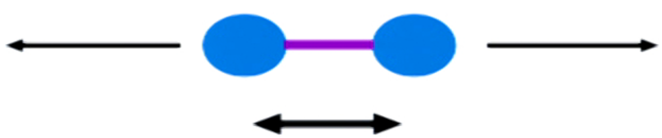
Orange Curve: l=2000, Green Curve: l=1000, Pink Curve: l= 500,
Blue Curve: l=100, Purple Curve: l=10, Red Curve: l=5

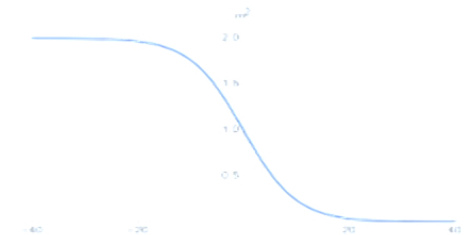


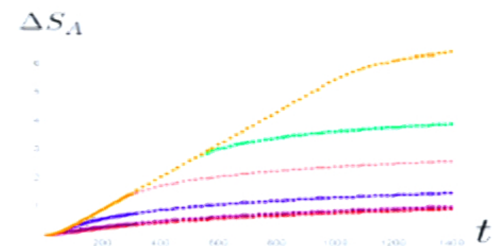
Slowly increase

Entangled Particle Interpretation

- **As in sudden quenches, around t=0, entangled quasi-particle are created everywhere.**







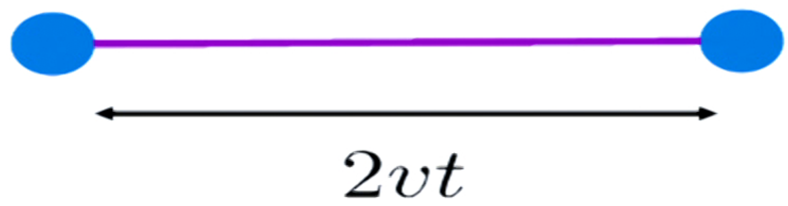
- Entangled pair is constructed two particles.

They propagate in the opposite directions with v :

If one of them is included in A and the other is out of A,

Entangled pair can contribute to entanglement entropy.

At l/2 > t > 1/m, the distance between entangled particles is given 2vt:



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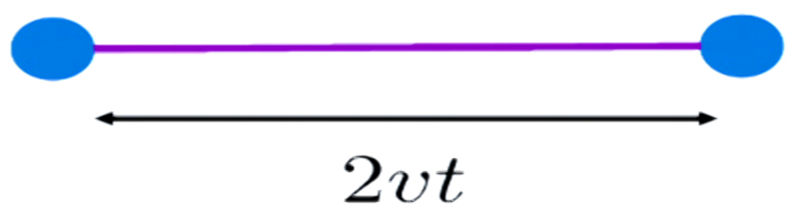
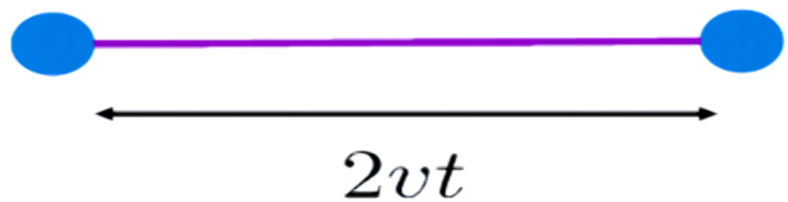


The particle created at the boundary at t=0 is at $x = vt$ or $x = -l - vt$.

At l/2 > t > 1/m, the distance between entangled particles is given 2vt:



The entangled pairs in the blue region can contribute to S_A .

# of entangled pair $\sim$ t

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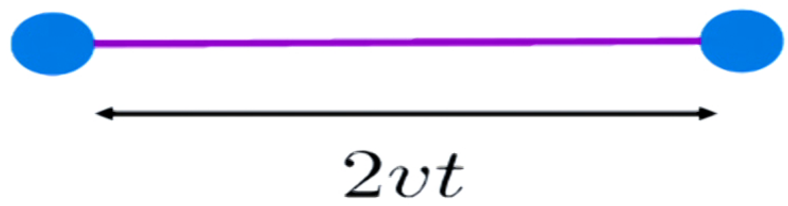
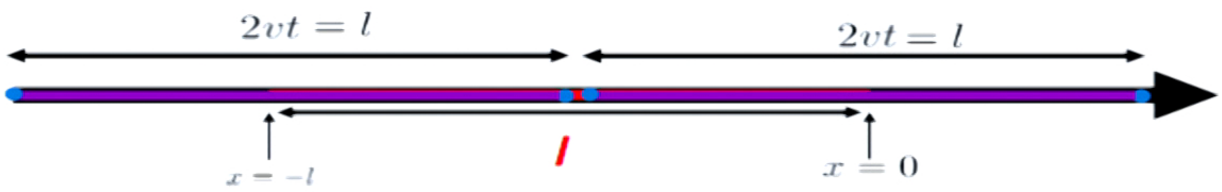


The entangled pairs in the blue region can contribute to S_A .

# of entangled pair $\sim$ t

S_A linearly grows with t

At t=l/2v, the distance between entangled particles is the subsystem size.

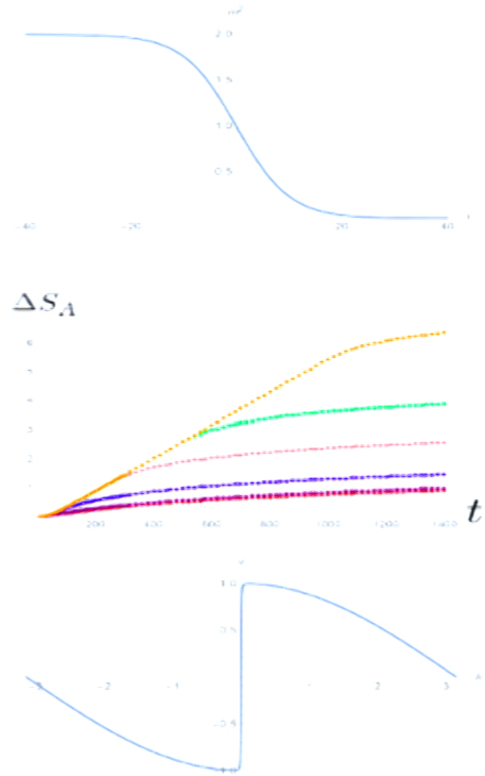


Entangled Particle Interpretation

- **As in sudden quenches, around t=0, entangled particle are created everywhere.**
- **Their speed is given by the group velocity around t=0**

Assumption

$$v_k = \frac{d\omega_k(t)}{dk}\ , \quad \omega_k(t) = \sqrt{4\sin^2\left(\frac{k}{2}\right) + m^2(t)}\ .$$



[Jordan-Mark-Mark-Mark, 16] [Andrea-Erik-Pasquale, 16]

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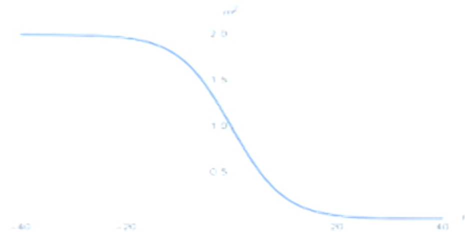
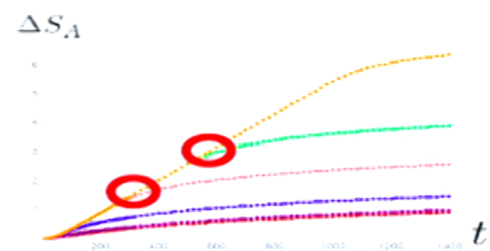
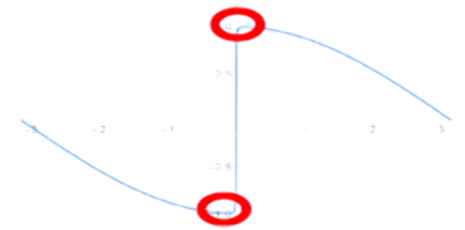
$|v_{max}| \sim 1$ ➡ Around t =l/2, the time evolution of ΔS_A changes.

[Jordan-Mark-Mark-Mark, 16] [Andrea-Erik-Pasquale, 16]







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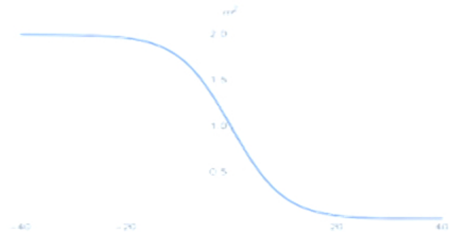
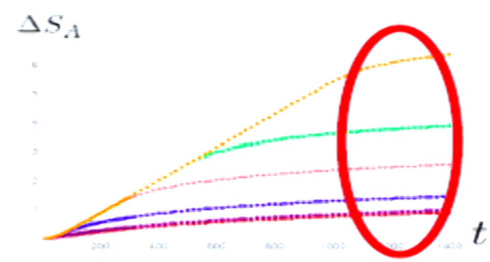
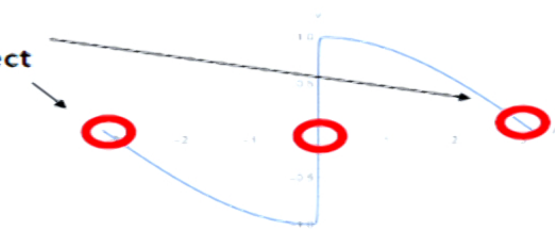
Slow mode ($\sim$ zero mode and large k mode)

→ Slowly increases in the late time

[Jordan-Mark-Mark-Mark, 16] [Andrea-Erik-Pasquale, 16]



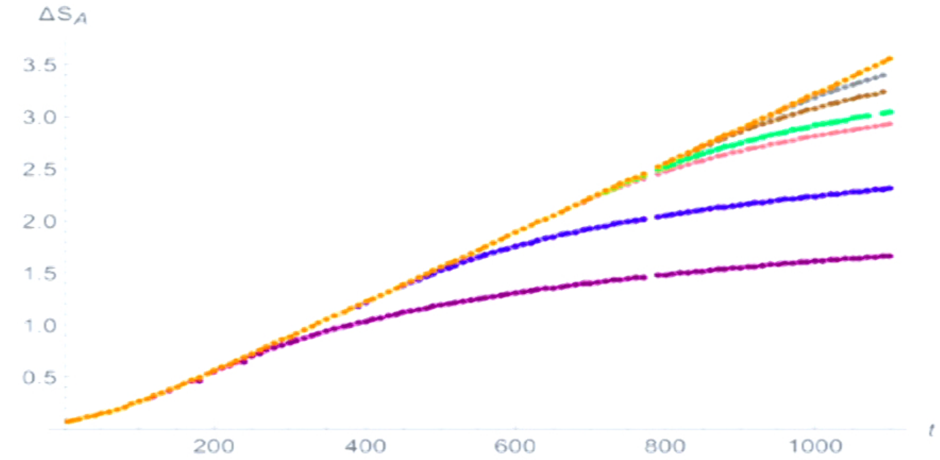




EE in slow ECP

Plot of EE in Slow ECP

Orange Curve: l=2000, Gray Curve: l=1000, Brown Curve: l= 800,
Green Curve: l=600, Pink Curve: l=500, Blue Curve: l=100, Purple Curve: l=10



$$(\xi, \delta t) = (1, 100)$$

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Orange Curve: l=2000, Gray Curve: l=1000, Brown Curve: l= 800,
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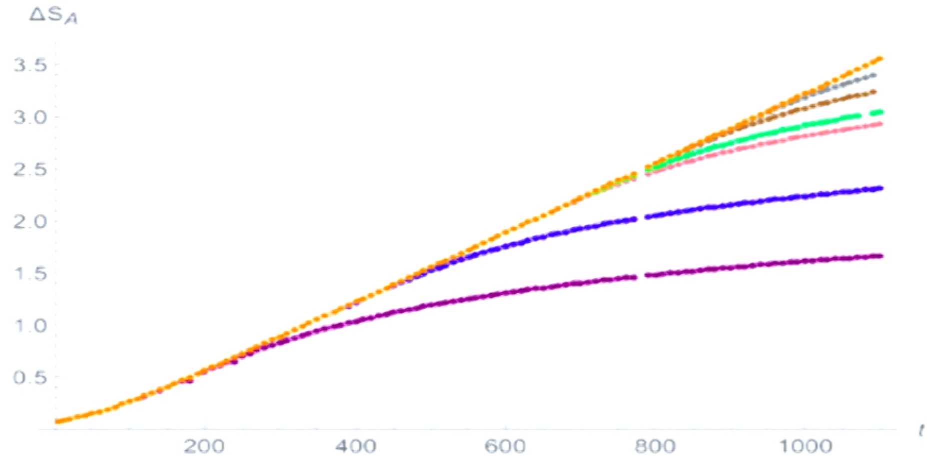
$$t_{kz} + \frac{l}{2} = \delta t \log \omega + \frac{l}{2} > t \gg \frac{1}{E_{kz}},$$

ΔS_A does not depend on *l*.



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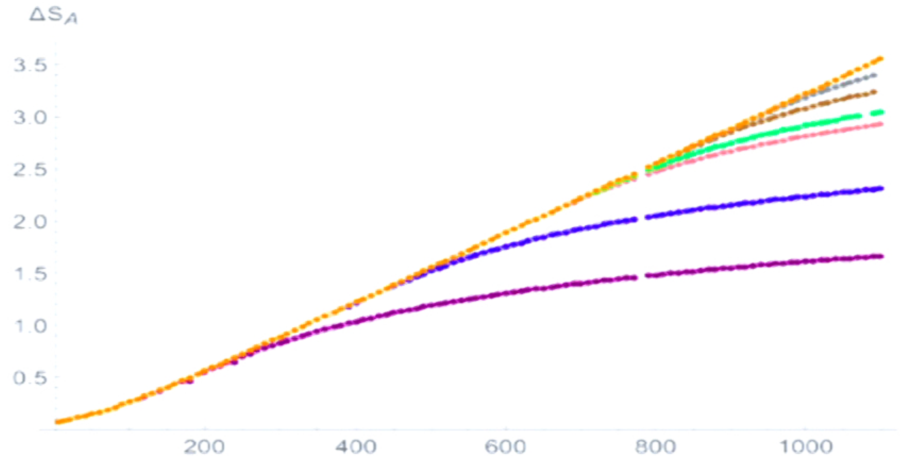
Adiabaticity breaks down.

$$t_{kz} + \frac{l}{2} = \delta t \log \omega + \frac{l}{2} > t \gg \frac{1}{E_{kz}},$$

$$\Delta S_A \sim \frac{1}{3} E_{kz} \cdot t$$

$$\frac{1}{\xi} \to E_{kz}$$



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Entangled Particle Interpretation

$$t_{kz} + \frac{l}{2} = \delta t \log \omega + \frac{l}{2} > t \gg \frac{1}{E_{kz}}, \quad \Delta S_A \sim \frac{1}{3} E_{kz} \cdot t$$

$$t \gg t_{kz} + \frac{l}{2}, \quad \Delta S_A \sim \frac{1}{6} E_{kz} \cdot l$$

Proportionality Coefficient

The proportionality coefficient of ***l*** or ***t*** is set by

an initial correlation length ξ in the fast limit,

a scale when adiabaticity breaks down, E_{kz} ***,***
in the slow limit.

CCP

EE in fast CCP

Plot of EE in fast CCP

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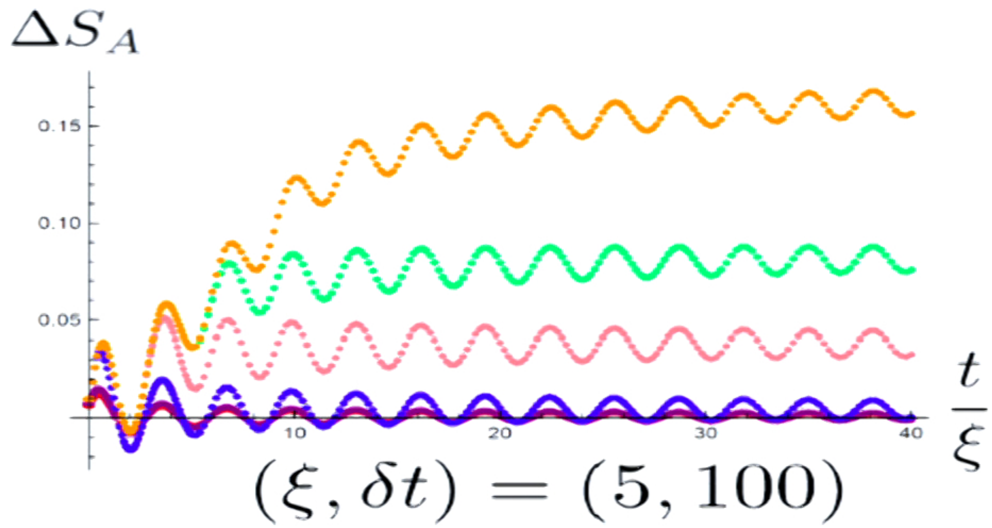

Plot of EE in fast CCP

Orange Curve: l=2000, Green Curve: l=1000, Pink Curve: l= 500,

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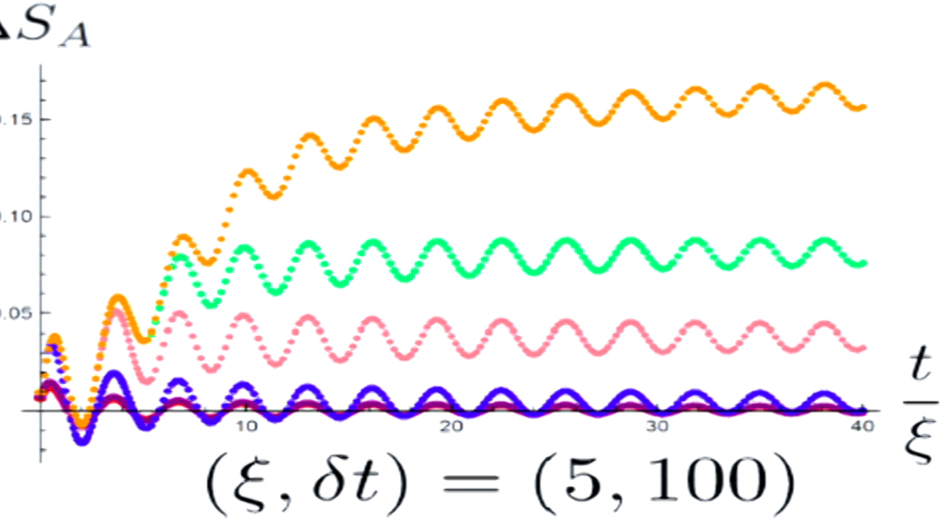
- **If** $t \gg l/2$,

$$\Delta S_A \sim -\omega^2 \log(\omega) \times \frac{l}{\xi}$$



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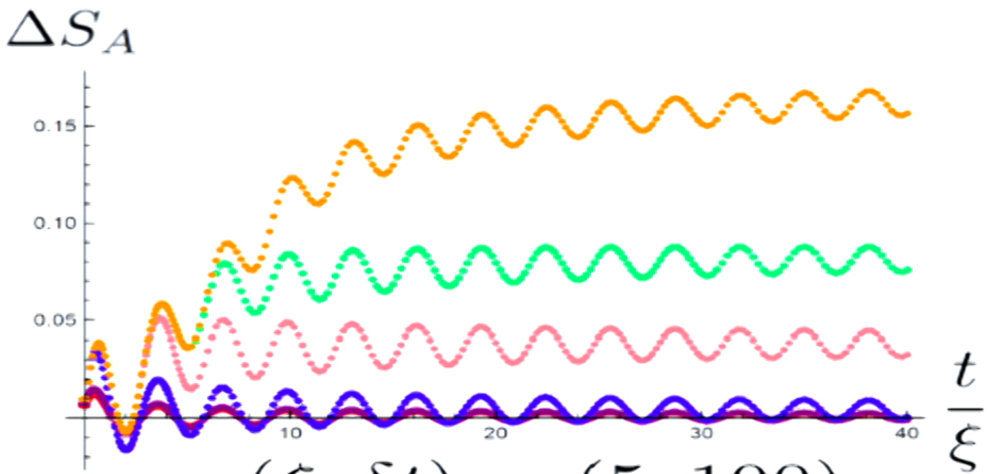
- If $t \gg l/2$,

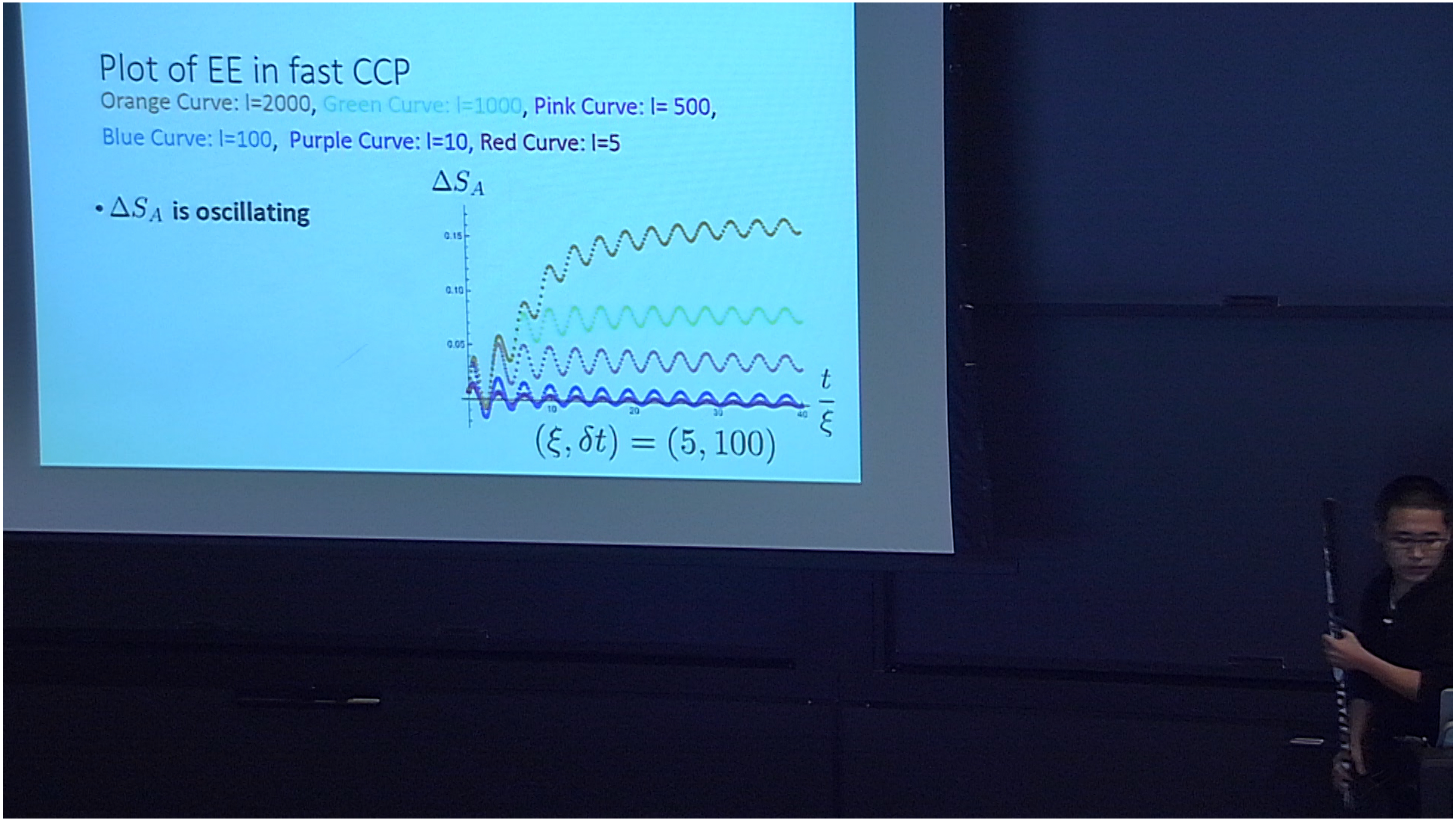
$$\Delta S_A \sim -\omega^2 \log(\omega) \times \frac{l}{\xi}$$

If ω ***decreases (*** ξ ***is fixed),***

$-\omega^2 \log(\omega)$ ***decreases.***



$(\xi, \delta t) = (5, 100)$


Plot of EE in fast CCP
Orange Curve: l=2000, Green Curve: l=1000, Pink Curve: l= 500,
Blue Curve: l=100, Purple Curve: l=10, Red Curve: l=5
• ΔS_A is oscillating
ΔS_A
0.15
0.10
0.05
10
20
30
40
$\frac{t}{\xi}$
$(\xi, \delta t) = (5, 100)$

Plot of EE in fast CCP

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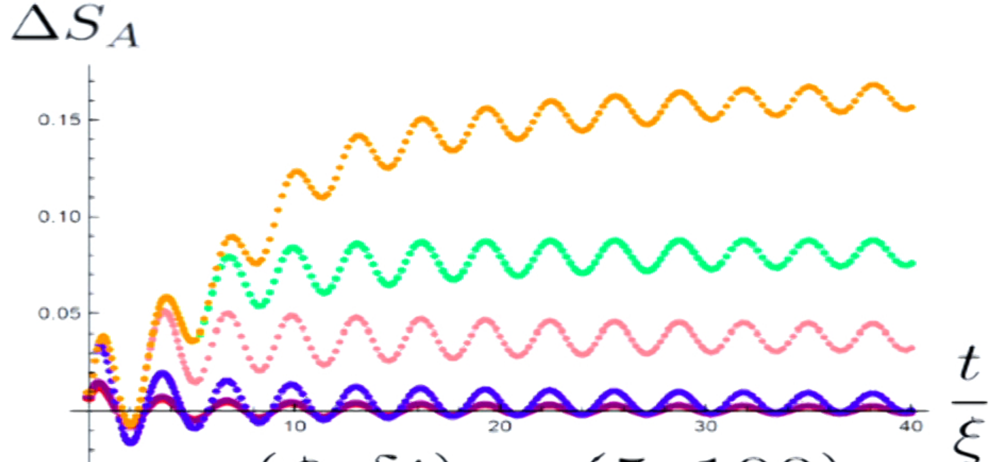

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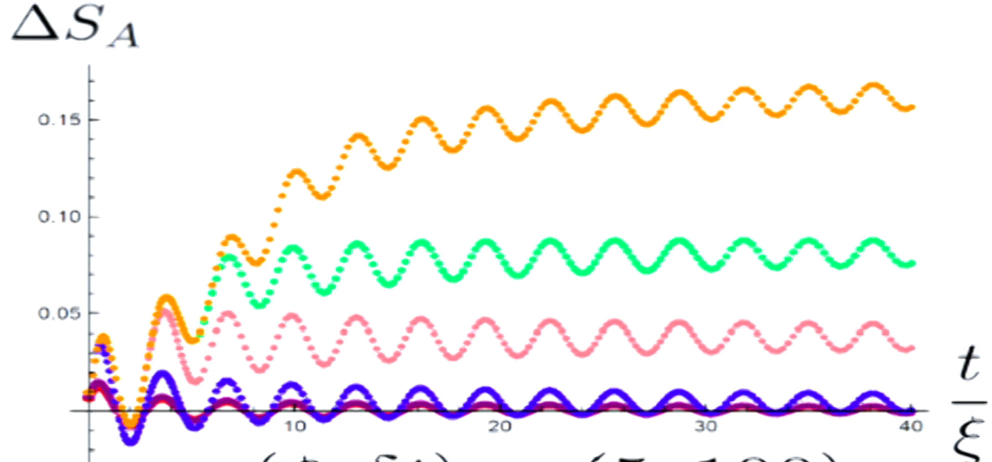
- **Frequency is determined by final mass.**

$$periodicity \sim \pi\xi$$



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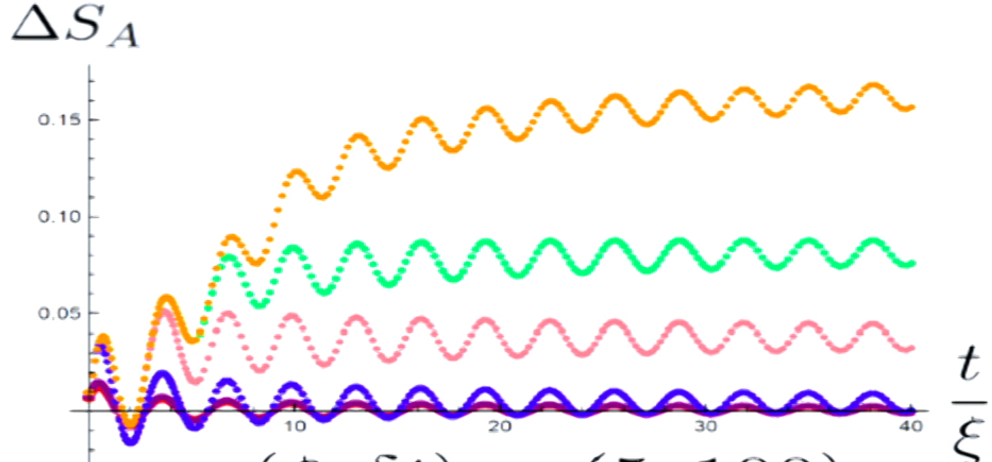
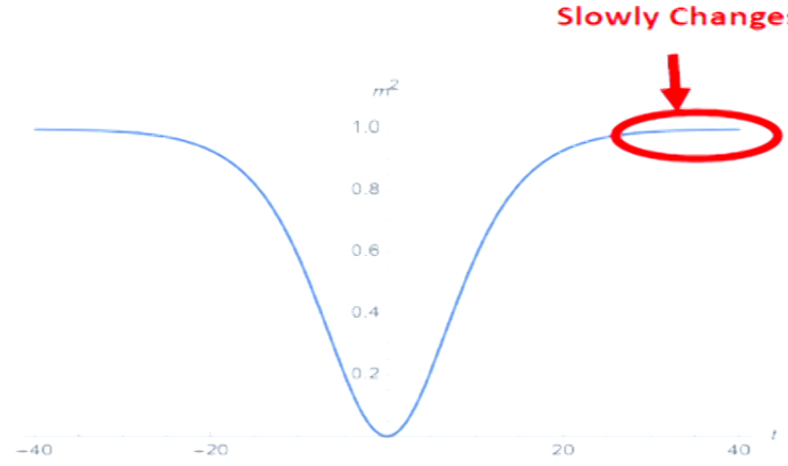


$$(\xi, \delta t) = (5, 100)$$

Oscillation

- **In the late time, the mass profile slowly changes.**

- **Physical quantities can be computed adiabatically.**



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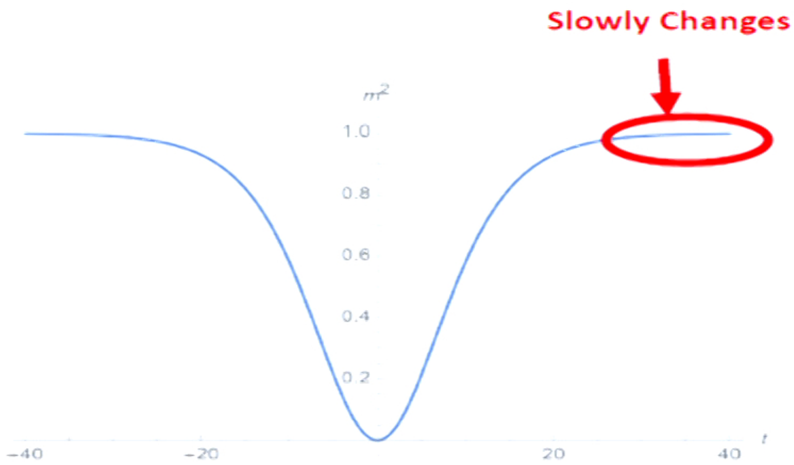
- **Physical quantities can be computed adiabatically.**

$$v_k = \partial_k \omega_k \qquad \omega_k = \sqrt{4\sin^2\left(\frac{k}{2}\right) + m_f^2}$$

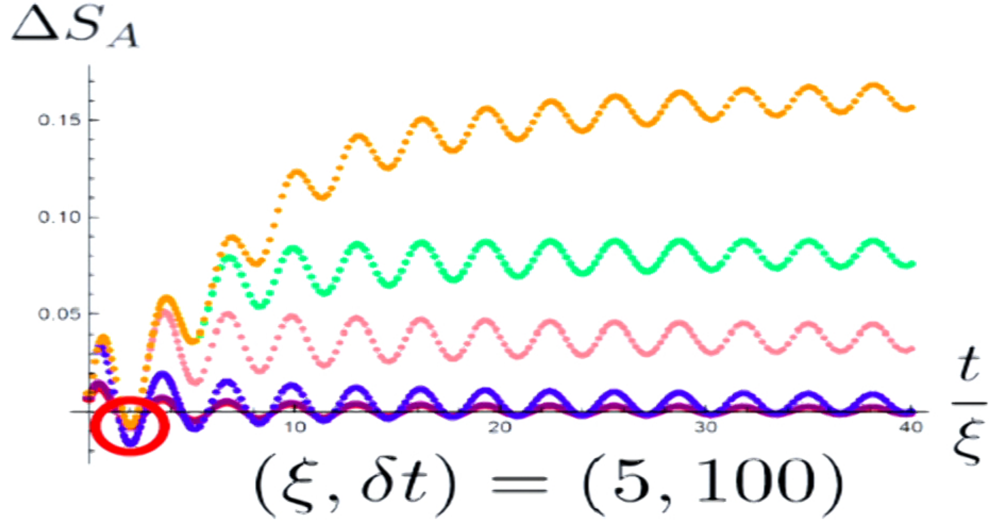
- **As in ECP, in the late time, slow mode (zero mode) contribute to ΔS_A.**

zero mode: $e^{-i\omega_k t} \sim e^{-im_f t}$



Minimum Value

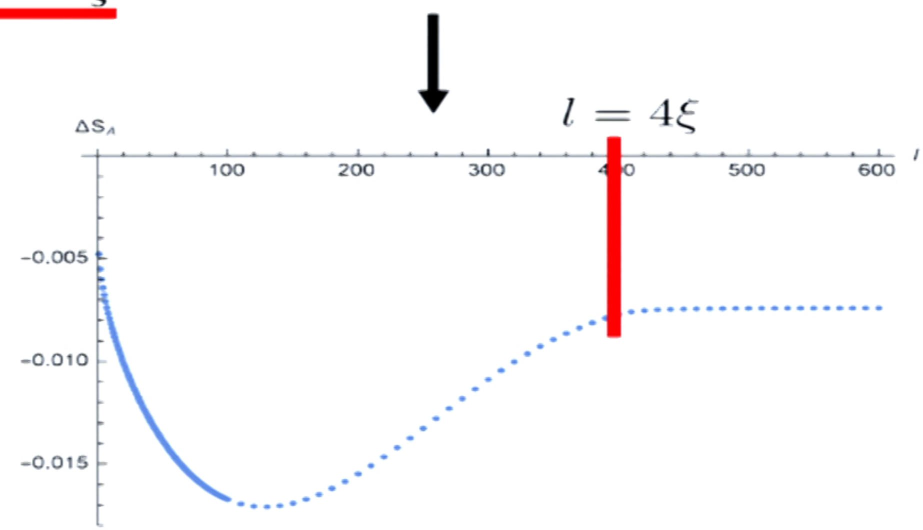
- Minimum value of ΔS_A is at $t = 2\xi$.



Minimum Value

- Minimum value of ΔS_A is at $t = 2\xi$.
- Around $l = \xi$,
  ΔS_A is ***minimized***.
- Around $l = 4\xi$,
  ΔS_A is constant.

The plot for *l-dependence of* ΔS_A *at* $t = 2\xi$



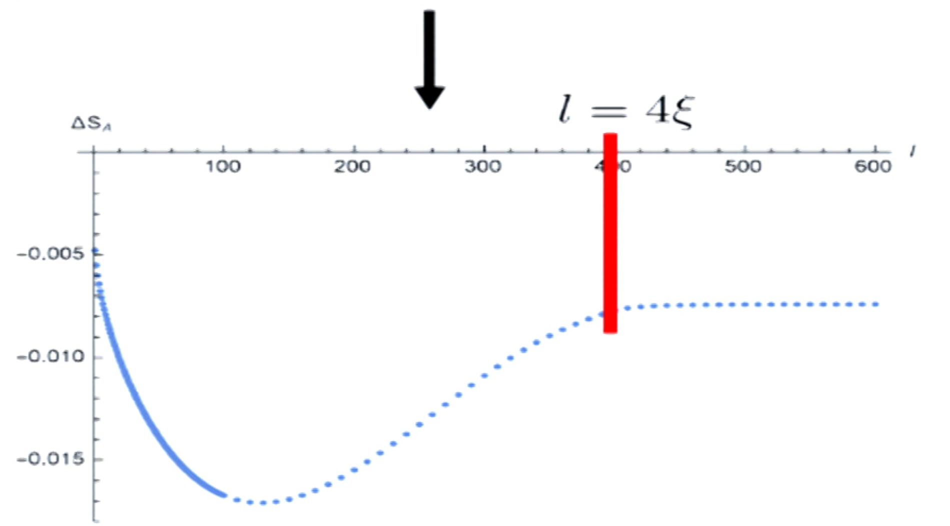
Minimum Value

- Around $l = 4\xi$, ΔS_A is constant.

Initially, the blue region of the subsystem ***A*** is entangled with the complemental region.

$l > \xi$, S_A is constant.

The plot for *l-dependence of* ΔS_A *at* $t = 2\xi$



Minimum Value

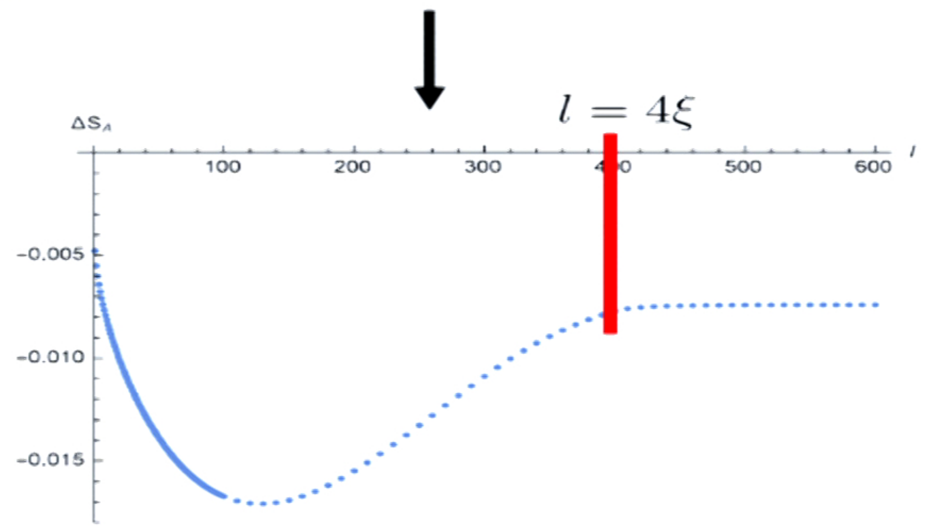
- Around $l = 4\xi$, ΔS_A is constant.

Entangled particle picture

At $t = 2\xi$, the blue region of the subsystem ***A*** is entangled with the complemental region.

$l > 4\xi$, ΔS_A is constant (<0).

The plot for *l-dependence of* ΔS_A *at* $t = 2\xi$



Minimum Value

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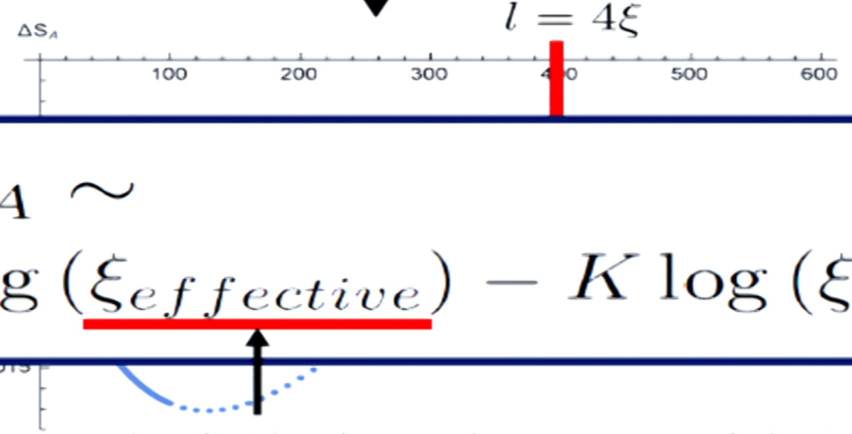

$$\Delta S_A \sim K \log\left(\xi_{effective}\right) - K \log\left(\xi\right)$$

related with a distance between entangled pair.

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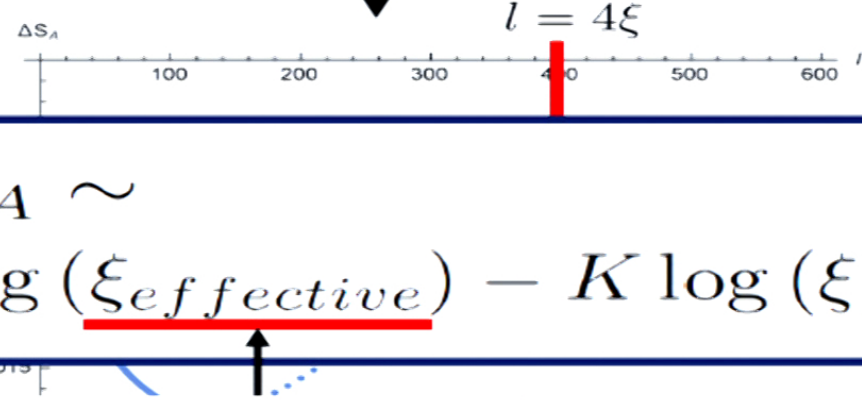


$$\Delta S_A \sim K \log(\xi_{effective}) - K \log(\xi)$$

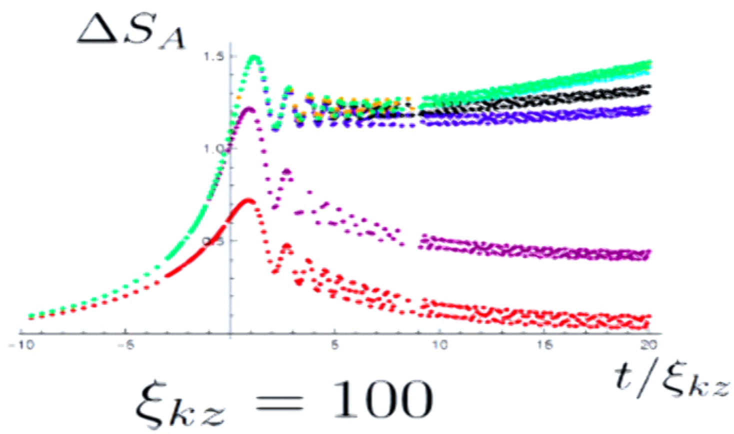
$$\xi_{effective} < \xi$$

EE in slow CCP

Plot of EE in Slow CCP

Green Curve: l=3000, Orange Curve: l=2500, Black Curve: l= 1000,

Blue Curve: l=500,Purple Curve: l=100, Red Curve: l=10



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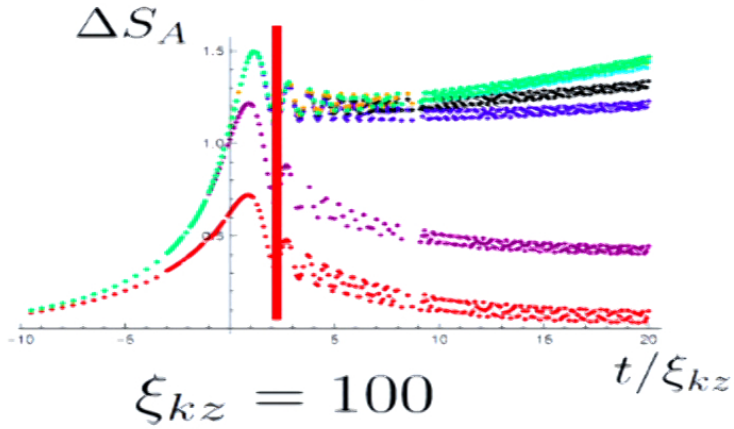
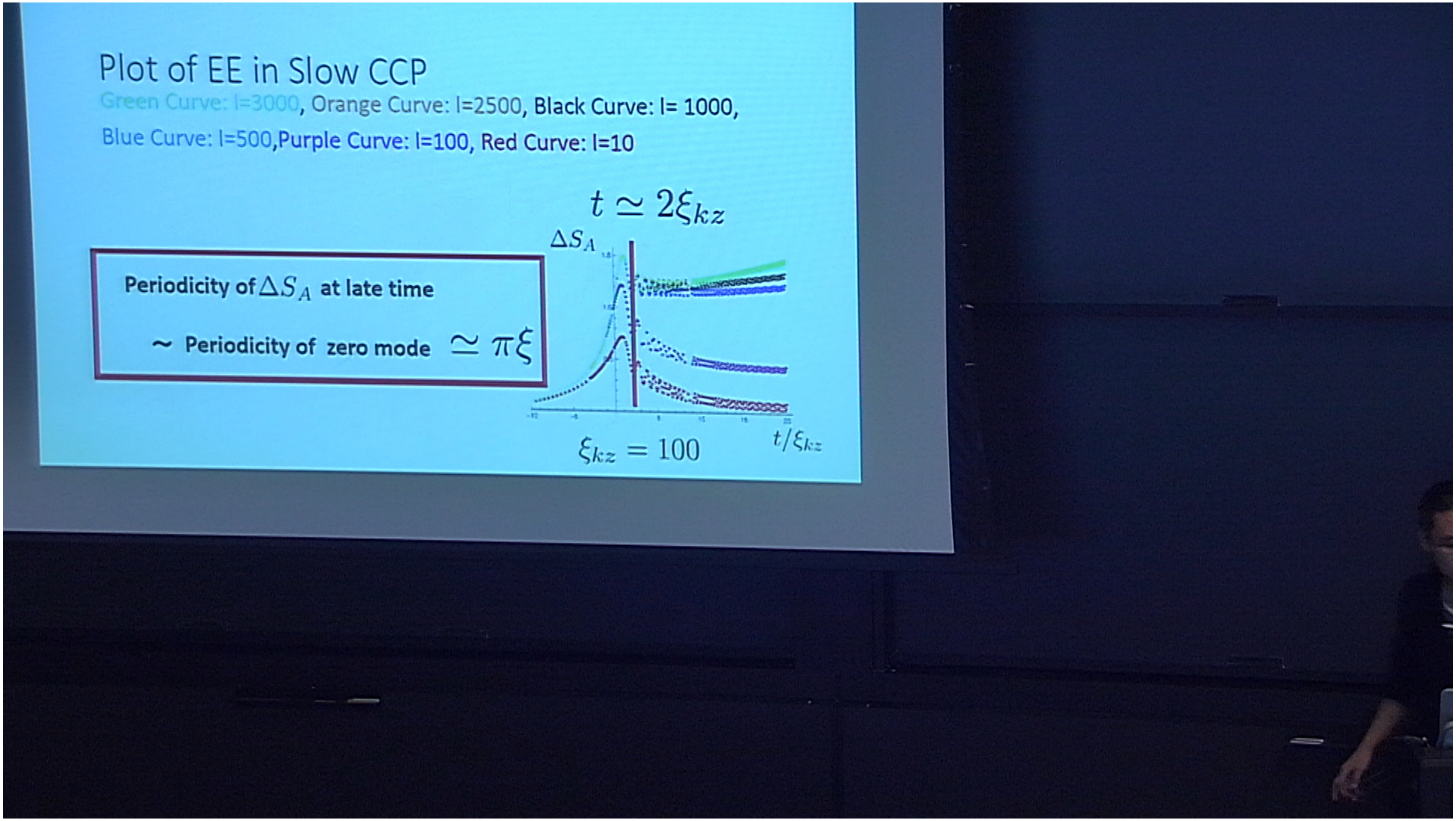
After $t = 2\xi_{kz}$ **,**

ΔS_A **starts to oscillate.**

$t \simeq 2\xi_{kz}$

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$t \simeq 2\xi_{kz}$
ΔS_A
Periodicity of ΔS_A at late time
~ Periodicity of zero mode $\simeq \pi\xi$
$\xi_{kz} = 100$
t/ξ_{kz}

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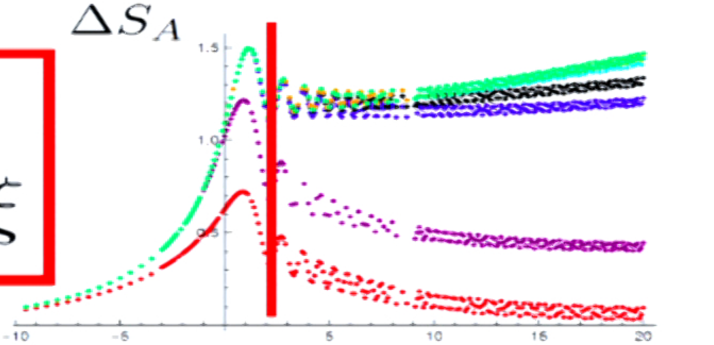


$\xi_{kz} = 100$

$$\Delta S_A(t = 2\xi_{kz})$$

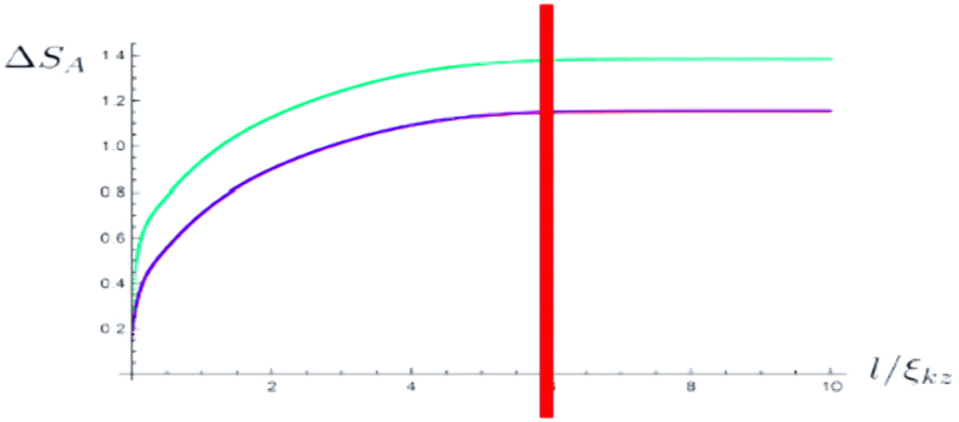
- $l > 6\xi_{kz}$

ΔS_A is a constant (>0).

Red: $(\omega, \xi_{kz}) = (100, 100)$

Blue: $(\omega, \xi_{kz}) = (100, 200)$

Green: $(\omega, \xi_{kz}) = (400, 200)$



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- Entangled particle interpretation

Adiabaticity breaks down.

@ $t \sim -t_{kz}$

Entangled particles are created.

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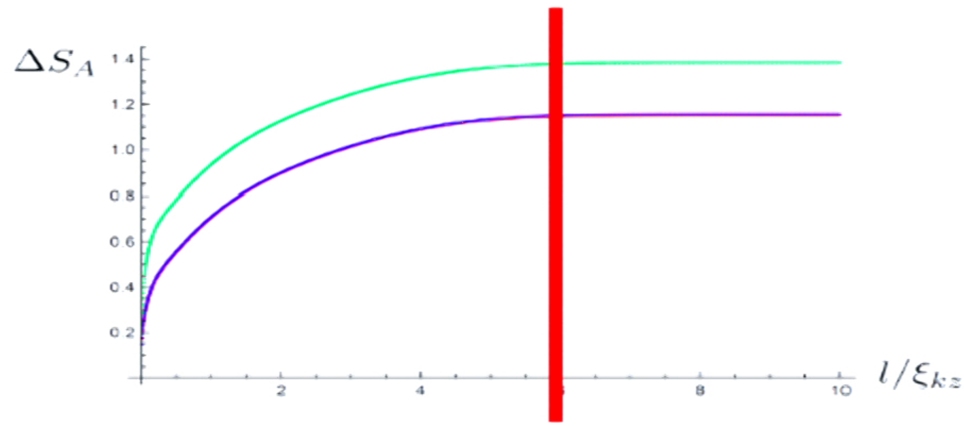

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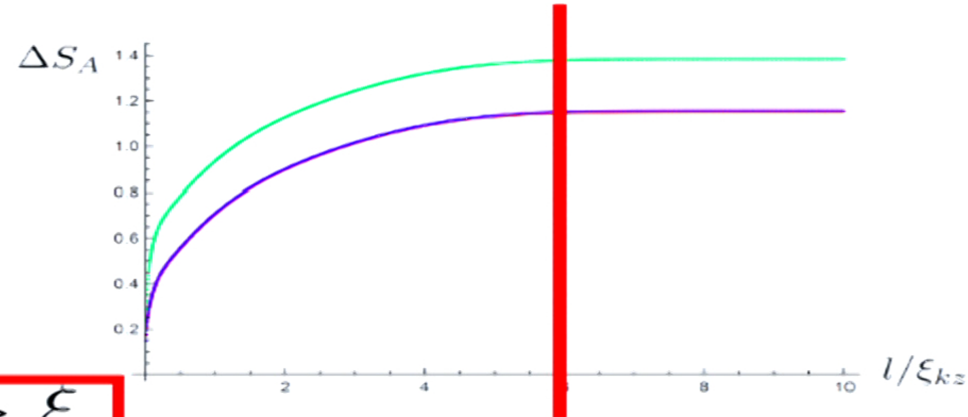
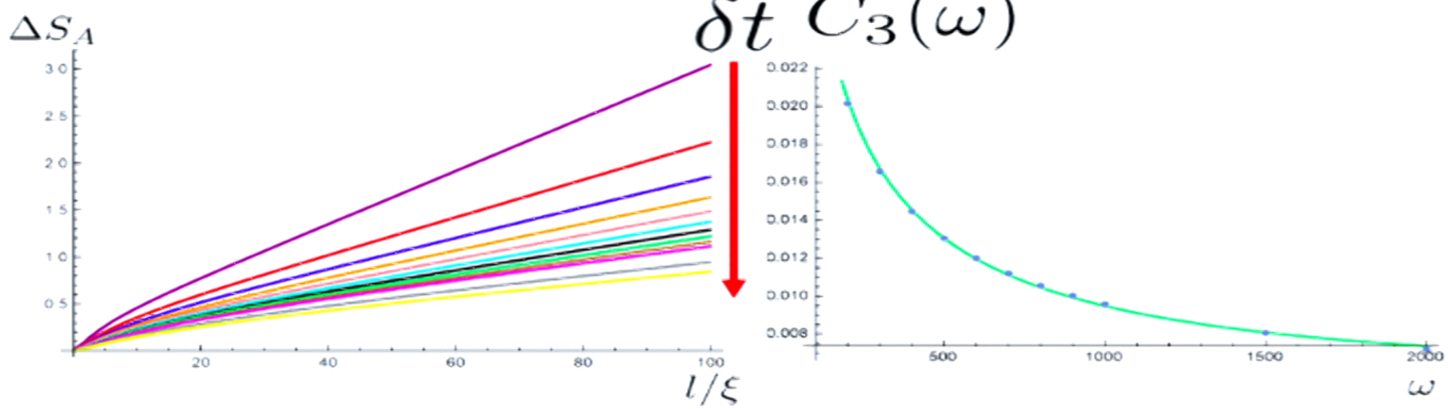


$$l = 6\xi_{kz}$$

Late time in CCP $(\xi, t) = (10, 1000000)$



ΔS_A Is fitted by $\Delta S_A \sim \frac{C_3(\omega)}{\xi} l + B$

Late time in CCP

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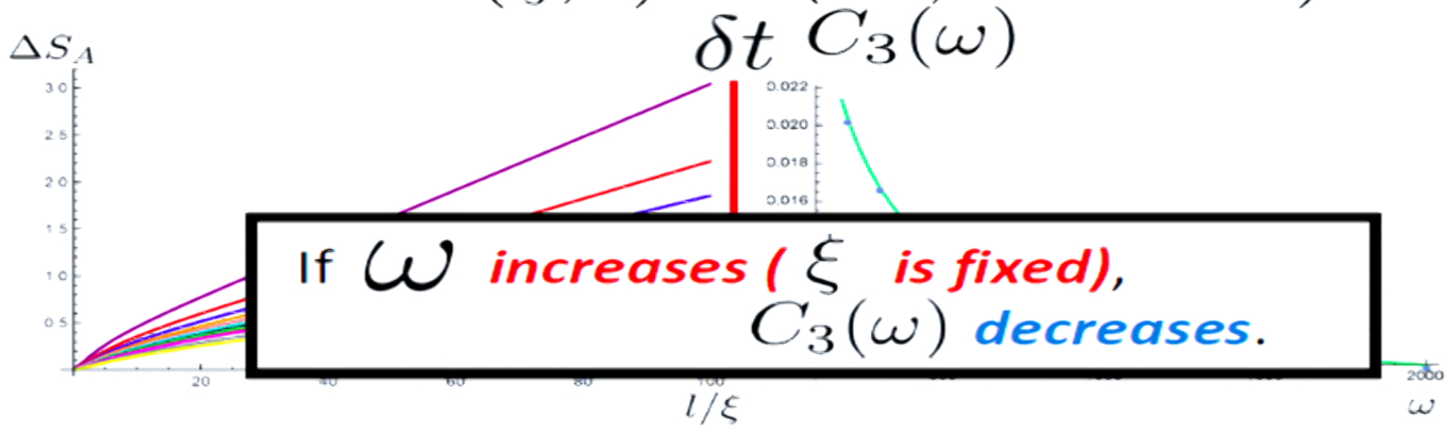


If ω ***increases (*** ξ ***is fixed),***
$C_3(\omega)$ ***decreases.***

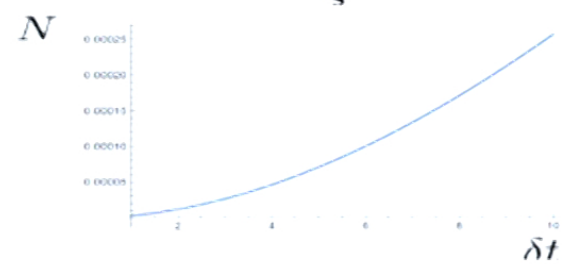
ΔS_A Is fitted by $\Delta S_A \sim \frac{C_3(\omega)}{\xi} l + B$

Volume law in Fast and slow limits

- In the fast limit , Fitting function: $\Delta S_A \sim -\omega^2 \log(\omega) \times \frac{l}{\xi}$
- In the slow limit, Fitting function: $\Delta S_A \sim \frac{C_3(\omega)}{\xi} l + B$

N is the number operator/ Volume at late time. $\xi = 100$

If ω ***decreases (ξ is fixed),***
$-\omega^2 \log(\omega)$ ***decreases.***



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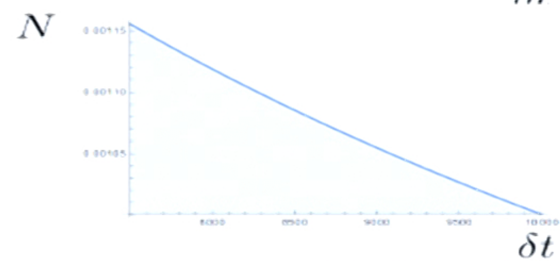

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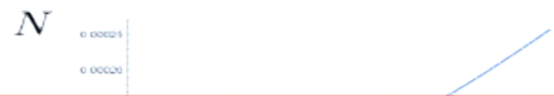
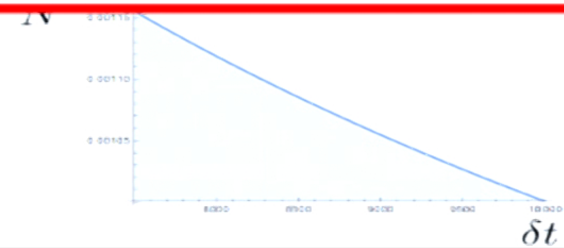


If ω *decreases (ξ is fixed),*

The behavior of entanglement entropy at late time is consistent with the behavior of number operator at late time.



If ω ***increases (ξ is fixed),*** $C_3(\omega)$ ***decreases.***

Summary

- We study what makes entangled particles.

→ ***Diadiabaticity plays an important role.***

- Scaling of EE depend on scales
  when adiabaticity breaks down.

- Late time behavior depends on slow mode (zero mode).

Future directions

- Why does change of EE oscillate after $t = 2\xi_{kz}$, $t = 2\xi$?
- Interacting theories
- Holographic Dual
- Floquet type potential