Title: The Big Data Approach to Quantum Gravity

Date: Dec 14, 2017 02:30 PM

URL: http://pirsa.org/17120015

Abstract: In both Causal Set Quantum Gravity as well as in the String Landscape, we face the challenging tasks of sifting through large state spaces and searching for the set of solutions which best model our physical universe. I demonstrate in this talk how efficient parallel algorithms can give us access to areas of physics previously unstudied due to computational barriers. I first present new methods to accelerate the evolution of causal set Markov chains, which enables us to look for the spontaneous emergence of manifoldlike structure. Then, I use a similar graph theoretic approach to show how a dynamic vacuum selection mechanism naturally emerges in F-Theory and Type IIB String Theory in the context of eternal inflation. We will discuss in detail relevant numerical aspects, including practical limitations and algorithms from graph theory, and how machine learning can be used in future work.

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The Big Data Approach to Quantum Gravity

Will Cunningham

Department of Physics Network Science Institute Northeastern University

December 14, 2017







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- Produce large data sets
- Analyze large data sets



Image: Wikipedia



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Basic Concepts

What is Big Data?

Big Data refers to several problems:

- Produce large data sets
- Analyze large data sets
- Machine learning to find patterns

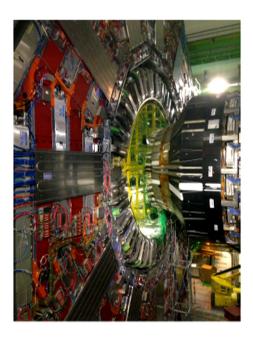


Image: Wikipedia



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Basic Concepts

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Big Data refers to several problems:

- Produce large data sets
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- Machine learning to find patterns
- 3Vs: Volume, Velocity, and Variety

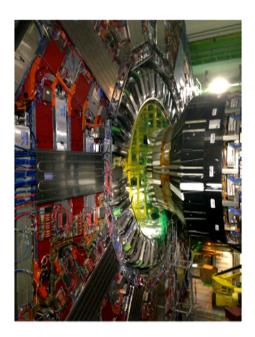


Image: Wikipedia



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Basic Concepts

What is Big Data?

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- Produce large data sets
- Analyze large data sets
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- 3Vs: Volume, Velocity, and Variety

Big graphs are interesting

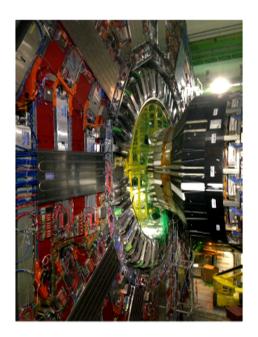


Image: Wikipedia



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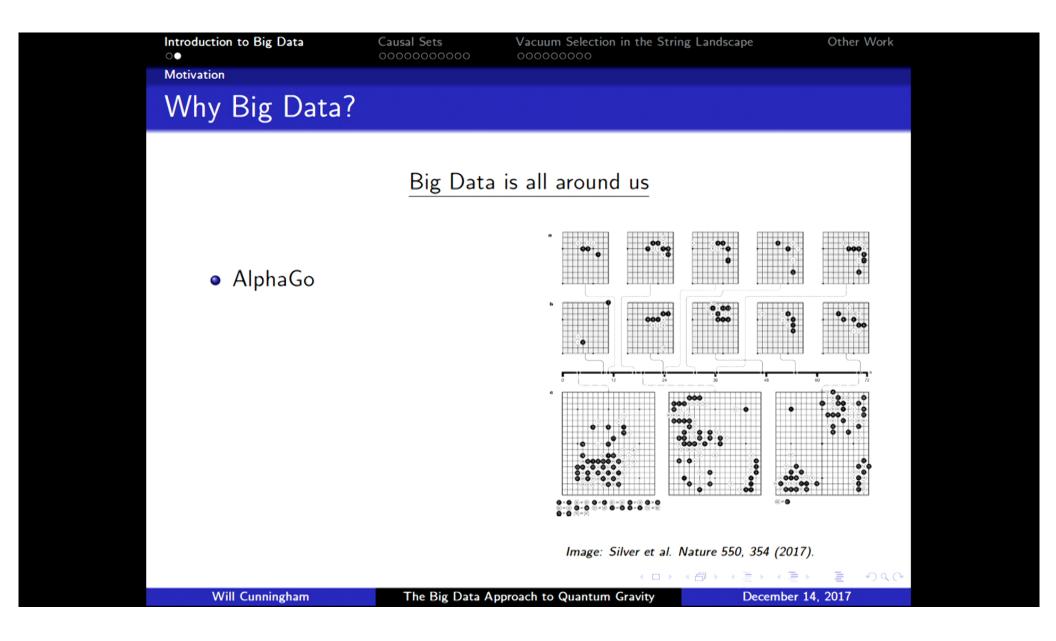
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• String Landscape

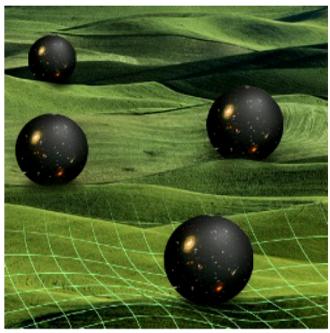


Image: Huffington Post

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Motivation

Why Big Data?

Big Data is all around us

- AlphaGo
- Tensor Networks
- Causal Sets
- String Landscape

We need efficient algorithms to study large complex systems



Image: Huffington Post



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Overview

Causal Set Quantum Gravity

Premise: Discrete spacetime with causal relations

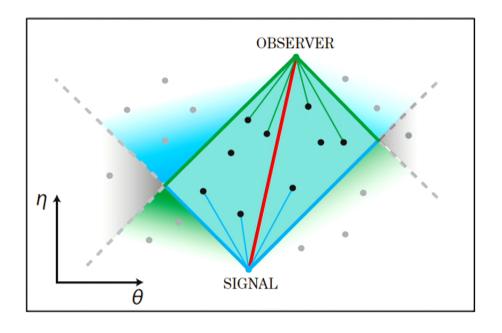


Image: Cunningham & Krioukov, arXiv:1709.03013 (2017).

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Overview

Path Integral Approach

Problem: How do we measure observables?

$$Z\sim\int \mathcal{D}[g_{\mu
u}]e^{iS[g_{\mu
u}]/\hbar}$$

Causal Sets:
$$\int \mathcal{D}[\mathsf{g}_{\mu\nu}]
ightarrow \sum_{\mathcal{C}}$$

$$Z = \sum_{\mathcal{C}} e^{iS(\mathcal{C})/\hbar}$$



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Overview

Path Integral Approach

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Causal Sets:
$$\int \mathcal{D}[\mathsf{g}_{\mu\nu}]
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$$Z = \sum_{\mathcal{C}} e^{iS(\mathcal{C})/\hbar}$$

New Problems:

- 1) How do we deal with $\sum_{\mathcal{C}}$?
- 2) What is S(C) for a causal set?



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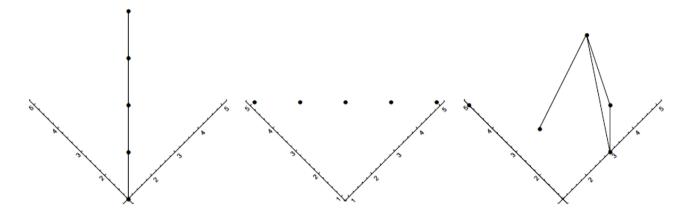
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Overview

2D Orders in M²

Consider 2D partial orders built from total orders U, V

$$U = \{u_1, \dots, u_N\}$$
 $V = \{v_1, \dots, v_N\}$



 $\Omega(N)$: All permutations of U and V

$$\sum_{\mathcal{C}} e^{iS(\mathcal{C})/\hbar}
ightarrow \sum_{\mathcal{C}} e^{-eta S(\mathcal{C})}$$

Brightwell, Henson & Surya, Class. Quantum Grav 25 (2008)

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Causal Sets

Vacuum Selection in the String Landscape

Other Work

Overview

The Causal Set Action

The Benincasa-Dowker action S_{BD} converges to the Einstein-Hilbert action S_{EH} as $N \to \infty$

- There's a different expression for each dimension
- ullet S_{BD} is a function of subgraphs in ${\cal C}$
- Computation is $O(N^3)$
- Boundary terms not included

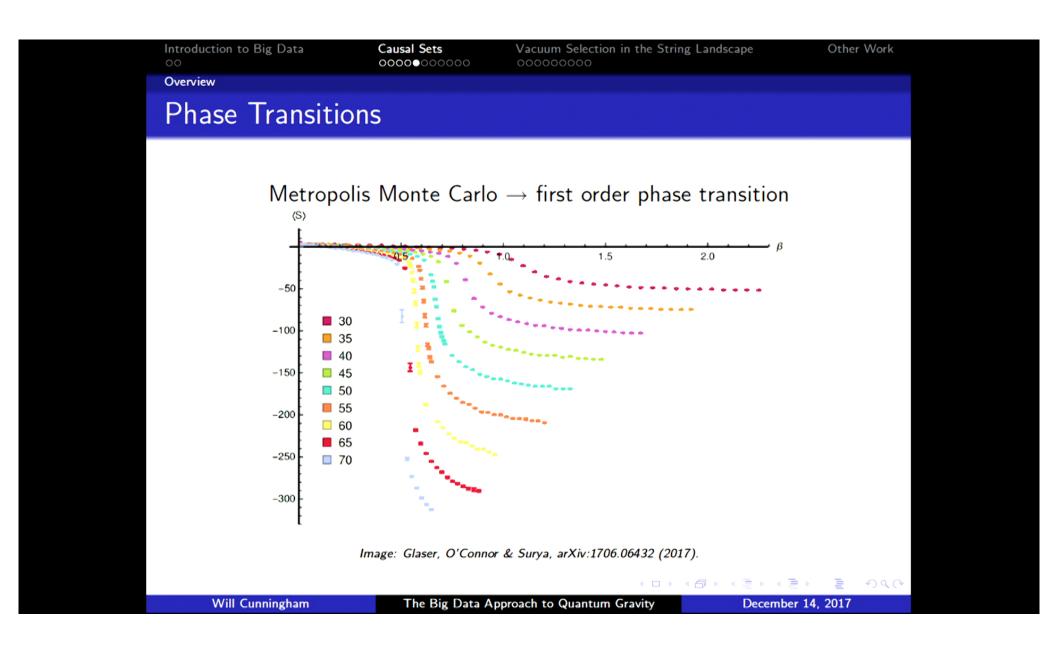
2D:
$$S_{BD} = 2(N - 2n_1 + 4n_2 - 2n_3)$$

Benincasa & Dowker, Phys. Rev. Lett. 104, 181301 (2010). Benincasa, Ph.D. Thesis, Imperial College London (2013).



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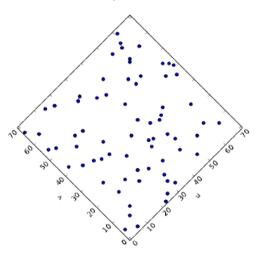
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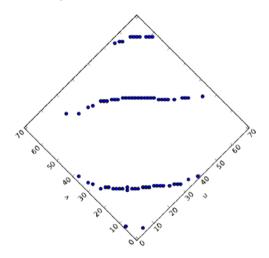
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Overview

Phase Transitions

Metropolis Monte Carlo \rightarrow first order phase transition





Claim: $\bar{\beta_c} \equiv \beta_c N = \text{constant}$

Image: Glaser, O'Connor & Surya, arXiv:1706.06432 (2017).



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Algorithmic Details

Details of the Action

$$Z(N, d, T) = \sum_{C} e^{-\beta S_d(C)/\hbar}$$
 $S_2(C) = 2(N - 2n_1 + 4n_2 - 2n_3)$

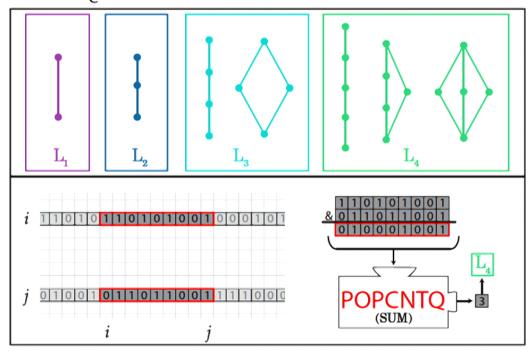


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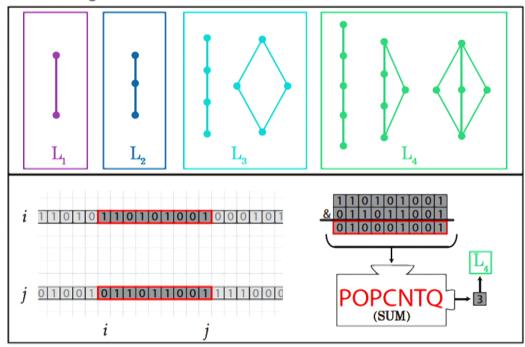


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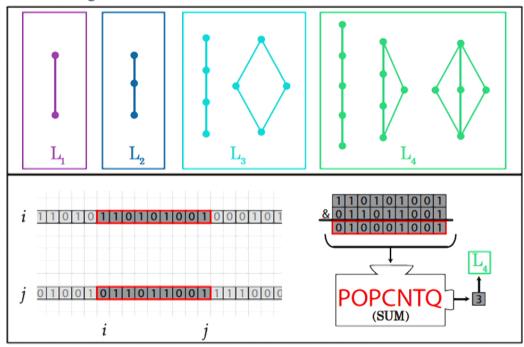


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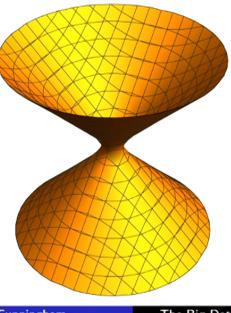
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Algorithmic Details

Graphs with Known Properties

Causal sets can be constructed as **random geometric graphs** in a compact region of spacetime

1+1 De Sitter Slab



Pick manifold, dimension, and compact region

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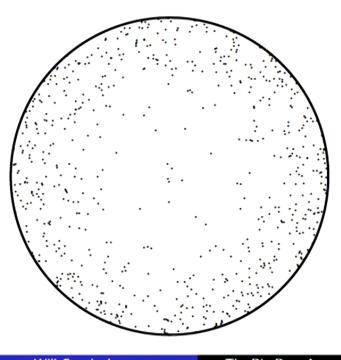
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Algorithmic Details

Graphs with Known Properties

Causal sets can be constructed as **random geometric graphs** in a compact region of spacetime



- Pick manifold, dimension, and compact region
- 2 Poisson sprinkle elements

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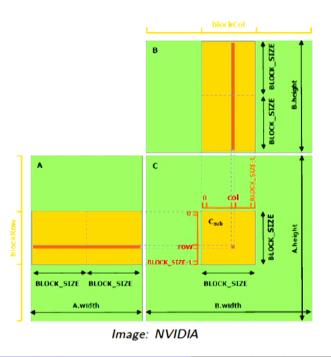
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Algorithmic Details

Graphs with Known Properties

Causal sets can be constructed as **random geometric graphs** in a compact region of spacetime



- Pick manifold, dimension, and compact region
- Poisson sprinkle elements
- Identify links (CUDA) (Use 48KB L1 cache for 10x speedup)

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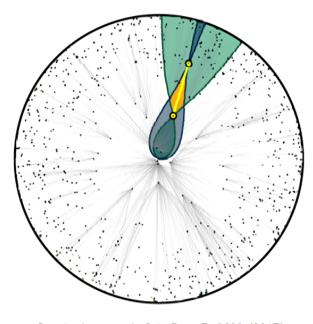
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Algorithmic Details

Graphs with Known Properties

Causal sets can be constructed as **random geometric graphs** in a compact region of spacetime



- Pick manifold, dimension, and compact region
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- Identify links (CUDA)
- Discard coordinates

Image: Cunningham et al. Sci. Rep. 7, 8699 (2017).



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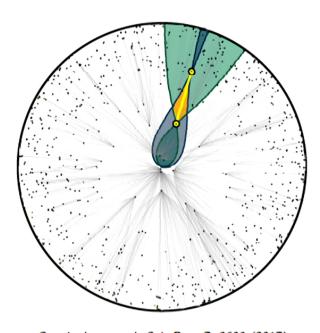
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Algorithmic Details

Graphs with Known Properties

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This is the control group

Image: Cunningham et al. Sci. Rep. 7, 8699 (2017).

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Image: Cunningham & Krioukov, arXiv:1709.03013 (2017).

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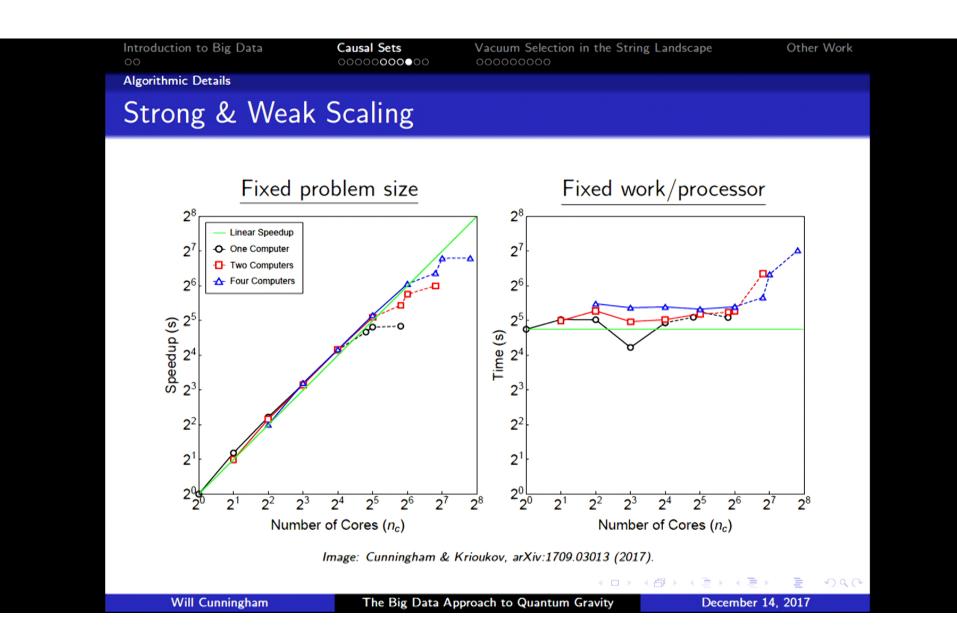
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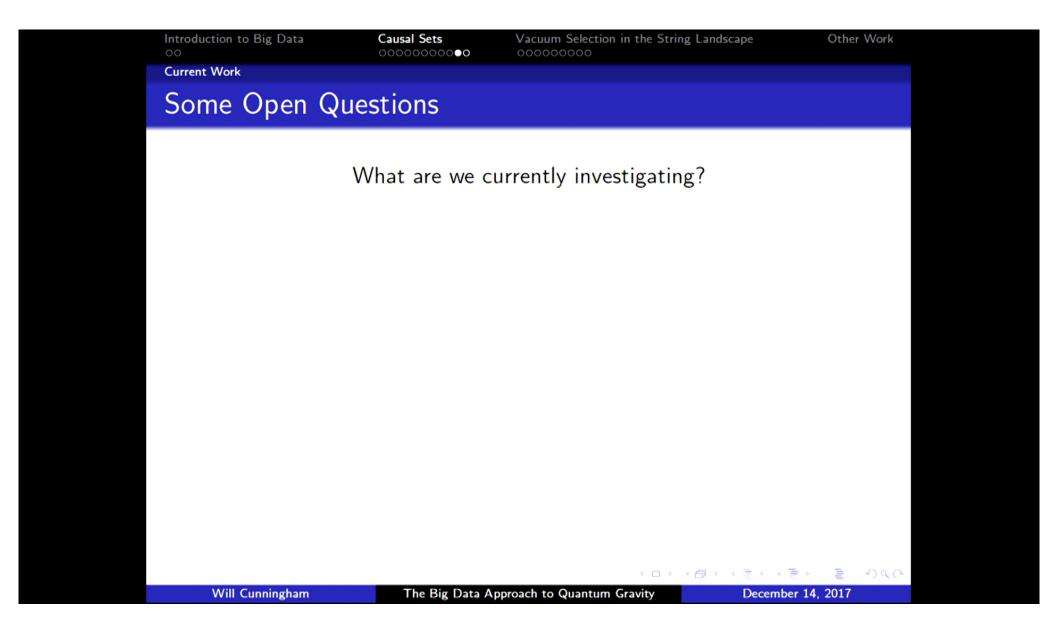
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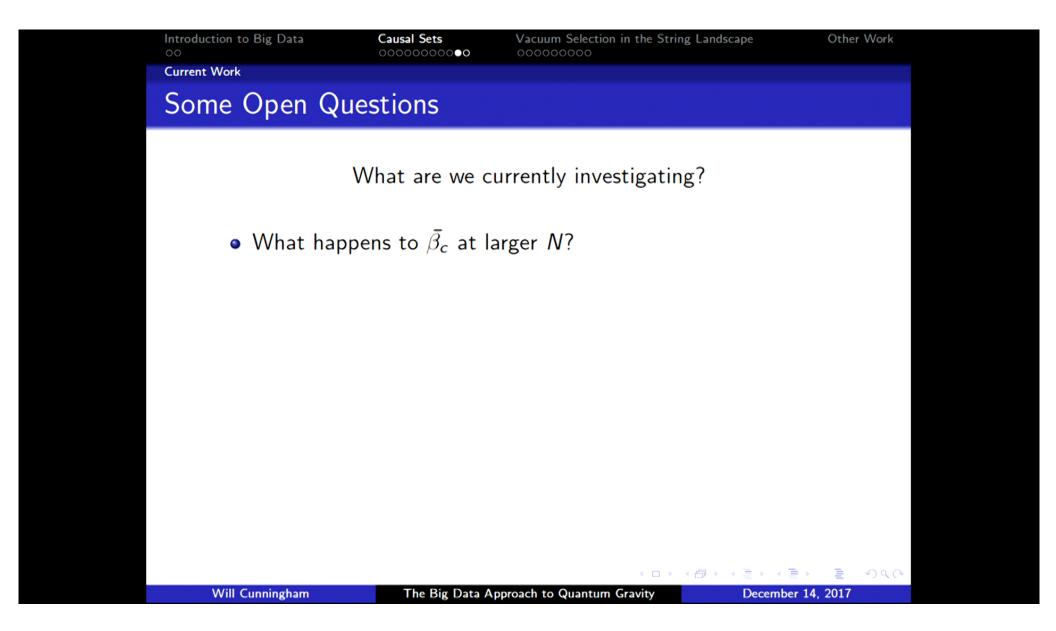
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Other Work

What are we currently investigating?

- What happens to $\bar{\beta_c}$ at larger N?
- How "manifoldlike" are causal sets in the high-temperature phase? How do we even answer this question?
- Do we find transitions for different topologies e.g. cylinder?



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Causal Sets

Vacuum Selection in the String Landscape

Other Work

Current Work

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Cannot answer these without high performance algorithms to compute the action!

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Vacuum Selection in the String Landscape

Other Work

Current Work

Machine Learning

Newer: Classification of Geometric Structure

Supervised Learning:

- Training data → Random geometric graphs
- Construct a deep neural network (DNN)
- ullet Classify according to spacetime dimension in \mathbb{M}^d
- Use DNN to monitor Markov chain properties efficiently

Reinforcement Learning:

- Grow causal sets while minimizing the action
- Use SL network to monitor growth
- Relate to Classical Sequential Growth
- Develop equations of motion

Collaboration: Benincasa, [Charles River Analytics], [Evans], [Northeastern]

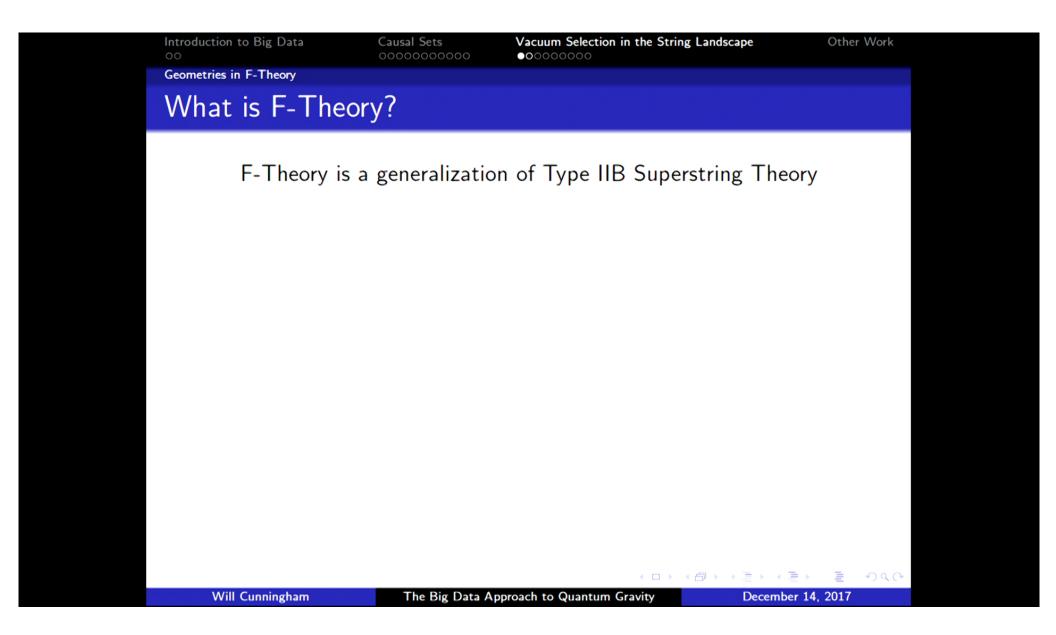
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Geometries in F-Theory

What is F-Theory?

F-Theory is a generalization of Type IIB Superstring Theory

- \bullet 12-D algebraic variety compactified on a 2-torus \rightarrow 10-D Type IIB String Theory
- This leaves 6 compactified spatial dimensions (Calabi-Yau elliptic fibrations over toric 3-fold)

$$y^2 = x^3 + f(z_1, z_2)x + g(z_1, z_2)$$



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ullet 3-folds (\mathbb{C}^3) are triangulated 3-D reflexive polytopes (4319)

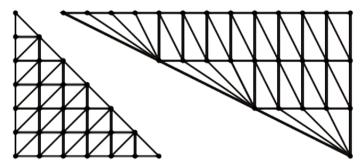


Image: Halverson et al. arXiv:1706.02299 (2017).

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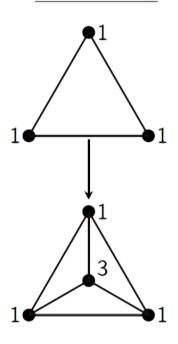
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Geometries in F-Theory

"Simple" Topological Transitions

Two 3-folds are related by a **blowup**; # **cones** += 2 Face Blowup



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Introduction to Big Data

Causal Sets

Other Work

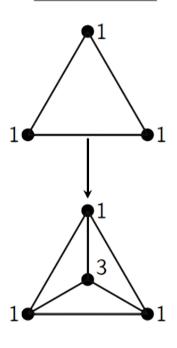
Geometries in F-Theory

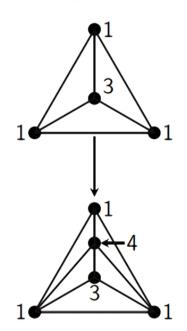
"Simple" Topological Transitions

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Face Blowup

Edge Blowup





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Causal Sets

Other Work

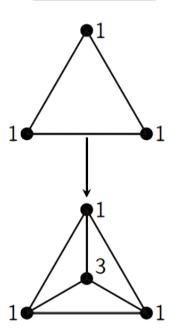
Geometries in F-Theory

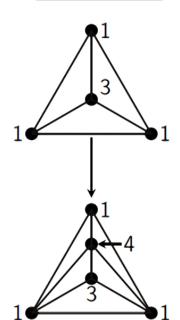
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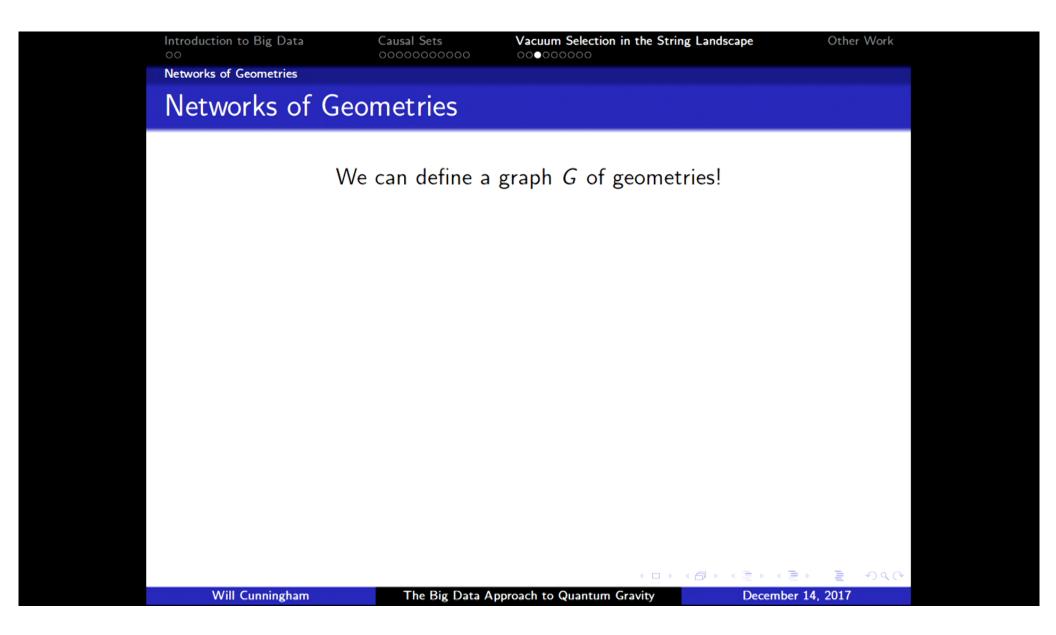


Height Restriction: $H \leq 6$

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Networks of Geometries

Networks of Geometries

We can define a graph G of geometries!

- A single face permits 41,873,645 geometries (3-folds)
- An edge between faces permits 82 geometries
- Combinatorics implies $\frac{4}{3} \times 2.96 \times 10^{755}$ is the lower bound¹

¹Halverson, Long & Sung, arXiv:1706.02299 (2017)



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Causal Sets

Vacuum Selection in the String Landscape ○●○○○○

Other Work

Networks of Geometries

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- Combinatorics implies $\frac{4}{3} \times 2.96 \times 10^{755}$ is the lower bound¹
- The entire network is a cartesian product of smaller ones
- Our adjacency matrix A contains transition rates Γ
- Transitions make sense when we think about bubble cosmology in eternal inflation

¹Halverson, Long & Sung, arXiv:1706.02299 (2017)



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Causal Sets

Vacuum Selection in the String Landscape ○○○●○○○

Other Work

Relation to Cosmology

Modeling Bubble Nucleation

Question: What is the distribution of vacua at $t \to \infty$?

Nucleation Rate: $\frac{d\mathbf{N}}{dt} = \mathbf{\Gamma}\mathbf{N}$

Solution: $\mathbf{N} = e^{\mathbf{\Gamma}t} \mathbf{N}_0 = \sum_{p} a_p e^{\gamma_p t} \mathbf{v}_p$

Late-Time Solution: $\mathbf{N} \to a_0 e^{\gamma_0 t} \mathbf{v}_0$ Normalize: $\mathbf{p} = \mathbf{N}/|\mathbf{N}|$

Toy Model: $\Gamma = A$

Perron-Frobenius Theorem: $p_i > 0 \ \forall i \text{ if } G \text{ is connected}$



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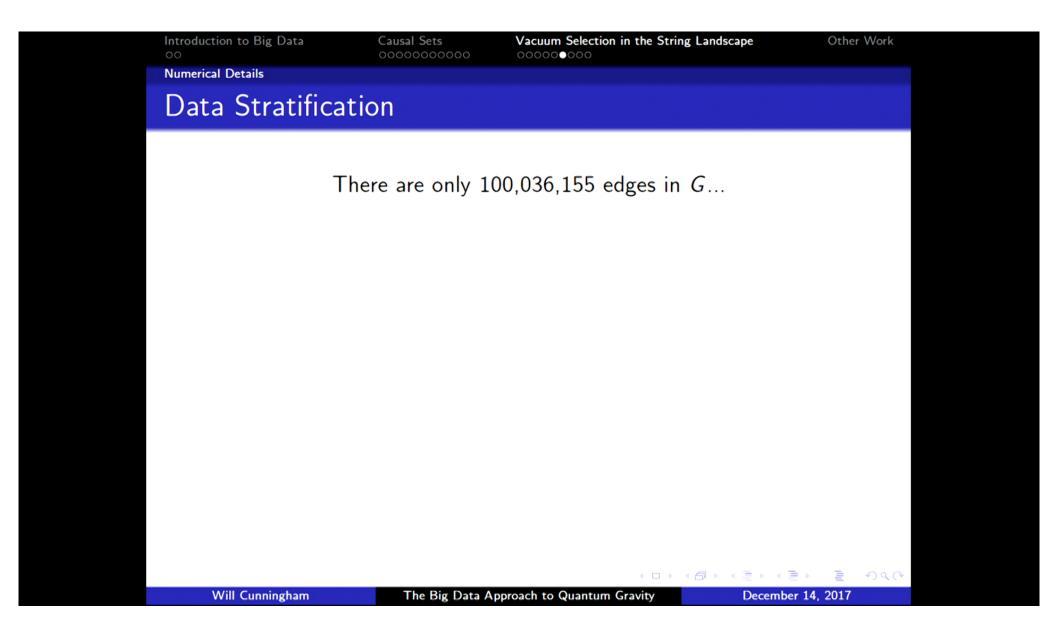
But to use **A** would require \sim 210 TB memory...



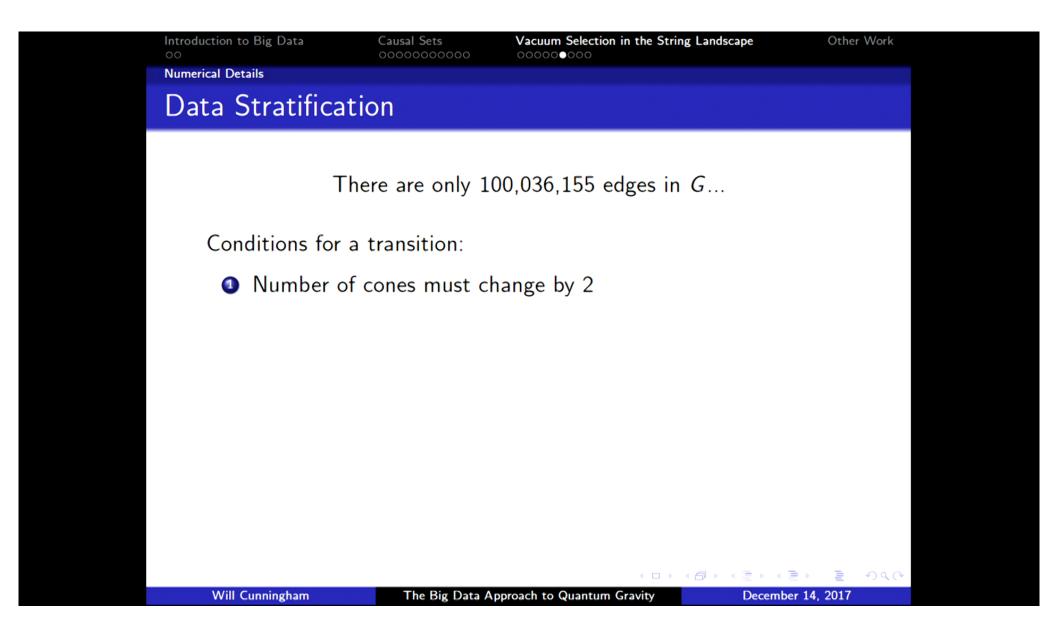
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Allowed blowups stored in a lookup table



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Numerical Details

Data Stratification

There are only 100,036,155 edges in G...

Conditions for a transition:

- Number of cones must change by 2
- Vertex sets differ by a single element
- Outer vertices modified appropriately
- 4 Allowed blowups stored in a lookup table

Speedup using C with OpenMP + AVX + Assembly: Over 100x 3.5 months \rightarrow 22 hours for the largest subset of data (20%)



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Vacuum Selection in the String Landscape ○○○○●○○

Other Work

Numerical Details

Data Stratification

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Same low-level code used here and for the causal set action!



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The Selection Mechanism

The Eigenvector Centrality

We compute the eigenvector centralities of the (sparse) graph

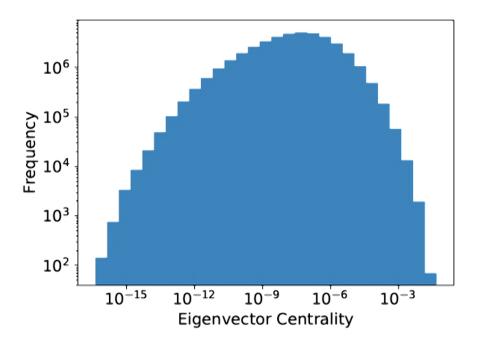


Image: Carifio, Cunningham, Halverson, Krioukov, Long & Nelson, arXiv:1711.06685 (2017)

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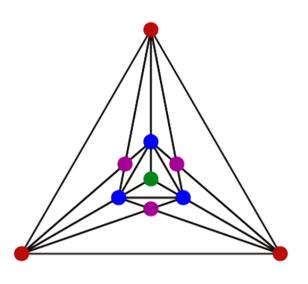
990

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Other Work

The Selection Mechanism

The Selected Vacuum State



Selected Gauge Group: $E_8^3 \times G_2 \times SU(2)^3$

Full 10^{755} Network: $E_8^{37} \times F_4^{85} \times G_2^{220} \times SU(2)^{320}$

Image: Carifio, Cunningham, Halverson, Krioukov, Long & Nelson, arXiv:1711.06685 (2017)



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The Selection Mechanism

Current Work

How good is a toy model?

- It doesn't exclude the MSSM
- The emphasis is on the selection mechanism, evidenced by a non-uniform eigenvector centrality distribution
- We want to include information about fluxes to get real physical answers
- The full solution requires new work in math and physics...
 we can still add information about the number of fluxes –
 possibly up to 10^{272,000} [Taylor & Wang, JHEP 12(2015)164]



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Vacuum Selection in the String Landscape ○○○○○○●

Other Work

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- We might also want to consider bubble collisions/decays
- This is the biggest Big Data problem ever
- We can use deep neural networks to search for the MSSM (Ruehle, Halverson & Nelson)

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