Title: Random Tensor Networks and holographic behaviour in Group Field Theory - Goffredo Chirco

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Abstract: <p>TBA In the framework set by the AdS/MERA conjecture, we investigate a generalisation of the Tensor Network description of bulk geometry in the language of Group Field Theories, a promising convergence of insights and results from Matrix Models, Loop Quantum Gravity and simplicial approaches. We establish a first dictionary between Group Field Theory and Tensor Network states. With such a dictionary at hand, we target the calculation of the Ryu-Takayanagi formula recently derived for Random Tensor Networks in the quantum gravity formalism.</p>

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Random Tensor Networks and holographic behaviour in Group Field Theory

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MAX-PLANCK-GESELLSCHAFT

inclusive framework

in AdS/CFT, the Ryu and Takayanagi (RT) formula, computing entanglement entropy in the CFT by the area of a certain minimal surface in the bulk geometry, has provided an intriguing connection between geometry and entanglement

S. Ryu and T. Takayanagi (2006) V. E. Hubeny, M. Rangamani, and T. Takayanagi, (2007) M. Headrick and T. Takayanagi, (2007) M. V. Raamsdonk, (2009,2010), P. Hayden, M. Headrick, and A. Maloney, (2013), A. Lewkowycz and J. Maldacena, (2013) 090. J. Maldacena and L. Susskind, (2013), N. Lashkari, M. McDermott, and M. Van Raamsdonk, (2014) .G. Vidal, (2003).

interesting new perspectives toward a structural understanding of such connection recently come from a convergence of techniques and insights from holographic duality, condensed matter theory, Quantum Information Theory and discrete approaches to quantum gravity

an important impulse in this sense has been played by an improved understanding of quantum entanglement in the condensed matter physics community and by the use of tensor networks techniques developed to efficiently represent quantum many-body states

G. Vidal, (2007), G. Vidal, (2008) G. Evenbly and G. Vidal, (2009)

in the AdS/CFT correspondence, the emergent radial direction can be regarded as a renormalization scale, and spatial slices have a hyperbolic geometry resembling the exponentially growing tensor networks of MERA. This similarity between AdS/CFT and MERA suggests that some physics of the AdS/CFT correspondence can be modeled by a MERA-like tensor network where quantum entanglement in the boundary theory is regarded as a building block for the emergent bulk geometry

B. Swingle, (2012)

inclusive framework

beyond the AdS/MERA, a new paradigm for studying holographic systems

recent works have introduced toy models of gauge/gravity duality based on quantum error correcting codes, which have been shown to exhibit key properties of the AdS/CFT correspondence such as bulk reconstruction and the Ryu-Takayanagi (RT) relation.

A. Almheiri, (2015) F. Pastawski, B. Yoshida, D. Harlow, and J. Preskill, (2015)
P. Hayden, S. Nezami, X-L. Qi, N. Thomas, M. Walter, and Z.Yang, (2016), D. Harlow, (2016) W. Donnelly, B. Michel, D. Marolf and J. Wien (2017)

such generalised framework for the geometry/entanglement correspondence has raised a lot of interest in the context of non-perturbative quantum gravity where entanglement is supposed to be responsible for the very architecture of space-time at the quantum scale.

Bianchi Myers (2012), Girelli Livine,Terno (2005-08), Donnelly (2010), GC Rovelli Haggard Riello Ruggiero (2014-15) GC Anzà (2016), Han et al. (2016), Yokomizo (2016)

=> recent new efforts to provide an understanding of holographic dualities from a non-perturbative quantum gravity perspective

B.Dittrich,C. Goeller, E. R. Livine, and A. Riello, (2017), Delcamp Dittrich Riello, Geiller (2016-17)

in general, the type of mathematical structures identified by quantum gravity approaches and used in the theory of tensor networks are very similar. consequently, it is very natural to try to put the two frameworks in more direct contact. This is the main goal of this talk why trying to include tensor network formalism in the non-pert quantum gravity formal framework?

> from the quantum gravity point of view,

tensor networks are very effective in controlling the entanglement properties of quantum states in many-body systems. This is exactly the language in which GFT deals with quantum gravity states, the very connectivity of spin network states being associated with entanglement between the fundamental quanta constituting them (associated to nodes).

tensor networks may become central tools in the renormalization analysis of QG models. such renormalization analyses are the main avenue for solving the crucial problem of the continuum limit in such formalism.

S. Carrozza, D. Oriti, and V. Rivasseau, (2014), D. Benedetti, J. Ben Geloun, and D. Oriti, (2015),S. Carrozza and V. Lahoche, (2016), V. Lahoche and D. Oriti, (2017), B. Bahr, B. Dittrich, F. Hellmann, and W. Kaminski, (2013), B. Bahr and S. Steinhaus, (2016).

further, the identification of the true (interacting) vacuum state of a quantum gravity theory, in absence of any space-time background is an open issue: one possible criterion is to look for states which maximise some measure of entanglement.

> from the perspective of the theory of tensor network

exporting a number of key results obtained via tensor network techniques, holographic mappings and possibly indications of new topological phases in many-body systems in the NPQG framework

GFT “dynamic” dressing of tensor network maybe useful for controlling sum over topologies, symmetry restoration(?)

outline

- definitions
  - tensor networks
  - group field theory states
    - dictionary
- application
  - group field random tensor network
  - RT formula
- conclusions

definitions

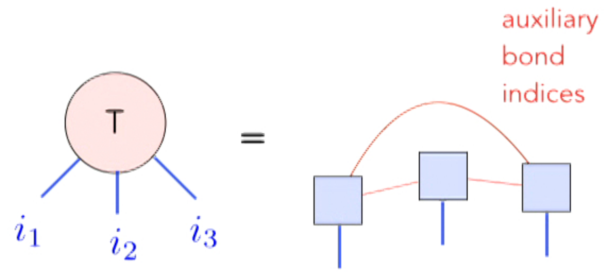
Tensor Networks

Singh, Robert N. C. Pfeifer, and Guifre Vidal (2010)

a tensor network N is a set of tensors whose indices are connected according to a network pattern.

given a tensor network N , a single tensor T can be obtained by contracting all the indices that connect the tensors in N. The indices of T correspond then to the open indices of the tensor network N.

the network N is a tensor network decomposition of T. from a tensor network decomposition N for a tensor T, another tensor network decomposition for the same tensor T can be obtained in many ways by a sequence of primitive operations



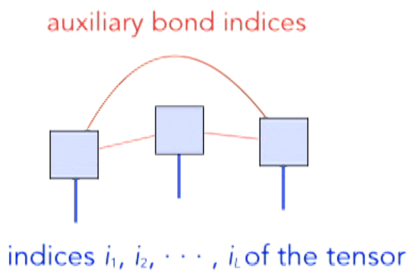
tensor networks are used as a means to represent the wave-function of certain quantum many-body systems on a lattice. consider a lattice made of L sites, each described by a complex vector space V of dimension d. A generic pure state $|\Psi\rangle \in \mathbb{V}^{\otimes N}$ can always be expanded as

$$|\Psi\rangle = \sum_{i_1, i_2, \ldots, i_N = 1}^{d} (\psi)_{i_1, i_2, \ldots, i_N} |i_1, i_2, \ldots, i_N\rangle$$

where $|i_s\rangle$ denotes a basis of $\mathbb{V}$ for site s in the lattice

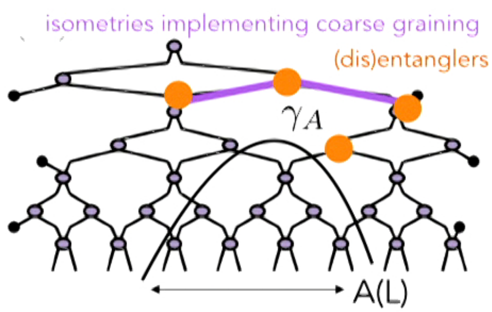
definitions

tensor (Ψ), with components $i_1 i_2 \cdots i_L$, contains d^L complex coefficients: the number that grows exponentially with the size L of the lattice. Thus, the representation of a *generic* pure state in $V^{\otimes L}$ is *inefficient*. an *efficient* representation of *certain* pure states can be obtained by expressing tensor in terms of a $O(L)$ tensor network.



if p is the rank of the tensors in one of these tensor networks, and N is the size of their indices, then the tensor network depends on $O(LN^p)$ complex coefficients. for a fixed value of N this number grows linearly in L (more efficient).

such tensor network states are used as variational ansätze to approximately describe the ground states of lo- cal Hamiltonians H of lattice models, with the $O(LN^p)$ complex coefficients as the variational parameters (e.g. for optimisation so as to minimise the expectation value of the energy of some system, etc…)



Multiscale Entanglement Renormalisation Ansatz (MERA) states provide an efficient approximation of wave functions with long-range entanglement of the type exhibited by ground states of local scale-invariant Hamiltonians.

=> contact with critical systems and *conformal field theories*

diagram geometry and gravity?

definitions

Group Field Theories (generalisation of tensor models, higher order matrix model, endowed with extra geometric info, diff, Lorentz sym)

L.Freidel, R.Gurau,D.Oriti (2009) A. Baratin and D. Oriti (2012), D. Oriti, (2015), R. Gurau and J. P. Ryan, (2012), V. Rivasseau, (2016), S.Carrozza, (2016).....

a Group Field Theory is a quantum field theory for a (complex) field φ defined on d copies of a (compact) Lie group manifold G, with combinatorially non local interaction

$$\varphi : G^d \to \mathbb{C}$$

$$g_i \mapsto \varphi(g_i)$$

$$\mathcal{Z} = \int \mathcal{D}\varphi \mathcal{D}\bar{\varphi} \exp(-\bar{\varphi}\, \mathcal{K}\, \varphi + \sum_{\{\mathcal{V}\}} \lambda_{\mathcal{V}} (\bar{\varphi}\varphi)^{p_{\mathcal{V}}})$$

polynomially perturbed Gaussian prob measure for the random tensor field

the GFT field can be seen as an infinite-dimensional tensor, transforming under the action of some (unitary) group $U^{\times d}$

$$\varphi(g_1, .., g_d) \to \int [\mathrm{d}g_i]\, U(g_1', g_1) \cdots U(g_d', g_d)\, \varphi(g_1, ..., g_d)$$

(d arguments of the GFT field to be labeled and ordered.

QFTs over a group manifold providing a 2nd quantised description for spin foam models with quanta corresponding to tensor maps associated to nodes of spin network graphs. the andom combinatorial structures, corresponding both to the elementary building blocks of quantum spacetime and to their interaction processes are defined by an action, at the classical level, and a partition function at the quantum level.

definitions

we focus on a class of GFT models based on the requirement that the Feynman diagrams of the theory are simplicial complexes, which in turn requires the interaction kernels to have the combinatorial structure of d-simplices.

in this simplicial case, the GFT action has the general form

$$S_d[\varphi] = \frac{1}{2}\int \mathrm{d}g_i \mathrm{d}g_i'\, \varphi(g_i)\, \mathcal{K}(g_i g_i'^{-1})\, \varphi(g_i') \; +$$
$$+\; \frac{\lambda}{d+1}\int \prod_{i\neq j=1}^{d+1} \mathrm{d}g_{ij}\, \mathcal{V}(g_{ij} g_{ji}'^{-1})\, \varphi(g_{1j}) \cdots \varphi(g_{d+1j})$$

a specific theory, with a specific related Feynman cellular complex, is completely defined by the choice of the kernels. Lets consider the simplest case, consisting in the choice

$$\mathcal{K}(g_i, g_i') = \int_G \mathrm{d}h \prod_i \delta(g_i g_i'^{-1}\, h),$$
$$\mathcal{V}(g_{ij} g_{ji}'^{-1}) = \int_G \prod_i \mathrm{d}h_i \prod_{i<j} \delta(h_i\, g_{ij} g_{ji}'^{-1}, h_j^{-1})$$

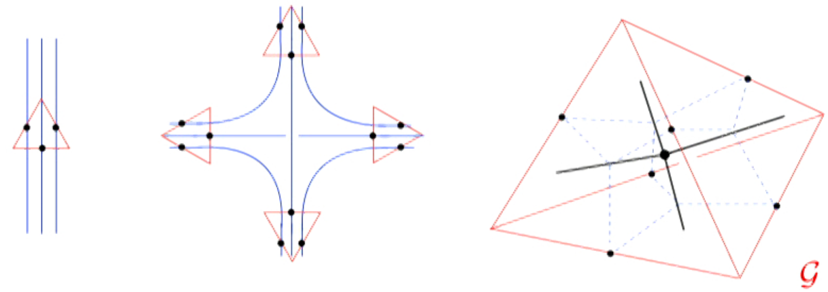
$\delta(\cdot)$ is the delta function on G

Boulatov model => Ponzano regge

definitions

$$\mathcal{Z} = \int \mathcal{D}\varphi\mathcal{D}\bar{\varphi}\, \exp(-\varphi\,\mathcal{K}\,\varphi + \sum_{\{\mathcal{V}\}} \lambda_{\mathcal{V}} (\bar{\varphi}\varphi)^{p_{\mathcal{V}}}) = \sum_{\mathcal{G}} \prod_i (\lambda_{\mathcal{V}_i})^{n_{\mathcal{V}_i}} \mathcal{A}_{\mathcal{G}}$$

restrict to the case of dimension d = 3



φ has three arguments, so each **edge** of a Feynman diagram comprises **three strands** running parallel to it.

the three strands running along the edges can be understood to be dual to a **triangle** and the propagator K gives a prescription for the gluing of two triangles.

four edges meet at each vertex and the form of the interaction kernel forces the strands to recombine. At the vertex, four triangles meet and their gluing via V form a **tetrahedron**.

GFT's Feynman diagrams define cellular complexes $\mathcal{G}$ weighted by amplitudes assigned to the faces, edges and vertices of the dual two-skeleton of a chosen triangulation of a d dimensional topological spacetime M_G

…1/N expansion from TM, universal properties of the statistical model for large I-dim, sum over topologies

definitions

GFT quantum states

the quantum states of the theory can be given a similar combinatorial characterization in terms of graphs and dual cellular complexes.

(Peter Weyl the field φ can be decomposed in terms of unitary irreps. (e.g. SU(2), V^j, labeled by the spin j ∈ N/2).

$$\varphi(g_1,g_2,g_3)=\sum_{\{j\}}\mathrm{Tr}\left[\varphi^{\{j\}}_{m_1,m_2,m_3}\left(\prod_i\sqrt{d_{j_i}}D^{j_i}_{m_i,n_i}(g_i)\right)\bar{i}^{\{j\}}_{n_1,n_2,n_3}\right]$$

d = 3, with G = SU(2)

spin network basis

$$\varphi^{\{j\}}_{\{m\}}=\sum_{\{k\}}\hat{\varphi}^{\{j\}}_{\{m\};\{k\}}\, i^{\{k\}}_{\{j\}}\prod_i\sqrt{d_{j_i}}$$

Fourier transformed GFT field tensors

$D^j(g)\ \in\ End(\mathbb{V}^j)$ group matrix element

$i^{\{j\}}_{n_1,n_2,n_3}\ \in\ \mathrm{Hom}_G(\mathbb{V}^{j_1}\otimes\mathbb{V}^{j_2}\otimes\mathbb{V}^{j_3},\mathbb{C})$ intertwiner operator

In particular, functions $\varphi(g_i)$ can also be understood as single particle wave functions for quanta corresponding to single open vertices of a spin network graph. Let us define these 'single-particle'quantum states in $H^{\otimes d}$

$$|\vec{\phi}\rangle=\int_{G^d}\mathrm{d}g_i\ \phi(g_i)|g_i\rangle\in\mathcal{H}^{\otimes d}\quad\text{for}\quad|g_i\rangle\in\mathcal{H}\simeq L^2[G]$$

$dg_i \equiv dg_1 dg_2 \ldots dg_d$ is the invariant Haar measure on the group manifold G^d; the vectors $|g_1\rangle \ldots |g_d\rangle$ provide a basis on the respective infinite dimensional spaces H ≃ $L^2[G]$.

definitions

Many-body from the one-particle Hilbert space $\mathbb{H} \equiv L^2(\mathbb{G} = G^d)$ one define a Fock space

$$\mathbb{F} \equiv \bigoplus_{n=0}^{\infty} \mathbb{H}^{\otimes n}$$

and field operators (choose bosonic statistics)

$$\widehat{\phi}(g_i) \equiv \widehat{\phi}(g_1, \cdots, g_d), \qquad \widehat{\phi}^\dagger(g_i) \equiv \widehat{\phi}^\dagger(g_1, \cdots, g_d)$$

$$[\widehat{\phi}(g_i), \widehat{\phi}^\dagger(g_i')] = \int \mathrm{d}h \; \delta^4\left(g_i h (g_i')^{-1}\right), \qquad [\widehat{\phi}(g_i), \widehat{\phi}(g_i')] = [\widehat{\phi}^\dagger(g_i), \widehat{\phi}^\dagger(g_i')] = 0$$

=> introduce the n-particle state in the group representation of the Fock space

$$|\mathbf{g}_1, \mathbf{g}_2, \cdots, \mathbf{g}_n\rangle = \frac{1}{\sqrt{n!}} \prod_{a=1}^{n} \widehat{\phi}^\dagger(\mathbf{g}_a)|0\rangle$$

basis in the Fock space defining a set of disconnected tensor states

=> GFT coherent states are eigenstates of the field operator and provide an over complete basis for the Fock space

$$|\phi\rangle \equiv \frac{1}{\mathcal{N}_\phi} \exp\left[\int \mathrm{d}\mathbf{g} \; \phi(\mathbf{g})\widehat{\phi}^\dagger(\mathbf{g})\right]|0\rangle \equiv \frac{1}{\mathcal{N}_\phi} \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{a}^{n} \left[\int \mathrm{d}\mathbf{g}_a \; \phi(\mathbf{g}_a)\widehat{\phi}^\dagger(\mathbf{g}_a)\right]|0\rangle$$

$$\equiv \frac{1}{\mathcal{N}_\phi} \sum_{n=0}^{\infty} \frac{1}{\sqrt{n!}} \prod_{a}^{n} \left[\int \mathrm{d}\mathbf{g}_a \; \phi(\mathbf{g}_a)\right] |\mathbf{g}_1, \cdots, \mathbf{g}_n\rangle$$

definitions

=> define observables $\widehat{\mathcal{O}}[\widehat{\phi}, \widehat{\phi}^\dagger]$ in Fock space are expanded as series of n-body operators:

action operator

$$\widehat{S}[\widehat{\phi}, \widehat{\phi}^\dagger] = \int \mathrm{d}\mathbf{g}\mathrm{d}\mathbf{g}' \; \widehat{\phi}^\dagger(\mathbf{g})\mathcal{K}(\mathbf{g}, \mathbf{g}')\widehat{\phi}(\mathbf{g}') + \lambda \widehat{S}_{\mathrm{int}}[\widehat{\phi}, \widehat{\phi}^\dagger]$$

graph state operator with n nodes and L links

$$\begin{aligned}\widehat{\Psi_\Gamma}[\widehat{\phi}] &= \int \prod_{a=1}^{n} \mathrm{d}\mathbf{g}_a \; \widehat{\phi}(\mathbf{g}_a)\Psi_\Gamma(\mathbf{g}_1, \cdots, \mathbf{g}_n) \\ &= \int \prod_{a=1}^{n} \mathrm{d}\mathbf{g}_a \prod_{\ell\in\Gamma}^{L} \mathrm{d}h_\ell \; \widehat{\phi}(\mathbf{g}_a) \prod_{\ell\in\Gamma} M_\ell \left(h_{s(\ell)} h_\ell h_{t(\ell)}^{-1}\right) \Psi''_\Gamma(\cdots, h_\ell, \cdots)\end{aligned}$$

acts on the Fock space and creates a network with a certain combinatorial structure

tensor network wave-function

in terms of the coherent states basis we then recover the one-particle and many-particle wave functions,by taking the expectation values of the field operators in the Fock space

$\langle\phi|\widehat{\phi}(\mathbf{g})|\phi\rangle = \phi(\mathbf{g})$ tensor wave function

$$\langle\phi|\widehat{\Psi_\Gamma[\widehat{\phi}]}|\phi\rangle = \int\prod_{a=1}^{n} \mathrm{d}\mathbf{g}_a\ \phi(\mathbf{g}_a)\Psi_\Gamma(\mathbf{g}_1,\cdots,\mathbf{g}_n) =$$

tensor network wave function

$$= \int\prod_{a=1}^{n}\mathrm{d}\mathbf{g}_a\prod_{\ell\in\Gamma}^{L}\mathrm{d}h_\ell\ \phi(\mathbf{g}_a)\prod_{\ell\in\Gamma} M_\ell\left(h_{s(\ell)}h_\ell h_{t(\ell)}^{-1}\right)\Psi''_\Gamma(\cdots,h_\ell,\cdots) = \Psi_\Gamma(\mathbf{g}_\ell)$$

If Γ is a closed graph, $\Psi_\Gamma(\mathbf{g}_\ell)$ gives the geometry of a closed spacial slice of the space-time. In loop quantum gravity, this is a cylindrical function, which can be understood as a state in the kinematical Hilbert space of the theory.

e.g. special choice of N body wave function and connectivity

$$\Psi''_\Gamma = \prod_{\ell\in\Gamma}^{L}\delta(h_\ell g_\ell^{-1}) \qquad M_\ell = \delta\left(h_{s(\ell)}h_\ell h_{t(\ell)}^{-1}\right)$$

factorised state — delta convolution link function

$$=> \quad = \int\prod_{a=1}^{n}\mathrm{d}\mathbf{g}_a\ \phi(\mathbf{g}_a)\prod_{\ell\in\Gamma}\delta\left(h_{s(\ell)}g_\ell h_{t(\ell)}^{-1}\right) \equiv \Psi_\Gamma[\varphi](\mathbf{g}_\ell) \equiv \Psi_\Gamma[\varphi](\mathbf{g}_\ell,\mathbf{g}_\partial)$$

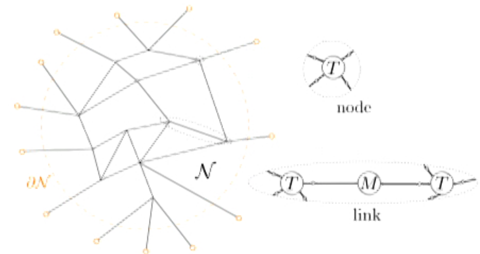
tensor network wave-function

in more familiar TN terms, we can define a link state

$$\langle M_\ell| = \int \mathrm{d}h_{s(\ell)}\mathrm{d}h_{t(\ell)}\delta\left(h_{s(\ell)}h_\ell h_{t(\ell)}^{-1}\right)\langle h_{s(\ell)}| \otimes \langle h_{t(\ell)}|$$

and write the TN state as a contraction

$$|\Psi_\Gamma\rangle \equiv \bigotimes_{\ell\in\Gamma}\langle M_\ell| \bigotimes_{n\in\Gamma}|\Phi\rangle_n \equiv \int \mathrm{d}\mathbf{g}_\partial \Psi_\Gamma(\mathbf{g}_\partial, \mathbf{g}_{\ell\in\Gamma})|\mathbf{g}_\partial\rangle$$



extra ingredient: dynamics

now we can use the partition function of our field theory to calculate averages

$$\mathbb{E}\left(\widehat{\Psi_\Gamma}[\widehat{\phi}]\right) \equiv \frac{1}{Z_0}\mathrm{Tr}\left(:\widehat{\Psi_\Gamma}[\widehat{\phi}]\ e^{-\widehat{S}[\widehat{\phi},\widehat{\phi}^\dagger]}:1_{\mathbb{F}}\right)$$

$$= \frac{1}{Z_0}\int \mathcal{D}\phi\mathcal{D}\overline{\phi}\ \delta_{\mathbb{C}}[\phi - \phi_{\mathrm{GF}}]\ \Psi_\Gamma[\phi]\ e^{-S[\phi,\overline{\phi}]}$$

encodes the dynamics of the tensorial d.o.f. bounded by the spacial slice. If we perturbatively expand this amplitude in terms of the small coupling constant λ in the action, we obtain a series of Feynman graphs corresponding to spin-foam amplitudes

dictionary

Table A	Group Fields	Tensors	
group basis	$\lvert g_i\rangle \in \mathbb{H} \simeq L^2[G]$	$\lvert\lambda_i\rangle$, $\lambda_i = 1,\dots,D$ in $\mathbb{H}_D$	index basis
one particle state	$\lvert\varphi\rangle = \int_{G^d} \mathrm{d}g_i\ \varphi(g_i)\,\lvert g_i\rangle$	$\lvert T_n\rangle = \sum_{\{\lambda_i\}} T_{\{\lambda_i\}}\lvert\lambda_i\rangle \in \mathbb{H}_n = \mathbb{H}_D^{\otimes d}$	tensor state
gluing functional	$\langle M_{g_\ell}\rvert = \int \mathrm{d}g_1\mathrm{d}g_2\ M(g_1^\dagger g_\ell g_2)\ \langle g_1\rvert\ \langle g_2\rvert \in \mathbb{H}^{*\otimes 2}$	$\lvert M\rangle = M_{\lambda_1\lambda_2}\,\lvert\lambda_1\rangle \otimes \lvert\lambda_2\rangle \in \mathbb{H}_\ell = \mathbb{H}_D^{\otimes 2}$	link state
multiparticle state	$\lvert\Phi_\Gamma\rangle \in \mathbb{H}_V \simeq L^2[G^{d\times V}/G^V]$	$\lvert\Psi_\mathcal{N}\rangle$	tensor network state
product state convolution	$\lvert\Phi_\Gamma^{g_\ell}\rangle \equiv \bigotimes_{\ell\in\Gamma} \langle M_{g_\ell}\rvert \bigotimes_n^V \lvert\varphi_n\rangle = \int \mathrm{d}g_\partial\, \Phi_\Gamma(g_\ell, g_\partial)\,\lvert g_\partial\rangle$	$\lvert\Psi_\mathcal{N}\rangle \equiv \bigotimes_\ell^L \langle M_\ell\rvert \bigotimes_n^N \lvert T_n\rangle \in \mathbb{H}_{\partial\mathcal{N}}$	tensor network decomposition
randomness	$\frac{1}{Z}d\nu(\varphi)$ field theory probability measure	$T_\mu^U \equiv (UT^0)_\mu$; $T_\mu^0 \equiv T^0_{\lambda_1\cdots\lambda_d} \in \mathbb{H}_T,\ U \in \mathrm{U}(\dim(\mathbb{H}_T))$	random tensor state

Group Field Random Tensor Networks

Because of the field theoretic description, we can see our group field network states as a random tensor networks

$$|\Psi\rangle \equiv \bigotimes_{<ij>} \langle M_{ij}| \bigotimes_{v}^{N} |T_v\rangle$$

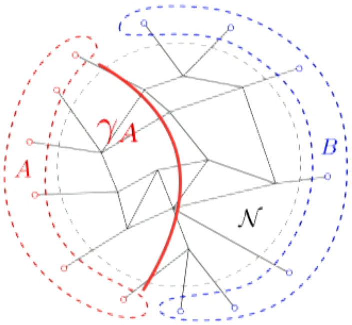
1 - maximally entangled link states $\qquad |M\rangle = \frac{1}{\sqrt{D}} \delta_{\lambda_1 \lambda_2} |\lambda_1\rangle \otimes |\lambda_2\rangle$

2 - tensors chosen independently at random from their respective Hilbert spaces.

(for arbitrary reference state $|0_v\rangle$ define $|T_v\rangle = U|0_v\rangle$ with U unitary)

3 - In the large bond dimension limit, RTS saturate the TN entropy bound, reproducing the holographic Ryu Takayanagi entropy formula

$$S(A) \simeq \log(D) \min |\gamma_A|$$



Hayden et al.arXiv:1601.01694v1

Group Field Random Tensor Networks

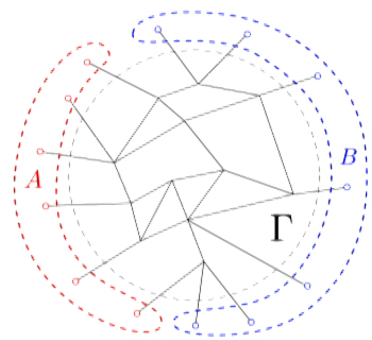
goal — use the established dictionary to investigate the holographic entanglement properties of the GFT network along the lines used for Random Tensor Networks

given the boundary state $|\Psi_\Gamma\rangle \in \mathbb{D}_\partial$ associated to the open network graph Γ

$$|\Psi_\Gamma\rangle \equiv \bigotimes_{\ell\in\Gamma}\langle M_\ell| \bigotimes_{n\in\Gamma}|\Phi\rangle \equiv \int \mathrm{d}\mathbf{g}_\partial \Psi_\Gamma(\mathbf{g}_\partial, \mathbf{g}_{\ell\in\Gamma})|\mathbf{g}_\partial\rangle$$

consider a Hilbert space factorisation $\mathbb{D}_\partial = \mathbb{D}_A \otimes \mathbb{D}_B$

associated to the definition of two, a priori non adjacent, subregions A and B of the boundary



$$\rho \equiv |\Psi_\Gamma\rangle\langle\Psi_\Gamma| = \mathrm{Tr}_\ell\left[\bigotimes_\ell \rho_\ell \bigotimes_n \rho_n\right]$$

assuming factorised state

measure of the entanglement between A or B, defined by first partial tracing over the full system Hilbert space:

$$S(A) = -\mathrm{Tr}\widehat{\rho}_A \ln \widehat{\rho}_A \quad \text{where} \quad \hat{\rho}_A = \mathrm{tr}_B[\rho]/\mathrm{tr}[\rho]$$

18/30

Group Field Random Tensor Networks

first trick to compute the entropy is so-called Rényi entropy

$$S_N(A) = \frac{1}{1-N} \ln \mathrm{Tr}\widehat{\rho}_A^N = \frac{1}{1-N} \ln \frac{\mathrm{Tr}\rho_A^N}{(\mathrm{Tr}\rho)^N} \equiv \frac{1}{1-N} \ln \frac{\mathrm{Tr}(\rho^{\otimes N}\mathcal{P}_A)}{\mathrm{Tr}(\rho^{\otimes N})}$$

where $\mathcal{P}_A$ is the 1-cycle permutation operator in SN acting on $\mathbb{D}_A$

given the random character of the network we look for the typical value of the entropy. In particular, the variables Z_N and Z_0 are easier to average than the entropy, since they are quadratic functions of the network density matrix.

Expand $e^{-S_N(A)} = \mathrm{tr}[\rho_A^N]/(\mathrm{tr}[\rho])^N \equiv Z_A/Z_0$ in powers of fluctuations:

$$\overline{S_N(A)} = -\overline{\log \frac{Z_A + \delta Z_A}{Z_0 + \delta Z_0}} = -\log \frac{\overline{Z_A}}{\overline{Z_0}} + \left(\sum_n^\infty \frac{(-1)^{n-1}}{n} \frac{\overline{\delta Z_0^n}}{\overline{Z_0^n}} - \frac{\overline{\delta Z_A^n}}{\overline{Z_A^n}} \right)$$

Hayden et al.arXiv:1601.01694v1

very general: random states in high-dimensional bipartite systems: "concentration of measure" phenomenon applies, meaning that on a large-probability set macroscopic parameters are close to their expectation values (bond dimension, => continuum limit)

Group Field Random Tensor Networks

first trick to compute the entropy is so-called Rényi entropy

$$S_N(A) = \frac{1}{1-N} \ln \mathrm{Tr} \widehat{\rho}_A^N = \frac{1}{1-N} \ln \frac{\mathrm{Tr}\rho_A^N}{(\mathrm{Tr}\rho)^N} \equiv \frac{1}{1-N} \ln \frac{\mathrm{Tr}(\rho^{\otimes N} \mathcal{P}_A)}{\mathrm{Tr}(\rho^{\otimes N})}$$

where $\mathcal{P}_A$ is the 1-cycle permutation operator in SN acting on $\mathbb{D}_A$

given the random character of the network we look for the typical value of the entropy. In particular, the variables Z_N and Z_0 are easier to average than the entropy, since they are quadratic functions of the network density matrix.

Expand $e^{-S_N(A)} = \mathrm{tr}[\rho_A^N]/(\mathrm{tr}[\rho])^N \equiv Z_A/Z_0$ in powers of fluctuations:

$$\overline{S_N(A)} = -\overline{\log \frac{Z_A + \delta Z_A}{Z_0 + \delta Z_0}} = -\log \frac{\overline{Z_A}}{\overline{Z_0}} + \left(\sum_n^\infty \frac{(-1)^{n-1}}{n} \frac{\overline{\delta Z_0^n}}{Z_0^n} - \frac{\overline{\delta Z_A^n}}{Z_A^n} \right)$$

Hayden et al.arXiv:1601.01694v1

very general: random states in high-dimensional bipartite systems: "concentration of measure" phenomenon applies, meaning that on a large-probability set macroscopic parameters are close to their expectation values (bond dimension, => continuum limit)

Group Field Random Tensor Networks

first trick to compute the entropy is so-called Rényi entropy

$$S_N(A) = \frac{1}{1-N} \ln \mathrm{Tr}\widehat{\rho}_A^N = \frac{1}{1-N} \ln \frac{\mathrm{Tr}\rho_A^N}{(\mathrm{Tr}\rho)^N} \equiv \frac{1}{1-N} \ln \frac{\mathrm{Tr}(\rho^{\otimes N}\mathcal{P}_A)}{\mathrm{Tr}(\rho^{\otimes N})}$$

where $\mathcal{P}_A$ is the 1-cycle permutation operator in SN acting on $\mathbb{D}_A$

given the random character of the network we look for the typical value of the entropy. In particular, the variables Z_N and Z_0 are easier to average than the entropy, since they are quadratic functions of the network density matrix.

Expand $e^{-S_N(A)} = \mathrm{tr}[\rho_A^N]/(\mathrm{tr}[\rho])^N \equiv Z_A/Z_0$ in powers of fluctuations:

$$\overline{S_N(A)} = -\overline{\log \frac{\overline{Z_A} + \delta Z_A}{\overline{Z_0} + \delta Z_0}} = -\log \frac{\overline{Z_A}}{\overline{Z_0}} \simeq S_N(A)$$

fluctuations are suppressed in the limit of large bond dimension

Hayden et al.arXiv:1601.01694v1

Group Field Random Tensor Networks

permutation operator acting on the states in A

now $$\frac{\overline{Z_A}}{\overline{Z_0}} = \frac{\mathbb{E}(\mathrm{tr}\rho_A^N)}{\mathbb{E}(\mathrm{tr}\rho)^N} = \frac{\mathbb{E}\mathrm{tr}[\rho^{\otimes N}\mathbf{P}(\pi_A^0; N, d)]}{\mathbb{E}(\mathrm{tr}\rho)^N}$$

+ assuming factorised state

$$= \frac{\mathrm{tr}\left[\bigotimes_\ell \rho_\ell^N \bigotimes_n \mathbb{E}(\rho_n^N)\mathbf{P}(\pi_A^0; N, d)\right]}{\mathrm{tr}\left[\bigotimes_\ell \rho_\ell^N \bigotimes_n \mathbb{E}(\rho_n^N)\right]} \simeq S_N(A)$$

we get SN (A) by computing the expectation values on the single tensor node states

$$\mathbb{E}(\rho_n^N) = \mathbb{E}[(|\Phi_n\rangle\langle\Phi_n)^N] = \mathbb{E}\left[\left(\int \prod_a^N \mathrm{d}\mathbf{g}_a \mathrm{d}\mathbf{g'}_a \, \phi_n(\mathbf{g}_a)\overline{\phi_n(\mathbf{g'}_a)} |\mathbf{g}_a\rangle\langle\mathbf{g'}_a|\right)\right]$$

linear function of

in the standard field theory formalism we define the averaging via the path integral of some GFT model

$$\mathbb{E}\left[f[\phi,\overline{\phi}]\right] \equiv \int [\mathcal{D}\phi][\mathcal{D}\overline{\phi}]\, f[\phi,\overline{\phi}]\, e^{-S[\phi,\overline{\phi}]}$$

because ZA, Z0 are polynomial of the fields, the average over the N-replica of the wave functions associated to each network vertex eventually reduces to calculate the N-point correlation functions of the GFT field

Group Field Random Tensor Networks

we take the case $S[\phi,\overline{\phi}] = \int \mathrm{d}\mathbf{g}\mathrm{d}\mathbf{g}'\, \overline{\phi(\mathbf{g})}\mathcal{K}(\mathbf{g},\mathbf{g}')\phi(\mathbf{g}') + \lambda S_{\mathrm{int}}[\phi,\overline{\phi}] + cc$

with $\mathcal{K}(\mathbf{g},\mathbf{g}') = \delta(\mathbf{g}^{\dagger}\mathbf{g}')$

λ <<1 and consider a perturbative expansion of the path integral in powers of λ

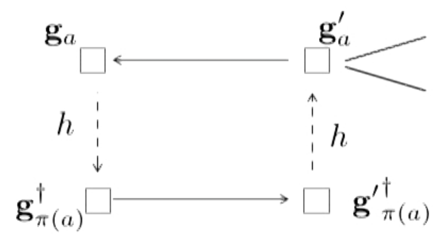
$$\mathbb{E}\left[\prod_a^N \phi(\mathbf{g}_a)\overline{\phi(\mathbf{g'}_a)}\right] \equiv \mathbb{E}_0\left[\prod_a^N \phi(\mathbf{g}_a)\overline{\phi(\mathbf{g'}_a)}\right] + \mathcal{O}(\lambda)$$

Wick theorem

$$\mathcal{C}\sum_{\pi\in\mathcal{S}_{NV_\Gamma}}\mathcal{P}(\pi)$$

where SNVΓ is the permutation group of NVΓ objects, which corresponding to the permutations of NVΓ nodes

with $\mathbb{P}_{\mathbf{h}}(\pi) = \prod_a^N \delta\left(h_a\mathbf{g}_a\mathbf{g}^{\dagger}_{\pi(a)}\right)$



the free theory N points correlation function traduces into a sum over all permutations among the group elements attached at each node

Group Field Random Tensor Networks

we take the case $S[\phi,\overline{\phi}] = \int \mathrm{d}\mathbf{g}\mathrm{d}\mathbf{g}'\, \overline{\phi(\mathbf{g})}\mathcal{K}(\mathbf{g},\mathbf{g}')\phi(\mathbf{g}') + \lambda S_{\text{int}}[\phi,\overline{\phi}] + cc$

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Wick theorem

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where $S_{NV\Gamma}$ is the permutation group of NV_Γ objects, which corresponding to the permutations of NV_Γ nodes

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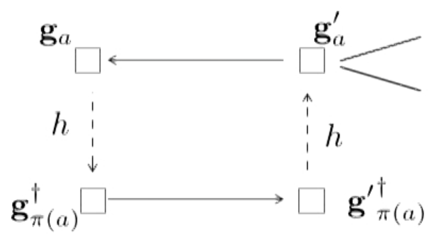


the free theory N points correlation function traduces into a sum over all permutations among the group elements attached at each node

Path integral averaging for the free theory

Z_A and Z_0 correspond to summations of the combinatorial networks $N_A(h_n, \pi_n)$ and $N_0(h_n,\pi_n)$

$$Z_N^A \approx \mathcal{C}^{V_\Gamma} \sum_{\boldsymbol{\pi_n} \in \mathcal{S}_N} \int \prod_n \mathrm{d}\mathbf{h}_n \, \mathrm{Tr}\left[\bigotimes_\ell \rho_\ell^N \bigotimes_n \mathbb{P}_{\mathbf{h}_n}(\pi_n) \mathbb{P}(\pi_A^0; N, d) \right]$$

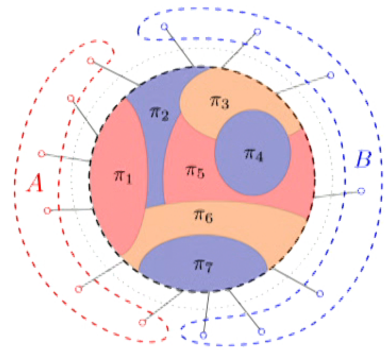
$$\equiv \mathcal{C} V_\Gamma \sum_{\boldsymbol{\pi_n} \in \mathcal{S}_N} \int \prod_n \mathrm{d}\mathbf{h}_n \, \mathcal{N}_A(\mathbf{h_n}, \boldsymbol{\pi_n})$$

at each node n we have a contribution $P_{h_n}(\pi_n)$

$$Z_0^N = \mathcal{C}^{V_\Gamma} \sum_{\boldsymbol{\pi_n} \in \mathcal{S}_N} \int \prod_n \mathrm{d}\mathbf{h}_n \, \mathrm{Tr}\left[\bigotimes_\ell \rho_\ell^N \bigotimes_n \mathbb{P}_{\mathbf{h}_n}(\pi_n) \right]$$

$$\equiv \mathcal{C}^{V_\Gamma} \sum_{\boldsymbol{\pi_n} \in \mathcal{S}_N} \int \prod_n \mathrm{d}\mathbf{h}_n \, \mathcal{N}_0(\mathbf{h_n}, \boldsymbol{\pi_n})$$

links contribution



the full (Feynman) networks get divided into several regions, with same π_n and h_n. The links which connect different regions identify boundaries between each pair of different regions or domain walls.

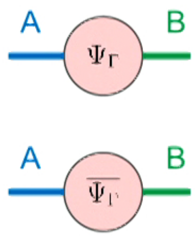
$$\int \prod_n \mathrm{d}\mathbf{h}_n \, \mathcal{N}_0(\mathbf{h_n}, \boldsymbol{\pi_n}) = \int \prod_n \mathrm{d}\mathbf{h}_n \prod_{\ell \in \Gamma} \left[\prod_i^{\chi(\varpi_\ell)} \delta\left(\prod_{k=1}^{\overleftarrow{r_i}} H_{\ell a_k^i} \right) \right] \prod_{\ell \in A} \left[\prod_i^{\chi(\mathcal{C}_0^{-1}\pi_{n\ell})} \delta\left(\prod_{k=1}^{\overleftarrow{r_i}} h_{\ell a_k^i} \right) \right] \prod_{\ell \in \overline{A}} \left[\prod_i^{\chi(\pi_{n\ell})} \delta\left(\prod_{k=1}^{\overleftarrow{r_i}} h_{\ell a_k^i} \right) \right]$$

Z_A and Z_0 are the amplitudes of a topological BF field theory, with given boundary condition, discretized on a specific 2-complex among the N replica of networks, with each different pattern P corresponding to a different 2-complex

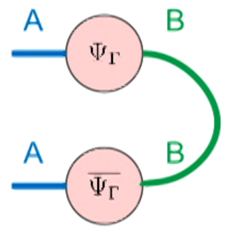
Group Field Random Tensor Networks

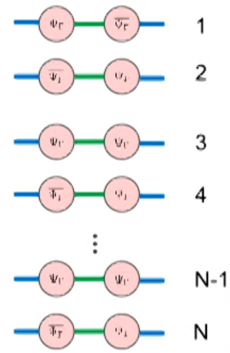
for large bond dimension, we then seek for the most divergent terms of the BF amplitudes

$\rho = |\Psi_\Gamma\rangle\langle\Psi_\Gamma|$



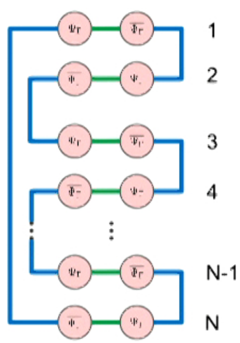
$\rho_A = \mathrm{tr}_B \rho$



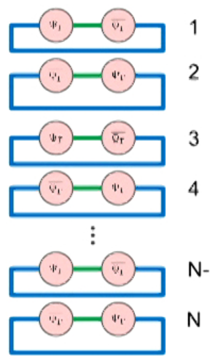


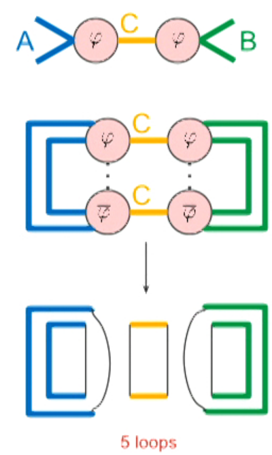
N replica

$\mathrm{tr}\rho_A^N$



$(\mathrm{tr}\rho)^N$





propagate among conjugate nodes only in the graph

assumptions and result

truncation: contributions coming from permutations involving couples of conjugate nodes, as if nodes were distinguishable

evaluation

Z0 : leading contribution for $\pi = \mathbb{1}$

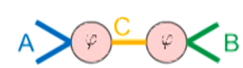
$$\mathbb{E}_0[\mathrm{Tr}(\rho^{\otimes N})] \approx \mathcal{C}\ \mathrm{Tr}\mathcal{P}(\mathbb{1}) = \mathcal{C}D(\Lambda)^{(L_\Gamma - L_T)N}$$

BF-like amplitudes

where L_T is the number of the branches of the minimal spanning tree of Γ, which is $V_\Gamma - 1$

ZA : Γ is split into two parts Γ_A and Γ_B by cutting some links inside of Γ, which forms a new boundary C such that $\partial\Gamma_A = A \cup C$ and $\partial\Gamma = B \cup C$

$$\Psi^{\Gamma}_{ab} = \Psi^{\Gamma_A}_{ac}\Psi^{\Gamma_B}_{cb}$$



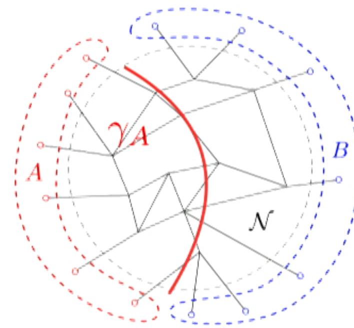
$$\mathbb{E}_0[\mathrm{Tr}(\rho^{\otimes N}\mathcal{P}_A)] \approx \mathcal{C}D(\Lambda)^{(L_\Gamma - L_T)N + (1-N)\min(L_C)}$$

if $\dim D_C$ is smaller than $\dim D_A$ and $\dim D_B$, the leading term in the large bond dimension $D(\Lambda)$ limit. where L_C is the number of links through boundary C

$$\mathbb{E}_0[S_N(A)] \approx \frac{1}{1-N}\ln\frac{\mathbb{E}_0[\mathrm{Tr}(\rho^{\otimes N}\mathcal{P}_A)]}{\mathbb{E}_0[\mathrm{Tr}(\rho^{\otimes N})]} = \min(L_C)\ln D(\Lambda)$$

assumptions and result

$$S_{\mathrm{EE}} = \min(\#_{\ell\in\partial_{AB}}) \ln \delta(\mathbb{1})$$



understood as the Ryu-Takayanagi formula in a GFT context,
with the same interpretation for the area of the minimal surface

box normalization on |h⟩ (quantum group) $\delta(\mathbb{1}) = D(\Lambda)$

$g_l = 1$ for all link $l \in \Gamma$. This assumption makes our state $|\Psi_\Gamma\rangle$ lying in the flat vacuum of loop quantum gravity. The cosmological constant Λ in the bond dimension makes our state a dS vacuum if $\Lambda > 0$ and AdS vacuum if $\Lambda < 0$.

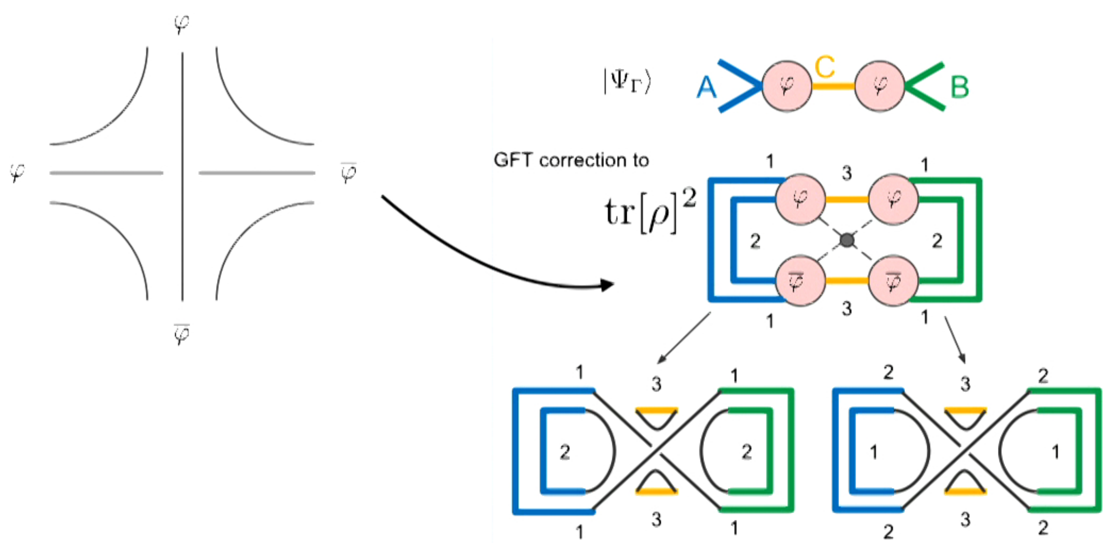
Only consider 3-valent nodes. We consider Ooguri GFT for 3D gravity for simpliticity. So there are 3-valent nodes in the network. The kinetic kernel $K(h,h')$ becomes $\delta(h^{-1}h')$

assumptions and result

the interaction kernel will generally lead to non-trivial bulk corrections to the RT formula

$$S_{\text{int}}[\varphi, \overline{\varphi}] = \int \prod_{a=1}^{6} \mathrm{d}h_a \varphi(h_1, h_2, h_3)\varphi(h_1, h_4, h_5)\overline{\varphi(h_6, h_2, h_5)\varphi(h_6, h_4, h_3)}$$

kernel action generates extra connectivity



comments and conclusions

remark

we are interested in is the leading term of Z_A and Z_0 , while taking the dimension D of the bond Hilbert space much larger than 1. This leads us to seek for the most divergent term of the amplitudes (bubble divergences) characteristic of BF-like amplitudes

this simple form of the various functions entering the calculation of the entropy, with the emergence of BF-like amplitudes, is not generic: it follows from the choice of GFT kinetic term, from the approximation used in the calculation of expectation values (neglecting GFT interactions) and from the special type of GFT tensor network chosen: bulk flatness $g_l = 1$

=> the divergence degree of BF amplitudes discretised on a lattice has been the subject of a number of works, both in the spin foam an GFT literature

Freidel, Gurau, Oriti (2009), Bonzom and Smerlak (2010-12)

comments and conclusions

- established a precise dictionary between GFT states and (generalized) random tensor networks. Such a dictionary also implies, under different restrictions on the GFT states, a correspondence between LQG spin network states and tensor networks, and a correspondence between random tensors models and tensor networks.

- compute the Rényi entropy and derived the RT entropy formula: using directly GFT and spin network techniques, first using a simple approximation to a complete definition of a random tensor network evaluation seen as a GFT correlation function, but still using a truly generalized tensor network seen as a GFT state

- AdS/MERA/CFT may be extended, beyond AdS/CFT, to a more general space/TNR/QFT correspondence? GFTs may play a role as auxiliary tensor field theories both fixing the entanglement structure of the boundary physical theory and providing a dual simplicial characterisation of the tensor network diagrams as discretised space

- dynamics induces entanglement: can we reproduce multi scale renormalisation techniques within the field theory framework of GFTs: coarse graining and disentangler from interaction vertices at each order in the perturbative expansion?

- the structural similarity had been noted before, and also exploited, in the context of renormalization of spin foam models treated as lattice gauge theories

M. Han and L.-Y. Hung, (2016) B. Dittrich, S. Mizera, and S. Steinhaus, C. Delcamp and B. Dittrich, B. Dittrich, F. C. Eckert, and M. Martin-Benito, B. Dittrich, E. Schnetter, C. J. Seth, and S. Steinhaus.

Thank You

(office 461)