Title: Fermion condensation and superconducting string-nets

Date: Dec 07, 2017 11:00 AM

URL: http://pirsa.org/17120009

Abstract: <p>A great deal of progress has been made toward a classification of bosonic topological orders whose microscopic constituents are bosons. Much less is known about the classification of their fermionic counterparts. In this talk I will describe a systematic way of producing fermionic topological orders using the technique of fermion condensation. Roughly, this can be understood as binding a physical fermion to an emergent fermion and condensing the pair. I will discuss the `super pivotal categories' that describe universal properties of these phases and use them to construct exactly solvable string-net models. These string-net models feature conventional anyons and two flavours of vortices. I will show that one of the vortex types is similar to a vortex in a p+ip superconductor binding a Majorana zero mode, and will mention some possible applications.</p>

Fermion condensation and superconducting string-nets

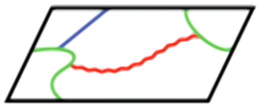
David Aasen — Caltech

Perimeter Institute December 6 2017

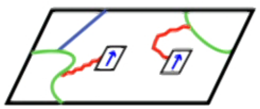
Ethan Lake
MIT

Kevin Walker
Station Q

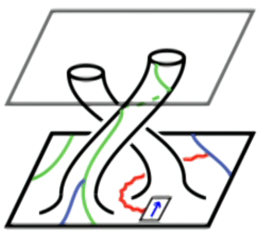
Outline:



introduce gapped (2+1)d topological phases and (braided) fusion categories $\mathcal{C}$



fermion condensation, Ising example and 1D 'string-net'/Majorana chain $\mathcal{C}/\psi$

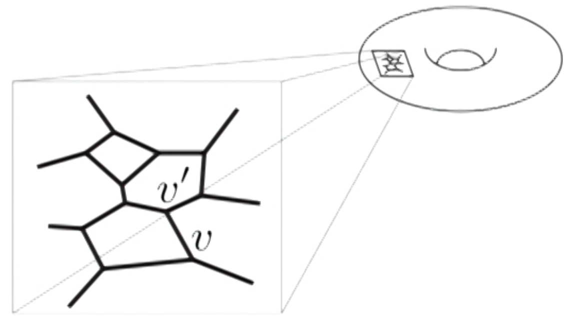


fermion condensation in string-net models and quasiparticle content $\mathrm{Tube}(\mathcal{C}/\psi)$

Talk is based on DA,Lake,Walker arXiv:1709.01941

Gapped phases in (2+1)d

$$\mathcal{H} = \otimes_v \mathcal{H}_v \qquad H = \sum_{v,v'} H_{v,v'}$$



microscopic constituents are either bosons or fermions

e.g., Toric code, Kitaev Honeycomb model, Levin-Wen string-nets,…

Kitaev 2003, 2006;
Levin,Wen 2005

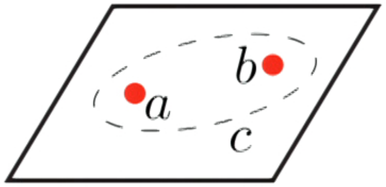
e.g., Fractional Quantum Hall effect, p+ip superconductor, fermionic toric code,…

Stormer, Tsui, Gossard 1982;
Read, Green 2000;
Gu, Wang, Wen 2014

Gapped bosonic phases are well understood

Universal low energy degrees of freedom are captured by a TQFT

Quasiparticles are anyons (= local excitation/ local operators)

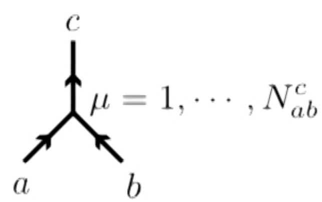


Mathematical description is given by a braided fusion category

simple objects $\{a, b, c, \cdots\}$ fusion rules $a \otimes b \cong \bigoplus_c N_{ab}^c c$

Moore, Read 1991; Kitaev 2006;
Review: Nayak, Simon, Stern,Freedman, Sarma RMP 2008

Operators such as fusion of anyons are represented diagrammatically



operator which fuses ‘a’ and ‘b’ into ‘c’

$\pitchfork$

V_{ab}^{c}

Labeled diagrams with open legs represent vector spaces

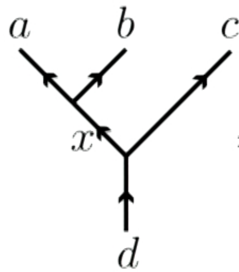
There are several physical constraints such as requiring vacuum/anti-particles:

identity/duals $\quad \mathbb{1} \otimes a \cong a \qquad a \otimes a^{*} \cong \mathbb{1} \oplus \cdots$

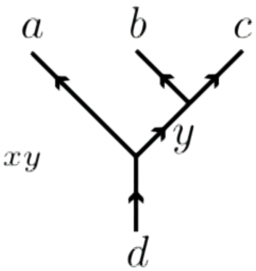
Using fusion or splitting operators, the same wave function can be created in multiple ways

$$\bigoplus_x V_x^{ab} \otimes V_d^{xc} \quad \cong \quad \bigoplus_y V_d^{ay} \otimes V_y^{bc} \quad \cong V_d^{abc}$$

$$\Cup \qquad\qquad\qquad\qquad \Cup$$

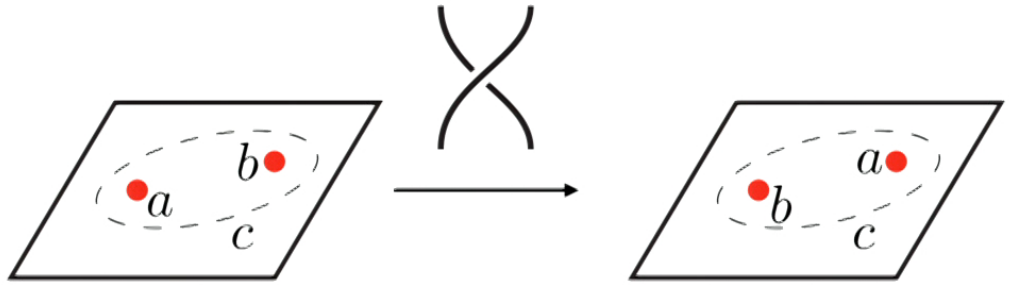


$$= \sum_y \left(F_d^{abc}\right)_{xy}$$

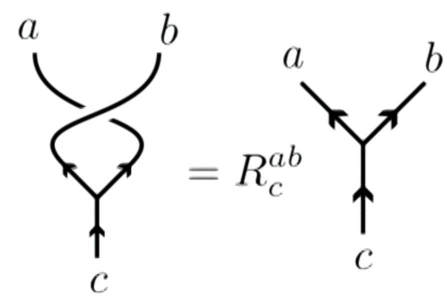


F-symbols are highly constrained by pentagon equation

There is also braiding:

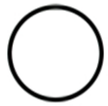


We keep track of braids using the R-symbols which are subject to the hexagon equation



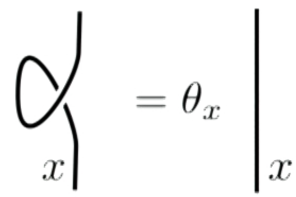
disclaimer: will drop draw arrows and multiplicities on diagrams

Physical quantities you might look at:

$d_x =$  x quantum dimension of x $\dim(V^{x\cdots x}) \sim (d_x)^n$

$$\mathcal{D} = \sqrt{\sum_x d_x^2}$$ total dimension

$S_{ab} = \frac{1}{\mathcal{D}}$ 



S-matrix encodes data about exchange statistics

Topological spin is the eigenvalue under a Dehn twist/full rotation

We would like to understand gapped fermionic phases on the same footing as their bosonic counterparts

Recently a lot of progress has been made in this direction

Commuting projector models

Gu, Wang, Wen 2014, 2015; Tarantino, Fidkowski 2016;
Ware, Son, Cheng, Mishmash, Alicea, Bauer 2016;
Bhardwaj, Gaiotto, Kapustin 2017,…

Algebraic theory

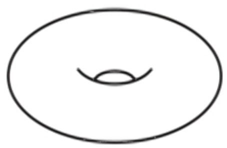
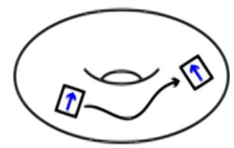
Walker 2014; Gaiotto, Kapustin 2016; Lan, Kong, Wen 2016;
Bruillard, Galnido, Hagge, Ng, Plavnik, Rowell, Wang 2017,…

Tensor networks

Fidkowski Kitaev 2011; Kapustin, Turzillo, You 2016;
Bultinck, Williamson, Haegeman, Verstraete 2016;
Wille, Buerschaper, Eisert 2016,…

conclusion is that we need to update/modify our formalism

Need to update bosonic formalism:

bosons		fermions
$\mathcal{H}_v \cong \mathbb{C}^n$	$\longrightarrow$	$\mathcal{H}_v \cong \mathbb{C}^{p\|q}$
tensor product		graded tensor product
surface		spin surface
	$\longrightarrow$	
anyons	$\longrightarrow$	anyons and vortices

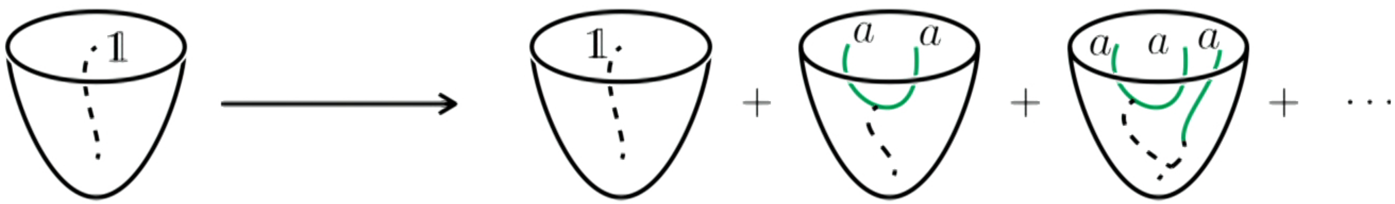
Fermion condensation provides a starting point to study fermionic theories in the same formalism as bosonic ones

Bose condensation:

Pick a boson in the spectrum
identify it with the vacuum

$$a \cong \mathbb{1}$$

Physically we think of the vacuum as a condensate of ‘a’



old vacuum

new vacuum

We have added morphisms to $\mathcal{C}$ $\quad \mathrm{mor}(\mathbb{1} \to a) \cong \mathbb{C}$

Bais, Slingerland 0808.0627; Eliens, Romers, Bais 1310.6001; Kong 1307.8244

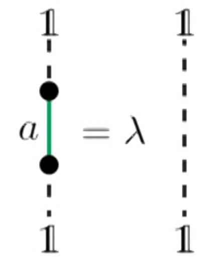
Bose condensation:

$$\mathcal{C} \xrightarrow{a \cong \mathbb{1}} \mathcal{C}/a$$

we denote the morphism by a dot:

$\longleftrightarrow \quad \mathbb{1} \cong a$

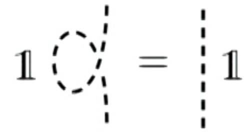
Creating ‘a’ out of the vacuum can be done locally in $\mathcal{C}/a$:



Particles that braid non-trivially with ‘a’ are confined in the condensed theory

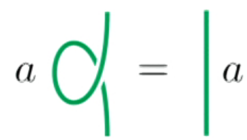
Bais, Slingerland 0808.0627; Eliens, Romers, Bais 1310.6001; Kong 1307.8244 cannot condense arbitrary ‘a’, must satisfy some constraints

The vacuum has trivial topological spin

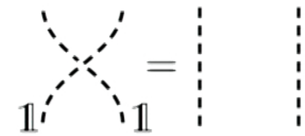


combining this with the fact that ‘a’ is identified with vacuum

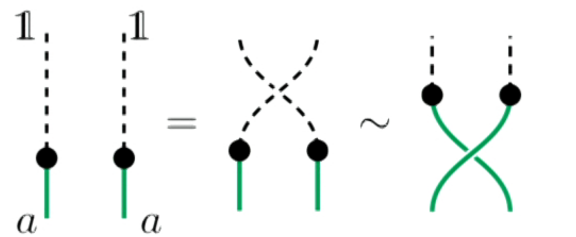
hence ‘a’ is required to have trivial topological spin



vacuum also has trivial braiding

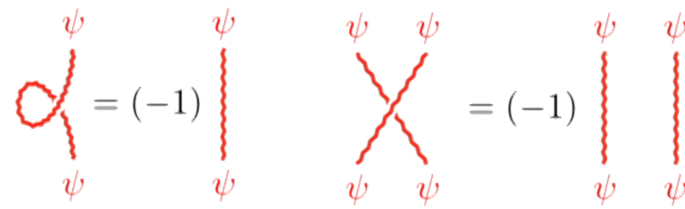


The particle we're condensing must also have trivial braiding



$(\text{assume } a \otimes a \cong \mathbb{1})$)

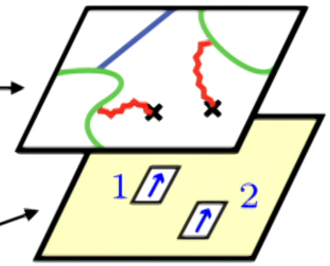
naive attempt to condense fermions is problematic



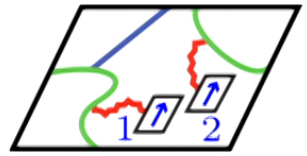
Fermion condensation compensates for these minus signs with physical fermions:

take bosonic phase with an emergent fermion

stack a gapped phase of physical fermions, e.g., a topological superconductor

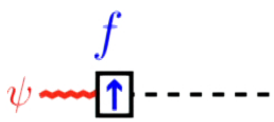


Add local interaction that binds the emergent fermion to the physical fermion

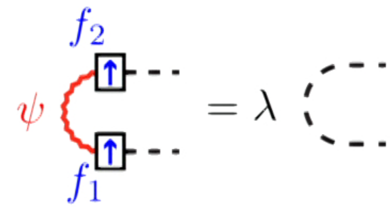


The composite particle is bosonic in its spin and statistics

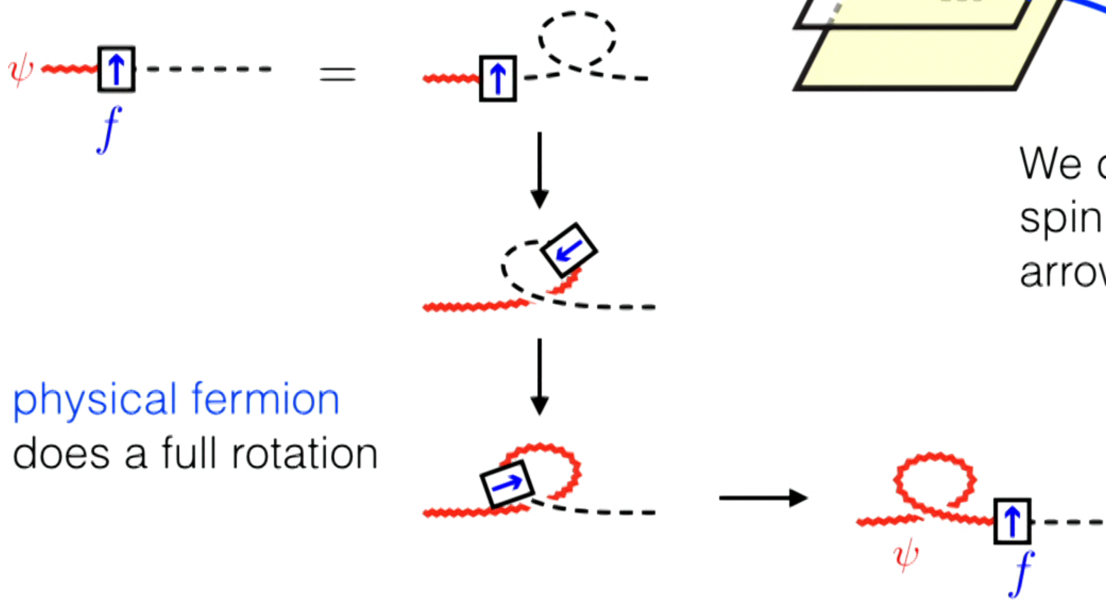
diagrammatically

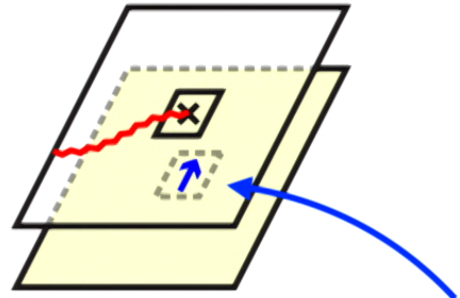


odd isomorphism



Walker 2014 Princeton; Gaiotto, Kapustin 2016; DA, Lake, Walker 2017; Wan, Wang 2017 topological spin

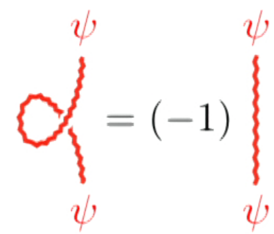




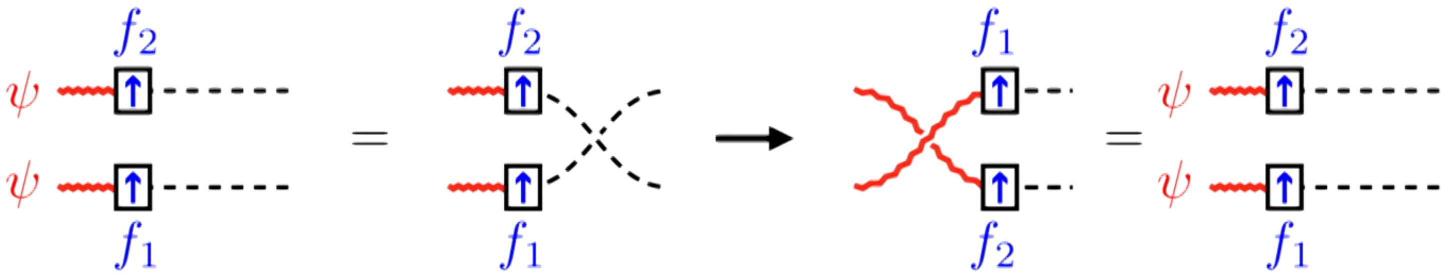
We denote the local spin framing by an arrow

physical fermion
does a full rotation

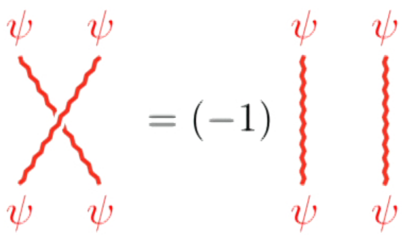
pick up one minus sign from the emergent fermion
and one from the physical fermion

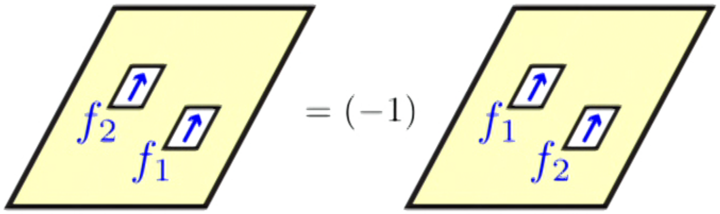


statistics:



We again get two minus signs, one from the emergent fermion and one from the Koszul ordering





We can now apply standard techniques of Bose condensation

Example: Ising

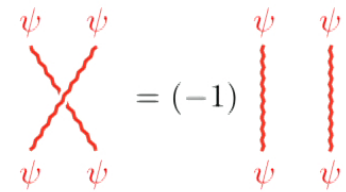
Simple objects

$$\{\mathbb{1}, \sigma, \psi\}$$

Fusion rules

$$\sigma \otimes \sigma = \mathbb{1} \oplus \psi$$
$$\psi \otimes \psi = \mathbb{1}$$
$$\psi \otimes \sigma = \sigma$$

emergent fermion



Example: Ising

Simple objects

$\{\mathbb{1}, \sigma, \psi\}$

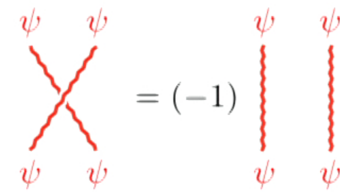
Fusion rules

$\sigma \otimes \sigma = \mathbb{1} \oplus \psi$

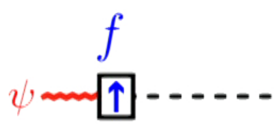
$\psi \otimes \psi = \mathbb{1}$

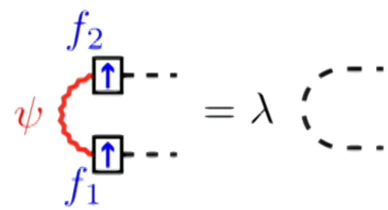
$\psi \otimes \sigma = \sigma$

emergent fermion



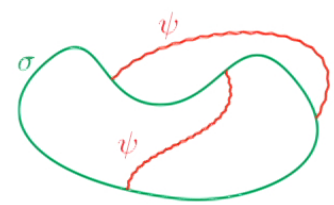
To condense ψ we add:



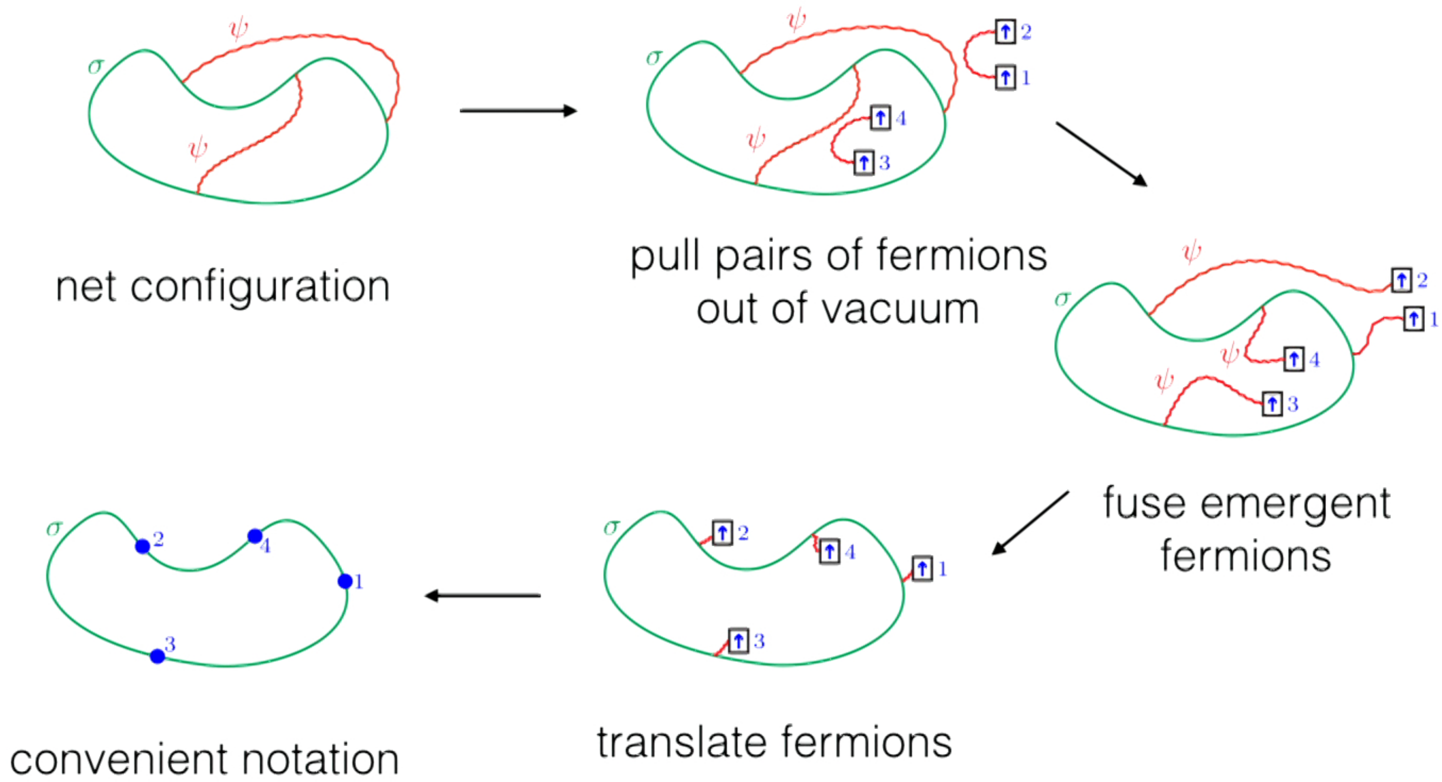


$\lambda \in \mathbb{C}$

Any net configuration can be reduced using these two relations to one that only has σ strands

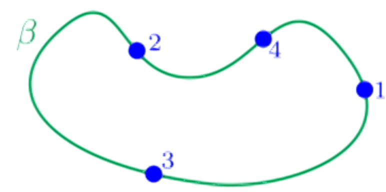


Example:


σ
ψ
ψ
net configuration
pull pairs of fermions out of vacuum
2
1
4
3
fuse emergent fermions
translate fermions
convenient notation

parent $\{\mathbb{1}, \sigma, \psi\}$ $\mathcal{C}$

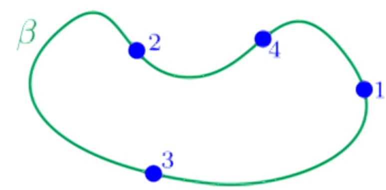
condensed $\{\mathbb{1} \cong \psi, \beta\}$ $\mathcal{C}/\psi$



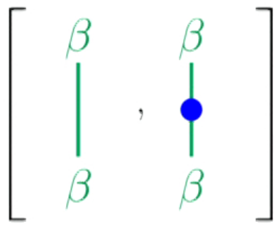
Fidkowski, Kitaev 1008.4138; Kapustin, Turzillo, You 1610.10075;
Bultinck et al. 1610.07849

parent $\{\mathbb{1}, \sigma, \psi\}$ $\mathcal{C}$

condensed $\{\mathbb{1} \cong \psi, \beta\}$ $\mathcal{C}/\psi$



β has an odd endomorphism

$\mathrm{End}(\beta) = \mathbb{C}$  $\cong \mathbb{C}\ell_1$

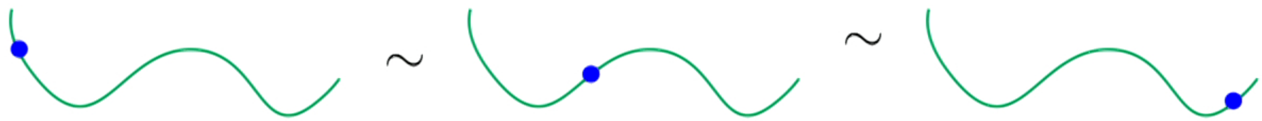
Clifford algebra

Objects of this type are intimately related to Kitaev chains and sometimes are referred to as Majorana objects

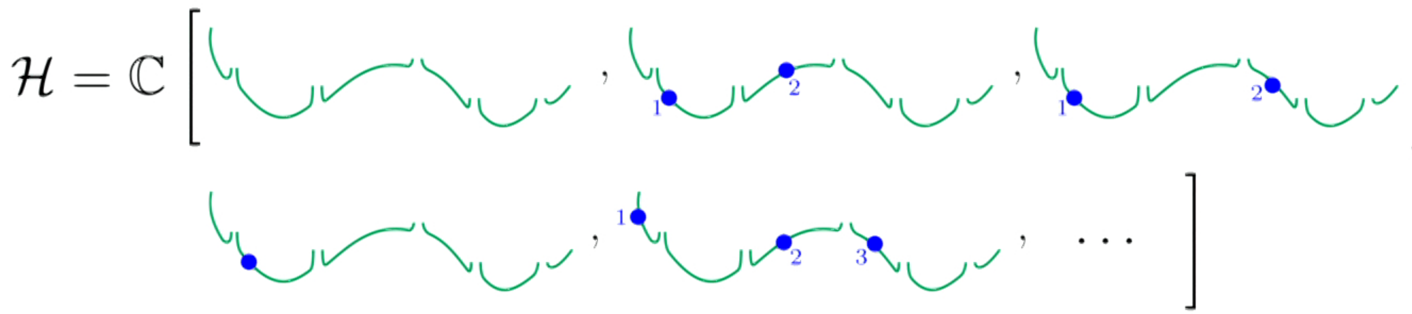
Fidkowski, Kitaev 1008.4138; Kapustin, Turzillo, You 1610.10075; Bultinck et al. 1610.07849

Can see this with a 1D “string-net”

Need a Hamiltonian which implements local relations such as:

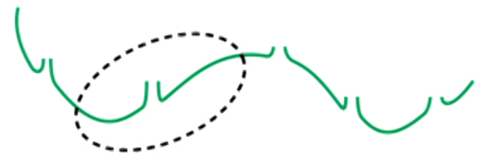


Cut the strand into many pieces and associate a vector space to possible labelings of each piece

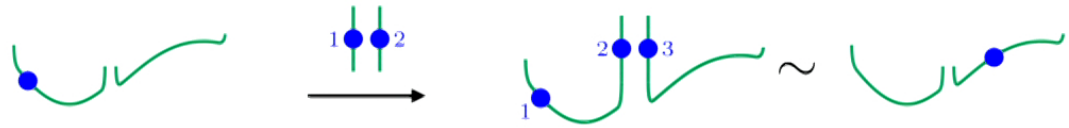


Each interval corresponds to a “site” and is labeled by its fermion parity

Local relations correspond to 'dot sliding' and 'cancellations'

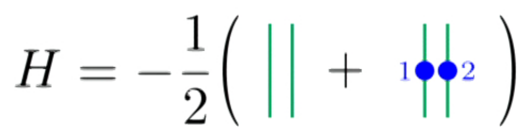


hopping



pairing

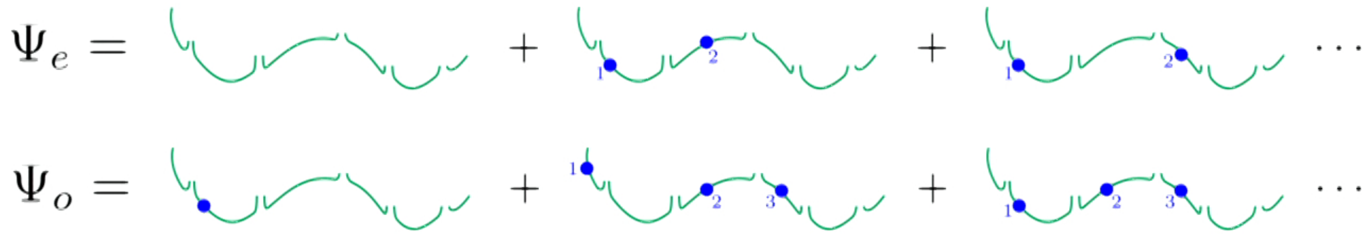
The Hamiltonian acts on the junctions by 'stacking'



This is just the 1D zero correlation length Kitaev wire

see DA, Lake, Walker for details

ground state wavefunctions are even(odd) parity super positions of fermions



can explicitly find Majorana zero mode operators

Satisfying the usual relations

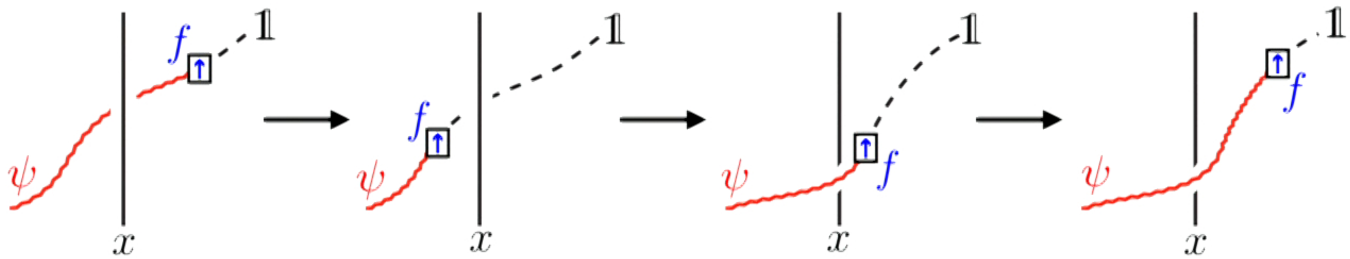
$$\gamma_L^2 \propto \gamma_R^2 \propto \mathrm{id} \qquad \{\gamma_L, F\} = \{\gamma_R, F\} = 0$$

$$\{\gamma_L, \gamma_R\} = 0 \qquad [\gamma_L, H] = [\gamma_R, H] = 0$$

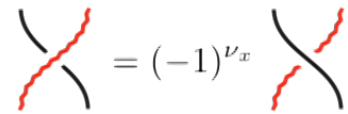
The same analysis carries through for any $\mathbb{C}\ell_1$ type object

Will shortly see similar results in (2+1)d

What about braiding in the condensed theory?
Consider the following isotopies:



Could differ by a minus sign if 'x' isn't transparent

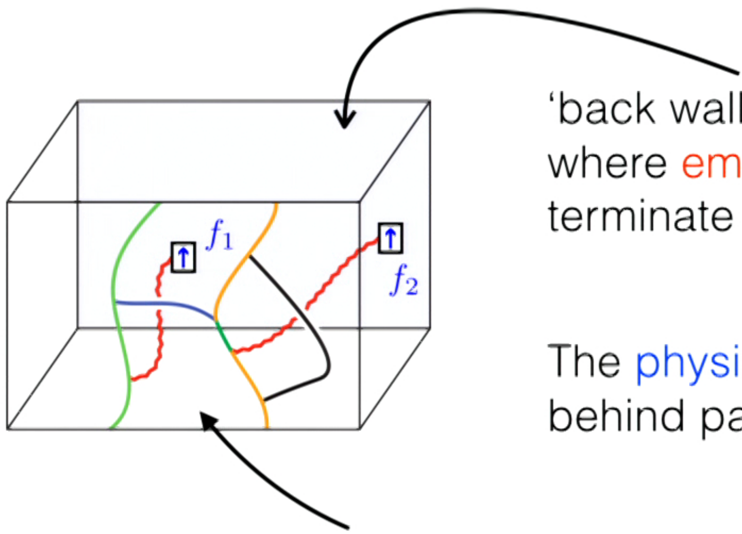


Three options:

1. ψ is transparent/particles with non-trivial braiding are confined
2. Require emergent vortices to bind physical vortices
3. Do 'back wall' condensation, only maintain a front braiding. This is equivalent to (2) on the Drinfeld center

'back wall' condensation: Technical tool for keeping track of fermion signs and spin structure data

We forget about all braiding data except for the emergent fermion



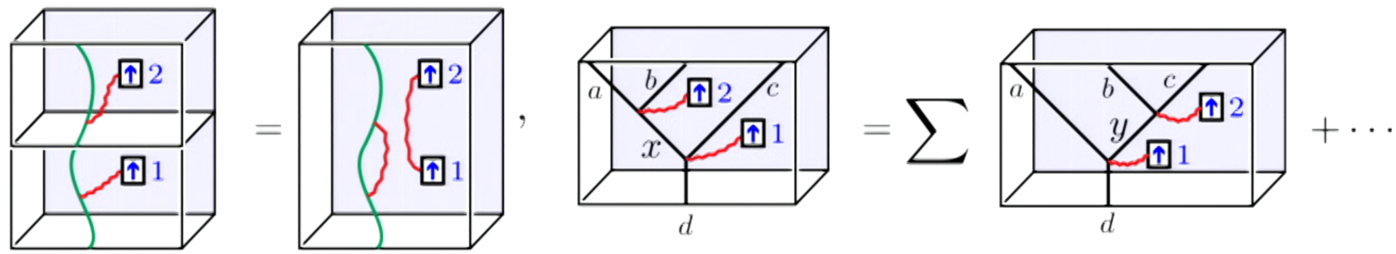
'back wall' is a defect to vacuum where emergent fermions can terminate on physical fermions

The physical fermions can only braid behind particles from $\mathcal{C}$

front contains net configurations of a fusion category

Can relax assumption of UBFC to UFC but require that $Z(\mathcal{C})$ contains an emergent fermion

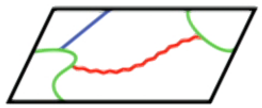
Composition is given by stacking, F-symbols are inherited from parent theory



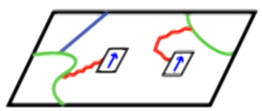
Remark:

- necessary to quotient by endomorphisms,
- i.e., allow dots to slide on $\mathbb{C}\ell_1$ type objects
- resulting fusion category is subject to various coherence conditions e.g., pivoting, F-symbols,…

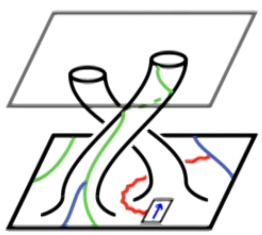
Outline:



introduce gapped (2+1)d topological phases and (braided) fusion categories $\mathcal{C}$



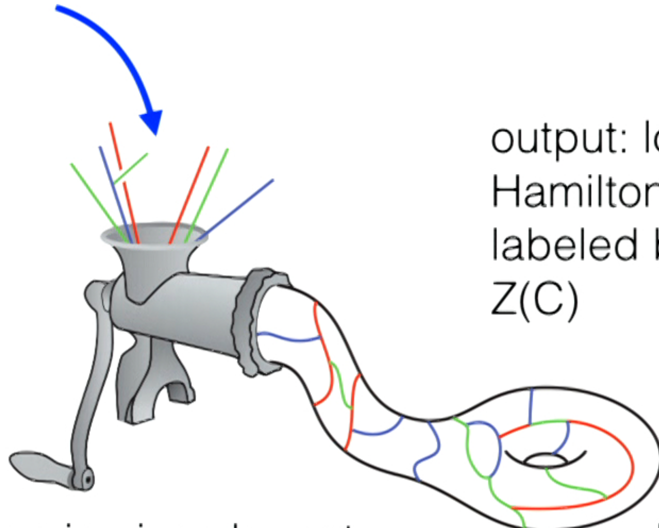
fermion condensation, Ising example and 1D ‘string-net’ $\mathcal{C}/\psi$



fermion condensation in string-net models and quasiparticle content $\mathrm{Tube}(\mathcal{C}/\psi)$

String-nets Levin, Wen 0404617

input: fusion category and surface



output: local gapped Hamiltonian; excitations labeled by simple objects in Z(C)

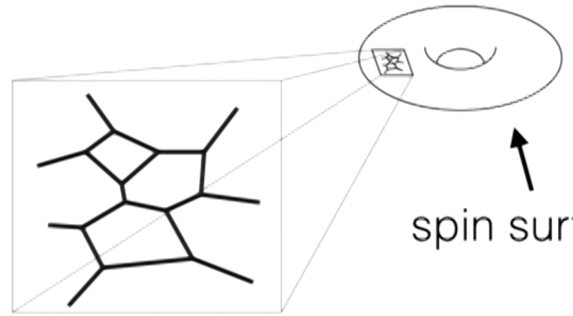
Hilbert space is given by net configurations/local relations

Hamiltonian implements local relations on nets

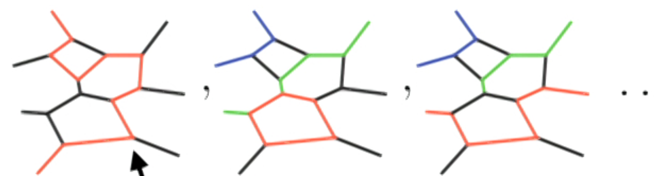
- require $\mathcal{Z}(\mathcal{C})$ to have an emergent fermion
- input is $\mathcal{C}/\psi$ and spin surface

Gu, Wang, Wen 1309.7032; Gu, Wang, Wen 1010.1517; Bhardwaj, Gaiotto, Kapustin 1605.01640

Lattice = trivalent graph, dual to a triangulation →



Hilbert space = collection of coloured graphs, `string-nets'



Each vertex is coloured by a basis vector of the fusion space

$$\mathcal{H} \cong \otimes_v \mathcal{H}_v \qquad \mathcal{H}_v \cong \bigoplus_{abc} V^{abc}$$

graded tensor product

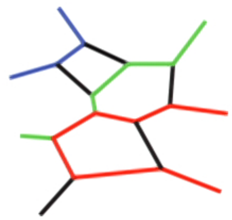
super vector space

The Hamiltonian is a sum of three projectors

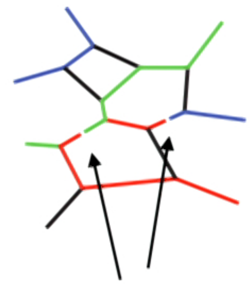
$$H = -J_1 \sum_p B_p - J_2 \sum_e D_e - J_3 \sum_v A_v$$

local relations (B_p, D_e) — Fusion rules (A_v)

Vertex term projects onto net configurations satisfying the fusion rules



ground space of vertex term

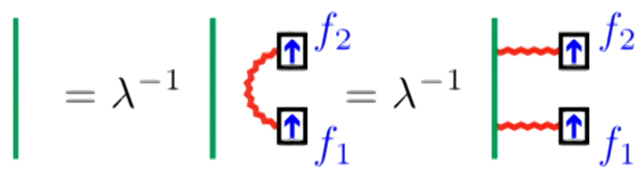


excitation

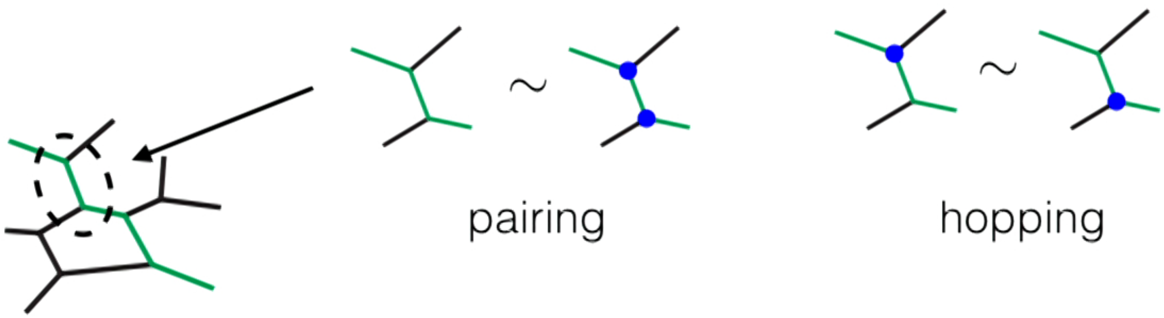
$$H = -J_1 \sum_p B_p - J_2 \sum_e D_e - J_3 \sum_v A_v$$

local relations (B_p, D_e)   Fusion rules (A_v)

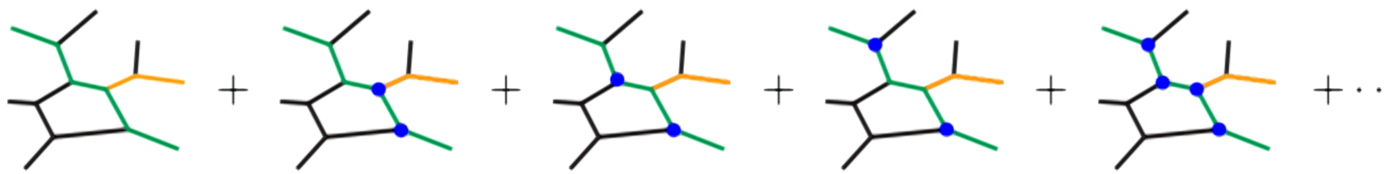
edge term enforces the extra linear relation coming from condensation



only non-trivial on edges with $\mathbb{C}\ell_1$-type objects, provides dynamics to the fermions



ground states of the vertex and edge term have equal weight superpositions of fermions along $\mathbb{C}\ell_1$ type edges



Coefficients are phases that depend on sign ordering and spin structure

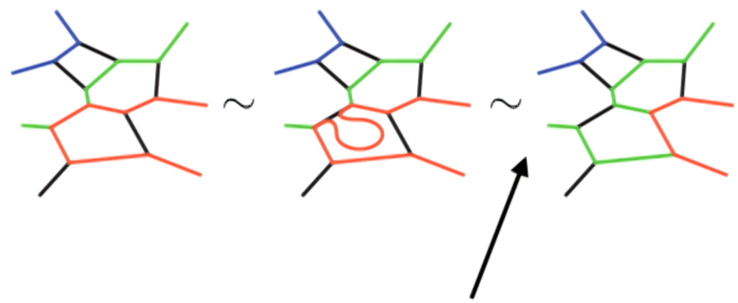
The edge terms can be written with dressed Majorana operators and describe a zero correlation length Kitaev/Majorana chain

Related models: Ware, Son, Cheng, Mishmash, Alicea, Bauer 2016
Tarantino, Fidkowski 2016

The plaquette term is an omega loop written in the basis of nets

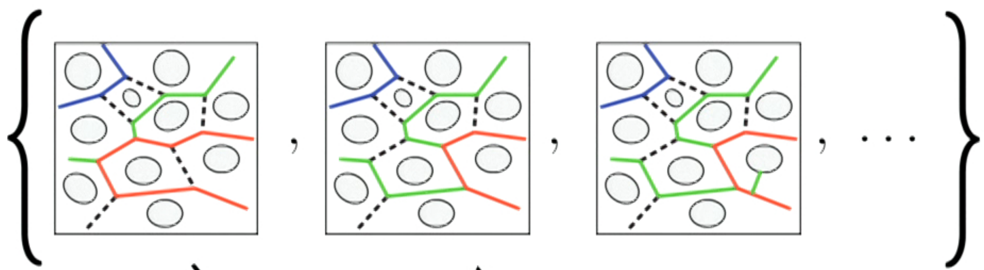
$$\bigcirc_{\omega} = \frac{1}{\mathcal{D}^2} \sum_{x \in \mathcal{C}/\psi} \frac{d_x}{n_x} \circlearrowright_{x} \qquad n_x = \begin{cases} 1 & \text{if } \mathrm{End}(x) \cong \mathbb{C} \\ 2 & \text{if } \mathrm{End}(x) \cong \mathbb{C}\ell_1 \end{cases}$$

action of plaquette term B_p on the nets



coefficients depend on F-symbols

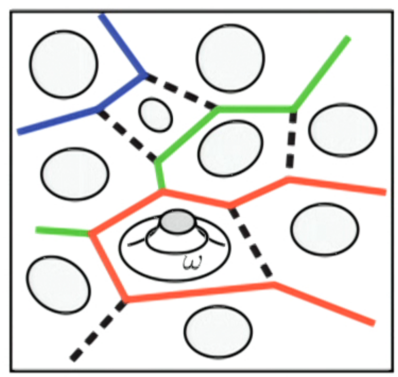
String-net Hilbert space/Hamiltonian can be written as

Basis 

$H = \sum \omega$ 

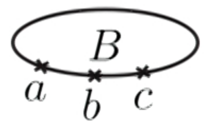
simplify using F-symbols

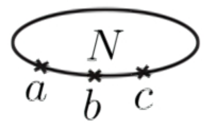
Hamiltonian is the trivial idempotent of the tube category



Excitations can be analyzed with a fermionic version of the tube category $\mathrm{Tube}(\mathcal{C}/\psi)$ Lan, Wen 1311.1784; Kirillov 1106.6033

objects are spin circles with marked points labeled by objects in $\mathcal{C}/\psi$

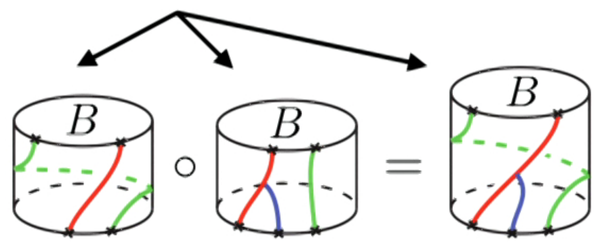




B = bounding = anti-periodic b.c.'s = non-vortex
N = non-bounding = periodic b.c.'s = vortex

morphisms are nets on tubes with fixed boundary conditions

composition is stacking



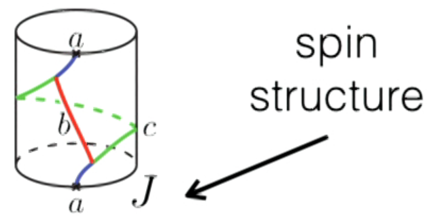
Ocneanu 1994

Isomorphism classes of excitations are in 1-1 correspondence with isomorphism classes of minimal idempotents of $\mathrm{Tube}(\mathcal{C}/\psi)$

In particular, the Hamiltonian projects onto the trivial idempotent

in general minimal idempotents can be written as:

$$e_i = \sum_{bc} t_{i,bc}$$



$$e_i \cdot e_i = \delta_{ij} e_j$$

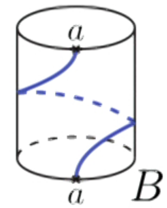
$$e_i \cdot \mathrm{Ann}(\mathcal{C}/\psi) \cdot e_i \cong \begin{cases} \mathbb{C}\ell_1 & \text{if } e_i \text{ has an odd endomorphism} \\ \mathbb{C} & \text{otherwise} \end{cases}$$

Provides a very convenient way to understand the excitations (fusion rules, F-symbols, braiding, S-matrix, etc..)

Analyze bounding spin structure first:

Natural question:
can the bounding idempotents be $\mathbb{C}\ell_1$ type?

Dehn twist with bounding spin structure anti-commutes with all odd operators, but commutes with minimal idempotents

$T =$ 

Implies that oddly isomorphic bounding idempotents will have twist eigenvalues that differing by a minus sign

$$Te_i = \theta_i e_i$$

$$T(\Gamma e_i \Gamma) = -\theta_i (\Gamma e_i \Gamma)$$

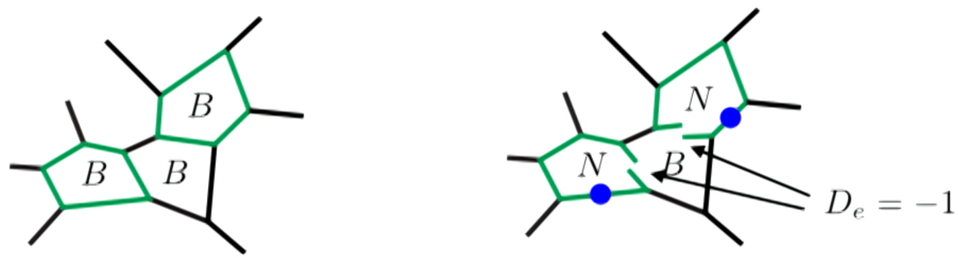
In particular, they can never have an odd endomorphism

Bounding idempotents are anyons

Now lets consider the non-bounding idempotents

These correspond to vortices (non-bounding spin stucture in the string-net)

pairs can be created from the vacuum by violating an edge term

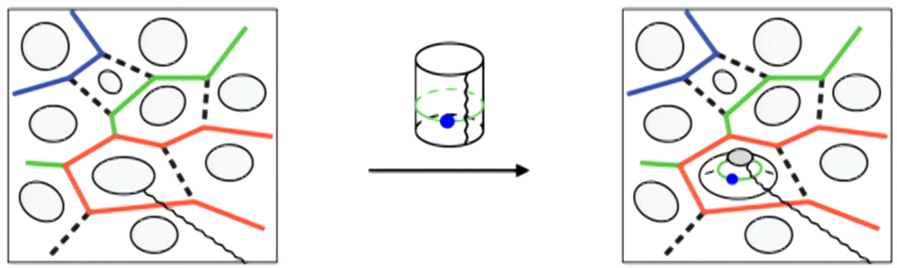


Energy to separate them grows linearly with distance (assuming there is atleast one $\mathbb{C}\ell_1$-type object in $\mathcal{C}/\psi$)

The vortices are confined

The vortices can be either $\mathbb{C}$ or $\mathbb{C}\ell_1$ type

$\mathbb{C}\ell_1$ type vortices are similar to vortices in a p+ip SC in that they harbour Majorana zero modes

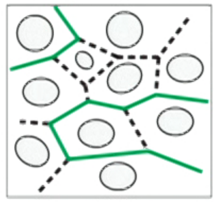
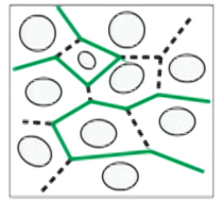
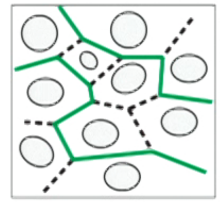


As usual, pairs of vortices always fuse to non-vortices

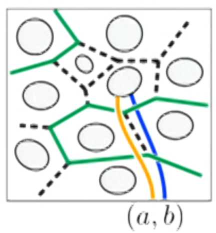
consequently two $\mathbb{C}\ell_1$ type objects always fuse to a non $\mathbb{C}\ell_1$ type object. This is not necessarily true in the fusion categories (example is $(E_6/2)/\psi$)

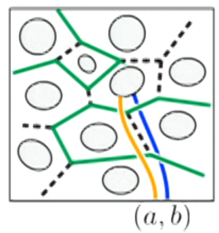
Example: Input: Ising/ψ two simple objects: $\mathbb{1}, \beta$

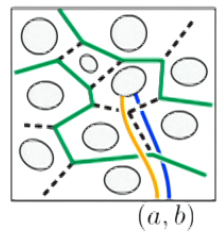
ground state is superposition of β loops

$\Psi = \cdots +$  $+$  $+$  $+ \cdots$

Can show that if $\mathcal{C}$ is modular then $\mathrm{Tube}(\mathcal{C}/\psi) \cong \mathcal{C} \times \mathcal{C}/\psi$

$\Psi' = \cdots +$ 

$+$ 

$+$ 

$+ \cdots$

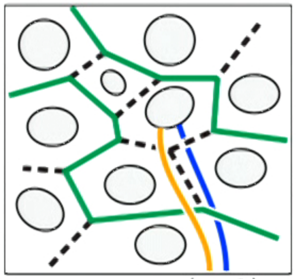
anyons: $(\mathbb{1}, \mathbb{1})$ $(\sigma, \mathbb{1})$ $(\psi, \mathbb{1})$

vortices: $(\mathbb{1}, \beta)$ (σ, β) (ψ, β)

See also: Ware, Son, Cheng, Mishmash, Alicea, Bauer 2016

Summary of excitations:

(Two types of objects) x (two spin structures)



Quasiparticles	non-vortices	vortices
$\mathbb{C}$ - type	deconfined	confined
$\mathbb{C}\ell_1$ - type	does not exist	confined

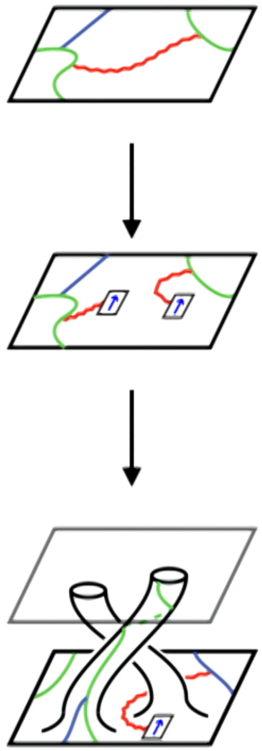
The non-vortices are deconfined anyonic excitations

Vortices are defects/confined

$\mathbb{C}\ell_1$ type objects are intimately related to Majorana zero modes

Can show that if $\mathcal{C}$ is modular then $\mathrm{Tube}(\mathcal{C}/\psi) \cong \mathcal{C} \times \mathcal{C}/\psi$

Outline:



introduce gapped (2+1)d topological phases and (braided) fusion categories $\mathcal{C}$

fermion condensation, Ising example and 1D 'string-net' $\mathcal{C}/\psi$

fermion condensation in string-net models and quasiparticle content $\mathrm{Tube}(\mathcal{C}/\psi)$