Title: Floquet phases of matter: time crystals and beyond

Speakers: Dominic Else

Series: Condensed Matter

Date: December 05, 2017 - 3:30 PM

URL: https://pirsa.org/17120006

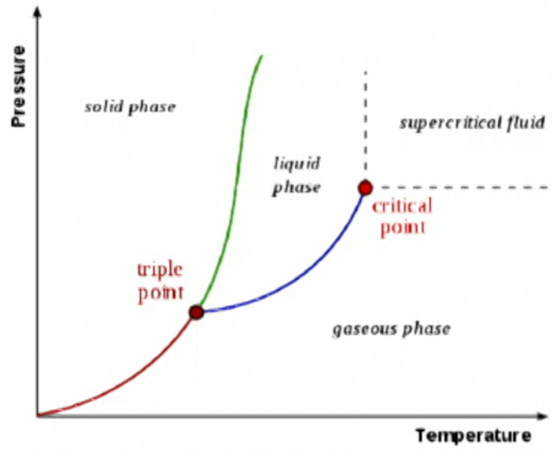
Abstract: Periodically driven (Floquet) systems can display entirely new many-body phases of matter that have no analog in stationary systems. One such phase is the Floquet time crystal, which spontaneously breaks a discrete time-translation symmetry. In this talk, I will survey the physics of these new phases of matter. I explain how they can be stabilized either through strong quenched disorder (many-body localization), or alternatively in clean systems in a "prethermal" regime which persists until a time that is exponentially long in a small parameter.

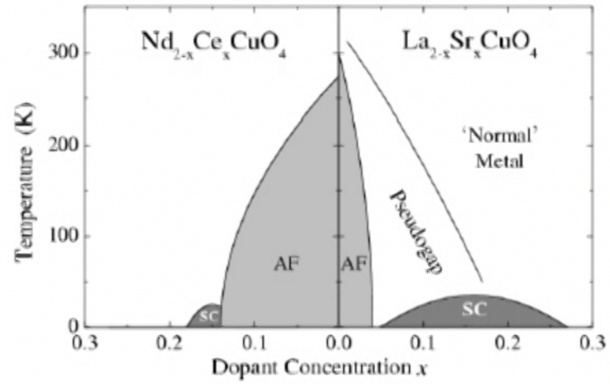
Floquet phases of matter: time crystals and beyond

Dominic V. Else

Department of Physics
University of California, Santa Barbara

Phases of matter



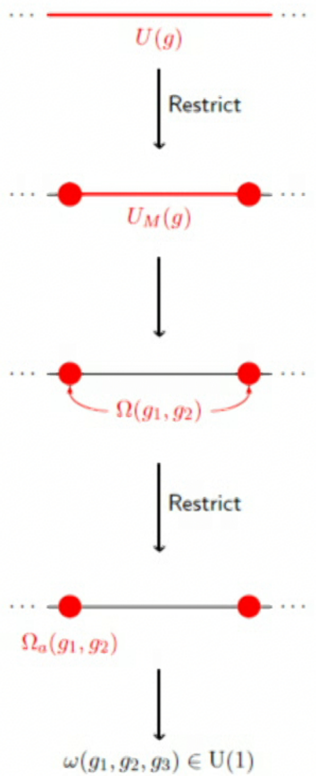


Damascelli, Hussain, Shen, RMP '03

- Landau phases: spontaneously broken symmetry
- Topological phases:
  - Fractional quantum Hall effect, spin liquids, topological insulators ...

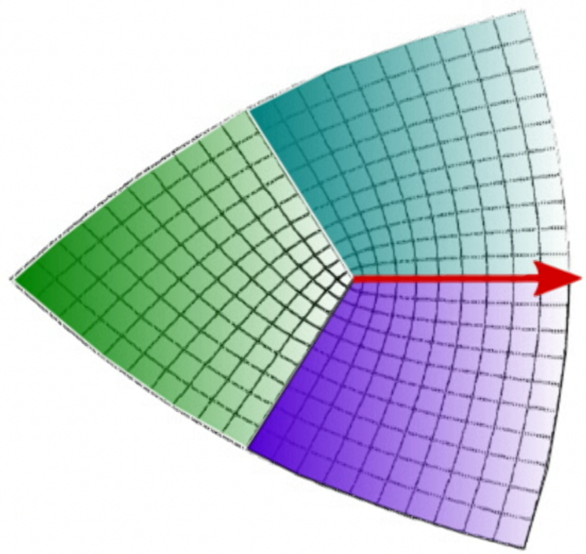
Aspects of topological phases

Symmetry action on the boundary



DVE and Nayak, PRB '14

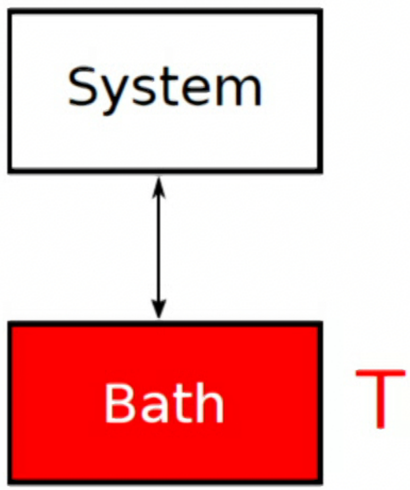
Understanding crystalline topological phases by "gauging" spatial symmetries.



Thorngren and DVE, 1612.00846

Systems out of equilibrium

Thermal equilibrium



$$\rho = \frac{1}{Z} e^{-\beta H}$$

Isolated systems

Isolated system

System

$$|\Psi(t)\rangle = e^{-itH}|\Psi(0)\rangle$$

Send $t \to \infty$, look at local observables

- Thermalization

$$\rho_{\text{steady}} = \frac{1}{Z}e^{-\beta H}$$

Deutsch, PRA '91
Srednicki, PRE '94, J. Phys. '99
Rigol et al, Nature '08

- Non-thermalization

$$\rho_{\text{steady}} \neq \frac{1}{Z}e^{-\beta H}$$

  - Integrable systems
  - Many-body localization
  - Prethermalization

Floquet systems

Isolated system

System

"Floquet"

$$i\frac{d|\Psi(t)\rangle}{dt} = H(t)|\Psi\rangle,$$

$$H(t+T) = H(t)$$

Send $t \to \infty$, look at local observables

- Thermalization

$$\rho_{\text{steady}} = \frac{1}{D}\mathbb{I}$$

- Non-thermalization

$$\rho_{\text{steady}} \neq \frac{1}{D}\mathbb{I}$$

  - Integrable systems
  - Many-body localization
  - Prethermalization
- Floquet systems have entirely new phases of matter!

Section 2

Floquet time crystals

Time crystals

Time crystals

Time crystal A system in which time translation symmetry is spontaneously broken.

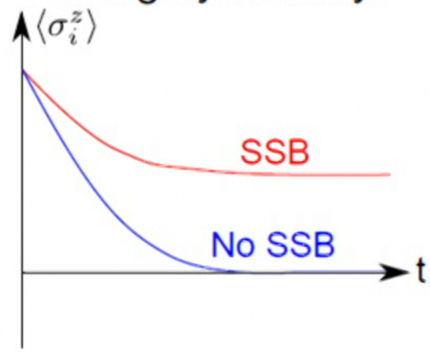
Wilczek, PRL '12
Shapere and Wilczek, PRL '15

- Time translation symmetry $\leftrightarrow$ Hamiltonian H is time-independent.
- Time crystal: In thermal equilibrium, the state of the system still oscillates in time.
- No-Go Theorem: Watanabe and Oshikawa, PRL '13

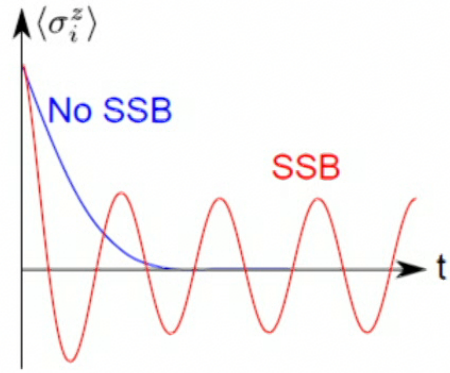
Spontaneous symmetry breaking out of equilibrium

- Consider the **long-time** ($t \to \infty$) state of an isolated many-body system.
- Spontaneous symmetry breaking (SSB)

Ising symmetry



Time-translation symmetry



Floquet time crystals

- "Floquet" system.

$$i\frac{d}{dt}|\psi\rangle = H(t)|\psi\rangle$$

$$H(t+T) = H(t)$$

- **Discrete** time-translation symmetry

Floquet time crystals

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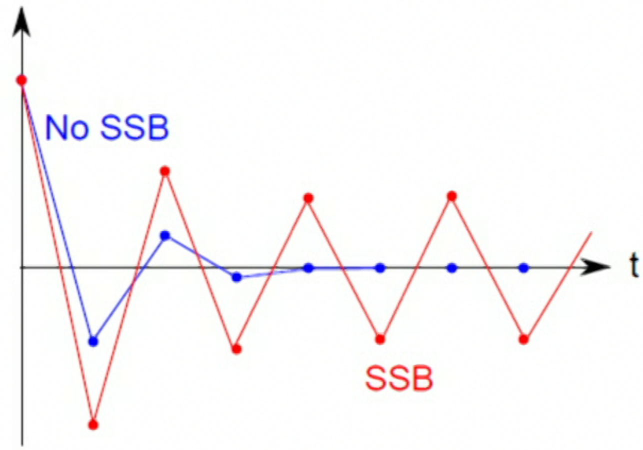

Key features of time crystals

- **Many-body**.
  - ~~Harmonic oscillator~~
- **Rigid Oscillations**
  - Infinite coherence time.
    - ~~Lasers~~
  - Stable to perturbations
- **Reversible**. (No entropy generation)
  - ~~Damped driven pendulum~~
- **Doesn't piggy-back on other spontaneously broken symmetries**.
  - ~~Frictionless pendulum~~
  - ~~AC Josephson effect~~.

Spontaneous symmetry breaking in quantum systems

- Transverse-field quantum Ising model

$$H = -J_{i,j} \sum_{\langle i,j \rangle} \sigma_i^z \sigma_j^z + h \sum_i \sigma_i^x, \qquad h \ll J$$

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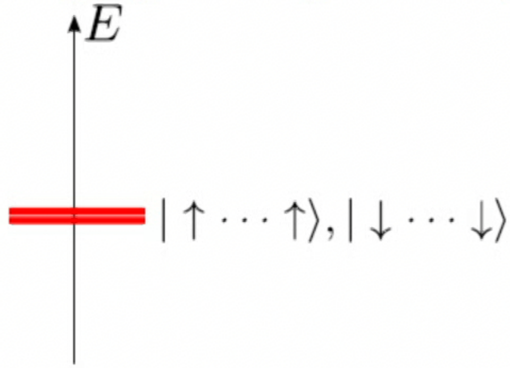

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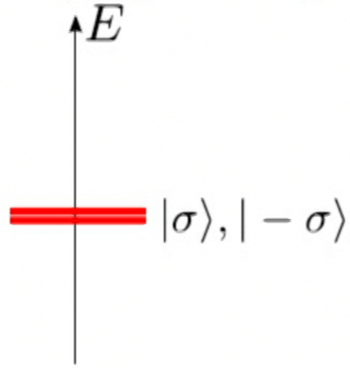

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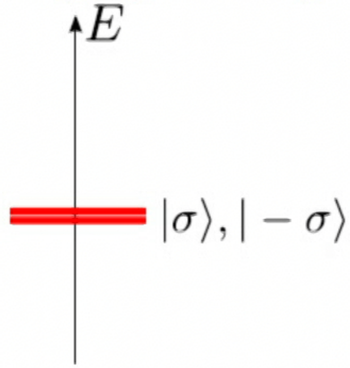

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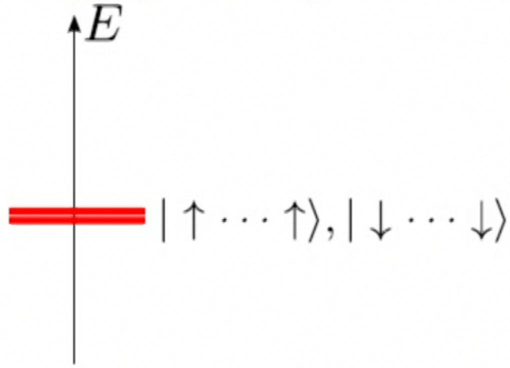

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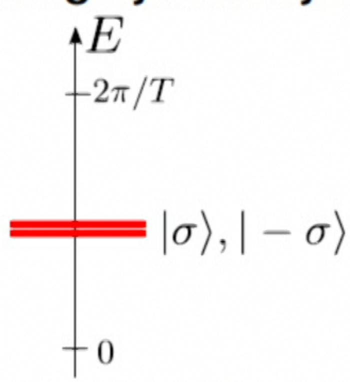
- Floquet systems

$$U_f = \mathcal{T}\left(-i \int_0^T H(t)dt\right)$$

- Eigenvalues of U_f are of the form $e^{i\omega T}$; ω is the *quasienergy*.

Ising symmetry-breaking



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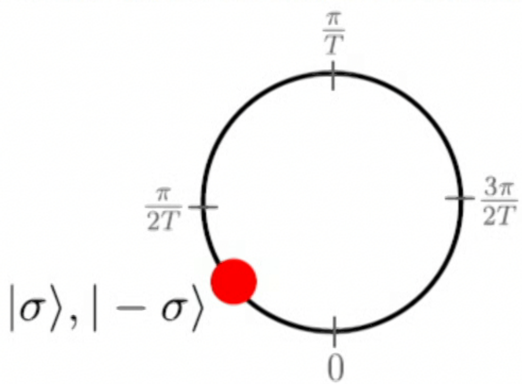

Spontaneous symmetry breaking in quantum systems

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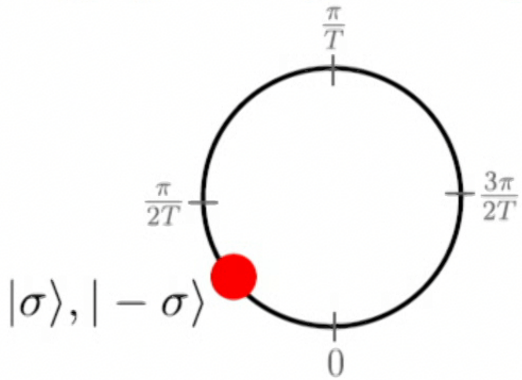
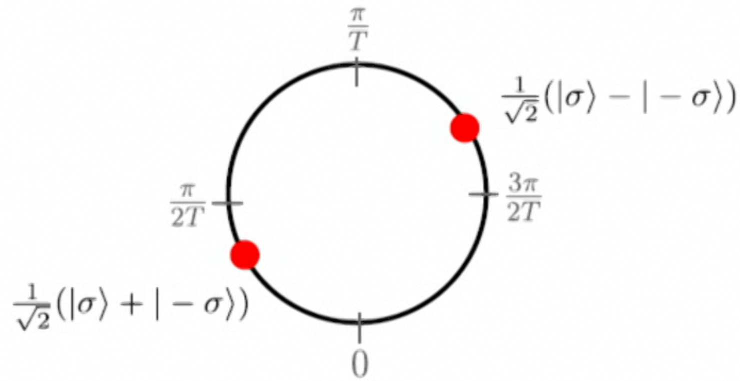
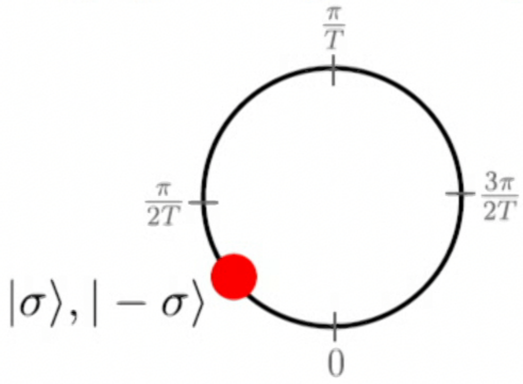


Time crystal



Spontaneous symmetry breaking in quantum systems

Ising symmetry-breaking



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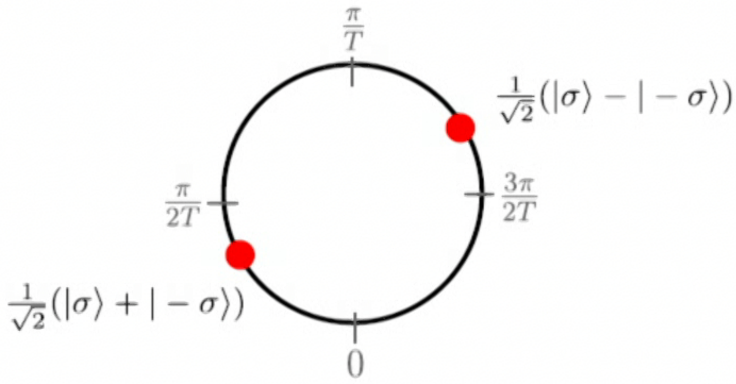
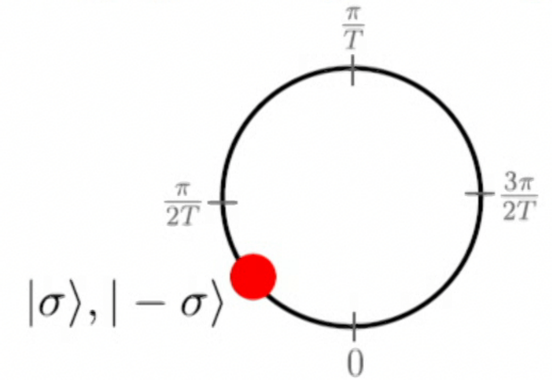
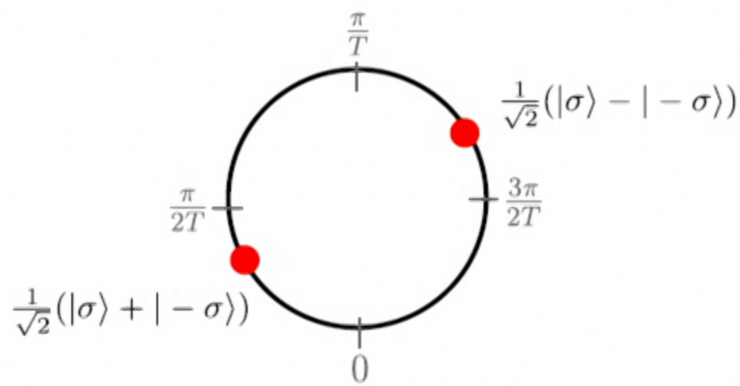


- **Time Crystal Paradox**: Eigenstates of U_f never exhibit period doubling.
- Resolution: Eigenstates are unphysical!

Spontaneous symmetry breaking in quantum systems





- **Time Crystal Paradox**: Eigenstates of U_f never exhibit period doubling.
- Resolution: Eigenstates are unphysical!
- $$U_f \propto |\sigma\rangle\langle-\sigma| + |-\sigma\rangle\langle\sigma| + \cdots$$

0-spin-glass and π-spin-glass

Khemani et al, PRL '16

- Take an MBL Hamiltonian D which has an Ising symmetry $X = \prod_i \sigma_i^x$ and spontaneously breaks it ("**spin glass**").

$$D = -\sum_{\langle i,j \rangle} J_{i,j} \sigma_i^z \sigma_j^z \qquad [H, X] = 0$$

0-spin-glass

$$H(t) = D$$

π-spin-glass

$$H(t) = \begin{cases} D & 0 < t < T/2 \\ \frac{\pi}{4T} \sum_i \sigma_i^x & T/2 < t < T \end{cases}$$

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Khemani et al, PRL '16

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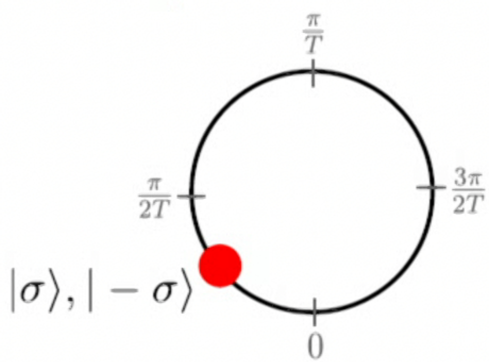
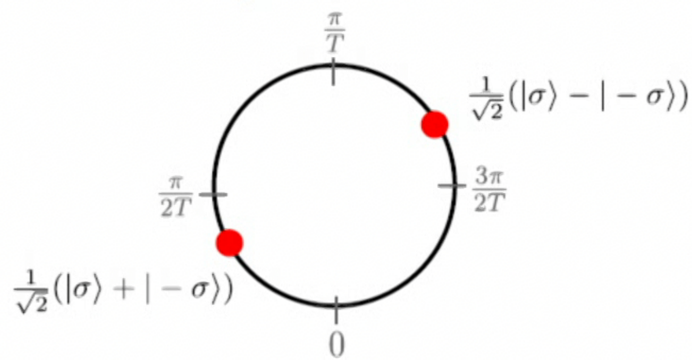
0-spin-glass

$U_f = e^{-iDT}$



π-spin-glass

$U_f = X e^{-iDT}$



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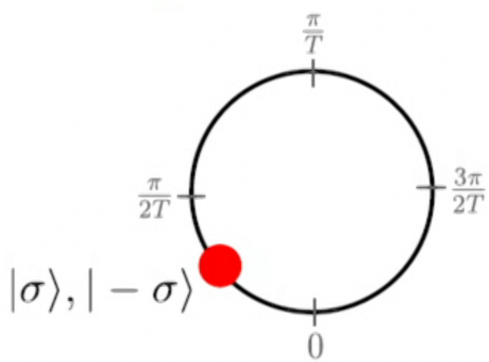

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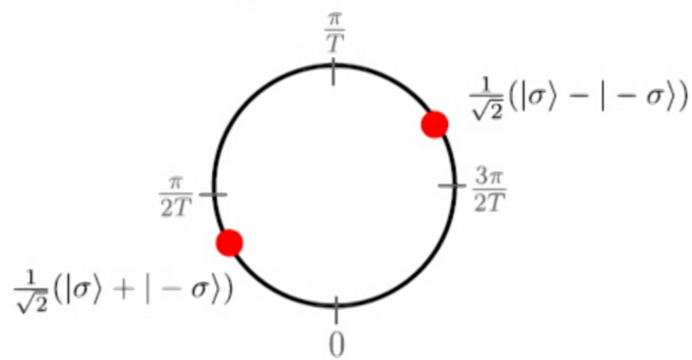


Numerical results

DVE, Bauer and Nayak, PRL '16

- $U_\mathrm{f} = e^{-it_0 H_0} X, \qquad H_0 = -\sum_i J_i \sigma_i^z \sigma_{i+1}^z - \sum_i h_i \sigma_i^z - \sum_i h_i^x \sigma_i^x$
  - $J_i \in [0.5, 1.5]$, $h_i \in [0, 1]$, $h_i^x \in [0, 0.3]$

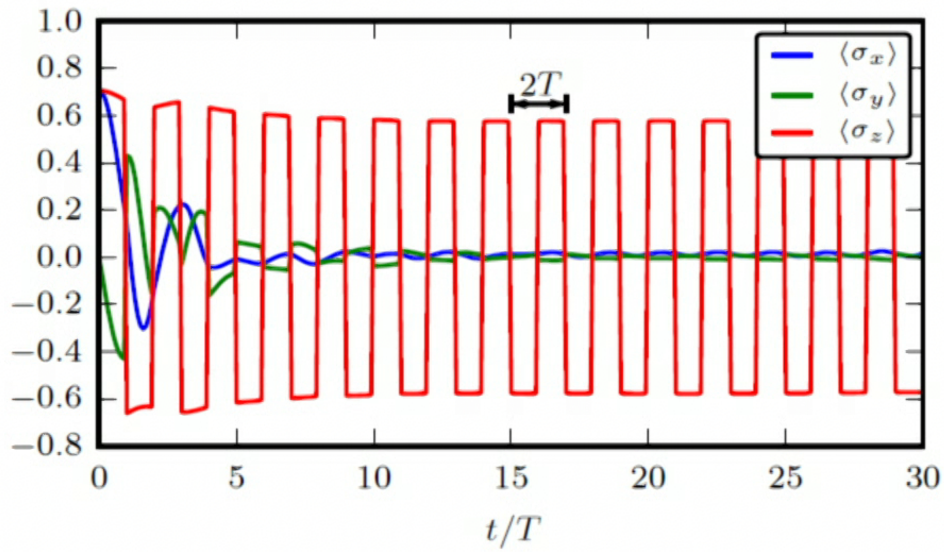


Figure: TEBD simulation for a 1-D chain of length 200. (Disorder- and spatially- averaged.)

Experimental realizations of Floquet time crystals

Theoretical proposal: Yao, Potter, Potirniche, Vishwanath, PRL '17

Experimental realization (in cold atoms):

LETTER

doi:10.1038/nature21413

Observation of a discrete time crystal

J. Zhang[1], P. W. Hess[1], A. Kyprianidis[1], P. Becker[1], A. Lee[1], J. Smith[1], G. Pagano[1], I.-D. Potirniche[2], A. C. Potter[3], A. Vishwanath[2,4], N. Y. Yao[2] & C. Monroe[1,5]

A different system (NV centers):

LETTER

doi:10.1038/nature21426

Observation of discrete time-crystalline order in a disordered dipolar many-body system

Soonwon Choi[1*], Joonhee Choi[1,2*], Renate Landig[1*], Georg Kucsko[1], Hengyun Zhou[1], Junichi Isoya[3], Fedor Jelezko[4], Shinobu Onoda[5], Hitoshi Sumiya[6], Vedika Khemani[1], Curt von Keyserlingk[7], Norman Y. Yao[8], Eugene Demler[1] & Mikhail D. Lukin[1]

Theoretical explanation ("critical time crystals"):

Ho, Choi, Lukin, Abanin, PRL '17

Section 3

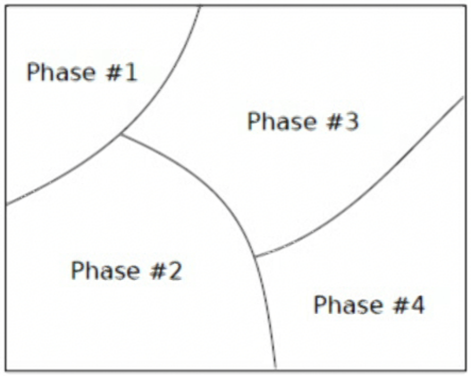
Floquet topological phases

A general picture: stationary phase

A (stationary) topological phase has a unique, gapped ground state.

- Implies short-ranged correlations, area law for entanglement entropy, ...)

Introduce a space Ω of "good" states.



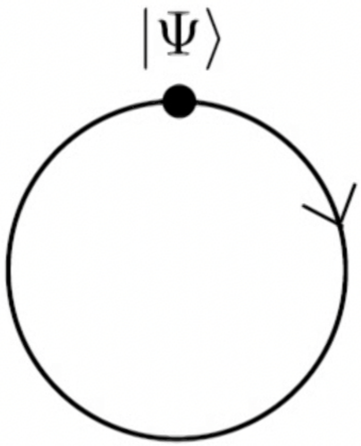
- Phase of matter $\leftrightarrow$ connected component of Ω.
- $\pi_0(\Omega)$.

A general picture: Floquet phases

Consider some eigenstate $|\Psi\rangle$ of the Floquet evolution operator

$$U_f = \mathcal{T} \exp\left(-i \int_0^T H(t) dt\right).$$

Then $|\Psi(t)\rangle$, $t \in [0, T]$ defines a *loop* in Ω.



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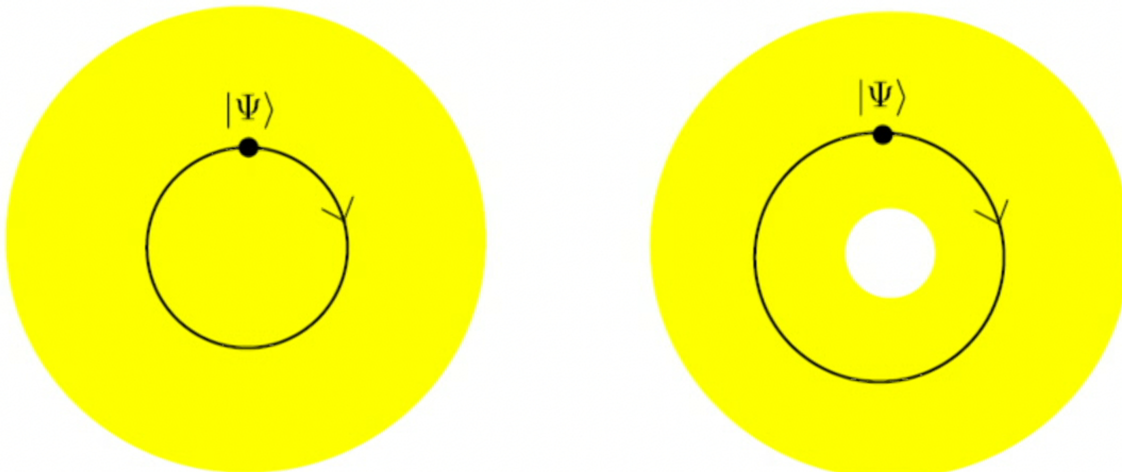


The Floquet equivalence principle

A general framework based on topological quantum field theory (TQFTs)

Thorngren and DVE, 1612.00846

Floquet equivalence principle

Floquet phases of matter

$\leftrightarrow$

Stationary phases with an internal $\mathbb{Z}$ symmetry

DVE and Nayak, PRB '16

Potter, Morimoto, Vishwanath, PRX '16

- Connection to *symmetry-protected topological* (SPT) or *symmetry-enriched topological* (SET) phases.

Floquet pumping

DVE and Nayak, PRB '16

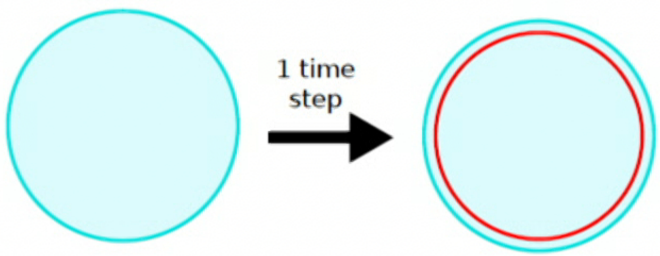
- Bosonic Floquet-SPT's (with unitary symmetries) classified by

$$H^{d+1}(\mathbb{Z} \times G, U(1)) = H^{d+1}(G, U(1)) \times H^{d}(G, U(1))$$

Stationary classification in *d* dimensions: ω_{d+1}

Stationary classification in *d*-1 dimensions: ω_d

- Interpretation: a lower-dimensional SPT gets *pumped* to the boundary at each time step.



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The Floquet bootstrap

DVE and Nayak, PRB '16

The "Floquet bootstrap" is a general way to construct models realizing Floquet phases of matter.

- H has an internal $\mathbb{Z}_N$ symmetry generated by X.
- $U_{\mathrm{f}} = X e^{-iHT}$.

Section 4

Prethermal Floquet phases

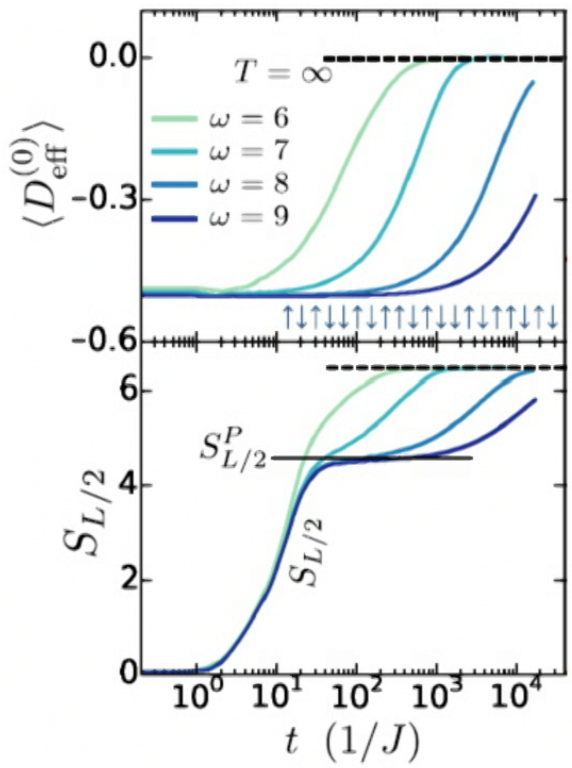
Prethermalization in Floquet systems

Can we stabilize Floquet phases without MBL?

Prethermalization:

Kuwahara et al, Ann. Phys. '16
Abanin et al, Commun. Math. Phys. '17

- If $\omega \gg J$, then there exists an *effective Hamiltonian* H_{eff} such that $U_{\text{f}} \approx e^{-iH_{\text{eff}}T}$.
- Approximation valid until times $t_* \sim e^{J/\omega}$.



Machado, Meyer, DVE, Nayak, Yao, 1708.01620

Prethermal stability of the Floquet bootstrap

The bootstrap construction:

$$U_f = X e^{-iDT}, \quad [X, D] = 1, \quad X^N = 1$$

This implies that $U_f^N = e^{-iNDT}$.

DVE, Bauer, Nayak, PRX '16

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DVE, Bauer, Nayak, PRX '16

- Now add a time-dependent perturbation

$$H(t) = H_0(t) + V(t)$$

.

  - Let J = local energy scale of D, λ = local energy scale of $V(t)$.

Prethermal stability of the Floquet bootstrap

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DVE, Bauer, Nayak, PRX '16

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- Let J = local energy scale of D, λ = local energy scale of $V(t)$.

Theorem

If $J \ll \omega$, then there exists a local unitary rotation $\mathcal{U}$ such that

$$\mathcal{U} U_f' \mathcal{U}^\dagger = X e^{-i\widetilde{D}T} + O(e^{-\omega/J})$$

where

$$\widetilde{D} = D + O(\lambda), \quad [\widetilde{D}, X] = 0$$

There exists a (time-independent) unitary $\mathcal{U}$ such that

$$\mathcal{U}\, U_{\mathrm{f}} \mathcal{U}^{\dagger} = X\, \mathcal{T} \exp\left(-i \int_0^T [D + E + V(t)] dt\right)$$

where D is local and $[D, X] = 0$; D, E are independent of time; and

$$\|V\|_{n_*} \leq \lambda \left(\frac{1}{2}\right)^{n_*}$$

$$\|E\|_{n_*} \leq \lambda \left(\frac{1}{2}\right)^{n_*}$$

The exponent n_* is given by

$$n_* = \frac{\lambda_0/\lambda}{[1 + \log(\lambda_0/\lambda)]^3}, \quad \lambda_0 = \frac{(\kappa_1)^2}{72(N+3)(N+4)T}$$

Furthermore,

$$\|D - \overline{V}\|_{n_*} \leq \mu(\lambda^2/\lambda_0), \quad \mu \approx 2.9,$$

where

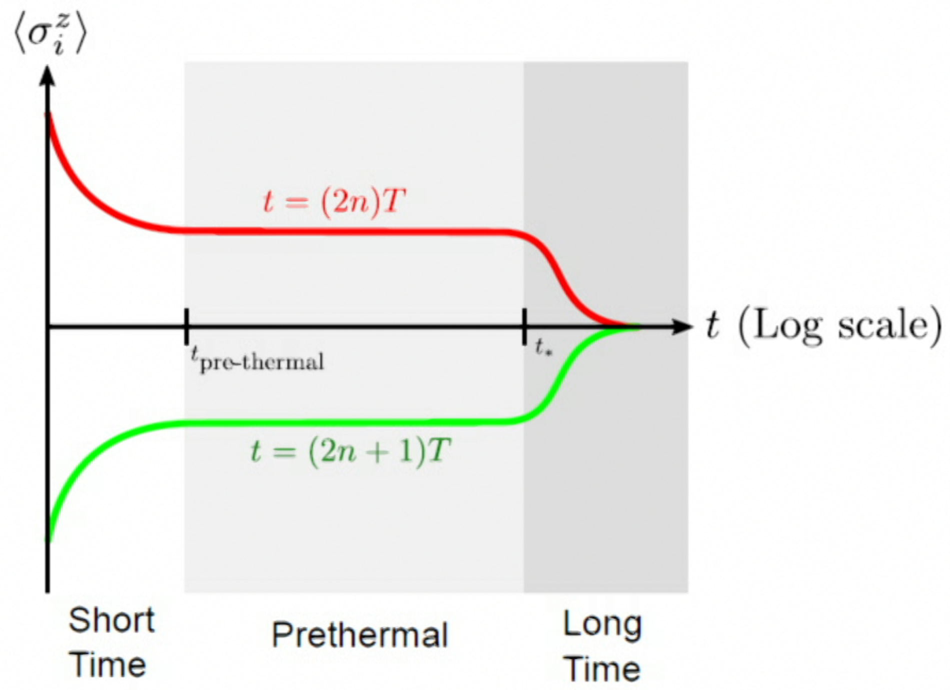
$$\overline{V} = \frac{1}{NT} \int_0^{NT} V^{\mathrm{int}}(t) dt$$

$$= \frac{1}{N} \sum_{k=0}^{N-1} X^{-k} \left(\frac{1}{T} \int_0^T V^{\mathrm{int}}(t) dt\right) X^k.$$

Prethermal Floquet time crystal

DVE, Bauer and Nayak, PRX '16

$$\mathcal{U} U_f' \mathcal{U}^\dagger = X e^{-i\tilde{D}T} + O(e^{\omega/J})$$



- t_* is exponentially long in JT.
- **(Prethermal) Floquet time crystal**

Thank you!

At Microsoft Station Q

Bela Bauer

Chetan Nayak

At Berkeley

Ryan Thorngren

Francisco Machado

Gregory Meyer

Norman Yao