Title: Beyond Topological Order: Fractons and their Field Theory

Date: Dec 01, 2017 11:00 AM

URL: http://pirsa.org/17120005

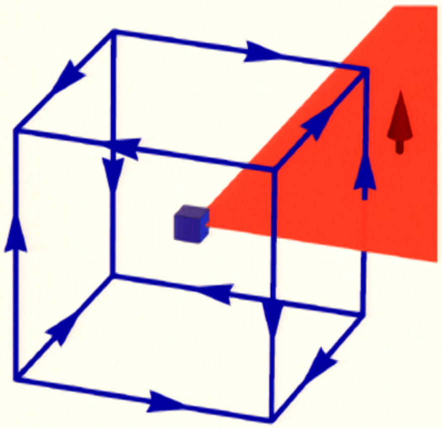
Abstract: <p>"Recently, exactly solvable 3D lattice models have been discovered for a new kind of phase, dubbed fracton topological order, in which the topological excitations are immobile or are bound to lines or surfaces. Unlike liquid topologically ordered phases (e.g. Z_2 gauge theory), which are only sensitive to topology (e.g. the ground state degeneracy only depends on the topology of spatial manifold), fracton orders are also sensitive to the geometry of the lattice. This geometry dependence allows for remarkably new physics which was forbidden in topologically invariant phases of matter.</p>

<p>
In this talk, I will review the X-cube model [1] of fracton order. I will then explain how geometry dependence allows for braiding of point-like particles in this 3D phase. I'll summary how the X-cube model can be described by a quantum field theory, which is analogous to a topological quantum field theory (TQFT). [3] We will see that the gauge invariance of the field theory results in the mobility restrictions of the topological excitations by imposing a new kind of geometric charge conservation. I will conclude by briefly discussing current work on the remarkable geometry-dependent phenomenology of fracton order. For example, I will explain why even on a manifold with trivial topology, spatial curvature can induce a robust ground state degeneracy.

[1] Vijay, Haah, Fu 1603.04442</p>

<p>[2] Slagle, Kim 1704.03870"</p>

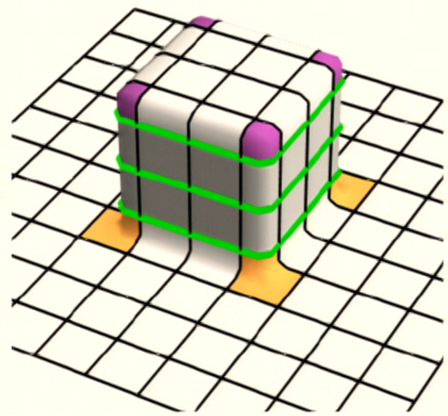
Beyond Topological Order:
Fractons and their Field Theory

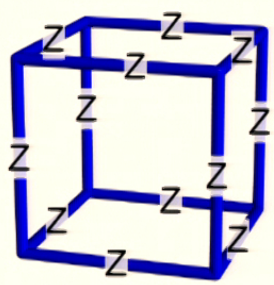


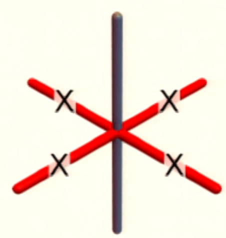
Kevin Slagle

University of Toronto

December 1, 2017

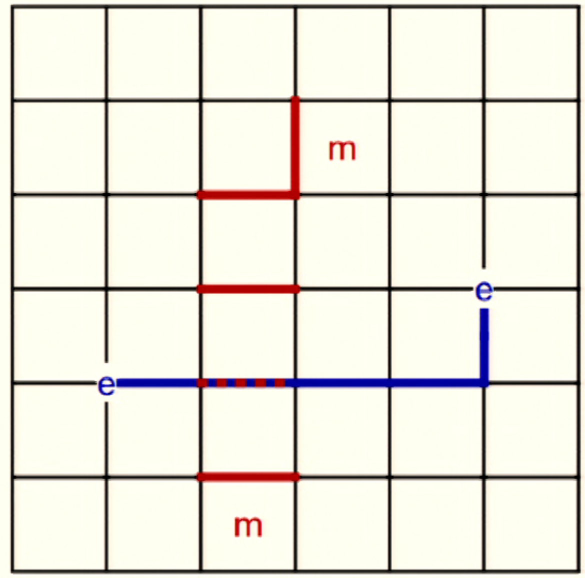


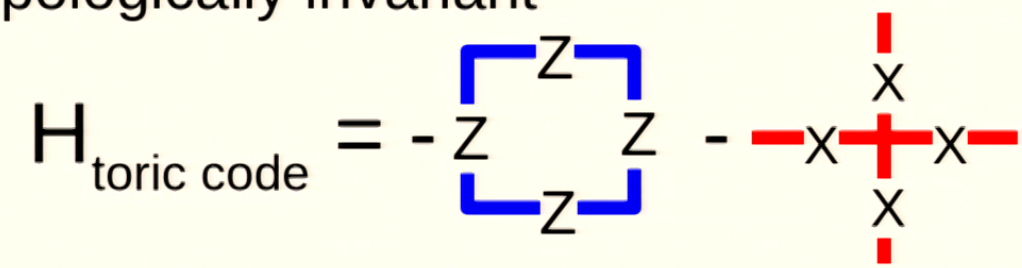




Toric code

- Gapped
- Exactly solvable
- Stable to excitations
- degen=4 on torus
- Topological excitations
- Topologically invariant





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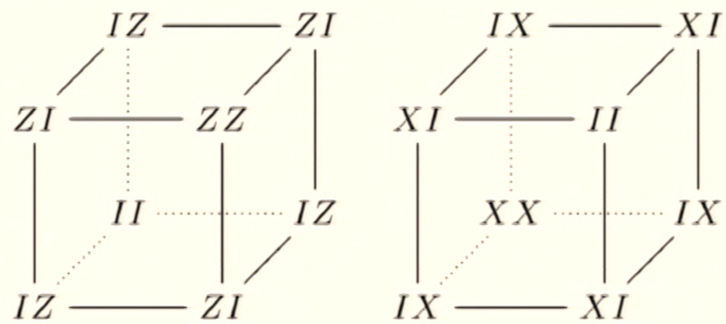
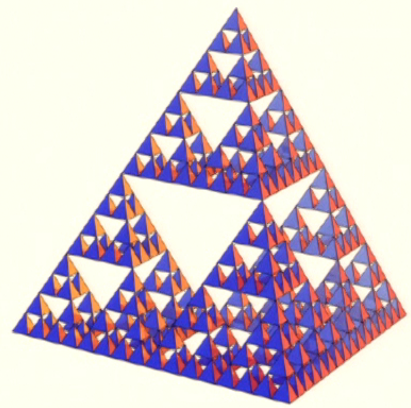
Haah's code

- Gapped
- Exactly solvable
- Stable to perturbations
- $1 \leq \text{degen} \leq 2^{(8L-2)}$
- Immobile fractons

  at corners of fractal operators
- E(r) ~ log(r)
- No topological invariance
- Quantum memory applications
- (type II fracton order)



Haah 2011



Yoshida 2013

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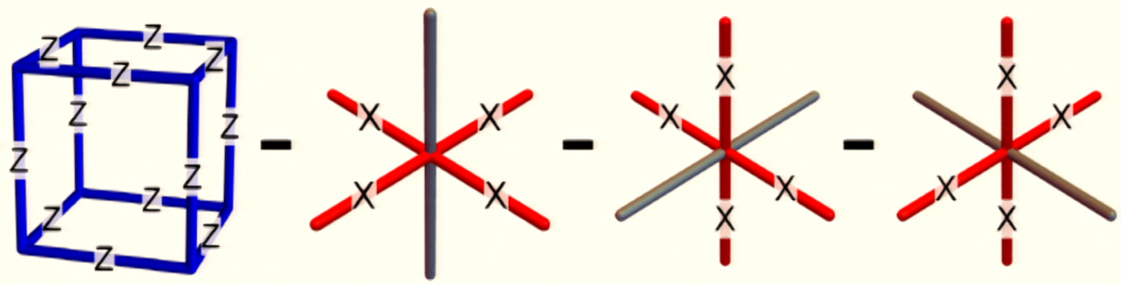
X-cube model

- (type I fracton order)
- Gapped
- Exactly solvable
- Stable to perturbations
- degen = 2^(6L-3)

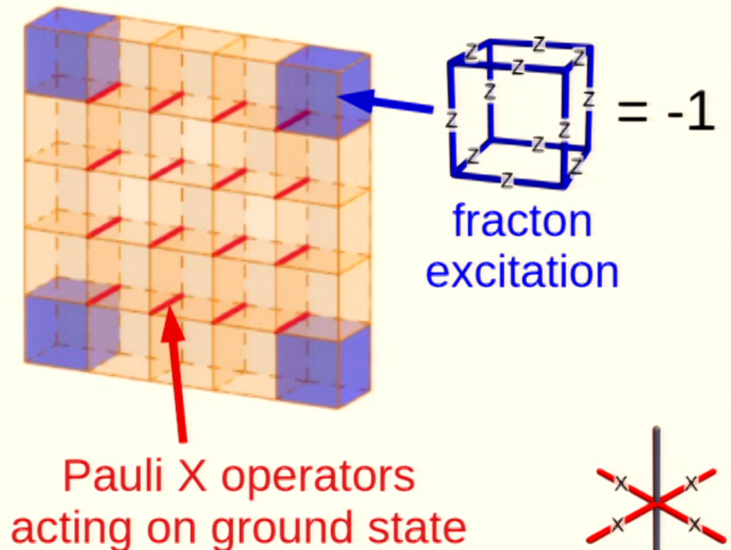
H = -

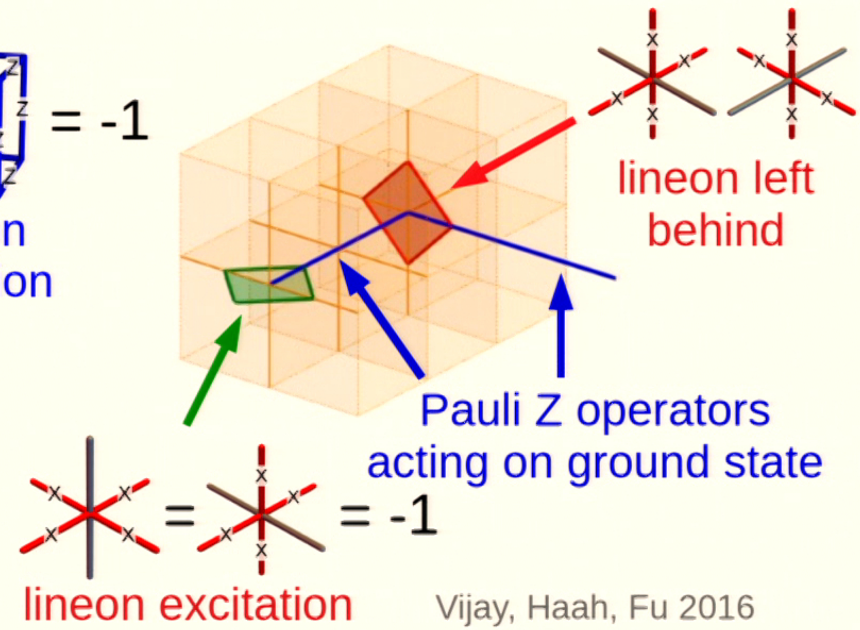


Vijay, Haah, Fu 2016

Subdimentional Excitations

- Create fractons at corners of rectangular operators
- Create dim-1 lineons at ends of line operators

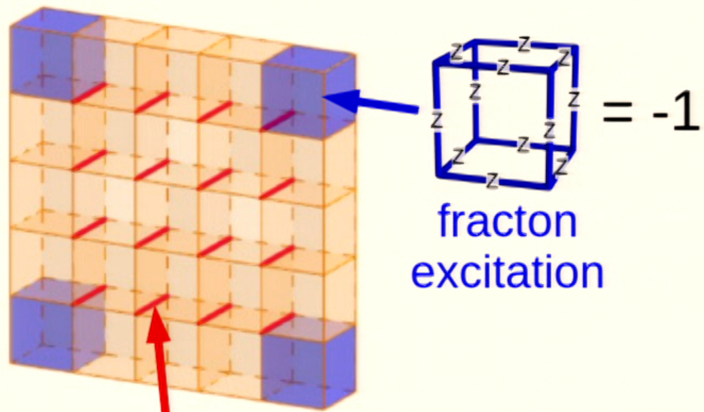


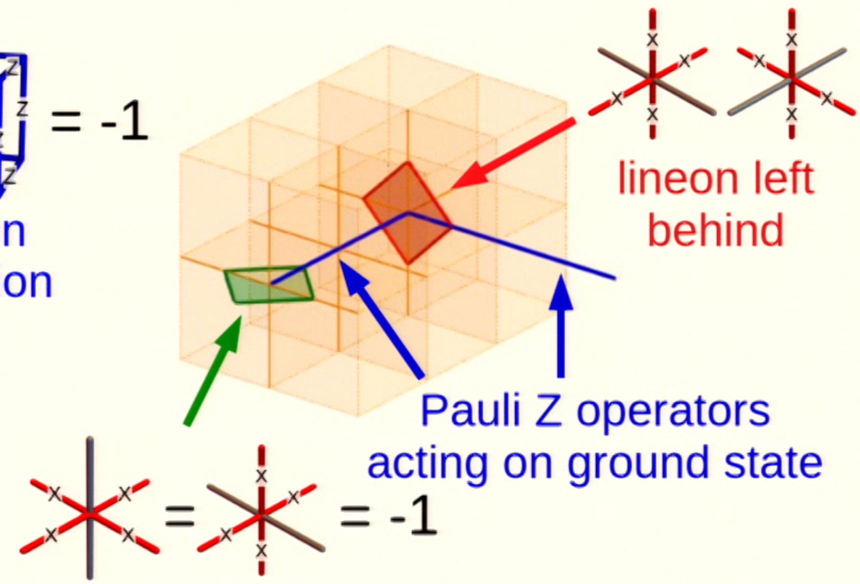


Vijay, Haah, Fu 2016

Questions?

- Create fractons at corners of rectangular operators
- Create dim-1 lineons at ends of line operators





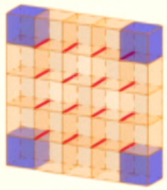
Vijay, Haah, Fu 2016

X-cube QFT?

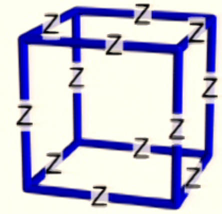
- QFT for X-cube model?
- Infinite degeneracy?
  - degen ~ 2^L → ∞ as L → ∞ ?
- How to write down the answer (Lagrangian)?
- Braiding?

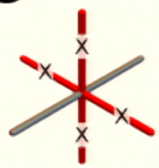
X-cube QFT?

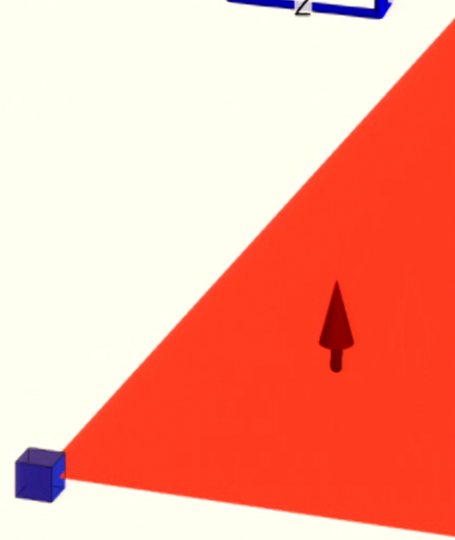
- QFT for X-cube model?
- Infinite degeneracy?
  - degen ~ 2^L → ∞ as L → ∞ ?
  - solution: add a cutoff! $L \sim l/a$
- How to write down the answer (Lagrangian)?
  - solution: inspiration from BF theory (toric code)
- Braiding?
  - next slide!



“braiding”: detect a fracton



- “braid” lineons! 



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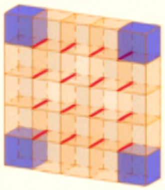

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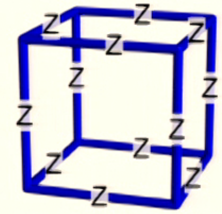
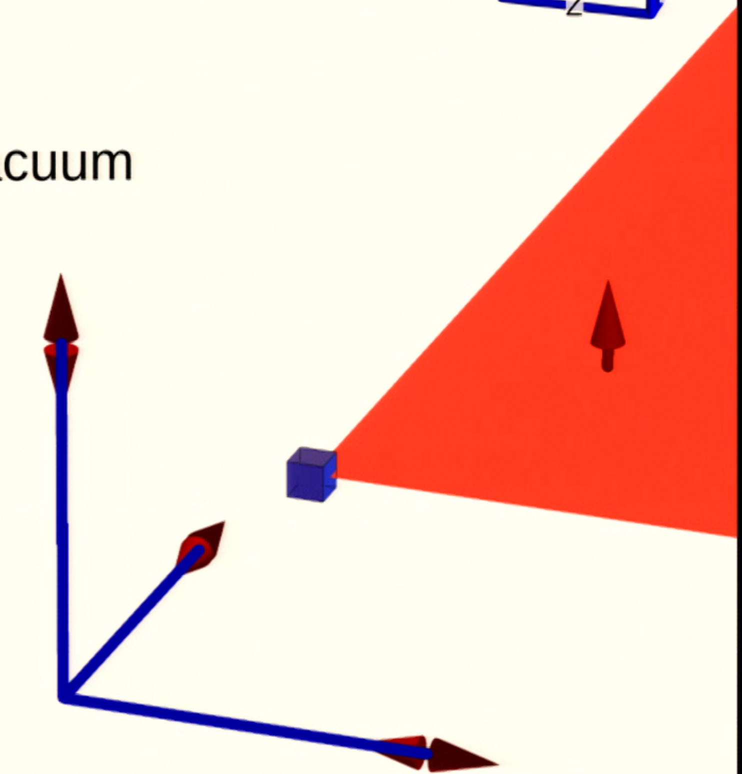
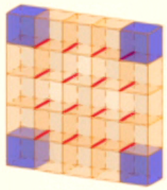


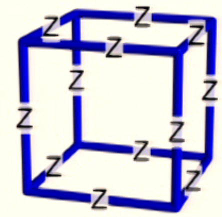
- “braid” lineons! 

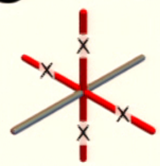
  - Create 3 lineons from vacuum





"braiding": detect a fracton

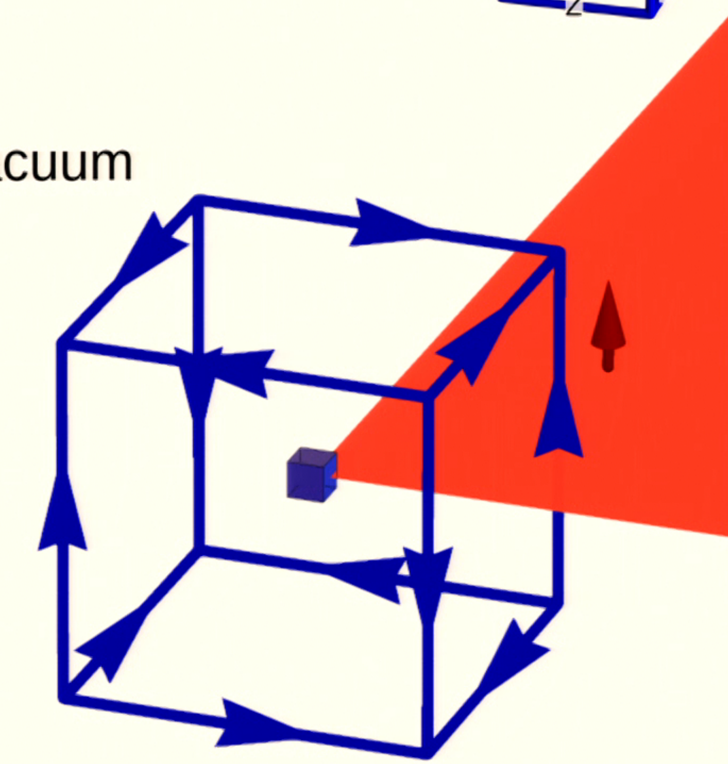


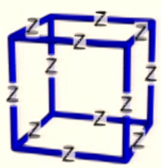
- "braid" lineons! 
  - Create 3 lineons from vacuum
  - Repeat x4
- Mobility restrictions

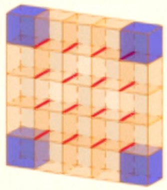
  ⇒

  Point particle

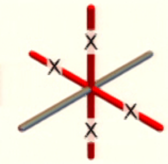
  braiding in 3D!

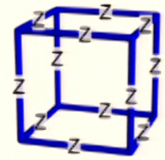


-  fracton operator

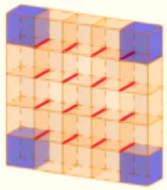


“braiding”: detect a lineon

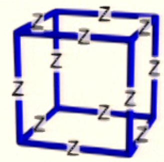


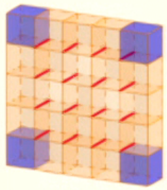
- “braid” fractons! 

- How to move “immobile” fractons?

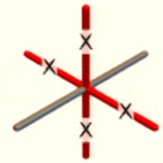


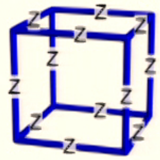
“braiding”: detect a lineon

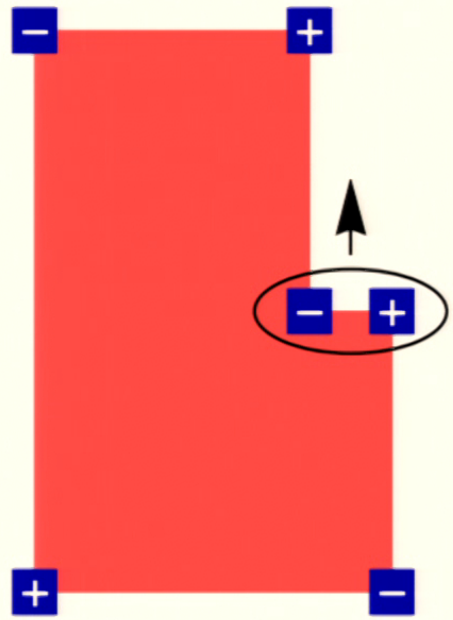
- “braid” fractons! 
- How to move “immobile” fractons:
  - start with fractons



"braiding": detect a lineon



- "braid" fractons! 
- How to move "immobile" fractons:
  - start with fractons
  - emit a fracton dipole



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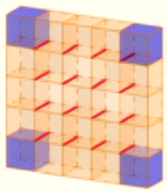
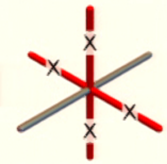
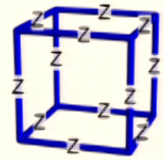


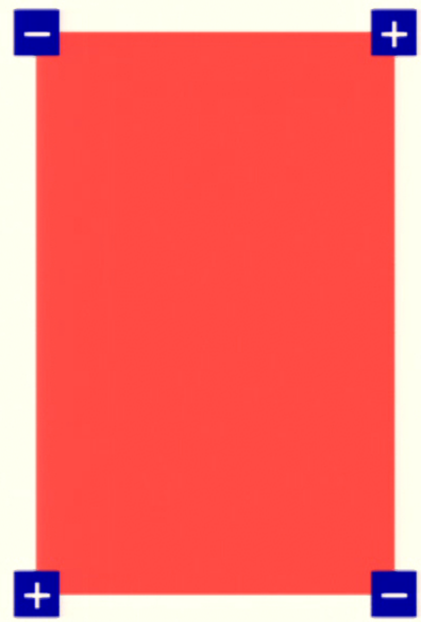
"braiding": detect a lineon

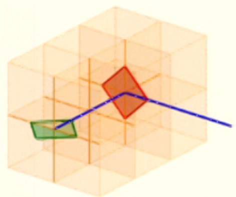


- "braid" fractons!



- How to move "immobile" fractons:
  - start with fractons
  - emit a fracton dipole
  - catch the dipole





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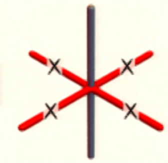

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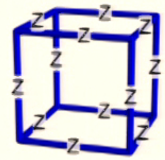
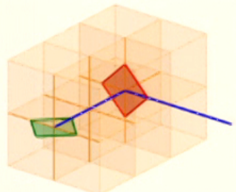
- "braid" fractons! 



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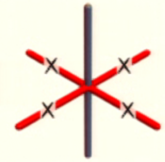
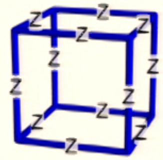
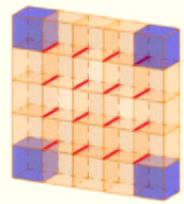
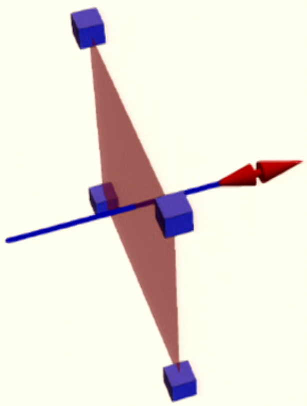


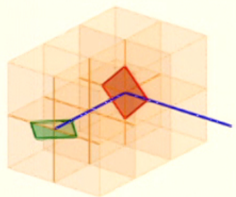
- “braid” fractons!



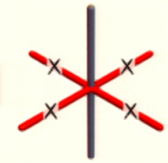
  – Create 4 fractons



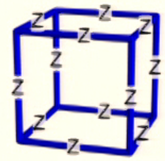




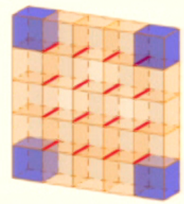
"braiding": detect a lineon

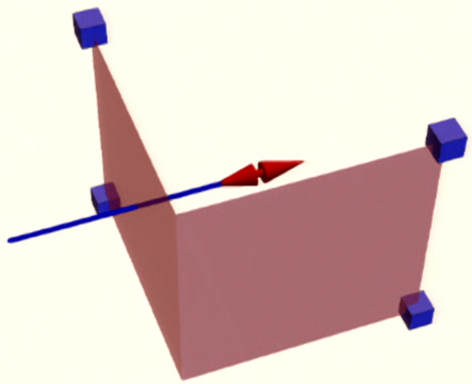


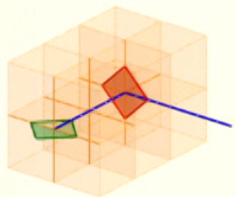
- "braid" fractons!



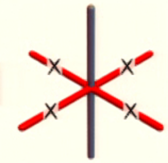
  - Create 4 fractons
  - Braid a pair around







“braiding”: detect a lineon



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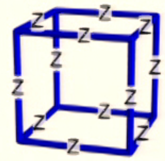

  - Create 4 fractons
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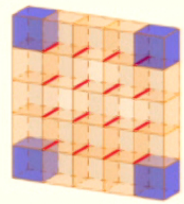
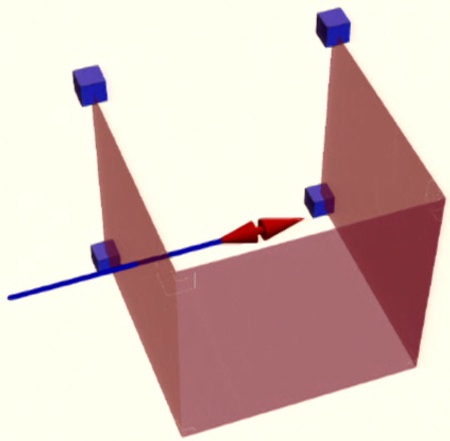
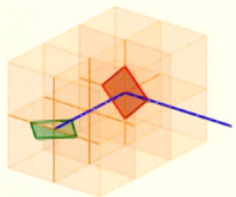
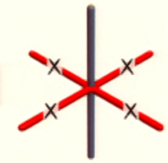
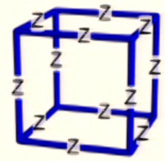
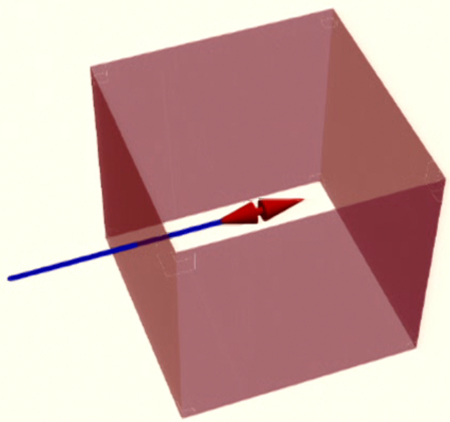


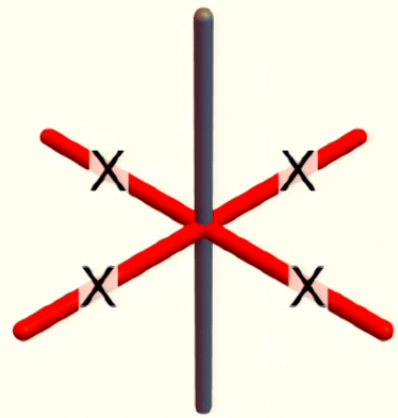


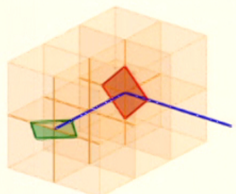


“braiding”: detect a lineon 

- “braid” fractons! 
  - Create 4 fractons
  - Braid a pair around
  - Same as lineon operator







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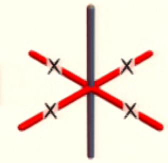
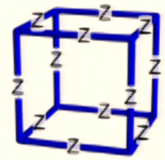


- “braid” fractons!



  - Create 4 fractons
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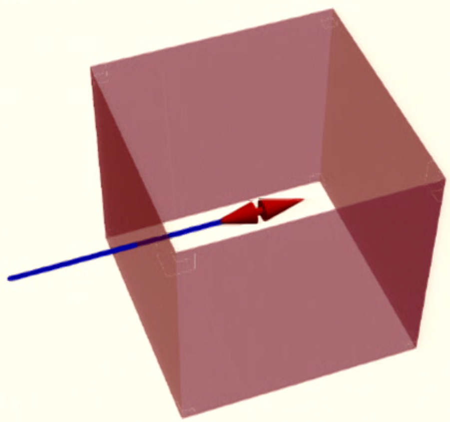
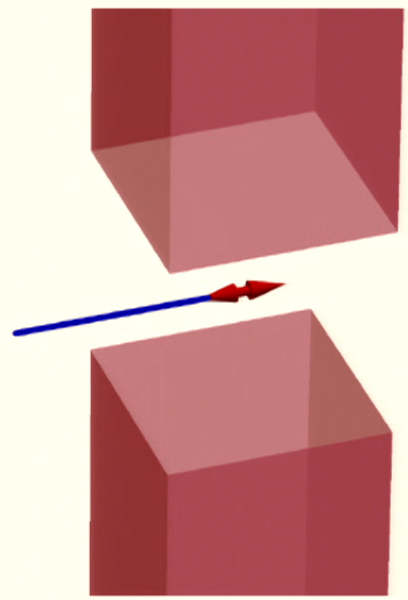


- Dipole current is essential



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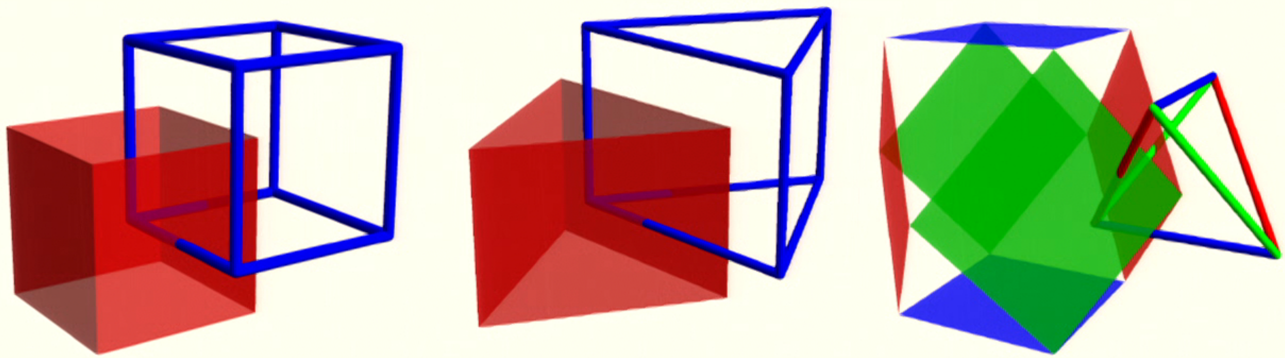
Geometric Braiding

- Different models allow different braiding geometries



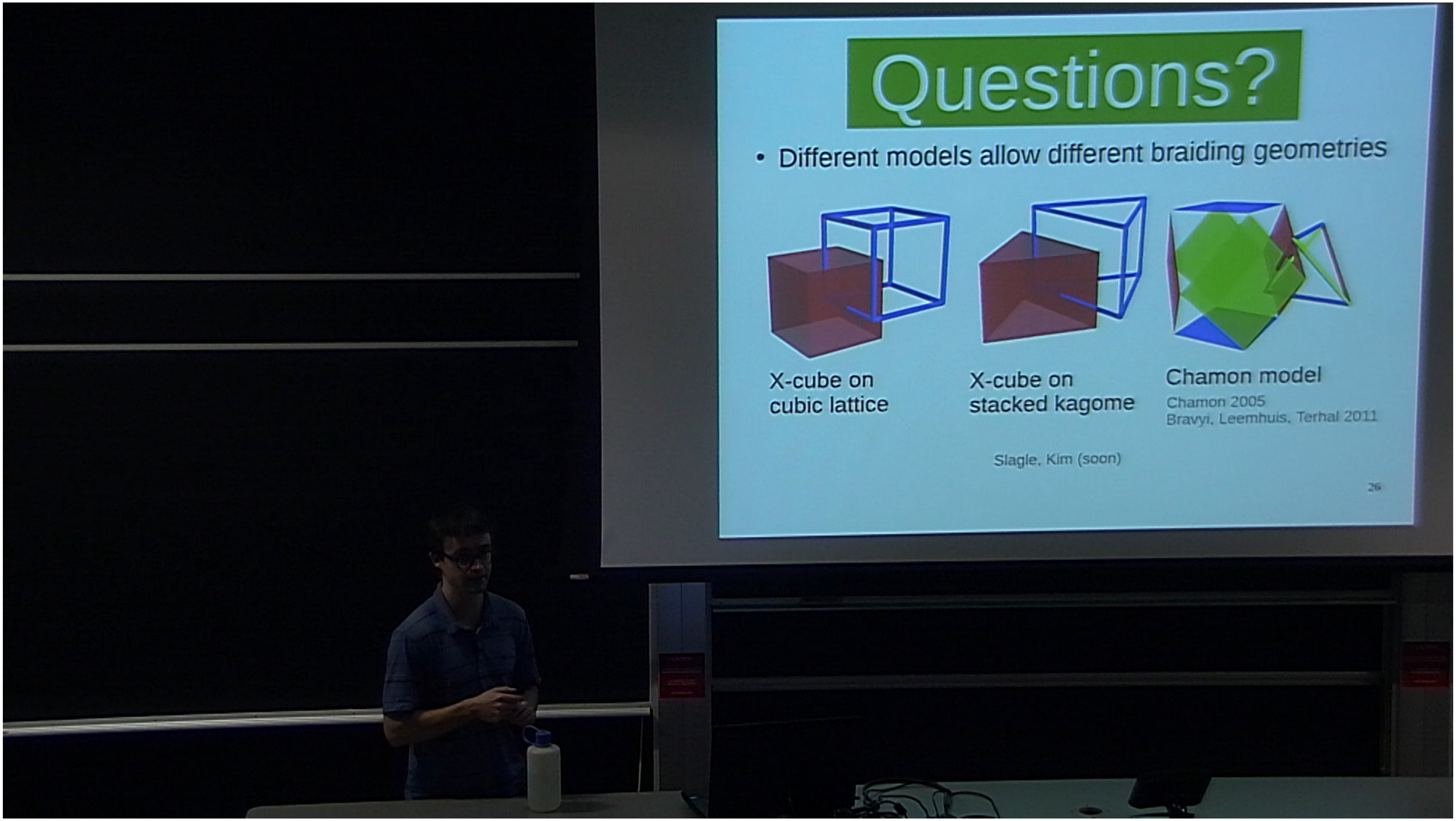
X-cube on cubic lattice

X-cube on stacked kagome

Chamon model

Chamon 2005
Bravyi, Leemhuis, Terhal 2011

Slagle, Kim (soon)


Questions?
• Different models allow different braiding geometries
X-cube on cubic lattice
X-cube on stacked kagome
Chamon model
Chamon 2005
Bravyi, Leemhuis, Terhal 2011
Slagle, Kim (soon)
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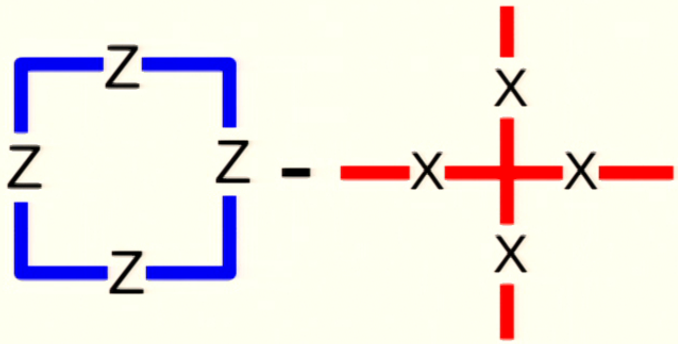
BF theory inspiration

- Z_N BF theory (Z_N toric code) $L_{\mathrm{BF}} = \frac{N}{2\pi}\epsilon^{\alpha\beta\gamma}B_\alpha \overbrace{\partial_\beta A_\gamma}^{F_{\beta\gamma}}$

$H_{\text{toric code}}$ = -



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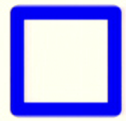
BF theory inspiration

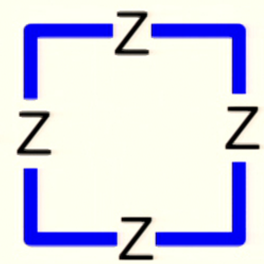
- Z_N BF theory (Z_N toric code) $L_{\rm BF} = \frac{N}{2\pi}\epsilon^{\alpha\beta\gamma}B_\alpha \overbrace{\partial_\beta A_\gamma}^{F_{\beta\gamma}}$

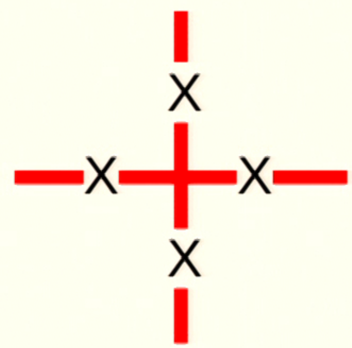
$$L_{\rm BF} = \underbrace{\frac{N}{2\pi}\epsilon^{a0c}B_a\partial_0 A_c}_{\text{conjugate fields}} + B_0 \underbrace{\frac{N}{2\pi}\epsilon^{0bc}\partial_b A_c}_{} + A_0 \underbrace{\frac{N}{2\pi}\epsilon^{ab0}\partial_a B_b}_{}$$



$\frac{N}{2\pi}\partial_b X^b$ 

$B_a = -\epsilon_{ab0}X^b$

$H_{\text{toric code}}$ = - 

- 

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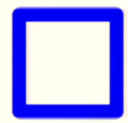
X-cube QFT!

- Z_N BF theory (Z_N toric code) $L_{\mathrm{BF}} = \frac{N}{2\pi}\epsilon^{\alpha\beta\gamma} B_\alpha \overbrace{\partial_\beta A_\gamma}^{F_{\beta\gamma}}$

$$L_{\mathrm{BF}} = \underbrace{\frac{N}{2\pi}\epsilon^{a0c} B_a \partial_0 A_c}_{\text{conjugate fields}} + B_0 \underbrace{\frac{N}{2\pi}\epsilon^{0bc}\partial_b A_c} + A_0 \underbrace{\frac{N}{2\pi}\epsilon^{ab0}\partial_a B_b}$$

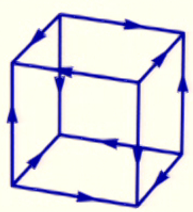


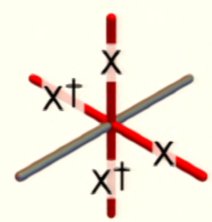
$\frac{N}{2\pi}\partial_b X^b$

$B_a = -\epsilon_{ab0} X^b$

- Z_N X-cube: $L_{\text{X-cube}} = \frac{N}{2\pi}|\epsilon^{0abc}|\frac{1}{2} B_{ab}\partial_0 A_c$

$$+ B_0 \underbrace{\frac{N}{2\pi}|\epsilon^{0abc}|\frac{1}{2}\partial_a\partial_b A_c} + A_{0;a} \underbrace{\frac{N}{2\pi}\epsilon^{0abc}\partial_c B_{ab}}$$



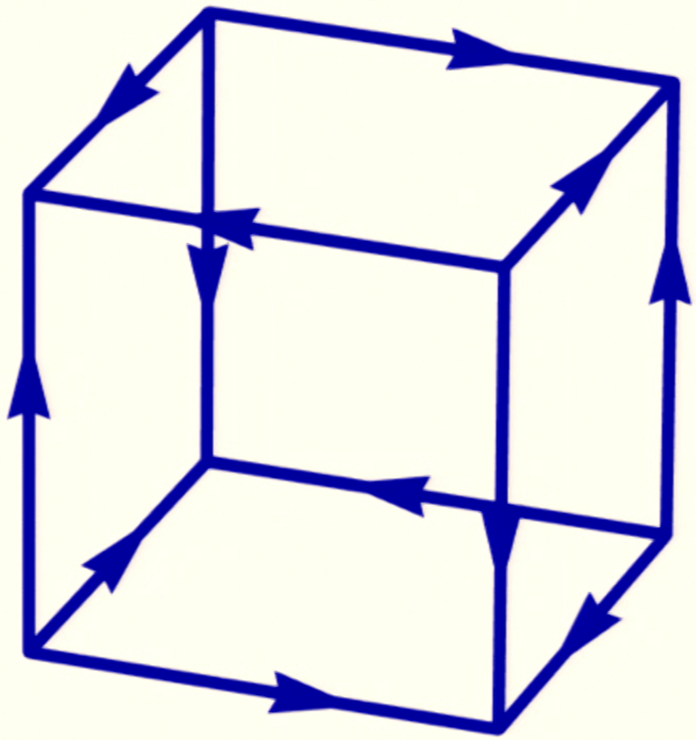


$\frac{N}{2\pi}\epsilon^{0abc}\partial_c X^c$

$B_{ab} = |\epsilon_{0abc}|\, X^c$

fracton operator

$$\underbrace{\frac{N}{2\pi}|\epsilon^{0abc}|\frac{1}{2}\partial_a\partial_b A_c}$$



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X-cube QFT!

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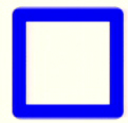

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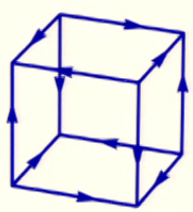

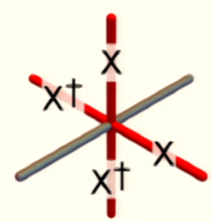

$\frac{N}{2\pi}\epsilon^{0abc}\partial_c X^c$

$B_{ab} = |\epsilon_{0abc}|\, X^c$

Conserved Charge

- Z_N BF theory (Z_N toric code)

$$L_{\mathrm{BF}} = \frac{N}{2\pi}\epsilon^{\alpha\beta\gamma} B_\alpha \partial_\beta A_\gamma - A_\alpha J^\alpha - B_\alpha I^\alpha$$

- gauge invariance:
  - since [, ] = 0

$$A_\alpha \to A_\alpha + \partial_\alpha \zeta$$

$$B_\alpha \to B_\alpha + \partial_\alpha \chi$$

- conservation of charge:

$$\partial_\alpha I^\alpha = \partial_\alpha J^\alpha = 0$$

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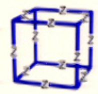
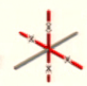
Mobility Constraints
(generalized charge conservation)

- Z_N X-cube: $L_{\text{X-cube}} = \frac{N}{2\pi}|\epsilon^{0abc}|\frac{1}{2}B_{ab}\partial_0 A_c$

$$+ B_0 \frac{N}{2\pi}|\epsilon^{0abc}|\frac{1}{2}\partial_a\partial_b A_c + A_{0;a}\frac{N}{2\pi}\epsilon^{0abc}\partial_c B_{ab}$$

- gauge invariance since [, ]:

$$A_{0;a} \to A_{0;a} + \partial_0\zeta_a \quad A_a \to A_a - \epsilon^{0abc}\partial_a\zeta_c$$

$$B_0 \to B_0 + \partial_0\chi \quad B_{ab} \to B_{ab} + \partial_a\partial_b\chi$$

- mobility constraint:
  - fracton: $\partial_0 I^0 - \frac{1}{2}\partial_a\partial_b I^{ab} = 0$
  - lineon: $\forall_a : \partial_0 J^{0;a} + \epsilon^{0abc}\partial_c J^c = 0$

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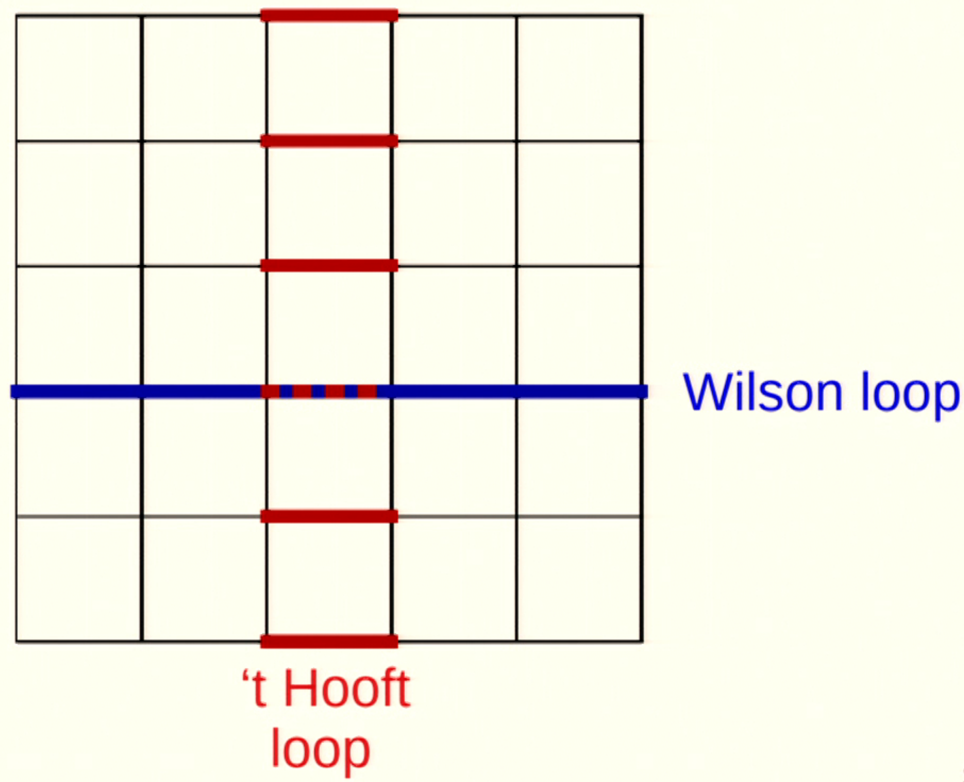
Toric Code
Ground State Degeneracy

- degen = 4

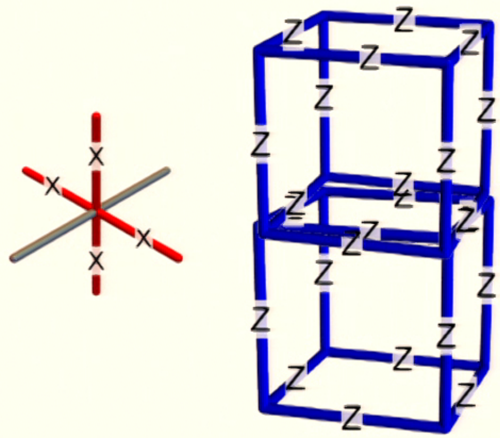


Ground State Degeneracy

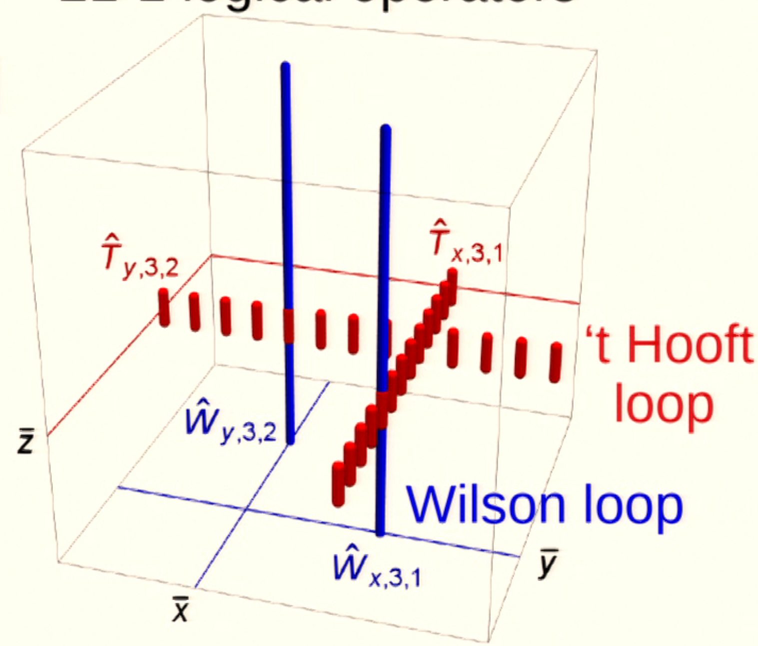
- degen = 2^(6L-3)

  = 2^[3*(2L-1)]
- Redundancy from

  X-cube operators

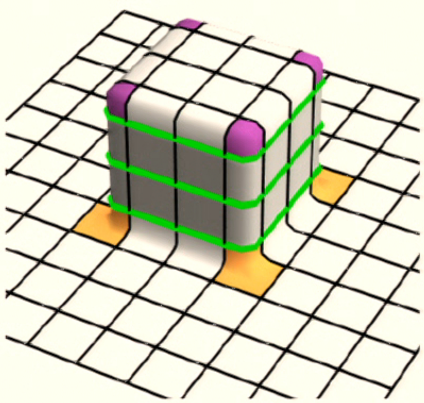


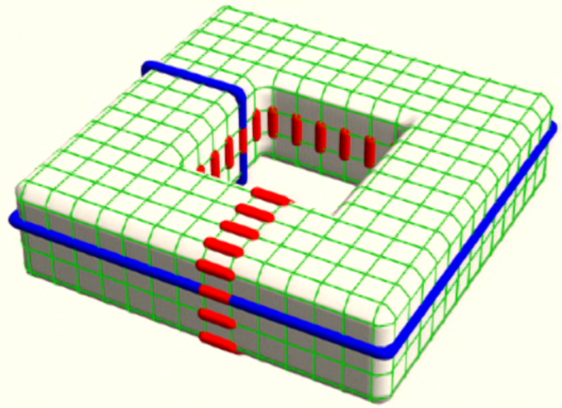
2L-1 logical operators



Robust Degeneracy from Geometry

- Degeneracy isn't topologically invariant
- Degeneracy depends on:
  - system size: degen = 2^(6L-3)
  - shifted boundary conditions
  - lattice geometry:





degen *= 4

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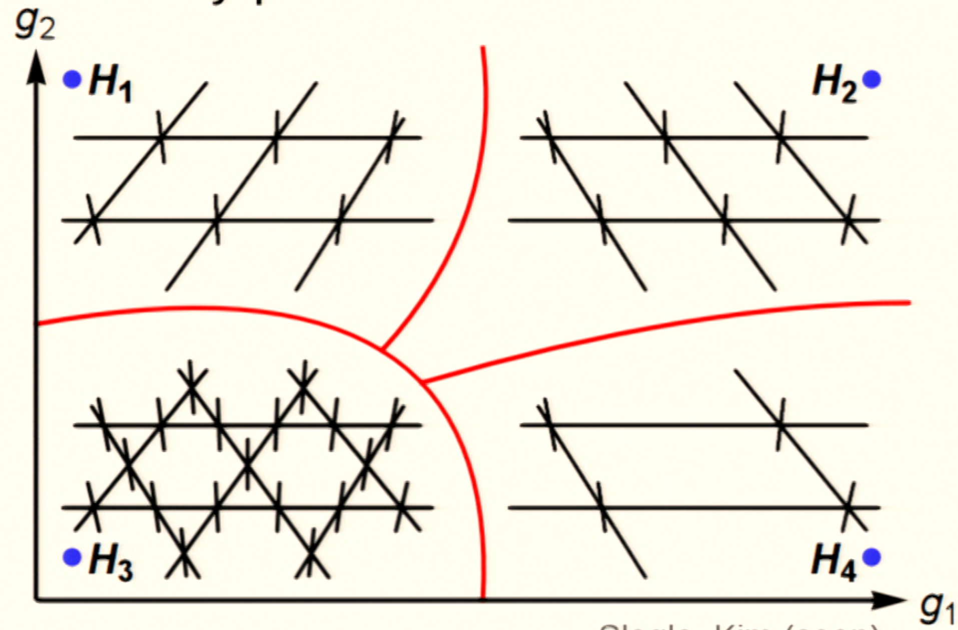
More Lattices!

- Each lattice is a new phase
  - Different subdimensional paticles and degeneracy
  - Separated by phase transitions



Slagle, Kim (soon)

More Lattices!

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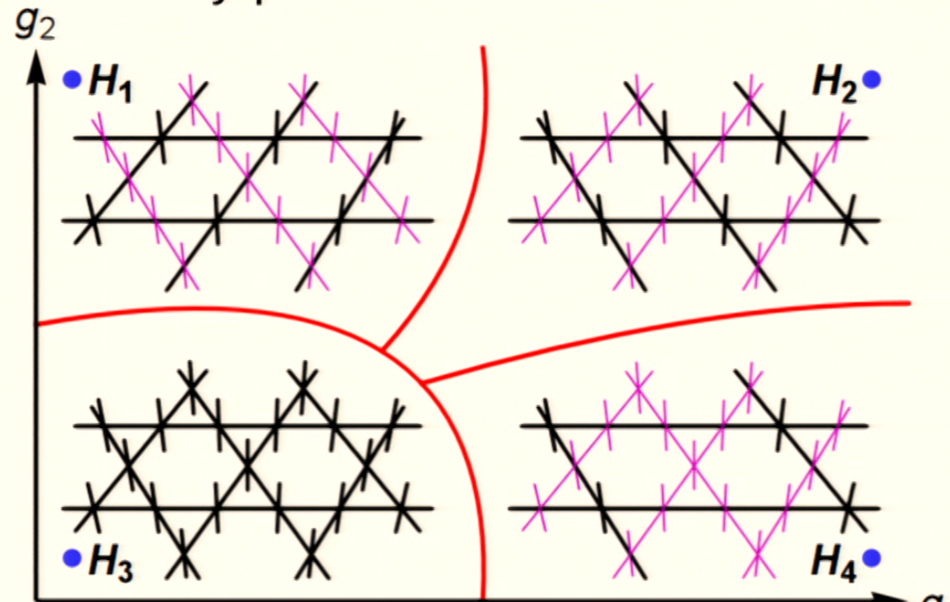
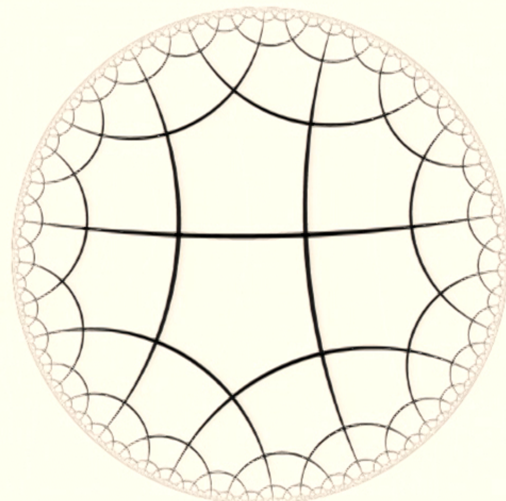


Slagle, Kim (soon)

Connections to Gravity-like Physics

- Linearized gravity → U(1) fracton models
  - Zheng-Cheng Gu, Xiao-Gang Wen (2006, 2009)
  - Cenke Xu (2006, 2006)
  - Cenke Xu, Petr Horava (2010)
- Mach's principle
  - Michael Pretko (2017)
- X-cube model on curved space
  - Lineons inherit lattice geometry

Slagle, Kim (soon)



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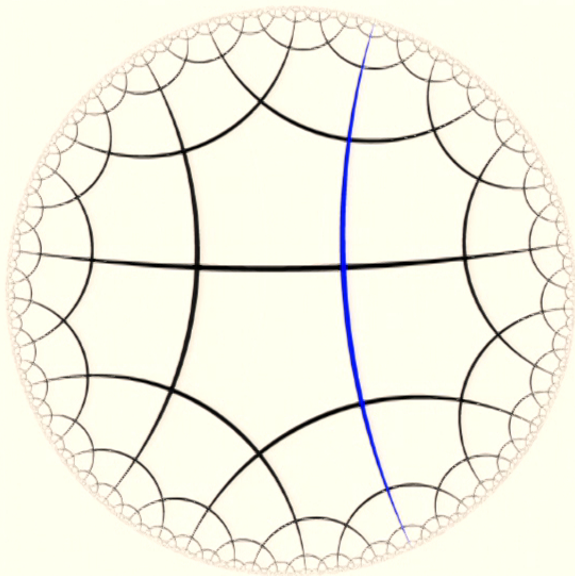
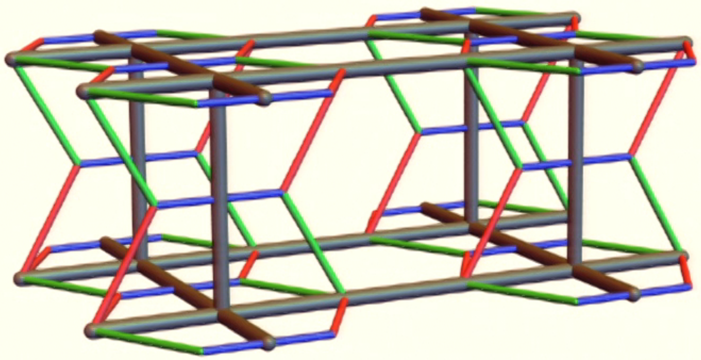
Simple AdS/CFT?

- Gapped geometric bulk
- Gapless boundary with long-range correlations?

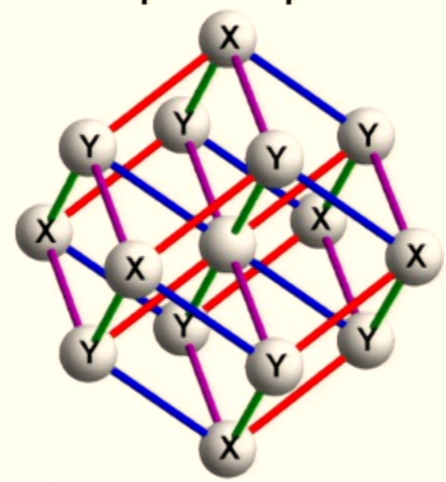


Two-Spin Models

- Coupled Kitaev honeycombs
- Coupled spin chains

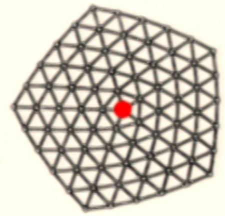
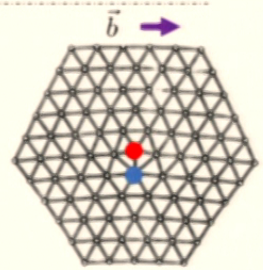


Slagle, Kim 2017



Halász, Hsieh, Balents 2017

Noncompact U(1) Fracton <-> Disclination Duality

Fracton $\partial_i \partial_j E^{ij} = \rho$	Disclination $\epsilon^{ik}\epsilon^{j\ell}\partial_i\partial_j u_{k\ell} = s$ 
Dipole	Dislocation 
Gauge Modes	Phonons
Electric Field E_{ij}	Strain Tensor u_{ij}
Magnetic Field B_i	Lattice Momentum π_i

Pretko, Radzihovsky (today)

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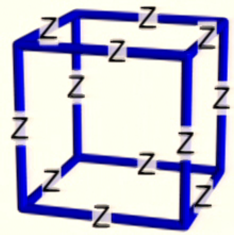
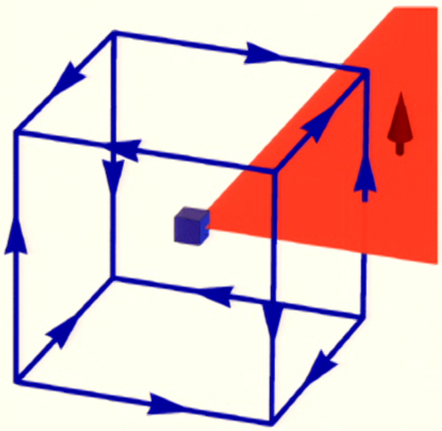
Beyond Topological Order:
Fractons and their Field Theory

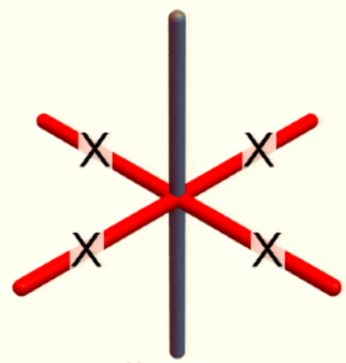


fracton

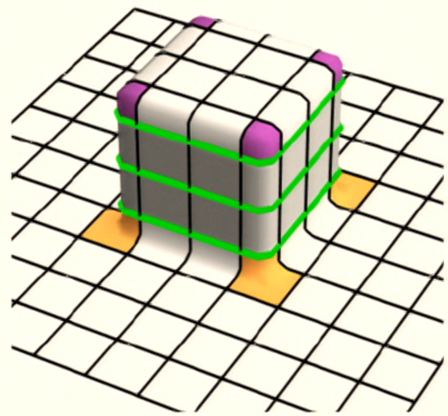


detect a fracton by
braiding lineons

Thank you!



lineon



robust degeneracy
from geometry