Title: De Sitter Space as a Tensor Network

Date: Nov 27, 2017 02:00 PM

URL: http://pirsa.org/17110143

Abstract: <p>Tensor network/spacetime correspondences explore the exciting idea that geometric information about a quantum state might be related to the actual geometry that the state describes in a quantum gravitational setting. I will give an overview of a new type of correspondence between global de Sitter spacetime and the MERA. This simple correspondence is already enough to see several features of de Sitter gravity emerge, such as cosmic no-hair and horizon complementarity. I will also comment on some more speculative topics like complexity = action and possible future directions.</p>

De Sitter Space as a Tensor Network

Ning Bao, Charles Cao, Sean Carroll, Aidan Chatwin-Davies

(1709.03513)

Tensor Network Seminar, Perimeter Institute

November 27, 2017

Outline

1. Introduction: spacetime/tensor network correspondences
2. Let's chat about de Sitter
3. The MERA as global de Sitter
4. So what?
   - *i.* Cosmic no-hair
   - *ii.* Horizon complementarity
   - *iii.* Complexity equals action
5. Future directions

Spacetime/tensor network correspondences...

Q: What?

A: Correspondence between a tensor network and a spacetime!

Q: Why?

→ TN: trades algebraic complexity for geometric information

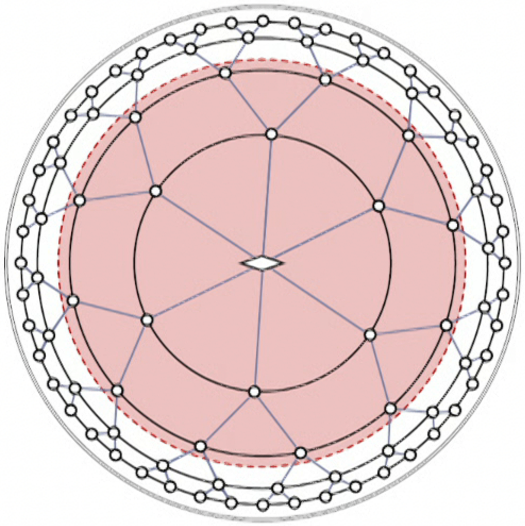
→ Maybe, in quantum gravity, geometry of TN has something to do with the *physical* geometry that the state describes

→ Exciting possibility: maybe also has something to do with the dynamics of the geometry as well

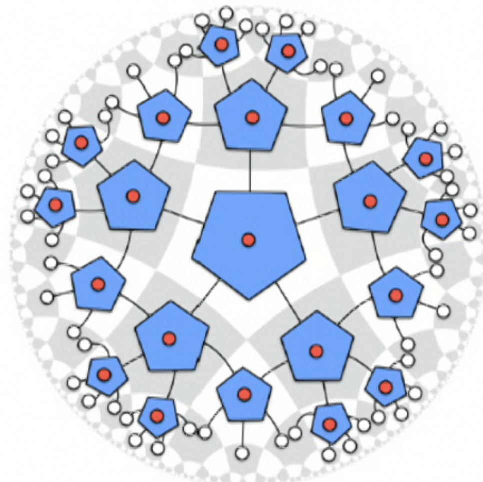
(In particular, please look forward to Charles' talk!)

ST/TN correspondences

Earliest examples: inspired by AdS/CFT



AdS/MERA, Tree Tensor Networks/Exact Holographic Mapping, ...



HaPPY code

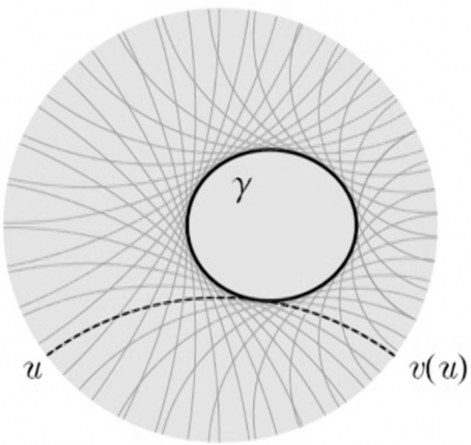
source: Pastawski, Yoshida, Harlow, & Preskill, 1503.06237

Getting closer to de Sitter

Bény: Causal structure of the MERA matches causal structure of de Sitter

Czech, Lamprou, McCandlish, Sully: space of boundary-anchored AdS geodesics, a.k.a. Kinematic Space $\simeq$ de Sitter

sources: 1110.4872, 1505.05515



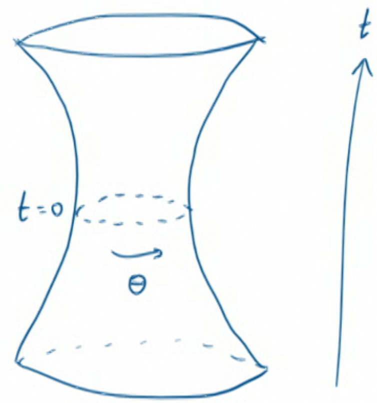
What is de Sitter spacetime?

- topology $\mathbb{R} \times S^d$, maximally symmetric
- homogeneous and isotropic spatial sections
- the prototypical cosmology with accelerated expansion

Ex: 1+1 dimensions

$$ds^2 = \ell_{\text{dS}}^2 \left(-dt^2 + \cosh^2 t \, d\theta^2\right)$$

$$t \in \mathbb{R}, \; \theta \in [0, 2\pi)$$



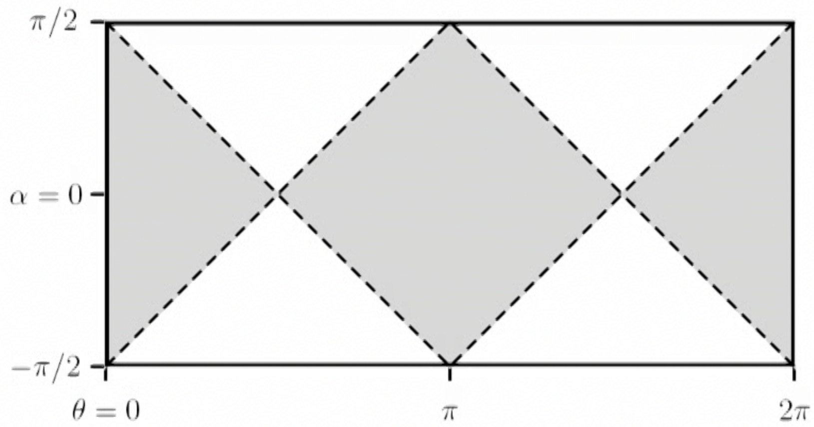
Penrose diagram for de Sitter

Convenient way to visualize spacetimes:

- Conformal transformation to new coordinates that take *finite values*...
- ... where the metric is *conformally flat*

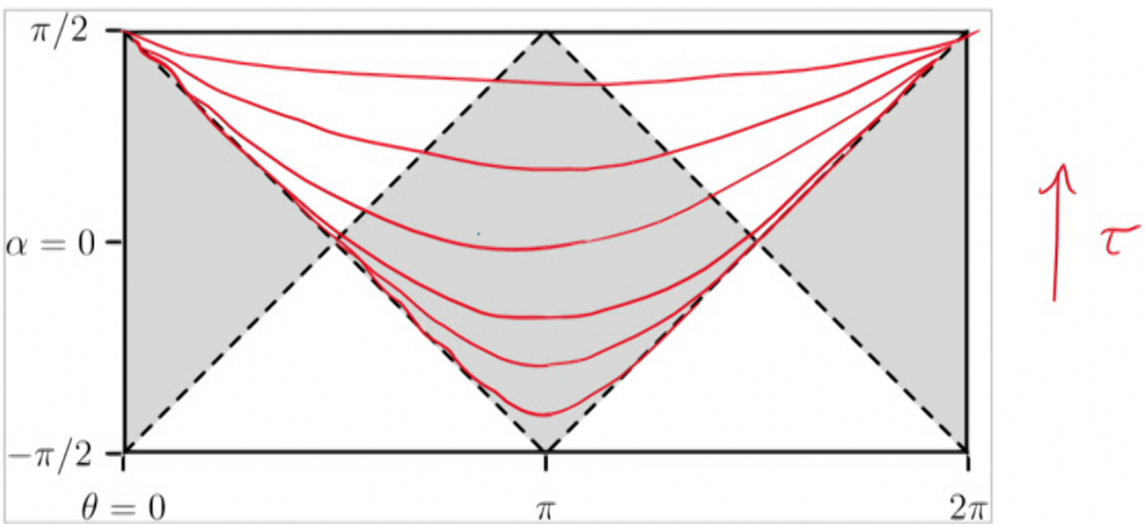
Ex: set $\cosh t = \sec\alpha$

$$ds^2 = \frac{\ell_{\text{dS}}^2}{\cos^2\alpha}\left(-d\alpha^2 + d\theta^2\right) \qquad \alpha \in [-\tfrac{\pi}{2}, \tfrac{\pi}{2}],\ \theta \in [0, 2\pi)$$



How you slice depends on who you are

e.g. this might look more familiar if you study the CMB:

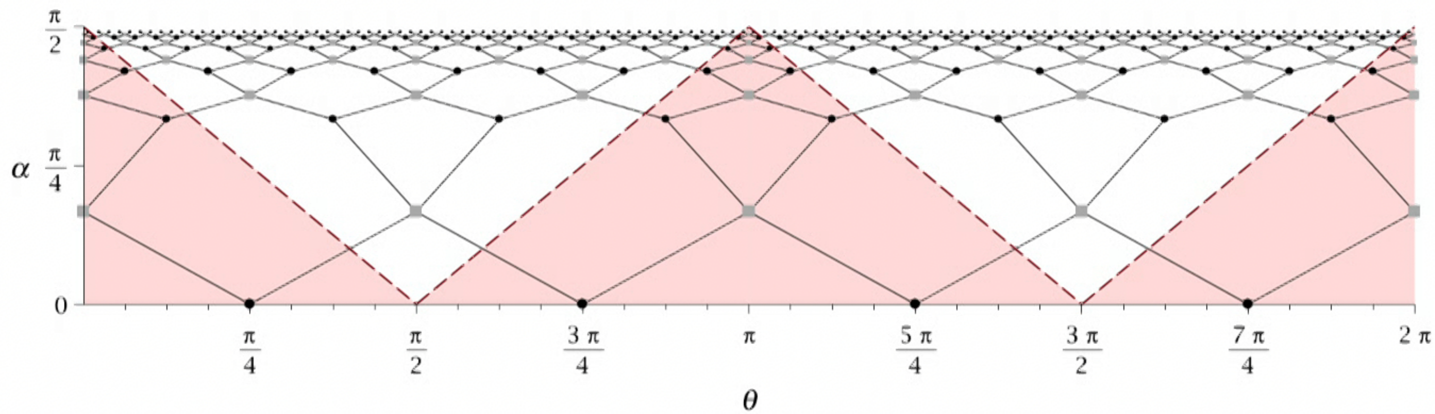


$$ds^2 = -d\tau^2 + e^{2H\tau} d\vec{x}^2$$

A global de Sitter correspondence

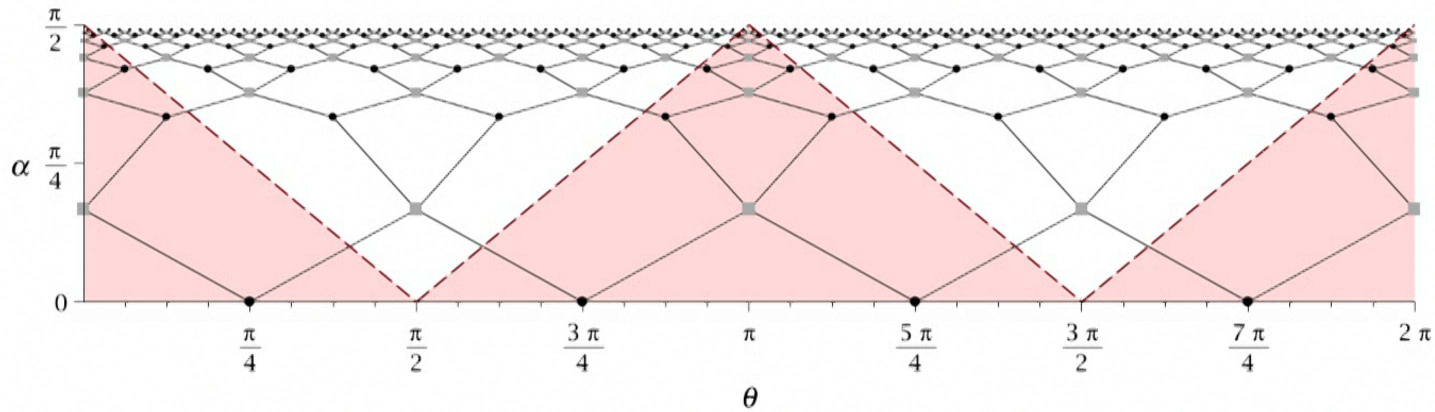
(keep working in 1+1 dimensions)

Ex: homogeneous periodic binary MERA



- at $t = 0$, place 4 equally spaced MERA sites
- MERA layers at constant t slices
- when volume of space doubles, add a MERA layer

Symmetry fixes the structure

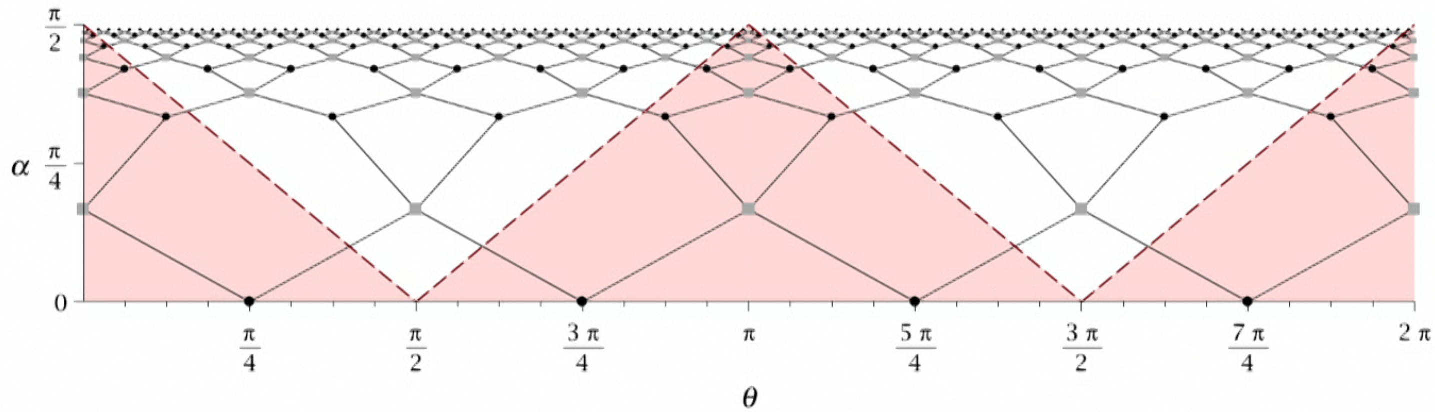


Q: why four sites?

- dS has two disjoint static patches
- stationary causal cone in a MERA has 2 sites per layer

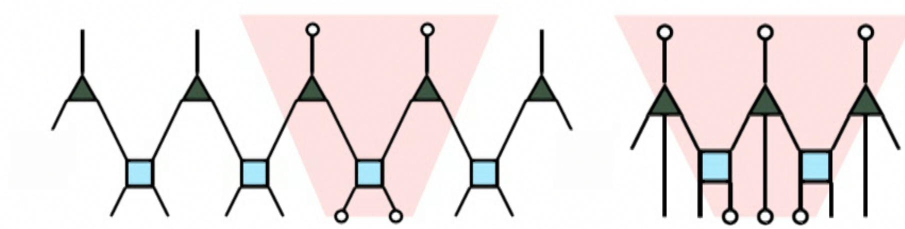
∴ need 4 sites to begin

Can you pack more sites into a patch?



No, you can't pack more in!

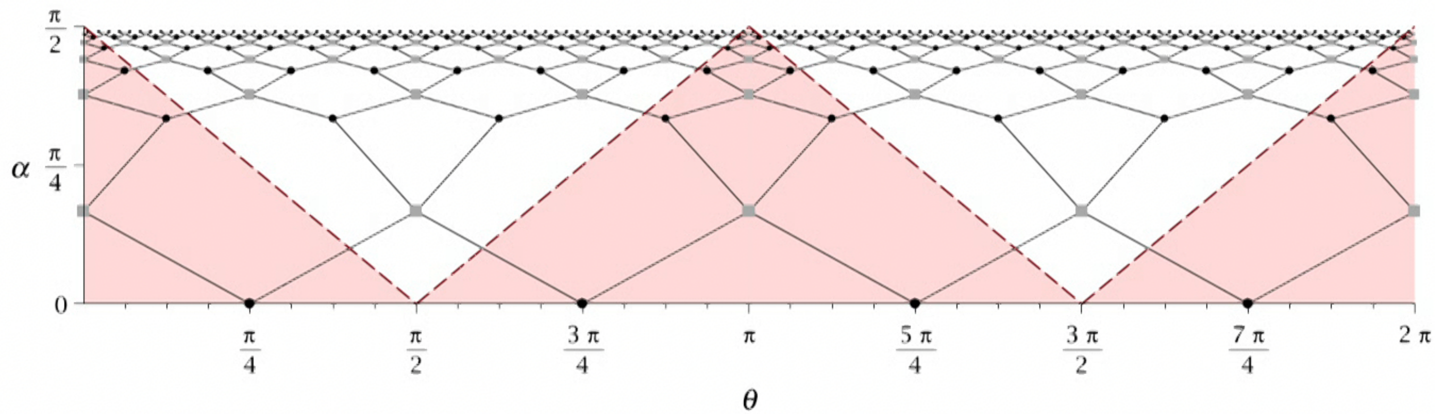
→ # sites per layer in stationary causal cone, is a property of the network



What is the correspondence, anyways?

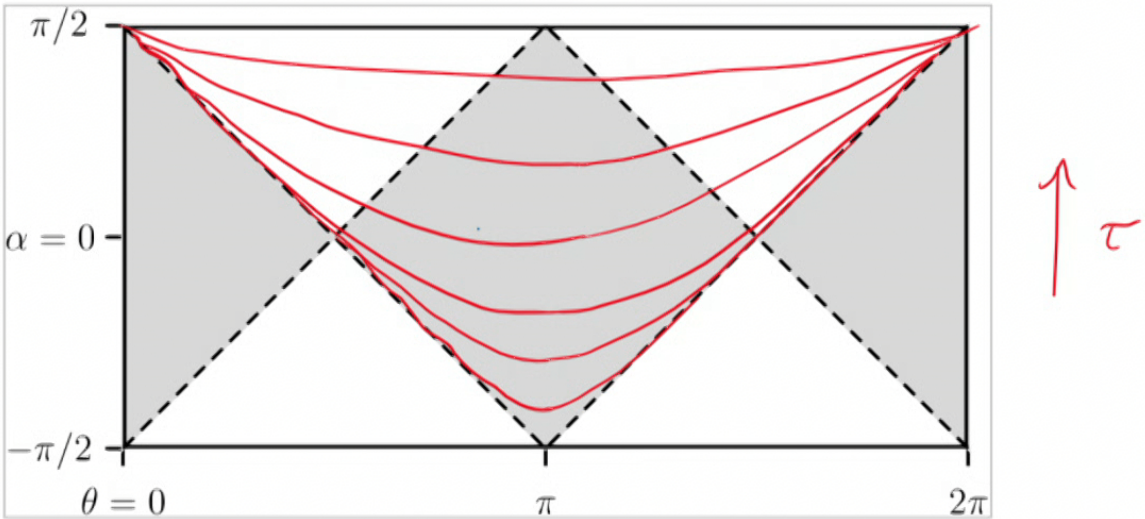
The idea:

- MERA-like circuit simulates effective quantum gravitational dynamics on super-Hubble scales
- but, only for states where it is appropriate to describe the geometry as de Sitter



How you slice depends on who you are

e.g. this might look more familiar if you study the CMB:



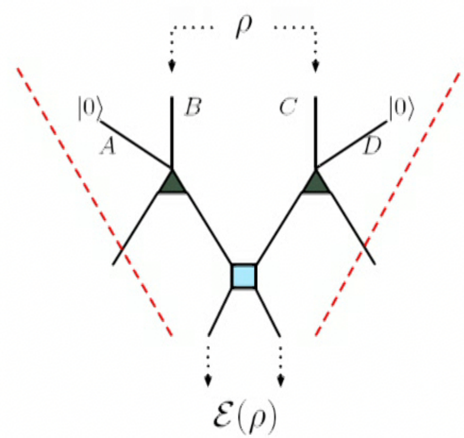
$$ds^2 = -d\tau^2 + e^{2H\tau} d\vec{x}^2$$

What is cosmic no-hair?

- Roughly, $\Lambda > 0 \quad \Leftrightarrow \quad$ asymptote to de Sitter
- Intuition: positive C.C. dilutes everything except C.C. itself

Quantum cosmic no-hair: states of fields approach de Sitter vacuum state

c.f. a single step in the MERA:



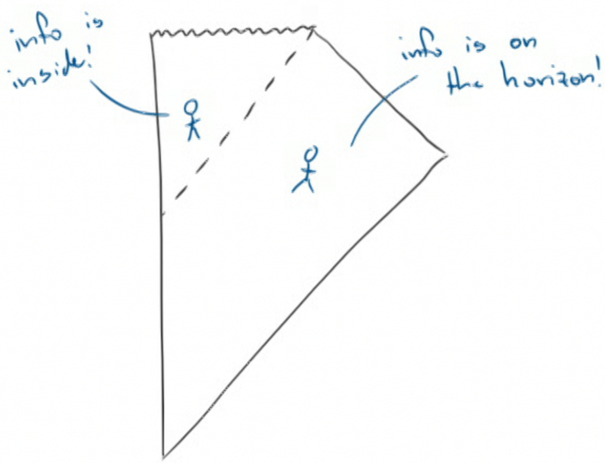
- information always flows out of the horizon $\rightarrow$ any perturbations will decay away

(more in Charles's talk, or come see my Strings talk on Friday for more about cosmic no-hair in general)

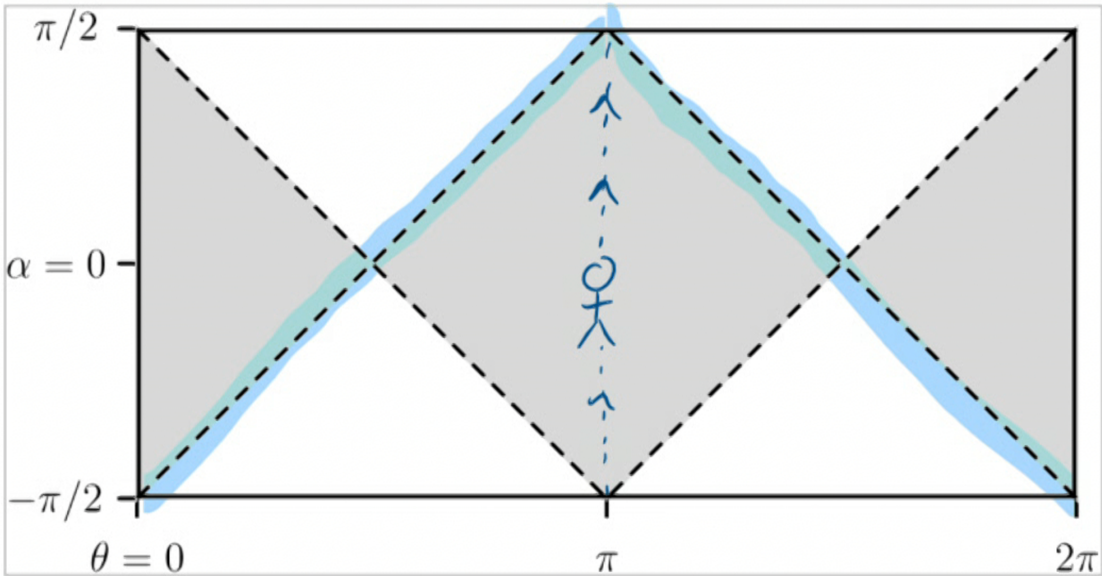
Really, we need to talk about 2 things first

1. Complementarity

- idea that two observers who are separated by a horizon can disagree on where quantum info is stored



Q: Is the state of the whole universe encoded on a cosmological horizon?



Really, we need to talk about 2 things first

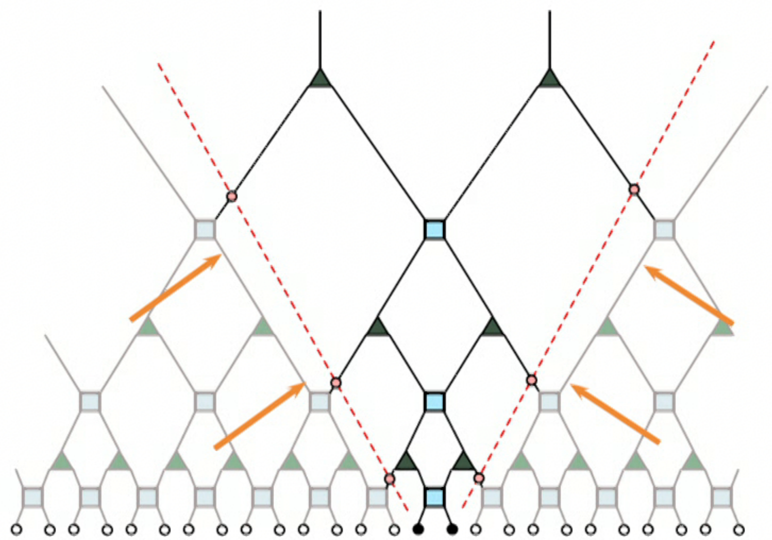
2. The Hilbert space of quantum gravity

Three logical options...

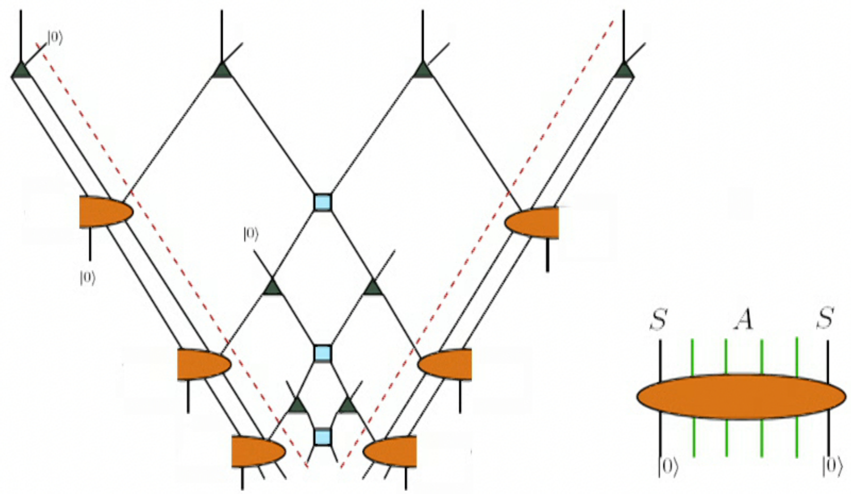
1. Locally infinite-dimensional
   - → "QFT all the way down"
2. Locally finite-dimensional
   - → QFT is emergent
3. Globally finite-dimensional
   - → weird, but motivated by horizon complementarity

Complementarity and the MERA

- dS/MERA by construction works in the locally finite case
- Q: can it accommodate globally finite dimension?

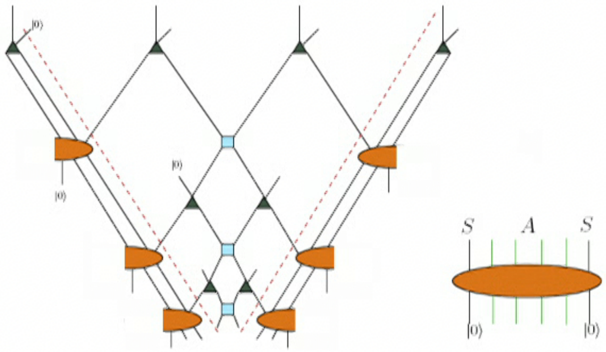


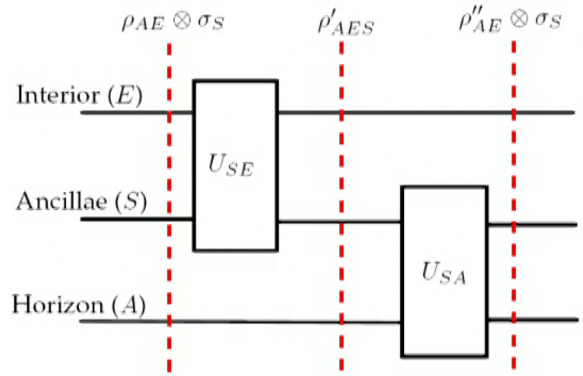
Strong complementarity and the MERA



- To not grow the overall $\dim\, \mathcal{H}_{\text{patch}}$, introduce a horizon *recovery tensor*
- In every step, feed in two ancillas $|0\rangle$, get them back out again

Useful to think of as recovery from deletion





Q: Codes that implement this recovery?

Recall, the Complexity = Action conjecture

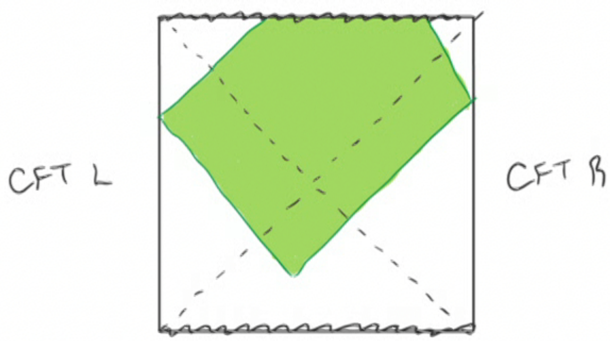
Definition:

Given a gate set $\mathcal{G}$, reference state $|\psi_0\rangle$, tolerance ϵ, the *complexity* of the target state $|\psi\rangle$ is

$$\mathcal{C}(|\psi\rangle) = \text{min \# of gates to make a state } \epsilon\text{-close to} |\psi\rangle \text{ from } |\psi_0\rangle$$

Conjecture:

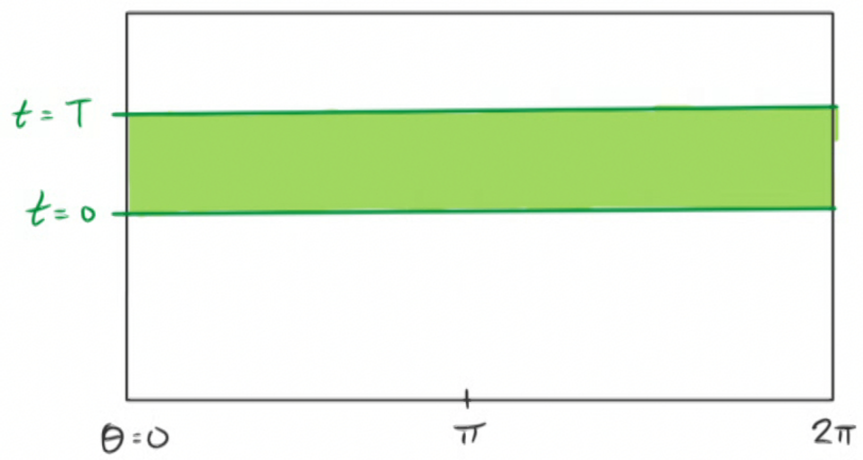
- In AdS/CFT, complexity of CFT state $\Leftrightarrow$ Einstein-Hilbert action of Wheeler-de Witt patch



The dS/MERA version is kind of trivial...

But hey! Why not?

→ Compare EH action of $0 \leq t \leq T$ region of dS to complexity of state prepared at $t = T$ layer



Rough estimates

Complexity:

- at $t = T$, # ancillas entangled is $N(T) = \sum_{j=0}^{T} 2^j$
- to even minimally entangle $N(T)$ ancillas using 2-local gates, need $N(T)/2$ gates (lower bound)
- MERA itself has $\sim N(T)$ gates (upper bound)

$$\therefore \; \mathcal{C} \sim \exp(T)$$

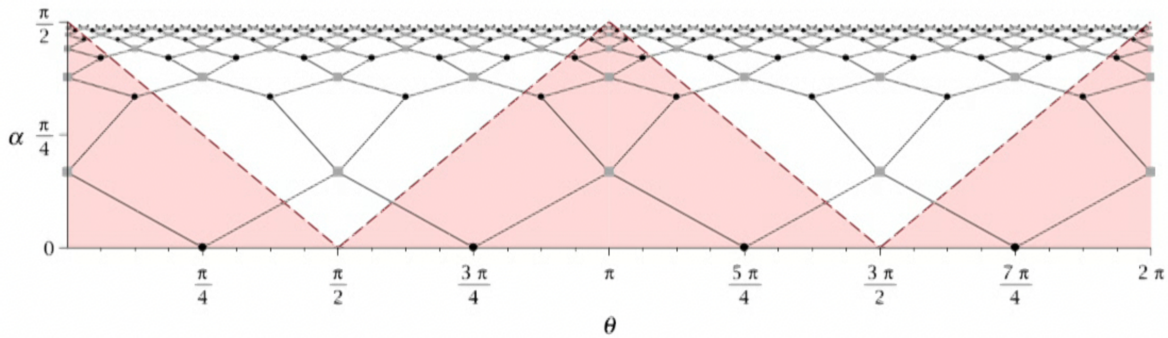
Action:

$$S_{EH}(T) = \frac{R\ell_{\mathrm{dS}}^{D}\mathcal{S}_{D-1}}{16\pi G}\frac{1}{(D-1)2^{D-1}}e^{(D-1)T} \; + \; \text{subleading}$$

Outline

1. Introduction: spacetime/tensor network correspondences
2. Let's chat about de Sitter
3. The MERA as global de Sitter
4. So what?
   - *i.* Cosmic no-hair
   - *ii.* Horizon complementarity
   - *iii.* Complexity equals action
5. Future directions

Some possible directions/questions

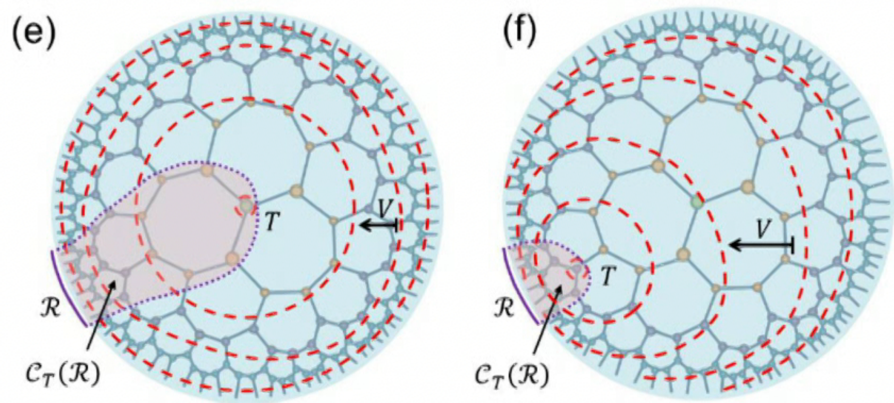


- What should the disentanglers and isometries be? (Can you make the correspondence explicit with, e.g., a field theory?)
- Examples of codes that realize strong complementarity?

Some possible directions/questions

- Different tensor networks or variations on the MERA?

E.g. hyperinvariant tensor networks?



source: Evenbly, 1704.04229