Title: Semi-classical evaluation of the 3D gravity path integral and quasi-local holography

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Abstract:


Semi-classical evaluation of the path integral for 3D quantum gravity and quasi-local holography
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with Christophe Goeller, Etera Livine, Aldo Riello
1710.04202 and 1710.04237
The path integral for gravity, PI, Nov 2017

Aim

A theory of quantum gravity:

Provides quantum amplitudes, here via evaluation of path integral, for all boundaries, including finite boundaries.

Holography (AdS/CFT):

Proposal for such amplitudes but only for a (small) subset of boundary conditions — those describing asymptotic boundaries.

How much holography survives for finite boundaries?

Investigate the properties of quantum amplitudes for regions with boundaries.
Are there (local) boundary theories?

3D gravity

Very special - no propagating degrees of freedom.

But is therefore
- 'solvable'
- suitable quantum theories are available
- ideal testbed for many questions.

3D gravity - approaches

Perturbative:
Einstein-Hilbert action - metric variables.
'Wick-rotated': Euclidean signature and statistical weights.

Continuum:

Path integral evaluation:
One-loop calculation for asymptotic boundaries.

[15 Barnich-Gonzalez-Maloney-Oblak]

Discretization:
With Regge (perfect) action and one-loop perfect measure: continuum bulk.
(Requires rotation of conformal mode.)

Path integral evaluation:
One-loop calculation for finite boundaries.
Identified linearized boundary theory, including bulk reconstruction.

[15 Bonzom-BD]

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3D gravity - approaches

Non-Perturbative:
Palatini action — equivalent to (SU(2)) BF action and (ISO(3)) Chern-Simon action.

Ponzano-Regge model:

Euclidean signature but quantum weights.
(Bulk) triangulation invariant.
(Boundary) triangulation invariant for fixed connection boundary conditions.

Equivalences

Combinatorical quantization:

Hamiltonian approach. Path integral given (in the cases under control) by Witten-Reshetikhin-Turaev models.

[Extensive body of work from mathematical physics, (loop) quantum gravity, condensed matter, …]

Other signatures and cosmological constant: possible but technically more involved.
Replace SU(2) with SL(2,R) or SU(2)q or SL(2,R)q.

Results for 3D gravity (for a solid torus)

[Barnich-Gonzalez-Maloney-Oblak `15]

Continuum metric gravity:
One-loop calculation for
asymptotic boundaries.

[Bonzom-BD `15]

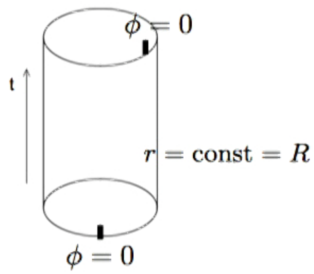
Regge gravity:
One-loop calculation for
finite boundaries.
Identified linearized boundary theory,
including bulk reconstruction.

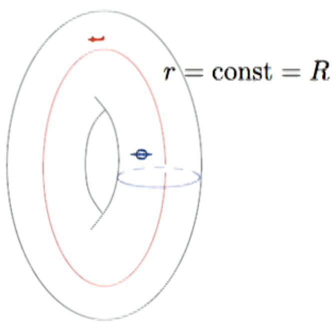
[BD-Goeller-Livine-Riello `17]

Ponzano Regge - non-perturbative path integral:
Determine exact boundary theories for finite boundaries.
Exact evaluation of path integral for "fuzzy square" boundary states.
Semi-classical evaluation for "coherent square" boundary states.
Provides also boundary theory and bulk reconstruction.

Classical solution:

Thermal spinning flat space





$$ds^2 = dx_1^2 + dx_2^2 + dx_3^2 = dt^2 + dr^2 + r^2 d\phi^2$$

periodic identification:

$$(t, r, \phi) \sim (t + \beta, r, \phi + \gamma)$$

inverse temperature and angular potential:
moduli parameter for the (boundary) torus

Want to study partition function:

$$Z(\beta, \gamma) = \int \mathcal{D}g \exp(-\frac{1}{\hbar} S_E)$$

One loop evaluation: asymptotic boundaries

[Barnich-Gonzalez-Maloney-Oblak 15]

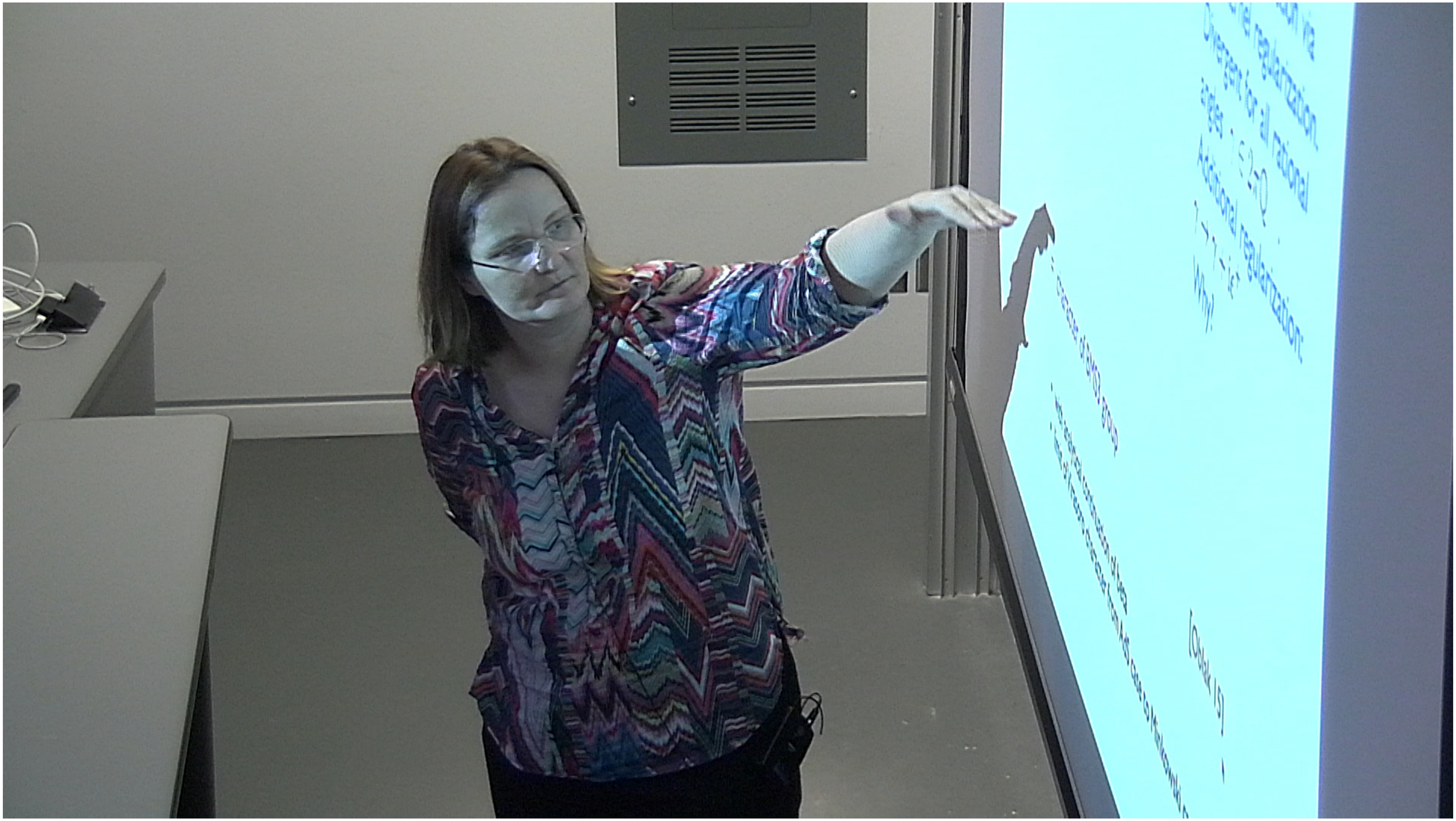
$$Z \sim \exp\left(\frac{(2)\pi\beta}{\ell_P}\right) \times \prod_{p\geq 2} \frac{1}{2 - 2\cos(p\gamma)}$$

Classical action given by (1/2) GHY boundary term.

One loop correction via heat kernel regularization. Divergent for all rational angles $\gamma \in 2\pi\mathbb{Q}$.
Additional regularization:
$\gamma \to \gamma + i\epsilon^+$
Why?

= character of BMS3 group [Oblak 15]

- with analytical continuation of beta
- limit of Virasoro character from AdS case to Minkowski case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One loop evaluation: asymptotic boundaries

[Barnich-Gonzalez-Maloney-Oblak 15]

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Product over (angular) Fourier modes starts with p=2. Why?

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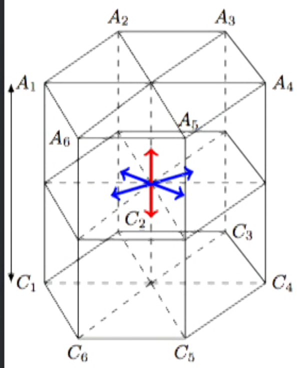
One loop result: finite boundaries

[Bonzom, BD JHEP 16]



Due to perfect action and one-loop measure [BD, Steinhaus 11] :
continuum result with mode truncation on boundary:

$$Z \sim \exp\left(\frac{2\pi\beta}{\ell_{\mathrm{P}}}\right) \exp\left(-S_{\mathrm{L}}(\delta l_{\mathrm{bound}})\right) \prod_{p\geq 2}^{\frac{1}{2}(N_x-1)} \frac{1}{2-2\cos(p\gamma)}$$

Scale 'symmetry' for 3D gravity maps solutions into solutions.
Regain asymptotic result.

One loop result: finite boundaries

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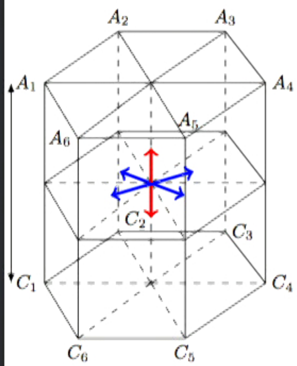

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Scale 'symmetry' for 3D gravity maps solutions into solutions. Regain asymptotic result.

(Linearized) boundary action for boundary fluctuations

Product starts with p=2:

- remnant of diffeomorphism symmetry given by vertex displacements
- p=0,+1,-1 correspond to diffeomorphism modes and need to be gauge fixed

Product is divergent for $\text{GCD}(N_x, N_\gamma) > 1$

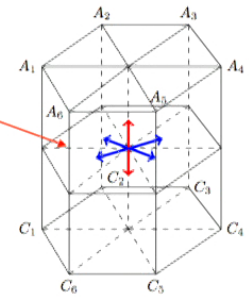
Linearized EOM admit ambiguities in solutions - resulting from needing more than one winding curve to cover boundary. Results in degenerate saddle points. Approximation artifact for finite boundaries.

Dual boundary field theory

[Bonzom, BD JHEP 16]

$$\exp\left(-S_{\mathrm{L}}(\delta l_{\mathrm{bound}})\right)$$

Boundary field gives geodetic length
from boundary to central axis:
boundary converts (radial) diffeomorphism
dof into physical (boundary) dof.
[Carlip 05]



- Find a Liouville type theory:

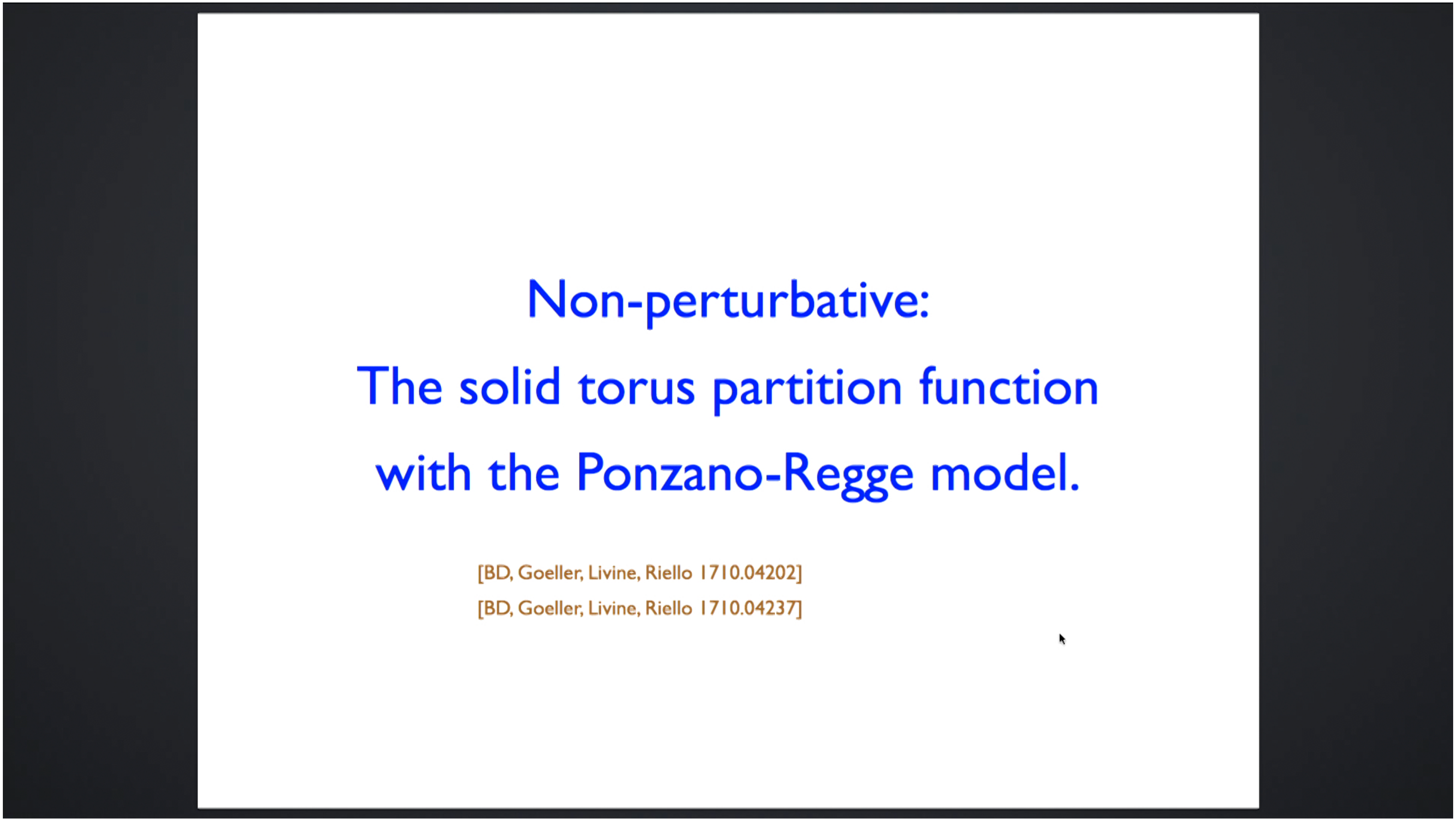
but: no angular derivatives

$$S_L \;=\; \int d^2y\sqrt{h}\left(h^{ab}\varphi\partial_a\partial_b\varphi + c\;{}^{2D}R\,\varphi\right)$$

similar actions in
[Barnich, Gomberoff, Gonzales 13;
Carlip 16: in Lorentzian signature]

Bulk reconstruction for free!

One-point function: geodesic lengths from boundary to a bulk point.
Two-point function: geodesic lengths from boundary to boundary point.


Non-perturbative:
The solid torus partition function
with the Ponzano-Regge model.
[BD, Goeller, Livine, Riello 1710.04202]
[BD, Goeller, Livine, Riello 1710.04237]

Ponzano-Regge: path integral for 3D gravity

[Ponzano-Regge 1968]

path integral for Palatini action $\rightsquigarrow$ path integral over flat connections

$$Z = \int \mathcal{D}[e]\mathcal{D}[\omega]\, e^{\frac{i}{2\ell_{\mathrm{Pl}}}\int \mathrm{Tr}(e\wedge F[\omega])} \rightsquigarrow Z = \int \mathcal{D}[\omega]\,\delta(F[\omega])$$

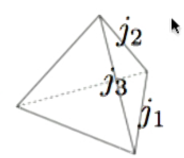
Regularize on a lattice (dual to a triangulation):

$$Z_{\text{PR-group}} = \left[\prod_l \int_{\mathrm{SU}(2)} \mathrm{d}g_l\right] \prod_f \delta\left(\overleftarrow{\prod}_{l\ni f} g_l^{\epsilon(l,f)}\right),$$

Variable transformation via (group) Fourier transform:

$$Z_{\text{PR-spin}} = \sum_{\{j_e\}} \prod_e v_{j_e}^2 \prod_t (-1)^{\sum_{e'\in t} j'_e} \prod_\sigma \left\{6j_{e''\in\sigma}\right\}.$$

6j symbols associated to tetrahedra



spin (SU(2) representations) give length of edges

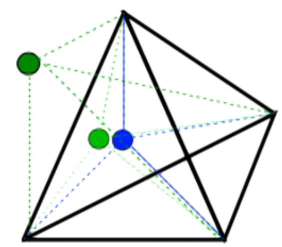
Ponzano-Regge: path integral for 3D gravity

$$\{6j_{e\in\sigma}\} \stackrel{j_e\to\infty}{\longrightarrow} \exp\left(iS_{\text{Regge}}\right) + \exp\left(-iS_{\text{Regge}}\right)$$

Large spin limit gives Regge (=discrete gravity) action for tetrahedron.

But amplitude is a sum over both orientations of for the tetrahedron.

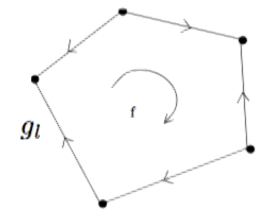
Ensures complete diffeomorphism (= vertex translation) symmetry and thus triangulation invariance.



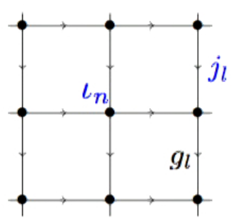
Meaning:
A boundary geometry can be either filled on the inside or on the outside.

Ponzano-Regge: path integral over flat connections

$$Z_{\text{PR-group}} = \left[\prod_l \int_{\text{SU}(2)} \mathrm{d}g_l\right] \prod_f \delta\left(\overleftarrow{\prod}_{l \ni f} g_l^{\epsilon(l,f)}\right),$$



Possible boundary states: spin networks

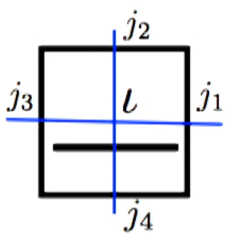


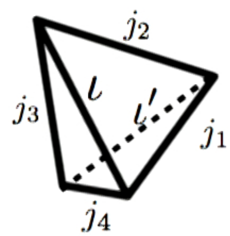
$$\psi_{\{j_l;\iota_n\}}(g_l)$$

labels:
SU(2) spins,
intertwiners
encode boundary
geometry

arguments:
SU(2) holonomies

Boundary spin networks = quantum boundaries





Spins and intertwiners
encode intrinsic and part of the
extrinsic geometry:
fuzzy geometry: the two diagonal lengths
do not commute

Boundary state determines boundary theory - statistical models like vertex models or spin chains.

Example: Spin 1/2 with general intertwiner leads to 6-vertex model (Heisenberg spin chain).

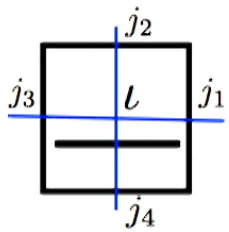
→ Interesting connection to integrable models.

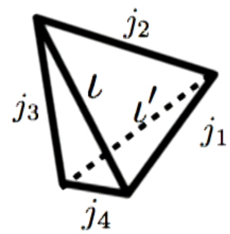
Semi-classical evaluation

of non-perturbative path integral

[used heavily in loop quantum gravity: Barrett et al, Freidel et al, Dowdal, Hellmann, Gomez, Han et al, …
but mostly for studying the large spin asymptotics for one building block]

Boundary spin networks = quantum boundaries





Spins and intertwiners encode intrinsic and part of the extrinsic geometry:
fuzzy geometry: the two diagonal lengths do not commute

Coherent intertwiners: geometry peaked on both diagonals. [Livine, Speziale 07]

$$|\iota\rangle = \int \mathrm{d}G \bigotimes_{l=1}^{4} D^{j_l}(G)|j_l, \xi_l\rangle$$

restoring gauge invariance

Spins giving length of edges.

Spinors encoding direction of edges.

Allows us to consider coherent squares.

Boundary theory for coherent states

Integrating out the flat bulk connection leaves us with a boundary theory:

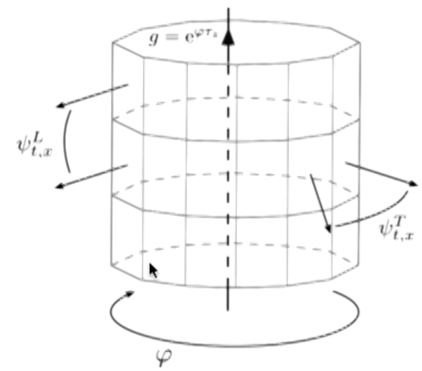
$$\langle Z|\Psi_{\mathrm{coh}}\rangle = \int \mathrm{d}\varphi \sin^2\frac{\varphi}{2}\left[\prod_v \int \mathrm{d}G_v\right] e^{\sum_v 2j_t \ln\langle\uparrow|G^{-1}_{v+x}G_v|\uparrow\rangle + 2j_x \ln\langle +|G^{-1}_{v+t}\, e^{\frac{\varphi}{N_t}\tau_z} G_v|+\rangle}$$

Class angle
for non-trivial
boundary holonomy

Boundary variables:
potential for a
flat connection

Boundary theory action:
Sigma model like.

Spinors
describing edge
of square.



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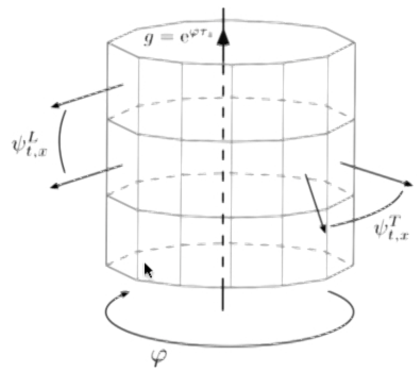

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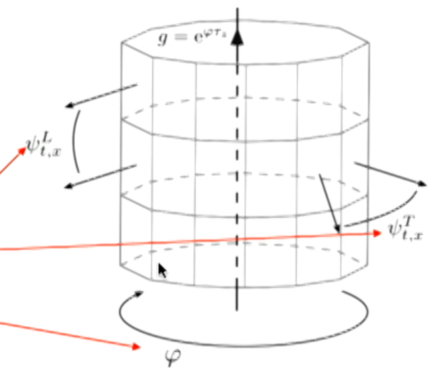
We perform a semi-classical evaluation (large spin) for the path integral of the boundary theory.

Saddle point equations determine the extrinsic curvature angles and the value for the non-trivial boundary holonomy.



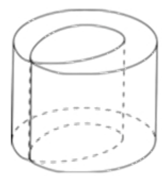
This provides a reconstruction of the bulk geometry.

Semi-classical evaluation: large spin asymptotics

[BD, Goeller, Livine, Riello 1710.04237]

Equation of motion reconstruct the extrinsic geometry.

Find expected 'background' geometry but also additional 'quantum backgrounds'.



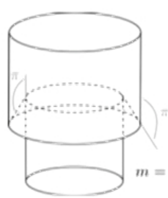
$n = 2$

Horizontal winding labelled by integer n:
Reason: Compactification of curvature angles to mod 2pi.
n<0: bulk is "outside" - global orientation change.

$N_x = 7,\ K = 1$

$N_x = 20,\ K = 4$

For $\mathrm{GCD}(N_x, N_\gamma) > 1$
Non-uniqueness of solutions.



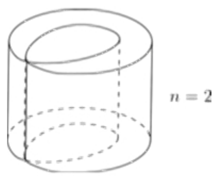
$m = 1$

Foldings.

Semi-classical evaluation: large spin asymptotics

[BD, Goeller, Livine, Riello 1710.04237]

$$\langle Z|\Psi_{\text{coh}}\rangle \overset{j\to\infty}{\rightarrow} \sum_{n\neq 0} e^{iS_0}\, \mathcal{A}(n)\, \mathcal{D}(n,\gamma)$$



windings

$$(-1)^{2N_t j_t} = e^{\mathrm{i}\frac{2\pi\beta n}{\ell_P}}$$

Exp action reduces to a sign.

$$\mathcal{A}(n) = \overline{\mathcal{A}(-n)}$$

Amplitude is real.
Dominated by
n=1 and

$$n = (N_x - 1)/2$$

(Due to an approximate symmetry.)

$$\mathcal{D}(n,\gamma) = \prod_{\substack{p>0\\ p\neq n}}^{(N_x-1)/2} \frac{1}{2-2\cos(p\gamma)}$$

For 'classical' background n=1
regain Regge/ continuum result.

n>1: particle with (n-1)xPlanck mass

Results

Semi-classical evaluation of fully non-perturbative path integral

- reproduces continuum result on 'classical background' and BMS3 character
- find the one-loop correction to start at p=2 (confirming diffeomorphism symmetry)
- dual boundary field theory given by sigma like model: linearization reproduces Regge results
- despite path integral being finite for finite boundaries reproduce divergencies
- divergencies for asymptotic boundary confirmed by exact evaluation for fuzzy squares

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Contributions from additional 'quantum' backgrounds

- winding due to compactification of connection to holonomies
- sum over orientations reduces to a global one!
- foldings

Can these be suppressed by modifying boundary state?

- add extrinsic curvature information
- boundary states: fully coherent states via "complexifier"

Conclusions

Aim:

Concrete model for holography, allowing for finite boundaries and explicit bulk reconstruction, connecting perturbative and non-perturbative frameworks.

Some features:

Interesting connections to
-integrable models (expected to describe only a subset of boundary conditions)
-condensed matter (edge modes, phase transitions of boundary theories etc.)
-BMS singularity structure arise from dual boundary fields described by spin chains.

Many open directions:

-Understand role of BMS group in dual boundary field theories, e.g. spin chains.
-Include massive and spinning particles: can we still perform bulk reconstruction.
-Holographic renormalization.