Title: Gravity and Higgs

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Abstract:

Gravity and The Higgs

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PI, 15/11/17

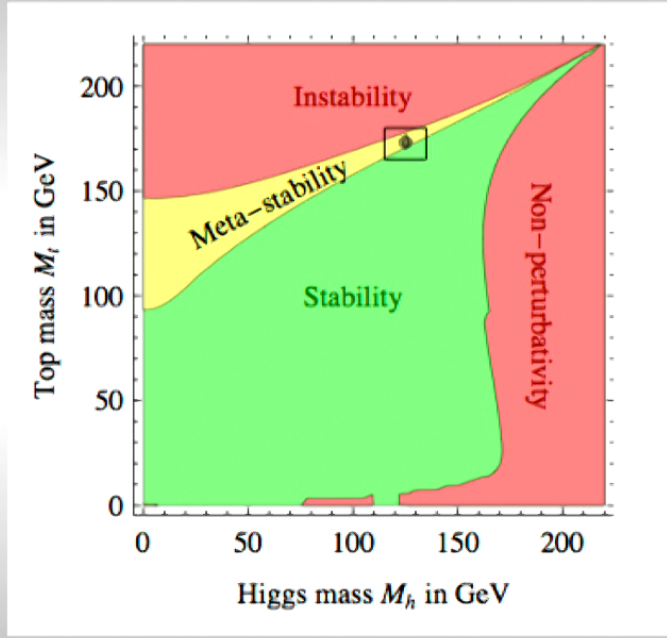
Ian Moss and Ben Withers, 1401.0017
Philipp Burda, Ian Moss 1501.04937, 1503.07331, 1601.02152

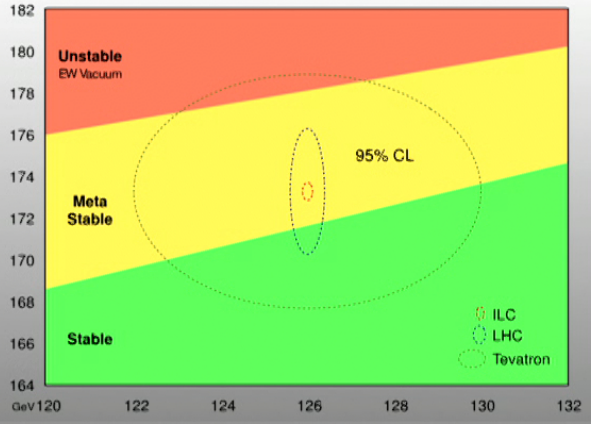
Outline

- How dependent are “classic” results on symmetry?
- Use standard semi-classical Euclidean approach for tunneling
- Breaking symmetry with black holes
- Keeping it analytic (for as long as possible!)
- Problems with primordial black holes

How Stable Is Our Vacuum?

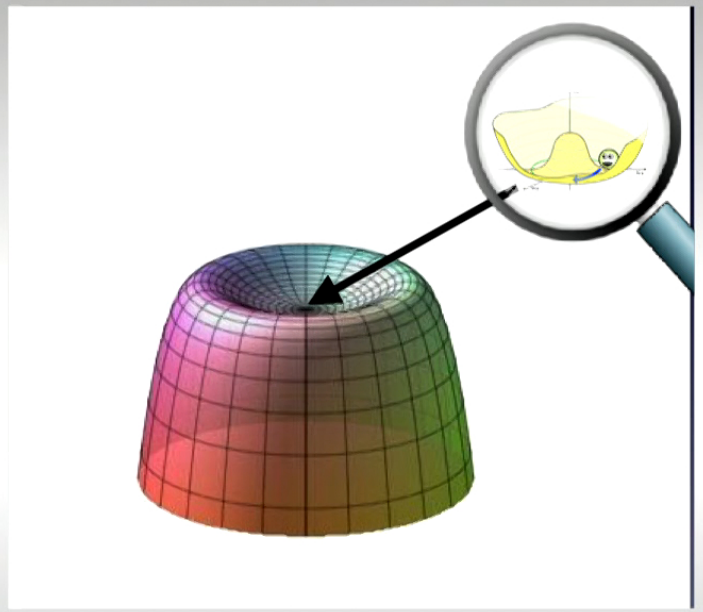
Calculating the running of the Higgs coupling tells us that we seem to be in a sweet spot between stability and instability – metastability.



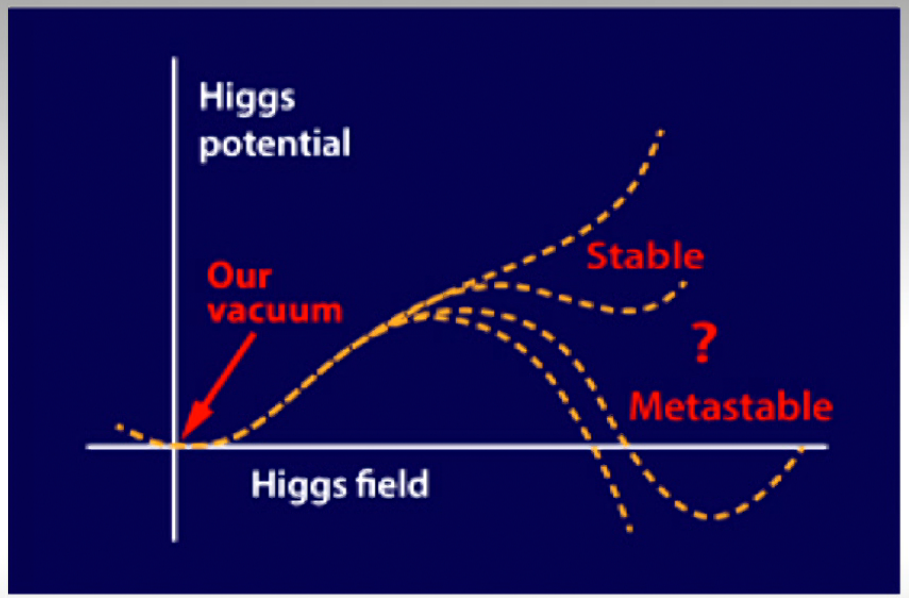


Higgs Potential

At high energies, the Higgs self-coupling becomes negative, opening the possibility that our universe may be….



….not entirely stable!



Coleman de Luccia say that the half-life of the universe is hundreds of billions of years.

Euclidean Trick

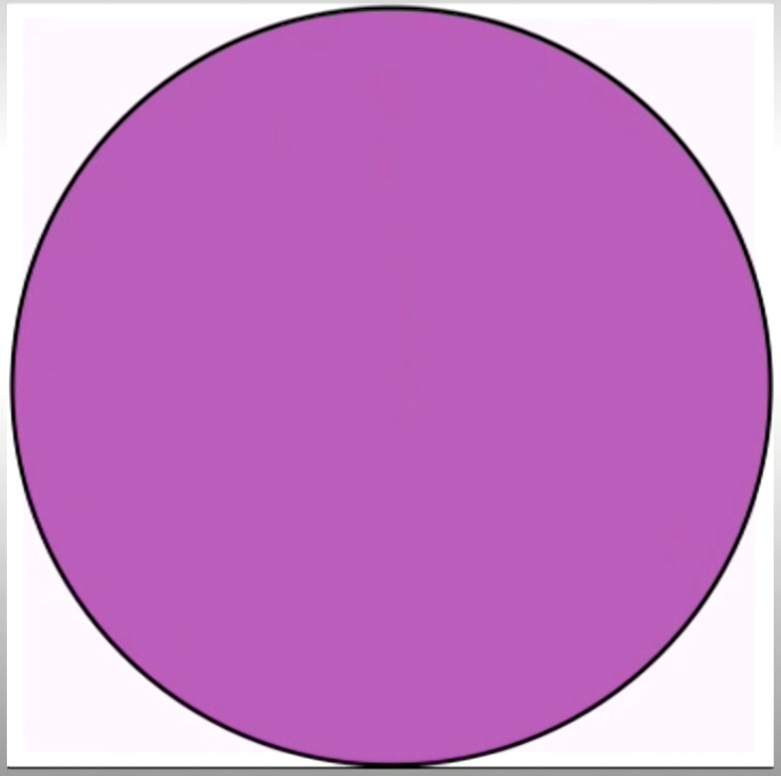
Using the idea that the probability of a tunelling process is roughly

$$\Gamma \sim e^{-S_E/\hbar}$$

Where S_E is the action of a classical process moving in an inverted potential. The particle rolls from the (now) unstable point to the "exit" and back again – a "bounce".

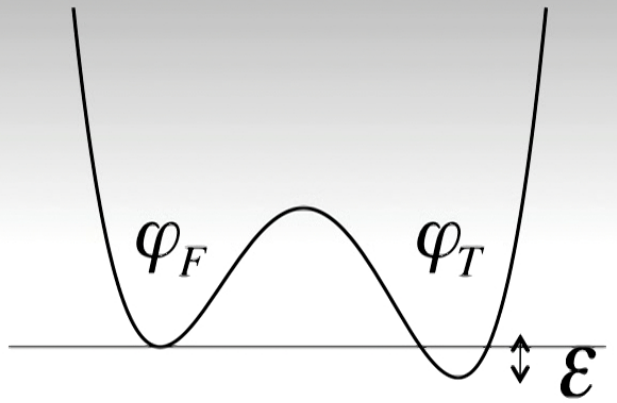
This gives a nice way of computing tunneling probability that generalises to field theory.

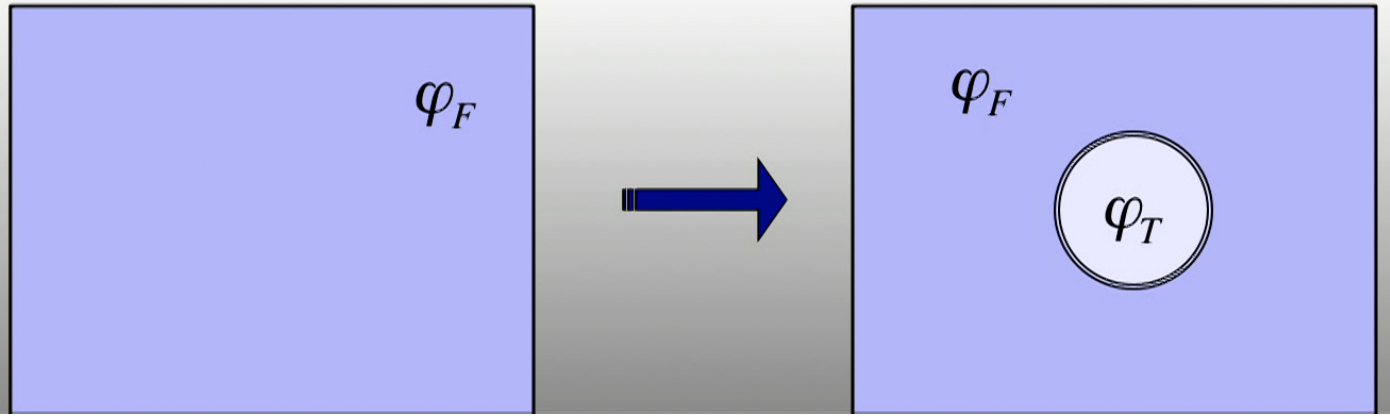
Lorentzian Picture



COLEMAN

Original work of Coleman considered a field theory with false vacuum, showed that in limit of small energy difference (relative to barrier) transition modelled by a "thin wall" bubble.





Coleman

Solving the Euclidean field equations should give the saddle point approximation for the tunneling solution.

$$\frac{d^2\phi}{d\tau^2} + \nabla^2\phi = -\frac{\partial V}{\partial \phi} = 2\lambda\phi(\phi^2 - \eta^2) + \mathcal{O}(\epsilon)$$

Original work of Coleman took a field theory with a "false" vacuum: in limit of small energy difference (relative to barrier) transition modeled by a "thin wall" bubble.

$$\phi'' + \frac{3}{\rho}\phi' = 2\lambda\phi(\phi^2 - \eta^2) \qquad [\rho^2 = \tau^2 + \mathbf{x}^2]$$

$$\phi \approx \eta \tanh[\sqrt{\lambda}\eta(\rho - \rho_0)]$$

Euclidean Action

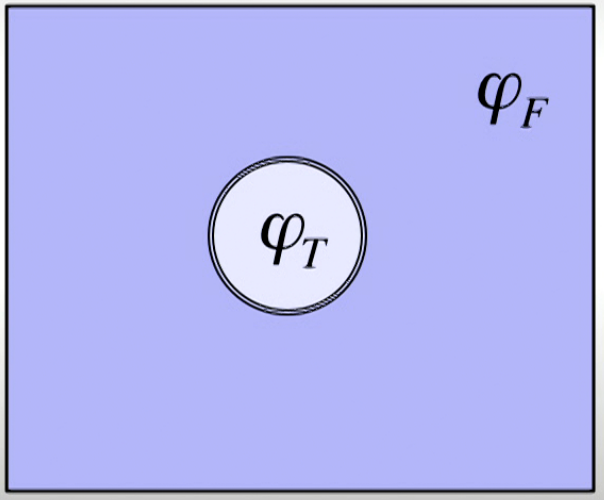
Amplitude determined by action of Euclidean tunneling solution: "The Bounce"

$$\mathcal{B} = \varepsilon \int d^4x\sqrt{g} - \sigma \int d^3x\sqrt{h}$$

$$\sim \frac{\pi^2}{2}\varepsilon R^4 - 2\pi^2\sigma R^3$$

GAIN FROM VACUUM

COST OF WALL



Coleman

Since the bounce is a solution to eqns of motion, it should be stationary under variation of R:

$$R = \frac{3\sigma}{\varepsilon} \quad , \quad \mathcal{B} = \frac{27\pi^2\sigma^4}{2\varepsilon^3}$$

Tunneling amplitude:

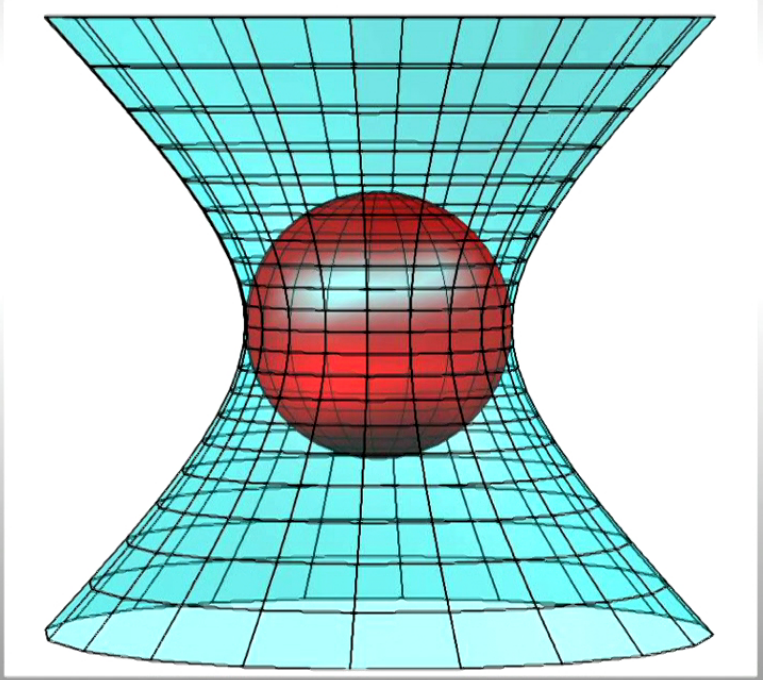
$$\mathcal{P} \sim e^{-\mathcal{B}/\hbar}$$

(Notice, R is big, so justifies use of the "thin wall" approximation.)

Add Gravity

Vacuum energy gravitates, so we must add gravity to our picture.

e.g. Lorentzian dS looks like a hyperboloid and Euclidean dS a sphere:



Our instanton must cut the sphere and replace it with flat space (true vacuum).

Coleman de Luccia (CDL)

Coleman and de Luccia showed how to do this with a bubble wall.

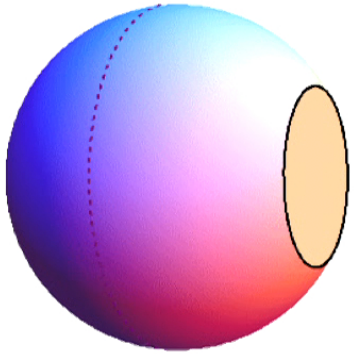
- The instanton is a solution of the Euclidean Einstein equations with a bubble of flat space separated from dS space by a thin wall.
- The wall radius is ***determined*** by the Israel junction conditions
- The action of the bounce is the difference of the action of this wall configuration and a pure de Sitter geometry.

Coleman and de Luccia, PRD21 3305 (1980)

CDL Instanton

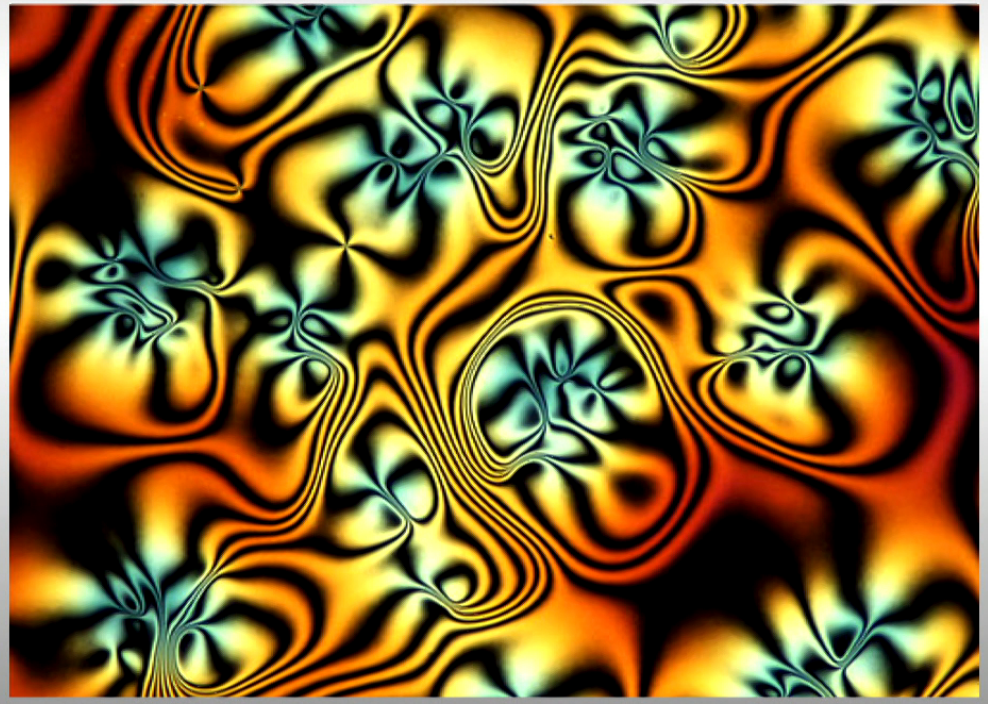
Euclidean de Sitter space is a sphere, of radius ℓ related to the cosmological constant. The true vacuum has zero cosmological constant, so must be flat.

$$\Lambda = \frac{3}{\ell^2}$$



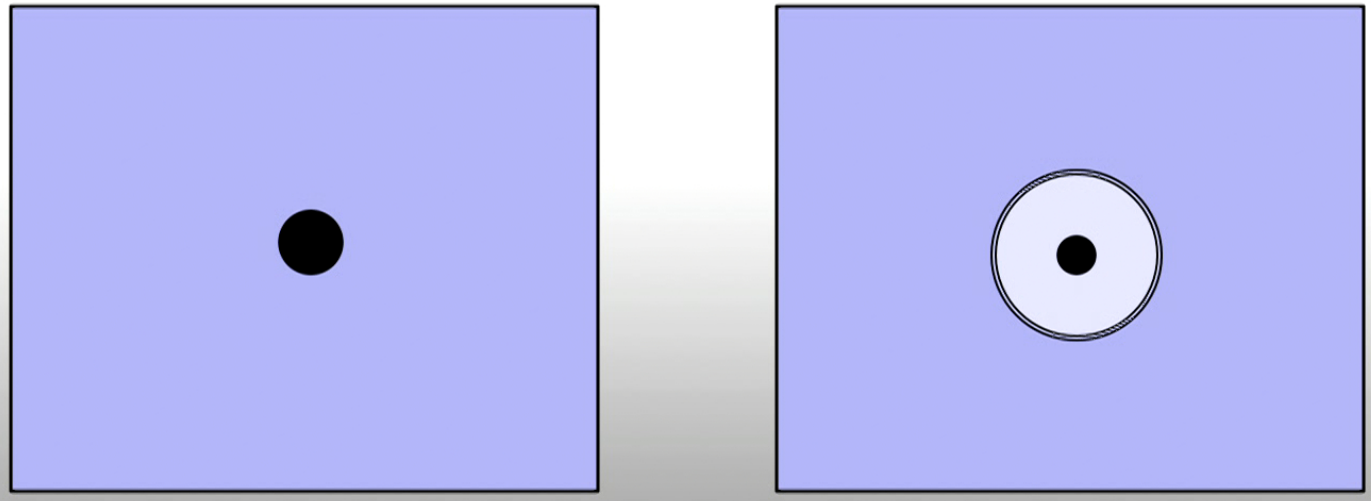
The bounce looks like a truncated sphere.

The universe is complex – so how dependent are our results on the assumptions of homogeneity and isotropy? Phase transitions in nature are more “dirty” – how does that affect modelling?



Tweaking CDL

The bubble of true vacuum has a spherical symmetry, so we can add a black hole at "minimal expense"!



RG, Moss & Withers, 1401.0017

A more general Thin Wall bubble

Straightforward to find solutions. Israel junction conditions determine the equation of motion of bubble wall with the black hole.

In each case we have to calculate the difference between the background black hole action and the effect of the bubble.

Need to deal with conical singularities (sometimes).

The general action with a black hole on each side is (details vary with Lambda):

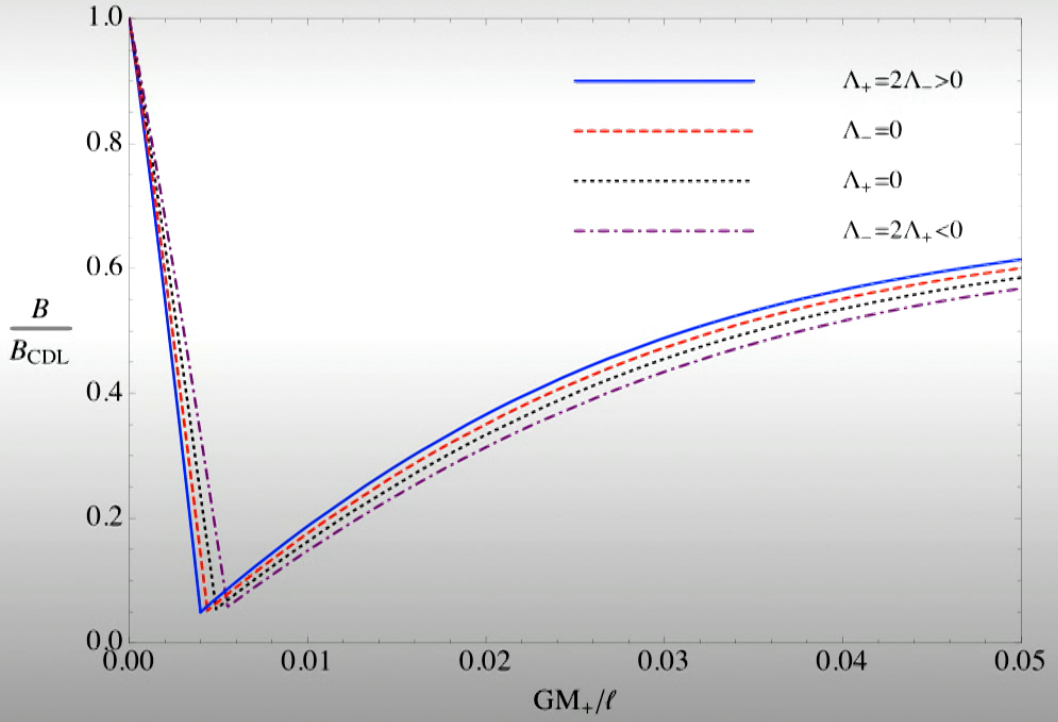
$$\mathcal{B} = \underbrace{\frac{\pi(r_+^2 - r_-^2)}{G}}_{\text{Geometry}} \underbrace{- \frac{\bar{\sigma}}{G}\int d\lambda R^2 - \frac{1}{4G}\int d\lambda R^2 \left(f'_+ \dot{\tau}_+ - f'_- \dot{\tau}_-\right)}_{\text{Bubble}}$$

General Bounce

- The general solution has a black hole inside the bubble (remnant) and a mass term outside (seed).
- The solution in general depends on time, but for each seed mass there is a unique bubble with lowest action.
- For small seed masses this is time dependent – a perturbed CDL – with no remnant black hole.
- For larger seed masses this is static and has a remnant black hole.
- For a special Mcrit, there is a static bubble with no remnant.
- Large range of solutions with $B<B_{CDL}$

Generic thin wall Tunneling

Main change is the value of lambda on each side, this changes the action ratio surprisingly little.



The fate of the black hole?

But that is not all that can happen! Black holes can also evaporate – so we must check which process wins. Compare the evaporation rate:

$$\Gamma_H \sim 3.6 \times 10^{-4} (G^2 M^3)^{-1}$$

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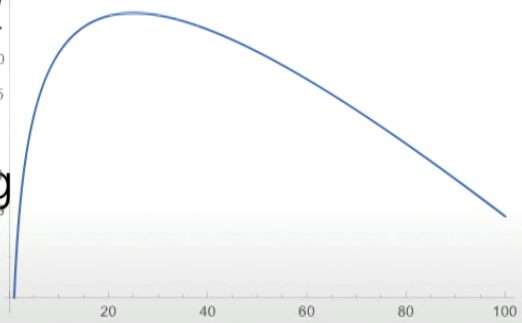
to our calculated tunneling rate

$$\Gamma_D \sim e^{-\Delta S}$$

The fate of the black hole?

The Euclidean action is the difference in horizon areas, related to difference in mass (which varies very slowly with M).

$$\Delta S \sim \frac{M}{M_p^2}\delta M$$



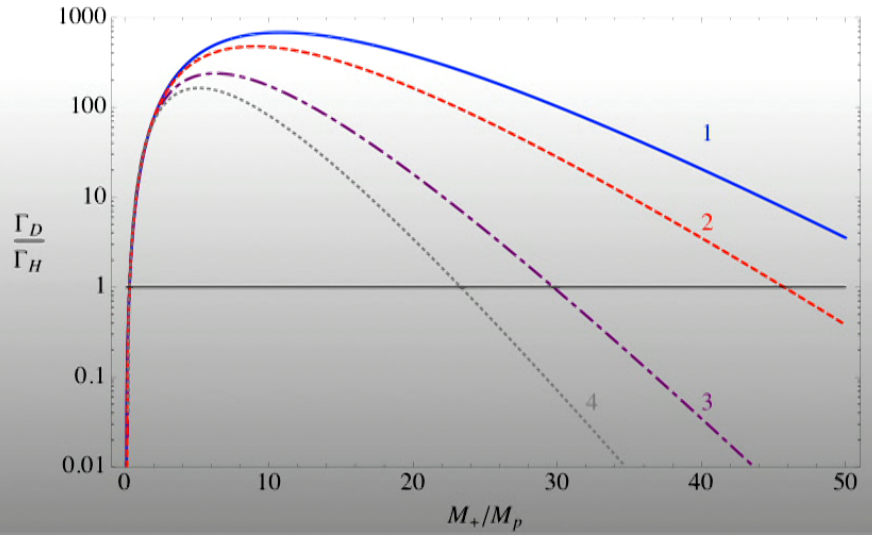
And estimate pre-factor as $(GM)^{-1/2}$, giving

$$\frac{\Gamma_D}{\Gamma_H} \sim \left(\frac{M}{M_p}\right)^{5/2} e^{-M\delta M/M_p^2}$$

Tunneling v Evaporation:

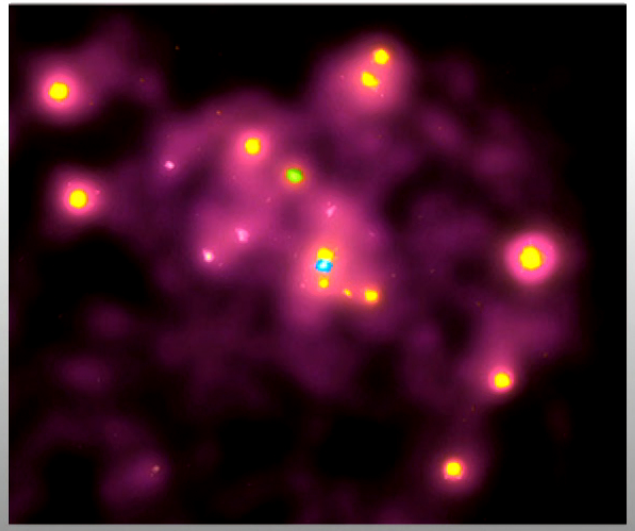
The actual factors calculated from thin walls shows that evaporation (perturbative) is much stronger than decay (nonperturbative) until the black holes are very small.

Decay NOT an issue for astrophysical black holes.



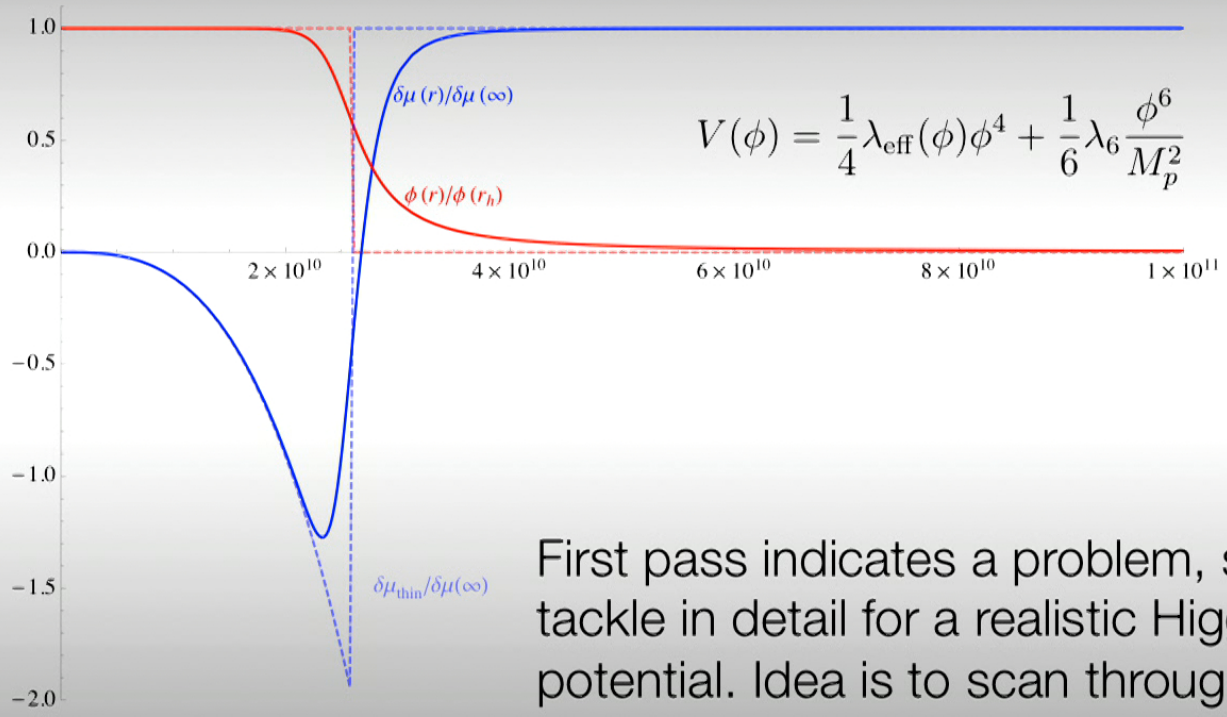
Primordial Black Holes

Primordial Black Holes are tiny black holes with masses of order a ton, so have a temperature above the CMB, so these do evaporate over time. Eventually, they become light enough that they hit the "danger range" for vacuum decay and WILL catalyse it.



For thin wall, parameter values push credulity – however – provide proof of principle.

Thin to Thick wall

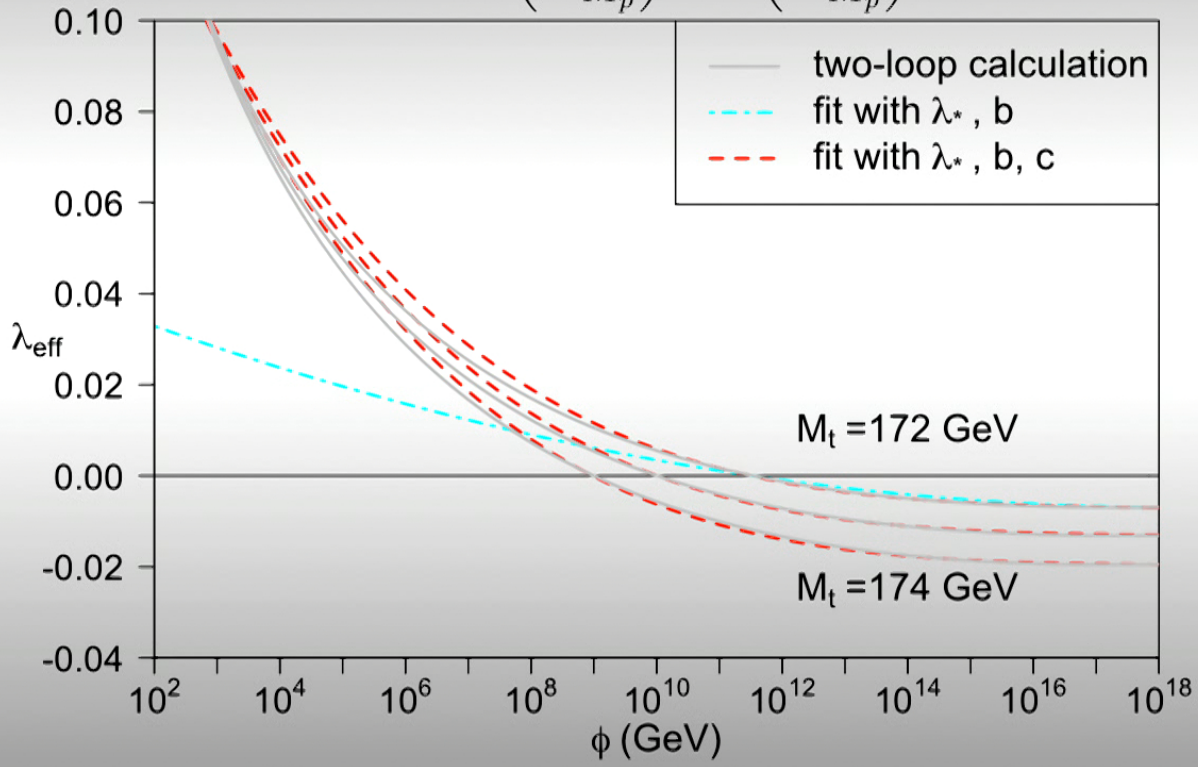


$$V(\phi) = \frac{1}{4}\lambda_{\rm eff}(\phi)\phi^4 + \frac{1}{6}\lambda_6\frac{\phi^6}{M_p^2}$$

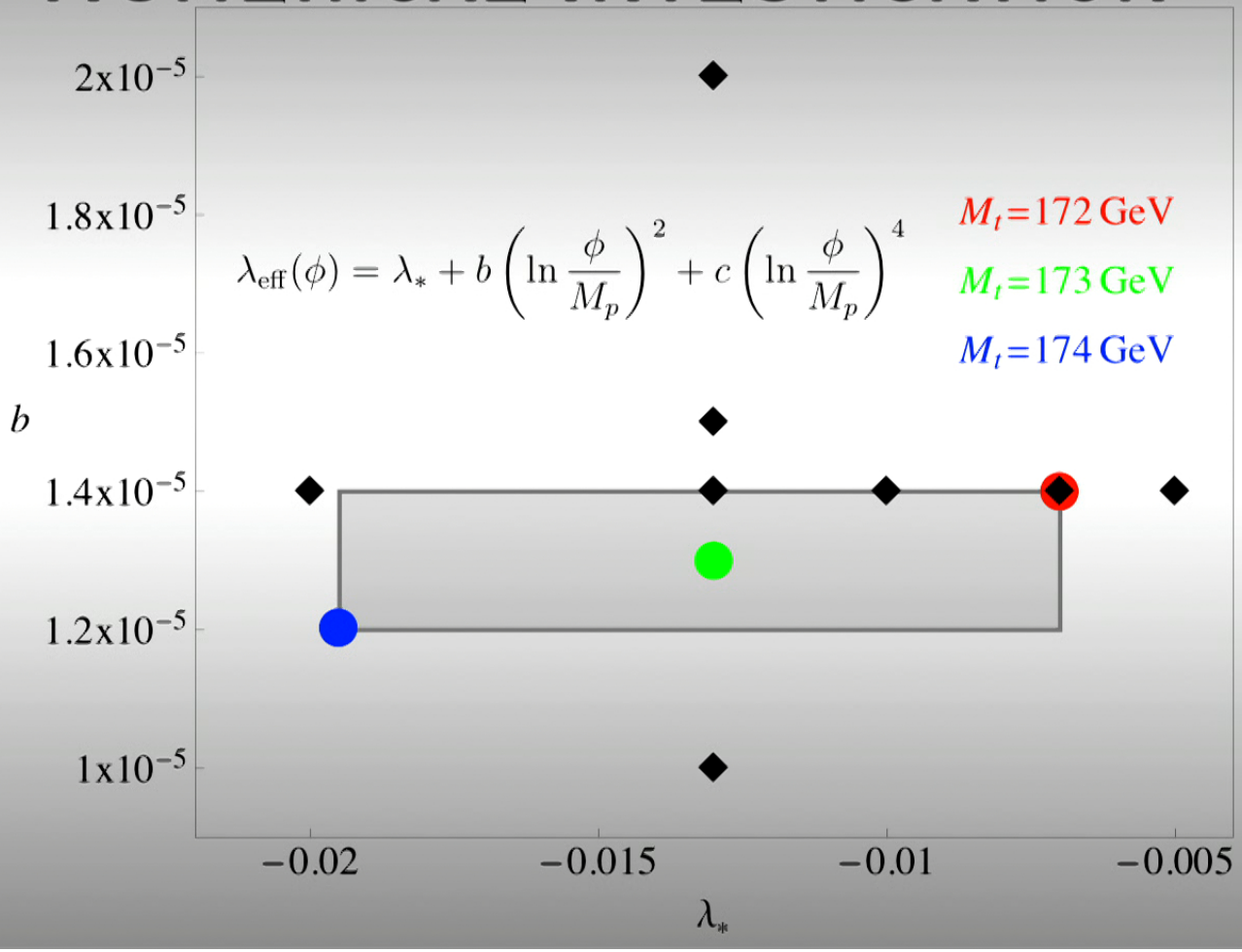
First pass indicates a problem, so tackle in detail for a realistic Higgs potential. Idea is to scan through parameter space (beyond standard model) to see how robust result it.

Fitting the Potential

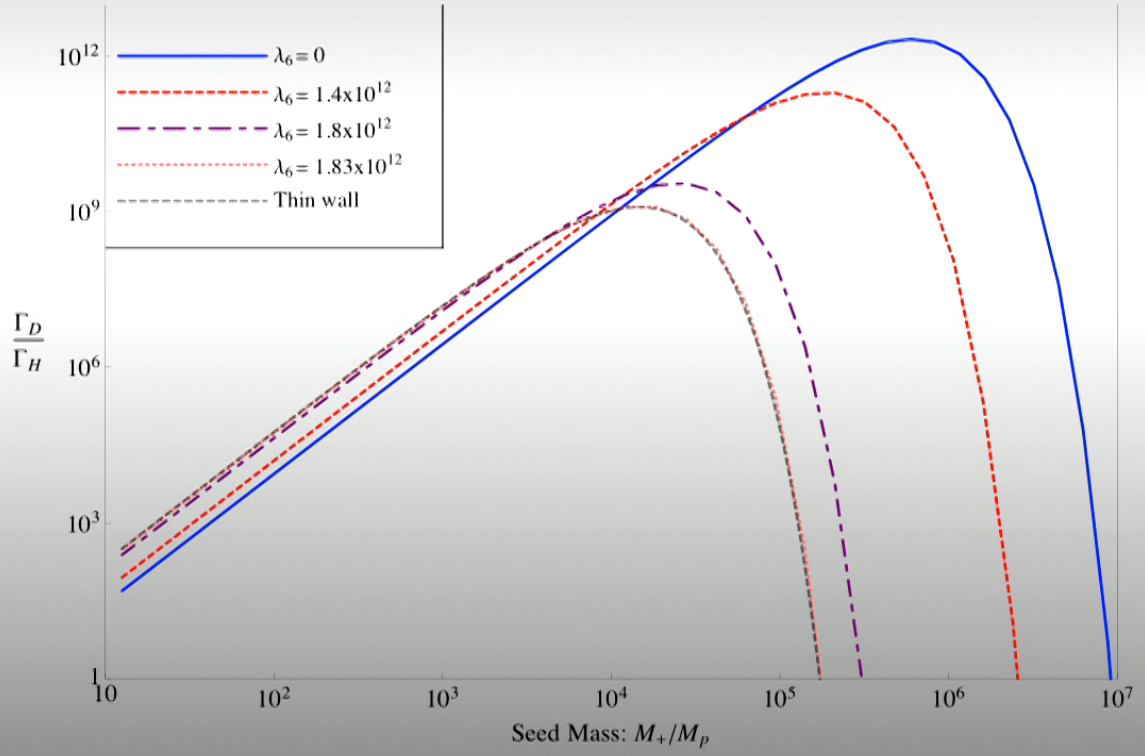
$$\lambda_{\rm eff}(\phi) = \lambda_* + b\left(\ln\frac{\phi}{M_p}\right)^2 + c\left(\ln\frac{\phi}{M_p}\right)^4$$



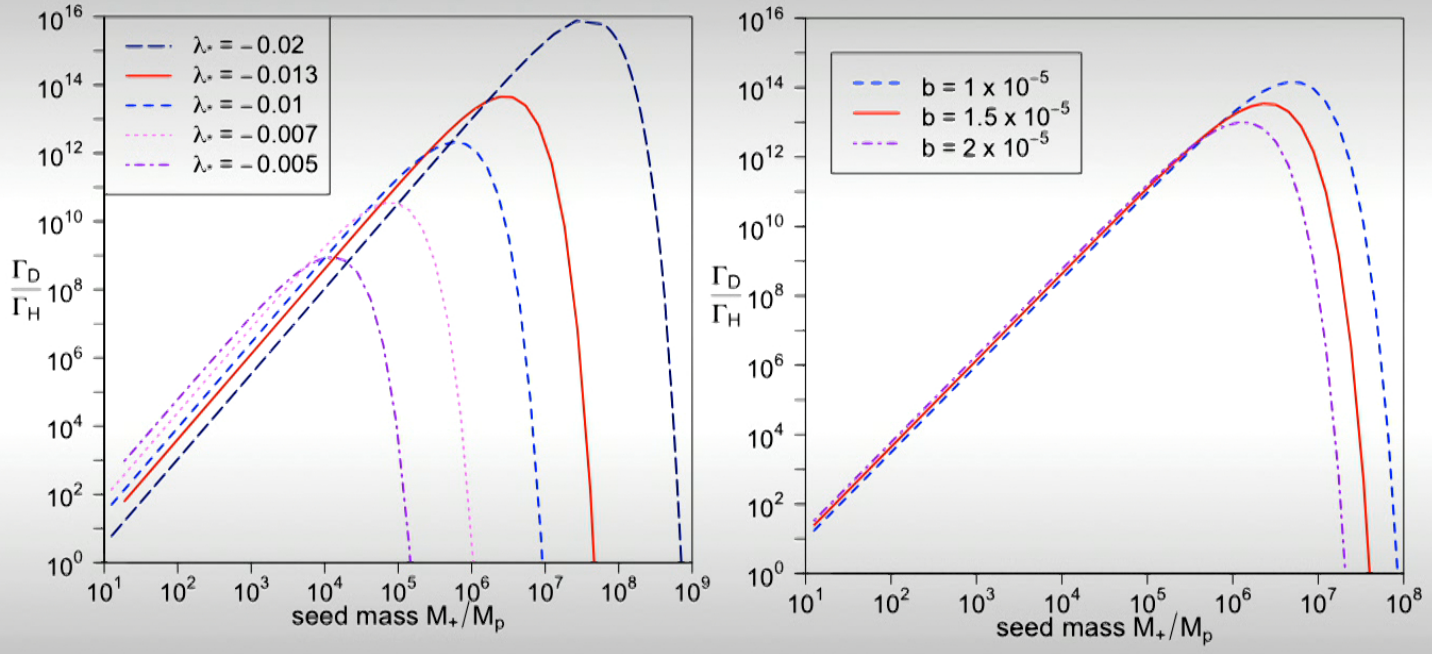
Numerical Investigation



First check thin wall, by increasing λ_6. Thickening the wall increases the effectiveness of the instanton – the primordial black hole will hit the danger zone much sooner, and the decay will proceed rapidly.



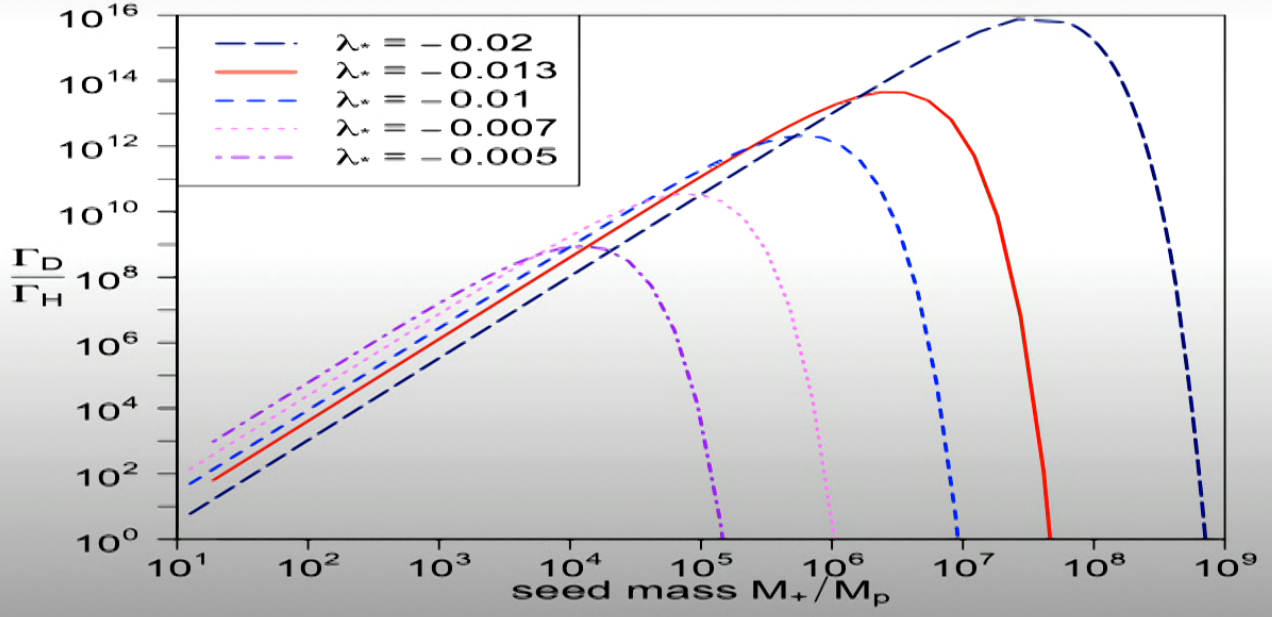
Scanning through parameter space for pure SM potential shows main dependence on λ_*:



And because we are at such extreme scales, the lifetime of the universe drops to around 10^{-17}s!

Primordial black holes start out with small enough mass to evaporate and will eventually hit these curves.

Can view as a constraint on PBH's or (weak) on corrections to the Higgs potential.



Summary

- Depending on higher energy physics, the Higgs vacuum may be unstable.
- We can construct an *instanton* to describe the decay process – including gravity.
- Tunneling amplitude significantly enhanced in the presence of a black hole – bubble forms around black hole and can remove it altogether.
- Very efficient for small black holes, so primordial black holes act as a trigger to change the state of the universe!

The Final BANG?

BUT – it's fair to say that we expect there is more to our standard model than we have currently seen…

…primordial black holes are not universally accepted....

...and the new phase would have to percolate.