Title: Consistency of a quantum cosmological bounce

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Abstract:

Consistency of a quantum cosmological bounce

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Outline

Introduction

Cosmological bounce models provide an alternative to the **initial Big Bang singularity** of classical general relativity with standard (NEC-preserving) matter.

An attractive alternative to modifying either general relativity or introducing additional non-standard matter in order to obtain a bounce is to explore the possibility of a **quantum bounce**, within "standard physics".

For models with radiation and minimally or conformally coupled scalar fields, there is the possibility of a **perfect bounce:** a quantum transition from a collapsing to an expanding universe which remains regular and unambiguous across the classical singularity.

Demanding consistency of this quantum picture can impose surprising constraints on cosmology, as I will show for the case of spatial curvature. Such constraints go beyond what can be understood within classical or semiclassical cosmology.

"Dirac Gravity" – Weyl invariant General Relativity

Certain cosmological singularities in classical general relativity can be resolved by a adding a "fake Weyl invariance" to GR: rewrite the Einstein-Hilbert action

$$S_{\rm EH}[\tilde{g}] = \frac{1}{16\pi G}\int d^4x\,\sqrt{-\tilde{g}}\;R[\tilde{g}]$$

in terms of new fields g and ϕ by setting $\tilde{g}_{\mu\nu} := (\frac{4}{3}\pi G)\phi^2\,g_{\mu\nu}$; then [Dirac 1973]

$$S_{\rm Dirac}[g,\phi] = \frac{1}{2}\int d^4x\,\sqrt{-g}\left((\partial\phi)^2 + \frac{1}{6}\phi^2 R[g]\right)$$

is invariant under $g \to \Omega^2 g,\;\; \phi \to \Omega^{-1}\phi$, for any nowhere vanishing $\Omega(x)$. All solutions with $\phi(x) \neq 0$ everywhere are solutions of GR (and vice versa).

Any conformally regular (e.g., conformally flat) metric can hence be made regular (flat) by using Weyl invariance to move all singularities into the field ϕ.

Cosmology in Dirac gravity

A (homogeneous, isotropic) FLRW metric can be written as

$$ds^2 = a^2(t)(-N_c^2(t)dt^2 + h_{ij}\,dx^i dx^j)$$

where N_c is a conformal lapse and h_{ij} is a metric of constant three-curvature $R^{(3)} = 6\kappa$.

We can exploit the symmetry under $a \to \Omega a,\ \phi \to \Omega^{-1}\phi$ to set $a(t) = \text{constant}$ with time-dependent ϕ. In this gauge, instead of an expanding universe, we have a static universe with time-dependent units as given by ϕ. ϕ can now cross zero, allowing the extension of solutions beyond the GR singularity $a = 0$.

This is the opposite of *Einstein gauge* in which

$$\phi^2(x) = \text{constant} =: \frac{3}{4\pi G}\,,$$

reducing Dirac gravity to GR, where the units set by G are time-independent.

Minisuperspace action with conformal matter

In the gauge in which the scale factor is constant, the minisuperspace action for Dirac gravity becomes ($V_0 \equiv \int d^3x \sqrt{h}$)

$$S[\phi] = V_0 \int dt \, [-\frac{1}{2N_c}\dot{\phi}^2 + N_c \frac{\kappa}{2}\phi^2] \, .$$

We now couple conformally invariant matter: a radiation fluid, characterised by an energy density ρ, and M conformally coupled scalar fields χ^i, $i = 1, \ldots, M$. The total action is then

$$S[\phi, \chi^i, \rho, \tilde{\varphi}] = V_0 \int dt \left[\frac{\sum_i (\dot{\chi}^i)^2 - \dot{\phi}^2}{2N_c} + N_c \left(\frac{\kappa}{2}(\phi^2 - \sum_i (\chi^i)^2) - \rho \right) - \tilde{\varphi}\dot{\rho} \right] \, .$$

In our gauge in which space does not expand, ρ is constrained to be constant in time. This is the action for a massive relativistic particle moving in $(M+1)$-dimensional Minkowski spacetime, subject to a quadratic potential.

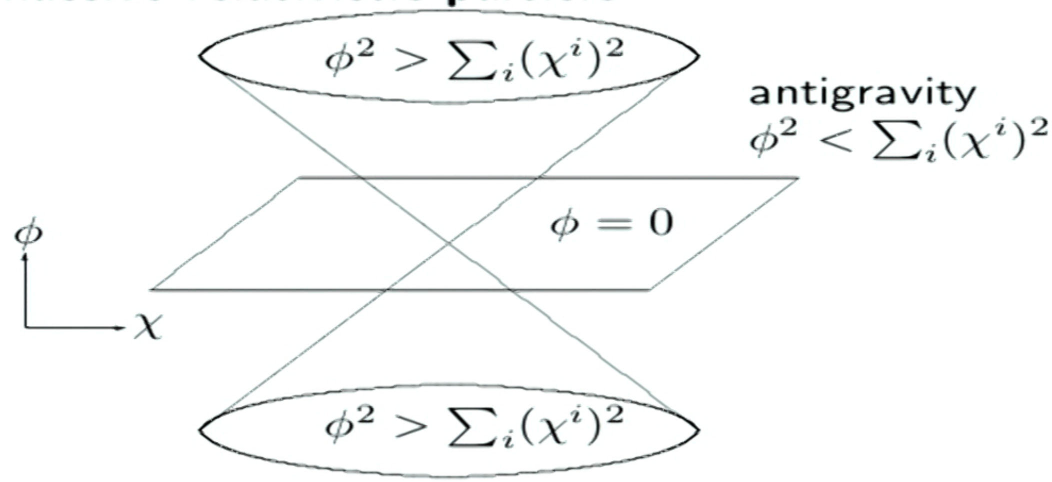
Cosmological evolution

The relative contribution of the three components (scalars, radiation, curvature) changes with cosmological evoution as usual,

- Near the would-be GR singularity $\phi = 0$, the kinetic energy in the scalar fields dominates. At very high energies, this growing kinetic energy drives the scalars to super-Planckian field values; thus the effective Newton's constant (the coefficient of R in the action) becomes negative ("antigravity").
- Radiation is the dominant matter component away from the singularity.
- If spatial curvature is present, it dominates at early/late times. Negative κ will contribute to faster expansion of the Universe, or growing ϕ in our gauge. Because of the nonminimal coupling of κ to the scalars, $\kappa < 0$ can also drive the scalars to super-Planckian values and hence into the antigravity regime.

We will see the last point becomes problematic in the quantum theory.

Universe as a massive relativistic particle



If we write the $M+1$ scalars and the radiation energy density as

$$x^\alpha := \frac{1}{\sqrt{2\rho}}(\phi, \chi^i)\,, \quad m := 2V_0\rho\,,$$

x^α are now Minkowski space coordinates, and m is the "mass" of the universe.

Quantum Transitions: Feynman Propagator

We are now interested in describing the quantum transition of a universe from a $\phi < 0$ initial state to a $\phi > 0$ final state.
The Feynman propagator implements a direction of time corresponding to the variable ϕ. Explicitly it is given by

$$
\begin{aligned}
G(x,m|x',m') &= \int \mathcal{D}x^\alpha\,\mathcal{D}P_\alpha\,\mathcal{D}m\,\mathcal{D}\varphi\,\mathcal{D}N_c \times \\
&\quad \exp\left(\mathrm{i}\int dt\left(\dot{x}^\alpha P_\alpha - \dot{m}\varphi - N_c\left(\frac{P_\alpha P^\alpha}{2m} + \frac{m}{2}(\kappa x_\alpha x^\alpha + 1)\right)\right)\right) \\
&= \mathrm{i}\delta(m-m')\int_0^\infty d\tau\,\left(\frac{m\sqrt{\kappa}}{2\mathrm{i}\pi\sin(\sqrt{\kappa}\,\tau)}\right)^{\frac{M+1}{2}} \times \\
&\quad \times \exp\left[\mathrm{i}\,\frac{m}{2}\left(\sqrt{\kappa}\,\frac{(x^2+x'^2)\cos(\sqrt{\kappa}\,\tau) - 2x\cdot x'}{\sin(\sqrt{\kappa}\,\tau)} - \tau\right)\right]\,.
\end{aligned}
$$

Feynman propagator for FLRW Universes

For $\kappa = 0$, we recover the Feynman propagator for a massive particle in $(M+1)$-dimensional Minkowski spacetime,

$$G^{\kappa=0}(x, m|x', m') = \frac{1}{2}\delta(m - m')(-\mathrm{i}m)^M (2\pi s)^{\frac{1-M}{2}} H^{(2)}_{\frac{M-1}{2}}(s) \,,$$

where $H^{(2)}_\alpha(x)$ is a Hankel function and $s := m\sqrt{-(x-x')^2 - \mathrm{i}\epsilon}$.

The propagator falls off for spacelike separations; this is consistent with the fact that classical solutions are always timelike for $m > 0$.

For flat FLRW universes, any classical solution starts and ends in a region in which $\phi^2 > \sum_i (\chi^i)^2$, i.e. the effective Newton's constant remains positive; quantum mechanically excursions into "antigravity" are exponentially suppressed.

Excursions into "antigravity"?

Choosing an open universe with $\kappa < 0$ means adding a repulsive quadratic potential, i.e., studying an "upside-down" relativistic harmonic oscillator.

Even for $m > 0$, there are then classical solutions that asymptotically remain in the "antigravity" region: the general solution is

$$x^\alpha(t) = \frac{x_1^\alpha}{\sqrt{-\kappa}} \exp\left(\sqrt{-\kappa}\int_0^t dt'\, N_c(t')\right) + \frac{x_2^\alpha}{\sqrt{-\kappa}} \exp\left(-\sqrt{-\kappa}\int_0^t dt'\, N_c(t')\right) ,$$

with $x_1 \cdot x_2 = \frac{1}{4}$, which can be satisfied for one or both of x_1^α and x_2^α chosen to be spacelike. For such solutions, repulsion dominates over the radiation mass m.

One would focus on solutions with timelike x_1^α and x_2^α, which start in the region $\phi^2 > \sum_i (\chi^i)^2$ for $\phi < 0$, eventually return for $\phi > 0$, and remain there.

Quantum consistent open FLRW?

As one might expect, the antigravity solutions for $\kappa < 0$ do contribute to quantum transitions as described by the propagator.

The Feynman propagator can no longer be computed analytically, but a saddle approximation for the proper time τ integral, for $x^2 \to +\infty$ at fixed x', gives

$$G(x, m|x', m') \sim |x|^{-1/2} e^{\mathrm{i}\frac{m}{2}\sqrt{-\kappa}x^2} .$$

The propagator becomes oscillatory at spacelike separations with a slowly decaying prefactor, so that, e.g., a wave packet centred around an initial state in the gravity region $\phi < 0$, is propagated to large spacelike distances into the antigravity region.

There is then no quantum consistent bounce from a contracting to an expanding universe that could be interpreted in semiclassical terms, in contrast to the well-behaved $\kappa = 0$ case.

Summary

- An alternative to the Big Bang singularity is a bounce, a transition from a collapsing to an expanding Universe. A quantum bounce can be described by the Feynman propagator in quantum cosmology.
- With conformally coupled scalar fields and radiation as matter, there is the possibility of an "antigravity" phase in which the scalars become super-Planckian, making gravity effectively repulsive.
- For a flat FLRW universe, while antigravity can exist for a finite amount of time, an inititally well-behaved collapsing Universe will make a transition to a well-behaved expanding Universe. This is also true quantum-mechanically.
- Adding negative curvature introduces a repulsive potential that can drive the Universe out into antigravity. This is avoidable classically for suitable initial conditions, but not in the quantum theory as the Feynman propagator only falls off slowly in spacelike directions.

Thank you!