

Title: TBA

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Abstract: <p>TBA</p>

The No-Boundary Wave Function: Alive and Well

w/ Nikolay Bobev, Juan Diaz Doronsorro, Jonathan Halliwell,
Jim Hartle, Stephen Hawking, Oliver Janssen, Yannick Vreys

[[arxiv:\(1111.6090\)](https://arxiv.org/abs/1111.6090), [1610.01497](https://arxiv.org/abs/1610.01497), [1705.05340](https://arxiv.org/abs/1705.05340), [1707.0772](https://arxiv.org/abs/1707.0772), 17..]

Perimeter Institute

Thomas Hertog

Institute for Theoretical Physics
KU Leuven



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[arxiv:(1111.6090), 1610.01497, 1705.05340, 1707.0772, 17..]

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No Boundary?

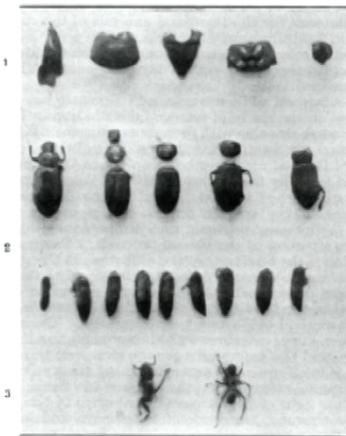
research fund established jointly by the Empire Marketing Board and the University of British Columbia. A detailed account of the experiments will appear shortly in the *Journal of Dairy Research*, Cambridge.

BLYTHE A. EAGLES.
WILFRID SADLER.

University of British Columbia,
Vancouver, Canada,
Mar. 27.

Insect Remains in the Gut of a Cobra, *Naja tripudians*.

THE accompanying photograph (Fig. 1) shows the remains of insects belonging to three orders, namely, Rhynchota (Heteroptera-Pentatomidae), Coleoptera, and Hymenoptera (Formicoidea), found in the gut of a cobra, *Naja tripudians*, brought to us in November 1928. The cobra, which was the black variety with no markings on the back of the hood but with white



patches on the throat, was captured at Banting, in the vicinity of Kuala Lumpur, Selangor, F.M.S. It was not a large specimen, since it measured only 3 ft. 7½ in. in length.

So far as it has been possible to ascertain, records of insects having been devoured by snakes do not appear to be abundant, the only other two which have

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The Beginning of the World from the Point of View of Quantum Theory.

SIR ARTHUR EDDINGTON¹ states that, philosophically, the notion of a beginning of the present order of Nature is repugnant to him. I would rather be inclined to think that the present state of quantum theory suggests a beginning of the world very different from the present order of Nature. Thermodynamical principles from the point of view of quantum theory may be stated as follows: (1) Energy of constant total amount is distributed in discrete quanta. (2) The number of distinct quanta is ever increasing. If we go back in the course of time we must find fewer and fewer quanta, until we find all the energy of the universe packed in a few or even in a unique quantum.

Now, in atomic processes, the notions of space and time are no more than statistical notions; they fade out when applied to individual phenomena involving but a small number of quanta. If the world has begun with a single quantum, the notions of space and time would altogether fail to have any meaning at the beginning; they would only begin to have a sensible meaning when the original quantum had been divided into a sufficient number of quanta. If this suggestion is correct, the beginning of the world happened a little before the beginning of space and time. I think that such a beginning of the world is far enough from the present order of Nature to be not at all repugnant.

It may be difficult to follow up the idea in detail as we are not yet able to count the quantum packets in every case. For example, it may be that an atomic nucleus must be counted as a unique quantum, the atomic number acting as a kind of quantum number. If the future development of quantum theory happens to turn in that direction, we could conceive the beginning of the universe in the form of a unique atom, the atomic weight of which is the total mass of the universe. This highly unstable atom would divide in smaller and smaller atoms by a kind of super-radioactive process. Some remnant of this process might, according to Sir James Jeans's idea, foster the heat of the stars until our low atomic number atoms allowed life to be possible.

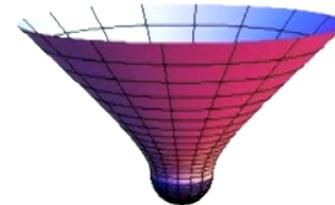
Clearly the initial quantum could not conceal in itself the whole course of evolution; but, according to the principle of indeterminacy, that is not necessary. Our world is now understood to be a world where something really happens; the whole story of the world need not have been written down in the first quantum like a song on the disc of a phonograph. The whole matter of the world must have been present at the beginning, but the story it has to tell may be written step by step.

G. LEMAÎTRE.

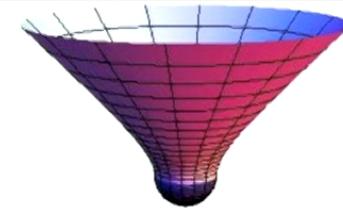
40 rue de Namur,

Louvain.

¹ NATURE, Mar. 21, p. 447.

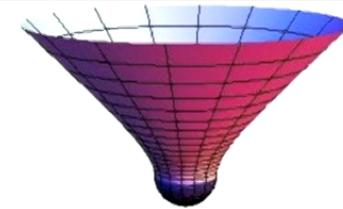


No Boundary



$$\Psi(h_f, \phi_f) = \int_{\mathcal{C}} \mathcal{D}N \int \mathcal{D}h \mathcal{D}\phi \ e^{iS[N,h,\phi]/\hbar}$$

No Boundary



$$\Psi(h_f, \phi_f) = \int_{\mathcal{C}} \mathcal{D}N \int \mathcal{D}h \mathcal{D}\phi e^{iS[N,h,\phi]/\hbar}$$

Requiring $C = [0^+, \infty[$ is meaningless principle

Lapse integration in quantum cosmology

Raymond Laflamme*

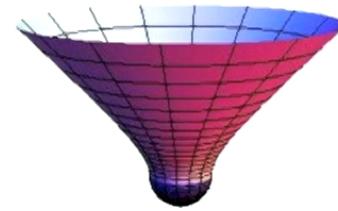
*Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Silver Street,
Cambridge CB3 9EW, United Kingdom
and Peterhouse College, Cambridge CB2 1RD, United Kingdom*

Jorma Louko[†]

*Theoretical Physics Institute, Department of Physics, University of Alberta, Edmonton, Alberta, Canada T6G 2J1
(Received 3 December 1990)*

We comment on the range of the lapse integration in quantum cosmology. We show by an explicit example that the range of the gauge-fixed lapse in the path integral need not be invariant under lapse rescalings in the action. In particular, the range appropriate for recovering a Green's function of the Wheeler-DeWitt operator may be smaller than semi-infinite. The importance of a careful treatment of the singularities in the lapse integral is emphasized.

Principle

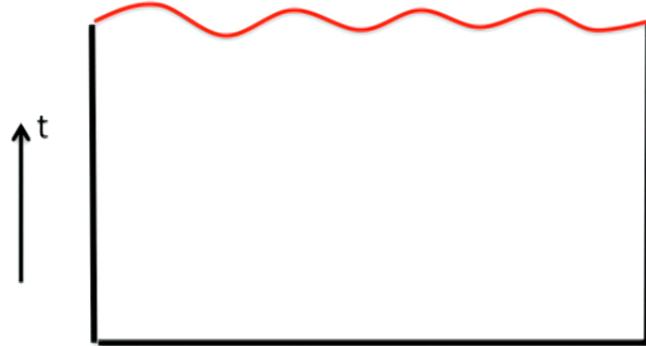


$$\Psi(h_f, \phi_f) = \int_{\mathcal{C}} \mathcal{D}N \int \mathcal{D}h \mathcal{D}\phi e^{iS[N, h, \phi]/\hbar}$$

A quantum state must be normalizable to be consistent with the predictive framework giving quantum probabilities.

Large fluctuations in eternal inflation

$$\Psi[\zeta] \propto \prod_n \exp\left(-\frac{\epsilon}{V} n^3 \zeta_n^2\right)$$

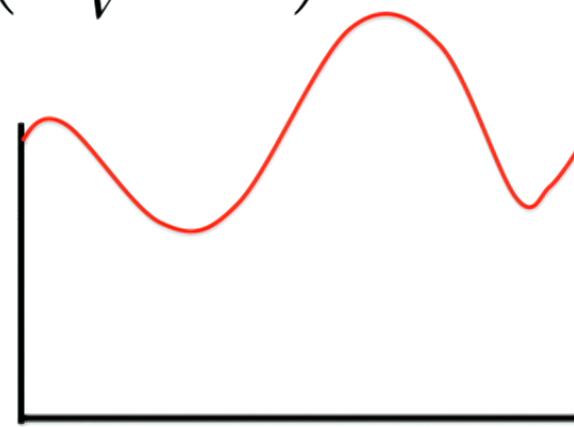


Slow roll

$$1 \gg \epsilon \gg V$$

Ψ narrowly peaked

fluctuations damped



Eternal inflation (near dS)

$$\epsilon \leq V$$

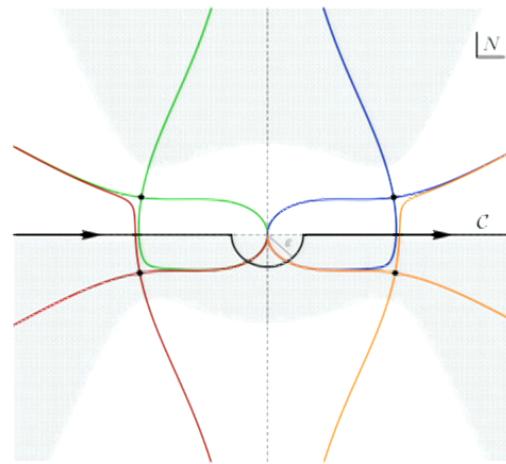
Ψ broadly distributed

fluctuations anti-damped

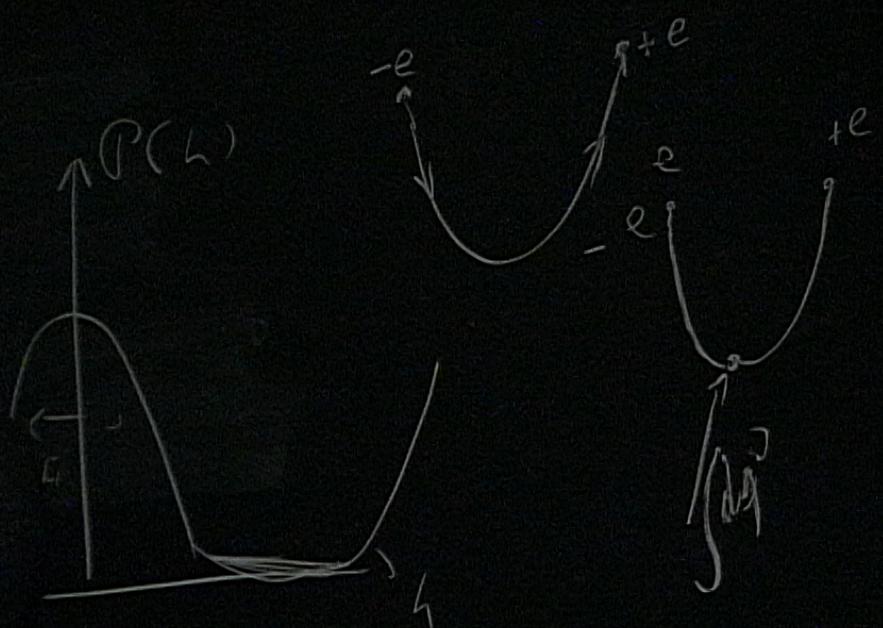
Gravitational waves in de Sitter

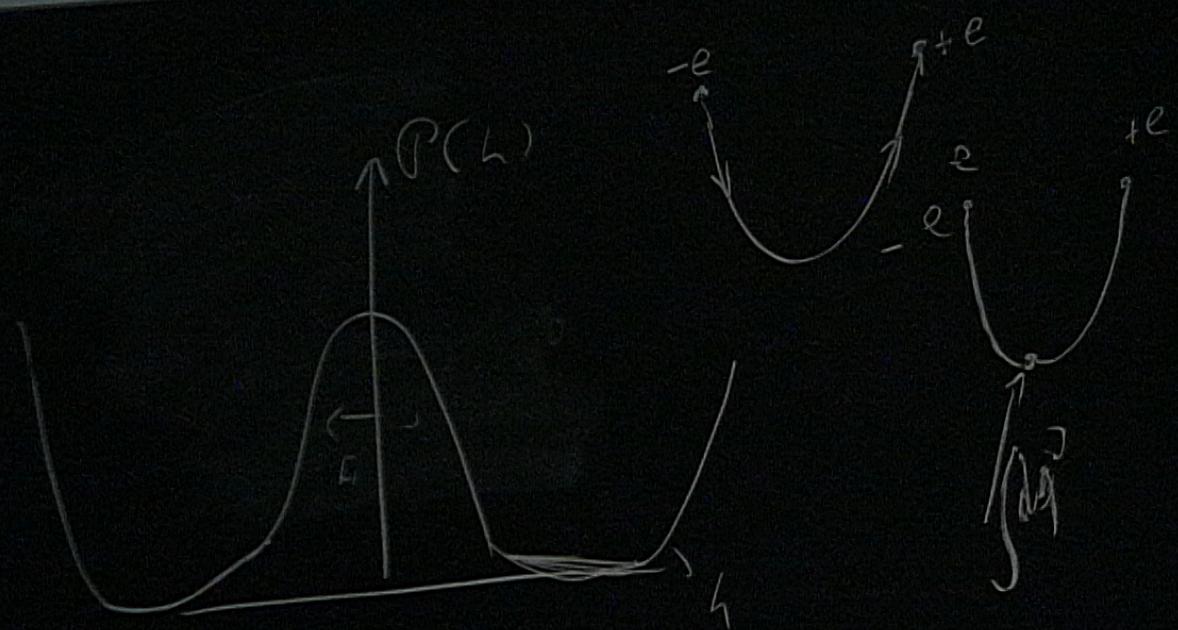
[Polyakov; Feldbrugge, Lehners, Turok]

$$|\Psi_{HH}| \sim e^{+1/\Lambda} \left(e^{-l^3 \phi_l^2 / \Lambda} + e^{-1/\Lambda} e^{+l^3 \phi_l^2 / \Lambda} + \dots \right)$$



$$\mathcal{C}_T = \langle \phi_l^2 \rangle l^3 = H^2 \ll 1$$





Anisotropic minisuperspace

$$ds^2 = -\frac{N^2}{q(t)}dt^2 + \frac{p(t)}{4}(\sigma_1^2 + \sigma_2^2) + \frac{q(t)}{4}\sigma_3^2$$

$$\Psi(p_1, q_1) = \int_{\mathcal{C}} \mathcal{D}N \mathcal{D}q \mathcal{D}p e^{iS[N, q, p]/\hbar}$$

$$S = 2\pi^2 \int_0^1 dt N \left[4 - \Lambda p - \frac{q}{p} - \frac{q\dot{p}^2}{4N^2 p} - \frac{\dot{p}\dot{q}}{2N^2} \right]$$

Anisotropic minisuperspace

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Boundary conditions at South Pole (t=0):

$$q_0 = p_0 = 0$$

$$\Psi = \Psi'$$

Anisotropic minisuperspace

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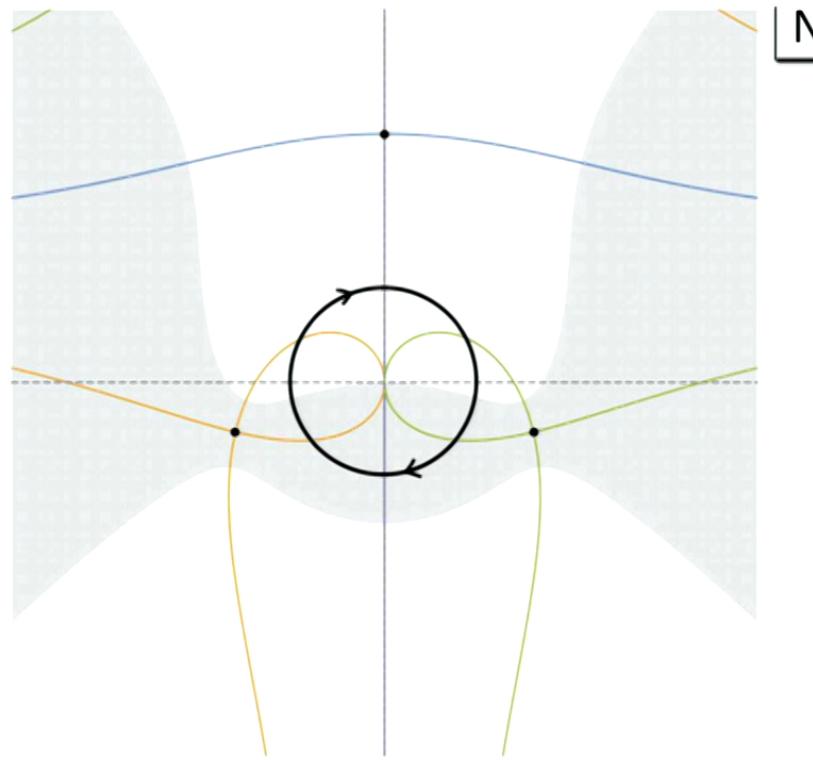
$$\Psi(p_1, q_1) = \int_{\mathcal{C}} \mathcal{D}N e^{i\bar{S}[N, q_1, p_1]/\hbar}$$

Reduced action:

$$\bar{S}[N, p_1, q_1] = 2\pi^2 \left[-\frac{p_1 q_1}{N} + iq_1 + \left(4 - \frac{p_1 \Lambda}{3} \right) N - \frac{i\Lambda N^2}{3} \right]$$

Anisotropic minisuperspace

$$\Psi(p_1, q_1) = \int_{\mathcal{C}} \mathcal{D}N e^{i\bar{S}[N, q_1, p_1]/\hbar}$$



Anisotropic minisuperspace

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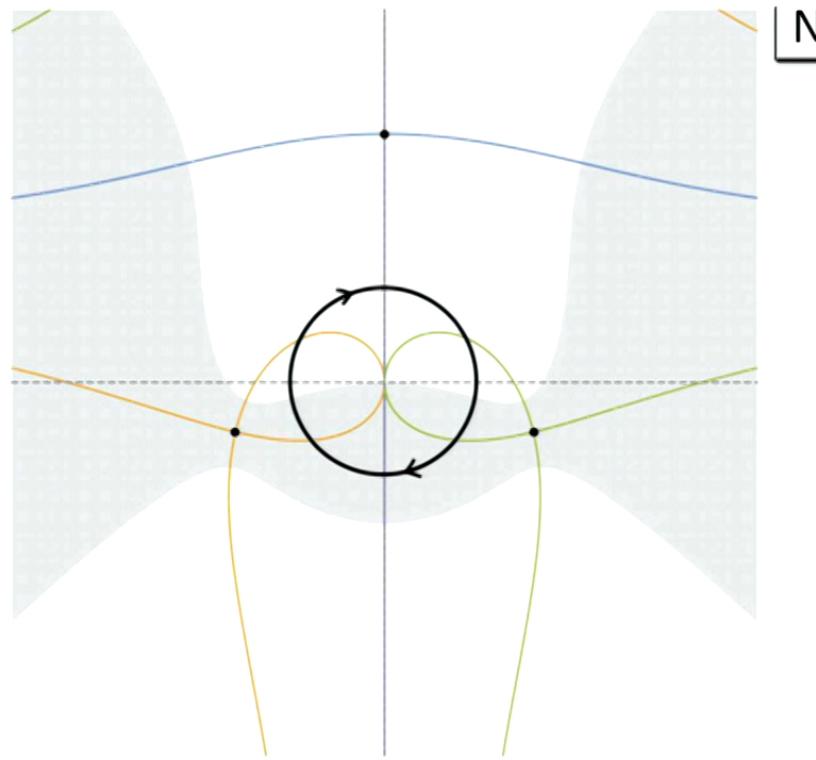
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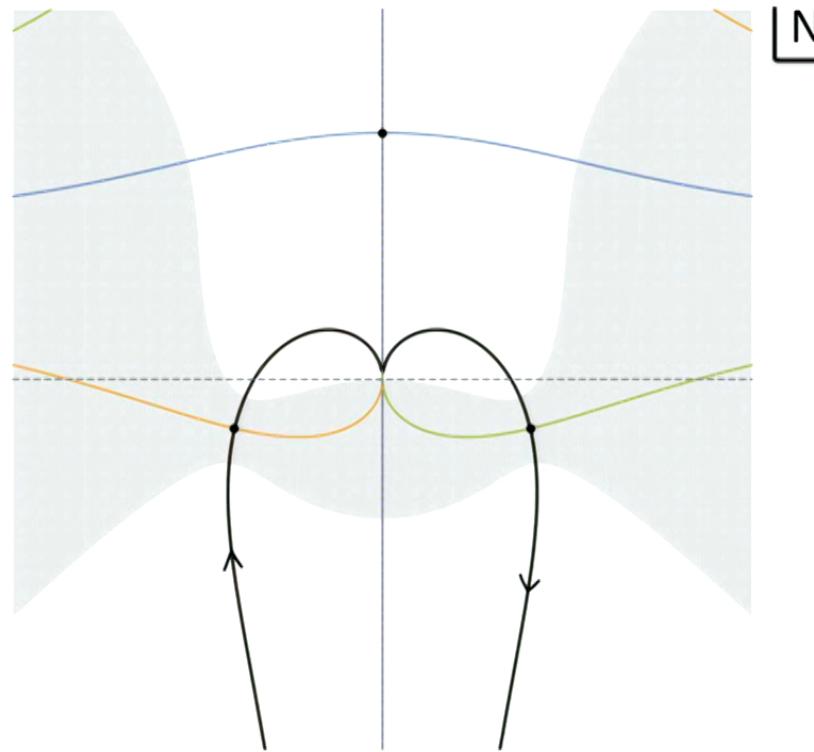
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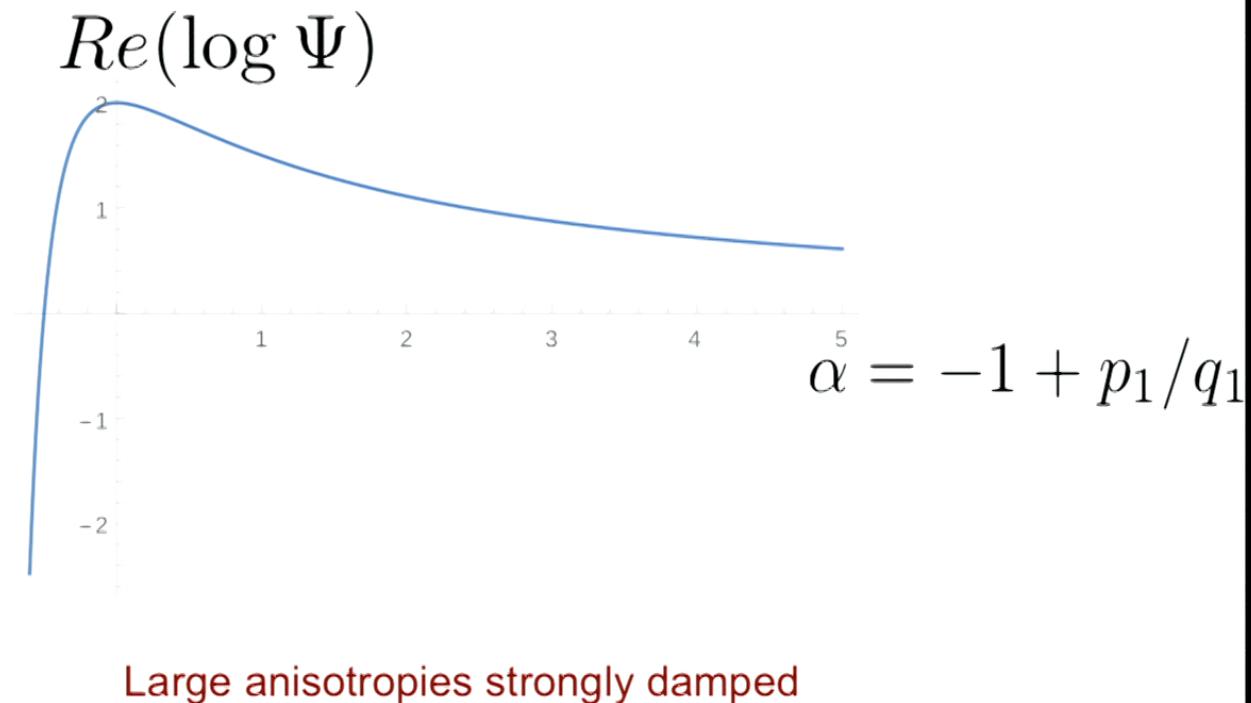
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Anisotropic minisuperspace

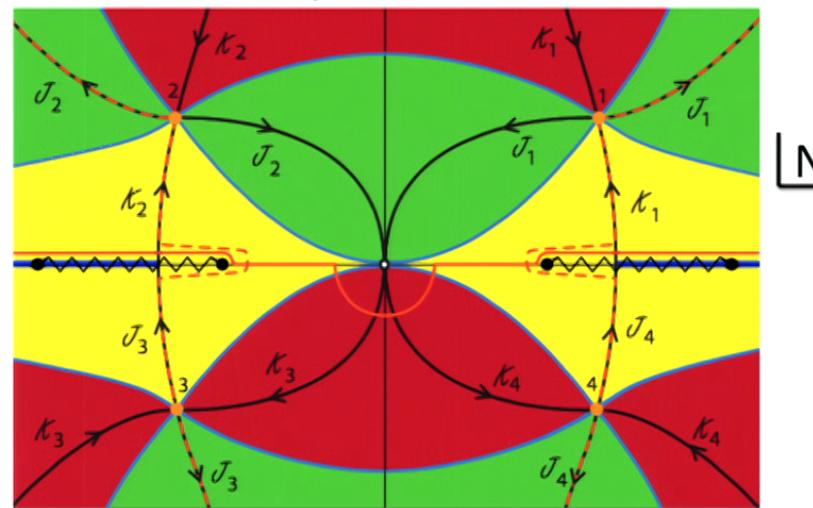
Probabilities for classical, anisotropic histories:



Picard-Lefschetz in Perturbation theory

[Feldbrugge, Lehners, Turok]

$$\Psi(q_1, \phi_1) = \int_{\mathcal{C}} \mathcal{D}N e^{i\bar{S}[N, q_1, \phi_1]/\hbar}$$



$$|\Psi_{HH}| \sim e^{+1/\Lambda} \left(e^{-l^3 \phi_l^2 / \Lambda} + e^{-1/\Lambda} e^{+l^3 \phi_l^2 / \Lambda} + \dots \right)$$

$\int \mathcal{D}h e^{\int h^2}$ $\stackrel{q(t)}{\longrightarrow}$
 $q(0) = 0$ $q(1) = q_1$
 $h_i < \frac{1}{l}$
 $g_{ij}^{S^3} + h_{ij}$
 $\nabla^2 + \sum R$
 $\underline{l > 0}$ $\frac{1}{q^2}$

$$N_k = \frac{\alpha}{H}$$

$$a(N)x + b(N)x^2$$

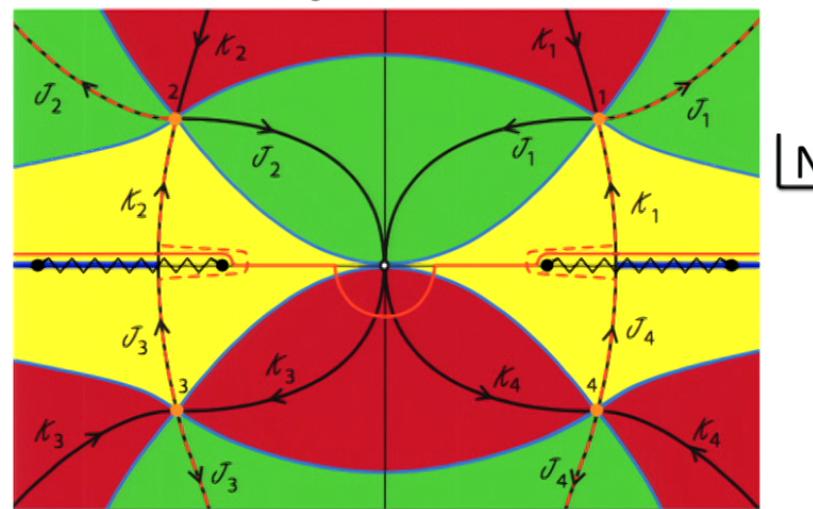
$$e^{\frac{1}{\pi k}(1-h^2)} + e^{\frac{1}{\pi k}(1-h_0^2)}$$

$|e^{\frac{1}{\pi k}}| < 1$ $\int \mathcal{D}\phi e^{\frac{1}{\pi k} S[\phi]}$
 $\phi(0) = 0$ $\phi(1) = \phi_1$

Picard-Lefschetz in Perturbation theory

[Feldbrugge, Lehners, Turok]

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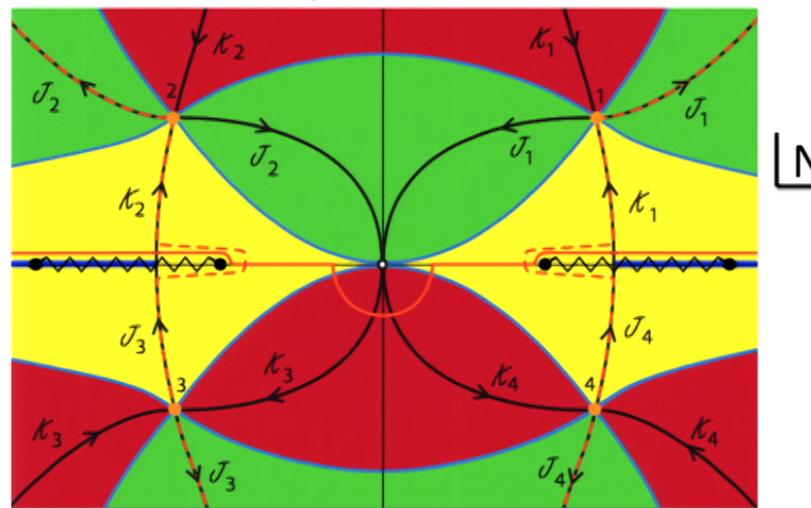
N

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Picard-Lefschetz in Perturbation theory

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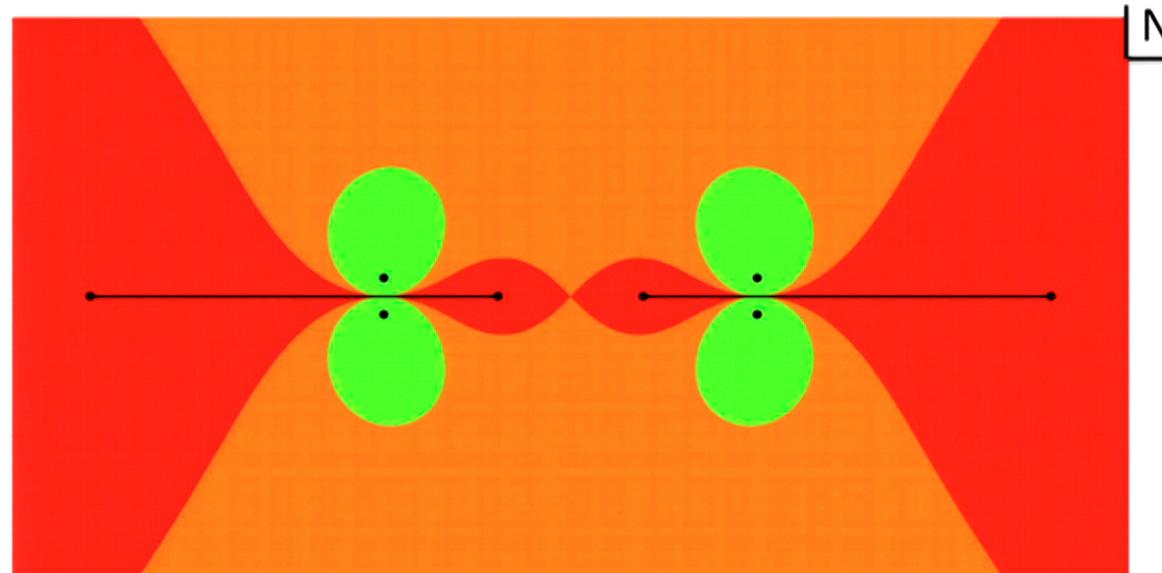
$$|\Psi_{HH}| \sim e^{+1/\Lambda} \left(e^{-l^3 \phi_l^2 / \Lambda} + e^{-1/\Lambda} e^{+l^3 \phi_l^2 / \Lambda} + \dots \right)$$

Assumption: $|\phi(t)| \ll 1$

Breakdown of perturbation theory

Perturbations diverge along deformed contour:

$$|\phi(t = 0)| \rightarrow \infty$$

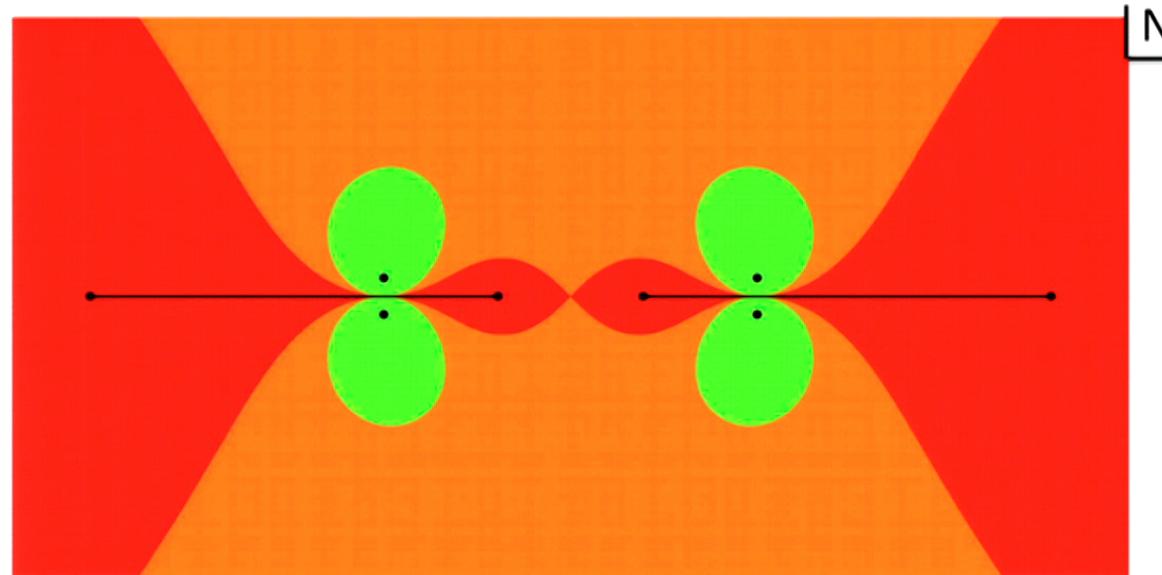


$$\ddot{q} - \frac{2N^2}{3}\Lambda + \frac{\dot{\phi}^2}{3\pi^2}q = 0 ,$$
$$\ddot{\phi} + 2\frac{\dot{q}}{q}\dot{\phi} + \frac{N^2}{q^2}l(l+2)\phi .$$

Breakdown of perturbation theory

Perturbations diverge along deformed contour:

$$|\phi(t = 0)| \rightarrow \infty$$



Include backreaction: no passage through (nearly) real N region

→ No evidence for large anisotropies in no-boundary state

Bianchi IX w/ Lorentzian contour

$$ds^2 = -\frac{N^2}{q(t)}dt^2 + \frac{p(t)}{4}(\sigma_1^2 + \sigma_2^2) + \frac{q(t)}{4}\sigma_3^2$$

$$\Psi(p_1, q_1) = \int_{\mathcal{C}} \mathcal{D}N \mathcal{D}q \mathcal{D}p e^{iS[N, q, p]/\hbar}$$

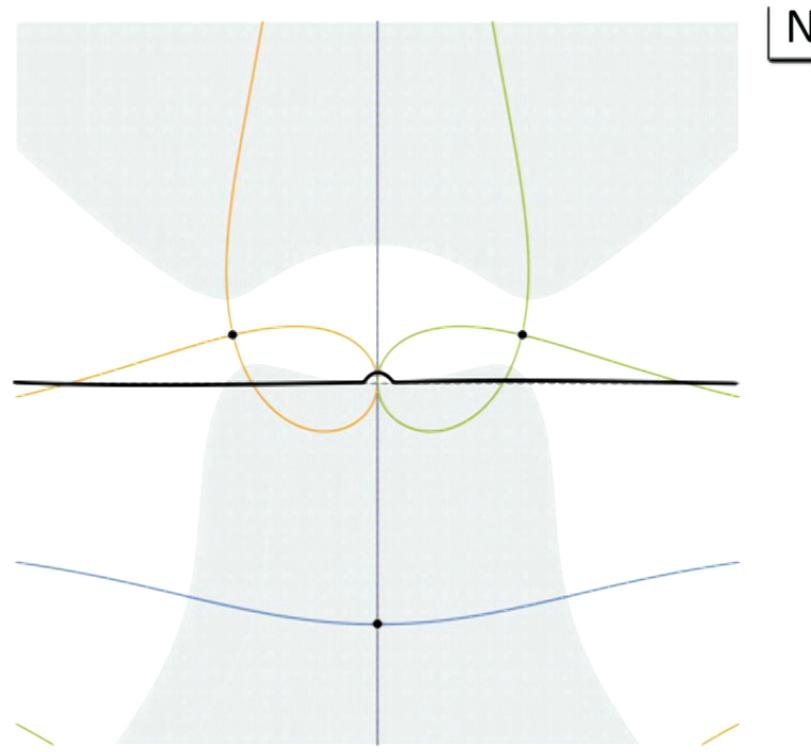
Different boundary conditions at South Pole ($t=0$):

$$q_0 = p_0 = 0$$

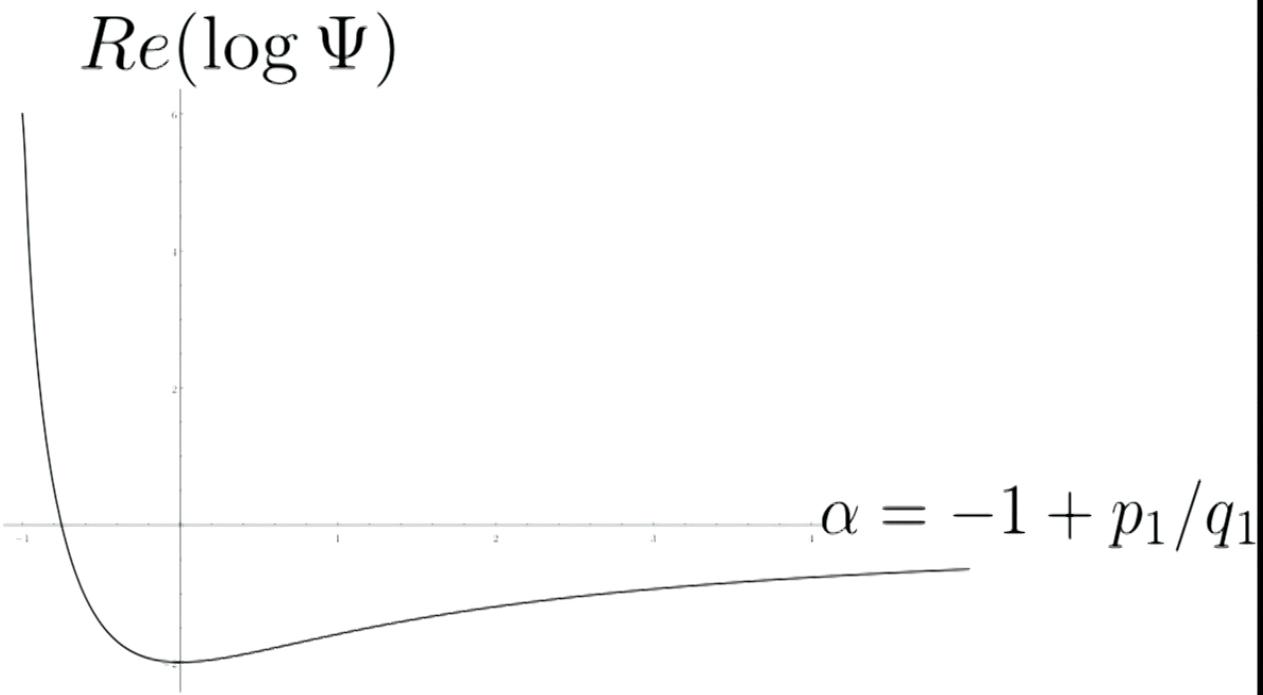
$$\Psi = -\Psi'$$

Bianchi IX w/ Lorentzian contour

$$\Psi(p_1, q_1) = \int_{\mathcal{C}} \mathcal{D}N e^{i\bar{S}[N, q_1, p_1]/\hbar}$$



Bianchi IX w/ Lorentzian contour



Not normalizable/no predictions

Toward a unique wave function?

A quantum state must be normalizable to be consistent with the predictive framework giving quantum probabilities.

- In Bianchi IX this selects a unique no-boundary wave function among those with an integral representation;
- This is in line with holography

Toward a unique wave function?

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- In Bianchi IX this selects a unique no-boundary wave function among those with an integral representation;
- This is in line with holography

$$\Psi_{HH}[h, \chi] = Z_{QFT}^{-1}[\tilde{h}, \tilde{\chi}] \exp(-iS_{loc}/\hbar)$$

[Horowitz & Maldacena ('04); Hartle & TH ('11); Anninos et al. ('12),...]

Holographic Measure on Bianchi IX

[Bobev, TH, Vreys (17); Anninos et al. (13); Hartnoll & Kumar (06)]

Evaluate $Z[A,B]$ of $O(N)$ vector models on **squashed spheres**:

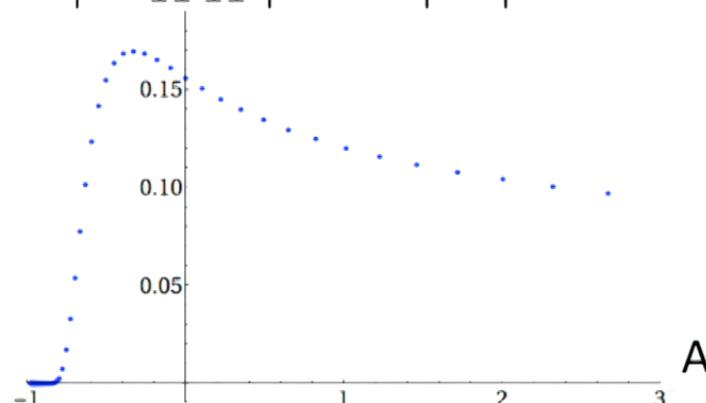
$$ds^2 = \frac{r_0^2}{4} \left((\sigma_1)^2 + \frac{1}{1+A} (\sigma_2)^2 + \frac{1}{1+B} (\sigma_3)^2 \right)$$

Holographic Measure on Bianchi IX

Evaluate $Z[A,B]$ of $O(N)$ vector models on **squashed spheres**:

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$$|\Psi_{HH}|^2 = |Z|^{-2}$$



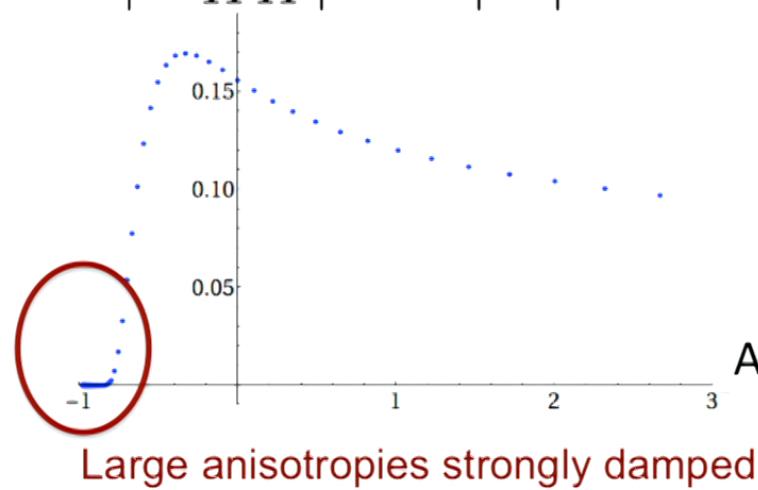
Large anisotropies strongly damped

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Large anisotropies strongly damped

Holographic Measure on eternal inflation

[Hawking, TH (17)]

Constant density surfaces in eternal inflation are thought to develop regions where

$$R(h) < 0$$

However on such boundary geometries one expects

$$Z(h) \rightarrow \infty$$

because the action includes a conformal coupling term $R\phi^2$.

Since

$$|\Psi_{HH}(h, \chi)| = Z_{QFT}^{-1}(\tilde{h}, \tilde{\chi})$$

→ Large fluctuations strongly suppressed.

Conclusion

- Fluctuations are damped in the no-boundary state
- Normalizability acts as a selection principle
- Agreement with holography

