Title: Quantum incompleteness of inflation - part I

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Abstract:

Quantum Incompleteness of Inflation – Part I

Jean-Luc Lehners
Max Planck Institute for Gravitational Physics
With Alice Di Tucci, Job Feldbrugge and Neil Turok
In preparation

Usual calculation of inflationary perturbations

- In conformal time $a = -\frac{1}{H\eta}$ $\quad -\infty < \eta < 0$
- Mode equation in Heisenberg picture:

$$v_k'' + v_k\left(k^2 - \frac{2}{\eta^2}\right) = 0$$

[Chibisov & Mukhanov; Bunch & Davies; …]

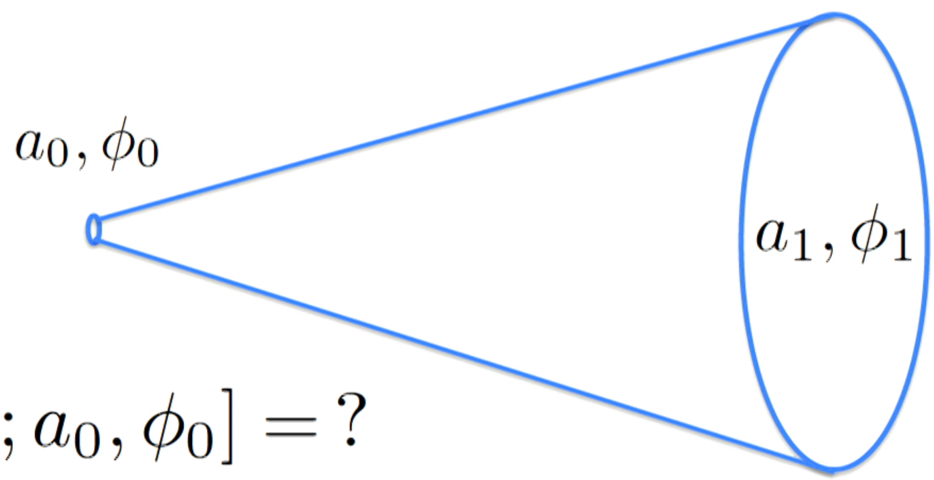
- Two solutions:

$$v_k = c_1 e^{-ik\eta}\left(1 - \frac{i}{k\eta}\right) + c_2 e^{ik\eta}\left(1 + \frac{i}{k\eta}\right)$$

- Then it is argued that at early times ($\eta \to -\infty$) the mode functions "feel" as in Minkowski spacetime and one should choose the positive frequency Minkowski vacuum, $c_2=0$, leading to a late-time, Gaussian distribution of classical fluctuations

Is this reproduced by quantum geometrodynamics?

- Here we want to go beyond QFT in curved spacetime, by quantizing the background too
- Hence the calculation we are interested in is the propagator (eventually in the flat slicing of de Sitter space) from a vanishingly small scale factor to a late time universe with fluctuations



$$G[a_1, \phi_1; a_0, \phi_0] = ?$$

Setting: minisuperspace plus perturbations

The propagator is given by

$$G[q_1, \phi_1; q_0, \phi_0] = \int_{0^+}^{\infty} \mathrm{d}N \int \mathcal{D}q \int \mathcal{D}\phi \, e^{iS[q,\phi,N]/\hbar}$$

with $S = S^{(0)} + S^{(2)}$

where $S^{(0)}$ is the action for gravity plus a cosmological constant, and we use the metric

$$ds^2 = -\frac{N^2}{q} dt^2 + q d\Omega_3^2 \qquad\qquad q = a^2$$

[Louko]

The spatial slices are spheres with curvature 6K

Setting: minisuperspace plus perturbations

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The perturbation action $S^{(2)}$ is (e.g. for a gravity wave mode with wavenumber l)

$$S^{(2)} = \frac{1}{2} \int N \mathrm{d}t \, \mathrm{d}^3 x \left[q^2 \left(\frac{\dot{\phi}}{N} \right)^2 - l(l+2)\phi^2 \right]$$

$$= \frac{1}{2} \left[\frac{q^2}{N} \phi \dot{\phi} \right]_0^1 \quad \text{(on-shell)}$$

Perturbative action

- In these variables, the off-shell solutions for the scale factor are simply quadratic in t:

$$\bar{q}(t) = H^2 N^2 (t-\alpha)(t-\beta)$$

$$\alpha, \beta = \frac{1}{2N^2}\left\{N^2 - N_{s-}N_{s+} \pm \sqrt{(N^2 - N_{s-}^2)(N^2 - N_{s+}^2)}\right\}$$

$$N_{s\pm} = \frac{\sqrt{q_1} \pm \sqrt{q_0}}{H}$$

Perturbative action

- One can explicitly find the solutions to the perturbation equation of motion (at the saddle points N_{s+} and N_{s-} they become the Bunch-Davies mode functions):

$$\phi(t) = \frac{af(t) + bg(t)}{\sqrt{\bar{q}(t)}}$$

$$f(t) = \left[\frac{t-\beta}{t-\alpha}\right]^{\mu/2}\left[(1-\mu)(\alpha-\beta) + 2(t-\alpha)\right]$$

$$g(t) = \left[\frac{t-\alpha}{t-\beta}\right]^{\mu/2}\left[(1+\mu)(\alpha-\beta) + 2(t-\alpha)\right]$$

$$\mu^2 = 1 - \frac{4k^2(\frac{N}{H^2})^2}{(N^2-N_{s-}^2)(N^2-N_{s+}^2)} \qquad l(l+2) \to k^2$$

Perturbative action

- Using these one can calculate the perturbative action $S^{(2)} = \frac{n}{d}$

$$n = \frac{k^2}{H^2 N}\Big\{ + \left[\frac{(1-\alpha)\beta}{(1-\beta)\alpha}\right]^{\mu/2} [(2-(\alpha+\beta)+\mu(\alpha-\beta))q_0\phi_0^2 + (\alpha+\beta+\mu(\alpha-\beta))q_1\phi_1^2]$$
$$- \left[\frac{(1-\beta)\alpha}{(1-\alpha)\beta}\right]^{\mu/2} [(2-(\alpha+\beta)-\mu(\alpha-\beta))q_0\phi_0^2 + (\alpha+\beta-\mu(\alpha-\beta))q_1\phi_1^2]$$
$$- 4\mu(\alpha-\beta)\sqrt{q_1 q_0}\phi_1\phi_0\Big\}$$

$$d = + \left[\frac{(1-\beta)\alpha}{(1-\alpha)\beta}\right]^{\mu/2} (2-(\alpha+\beta)-\mu(\alpha-\beta))(\alpha+\beta-\mu(\alpha-\beta))$$
$$- \left[\frac{(1-\alpha)\beta}{(1-\beta)\alpha}\right]^{\mu/2} (+2-(\alpha+\beta)+\mu(\alpha-\beta))(\alpha+\beta+\mu(\alpha-\beta))$$

- Recall: $\mu = \sqrt{1 - \frac{4k^2(\frac{N}{H^2})^2}{(N^2-N_{s-}^2)(N^2-N_{s+}^2)}}$

Various terms conspire such that the action has no branch cuts!

Vanishing q_0 limit

- Now we want to take the limit where q_0->0
- Note that ϕ_0 only appears in the combination $q_0\phi_0^2$ and the mode functions imply that this quantity vanishes when $q_0=0$
- Hence in the limit $q_0=0$ all dependence on ϕ_0 drops out – we do not need to specify initial perturbations!

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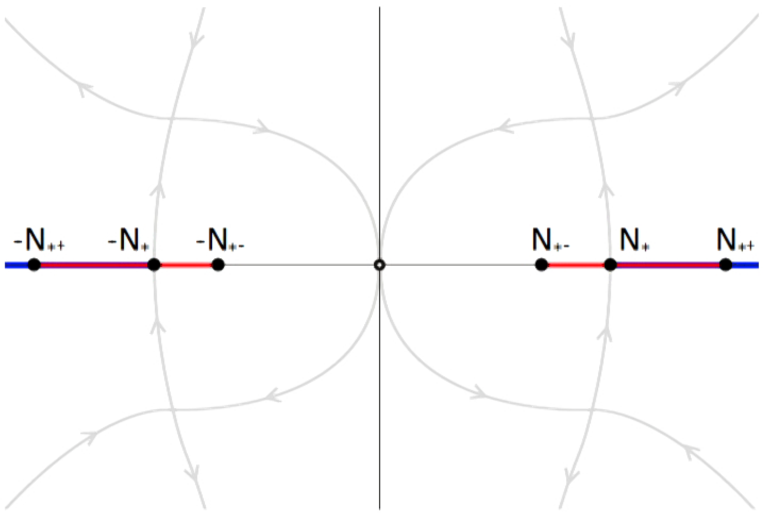
- Hence in the limit $q_0=0$ all dependence on ϕ_0 drops out – we do not need to specify initial perturbations!

Vanishing q_0 limit

- Second consequence: the action develops a branch cut:

$$S^{(2)} = -\frac{k^2 N q_1 \phi_1^2}{H^2} \frac{1}{N^2 + N_*^2 - \sqrt{(N^2 - N_+^2)(N^2 - N_-^2)}}$$

$$N_* = \frac{\sqrt{q_1}}{H}$$

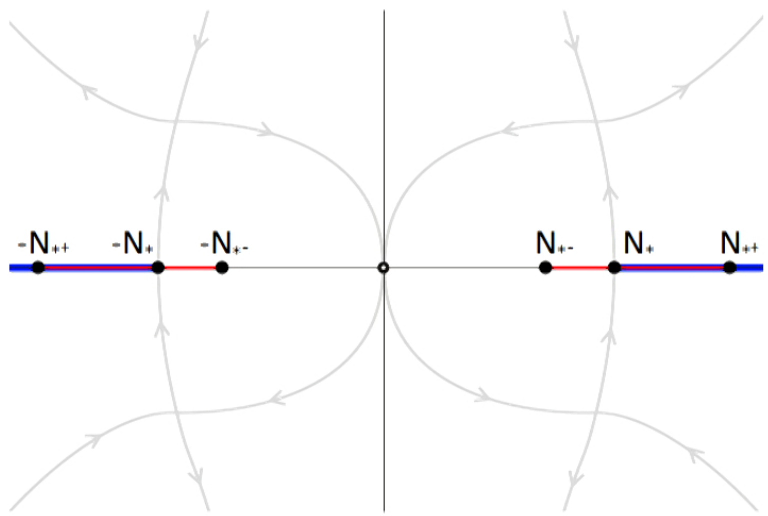


Vanishing q_0 limit

- Explicitly:

$$S_{pert,upper}(N) = \frac{k^2 q_1 \phi_1^2}{4N(3k^2 + q_1\Lambda)}\left(-3q_1 - N^2\Lambda - i\sqrt{N_+^2 - N^2}\sqrt{N^2 - N_-^2}\right)$$

$$S_{pert,lower}(N) = \frac{k^2 q_1 \phi_1^2}{4N(3k^2 + q_1\Lambda)}\left(-3q_1 - N^2\Lambda + i\sqrt{N_+^2 - N^2}\sqrt{N^2 - N_-^2}\right)$$



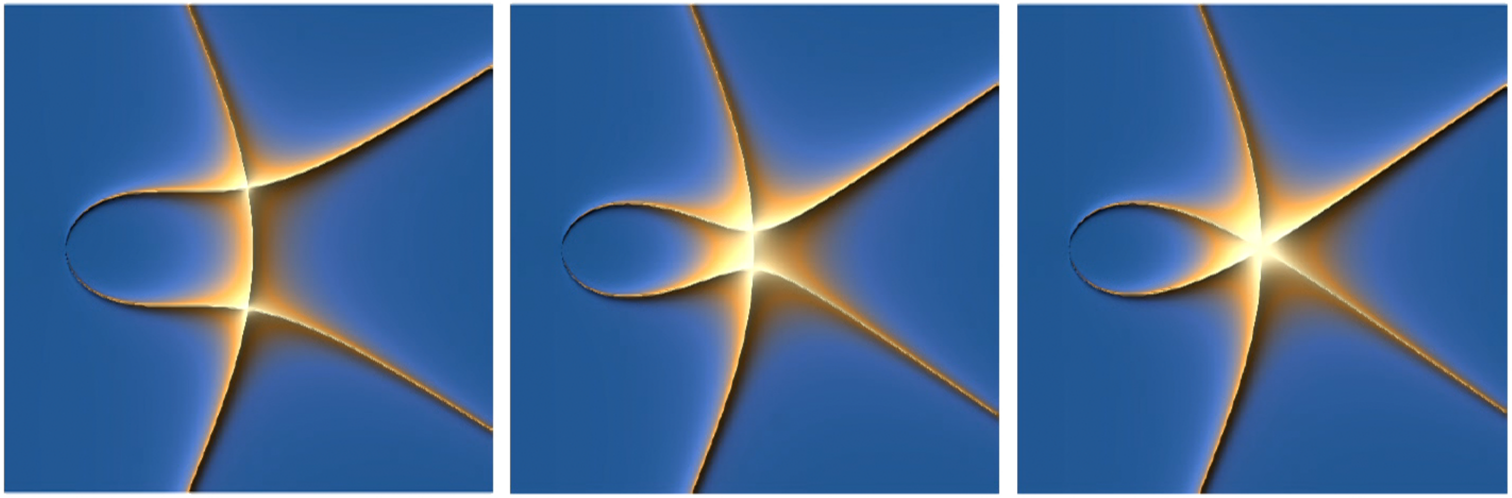
- Two ways to do the calculation
  - Assume K=0 (flat slicing) with non-zero q_0 then take the limit q_0->0
  - Assume q_0=0 from the outset and let K/q_1->0

- The saddle points reside at

$$N_{s\pm}^{(K)} = \frac{1}{H^2}\left[(H^2 q_1 - K)^{1/2} \pm (-K)^{1/2}\right]$$

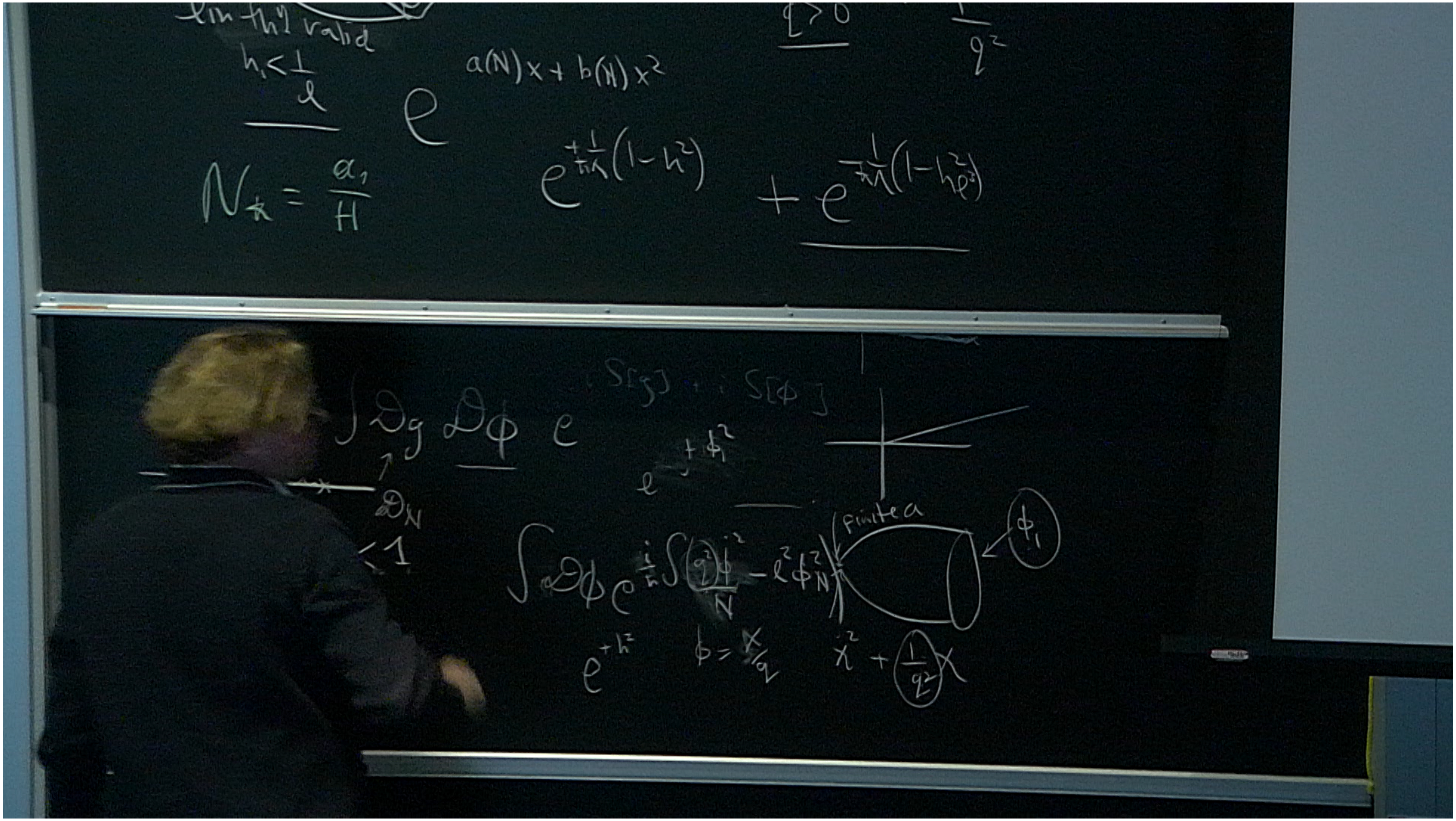
and will hence move towards the real N line

- The flow lines change as follows:



K=1 K=1/10 K=0

- The saddle points become degenerate at $N_{\star}$.



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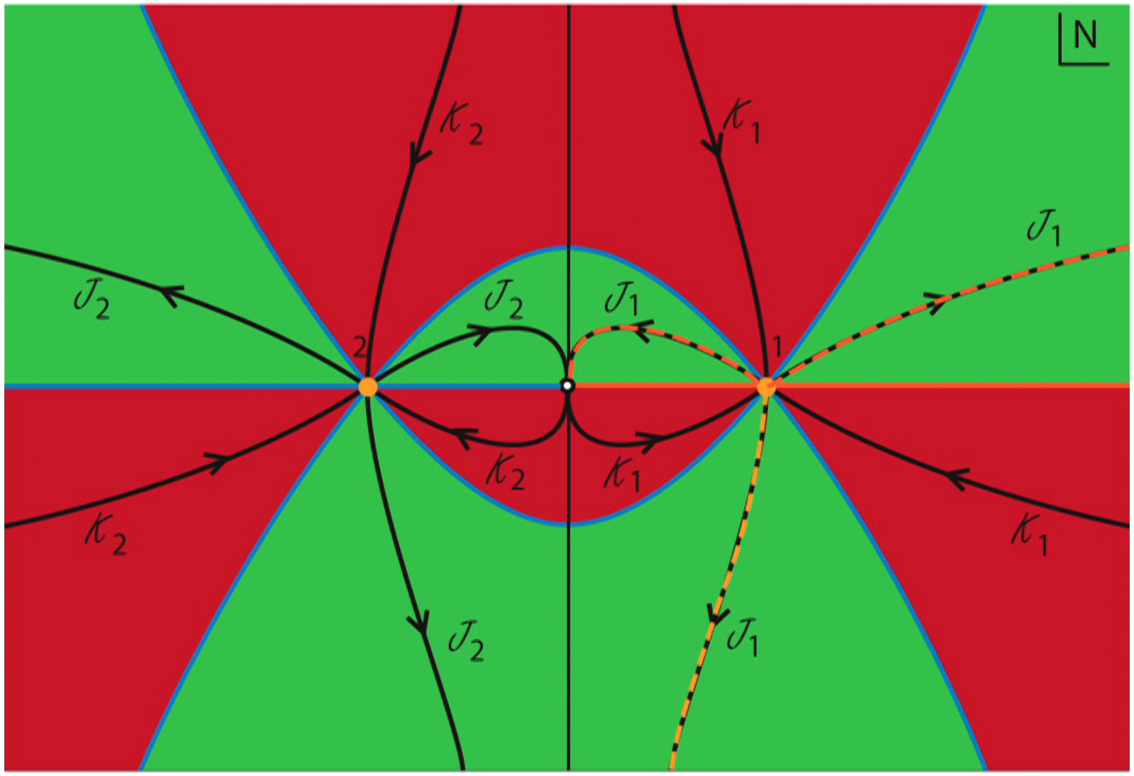


- Only the thimble in the upper half plane contributes to the propagator

Final Result

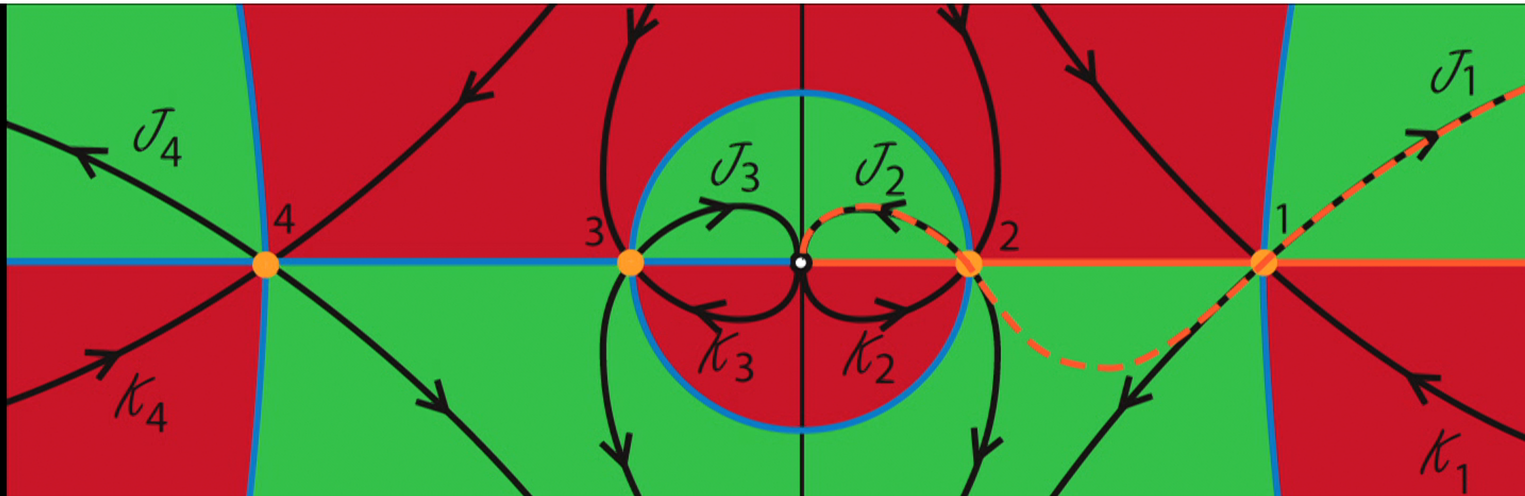
- The propagator can thus be estimated by the value at the saddle point N⋆, approached from above:

$$G[q_1, \phi_1; q_0 \to 0] \approx e^{\frac{1}{\hbar K^{3/2}}\left(-4\pi^2 i H q_1^{3/2} + \frac{N_\star}{i + k N_\star / q_1} \frac{k^2}{2} \phi_1^2\right)}$$

$$\approx e^{\frac{1}{\hbar K^{3/2}}\left(-4\pi^2 i H q_1^{3/2} - i \frac{\sqrt{q_1} k^2}{2H} \phi_1^2 + \frac{k^3}{2H} \phi_1^2\right)}$$

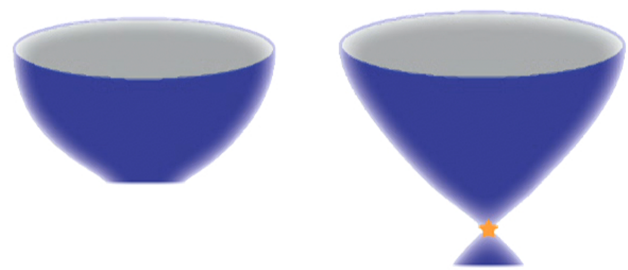
- *Unsuppressed* fluctuations, contrary to the usual assumptions!

- Performing the calculation the other way around, i.e. starting with the flat slicing and with classical boundary conditions, offers further insights:



Two saddle points contribute: N_2 and N_1

As the initial scale factor is reduced, they move toward each other
One of them has stable fluctuations and the other unstable

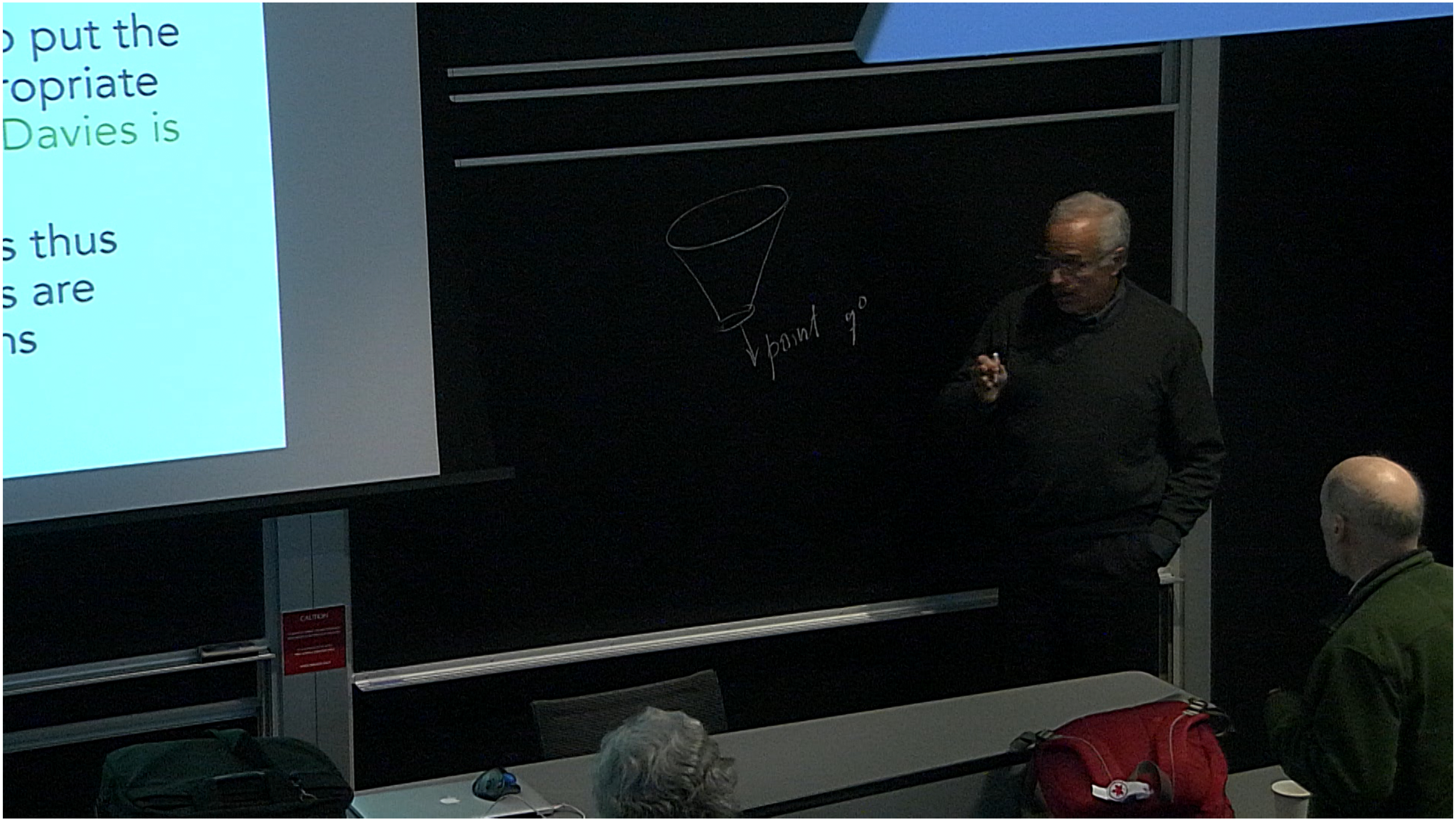


In the limit of small scale factor only the unstable one contributes

Talk by Job Feldbrugge

Quantum incompleteness of inflation

- This is a new initial conditions problem for inflation, at the quantum level
- Inflation requires a mechanism to put the quantum fluctuations in the appropriate "initial" quantum state – Bunch-Davies is not automatic
- At the quantum level, inflation is thus incomplete, and the fluctuations are highly sensitive to UV corrections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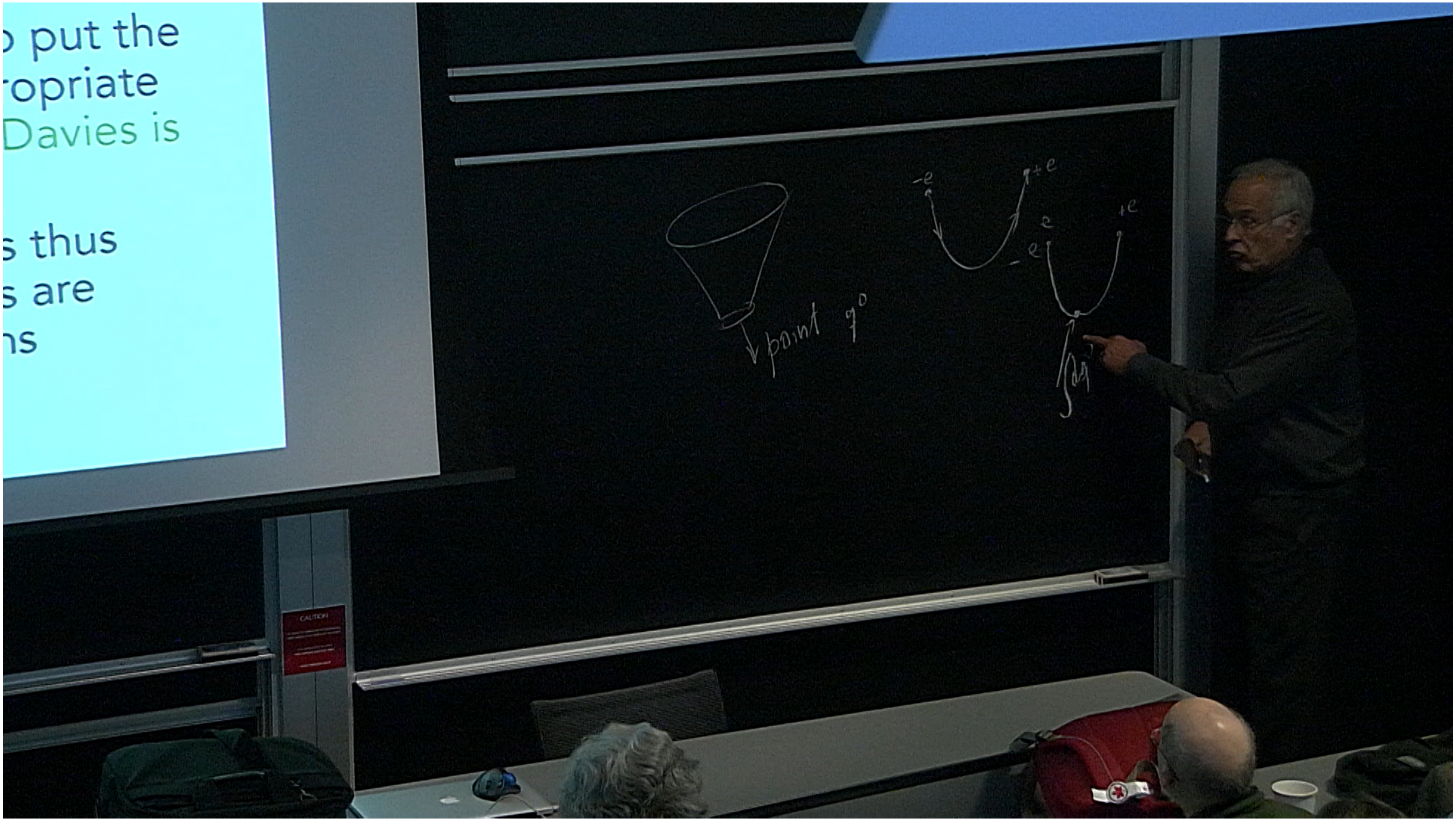
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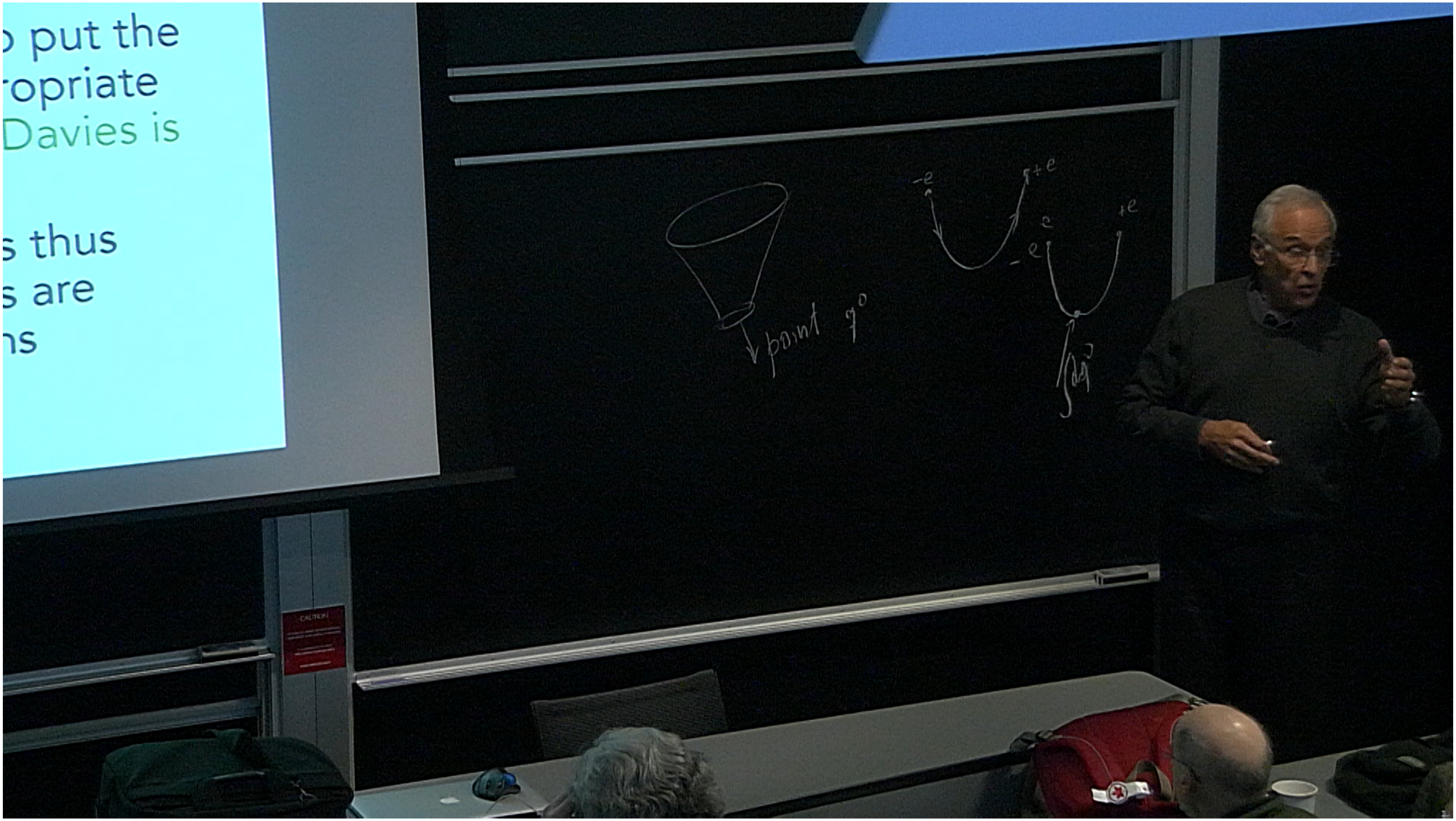
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