Title: An asymptotically safe point of view on the gravitational path integral

Speakers: Astrid Eichhorn

Series: Cosmology, Quantum Gravity

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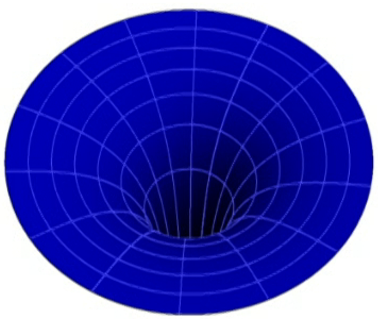
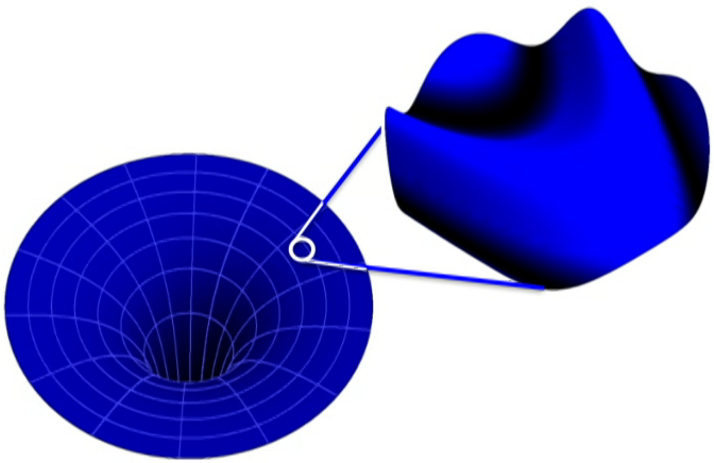
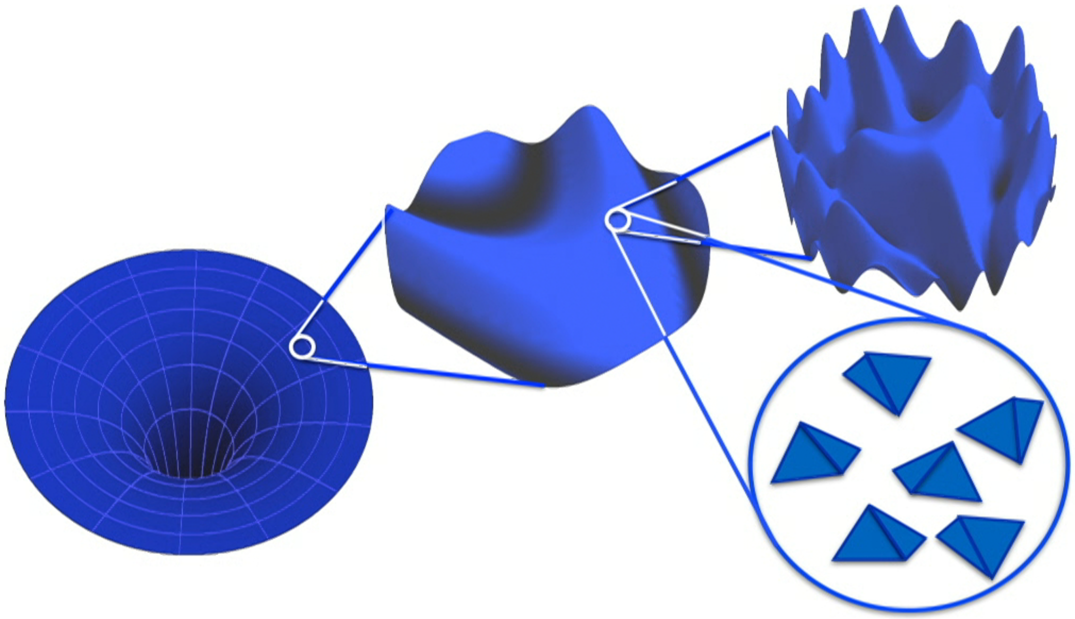
Renormalization Group flows of spacetime

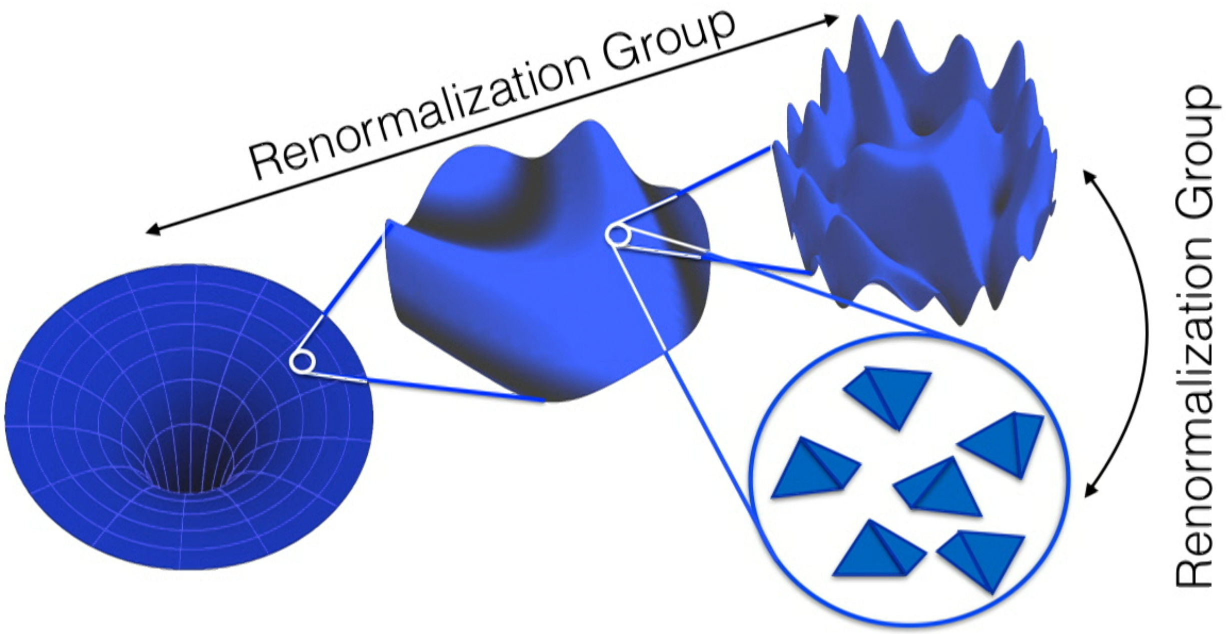
Astrid Eichhorn
University of Heidelberg

May 16th,
Perimeter Institute for Theoretical Physics
Workshop on Shape Dynamics








Renormalization Group
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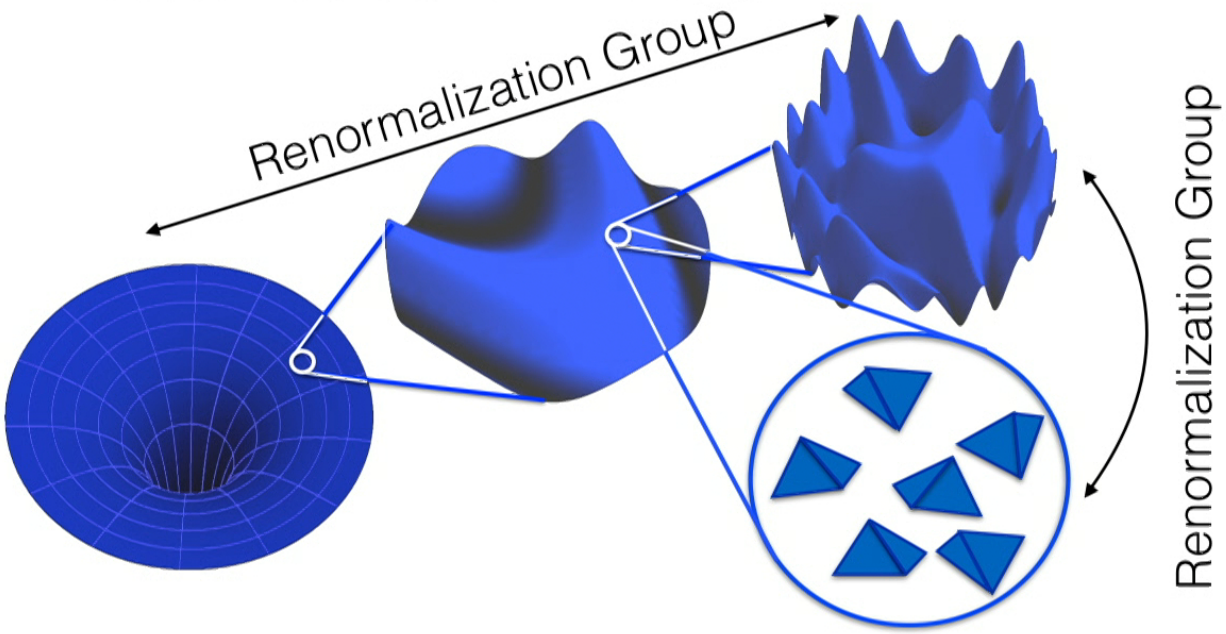
Outline

1) Functional RG: Probing the scale dependence of QFTs

2) RG for spacetime in the continuum:
   Setting a scale in Quantum Gravity

3) RG for discrete spacetimes:
   Scales in a pre-geometric setting?



Functional Renormalization Group

$$e^{-\Gamma[\phi]} = \int \mathcal{D}\varphi \, e^{-S[\varphi]}$$

$\phi = \langle \phi \rangle$

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$$e^{-\Gamma_k[\phi]} = \int \mathcal{D}\varphi \, e^{-S[\varphi] - \frac{1}{2}\int \varphi(-p) R_k(p) \varphi(p)}$$

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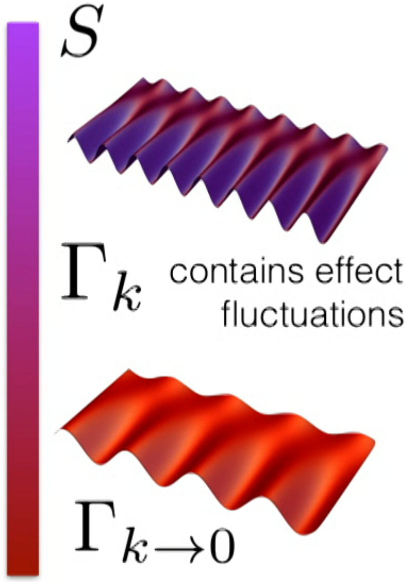
Wetterich '93; Morris, '94

scale- and momentum-dependent ``mass'': dials ``resolution scale''



S

Γ_k contains effect of quantum fluctuations above *k*

$\Gamma_{k\to 0}$

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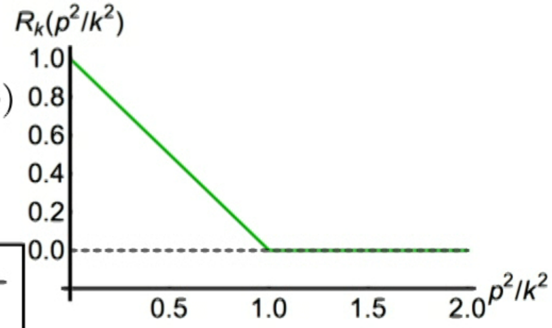
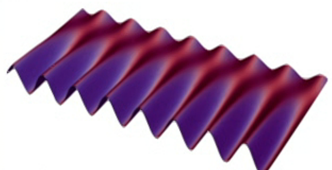


$$R_k(p^2/k^2) = (k^2 - p^2)\theta(k^2 - p^2)$$

Litim '01

S



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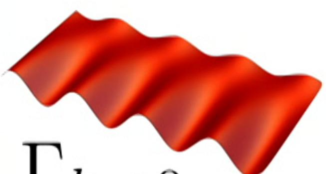

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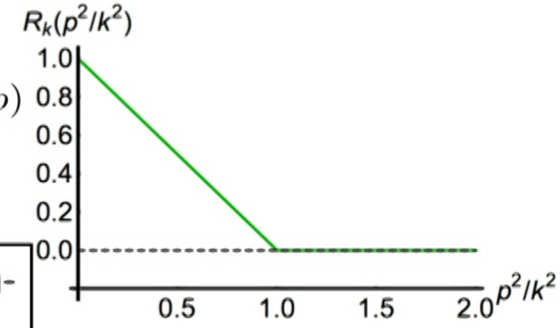

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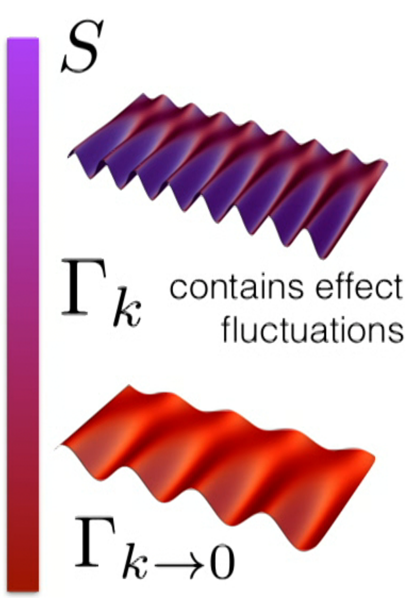

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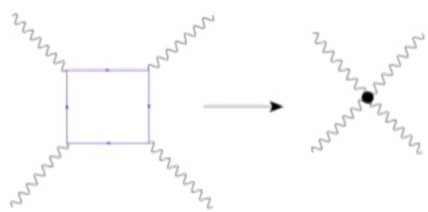

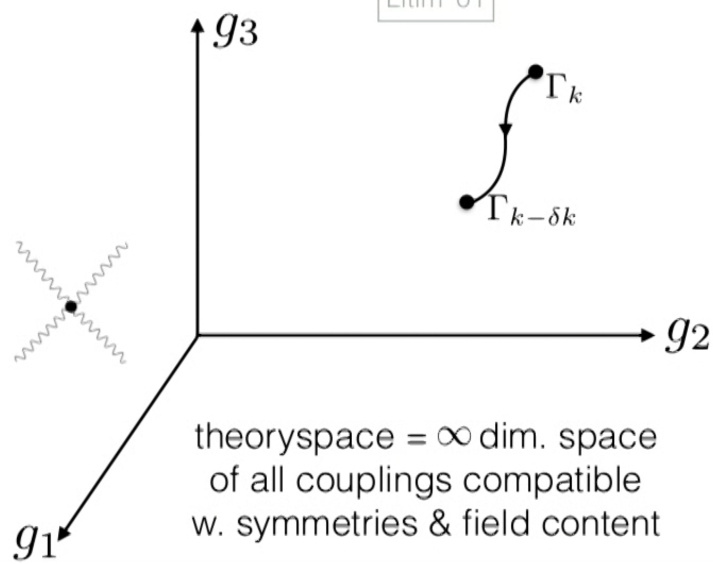

theoryspace = ∞ dim. space of all couplings compatible w. symmetries & field content

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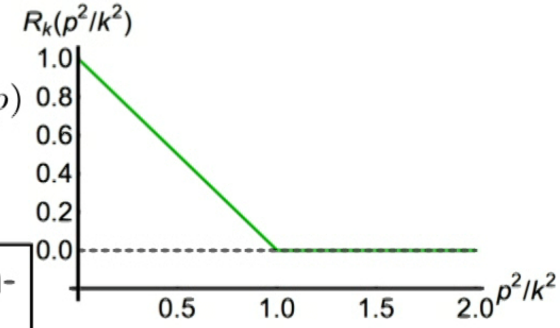

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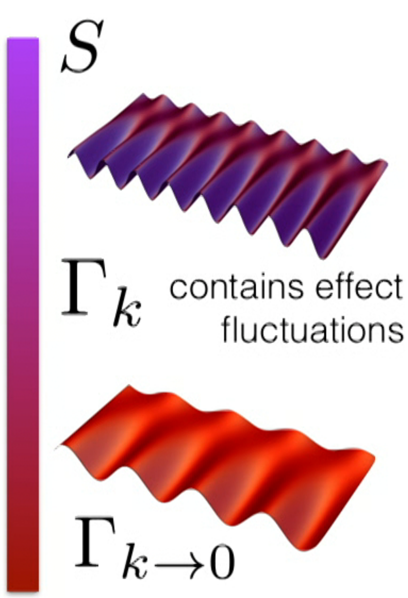

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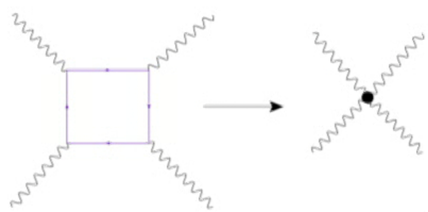

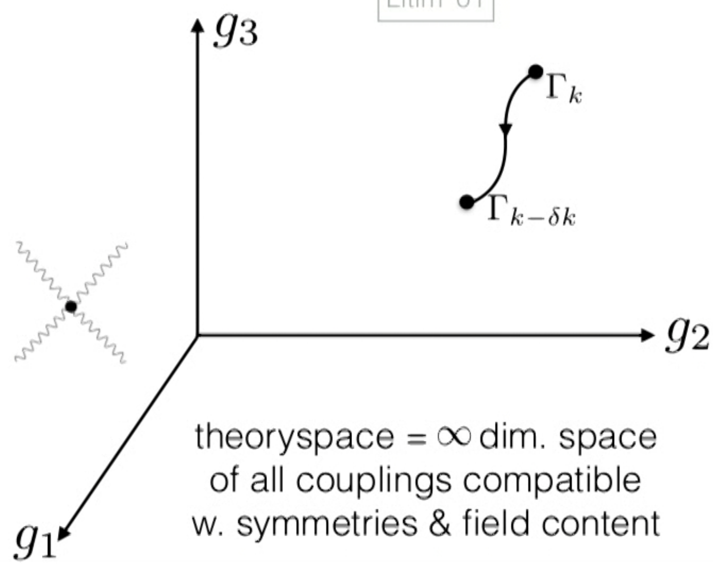

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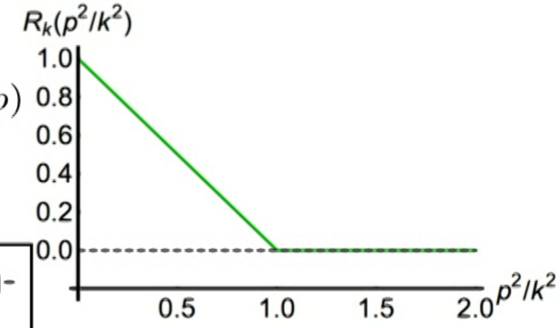

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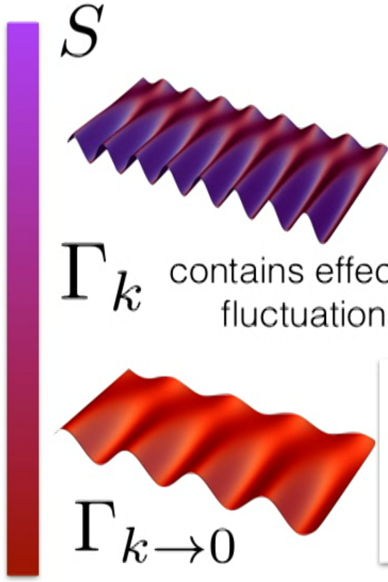

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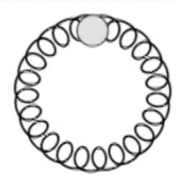

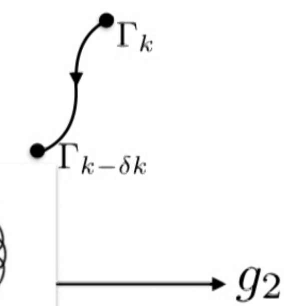

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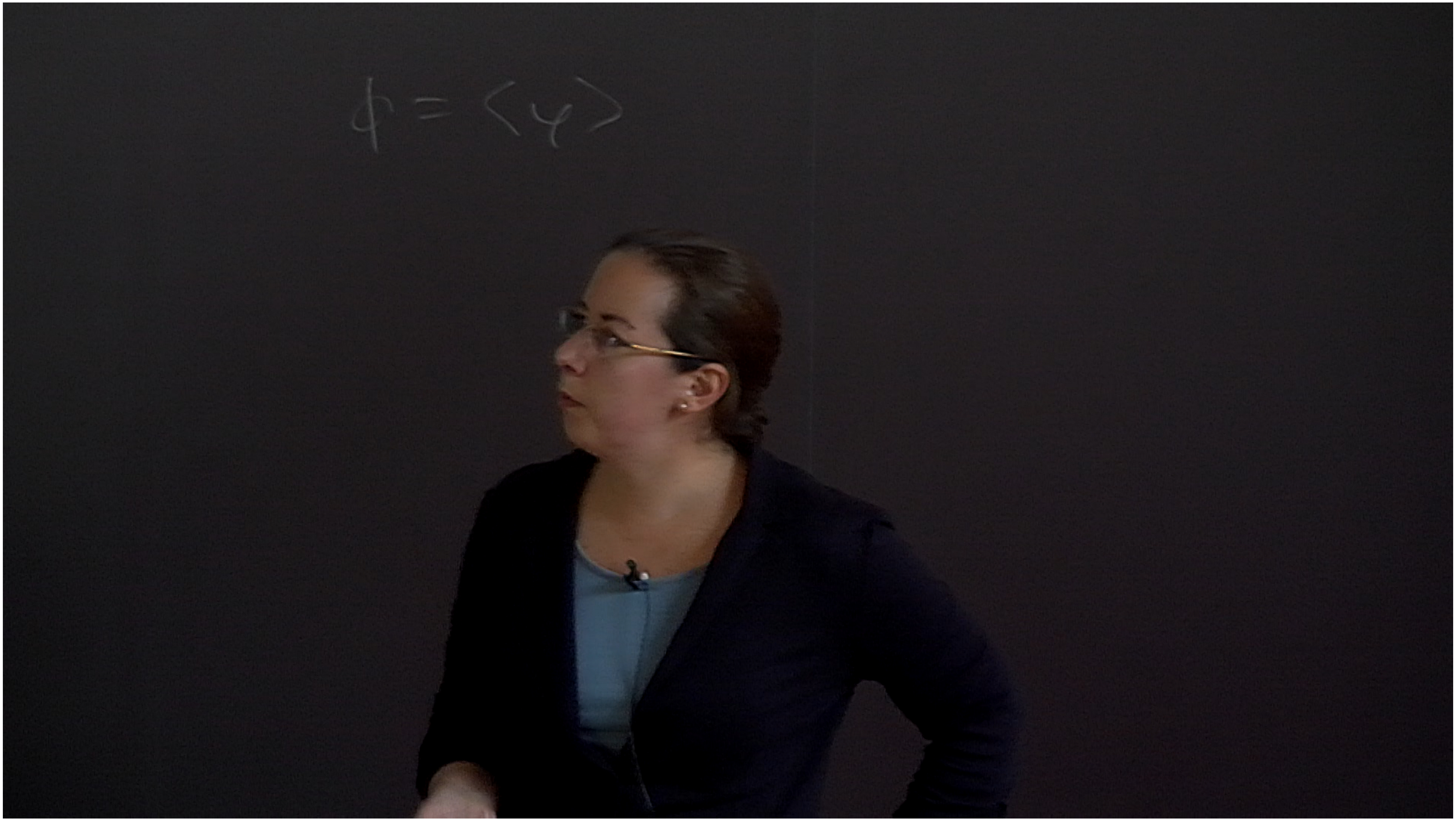
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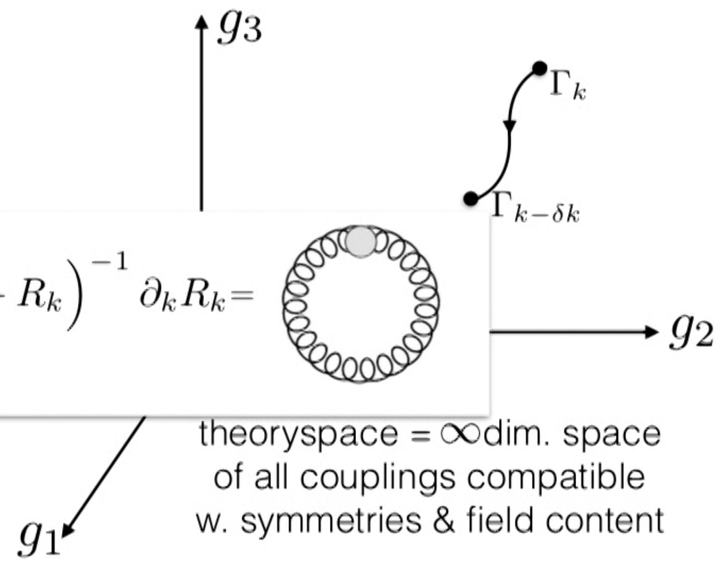

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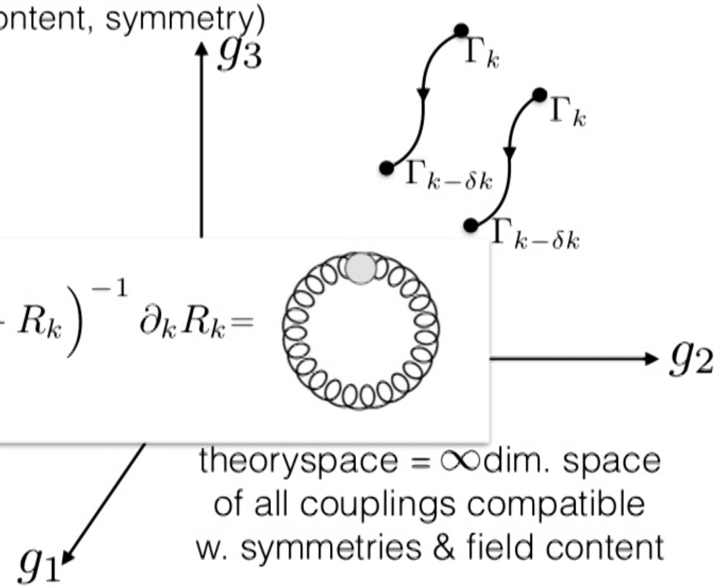

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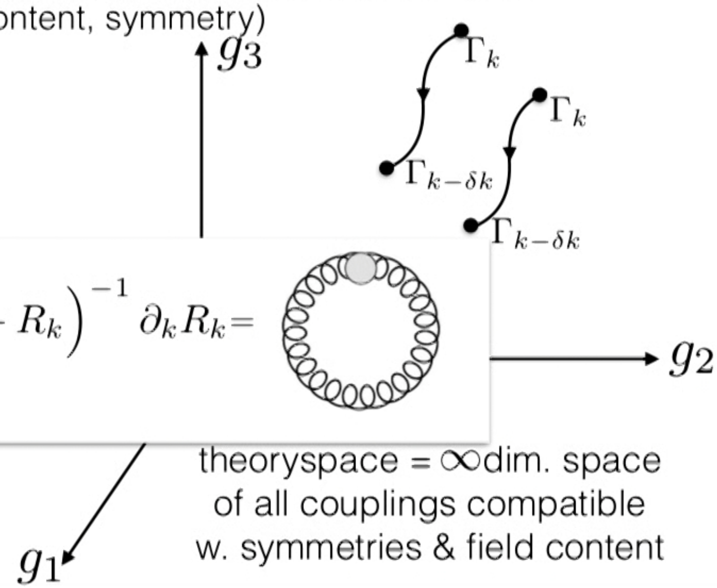

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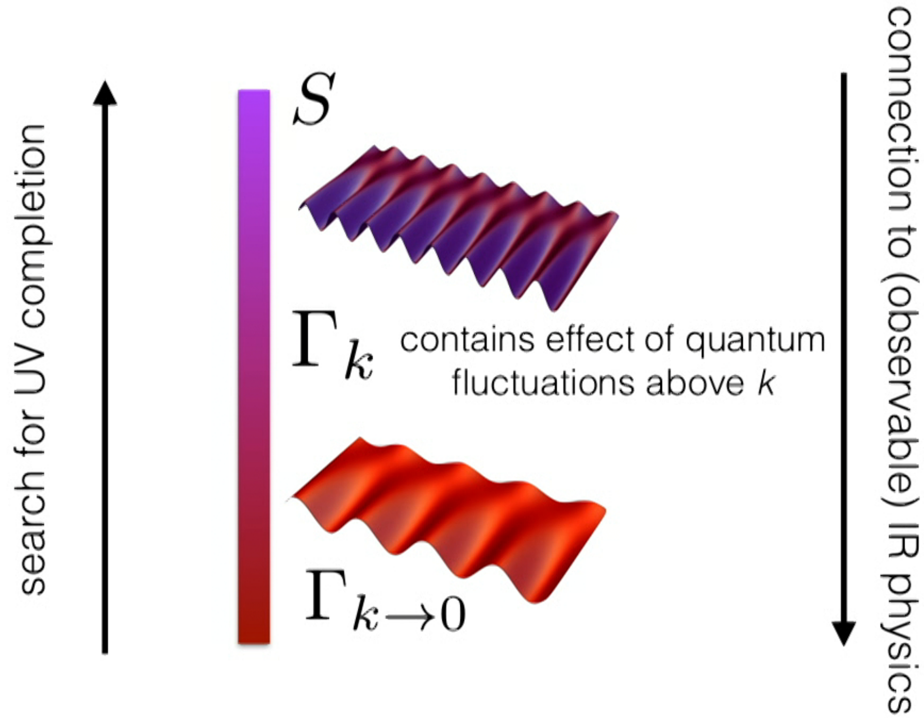
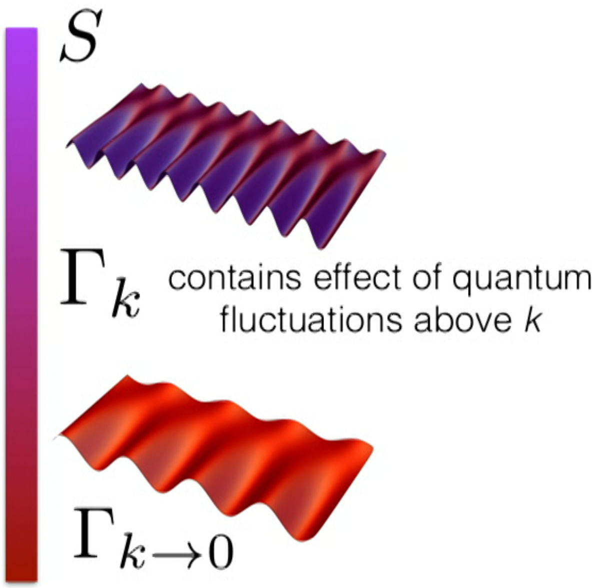
Renormalization Group: uses in Quantum Gravity



Search for UV completion in quantum gravity



Search for UV completion in quantum gravity

given a set of fields and symmetries
can explore whether model is
fundamental (RG fixed point)
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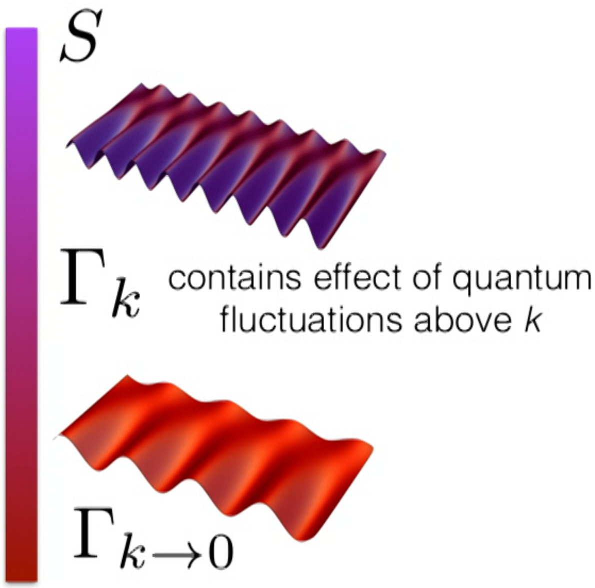

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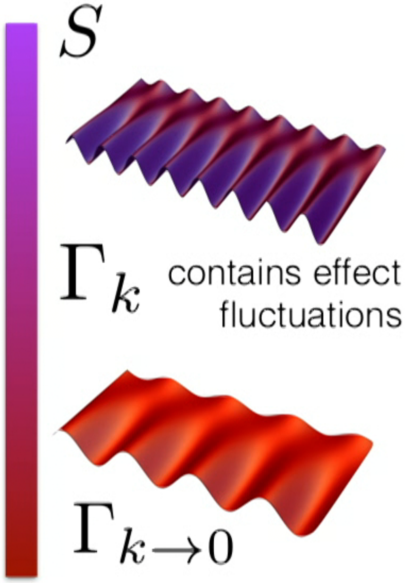

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$\Gamma_{k \to 0}$

effective theory:
new physics must
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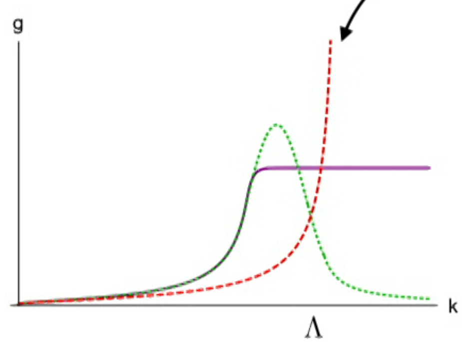

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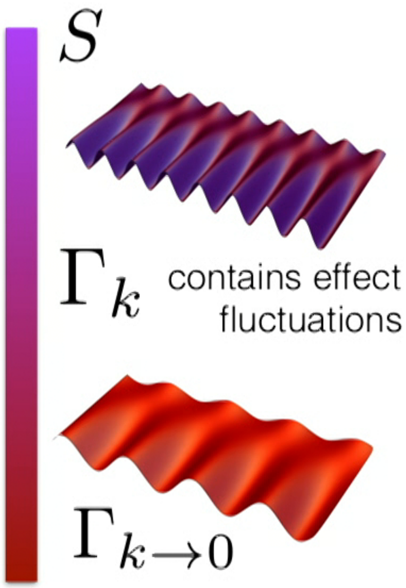

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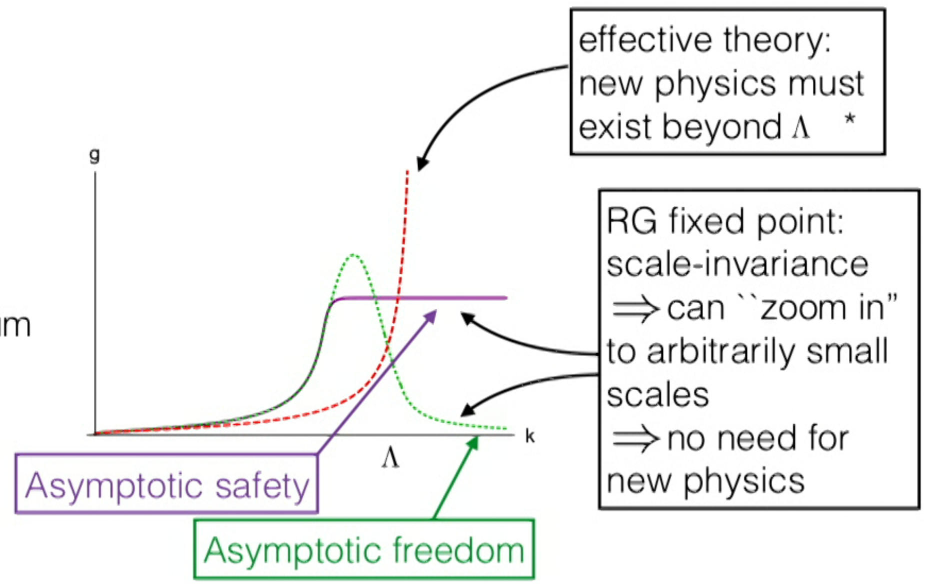

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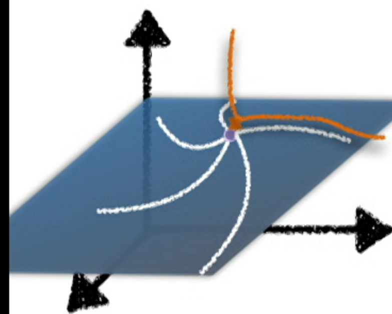

predictivity

UV-repulsive couplings
= predictions
≈ higher-order interactions

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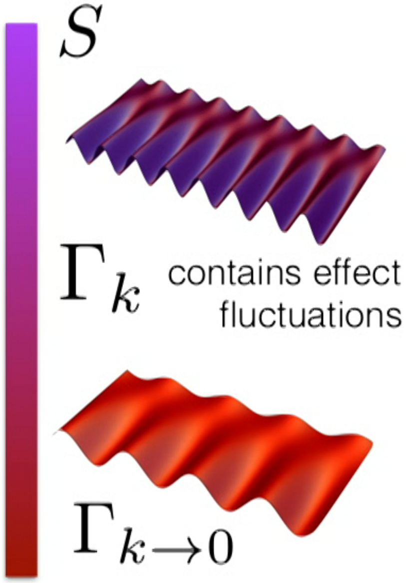

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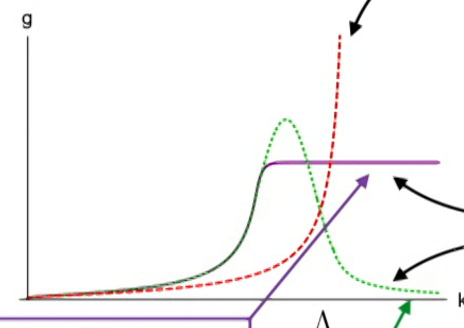

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RG fixed point:
scale-invariance
$\Rightarrow$ can ``zoom in"
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$\Rightarrow$ no need for
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Asymptotic safety

Asymptotic freedom

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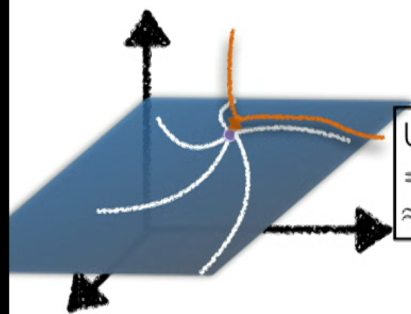

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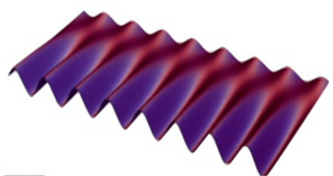

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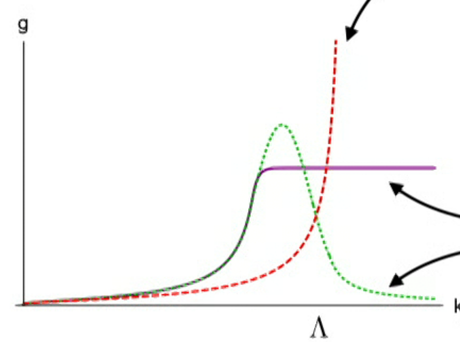

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symmetries matter!

e.g. unimodular asymptotic safety [A.E. '13, '15]

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Setting a scale in a geometric setting $\int \mathcal{D}g_{\mu\nu}$

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covariant generalization of momentum: (eigenvalues of) Laplacian

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$\rightarrow$ background field method

linear split: $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$ Reuter '96, …

exponential split: $g_{\mu\nu} = \bar{g}_{\mu\kappa} \left(\exp(h_{..})\right)^{\kappa}_{\nu}$

A.E. '13, Percacci, Vacca '15, …

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$h_{\mu\nu} R_k^{\mu\nu\kappa\lambda}(-\bar{D}^2) h_{\kappa\lambda} \rightarrow$ one- loop structure of flow equation intact

No free lunch

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shift symmetry: (``there is only one metric")

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$\to$ if the regulator breaks a symmetry, the flow no longer preserves the symmetry, even if the initial condition does

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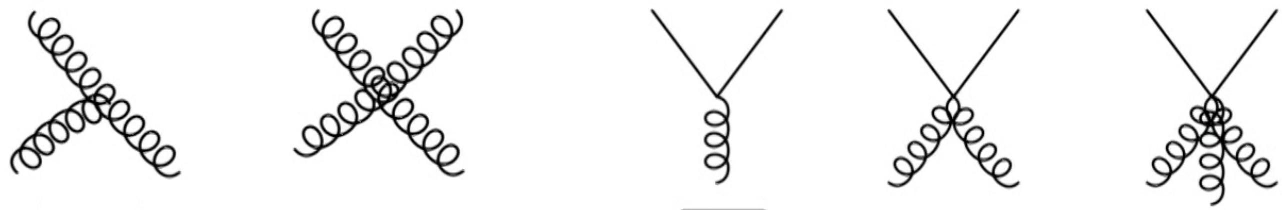
$\rightarrow$ enlarged theory space

$$S[g_{\mu\nu}] = -\frac{1}{16\pi G_N}\int d^4x\sqrt{g}\,R$$
$$= -\frac{1}{16\pi \bar{G}_N}\int d^4x\sqrt{\bar{g}}\,\bar{R}$$
$$-\frac{1}{16\pi G_1}\int d^4x\sqrt{\bar{g}}\,h_{\mu\nu}K^{\mu\nu}[\bar{g}] + \frac{1}{16\pi G_2}\int d^4x\sqrt{\bar{g}} h_{\mu\nu}h_{\kappa\lambda}K^{\mu\nu\kappa\lambda}[\bar{g}]$$
$$+\frac{1}{16\pi G_3}\int d^4x\sqrt{\bar{g}} h_{\mu\nu}h_{\kappa\lambda}h_{\rho\sigma}K^{\mu\nu\kappa\lambda\rho\sigma}[\bar{g}] + ...$$

$\beta_{\bar{G}_N} \neq \beta_{G_1} \neq \beta_{G_2}...$

How expensive is it?

"Newton couplings" in gravity-matter systems



$\sqrt{G_{(3,0)}}$ $G_{(4,0)}$ $\sqrt{G_{(1,2)}}$ $G_{(2,2)}$ $G_{(3,2)}$

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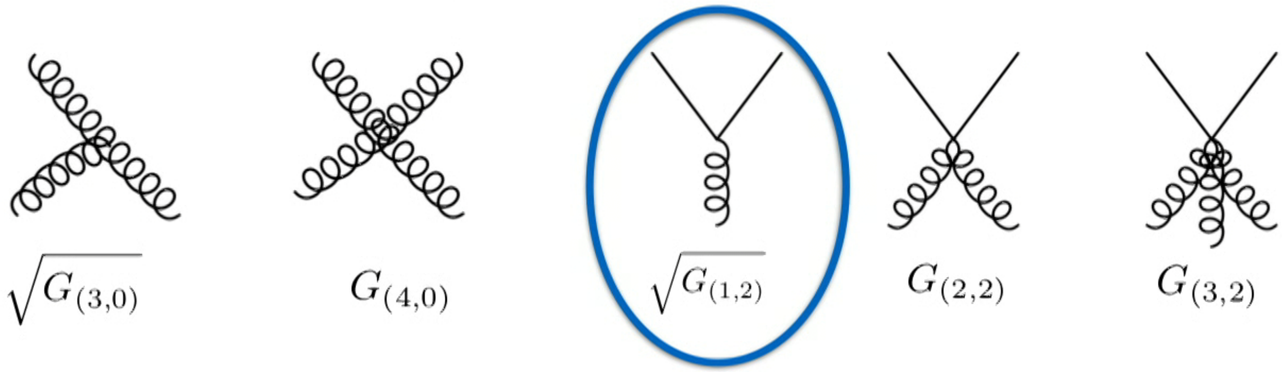


background Newton coupling:

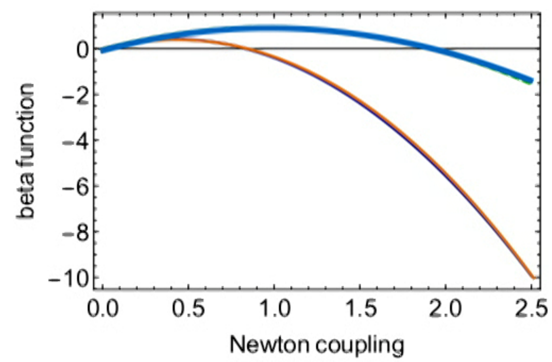
$$\beta_G = 2G + \frac{G^2}{6\pi}(-46 + N_S) + \ldots$$

[Dona, A.E., Percacci, '13]

``graviton"-scalar coupling:

$$\beta_{G_{(1,2)}} = 2G_{(1,2)} + \frac{G_{(1,2)}^2}{3\pi}\left(-10 + \frac{N_S}{8}\right) + \ldots$$

[Dona, A.E., Labus, Percacci, '15]



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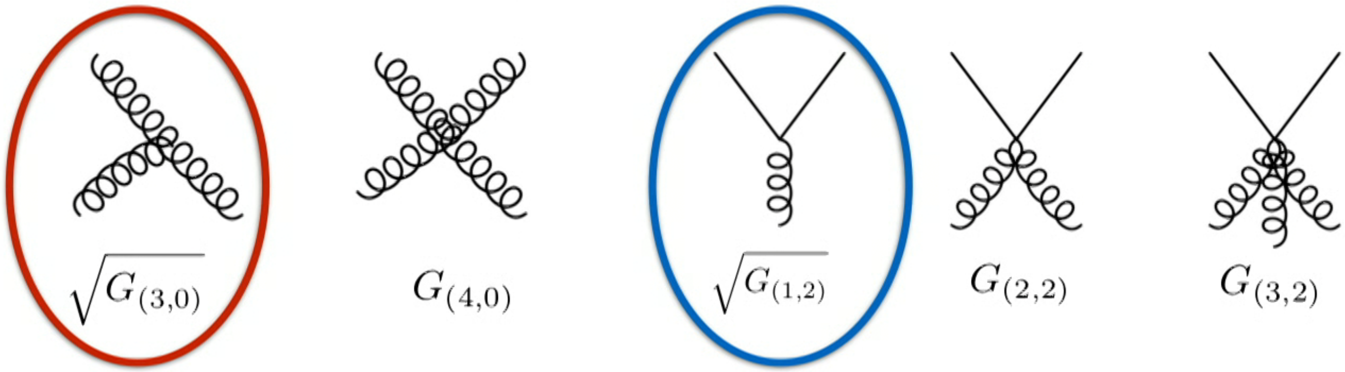

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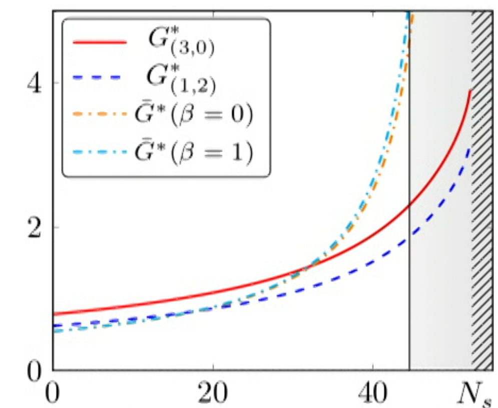
[Dona, A.E., Labus, Percacci, '15]

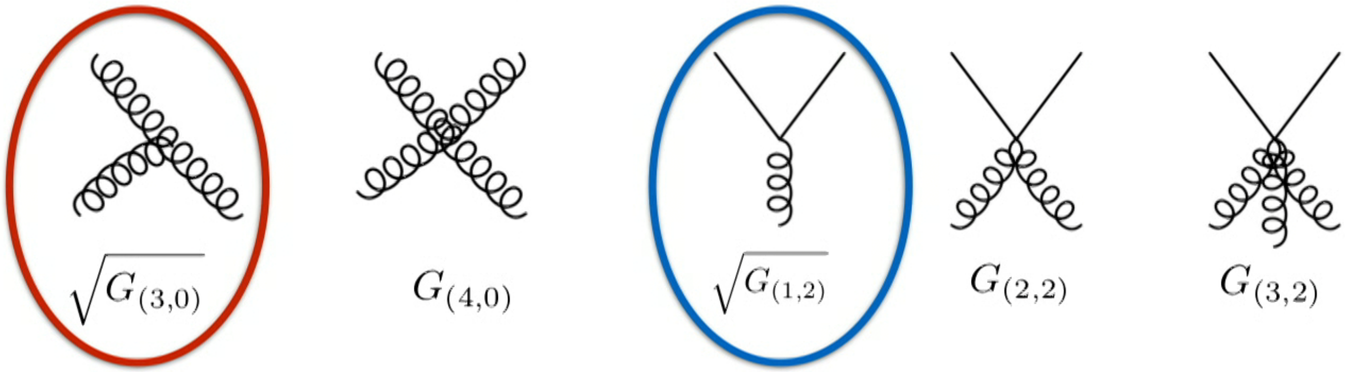


$\longrightarrow$ effective universality

[A.E., Labus, Pawlowski, Reichert, to appear]

How expensive is it?

"Newton couplings" in gravity-matter systems



background Newton coupling:

$$\beta_G = 2G + \frac{G^2}{6\pi}(-46 + N_S) + \ldots$$

[Dona, A.E., Percacci, '13]

``graviton"-scalar coupling:

$$\beta_{G_{(1,2)}} = 2G_{(1,2)} + \frac{G^2_{(1,2)}}{3\pi}\left(-10 + \frac{N_S}{8}\right) + \ldots$$

[Dona, A.E., Labus, Percacci, '15]

<u>quantitative difference but qualitative agreement</u>

scalars drive gravity into
the ``strong-gravity" regime
$\Rightarrow$ violation of weak-gravity bound

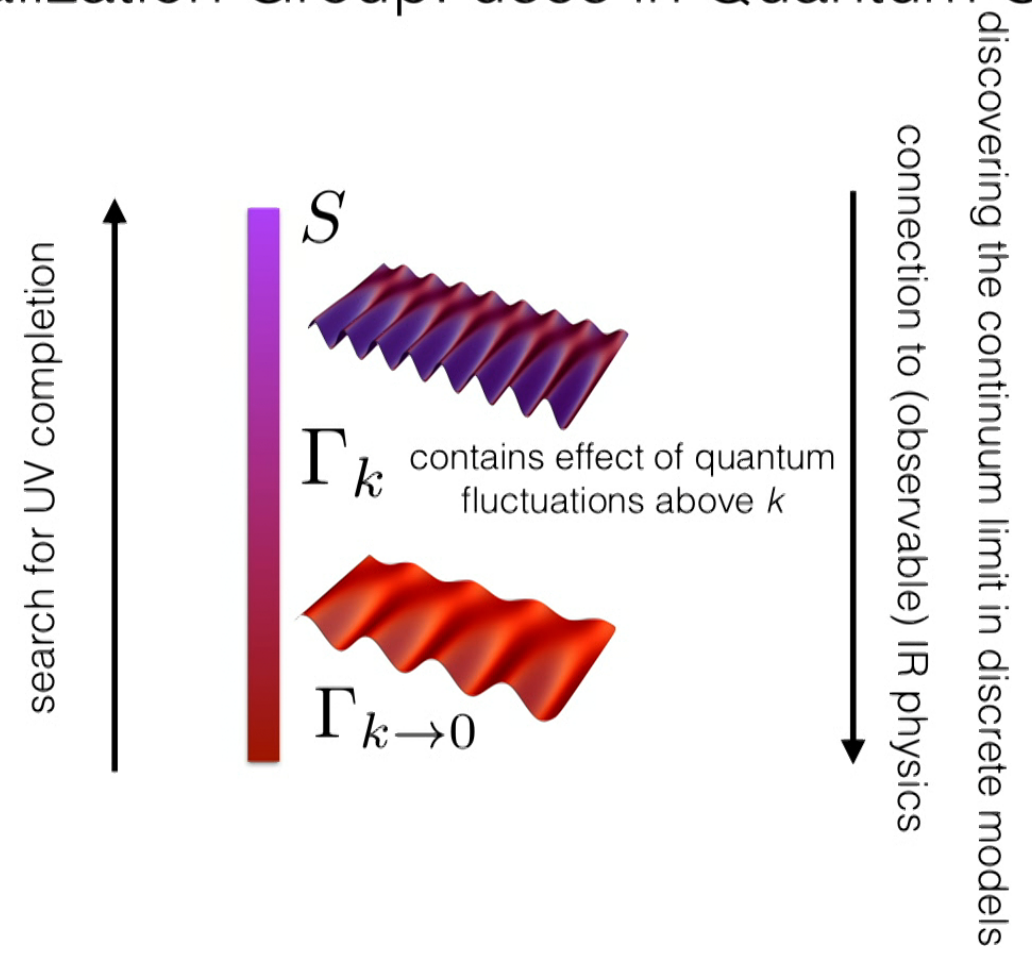
[A.E., Held, Pawlowski '16; Christiansen, A.E. '17, A.E., Held '17]

$\Rightarrow$ asymptotic safety only for $N_S < N_{S\,\mathrm{crit}}$

$\rightarrow$ contact of QG to BSM pheno.

$\rightarrow$ what about qm shape dynamics?

Renormalization Group: uses in Quantum Gravity



Setting a scale in a pre- geometric setting

RG flow: from many degrees of freedom to
fewer (coarser) degrees of freedom

Setting a scale in a pre- geometric setting

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example: $N \times ... \times N$ matrix/tensor models

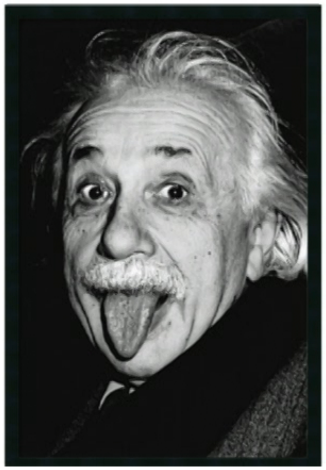
$$\int dT_{i_1 \ldots i_d} d\bar{T}_{i_1 \ldots i_d} e^{-S[\bar{T},T]}$$

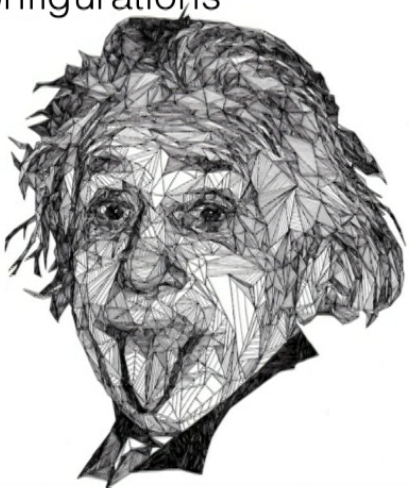
[Weingarten, Ambjorn, Durhuus, Frohlich, Kazakov, Migdal, Boulatov, 1980's]

[Sasakura '91, Gross '91, Ambjorn, Durhuus, Jonsson '91, Gurau '11, Gurau, Rivasseau '11, Gurau '12 …]

trace invariants: $T_{i_1 \ldots i_d} \bar{T}_{i_1 \ldots i_d} + T_{i_1 \ldots i_d} \bar{T}_{i_1 j_2 \ldots j_d} T_{j_1 \ldots j_d} \bar{T}_{j_1 i_2 \ldots i_d} + \ldots$

physics: discrete path integral over spacetime configurations





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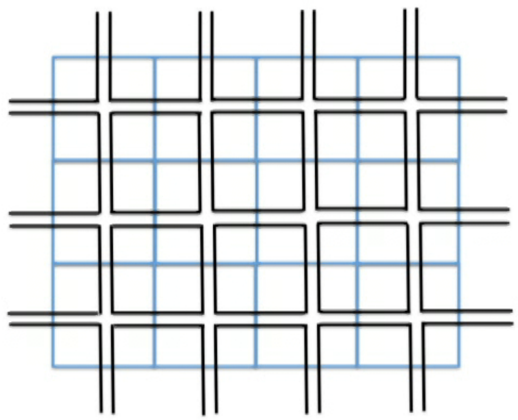
physics: discrete path integral over spacetime configurations

matrix model:

$$S = \mathrm{Tr} T^2 + g\, \mathrm{Tr} T^4$$

Feynman-diag. expansion
= all possible tessellations



continuum limit =
double-scaling limit:
$N \to \infty \quad g \to g_c$

$N \left(g - g_c\right)^{-\frac{1}{\theta}} = \mathrm{const}$

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universal scaling near RG fixed points:

$$\beta_g = \underbrace{\frac{\partial \beta_g}{\partial g}\Big|_{g=g_*}}_{= -\theta} (g - g_*) + ...$$

$$\to g(k) - g_* = \left(\frac{k}{k_0}\right)^{-\theta}$$

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double-scaling limit = universal scaling near RG fixed point in N

[Brezin, Zinn-Justin, '92]

No free lunch

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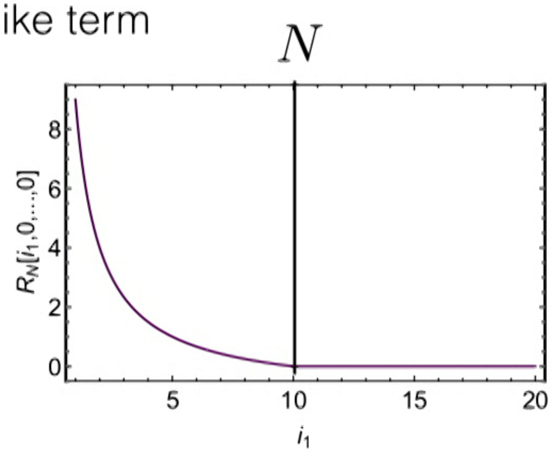
trace invariants: $T_{i_1 \dots i_d}\bar{T}_{i_1 \dots i_d} + T_{i_1 \dots i_d}\bar{T}_{i_1 j_2 \dots j_d} T_{j_1 \dots j_d}\bar{T}_{j_1 i_2 \dots i_d} + \dots$

has $U(N') \otimes ... \otimes U(N')$ invariance

Regulator: $R_N[i_1, ..., i_d] = \left(\frac{N}{i_1 + ... + i_d} - 1 \right) \theta(N - (i_1 + ... + i_d))$

index- dependent mass-like term



A.E., Koslowski '13, '14,'17

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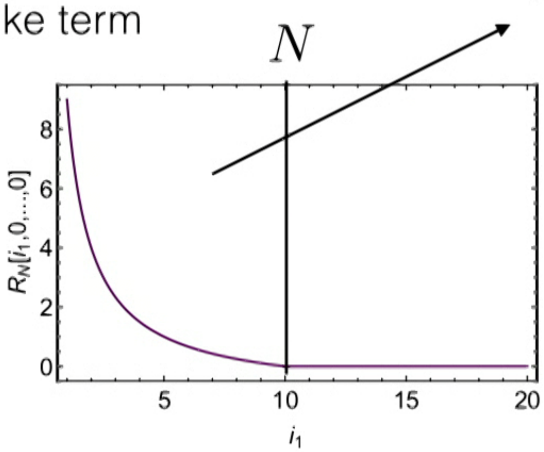
index- dependent mass-like term

$U(N') \otimes ... \otimes U(N')$ broken



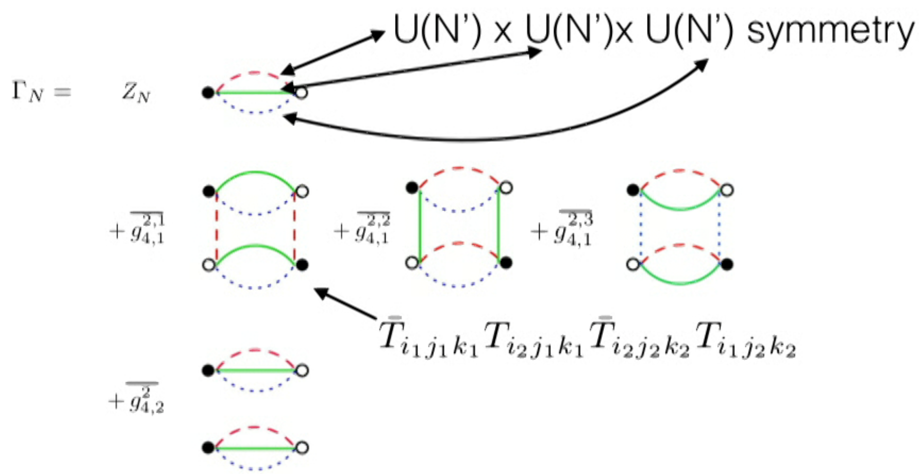
A.E., Koslowski '13, '14,'17

RG fixed points in rank 3 hermitian tensor models

the model

T : ○

$\bar{T}$: ●



scaling dimensionality: $\bar{g}_i = N^{d_{\bar{g}_i}} g_i$

consistently determined by RG equation (not just in tensor models)

$$\beta_{g_i} = -d_{\bar{g}_i} g_i + \#_1(N)\, g_i\, g_4 + \#_2(N) g_{i+2} + \ldots$$

$$\beta_{g_{i+2}} = -d_{\bar{g}_{i+2}} g_{i+2} + \#_3(N)\, g_{i+2}\, g_4 + \#_4(N) g_{i+4} + \ldots$$

[A.E., Koslowski '17, accepted in Annales de l'Institut Henri Poincaré D]

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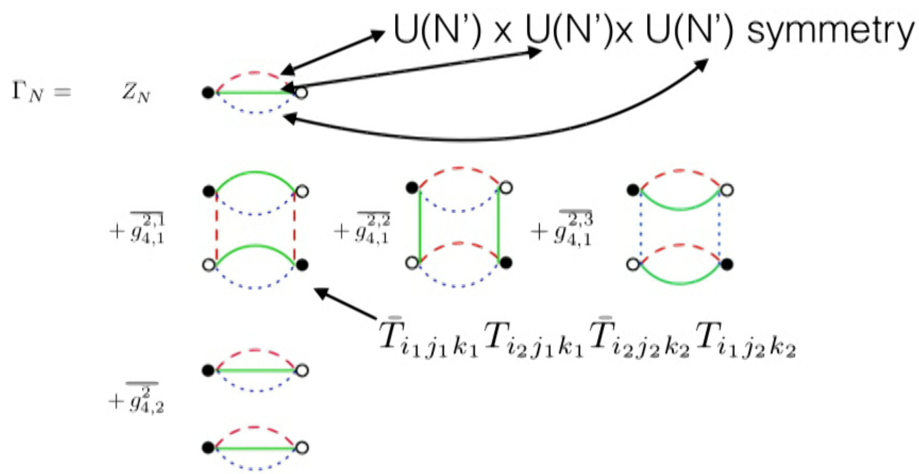

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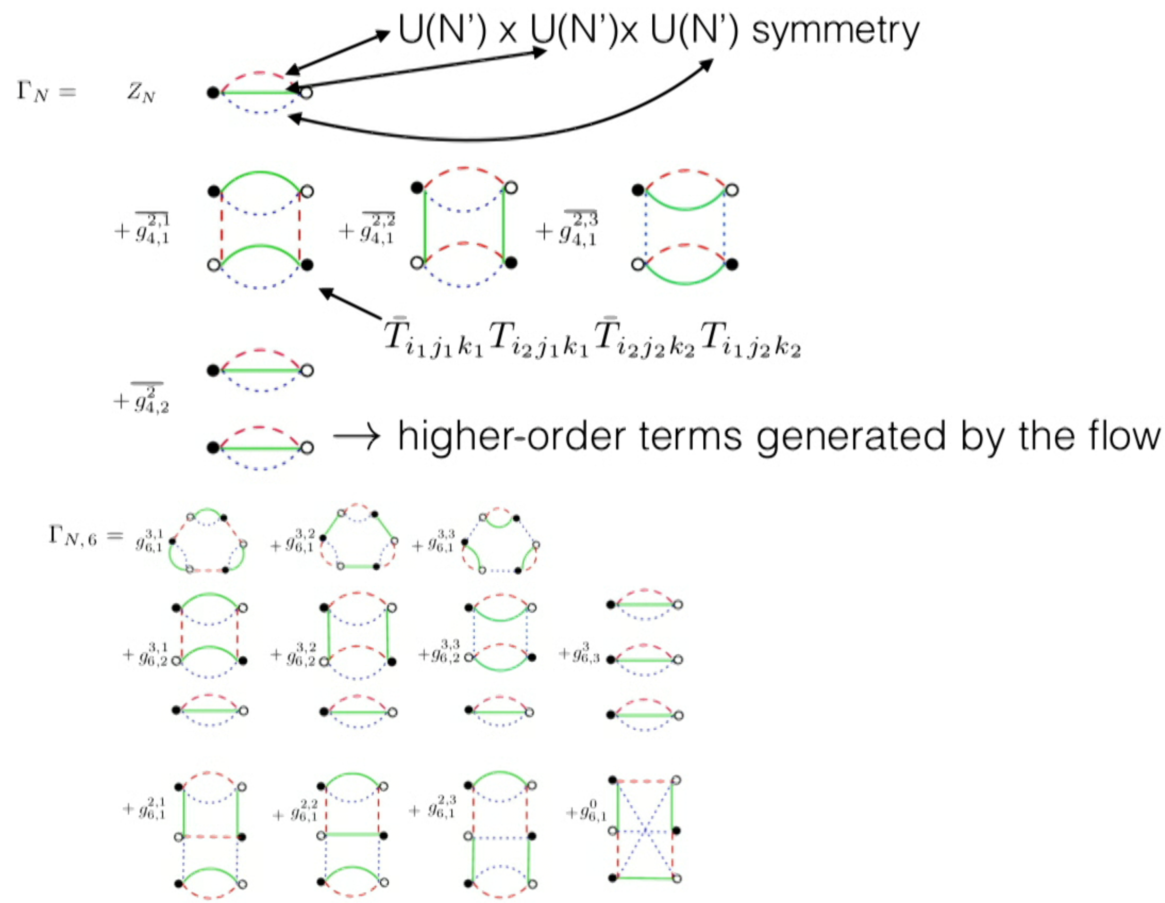
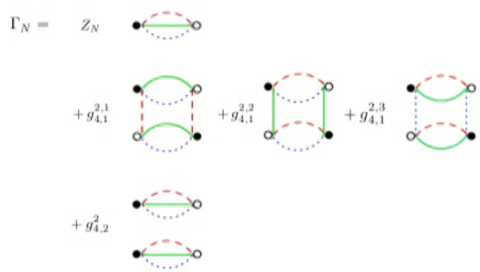
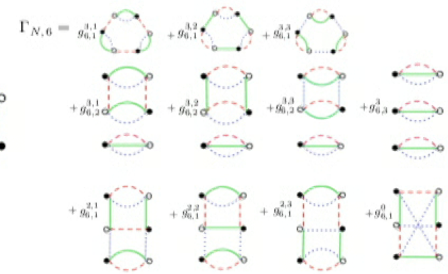


[A.E., Koslowski '17, accepted in Annales de l'Institut Henri Poincaré D]

Fixed-point results





scheme	$g^{2,1}_{4,1*}$	$g^{2}_{4,2*}$	$g^{3,1}_{6,1*}$	$g^{2,1}_{6,1*}$	$g^{3,1}_{6,2*}$	$g^{3}_{6,3*}$
full	-1.94	0	–	–	–	–
semi-pert	-2.14	0	–	–	–	–
pert	-4.62	0	–	–	–	–
full	-1.37	0	-2.14	0	0	0
semi-pert	-1.47	0	-2.46	0	0	0
pert	-2.14	0	-6.12	0	0	0

$$\theta = 1$$

[Bonzom, Gurau, Ryan, Tanasa '14
Dartois, Gurau, Rivasseau '13]

anomalous dimension

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scheme	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	θ_7	θ_8	θ_9	θ_{10}	θ_{11}	θ_{12}	θ_{13}	θ_{14}	η
full	2.21	-0.24	-0.93	-0.93	–	–	–	–	–	–	–	–	–	–	-0.54
semi-pert	2	-0.21	-0.68	-0.68	–	–	–	–	–	–	–	–	–	–	-0.54
pert	2	0.69	-2	-2	–	–	–	–	–	–	–	–	–	–	-0.54
full	2.14	-0.59	-1.26	-1.26	-2.24	-2.54	-2.89	-2.89	-3.15	-3.26	-3.26	-3.31	-3.31	-3.89	-0.37
semi-pert	2	-0.58	-1.26	-1.26	-2.24	-2.54	-2.90	-2.90	-3.13	-3.26	-3.26	-3.31	-3.31	-3.90	-0.37
pert	2	-0.35	-2	-2	-3.23	-3.37	-3.43	-4	-4	-4.07	-4.07	-4.14	-4.14	-5	0

scheme	θ'_1	θ'_2	θ'_3	θ'_4	θ'_5	θ'_6	θ'_7	θ'_8	θ'_9	θ'_{10}	θ'_{11}	θ'_{12}	θ'_{13}	θ'_{14}	η
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semi-pert	0.93	-0.21	-0.93	-0.93	–	–	–	–	–	–	–	–	–	–	-0.54
full	1.33	-0.59	-1.26	-1.26	-2.30	-2.54	-2.89	-2.89	-3.15	-3.26	-3.26	-3.31	-3.31	-3.89	-0.37
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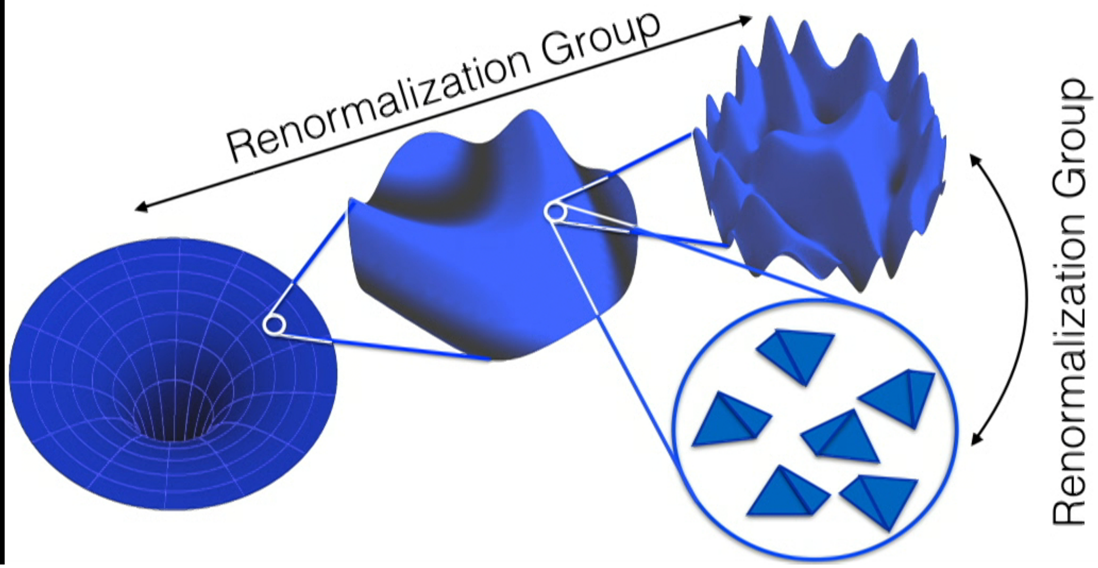
[A.E., Koslowski '17, accepted in Annales de l'Institut Henri Poincaré D]

Summary

1) RG: Probing the scale dependence of QFTs

2) RG for spacetime in the continuum: The price for setting a scale in Quantum Gravity → effective universality in asymptotically safe qg

3) RG for discrete spacetimes: The price for setting a scale in Quantum Gravity → reason behind large error in estimate of scaling exponent in tensor models?



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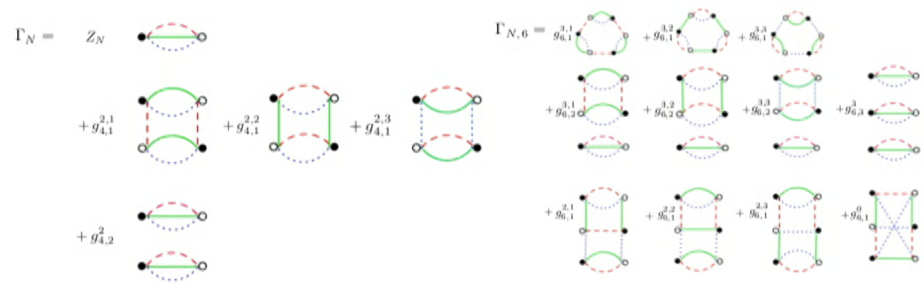

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$$\theta = 1$$

[Bonzom, Gurau, Ryan, Tanasa '14
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anomalous dimension

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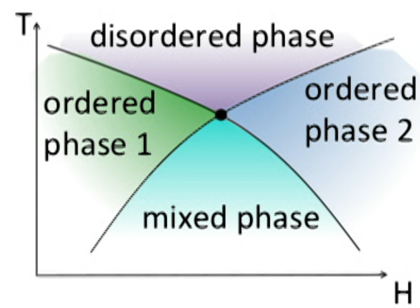
Observation:
Second scheme in accordance with scaling relation from ϵ expansion at multicritical points in systems with competing orders



[A.E., Mesterhazy, Scherer '13; Boettcher '14]

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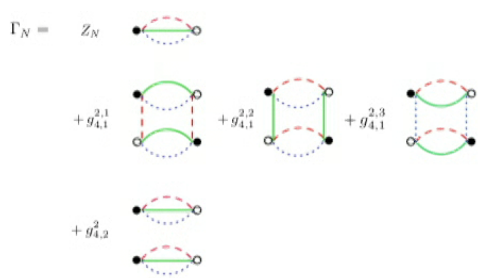

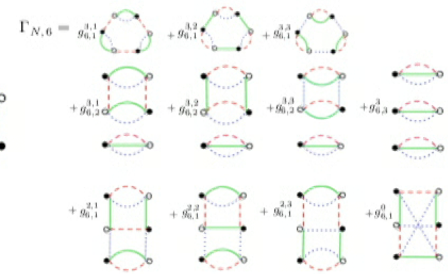

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[A.E., Koslowski '17, accepted in Annales de l'Institut Henri Poincaré D]

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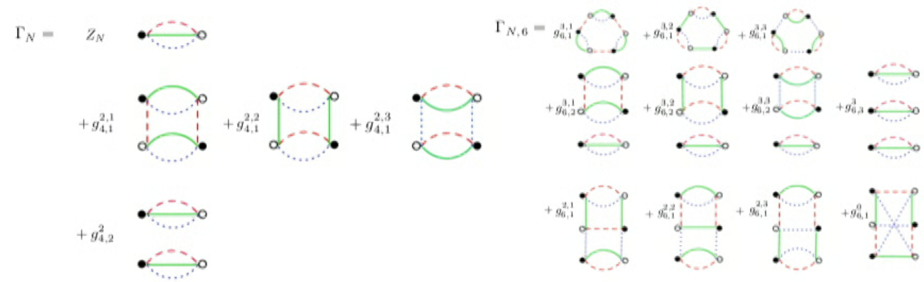

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$$\theta' : \quad \beta_g = \beta_g(g, \eta(g)_* = \text{const})$$

towards convergence in extended truncations?

scheme	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	θ_7	θ_8	θ_9	θ_{10}	θ_{11}	θ_{12}	θ_{13}	θ_{14}	η
full	2.21	-0.24	-0.93	-0.93	–	–	–	–	–	–	–	–	–	–	-0.54
semi-pert	2	-0.21	-0.68	-0.68	–	–	–	–	–	–	–	–	–	–	-0.54
pert	2	0.69	-2	-2	–	–	–	–	–	–	–	–	–	–	-0.54
full	2.14	-0.59	-1.26	-1.26	-2.24	-2.54	-2.89	-2.89	-3.15	-3.26	-3.26	-3.31	-3.31	-3.89	-0.37
semi-pert	2	-0.58	-1.26	-1.26	-2.24	-2.54	-2.90	-2.90	-3.13	-3.26	-3.26	-3.31	-3.31	-3.90	-0.37
pert	2	-0.35	-2	-2	-3.23	-3.37	-3.43	-4	-4	-4.07	-4.07	-4.14	-4.14	-5	0

scheme	θ'_1	θ'_2	θ'_3	θ'_4	θ'_5	θ'_6	θ'_7	θ'_8	θ'_9	θ'_{10}	θ'_{11}	θ'_{12}	θ'_{13}	θ'_{14}	η
full	0.93	-0.24	-0.93	-0.93	–	–	–	–	–	–	–	–	–	–	-0.54
semi-pert	0.93	-0.21	-0.93	-0.93	–	–	–	–	–	–	–	–	–	–	-0.54
full	1.33	-0.59	-1.26	-1.26	-2.30	-2.54	-2.89	-2.89	-3.15	-3.26	-3.26	-3.31	-3.31	-3.89	-0.37
semi-pert	1.33	-0.58	-1.26	-1.26	-2.31	-2.54	-2.90	-2.90	-3.13	-3.26	-3.26	-3.31	-3.31	-3.90	-0.37

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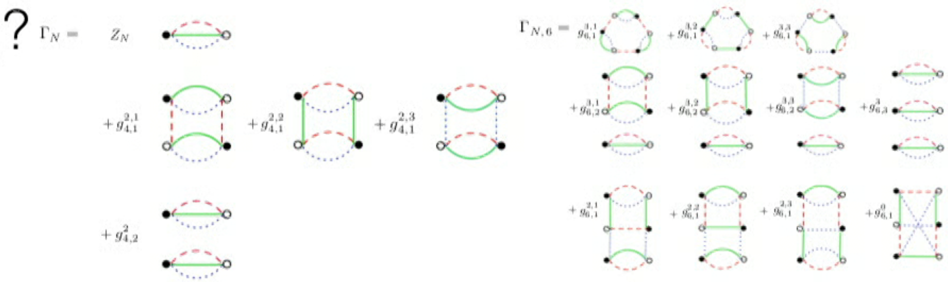
Beyond double scaling?



scheme	$g_{4,1}^{2,1}$	$g_{4,2}^{2}$	$g_{6,1}^{3,1}$	$g_{6,1}^{2,1}$	$g_{6,2}^{3,1}$	$g_{6,3}^{3}$
full	-1.05	-1.33	–	–	–	–
semi-pert	-1.58	-1.05	–	–	–	–
pert	-4.62	1.73	–	–	–	–
full	-0.53	-2.11	-0.14	0	-0.39	–
semi-pert	-0.63	-2.35	-0.20	0	-0.57	–
pert	-2.01	-1.64	-4.43	0	-4.84	–
full	-1.04	-0.84	-0.97	0	-0.64	-0.99
semi-pert	-1.15	-0.89	-1.21	0	-0.78	-1.14
pert	-2.10	-0.54	-5.54	0	-1.68	-0.47

scheme	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	θ_7	θ_8	θ_9	θ_{10}	θ_{11}	θ_{12}	θ_{13}	θ_{14}	η
full	2.64	0.14	-0.65	-0.65	–	–	–	–	–	–	–	–	–	–	-0.67
semi-pert	2.28	0.14	-0.68	-0.68	–	–	–	–	–	–	–	–	–	–	-0.66
pert	2	-0.69	-2	-2	–	–	–	–	–	–	–	–	–	–	0
full	3.10	0.25	-0.48	-0.48	-1.22 ± i 0.44		-1.53	-1.53	-1.72	-1.72	-2.46	-2.46	-2.72	–	-0.76
semi-pert	2.68	0.26	-0.51	-0.51	-1.24 ± i 0.45		-1.57	-1.57	-1.76	-1.76	-2.49	-2.49	-2.76	–	-0.75
pert	2.11	0.34	-2	-2	-2.99	-2.99	-3.54	-3.54	-4	-4	-4.13	-4.13	-5	–	0
full	2.56	0.44	-0.97	-0.97	-1.80 ± i 0.42		-2.45	-2.45	-2.63	-2.63	-2.95	-2.95	-3.06	-3.45	-0.52
semi-pert	2.30	0.44	-0.98	-0.98	-1.81 ± i 0.42		-2.47	-2.47	-2.65	-2.65	-2.97	-2.97	-3.06	-3.47	-0.51
pert	2.04	0.37	-2	-2	-2.68	-3.16	-3.87	-3.95	-3.95	-4	-4	-4.09	-4.09	-5	0

scheme	θ'_1	θ'_2	θ'_3	θ'_4	θ'_5	θ'_6	θ'_7	θ'_8	θ'_9	θ'_{10}	θ'_{11}	θ'_{12}	θ'_{13}	θ'_{14}	η
full	0.65	0.47	-0.65	-0.65	–	–	–	–	–	–	–	–	–	–	-0.67
semi-pert	0.68	0.42	-0.68	-0.68	–	–	–	–	–	–	–	–	–	–	-0.66
full	1.03	0.43	-0.48	-0.48	-1.23 ± i 0.46		-1.53	-1.53	-1.72	-1.72	-2.46	-2.46	-2.72	–	-0.76
semi-pert	1.04	0.44	-0.51	-0.51	-1.25 ± i 0.47		-1.57	-1.57	-1.76	-1.76	-2.49	-2.49	-2.76	–	-0.75
full	1.17	0.64	-0.97	-0.97	-1.82 ± i 0.42		-2.45	-2.45	-2.63	-2.63	-2.95	-2.95	-3.10	-3.45	-0.52
semi-pert	1.17	0.63	-0.98	-0.98	-1.83 ± i 0.42		-2.47	-2.47	-2.65	-2.65	-2.97	-2.97	-3.10	-3.47	-0.51

[A.E., Koslowski '17, accepted in Annales de l'Institut Henri Poincaré D]

Summary

1) RG: Probing the scale dependence of QFTs

2) RG for spacetime in the continuum: The price for setting a scale in Quantum Gravity → effective universality in asymptotically safe qg

3) RG for discrete spacetimes: The price for setting a scale in Quantum Gravity → reason behind large error in estimate of scaling exponent in tensor models?