Title: TBA

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Abstract:

Effective Theory of Gravity: Macroscopic Effects of the Conformal Anomaly

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Functional Integration over Geometries
Review: J. Math. Phys. 36 (5), 2470 (1995)

w. R. Vaulin, Phys. Rev. D 74, 064004 (2006)

w. M. Giannotti, Phys. Rev. D 79, 045014 (2009)

Review: Acta. Phys. Pol. B 41, 2031 (2010)

w. D. Blaschke & R. Caballo-Rubio, JHEP 1412 (2014) 153

Conformalon Scalar in Cosmology w. P. R. Anderson
Phys. Rev. D 80, 084005 (2009); 89, 104039 (2014)

Scalar Gravitational Waves: JHEP 1707, 043 (2017)

Outline

- The Covariant Functional Integral over Geometries
  - Conformal Anomaly required by General Covariance
- Effective Field Theory & Anomalies
  - Massless Scalar Poles in Anomaly Amplitudes
- Effective Theory of Low Energy Gravity
  - Role of the Conformal/Trace Anomaly
  - New Scalar Degree of Freedom from the Trace Anomaly
- Scalar Gravitational Waves in EFT of Gravity
  - Conformal Part of Metric becomes Dynamical
- Consequences for Astrophysics and Cosmology

Functional Integral over Geometries

Treat an arbitrary metric as a 'point'

$$X^a \leftrightarrow g_{\mu\nu}(x)$$

in the space of all metrics, and the one-form

$$dX^a \leftrightarrow \delta g_{\mu\nu}(x) \equiv h_{\mu\nu}(x)$$

in the cotangent space, with covariant inner product

$$g_{ab}(X)dX^a dX^b \leftrightarrow \langle h, h\rangle \equiv \int d^4x \sqrt{-g}\, h_{\mu\nu}(x) G^{\mu\nu\alpha\beta} h_{\alpha\beta}(x)$$

In local tangent frame vierbein notation

$$g_{\mu\nu} = e^m_\mu\, e^n_\nu\, \eta_{mn} \qquad e \equiv \det(e^m_\mu) = \sqrt{-g}$$

$$h_{\mu\nu} = \frac{1}{\sqrt{e}}\, e^m_\mu\, e^n_\nu\, \tilde{h}_{mn}$$

Gaussian Measure

Inner Product becomes

$$\langle h, h\rangle = \int d^4x\, \tilde{h}_{mn}(x)\, \tilde{G}^{mnrs}\, \tilde{h}_{rs}(x)$$

with the 'Supermetric'

$$\tilde{G}^{mnrs} = \frac{1}{2}\left(\eta_{mr}\eta_{ns} + \eta_{ms}\eta_{nr} + C\eta_{mn}\eta_{rs}\right)$$

The pointwise functional measure

$$[\mathcal{D}h_{\mu\nu}] = \Pi_x \Pi_{m\leq n}\, d\tilde{h}_{mn}(x) = \Pi_x e^{(D-4)(D+1)/4}(x)\, \Pi_{\mu\leq\nu} dh_{\mu\nu}(x)$$

is ill-defined, but can be fixed by defining

$$\int [\mathcal{D}h_{\mu\nu}]\, \exp\left(-\frac{i}{2}\langle h, h\rangle\right) = 1$$

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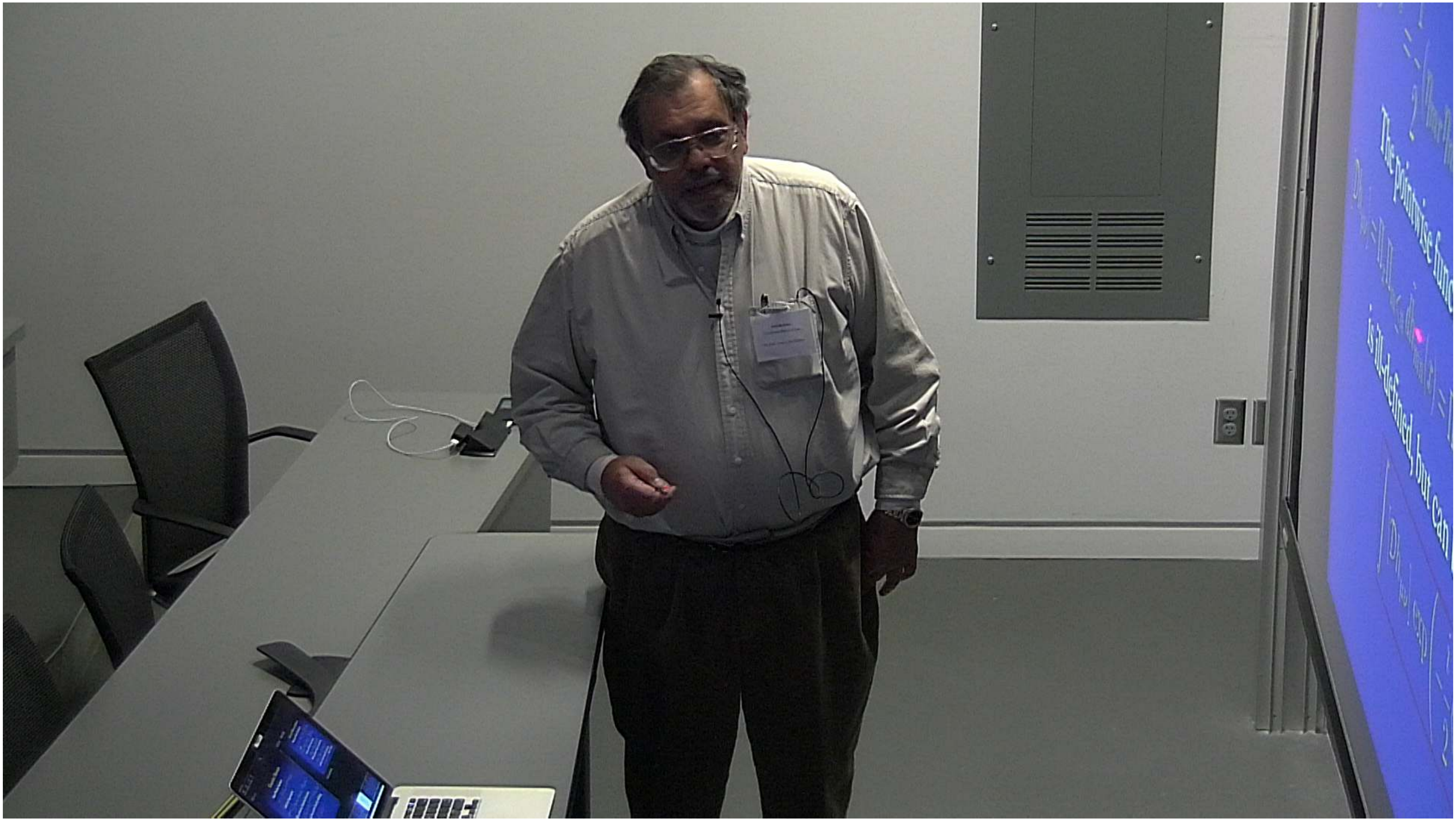

Extract Diffeomorphisms

Decompose Metric Perturbation

$$h_{\mu\nu} = h^{\perp}_{\mu\nu} + (L\xi)_{\mu\nu} + \left(2\sigma + \frac{2}{D}\nabla_\lambda \xi^\lambda\right) g_{\mu\nu}$$

with

$$(L\xi)_{\mu\nu} \equiv \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu - \frac{2}{D}\nabla_\lambda \xi^\lambda g_{\mu\nu}$$

Change variables in the measure

$$[\mathcal{D}h_{\mu\nu}] = J[\mathcal{D}h^{\perp}_{\mu\nu}][\mathcal{D}\xi_\mu][\mathcal{D}\sigma]$$

Diffeomorphism Volume eliminated by

$$[\mathrm{Vol}(\mathcal{G})]^{-1} \int [\mathcal{D}h_{\mu\nu}] = \int J\,[\mathcal{D}h^{\perp}_{\mu\nu}][\mathcal{D}\sigma]$$

Jacobian easily computed by Gaussians

$$J = (\det{}_V L^\dagger L)^{\frac{1}{2}} \quad \textbf{(Fadeev-Popov)}$$

General Invariant Path Integral

General coordinates of Metric

$$g_{\mu\nu}(x) = \frac{\partial X^\alpha}{\partial x^\mu}\frac{\partial X^\beta}{\partial x^\nu}\, \exp\left(2\sigma(X)\right) g^{\perp}_{\alpha\beta}(X)$$

gives

$$[\mathrm{Vol}(\mathcal{G})]^{-1}\int [\mathcal{D}h_{\mu\nu}]\exp\left(iS_{inv}[g]\right) = \int J[\mathcal{D}g^{\perp}_{\mu\nu}][\mathcal{D}\sigma]\exp\left(iS_{inv}\left[e^{2\sigma}g^{\perp}\right]\right)$$

- **Generally Covariant Definition of Path Integral over Lorentzian Geometries without Discretization**
- ***J*** **removes unphysical gauge modes, ghosts**
- **Agrees with Canonical Phase Space Defn.**
- **Gives Trace Anomaly in 2D Gravity (a.k.a. non-critical string theory) and in 4D (cf. Fujikawa)**

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2D Gravity

$$S_{cl}[g] = \int d^2x \sqrt{g}(\gamma R - 2\lambda)$$

has no local degrees of freedom in 2D, since

$$g_{ab} = \exp(2\sigma)\bar{g}_{ab} \to \exp(2\sigma)\eta_{ab}$$

(all metrics conformally flat) and

$$\sqrt{g}R = \sqrt{\bar{g}}\bar{R} - 2\sqrt{\bar{g}}\,\bar{\Box}\sigma$$

gives a total derivative in S_{cl}

Quantum Trace or Conformal Anomaly

$$\langle T^a_{\ a}\rangle = -\frac{c_m}{24\pi}R$$

$c_m = N_S + N_F$ for massless scalars or fermions

Linearity in σ in the variational eq.

$$\frac{\delta\Gamma_{WZ}}{\delta\sigma} = \sqrt{g}\,\langle T^a_{\ a}\rangle$$

determines the Wess-Zumino Action by *inspection*

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2D Anomaly Action

- Integrating the anomaly linear in **σ** gives

$$\Gamma_{WZ}[\bar{g},\sigma] = \frac{c_m}{24\pi}\int d^2x\sqrt{\bar{g}}\,\left(-\sigma\,\bar{\Box}\sigma + \bar{R}\,\sigma\right)$$

- This is local but **non-covariant**. Note kinetic term for **σ**
- By solving for **σ** the WZ action can be also written

$$\Gamma_{WZ}[\bar{g},\sigma] = S_{anom}[g = e^{2\sigma}\,\bar{g}] - S_{anom}[\bar{g}]$$

- Polyakov form of the action is covariant but **non-local**

$$S_{anom}[g] = -\frac{c}{96\pi}\int d^2x\sqrt{g}\int d^2x'\sqrt{g'}\;R_x\left(\Box^{-1}\right)_{x,x'}R_{x'}$$

- A covariant local form implies a **dynamical** **scalar** field

$$S_{anom}[g;\varphi] = \frac{c}{96\pi}\int d^2x\sqrt{g}\left[g^{ab}(\nabla_a\varphi)(\nabla_b\varphi) + 2R\varphi\right]$$

$$-\Box\varphi = R \qquad\qquad \varphi \leftrightarrow 2\sigma$$

2D Ward Identity and Massless Poles

Effects of Anomaly may be seen in flat space amplitudes

Conservation of T_{ab} Ward Identity in **2D** implies

$$\Pi_{abcd}(k) = (\eta_{ab}k^2 - k_a k_b)(\eta_{cd}k^2 - k_c k_d)\,\Pi(k^2)$$

Anomalous Trace Ward Identity in **2D** implies

$k^2\,\Pi(k^2) \neq 0$ at $k^2 = 0$ **massless boson pole**

Correlated Fermion Pair ➔Boson $\quad a_n^\dagger \sim \sum_{q=\frac{1}{2}}^{n-\frac{1}{2}} b_{n-q}^\dagger d_q^\dagger$

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Quantum Effects of 2D Anomaly Action

- Massless Pole in Anomalous Amplitude
- Effective Massless Boson in Two-Particle Intermediate State: 'Cooper Pair' of the Vacuum
- Anomalous Current Commutators are Canonical Commutators of the Composite Boson
- Modification of Classical Theory required by Quantum Fluctuations & Covariant Conservation of $\langle T^a{}_b \rangle$
- Metric conformal factor $e^{2\sigma}$ (was constrained) becomes dynamical & itself fluctuates freely $c\text{-}26 \rightarrow c\text{-}25$
- Gravitational 'Dressing' of critical exponents: long distance/IR macroscopic physics
- Additional non-local Infrared Relevant Operator in S_{EFT}

'New' Massless Scalar Degree of Freedom at low energy

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Effective Field Theory & Quantum Anomalies

- EFT = Expansion of Effective Action in Local Invariants
- Assumes Decoupling of Short (UV) from Long Distance (IR)
- But Massless/Light Fields do not decouple
- Massless Chiral, Conformal Symmetries are Anomalous
- Massless Poles in Anomalous Amplitudes
- Special Non-Local Contributions Low Energy EFT
- Equivalently, additional Low Energy Local Degrees of Freedom
- IR Macroscopic Effects from Quantum Fluctuations