

Title: The path integral for gravity and Fedosov quantization

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Abstract:

The path integral in gravity and Fedosov quantization

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based on Hamburg talk 30/07/12, [1603.09626] (G. Collini PhD thesis)

Perimeter Institute, November 2017

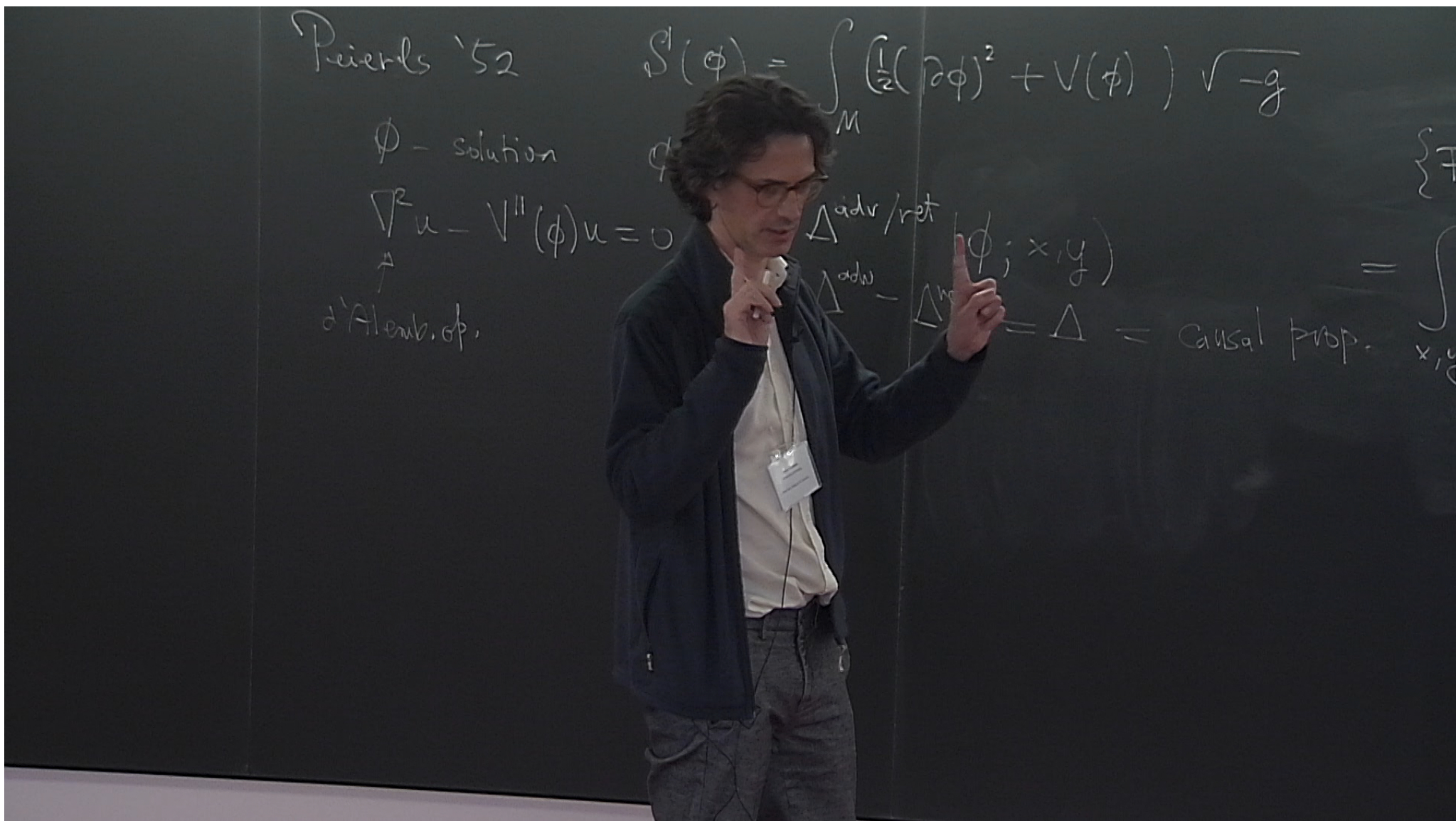
Peierls bracket

$$S(\phi) = \int_M \sqrt{-g} [(\nabla\phi)^2 + V(\phi)] dx$$



R. Peierls CMB, * 1907 † 1995

$$\{F_1, F_2\}(\phi) = \int dx dy \frac{\delta F_1}{\delta \phi(x)} \Delta(\phi; x, y) \frac{\delta F_2}{\delta \phi(y)}$$



Poisson brackets without space-time split

Peierls '52

$$S(\phi) = \int_M \left(\frac{1}{2} (\partial\phi)^2 + V(\phi) \right) \sqrt{-g}$$

$F(\phi)$ - observables

ϕ - solution $\phi + u + \dots$

$\{F, G\}(\phi)$ Peierls bracket

$$\nabla^2 u - V''(\phi)u = 0 \rightarrow \Delta^{\text{adv/ret}}(\phi; x, y)$$

$$= \int_{x, y} \frac{\delta F(\phi)}{\delta \phi(x)} \Delta(\phi; x, y) \frac{\delta G(\phi)}{\delta \phi(y)}$$

∇^2
d'Alemb. op.

$$\Delta^{\text{adv}} - \Delta^{\text{ret}} = \Delta = \text{causal prop.}$$

Gravity OK.

Deformation quantization:

Fancy maths: $\star$ -product:

$$\begin{aligned} F \star G &= FG + \mathcal{O}(\hbar) \\ F \star G - G \star F &= i\hbar \{F, G\} + \mathcal{O}(\hbar^2) \end{aligned} \quad \left. \vphantom{\begin{aligned} F \star G &= FG + \mathcal{O}(\hbar) \\ F \star G - G \star F &= i\hbar \{F, G\} + \mathcal{O}(\hbar^2) \end{aligned}} \right\} + \text{associativity}$$

Fedosov '94 (Kontsevich '02)

Free-scalar field $V(\phi) = \frac{1}{2} m^2 \phi^2$

Need: Wightman fct $W(x, y) = \frac{i}{2} \Delta(x, y) + \frac{1}{2} G(x, y) = \langle \phi(x) \phi(y) \rangle$

$F \star G = F \exp \left(\int_{x, y} \frac{\delta}{\delta \phi(x)} W(x, y) \frac{\delta}{\delta \phi(y)} \right)$

Free-scalar field $V(\phi) = \frac{1}{2} m^2 \phi^2$

Need: Wightman fct $W(x,y) = \frac{i}{2} \Delta(x,y) + \textcircled{G_w(x,y)} = \langle \phi(x) \phi(y) \rangle_w$

$$F \star G = F \exp \left(\hbar \int \overleftrightarrow{\frac{\delta}{\delta \phi(x)}} W(x,y) \overleftrightarrow{\frac{\delta}{\delta \phi(y)}} \right) \quad G = F G + \hbar \int \frac{\delta F}{\delta \phi} W \frac{\delta G}{\delta \phi} + \dots$$

quantization :

$\star$ -product :

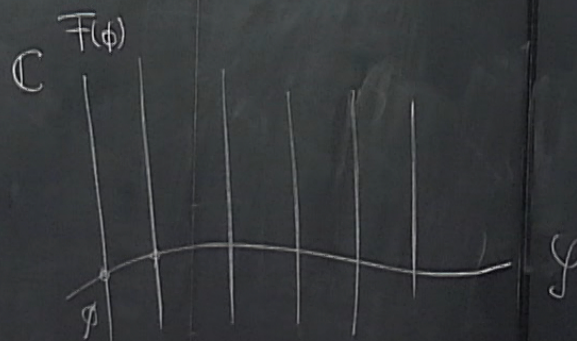
$$FG + \mathcal{O}(\hbar)$$

$$G \star F = i\hbar \{F, G\} + \mathcal{O}(\hbar^2) \quad \left. \vphantom{G \star F} \right\} + \text{associativity}$$

(Kontsevich '02)

Fedosov's idea

$\mathcal{L} = \text{class sol}^n$ s



Free-scalar field $V(\phi) = \frac{1}{2} m^2 \phi^2$ $\nearrow \sigma_{ij} + g_{ij}$

Need: Wightman fct $W(x,y) = \frac{i}{2} \Delta(x,y) + \frac{1}{2} G_w(x,y) = \langle \phi(x) \phi(y) \rangle_w$

$$\overline{F} \star G = \overline{F} \exp \left(\hbar \int \overrightarrow{\frac{\delta}{\delta \phi(x)}} W(x,y) \overrightarrow{\frac{\delta}{\delta \phi(y)}} \right) G = \overline{F} G + \hbar \int \frac{\delta \overline{F}}{\delta \phi} W \frac{\delta G}{\delta \phi} + \dots$$

Deformation quantization:

Fancy maths: $\star$ -product:

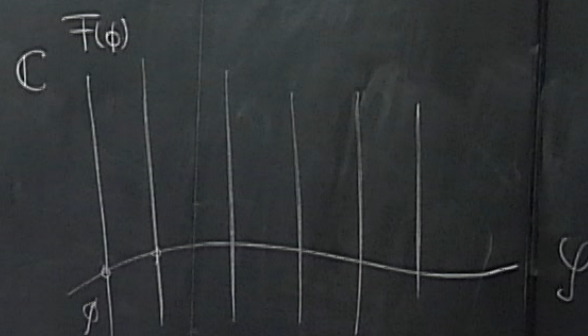
$$F \star G = FG + \mathcal{O}(\hbar)$$

$$F \star G - G \star F = i\hbar \{F, G\} + \mathcal{O}(\hbar^2) \quad \left. \vphantom{\begin{matrix} F \star G \\ F \star G - G \star F \end{matrix}} \right\} + \text{associativity}$$

Fedosov '94 (Kontsevich '02)

Fedosov's idea

$\mathcal{L} = \text{class sol}^n$ s



Free-scalar field $V(\phi) = \frac{1}{2} m^2 \phi^2$ $\nearrow \sigma_{ij} + g_{ij}$

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$\overline{F} \star G = \overline{F} \exp \left(\hbar \int \overleftrightarrow{\frac{\delta}{\delta \phi(x)}} W(x,y) \overleftrightarrow{\frac{\delta}{\delta \phi(y)}} \right) G = \overline{F} G + \hbar \int \frac{\delta \overline{F}}{\delta \phi} W \frac{\delta G}{\delta \phi} + \dots$

$$\mathcal{O}\left(\frac{1}{h}\right) \left\{ \begin{array}{l} + \text{associativity} \end{array} \right.$$

Idea

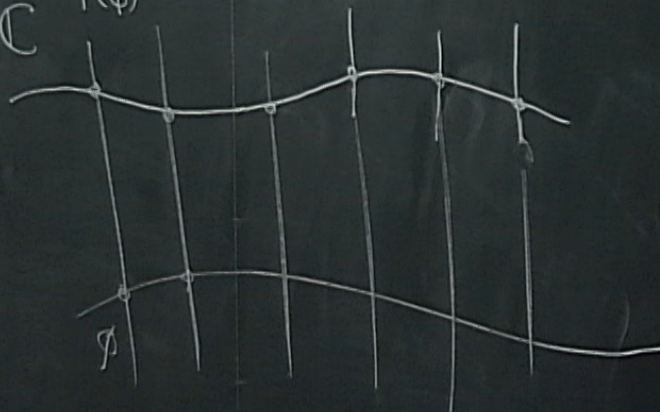
$$D F = 0$$

W_ϕ

$\uparrow$

$\mathbb{C}$

$\wedge \frac{1}{F(\phi)}$



Fedosov's idea

$$\mathcal{I} = \text{class. sol}^n_s$$

$$D_{\text{Fedosov}} = D + A$$

almost Kähler

$$\mathcal{I} \left\{ \begin{array}{l} (\Delta; \nabla) \end{array} \right.$$

Poisson brackets without space-time split

Peierls '52

$$S(\phi) = \int_M \left(\frac{1}{2} (\partial\phi)^2 + V(\phi) \right) \sqrt{-g}$$

$F(\phi)$ - observables

ϕ - solution

$\phi + u + \dots$

$$+ \int_M R \sqrt{-g} + \int_M K \sqrt{h} \{F, G\}(\phi)$$

Peierls bracket

$$\nabla^2 u - V''(\phi)u = 0 \rightarrow$$

$\Delta^{\text{adv/ret}}$

$(\phi; x, y)$

∂M

$$= \int \frac{\delta F(\phi)}{\delta \phi(x)} \Delta(\phi; x, y) \frac{\delta G(\phi)}{\delta \phi(y)}$$

$$\Delta(\phi; x, y) \frac{\delta G(\phi)}{\delta \phi(y)}$$

$\delta^2 \Gamma_{\text{emb.}} \phi$

$\Delta^{\text{adv}} - \Delta^{\text{ret}} = \Delta$

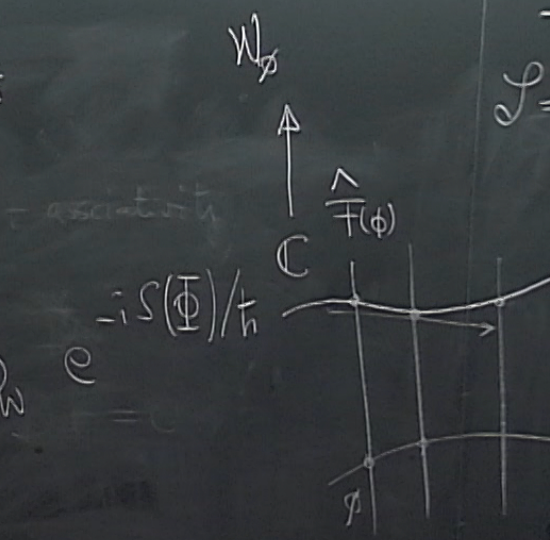
= causal prop.

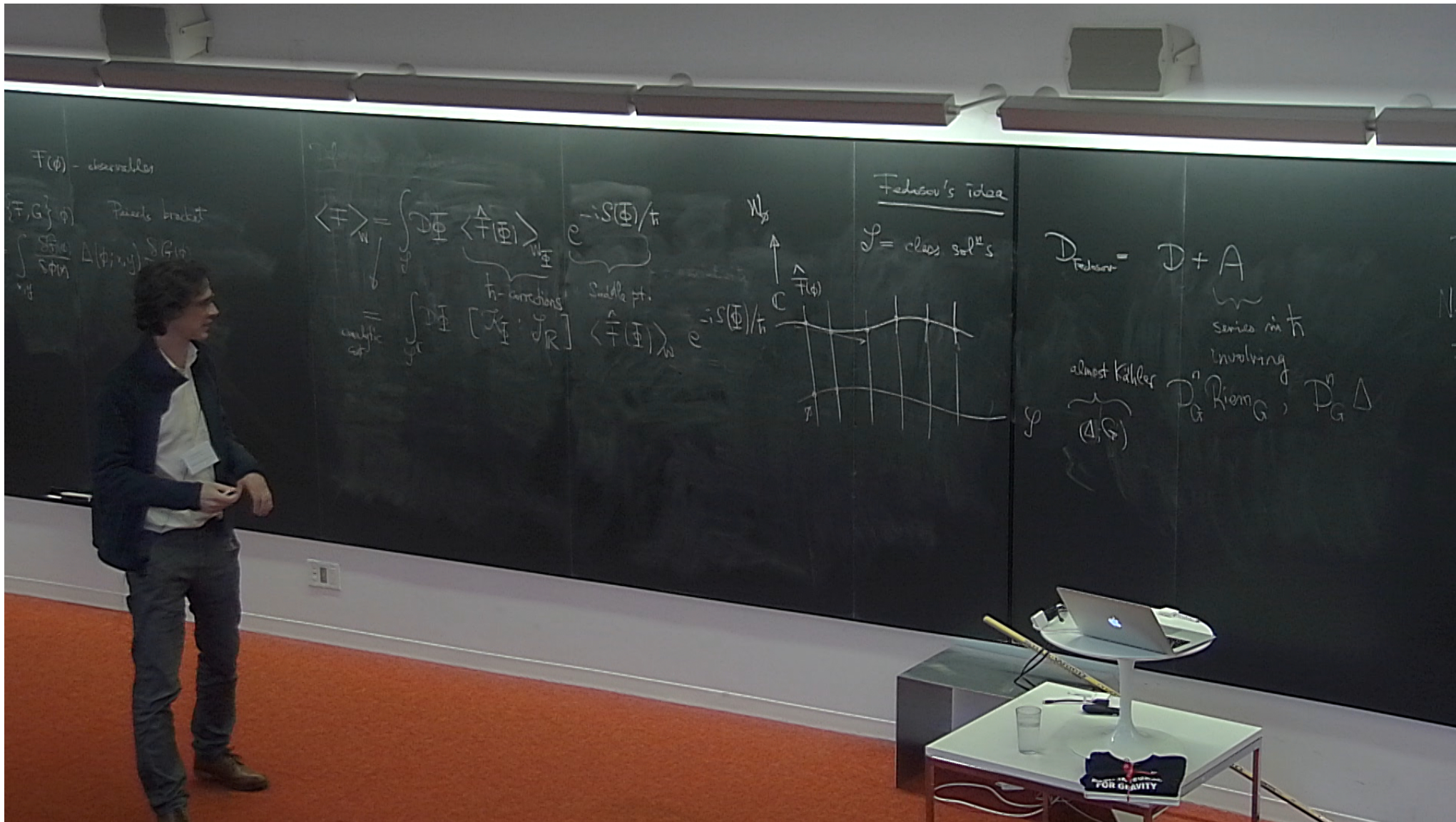
x, y

Gravity OK.

Deformation Quantization

$$\begin{aligned}
 \langle \hat{T} \rangle_W &= \int_{\mathcal{P}} \mathcal{D}\Phi \underbrace{\langle \hat{T}(\Phi) \rangle_{W_\Phi}}_{\substack{\hbar\text{-corrections} \\ \text{Saddle pt.}}} e^{-iS(\Phi)/\hbar} \\
 &\stackrel{\text{analytic cont.}}{=} \int_{\mathcal{P}_R} \mathcal{D}\Phi [\mathcal{K}_I : \mathcal{J}_R] \langle \hat{T}(\Phi) \rangle_W e^{-iS(\Phi)/\hbar}
 \end{aligned}$$





Poisson brackets without space-time split

Peierls '52

$$S(\phi) = \int_M \left(\frac{1}{2} (\partial\phi)^2 + V(\phi) \right) \sqrt{-g}$$

$F(\phi)$ - observables

ϕ - solution $\phi + u + \dots$

$$+ \int_M R \sqrt{-g} + \int_{\partial M} K \sqrt{h} \{F, G\}(\phi)$$

Peierls bracket

$$\nabla^2 u - V''(\phi)u = 0 \rightarrow$$

$$\Delta^{\text{adv/ret}}(\phi; x, y)$$

∂M

$$= \int_{x, y} \frac{\delta F(\phi)}{\delta \phi(x)} \Delta(\phi; x, y) \frac{\delta G(\phi)}{\delta \phi(y)}$$

∇^2 d'Alemb. op.

$$\Delta^{\text{adv}} - \Delta^{\text{ret}} = \Delta = \text{causal prop.}$$

Gravity OK.

$$\langle \hat{T}^{\dagger} \hat{T} \rangle \geq 0$$

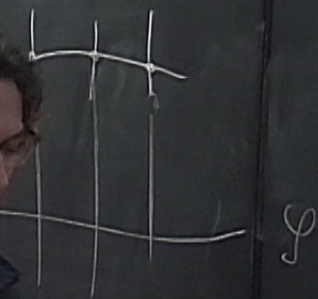
$$\langle \hat{T} \rangle_W = \int_{\mathcal{L}} \mathcal{D}\Phi \underbrace{\langle \hat{T}(\Phi) \rangle_{W_{\Phi}}}_{\hbar\text{-corrections}} e^{-iS(\Phi)/\hbar}$$

$$\stackrel{\text{analytic cont.}}{=} \int_{\mathcal{L}^{\text{st}}} \mathcal{D}\Phi \underbrace{[\mathcal{K}_{\Phi} : \mathcal{J}_{\mathbb{R}}]}_{\text{Saddle pt.}} \langle \hat{T}(\Phi) \rangle_W e^{-iS(\Phi)/\hbar}$$

Fedosov's idea

$\mathcal{L} = \text{class. sol}^n$ s

$$W_{\Phi} \uparrow \hat{T}(\Phi) \downarrow e^{-iS(\Phi)/\hbar}$$



Fedosov's idea

$$\mathcal{L} = \text{class sol}^n\text{'s}$$

$$D_{\text{Fedosov}} = D + A$$

series in $\hbar$
involving

almost Kähler $D_G^n \text{Riem}_G, D_G^n \Delta$

(Δ, G)

Free - scalar -

Need: Wightman

$$F \star G =$$

W_ϕ

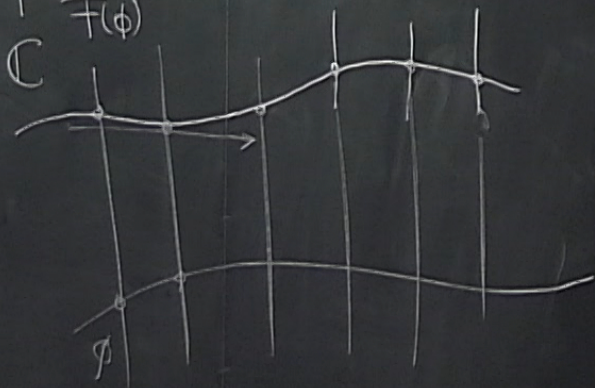
$\uparrow$

$\hat{F}(\phi)$

$\mathbb{C}$

$$-iS(\Phi)/\hbar$$

e



$\mathcal{Y}$

