Title: On singularities of Super Conformal Field Theories

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Abstract: <p>Carefully studying the singularity structure of the moduli space of vacua of super conformal field theories, provides an incredibly powerful tool to extract information about such theories. In particular I will outline a concrete, yet technically challenging, strategy to go about carrying out a classification for SCFTs with extended supersymmetry in 4d.

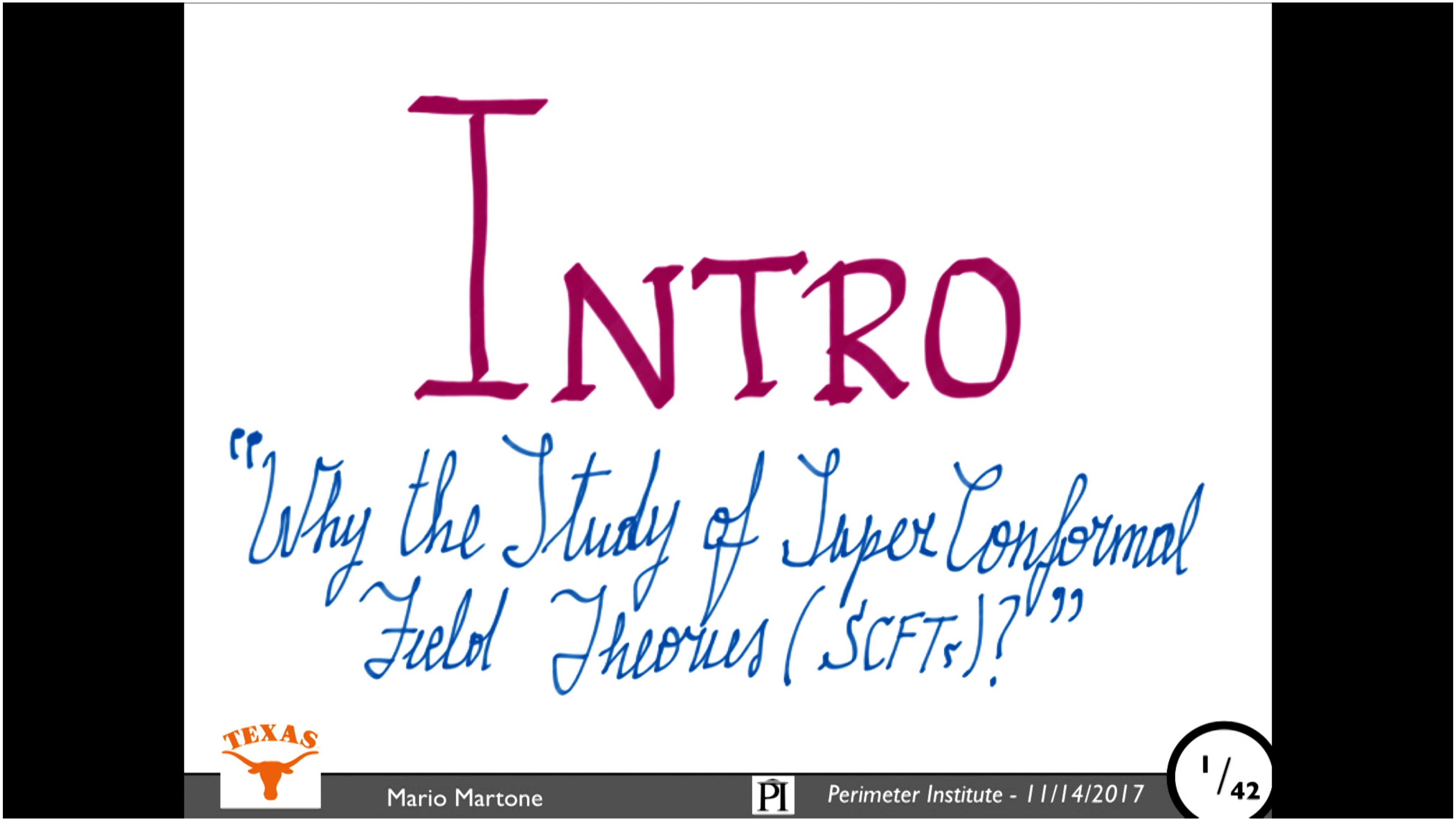
I will explain in details how this classification has been worked out successfully for N=2 SCFTs in rank-1 and it is very promising in rank-2. And how with even higher Supersymmetry (Nâ‰¥3) we can obtain a classification valid for all ranks making a few reasonable, yet key, assumptions.</p>

ON SINGULARITIES of Super Conformal Field Theories

Perimeter Institute 11/14/2017

PERIMETER INSTITUTE
FOR THEORETICAL PHYSICS

in collaboration with **P. Argyres, F. Bonetti, C. Meneghelli, C. Long** and **L. Rastelli.**



INTRO
"Why the Study of Super Conformal Field Theories (SCFTs)?"
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1. CFT's are "Fundamental Bricks" of Effective Field Theory.
2. We have MORE TOOLS at our disposal.
3. We have already learned a LOT about QFT.

Goal of the Talk

UNDERSTAND the SINGULARITY structure of the CB/MODULI SPACE of 4d SCFTs with rank HIGHER than 1 & 8 or MORE supercharges the study of SINGULARITIES can tell us a lot about the SPACE of EXISTING 4d SCFTs setting a TARGET for EXPLICIT constructions

PART I "Setting Things Up"

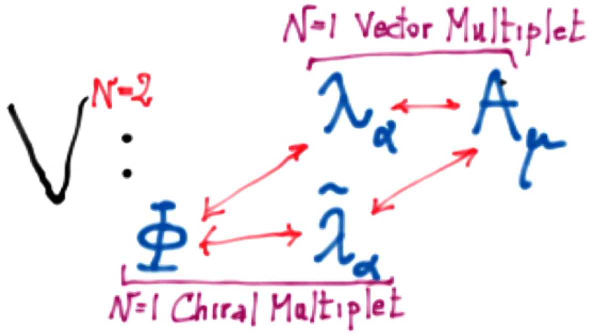
PART II "Recap On Rank-1"

PART III "Exploring Rank-2"

PART IV "An Example In Details"

PART V "$\mathcal{N}=3$ & $\mathcal{N}=4$ Susy"

VECTOR MULTIPLET



Φ is a scalar which can acquire a VEV:

$$\boxed{\Phi = \begin{pmatrix} a_1 & & & \\ & a_2 & & \\ & & \ddots & \\ & & & a_N \end{pmatrix}, \quad \sum_i^N a_i = 0}$$

$\Phi \in \mathbb{C}^{N-1}$ & on the CB $SU(N) \to U(1)^{N-1}$

In general $(a_1, \dots, a_N)$ are NOT "gauge" invariant & we need to impose further identifications (e.g. Weyl Groups)

- $\dim_{\mathbb{C}} \mathcal{CB} :=$ Rank of the Theory
- $(u_1, \ldots, u_r) :=$ Coulomb Branch coordinates.
- The geometry is Singular @ $\vec{u}$'s where massless states appear.

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$\mathbb{C}^*$ - Action

i] Φ has non-zero $U(1)_R$ charge.

ii] In the scale-invariant case $\exists$ $\mathbb{R}^+$ Action.

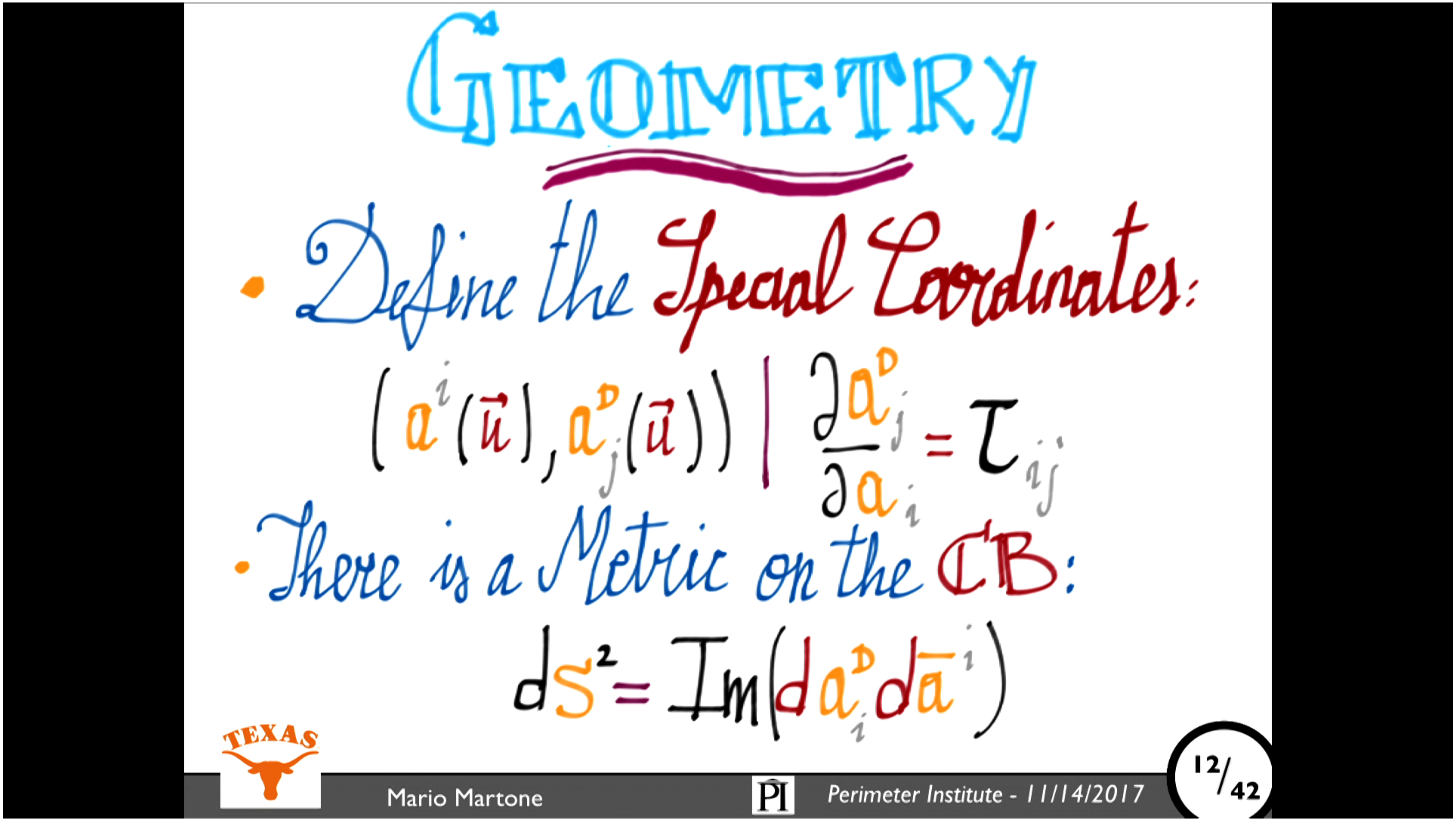
i & ii Combine into a $\mathbb{C}^*$-Action

Classification Steps

1. TOPOLOGY
2. GEOMETRY
3. DEFORMATION

TOPOLOGY

The SINGULAR Locus $\mathcal{V}$ HAS to be A Complex CO-DIMENSION 1 $\mathbb{C}^*$ INVARIANT Set.


GEOMETRY
• Define the Special Coordinates:
$(a^i(\vec{u}), a^D_j(\vec{u})) \mid \frac{\partial a^D_j}{\partial a_i} = \tau_{ij}$
• There is a Metric on the CB:
$ds^2 = \mathrm{Im}(da^D_i d\bar{a}^i)$
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- Imposing SPECIAL KÄHLER conditions: $\boxed{\mathrm{Im}\,\tau_{ij} > 0,\ \tau_{ij} = \tau_{ji}}$

- Looping around the singularity:

$$\begin{pmatrix} \vec{a} \\ \vec{a}^D \end{pmatrix} \xrightarrow[\gamma]{} M \begin{pmatrix} \vec{a} \\ \vec{a}^D \end{pmatrix},\ M \in Sp(2r, \mathbb{Z})$$

DEFORMATION

HARDEST of All.

Listing the set of PHYSICALLY Allowed MASS Deformations

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PART II

"Recap On Rank-1"

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[hep-th/1609.04404] [hep-th/1611.08602] [hep-th/1704.05110]

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Rank-1 Recap

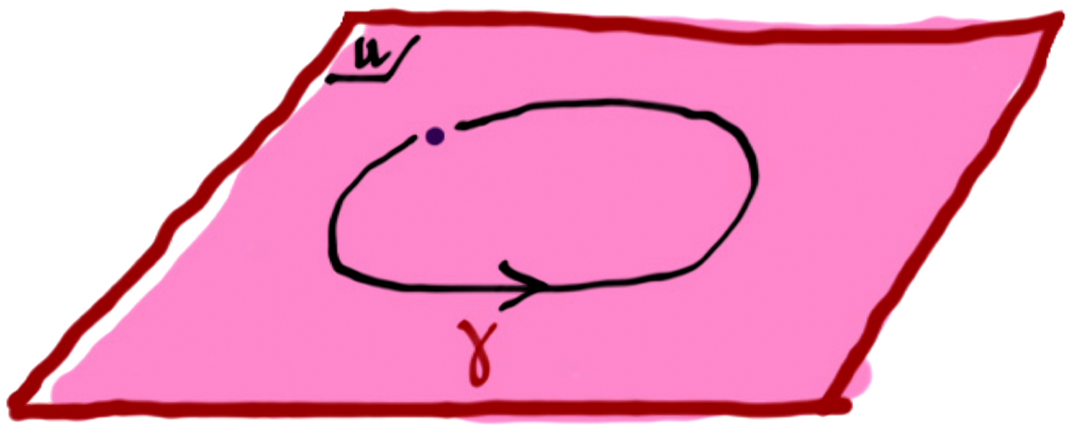
for SU(2) $\Phi = \begin{pmatrix} a & \\ & -a \end{pmatrix}$

- a is not gauge invariant X
- $u = \frac{1}{2}\langle \mathrm{tr}\, \Phi^2 \rangle = a^2$ is ✓

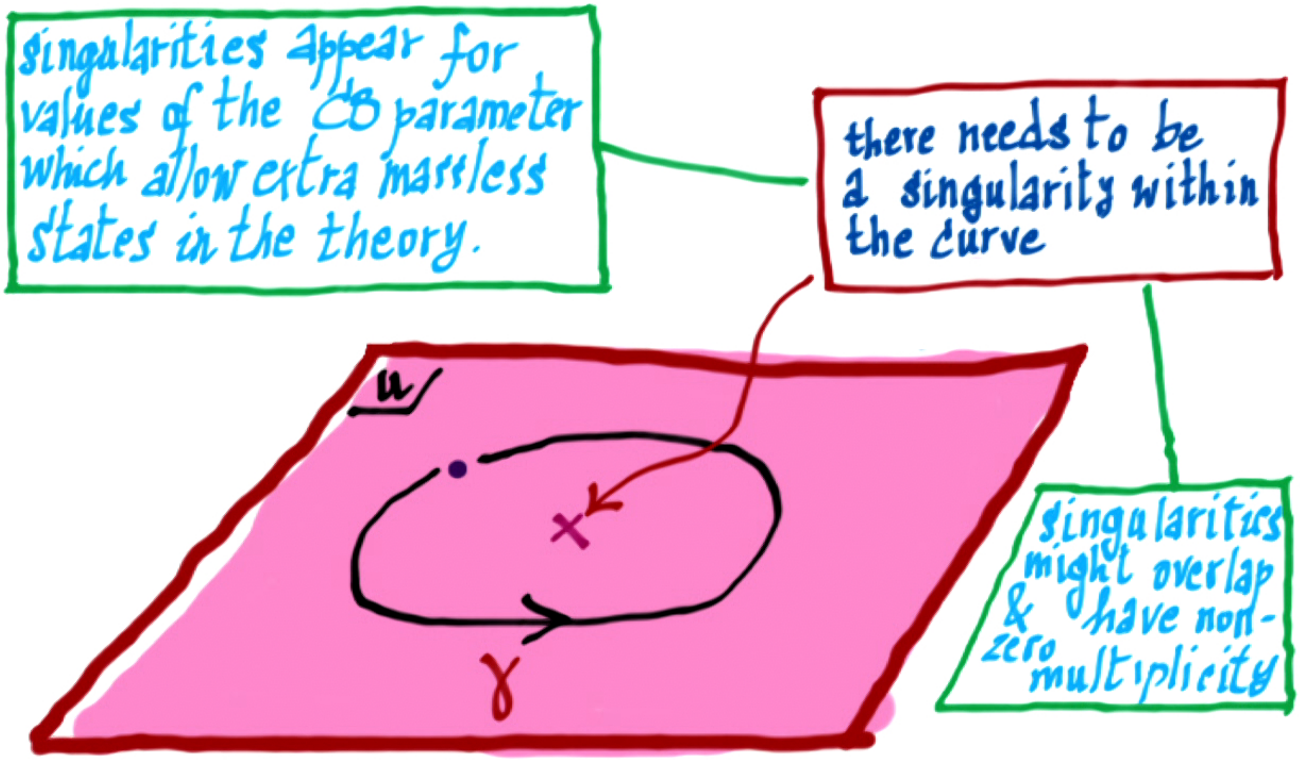
The CB is 1 Complex Dimensional

Rank-1 Recap

$$\begin{pmatrix} a \\ a^D \end{pmatrix} \xrightarrow[\gamma]{SL(2,\mathbb{Z})} M \begin{pmatrix} a \\ a^D \end{pmatrix} \quad \Bigg| \quad M = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2,\mathbb{Z})$$

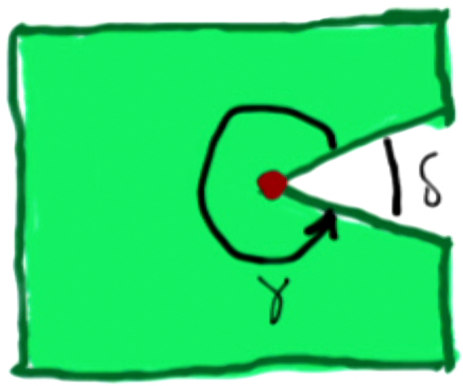


Rank-1 Recap



singularities appear for values of the CB parameter which allow extra massless states in the theory.
there needs to be a singularity within the curve
u
γ
singularities might overlap & have non-zero multiplicity

Requiring $\mathbb{C}^*$-Invariance: $\boxed{\mathcal{V} \equiv \{0\}}$



$\mathbb{C}$ w/ deficit angle δ

- Special Kähler conditions are trivially satisfied.
- E-M Duality only Allows 7 Values for the opening angle:

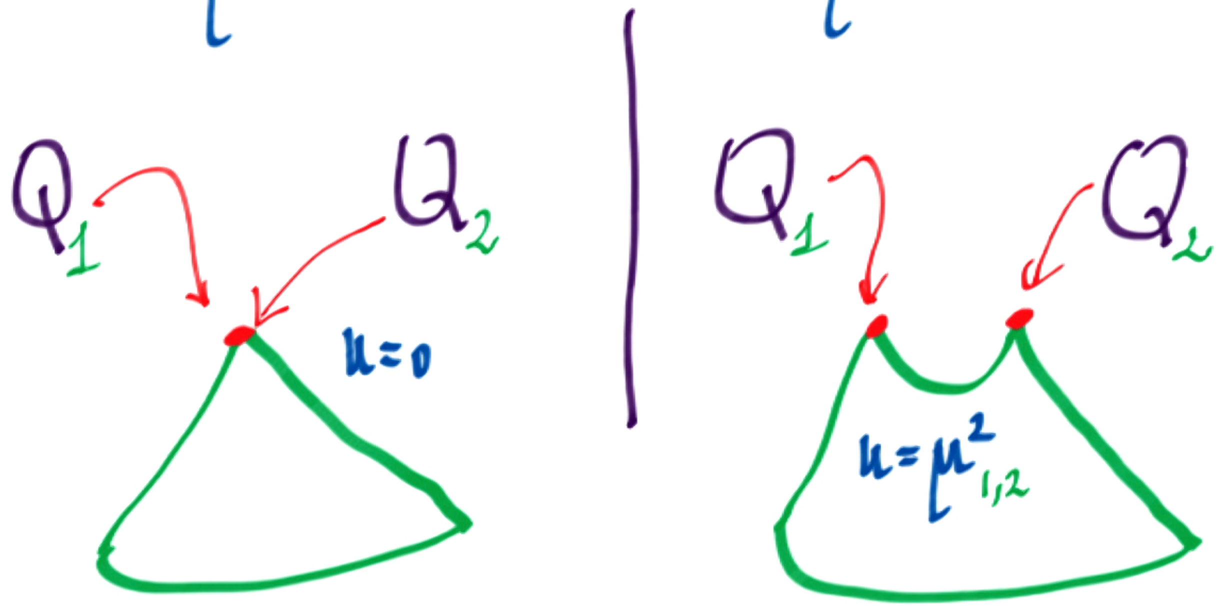
δ	$\pi/3$	$\pi/2$	$2\pi/3$	π	$4\pi/3$	$3\pi/2$	$5\pi/3$	2π (cusp)
$\Delta(a)$	6	4	3	2	$3/2$	$4/3$	$6/5$	$\mathbb{N}$

They correspond to the KODAIRA class.

Name	curve ($y^2=\ldots$)	$\Delta(u)$	M_0	#
II^*	x^3+u^5	6	ST	10
III^*	x^3+u^3x	4	S	9
IV^*	x^3+u^4	3	$-(ST)^{-1}$	8
I_0^*	$x^3+u^2x+gu^2$	2	$-\mathbb{1}$	6
IV	x^3+u^2	$3/2$	$-ST$	4
III	x^3+ux	$4/3$	S^{-1}	3
II	x^3+u	$6/5$	$(ST)^{-1}$	2
$I_{n>0}^*$	$x^3+ux^2+\Lambda^{-2n}u^{n+3}$	2	$-T^{-n}$	$n+6$
$I_{n>0}$	$(x-1)(x^2+\Lambda^{-n}u^n)$	1	T^{-n}	n

MASS DEFORMATIONS

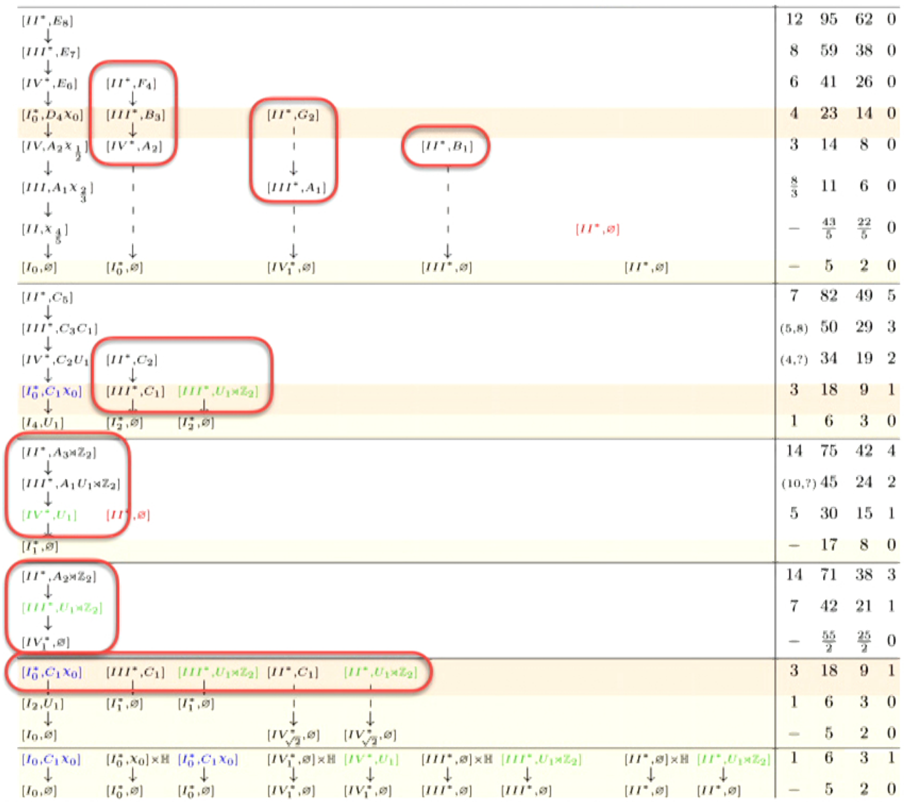
$\mu = 0$ | $\mu \neq 0$



Classification of Rank-1 $\mathcal{N}$=2 SCFTs I

– Through a systematic analysis of mass def^{ns} of scale inv. 1 dim. special Kähler geometries: –

We discover over 10 new $\mathcal{N}$=2 SCFTs



IMPORTANT

- Most theories are Non-Lagrangian
- Many don't have an S-class description!

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Rank 2: $\dim \mathcal{CB} = 2$, $\dim \mathcal{V} = 1$

$$(u,v) \rightarrow (\lambda^{\Delta_u} u, \lambda^{\Delta_v} v)$$

Thus $\mathcal{V}$ can be:

- $u = 0$
- $v = 0$
- $u^p / v^q = \omega = \omega_0$

$$\boxed{\begin{array}{c} \Delta_u = rp \quad \Delta_v = rq \\ \hline \gcd(p,q) = 1,\ r \in \mathbb{Q} \end{array}}$$

To help visualizing:

$$X_\rho := V \cap S_\rho\,, \quad S_\rho := \{|u|^p + |v|^p = 2\epsilon^\rho\}$$

e.g.: • $\{u=0\} \cap S_0 := K_0(u,v)$

$$K_0(u,v) := \{(u,v) \in \mathbb{C}^2 \,|\, u=0,\ v=e^{i\phi}\}$$

Define:

- $\{u^p/v^q = 1\} \cap S_0 := K_{(p,q)}(u,v)$

On $K_{(p,q)}(u,v) \longrightarrow |u| = |v| = 1$

Thus:

$$K_{(p,q)} := \left\{ u = e^{i\theta}, v = e^{i\varphi} \,\middle|\, p\theta = q\varphi \right\}$$

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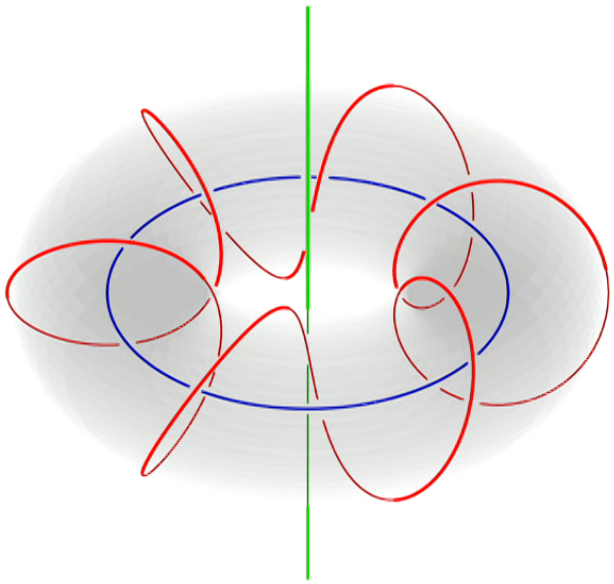 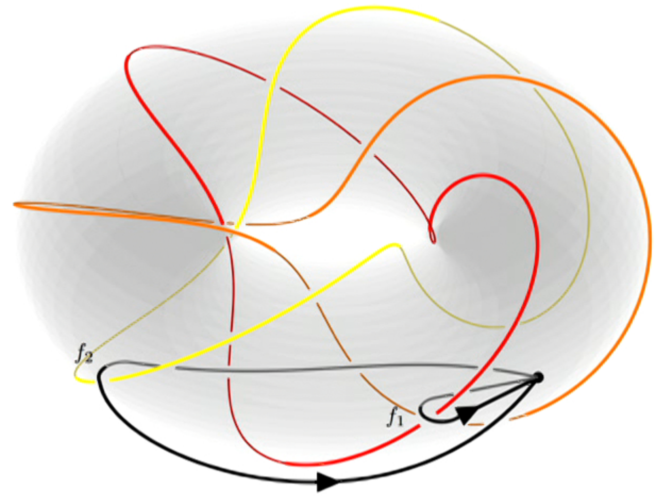

$K_{(p,q)}$ is a Torus (p,q) link.
with Unknots.

Special Kähler conditions & the way $Sp(4,\mathbb{Z})$ monodromies tie with scale invariance, strongly constrain the Analytic behaviour of $(\vec{a}, \vec{a}_D)$

Assuming no Unknots

(u,v) can only have one of the following Scaling Dimensions

$$\left\{1, \frac{12}{11}, \frac{10}{9}, \frac{8}{7}, \frac{6}{5}, \frac{5}{4}, \frac{4}{3}, \frac{10}{7}, \frac{3}{2}, \frac{8}{5}, \frac{5}{3}, \frac{12}{7}, 2, \frac{12}{5}, \frac{5}{2}, \frac{8}{3}, 3, \frac{10}{3}, 4, 5, 6, 8, 10, 12\right\}$$

SU(3)

$$\Phi = \begin{pmatrix} a & & \\ & b & \\ & & c \end{pmatrix}, \quad a+b+c=0$$

Invariant Coordinates

$$u = ab+bc+ac$$

$$v = abc$$

Where are the Singularities?

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SU(3) w/ 1 adj. [$\mathcal{N}=4$]

Unbroken SU(2): $\Phi = \begin{pmatrix} a & & \\ & \alpha & \\ & & -a \end{pmatrix}$

$$\boxed{u^3 = v^2}$$

In the $\mathcal{N}=4$ case the singularity is a Single (3,2) knot.

SU(3) w/ 6 Flavors.

i] Unbroken SU(2) @ $u^3 = v^2$

ii] Massless Quarks @ $v = 0$

What about Non Perturbative Effects?

i] In the Deep-IR Quarks decouple.

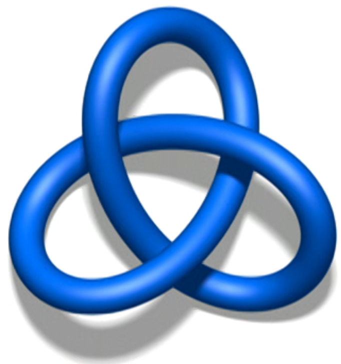
ii] The SU(2) has a Monopole-Dyon singularity!

iii] The Splitting Depends on the scale a!

$$\boxed{u^p = v^q \longrightarrow u^p = \omega_{1,2}\, v^q}$$

IMPORTANT

i] $\mathcal{N}=4$ $\mathcal{V}$ IS An Orbifold

 $\simeq \mathbb{C}^2/S_3$

ii] $\mathcal{N}=2$ $\mathcal{V}$ IS NOT!

PART V

"$\mathcal{N}=3$ & $\mathcal{N}=4$ Susy"

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For 9 or more supercharges the Moduli space $\mathcal{M}$ is FLAT.

Can we prove that

$$\mathcal{M} \simeq \mathbb{C}^{3r}/\Gamma$$

Let's Assume it for now...

On a $\mathcal{N}=2$ Coulomb Branch Slice

- Freely generated CB Chiral Ring:

  Γ COMPLEX REFLECTION

- Flatness $\Rightarrow \begin{pmatrix} \vec{a} \\ \vec{a}_D \end{pmatrix} \sim \begin{pmatrix} \vec{z} \\ \vec{z}' \end{pmatrix}, (z,z') \in \mathbb{C}^{2r}$

- Monodromies in $Sp(4,\mathbb{Z})$:

  Γ CRYSTALLOGRAPIC

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CRYSTALLOGRAPIC COMPLEX REFLECTION GROUPS

Have been Fully classified so we have a full list of $\mathcal{N}=3$ Geometries!

- We find many NEW Geometries already at Rank-2.

Still working out the physical properties.

STAY TUNED.

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CONCLUSIONS

1. The classification of $\mathcal{N}=2$ CB seems to be extensible to Higher Ranks.
2. For $\mathcal{N}=3$ & $\mathcal{N}=4$ there seems to be a Full answer for All Ranks.
3. Getting away from the scale-invariant limit is the Hard part.

THANK YOU!