Title: Schwarzschild's Interior Solution, Gravastars and Echoes

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Abstract:

Gravitational Condensate Stars

Negative Pressure & Surface Tension of the Interior Schwarzschild Solution
Echoes, GWs and Imaging

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➔ Class. Quant. Grav. 32, 215024 (2015)

Proc. Natl. Acad. Sci., 101, 9545 (2004)

arXiv:gr-qc/0109035

Acta Phys. Pol. B 41, 2031 (2010)

Irreducible Mass and 'No Hair' Theorem

- All Gen. Rel. Black Holes specified by their mass, angular momentum and electric charge: M, J, Q
- Rotating Kerr Black Holes have all higher multipoles ***determined completely*** by M, J (no "hair")
- Irreducible Mass M_{irr} ***increases monotonically classically*** (Christodoulou, 1972)

$$M^2 = (M_{irr} + Q/4M_{irr})^2 + J^2/4M_{irr}^2$$

$$M_{irr}^2 = (\text{Area})/16\pi G \qquad \Delta M_{irr}^2 \geq 0$$

- **Smarr Formula:** ***Purely Classical***

$$dM = \frac{\kappa}{8\pi G}\, dA + \Omega\, dJ + \Phi\, dQ$$

Surface Gravity $\kappa = \frac{1}{4GM}\left\{1 - \frac{16\pi^2 G^2}{A^2}\left(Q^4 + 4J^2\right)\right\}$

Black 'Holes'… or Not
Singularity Theorems

Black Holes 'inevitable' in Gen. Rel. if

- A Trapped Surface forms
- One of Energy Conditions:

- **Weak** Energy Condition (Penrose 1965)

$$\rho + p_i \geq 0 \qquad i = 1, 2, 3$$

Violated by Quantum Fields, e.g. by Casimir Effect

- **Strong** Energy Condition (Hawking-Penrose 1970)

$$\rho + \sum_{i=1}^{3} p_i \geq 0$$

Violated by Hadronic 'Bag', Cosmological Dark Energy,

Inflation V(φ): $p_i = -\rho < 0$

Negative Pressure → Defocusing → Effective **Repulsion**

Gravitational Vacuum Condensates

- Gravity is a theory of spin-2 bosons
- Its interactions are attractive
- The interactions become strong near $r = R_s$
- Energy of any scalar order parameter must couple to gravity with the vacuum eq. of state,
  $$p_V = -\rho_V = -V(\phi)$$
- Relativistic Entropy Density s is (for $\mu = 0$),
  $$Ts = p + \rho = 0 \text{ if } p = -\rho$$
- Zero entropy density for a single macroscopic quantum state, $k_B \ln \Omega = 0$ for $\Omega = 1$
- This eq. of state violates the energy condition, $\rho + 3p \geq 0$ (if $\rho_V > 0$) needed to prove the classical singularity theorems
- Dark Energy acts as a repulsive core

A GBEC phase transition can stabilize
a high density, compact cold stellar
remnant to further gravitational collapse

Gravitational Vacuum Condensate Star Proposed (2001)

Today: It's Realized in Schwarzschild Soln.II (1916)

Static, Spherical Symmetry

- 2 Metric Fns.

$$f(r),\ h(r) \equiv 1 - \frac{2Gm(r)}{r}$$ ← Misner-Sharp Mass

- 3 Stress Tensor Fns.

$$T^{\mu}{}_{\nu} = \begin{pmatrix} -\rho & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p_\perp & 0 \\ 0 & 0 & 0 & p_\perp \end{pmatrix}$$

- 2 Einstein Eqs.

$$\frac{dm}{dr} = 4\pi\, r^2 \rho \qquad \frac{h}{2f}\frac{df}{dr} = \frac{Gm}{r^2} + 4\pi G p r$$

- 1 Conservation Eq.

$$\nabla_\mu T^{\mu}{}_{r} = \frac{dp}{dr} + \frac{\rho + p}{2f}\frac{df}{dr} + \frac{2\,(p - p_\perp)}{r} = 0$$

Buchdahl Bound (1959)

Assuming classical Einstein eqs. &

- Static Killing time:
- Spherical Symmetry:

$$K^\mu \frac{\partial}{\partial x^\mu} = \frac{\partial}{\partial t}$$

$$ds^2 = -f(r)\, dt^2 + \frac{dr^2}{h(r)} + r^2\, d\Omega^2$$

- Isotropic Pressure: $p_i = p(r)$
- Positive Monotonically Decreasing Density: $\frac{d\rho}{dr} \leq 0$
- Metric Continuity at Surface of Star r=R
- Then $R > \frac{9}{8} R_s = \frac{9}{4} GM$

or the pressure must diverge in the Interior

Note this ***R*** is outside horizon

Schwarzschild Interior (1916)

- Importance of Buchdahl Bound is:

  Under **Adiabatic Compression Something Happens Inside**-- Before the Event Horizon is Reached

- Holds for **Any** isotropic equation of state $p_\perp = p$
- Bound is Saturated by Schwarzschild Interior Soln.
- Constant Density

$$\frac{d\rho}{dr} = 0 \qquad\qquad \rho(r) = \bar{\rho} \equiv \frac{3M}{4\pi R^3}$$

- Solve for Pressure *p(r)* , Metric Functions $f(r), h(r)$

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Schwarzschild Interior Solution

- Constant Density

  $$\rho' = 0$$

  Saturates Buchdahl Bound

$$m(r) = \frac{4\pi}{3}\bar{\rho}r^3 = \frac{M}{R^3}r^3$$

$$h(r) = 1 - H^2r^2$$

$$H^2 = \frac{8\pi G}{3}\bar{\rho} = \frac{2GM}{R^3}$$

- Pressure

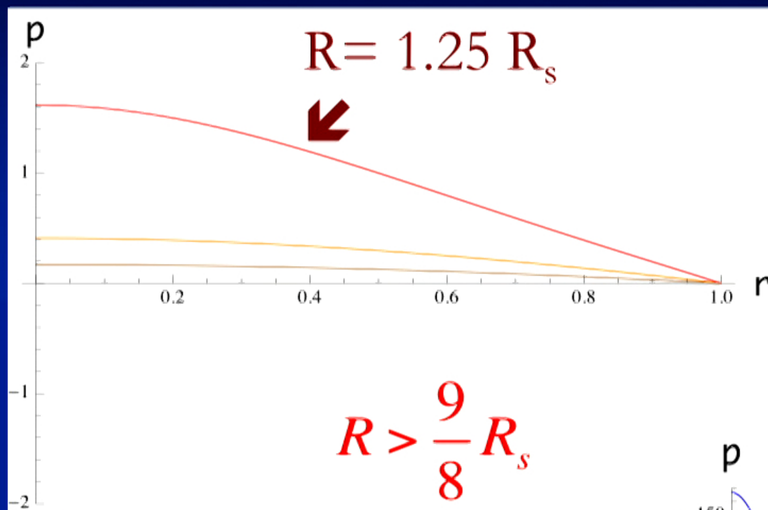
$$p(r) = \bar{\rho}\left[\frac{\sqrt{1-H^2r^2} - \sqrt{1-H^2R^2}}{3\sqrt{1-H^2R^2} - \sqrt{1-H^2r^2}}\right]$$

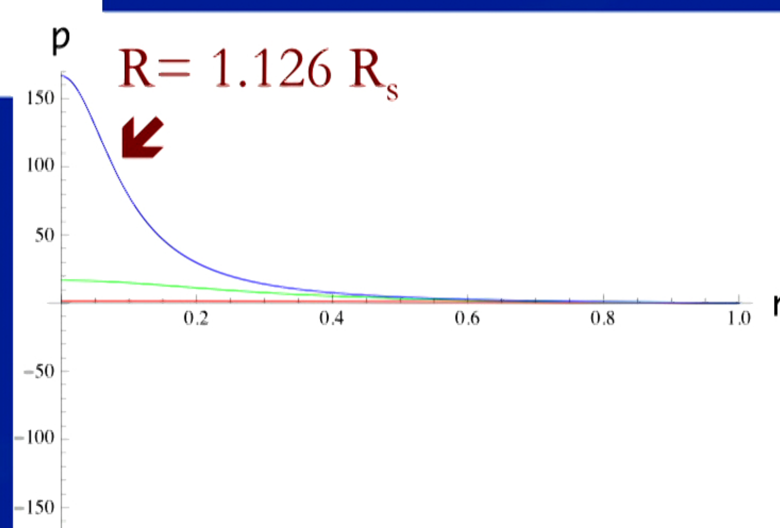
- Diverges at $R_0 = 3R\sqrt{1 - \frac{8}{9}\frac{R}{R_s}}$ iff $R < \frac{9}{8}R_s = \frac{9}{4}GM$

- $f(r) = \frac{1}{4}\left[3\sqrt{1-H^2R^2} - \sqrt{1-H^2r^2}\right]^2$ Vanishes at R_0

- Pressure becomes negative for $0 < r < R_0$

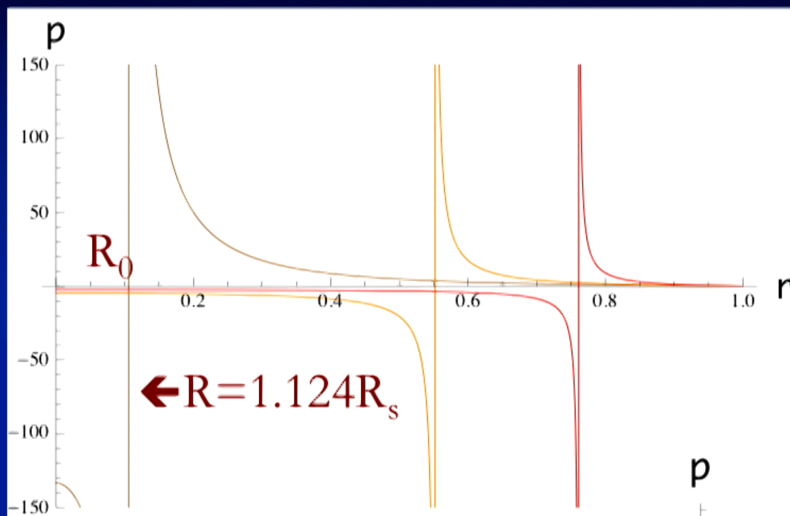
Interior Pressure





As $R \rightarrow 9/8\ R_s$ from above central pressure diverges

Interior Pressure

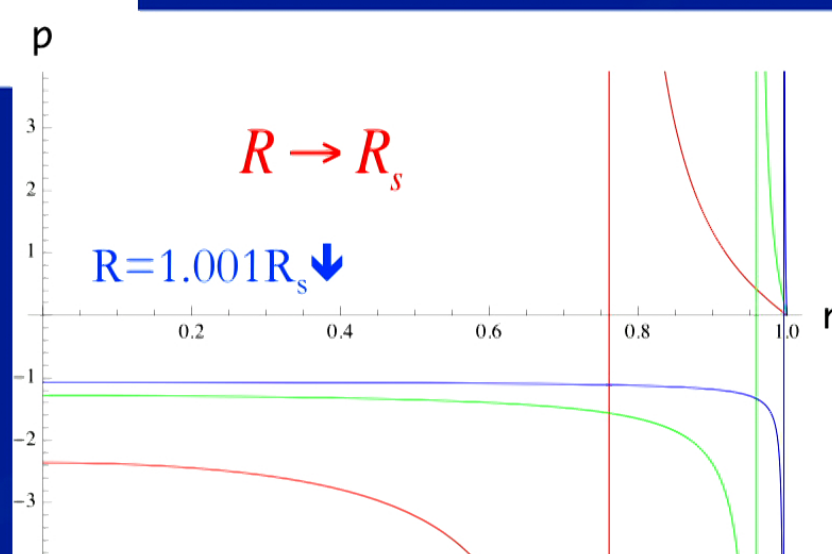


$$R < \frac{9}{8} R_s$$

Negative Pressure soln. opens up for $R < R_0$

As $R \to R_s$ from above $R_0 \to R_s$ from below and negative pressure region fills entire interior with

$p = -\rho$, $w = -1$



Schwarzschild Interior Solution

- Constant Density

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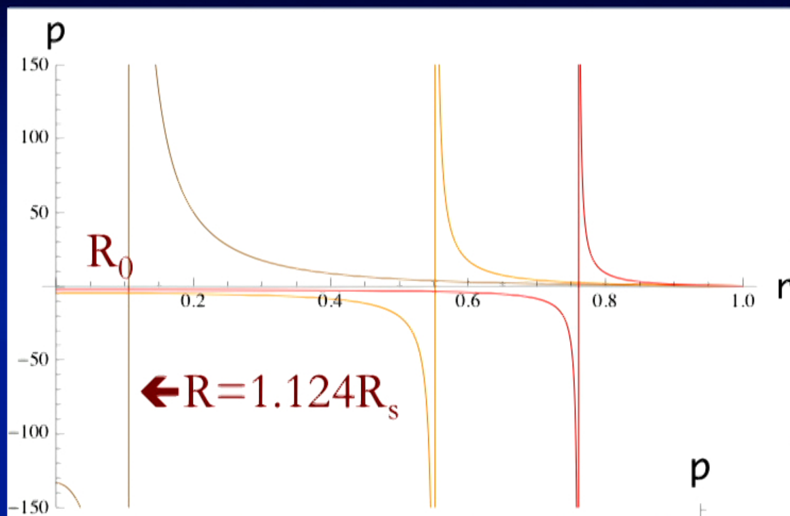

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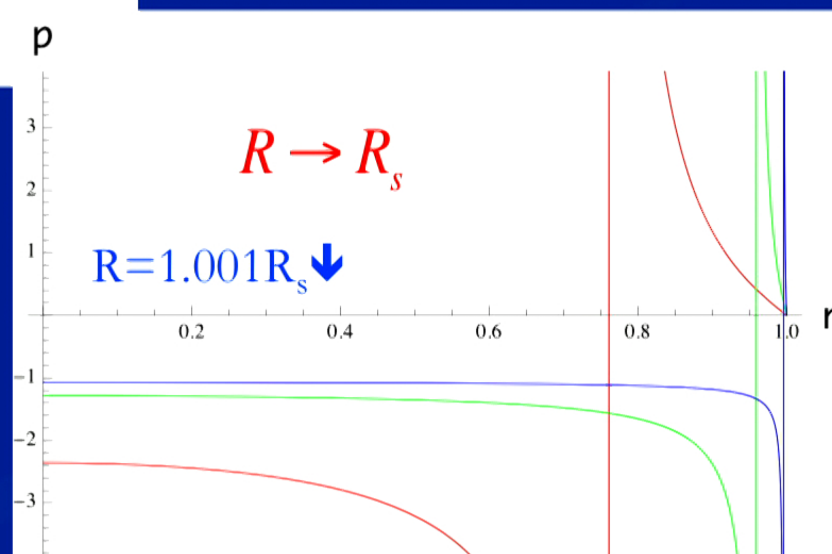
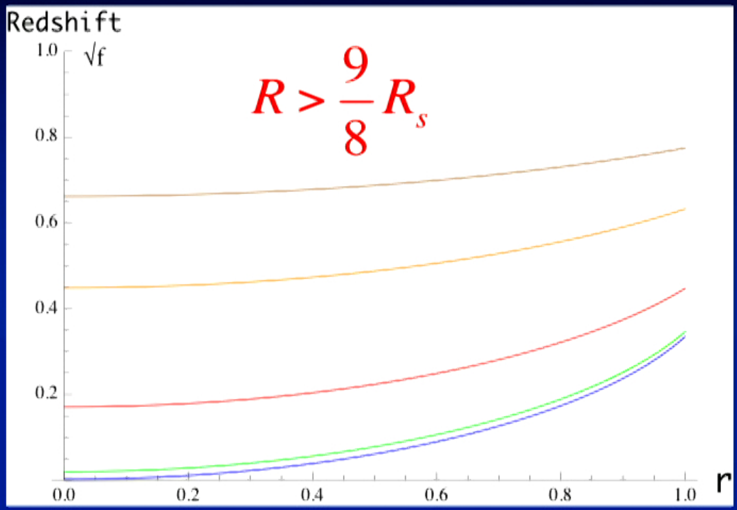
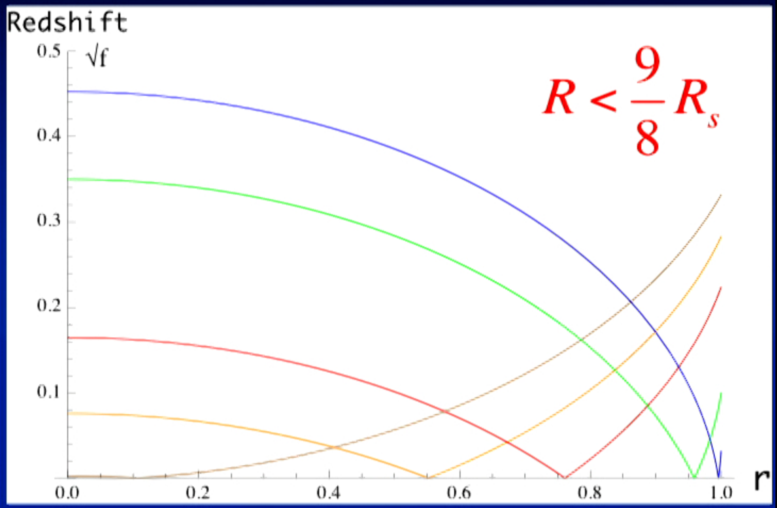


Interior Redshift





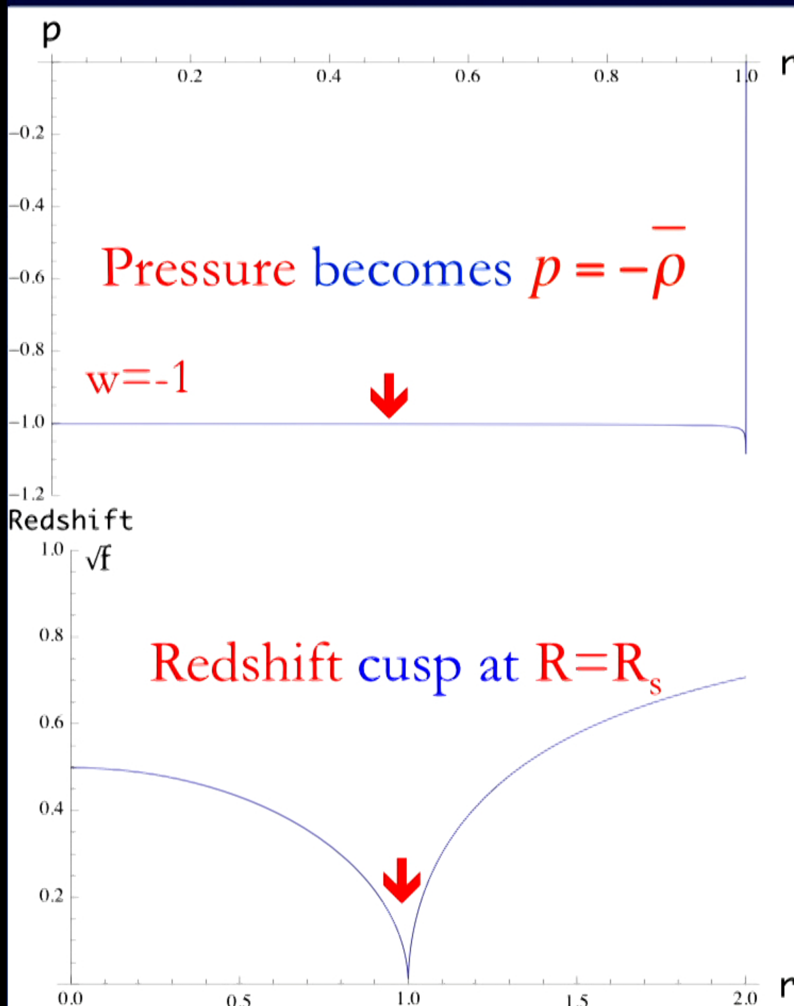
$f(r) = \frac{1}{4} D^2 \geq 0$ Non-negative (no trapped surface)

$D \equiv 3\sqrt{1 - R_s/R} - \sqrt{1 - R_s r^2/R^3}$ vanishes at **same** radius

$R_0 = 3R\sqrt{1 - \frac{8}{9}\frac{R}{R_s}}$ where p diverges $\longrightarrow$ **Integrable**

Redshift $\sqrt{f(r)} = \frac{1}{2}|D|$ has cusp-like behavior

$R=R_s$ Limit is Grav. Condensate Star (2001)



No divergence in p

$$p = -\bar{\rho}$$

$$h(r) = 1 - H^2 r^2$$

$$f(r) = \frac{1}{4}\, h(r)$$

$$H = 1/R_s$$

but non-analytic cusp
Discontinuity (classically)
Interior is de Sitter space in
(modified) static coordinates
(Time runs slower inside)

Komar Mass-Energy Flux (1959-62)

$$\frac{1}{G}\frac{d}{dr}\left(r^2\kappa\right) = 4\pi\sqrt{\frac{f}{h}}\; r^2\left(\rho + p + 2p_\perp\right)$$

$$\kappa(r) = \frac{1}{2}\sqrt{\frac{h}{f}}\;\frac{df}{dr} \to \frac{GM}{r^2}$$ Surface Gravity

Total Mass: 'Gauss' Law' for Static Gravity

$$M = 4\pi\int_0^{R_s} dr\,\sqrt{\frac{f}{h}}\; r^2\left(\rho + p + 2p_\perp\right)$$

Transverse Pressure

Cusp in Redshift & Step produces Transverse Pressure

$$r\frac{d}{dr}\left[(p+\bar{\rho})f^{\frac{1}{2}}\right] = 2\,(p_\perp - p)\,f^{\frac{1}{2}}$$

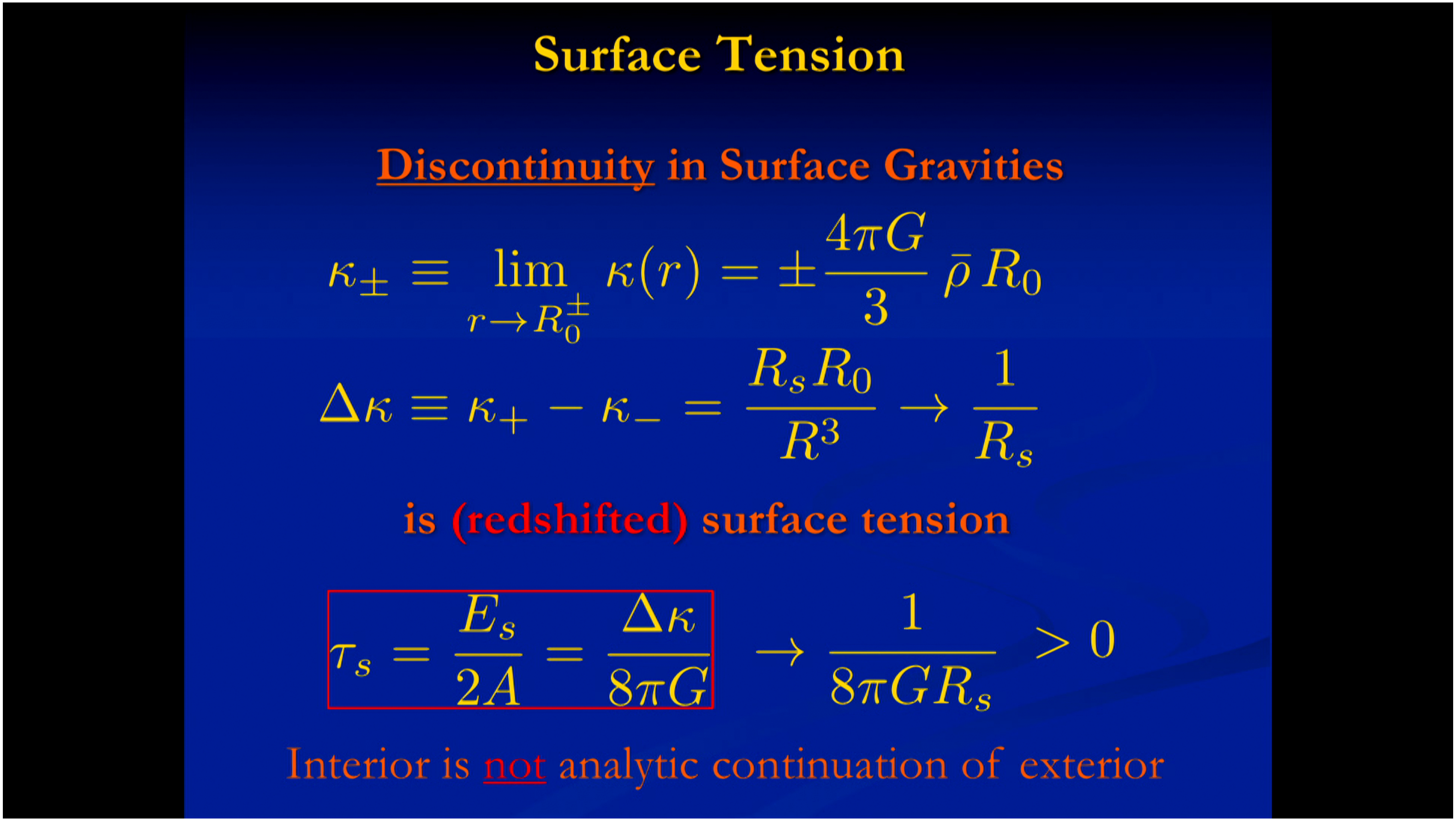
δ Localized at $r = R_0$ Surface

$$8\pi\sqrt{\frac{f}{h}}\;r^2\,(p_\perp - p) = \frac{8\pi}{3}\,\bar{\rho}\,{R_0}^3\,\delta(r - R_0)$$

Integrable Surface Energy

$$E_s = \frac{8\pi}{3}\,\bar{\rho}\,{R_0}^3 = 2M\left(\frac{R_0}{R}\right)^3 \to 2M$$

$$M = E_v + E_s$$


Surface Tension
Discontinuity in Surface Gravities
$\kappa_\pm \equiv \lim_{r\to R_0^\pm} \kappa(r) = \pm\frac{4\pi G}{3}\,\bar\rho\,R_0$
$\Delta\kappa \equiv \kappa_+ - \kappa_- = \frac{R_s R_0}{R^3} \to \frac{1}{R_s}$
is (redshifted) surface tension
$\tau_s = \frac{E_s}{2A} = \frac{\Delta\kappa}{8\pi G} \to \frac{1}{8\pi G R_s} > 0$
Interior is not analytic continuation of exterior

First Law

Classical Mechanical Conservation of Energy

$$dM = dE_v + \tau_s\, dA$$

Gibbs Relation

$$p + \rho = s\,T + \mu\, N = 0$$

Schw. Interior Soln. in $R \to R_s$ Limit describes

a **Zero Entropy/Zero Temperature Condensate**

Discontinuity in κ implies non-analytic behavior

No Trapped Surface, Truly Static, t is a Global Time

Surface Area is Surface Area not Entropy

Surface Gravity is Surface Tension not Temperature

Rindler-like $\mathcal{C}^1$ Coordinates

$$ds^2 = -\frac{\xi^2}{4R_s^2}\,dt^2 + q^2(\xi)\,d\xi^2 + r^2(\xi)\,d\Omega^2$$

$$q(\xi) = \begin{cases} \dfrac{R_s}{r} = \left(1 - \dfrac{\xi^2}{R_s^2}\right)^{-\frac{1}{2}} & -R_s < \xi \leq 0 \\[2ex] \dfrac{r^2}{R_s^2} = \left(1 - \dfrac{\xi^2}{4R_s^2}\right)^{-2} & 0 \leq \xi < 2R_s \end{cases}$$

Surface is at $\xi = 0$ $\quad \xi^2 \geq 0 \quad$ ξ remains real

Generalizes Israel Jcn. Condition to Null Boundary

Faithful to Einstein Equivalence Principle as originally conceived for real coordinate transformations

Surface Oscillations

$$dM = dE_v + \tau_s \, dA$$

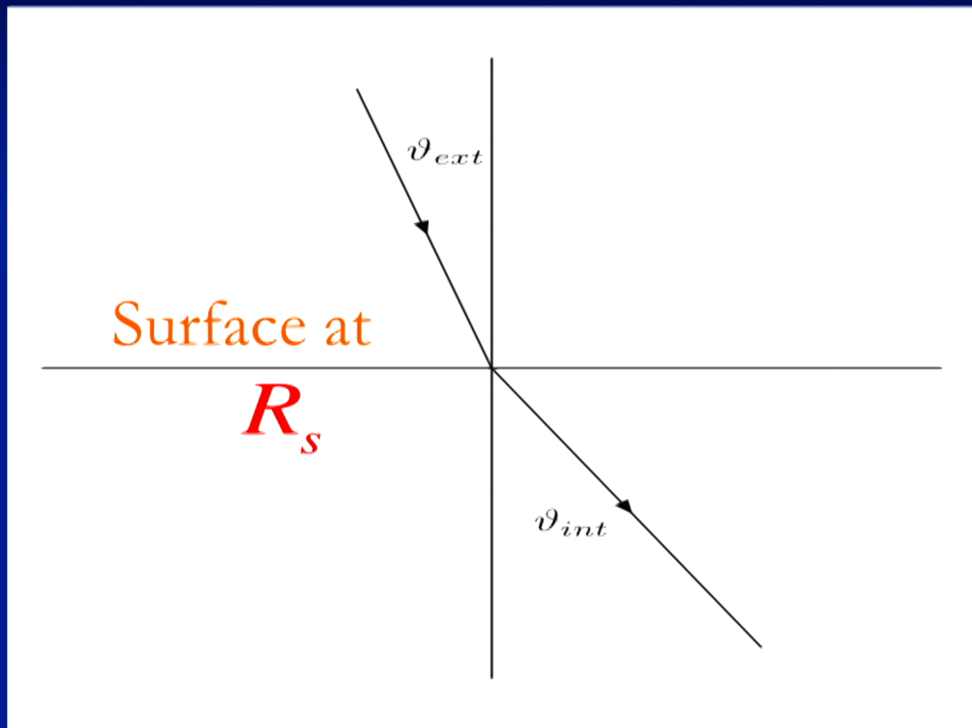
- Energy Minimized by minimizing A for fixed Volume
- Surface Tension acts as a restoring force
- Surface Oscillations are **Stable**
- Surface Normal Modes are **Discrete**
- Characteristic Frequency

$$\omega \sim \frac{c}{4\pi R_s} = 8.1 \left(\frac{M_\odot}{M}\right) \text{kHz}$$

- **Discrete Gravitational Wave Spectrum**
- Striking Signature for LIGO/VIRGO for

$$M \sim 10^{1-2} M_\odot$$

Refraction of Null Rays at Surface



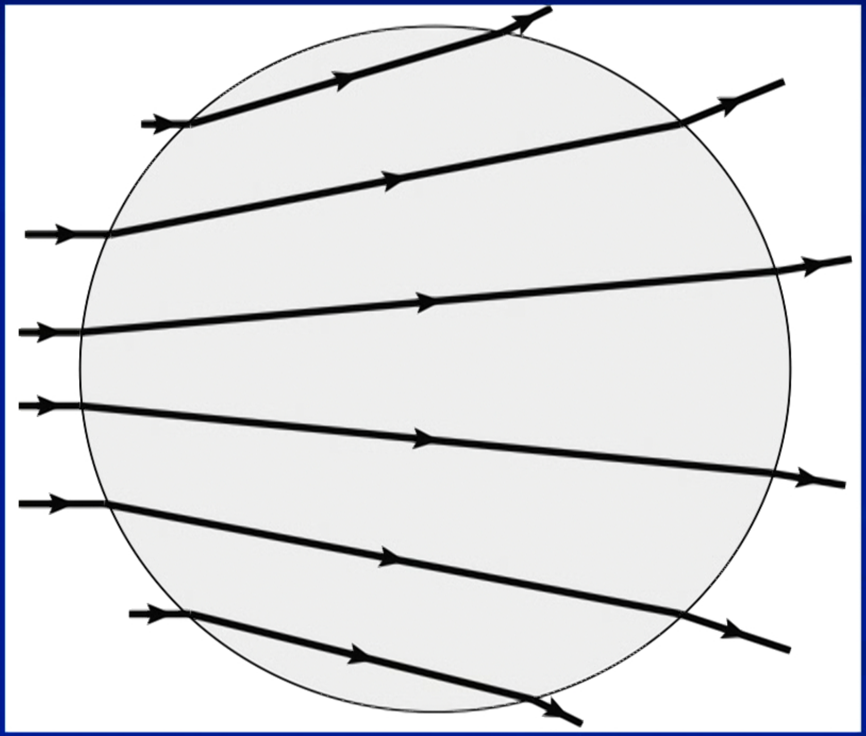
Solve
Geodesic Eq.
→ Snell's Law
$n < 1$

Impact
Parameter
b

$$\sin\vartheta_{ext}\sqrt{1+\frac{4R_s^2}{b^2}} = \sin\vartheta_{int}\sqrt{1+\frac{R_s^2}{b^2}}$$

Defocusing of Null Rays

No Horizon ➔ Light Rays Penetrate Interior



Completely Different Imaging from a Black Hole

Remarks

- Non-Analyticity Characteristic of a Phase Boundary

  Freezing of Time ➔ Critical Slowing Down

$$-g_{tt} = -K_\mu K^\mu = f(r) = c^2_{eff} \geq 0$$

- K_μ is everywhere timelike: Hamiltonian exists & is Hermitian w. proper b.c. at 0 and ∞
- Quantum State of Test Field ~ Boulware State (No Flux)
- Energy Density $p = \frac{\rho}{3} \to -\frac{C}{M^4 f^2} < 0$
- Violates Both Weak & Strong Energy Conditions
- Significant Backreaction when $f(r) \geq \epsilon \sim \frac{L_{pl}}{R_s}$
- Occurs at Physical Length
- Also time delay ~ ln (ε)
- All Regulated by finite ε

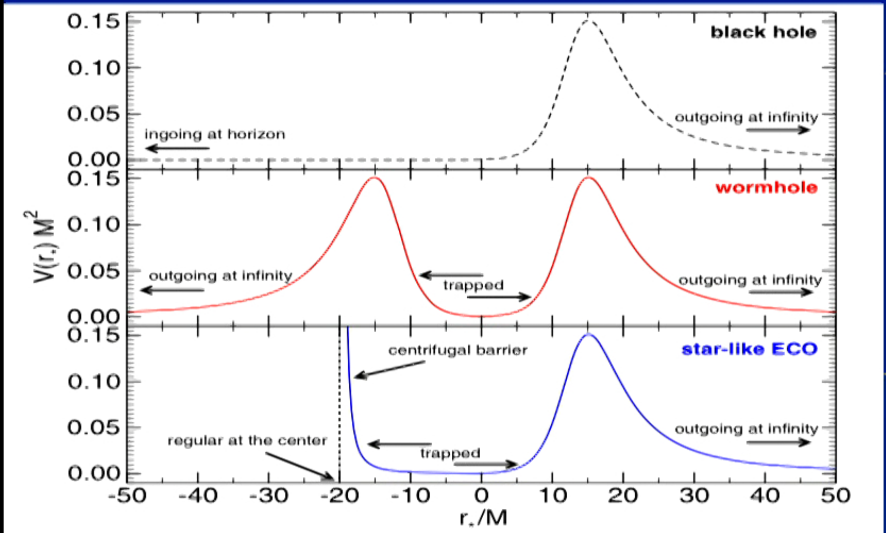
$$\ell \sim \frac{\Delta r}{\sqrt{h(r)}} \sim \sqrt{L_{pl} R_s}$$

Gravastar Interior Echoes

- Transmission (with log time delay) through surface
- Regge-Wheeler radial coordinate $dr^* = \frac{dr}{\sqrt{fh}}$
- Scalar Wave or Grav. Wave Eq.

$$\left(-\frac{d^2}{dr^{*\,2}} + V_\ell\right)\phi_{\omega\ell} = \omega^2 \phi_{\omega\ell}$$

- Reflection from de Sitter interior barrier: 'Echo'



$$V_\ell = \frac{1}{4}(1 - H^2 r^2)\left(\frac{\ell(\ell+1)}{r^2} - 2H^2\right)$$

$$r < 2M = H^{-1}$$

$$V_\ell = \left(1 - \frac{2M}{r}\right)\left(\frac{\ell(\ell+1)}{r^2} + \frac{2M}{r^3}\right)$$

$$r > 2M$$

Summary

- Buchdahl Bound → Interior Pressure Divergence Develops (inside out) **before** Event Horizon forms
- Constant Density Interior Schwarzschild Solution $R > \frac{9}{8}R_s = \frac{9}{4}GM$ **Saturates Bound** & illustrates the generic behavior
- Pressure Singularity is **Integrable-Sensible Boundary Layer**
- Implies Formation of a **δ-fn.** Surface & **Surface Tension** & a Non-Singular (de Sitter $p = -\rho$) Interior
- **Gravitational Condensate Star negative pressure** already realized/inherent in Classical Gen. Rel. (1916)
- **Cold** Quantum **Final State** of Gravitational Collapse
- No Thermal Radiation/ Not a Firewall

Summary

- Area Term is Classical **Mechanical Surface Energy** **not Entropy**
- QM, Unitarity ✓ **No 'Information Paradox'**
- **No large number of microstates:** One Vacuum State
- Both Echoes and Discrete Surface Modes
- Transmission/Refraction→ different imaging for EHT
- Has been extended to slow rotation C. Posada-Aguirre, MNRAS 468 (2017) 2128
- Full Non-Singular Soln. Requires Quantum Effective Theory of the Conformal Anomaly
- Dynamical Vacuum Condensate Energy
- Regulated Finite Thickness Boundary layer