Title: A Self-consistent Model of Evaporating Black Holes

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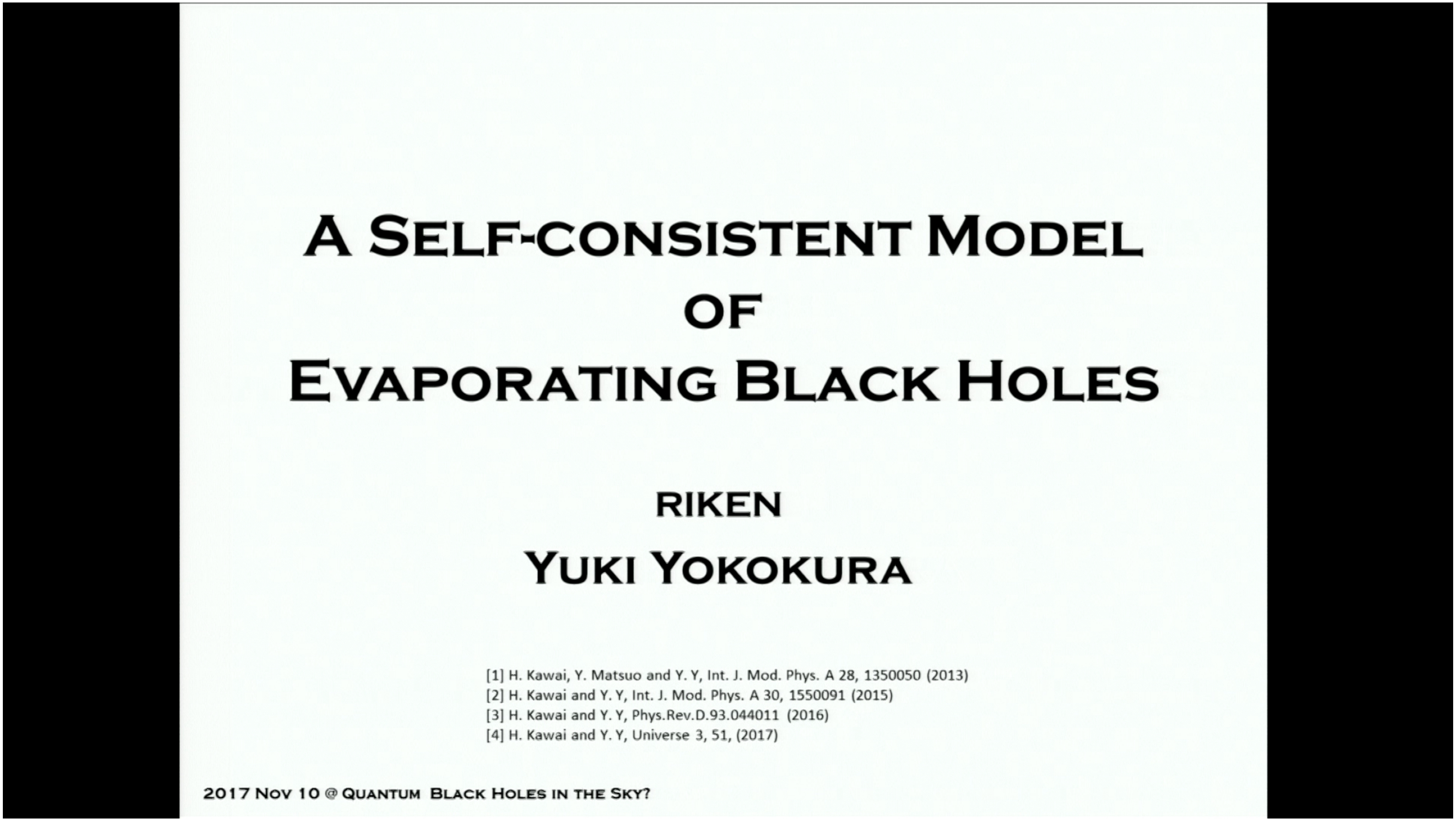
Abstract: We analyze the time evolution of a spherically-symmetric collapsing matter from the point of view that black holes evaporate by nature. We obtain a self-consistent solution of the semi-classical Einstein equation. The solution indicates that the collapsing matter forms a dense object and evaporates without horizon or singularity, and it has a surface but looks like an ordinary black hole from the outside. Any object we recognize as a black hole should be such an object. In the case of stationary black holes that are formed adiabatically in the heat bath, the area law is reproduced by integrating the entropy density over the interior volume. This result implies that the information is stored inside the object.



A SELF-CONSISTENT MODEL OF EVAPORATING BLACK HOLES

RIKEN

YUKI YOKOKURA

[1] H. Kawai, Y. Matsuo and Y. Y, Int. J. Mod. Phys. A 28, 1350050 (2013)
[2] H. Kawai and Y. Y, Int. J. Mod. Phys. A 30, 1550091 (2015)
[3] H. Kawai and Y. Y, Phys.Rev.D.93.044011 (2016)
[4] H. Kawai and Y. Y, Universe 3, 51, (2017)

2017 NOV 10 @ QUANTUM BLACK HOLES IN THE SKY?

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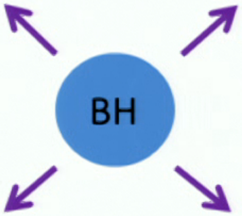
2017 Nov 10 @ Quantum Black Holes in the Sky?

Motivation

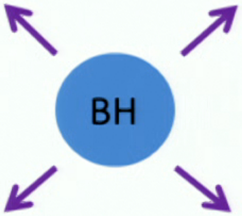
- Black holes evaporate.



- What is the black hole in quantum mechanics?

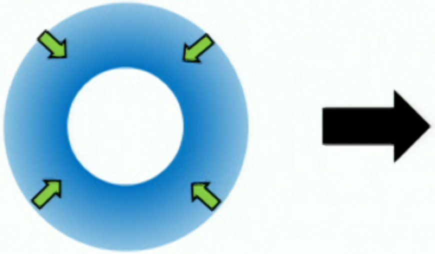
Motivation

- Black holes evaporate.



- What is the black hole in quantum mechanics?

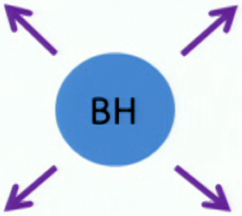
⇒Reconsider the time evolution of a 4D spherically-symmetric collapsing matter.



What happens?
Horizon is formed or not?

Motivation

- Black holes evaporate.



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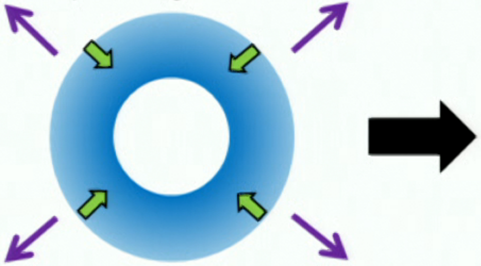


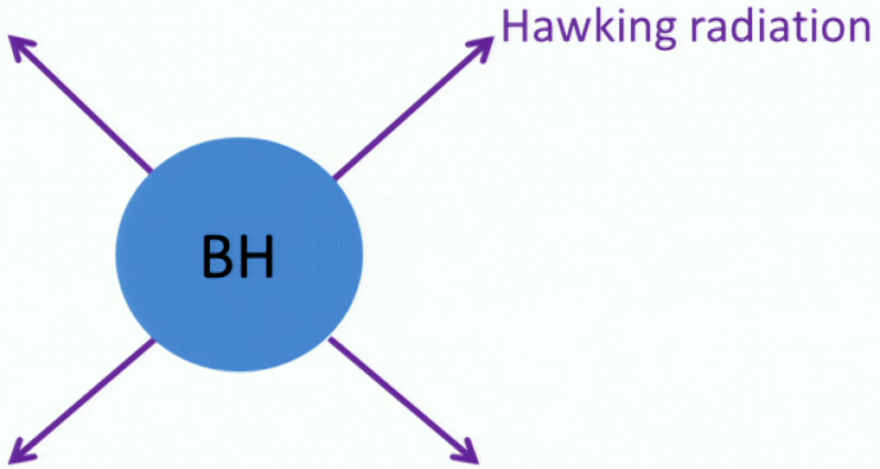
What happens?
Horizon is formed or not?

Generically, particle creation occurs in a time-dependent metric.
⇒ We need to include the back reaction of evaporation in the formation process.

⇒Our basic idea by 3 steps (self-consistent)

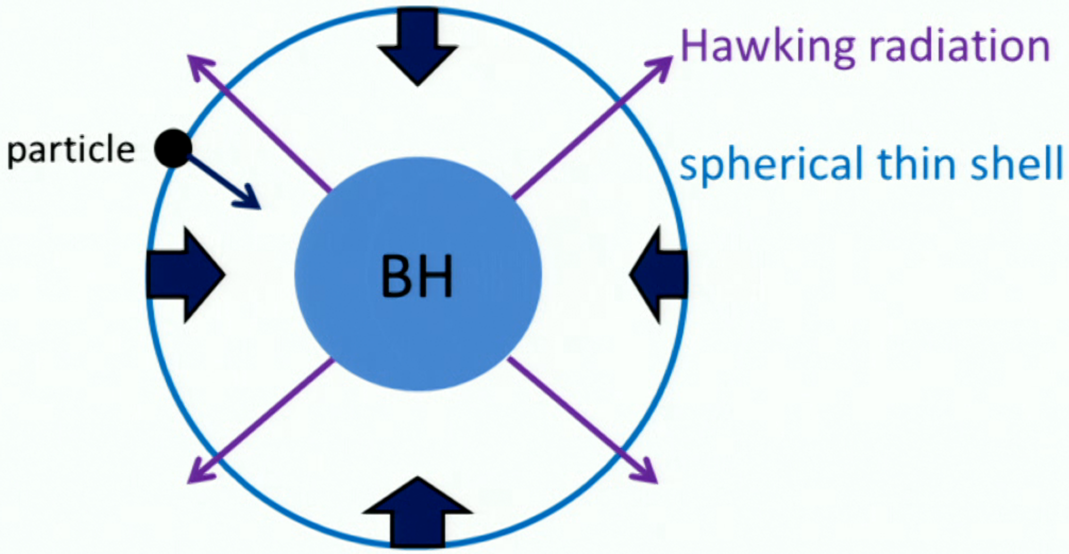
Basic idea: step1

Imagine that a spherically-symmetric BH is evaporating.



Basic idea: step1

Imagine that a spherically-symmetric BH is evaporating.
Add a spherical thin shell (or a particle) to it.



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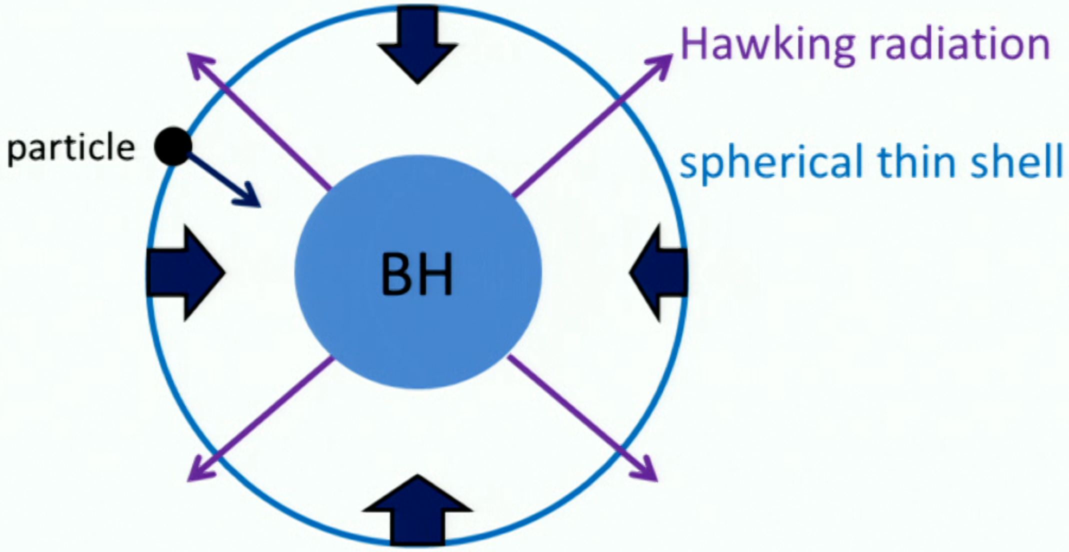
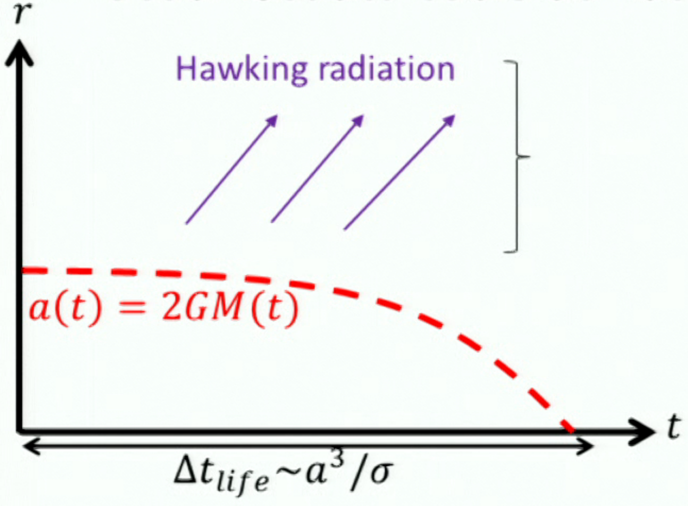


What happens if both dynamics of the matter and spacetime is considered?

⇒The shell will never reach “the horizon”.

The shell never crosses "the horizon".



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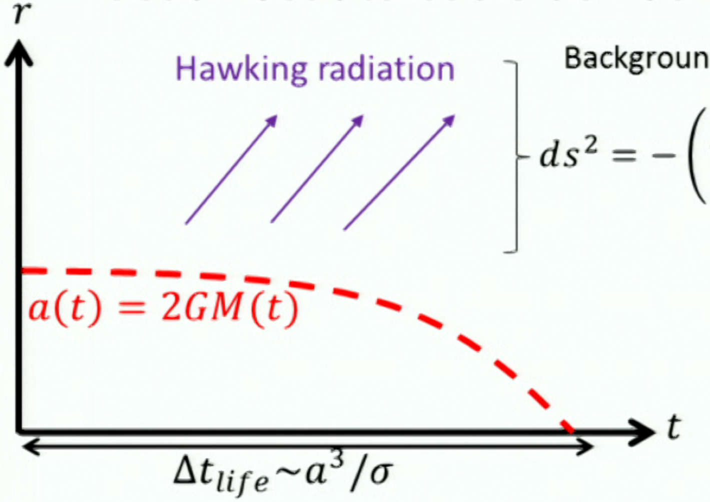

Background ≈ time-dependent Schwarzschild metric

$$ds^2 = -\left(1 - \frac{a(t)}{r}\right)dt^2 + \left(1 - \frac{a(t)}{r}\right)^{-1} dr^2 + r^2 d\Omega^2$$

$$\frac{d}{dt}a(t) = -\frac{2\sigma}{a(t)^2}$$

←Stephan-Boltzmann law of $T_H = \frac{\hbar}{4\pi a}$

$a \equiv 2GM$

intensity: $\sigma = O(1) \sim \hbar G N$

(N=d.o.f. of fields)

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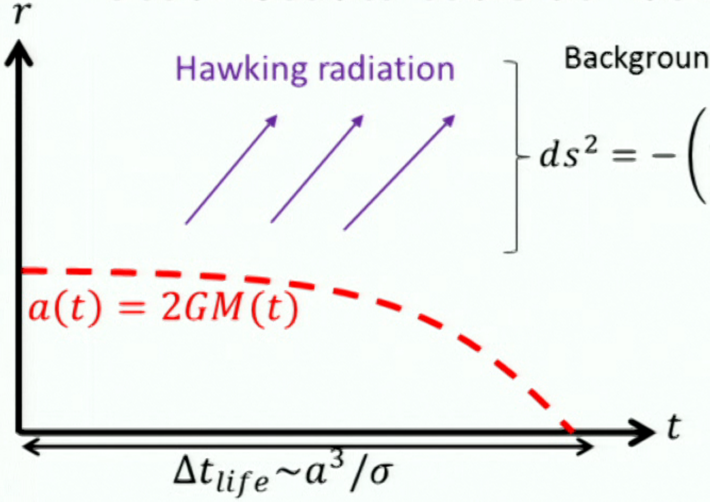

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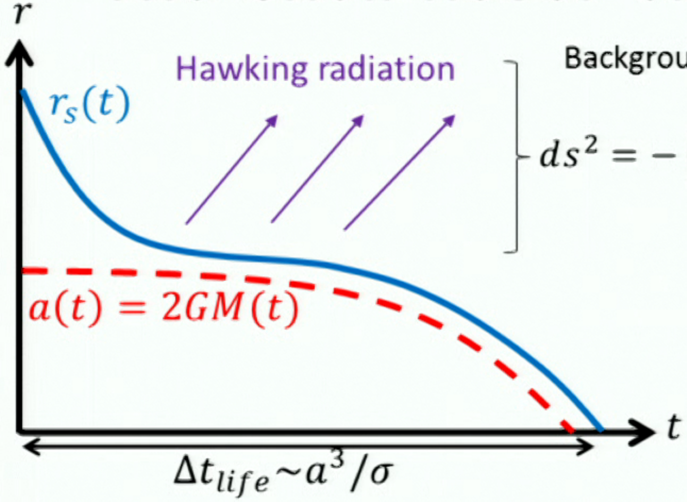

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For $r_s \sim a$, a particle with any (l, m) behaves lightlike:

$$\frac{dr_s(t)}{dt} = -\frac{r_s(t) - a(t)}{r_s(t)}.$$

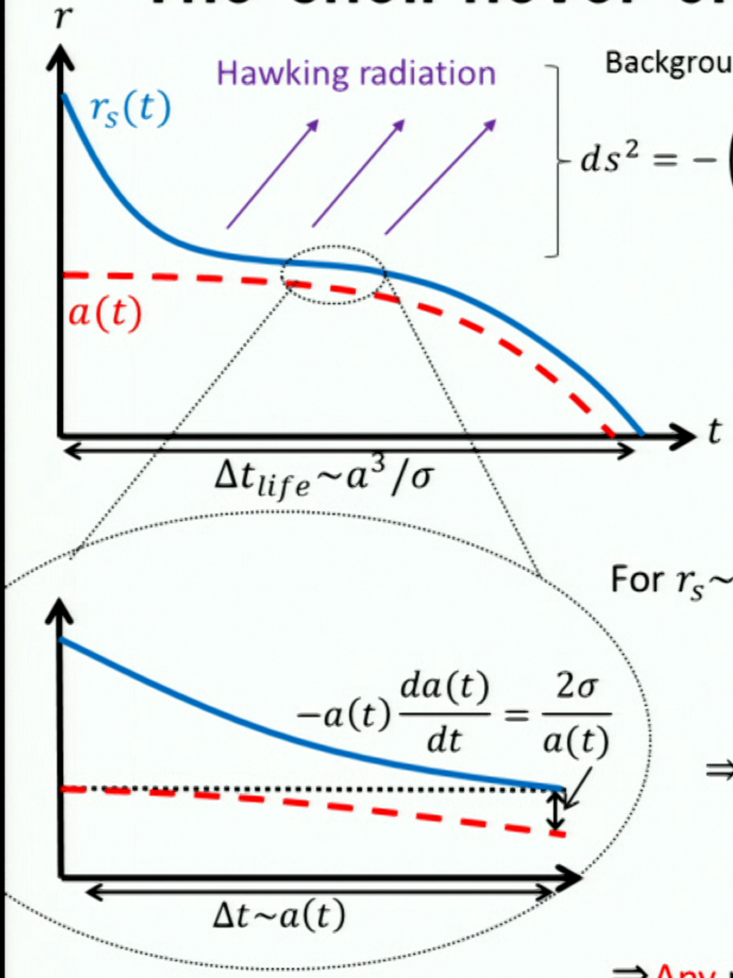
$$\Rightarrow r_s(t) \simeq a(t) - a(t)\frac{da(t)}{dt} + Ca(t)e^{-\frac{t}{a(t)}}$$

←Effect of back reaction

$$\rightarrow a(t) + \frac{2\sigma}{a(t)}$$

⇒Any particle will approach $a(t) + \frac{2\sigma}{a(t)}$.

The shell never crosses "the horizon".



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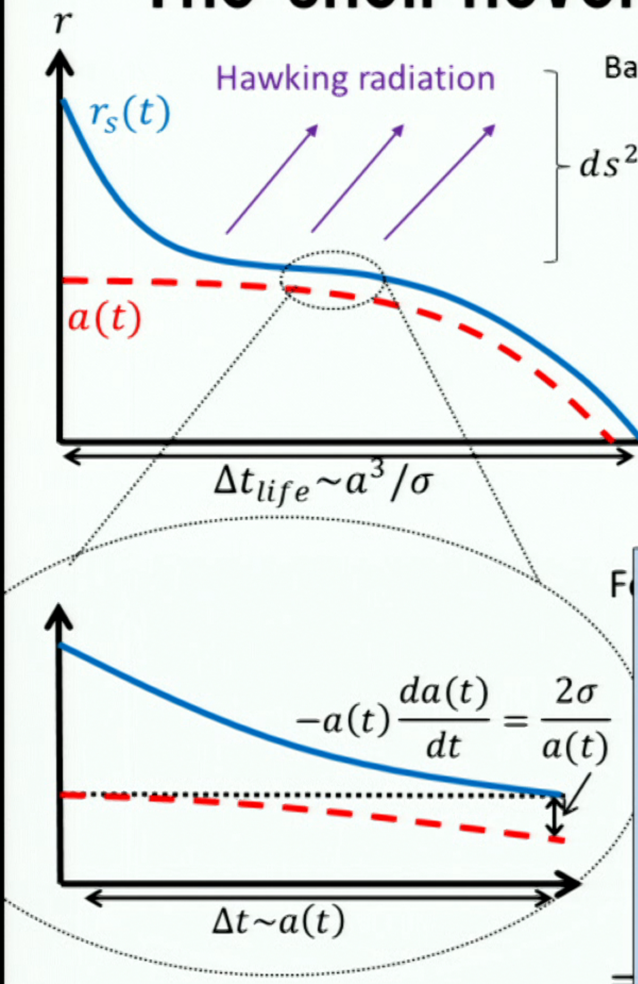

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$a \equiv 2GM$
intensity: $\sigma = O(1) \sim \hbar G N$
(N=d.o.f. of fields)

- The proper length of $\Delta r = \frac{2\sigma}{a}$ is physical:

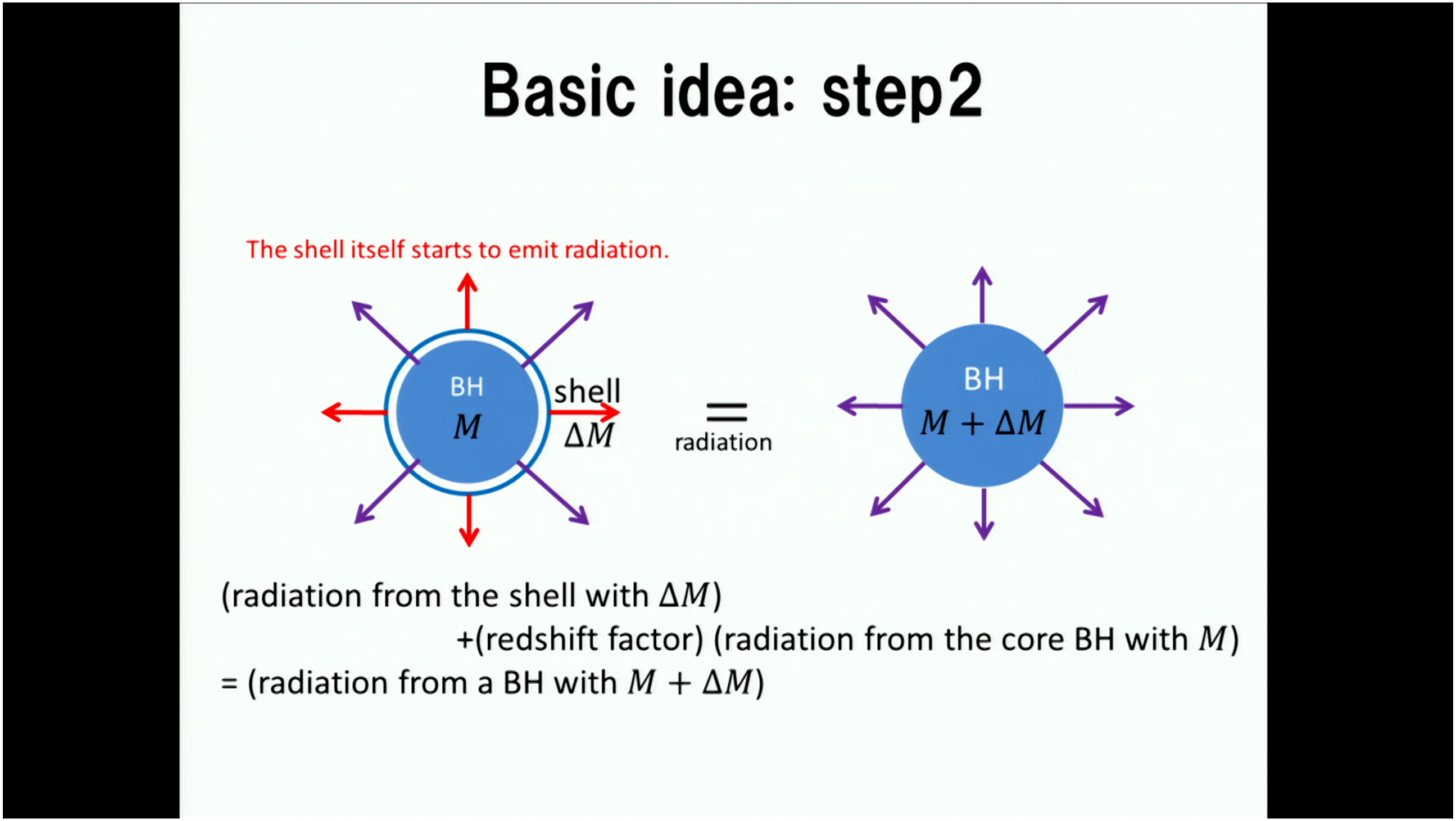
$$\sqrt{g_{rr}}\frac{2\sigma}{a} \approx \sqrt{\sigma} \sim \sqrt{N} l_p \gg l_p$$

if there are many d.o.f. of fields:

$$N \gg 1.$$

⇒The shell is physically outside the "BH".

(Note: The (t, r)-coordinates will be complete to describe the whole space time.)



Basic idea: step2

The shell itself starts to emit radiation.

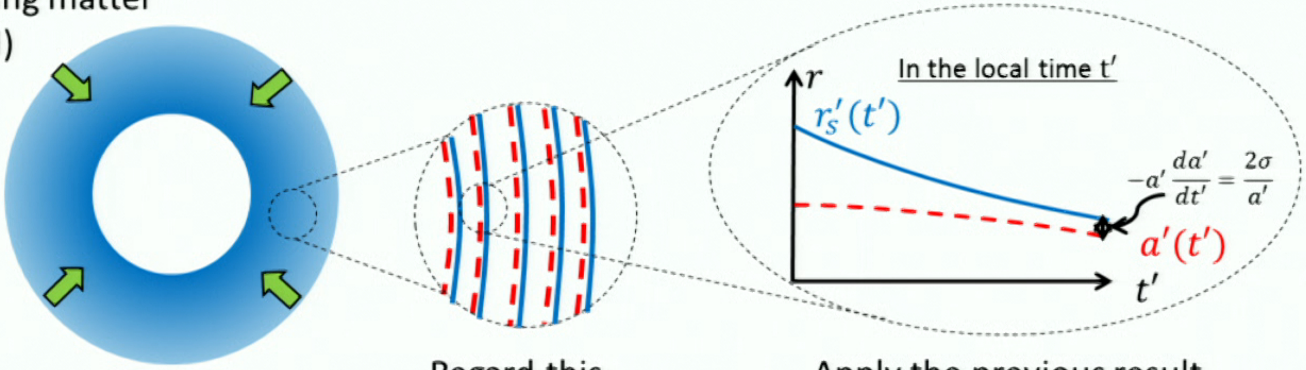
(radiation from the shell with ΔM)

+(redshift factor) (radiation from the core BH with M)

= (radiation from a BH with $M + \Delta M$)

Basic idea: step3

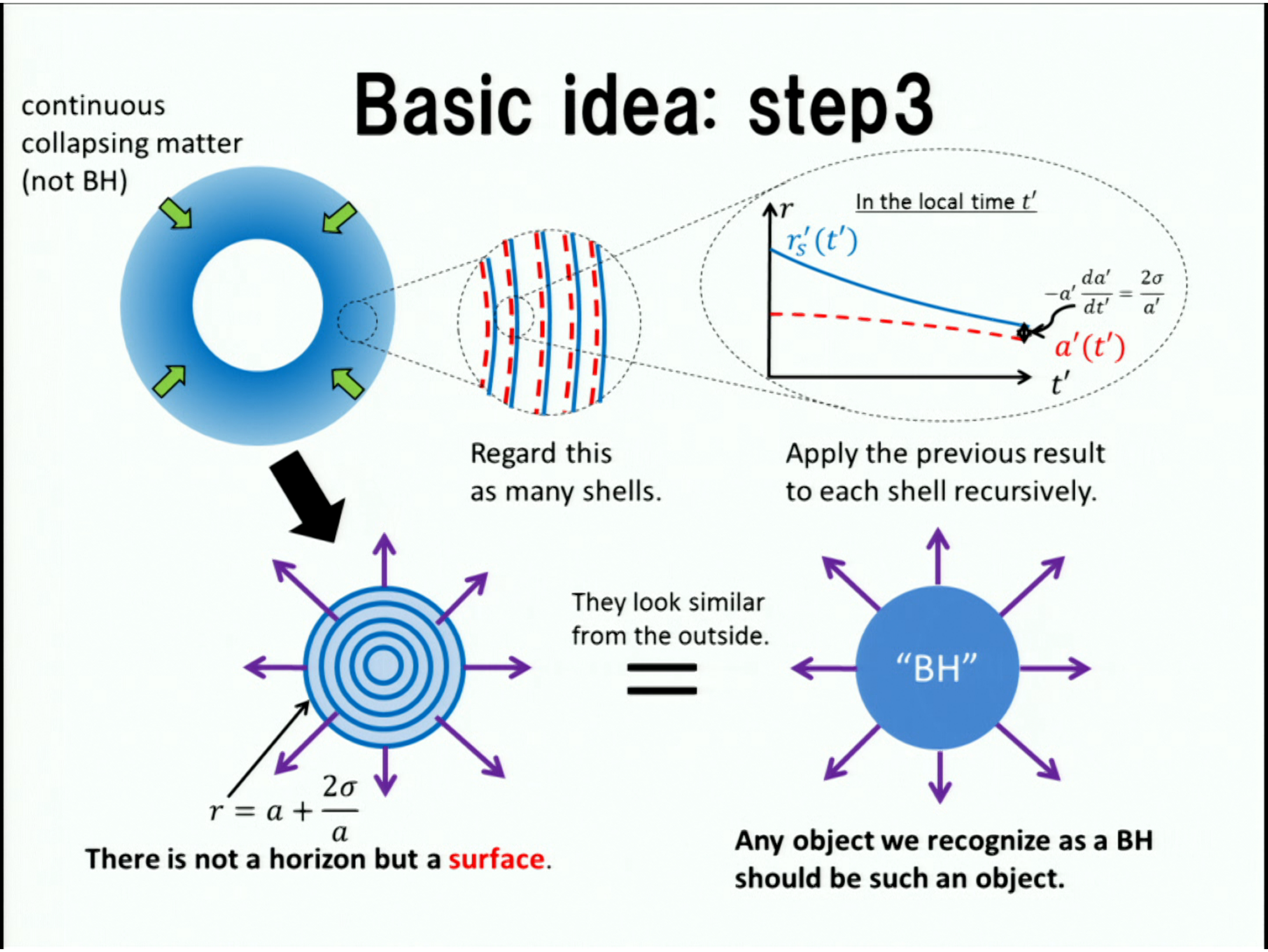
continuous
collapsing matter
(not BH)



Regard this
as many shells.

Apply the previous result
to each shell recursively.

Basic idea: step3



continuous
collapsing matter
(not BH)

Regard this
as many shells.

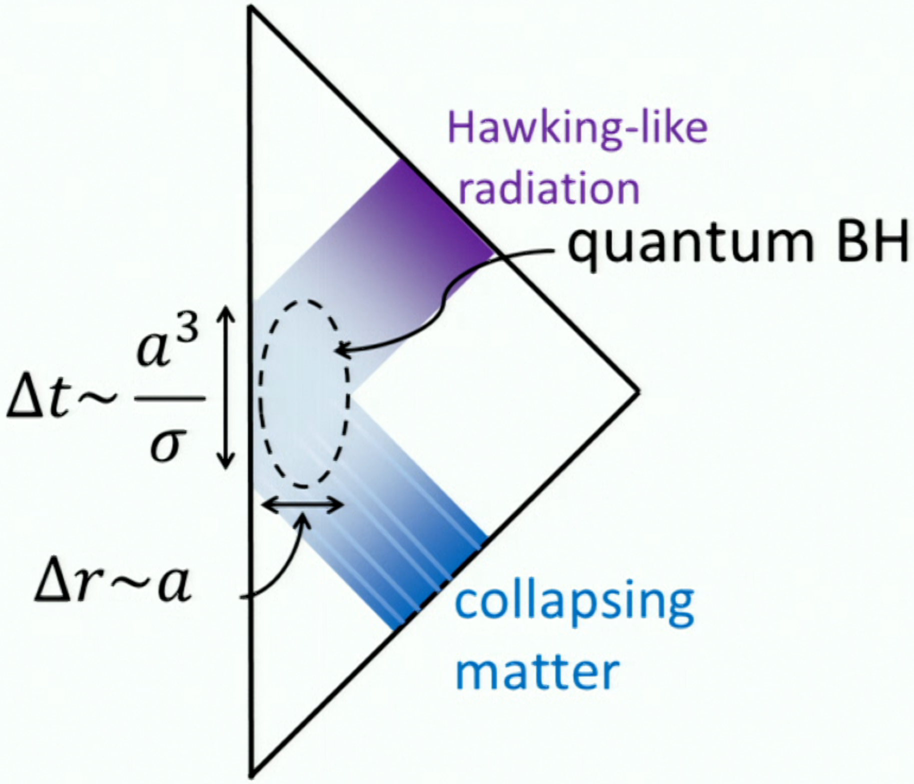
Apply the previous result
to each shell recursively.

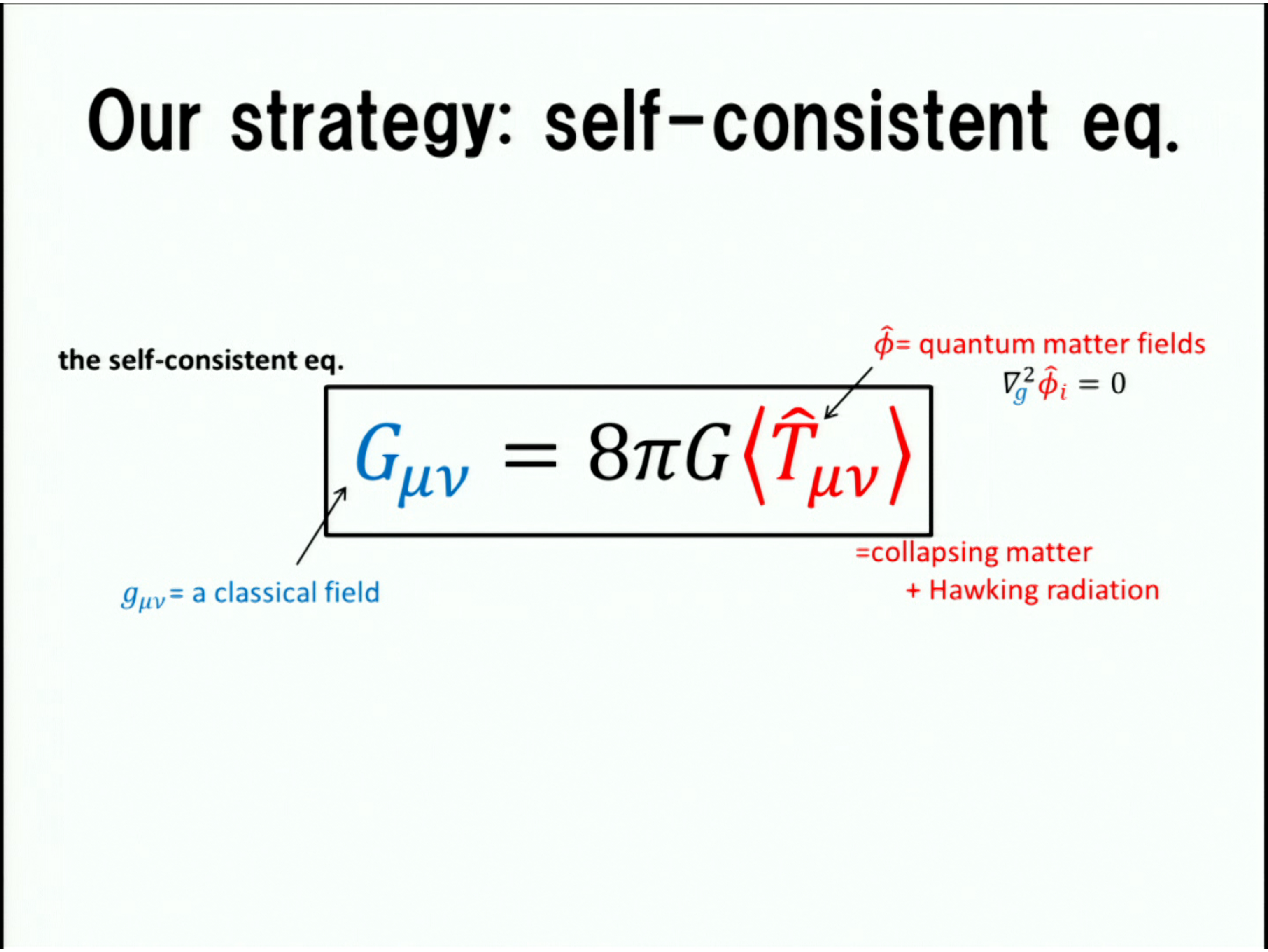
They look similar
from the outside.

There is not a horizon but a surface.

Any object we recognize as a BH should be such an object.

Penrose diagram





Our strategy: self-consistent eq.

the self-consistent eq.

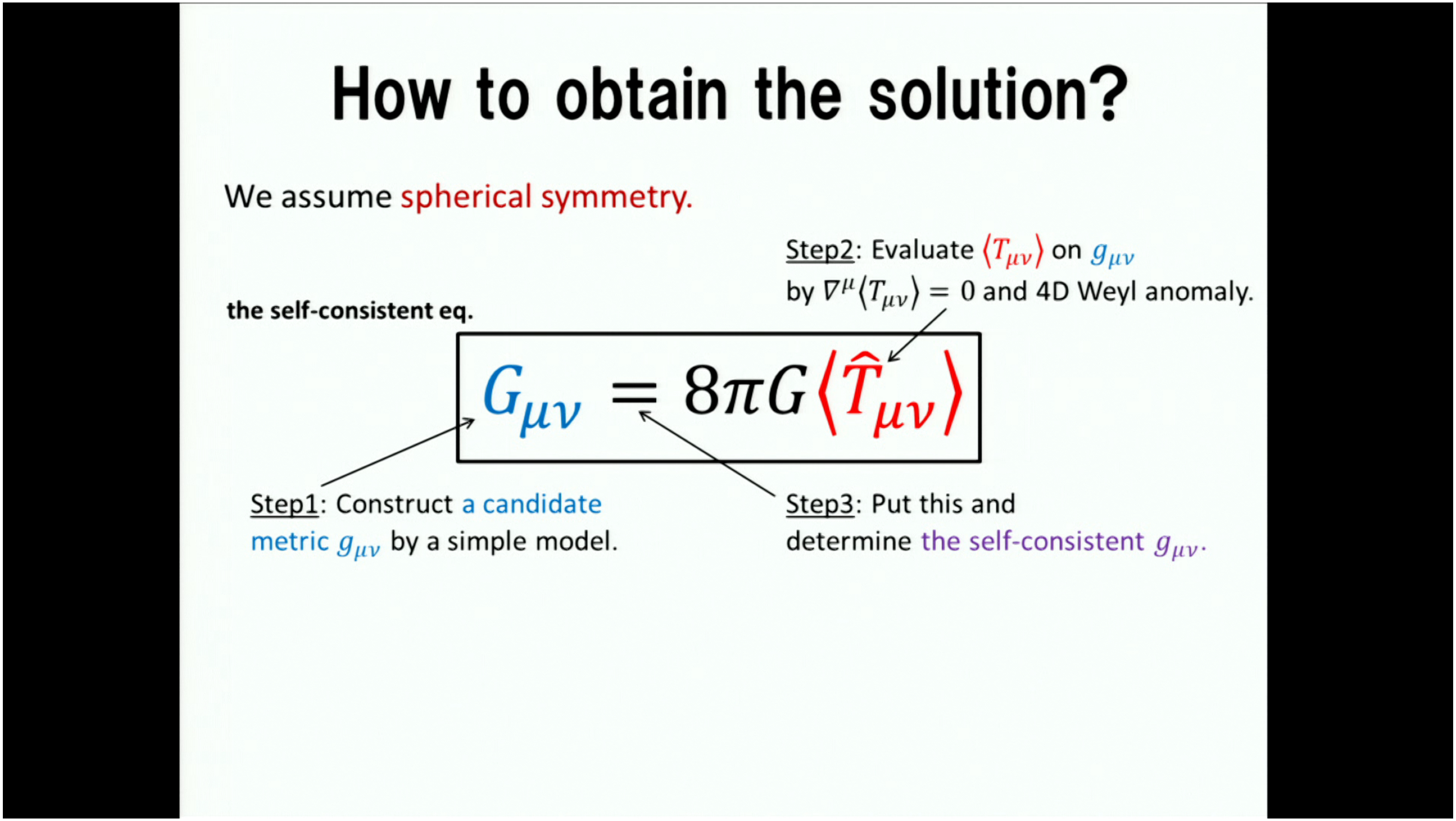
$$\boxed{G_{\mu\nu} = 8\pi G\langle\hat{T}_{\mu\nu}\rangle}$$

$\hat{\phi}$= quantum matter fields

$\nabla_g^2\hat{\phi}_i = 0$

=collapsing matter
+ Hawking radiation

$g_{\mu\nu}$= a classical field



How to obtain the solution?

We assume spherical symmetry.

Step2: Evaluate $\langle T_{\mu\nu} \rangle$ on $g_{\mu\nu}$
by $\nabla^{\mu}\langle T_{\mu\nu} \rangle = 0$ and 4D Weyl anomaly.

the self-consistent eq.

$$\boxed{G_{\mu\nu} = 8\pi G \langle \hat{T}_{\mu\nu} \rangle}$$

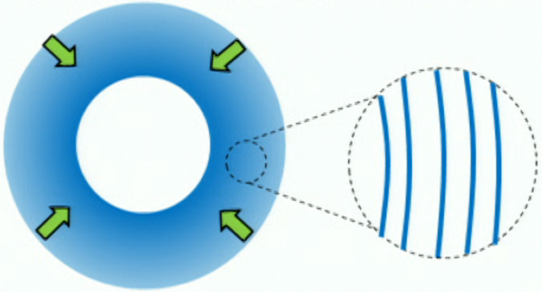
Step1: Construct a candidate
metric $g_{\mu\nu}$ by a simple model.

Step3: Put this and
determine the self-consistent $g_{\mu\nu}$.

Step1: Construction of a candidate $g_{\mu\nu}$ (1/3)

A multi-shell model

Consider a continuous spherical matter.
⇒Model this as many null shells.



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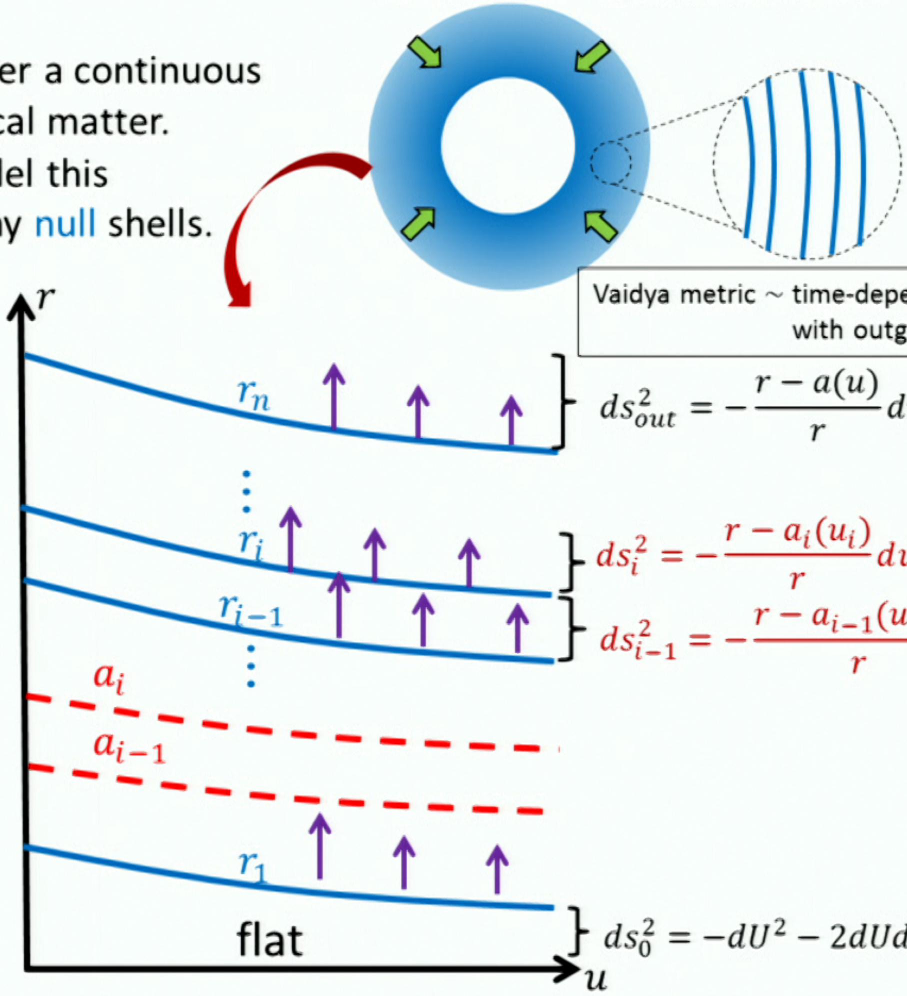


Vaidya metric ~ time-dependent Schwarzschild metric with outgoing null energy flow, $G_{uu} = -\dot{a}/r^2$

$$ds_{out}^2 = -\frac{r-a(u)}{r}du^2 - 2dudr + r^2 d\Omega^2$$

$$ds_i^2 = -\frac{r-a_i(u_i)}{r}du_i^2 - 2du_i dr + r^2 d\Omega^2$$

$$ds_{i-1}^2 = -\frac{r-a_{i-1}(u_{i-1})}{r}du_{i-1}^2 - 2du_{i-1} dr + r^2 d\Omega^2$$

$$ds_0^2 = -dU^2 - 2dUdr + r^2 d\Omega^2$$

Step1: Construction of a candidate $g_{\mu\nu}$ (2/3): Self-consistent ansatz

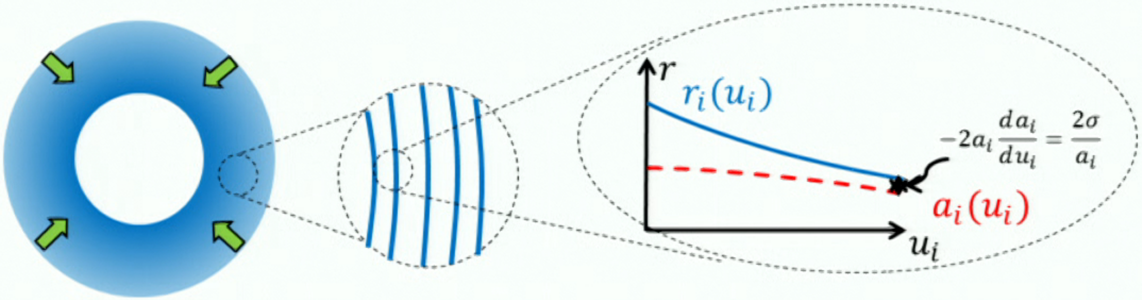
Ansatz:

Each shell behaves like the ordinary evaporating BH:

$$\frac{da_i}{du_i} = -\frac{\sigma}{a_i^2},$$

and that each shell has already come close to

$$r_i = a_i + \frac{2\sigma}{a_i}$$



⇒After taking continuum limit ($\Delta a_i \equiv a_i - a_{i-1} \to 0$), we obtain....

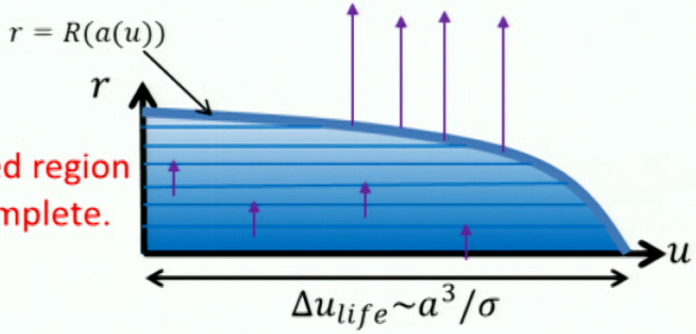
Step1: Construction of a candidate $g_{\mu\nu}$ (3/3): the metric

$$ds^2 = \begin{cases} -\left(1-\frac{a(u)}{r}\right)du^2 - 2dudr + r^2 d\Omega^2 & \leftarrow\text{time-dep} \\ -\frac{2\sigma}{r^2} e^{-\frac{1}{2\sigma}\left(R(a(u))^2 - r^2\right)} du^2 - 2e^{-\frac{1}{4\sigma}\left(R(a(u))^2 - r^2\right)} dudr + r^2 d\Omega^2 & \leftarrow\text{static} \end{cases}$$

Here

$$\frac{da(u)}{du} = -\frac{\sigma}{a(u)^2}, \qquad R(a) \equiv a + \frac{2\sigma}{a}$$

Note: At this stage, σ is not determined.



No horizon or trapped region
⇒(u, r) coord. is complete.

Step1: Construction of a candidate $g_{\mu\nu}$ (3/3): the metric

$$ds^2 = \begin{cases} -\left(1-\frac{a(u)}{r}\right)du^2 - 2dudr + r^2 d\Omega^2 & \leftarrow\text{time-dep} \\ -\frac{2\sigma}{r^2}e^{-\frac{1}{2\sigma}\left(R(a(u))^2-r^2\right)}du^2 - 2e^{-\frac{1}{4\sigma}\left(R(a(u))^2-r^2\right)}dudr + r^2 d\Omega^2 & \leftarrow\text{static} \end{cases}$$

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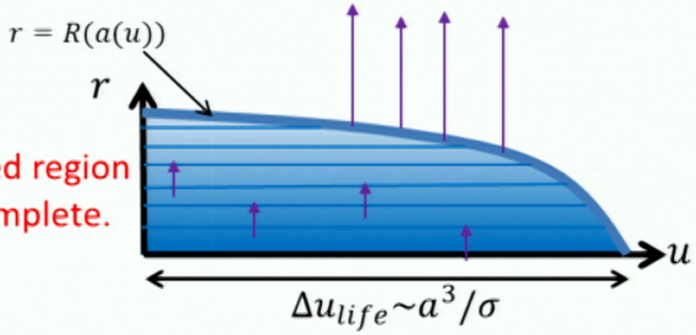
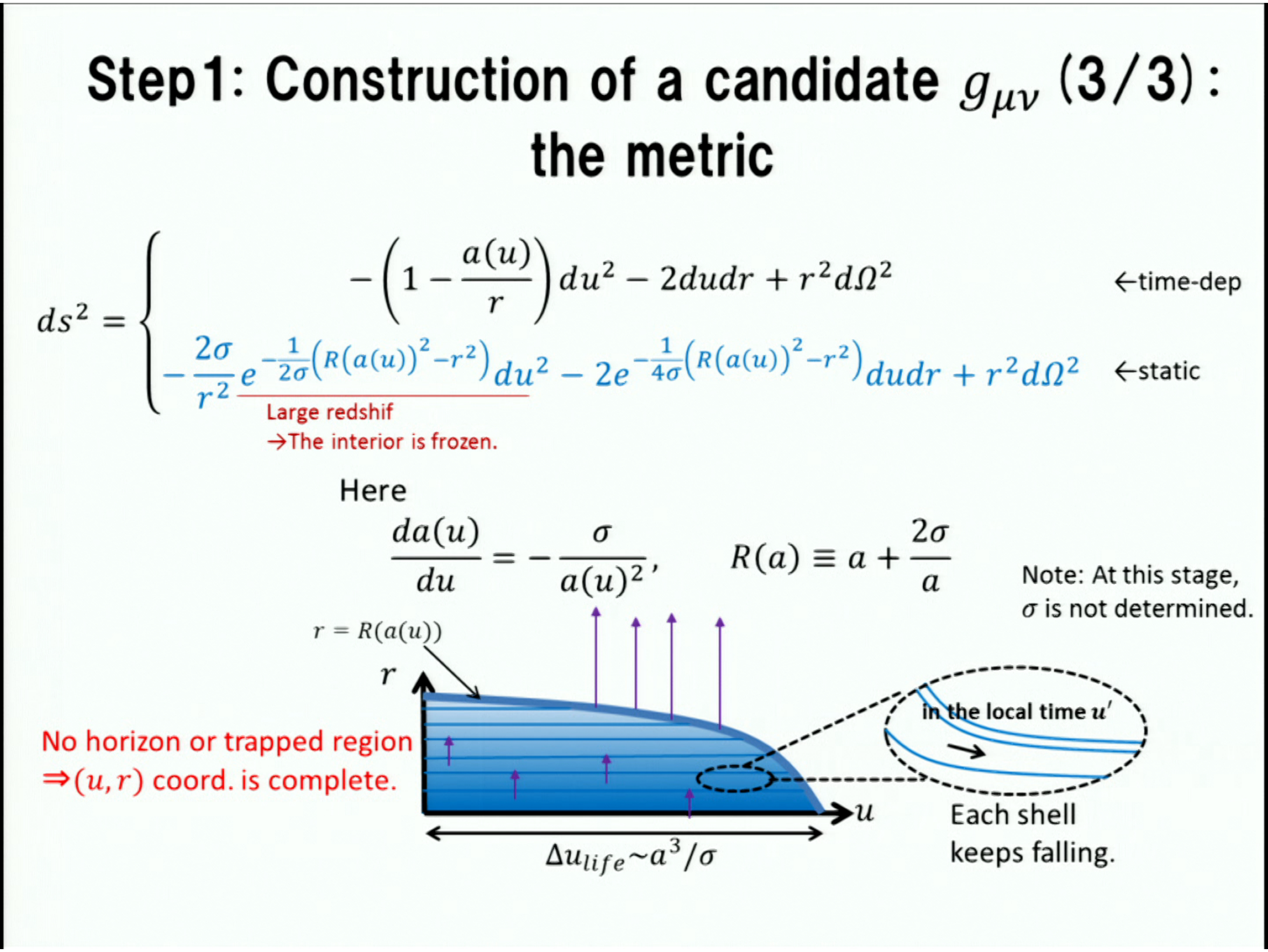


No horizon or trapped region
$\Rightarrow (u, r)$ coord. is complete.



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Large redshif

→The interior is frozen.

Here

$$\frac{da(u)}{du} = -\frac{\sigma}{a(u)^2}, \qquad R(a) \equiv a + \frac{2\sigma}{a}$$

Note: At this stage, σ is not determined.

No horizon or trapped region

⇒(u, r) coord. is complete.

Each shell keeps falling.

Step2: Evaluation of $\langle T_{\mu\nu} \rangle$ (1/3): Setup

Consider the interior region.

The background metric is static:

$$ds^2 = -\frac{2\sigma}{r^2} e^{\frac{r^2}{2\sigma}} dU^2 - 2e^{\frac{r^2}{4\sigma}} dU dr + r^2 d\Omega^2$$
$$= -e^{\varphi(r(U,V))} dU dV + r(U,V)^2 d\Omega^2,$$

$\Rightarrow \langle T_{\mu\nu} \rangle$ also should be static:

$$\langle T_{\mu\nu} \rangle = \langle T_{\mu\nu}(r) \rangle, \qquad \langle T_{UU} \rangle = \langle T_{VV} \rangle$$

Step2: Evaluation of $\langle T_{\mu\nu}\rangle$ (1/3): Setup

Consider the interior region.

The background metric is static:

$$ds^2 = -\frac{2\sigma}{r^2} e^{\frac{r^2}{2\sigma}} dU^2 - 2e^{\frac{r^2}{4\sigma}} dU dr + r^2 d\Omega^2$$
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⇒$\langle T_{\mu\nu}\rangle$ also should be static:

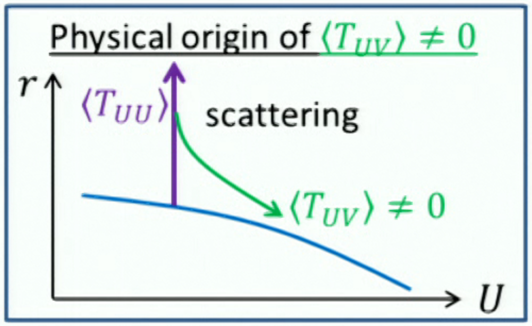
$$\langle T_{\mu\nu}\rangle = \langle T_{\mu\nu}(r)\rangle, \qquad \langle T_{UU}\rangle = \langle T_{VV}\rangle$$

⇒Use of Vaidya metric (only $G_{uu} \neq 0$) means

$$\langle T_{UV}\rangle = 0,$$

we have to determine only

$$\langle T_{UU}\rangle, \qquad \langle T^{\theta}_{\theta}\rangle.$$



(⇒We can remove this artificial assumption and generalize it to $\langle T_{UV}\rangle \neq 0$.)

Step2: Evaluation of $\langle T_{\mu\nu}\rangle$ (2/3): The relations of $\langle T_{\mu\nu}\rangle$

・1st eq.

$$\langle T^{\mu}_{\mu}\rangle = 2g^{UV}\underbrace{\langle T_{UV}\rangle}_{=0} + 2\langle T^{\theta}_{\theta}\rangle \text{ leads to}$$

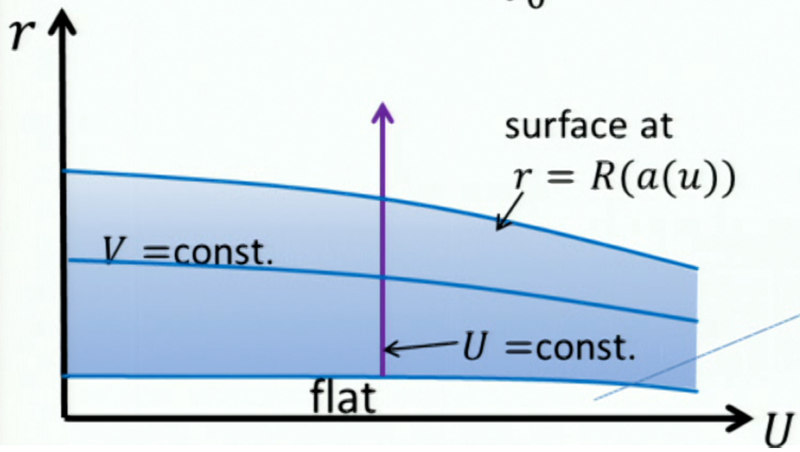
$$\langle T^{\theta}_{\theta}\rangle = \frac{1}{2}\langle T^{\mu}_{\mu}\rangle$$

All components are expressed in terms of $\langle T^{\mu}_{\mu}\rangle$.

・2nd eq.

$\nabla^{\mu}\langle T_{\mu U}\rangle = 0$ $(\Leftrightarrow \nabla^2\phi = 0)$ provides

$$r^2\langle T_{UU}\rangle = \frac{1}{2}\int_0^r dr' r' e^{\varphi(r')}\langle T^{\theta}_{\theta}(r')\rangle + \underbrace{[r^2\langle T_{UU}\rangle]_{r=0}}_{=0},$$



Boundary condition:

$$\langle T_{\mu\nu}(r=0)\rangle = 0$$

(⇔initial condition that the system started from the collapsing matter.)

Step2: Evaluation of $\langle T_{\mu\nu} \rangle$ (3/3) : 4D Weyl anomaly

For simplicity, consider conformal matters.

$\Rightarrow \langle T^{\mu}_{\mu} \rangle$ is determined by the 4D Weyl anomaly:

$$\langle T^{\mu}_{\mu} \rangle = \hbar c_w \mathcal{F} - \hbar a_w \mathcal{G}$$ ←state-independent

where

$$\mathcal{F} \equiv C_{\mu\nu\alpha\beta} C^{\mu\nu\alpha\beta}, \qquad \mathcal{G} \equiv R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} - 4 R_{\mu\nu} R^{\mu\nu} + R^2$$

$\Rightarrow$The metric determines

$$\langle T^{\mu}_{\mu} \rangle = \frac{\hbar c_w}{3\sigma^2}$$

Step3: Check of $G_{\mu\nu} = 8\pi G\langle T_{\mu\nu}\rangle$ (1/2): The $g_{\mu\nu}$ is the self-consistent solution.

Thus , we have obtained

$$\langle T^{\theta}_{\theta}\rangle = \frac{\hbar c_W}{6\sigma^2}, \qquad \langle T_{UU}\rangle = \langle T_{VV}\rangle = \frac{\hbar c_W}{3r^4} e^{\frac{1}{2\sigma}r^2}, \qquad \langle T_{UV}\rangle = 0$$

On the other hand, the metric gives

$$G^{\theta}_{\theta} = \frac{1}{2\sigma}, \qquad G_{UU} = G_{VV} = \frac{\sigma}{r^4} e^{\frac{1}{2\sigma}r^2}, \qquad G_{UV} = 0$$

$\Rightarrow G_{\mu\nu} = 8\pi G\langle T_{\mu\nu}\rangle$ is satisfied if we identify

$$\sigma = \frac{8\pi G\hbar}{3} c_W.$$

← Hawking radiation ∝ c_W

$$\frac{da(u)}{du} = -\frac{\sigma}{a(u)^2}$$

Consistency check (1/2)
Energy condition of $\langle T_{\mu\nu}\rangle$

$$4\pi \int_{\sim\sqrt{c_W}\,l_p}^{R(a)} dr\, r^2 (-\langle T^t_t\rangle) \approx \frac{a}{2G} = M$$

$$0 < -\langle T^t_t\rangle = \frac{1}{8\pi G}\frac{1}{r^2} = \langle T^r_r\rangle$$

The dominant energy condition ($\rho \geq p_i > 0$) $\overset{\wedge}{\wedge}$ is broken.

⇗ Not a fluid

$$\langle T^\theta_\theta\rangle = \frac{1}{8\pi G}\frac{3}{16\pi c_W l_p^2}$$

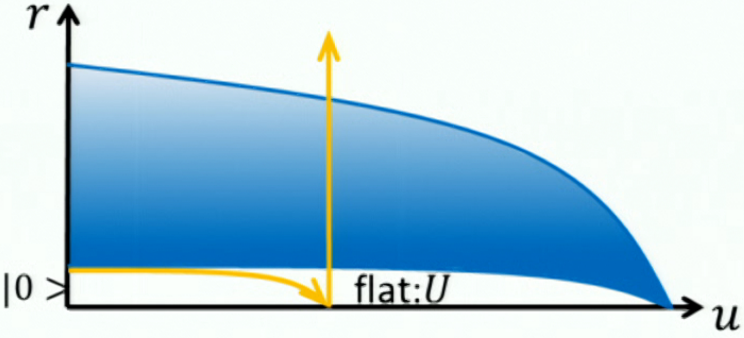
A large angular pressure supports the object, which is consistent with the anomaly.

Consistency check（2/2）: Hawking radiation

- Hawking radiation appears self-consistently:

By a similar manner to Hawking's derivation, we can show

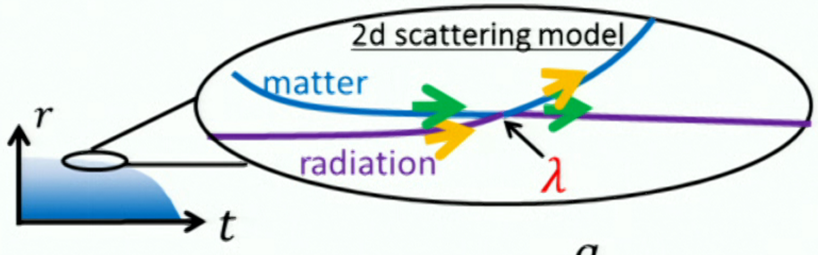
$$\langle 0|\widehat{N}_\omega|0\rangle = \frac{1}{e^{\hbar\omega/T}-1}, \qquad T = \frac{\hbar}{4\pi a(u)}$$



$$\frac{da(u)}{du} = -\frac{\sigma}{a(u)^2}$$

Information recovery and BH entropy

The matter and radiation interact inside the BH.



$$\Delta t_{scat} \sim a \log \frac{a}{\lambda c_W l_p}$$ ~ scrambling time

⇒information recovery (state-dependence)?

a precise analysis of echo signal?

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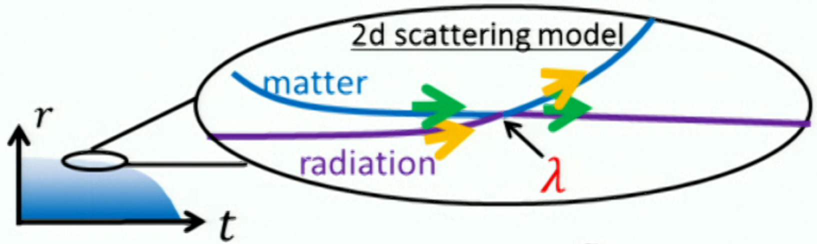


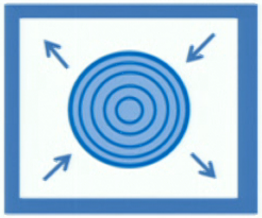
$$\Delta t_{scat} \sim a \log \frac{a}{\lambda c_W l_p}$$ ~ scrambling time

⇒information recovery (state-dependence)?

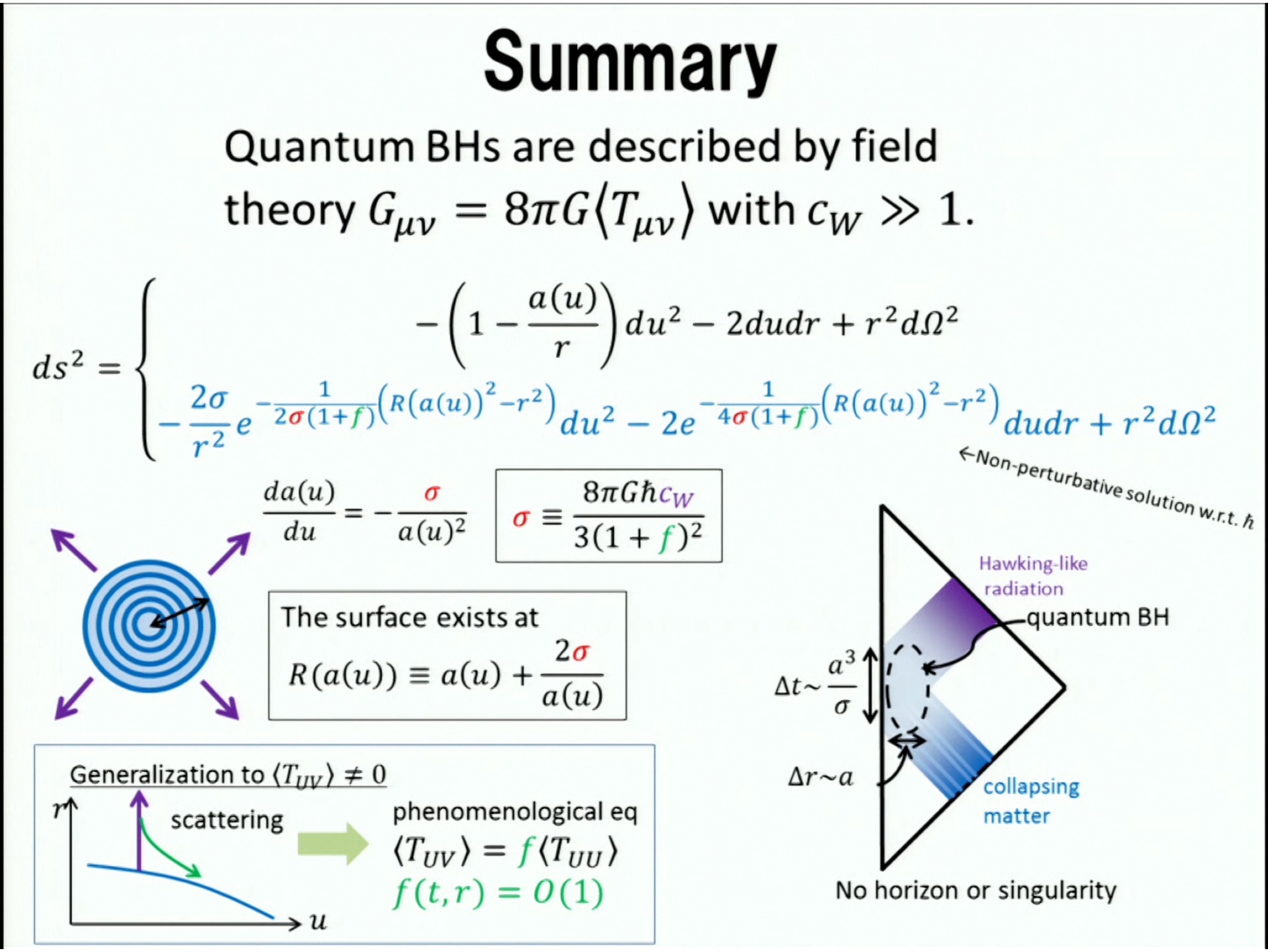
a precise analysis of echo signal?

Reproduce area law by integrating entropy density s over volume:

$$S = \int d^3x \sqrt{h}\, s = \frac{A}{4 l_p^2}$$



⇒The information should be stored inside the BH!



Summary

Quantum BHs are described by field theory $G_{\mu\nu} = 8\pi G\langle T_{\mu\nu}\rangle$ with $c_W \gg 1$.

$$ds^2 = \begin{cases} -\left(1-\frac{a(u)}{r}\right)du^2 - 2dudr + r^2 d\Omega^2 \\ -\frac{2\sigma}{r^2}e^{-\frac{1}{2\sigma(1+f)}\left(R(a(u))^2-r^2\right)}du^2 - 2e^{-\frac{1}{4\sigma(1+f)}\left(R(a(u))^2-r^2\right)}dudr + r^2 d\Omega^2 \end{cases}$$

←Non-perturbative solution w.r.t. $\hbar$

$$\frac{da(u)}{du} = -\frac{\sigma}{a(u)^2} \qquad \boxed{\sigma \equiv \frac{8\pi G\hbar c_W}{3(1+f)^2}}$$

The surface exists at

$$R(a(u)) \equiv a(u) + \frac{2\sigma}{a(u)}$$

Generalization to $\langle T_{UV}\rangle \neq 0$

phenomenological eq

$$\langle T_{UV}\rangle = f\langle T_{UU}\rangle$$

$$f(t,r) = O(1)$$

No horizon or singularity