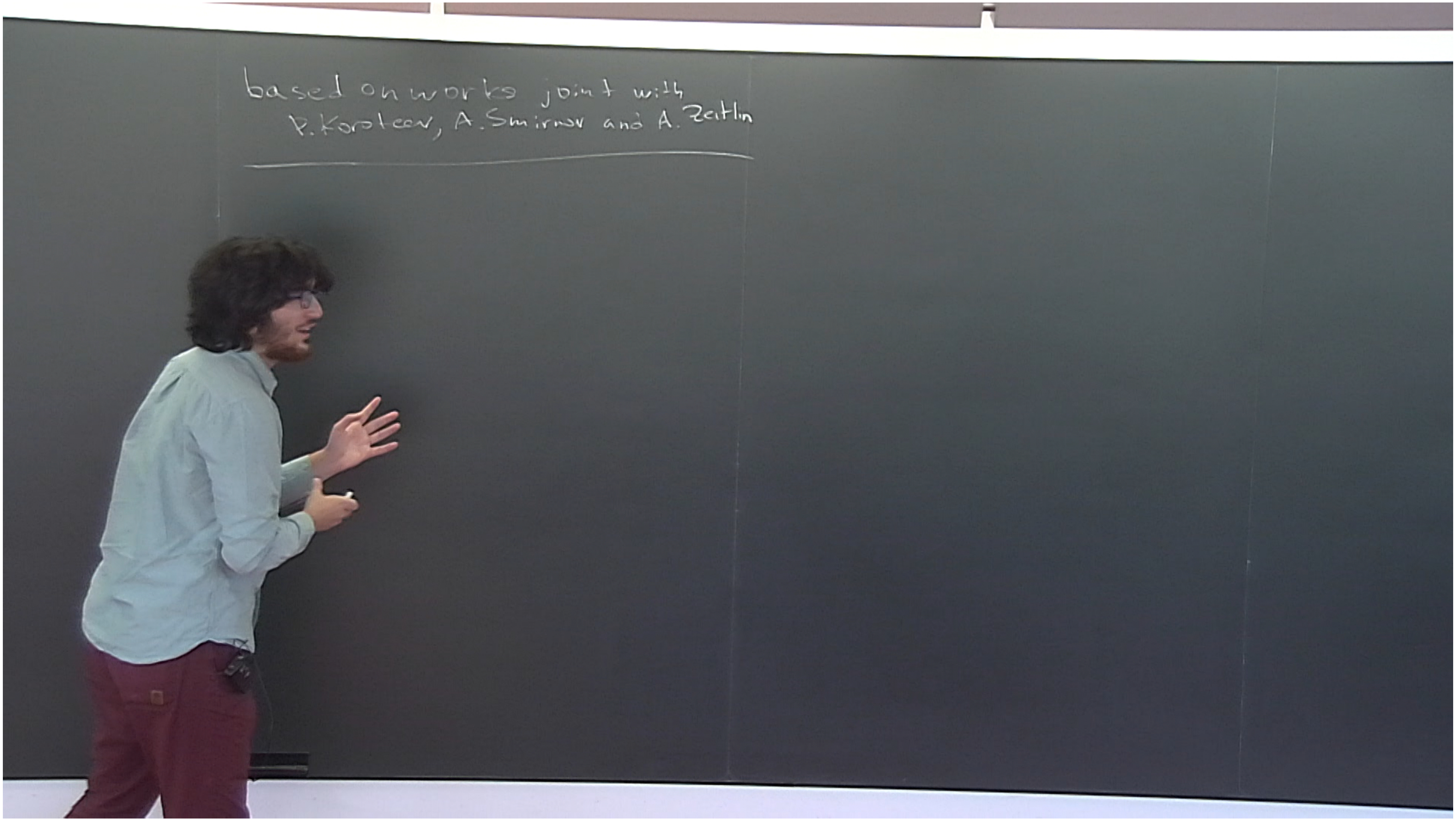
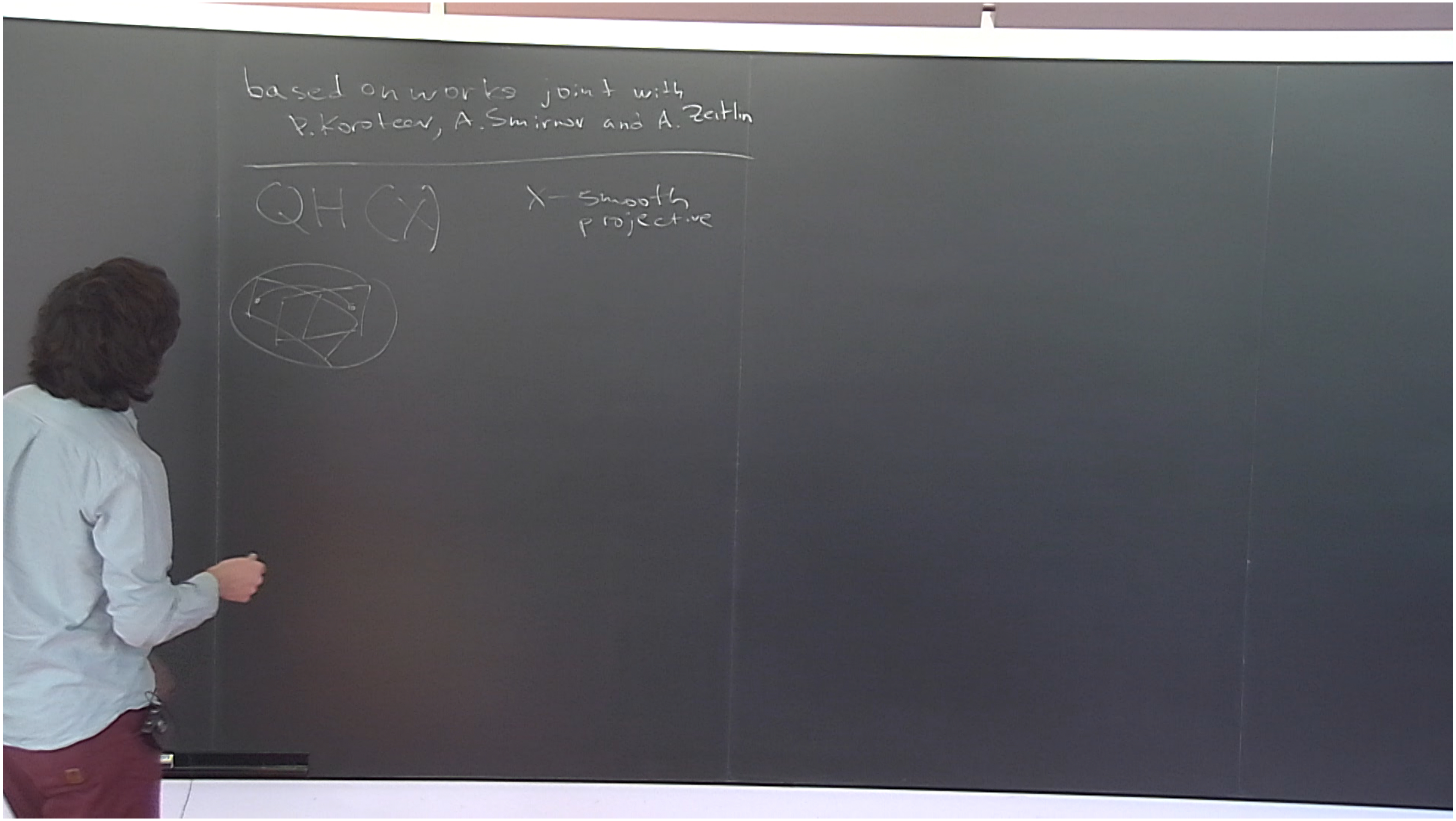
Title: Quantum K-theory of quiver varieties and quantum integrable systems

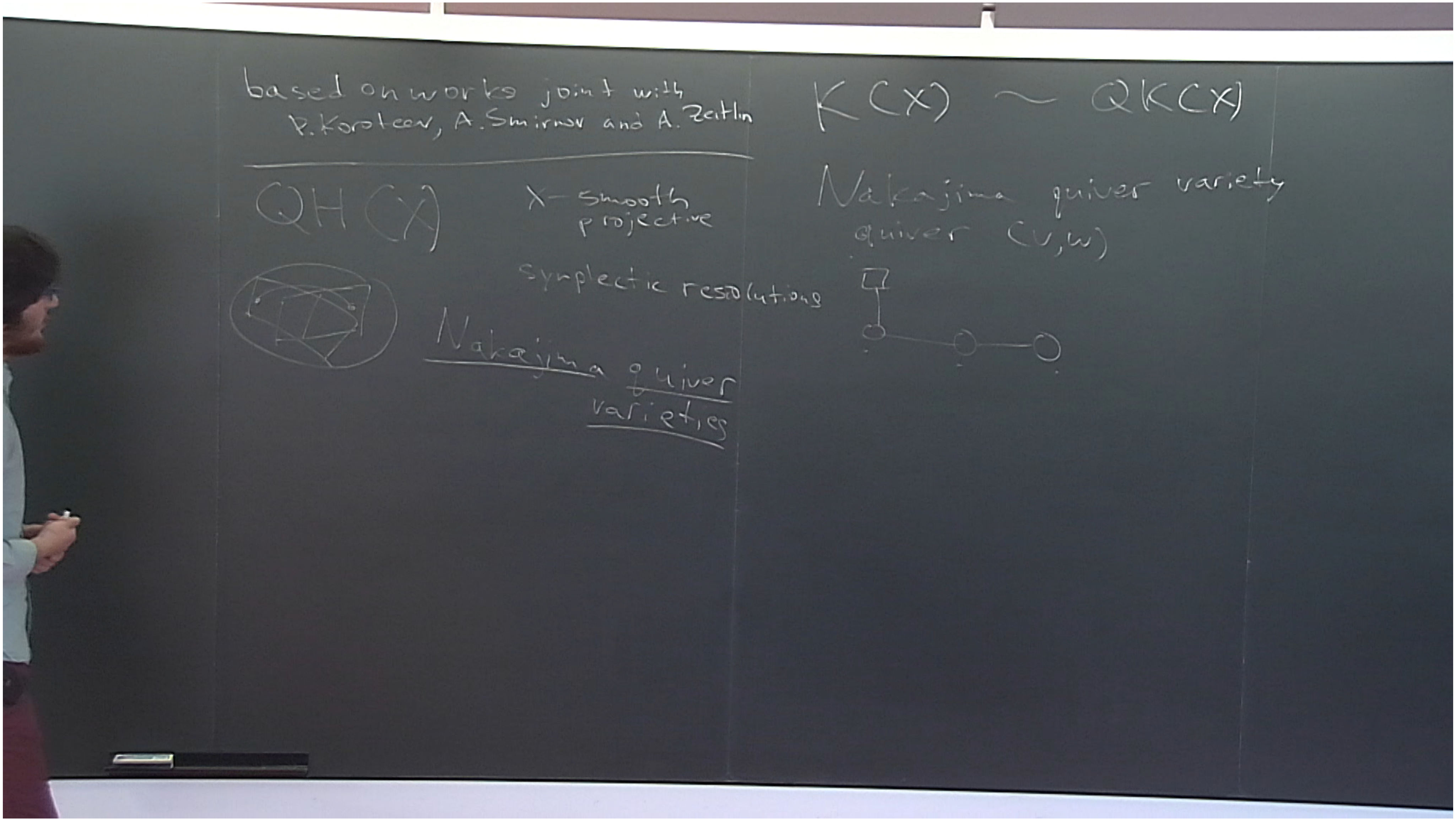
Date: Nov 06, 2017 01:00 PM

URL: http://pirsa.org/17110068

Abstract: <p>In this talk I will define the quantum K-theory of Nakajima quiver varieties and show its connection to representation theory of quantum groups and quantum integrable systems on the examples of the Grassmannian and the flag variety. In particular, the Baxter operator will be identified with operators of quantum multiplication by quantum tautological classes via Bethe equations. Quantum tautological classes will also be constructed and, time permitting, an explicit universal combinatorial formula for them will be shown.
Based on joint works with P.Koroteev, A.Smirnov and A.Zeitlin</p>


based on works joint with
P. Koroteev, A. Smirnov and A. Zeitlin


based on works joint with
P. Koroteev, A. Smirnov and A. Zeitlin
QH(X)
X - smooth
projective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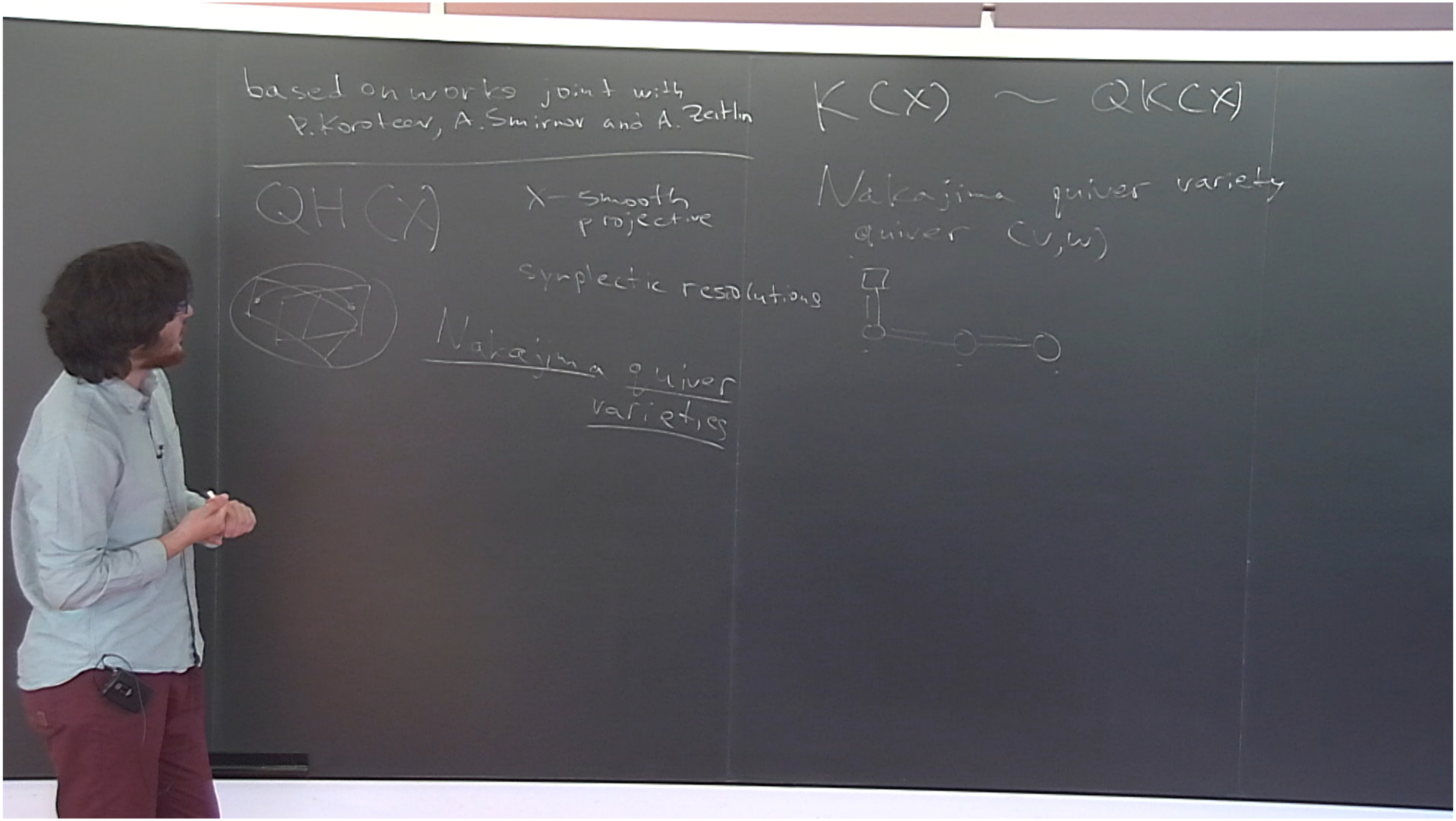
QH(X)
X - smooth projective
symplectic resolutions
Nakajima quiver varieties
K(X) ~ QK(X)
Nakajima quiver variety
quiver (v,w)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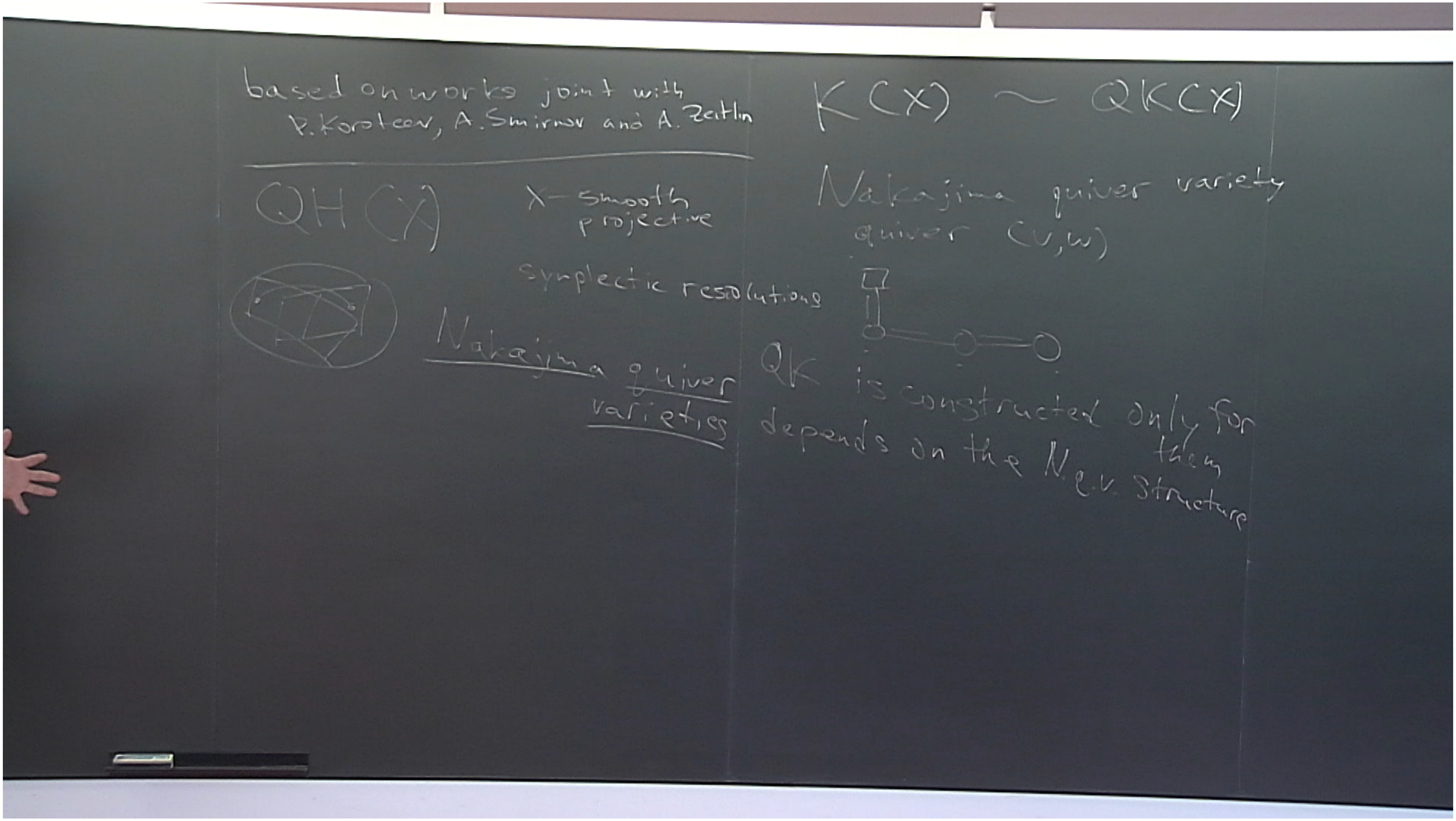
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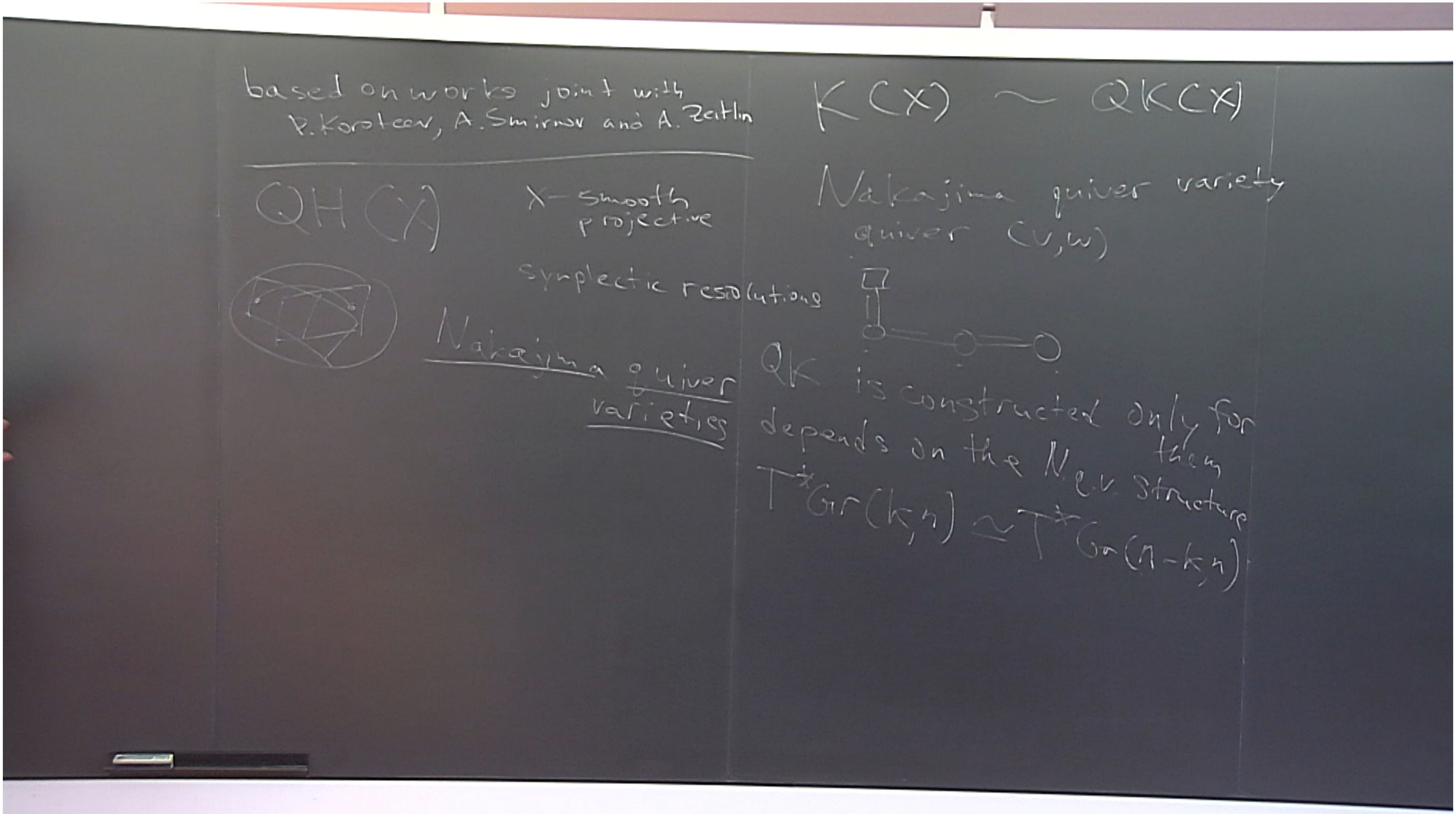
QK is constructed only for them
depends on the N.q.v. structure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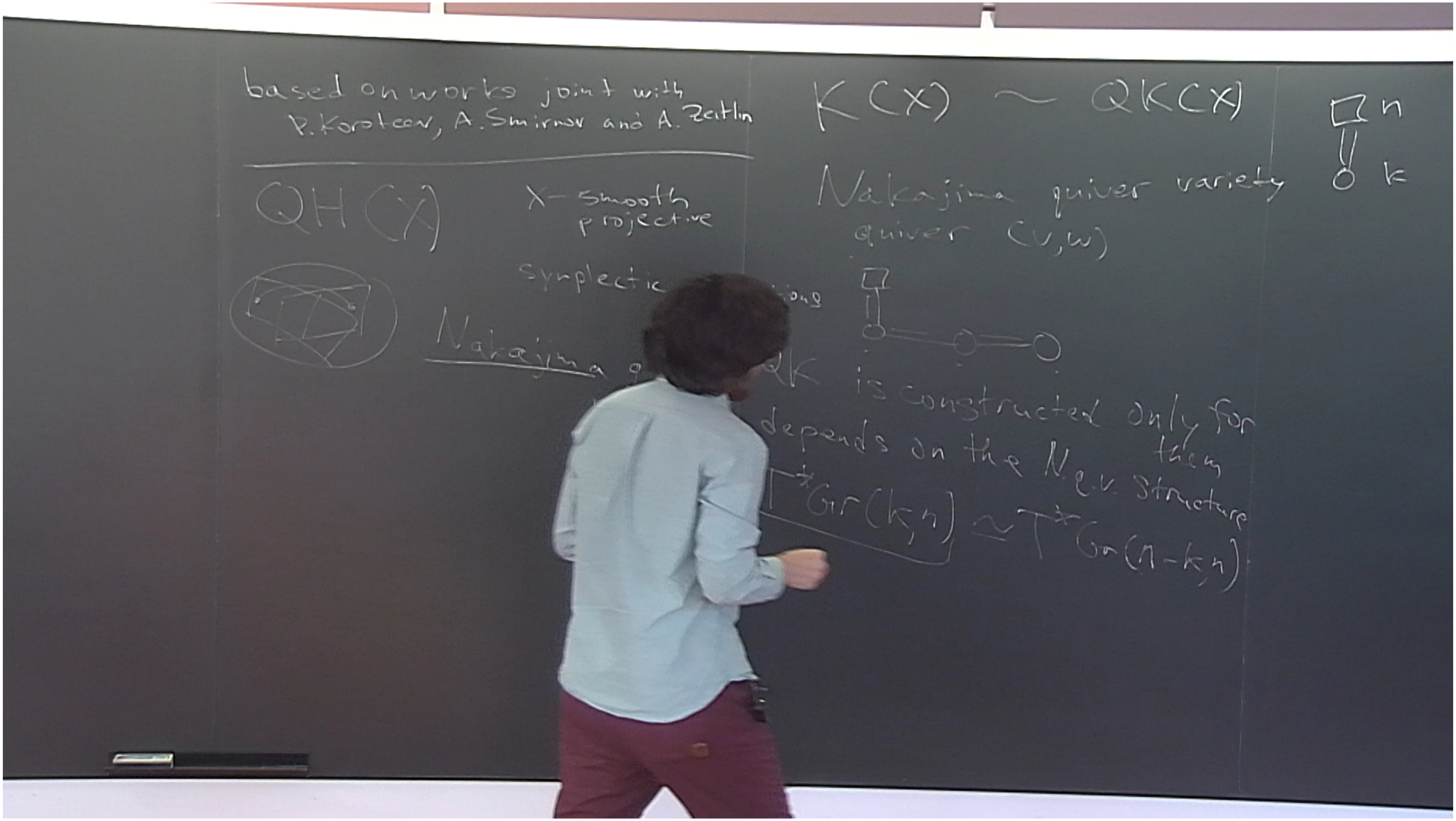
T*Gr(k,n) ≃ T*Gr(n-k,n)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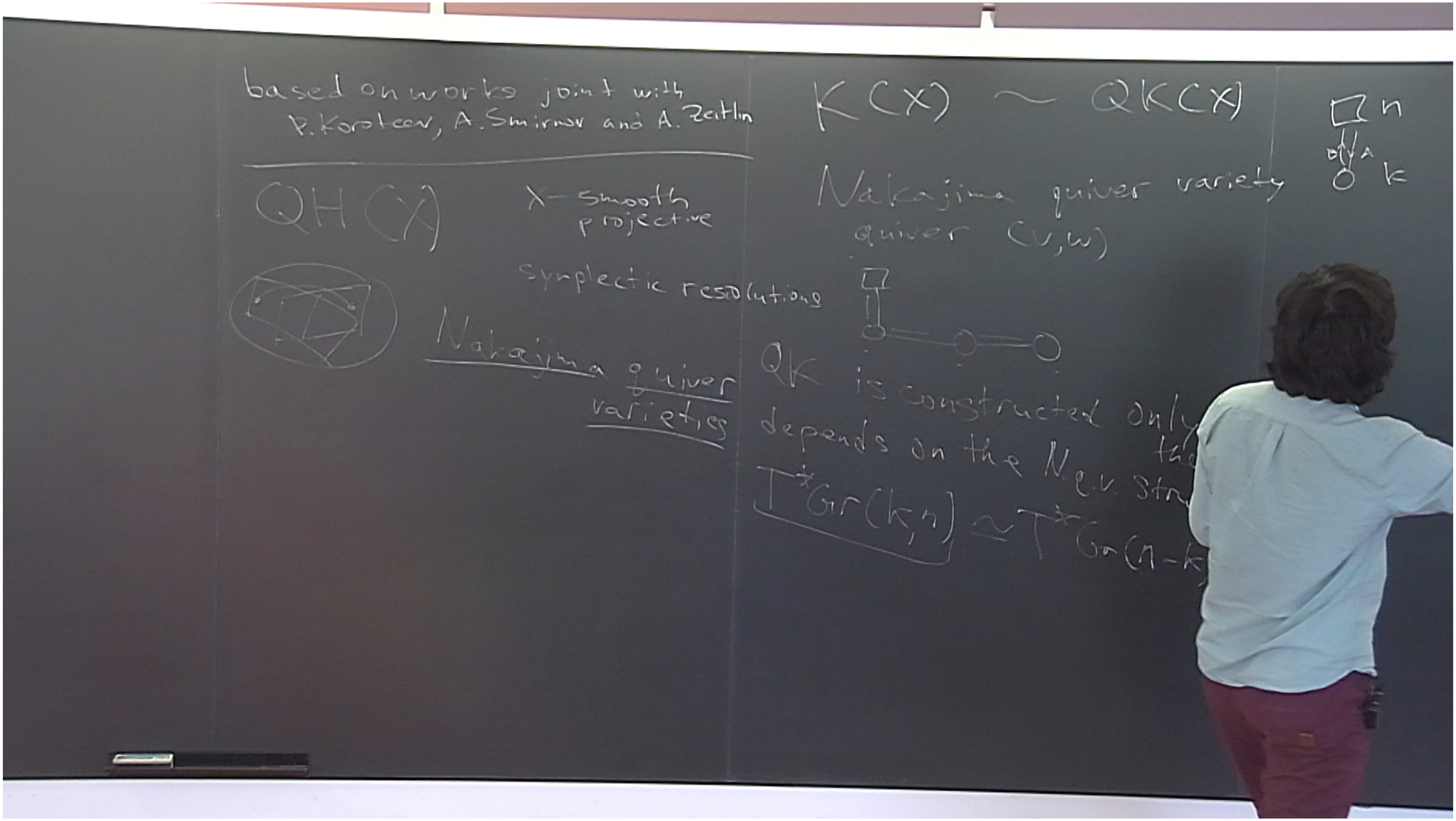
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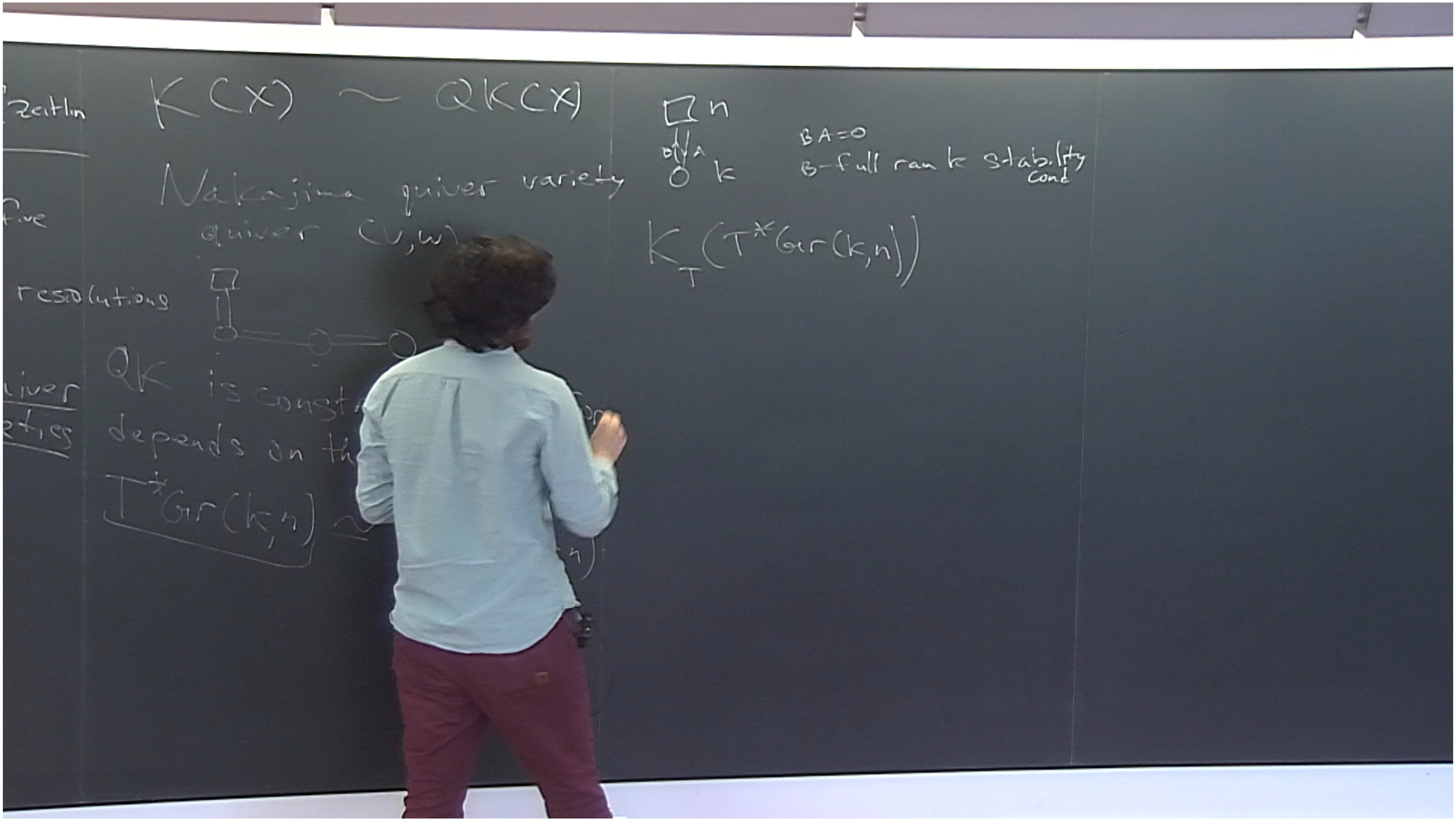
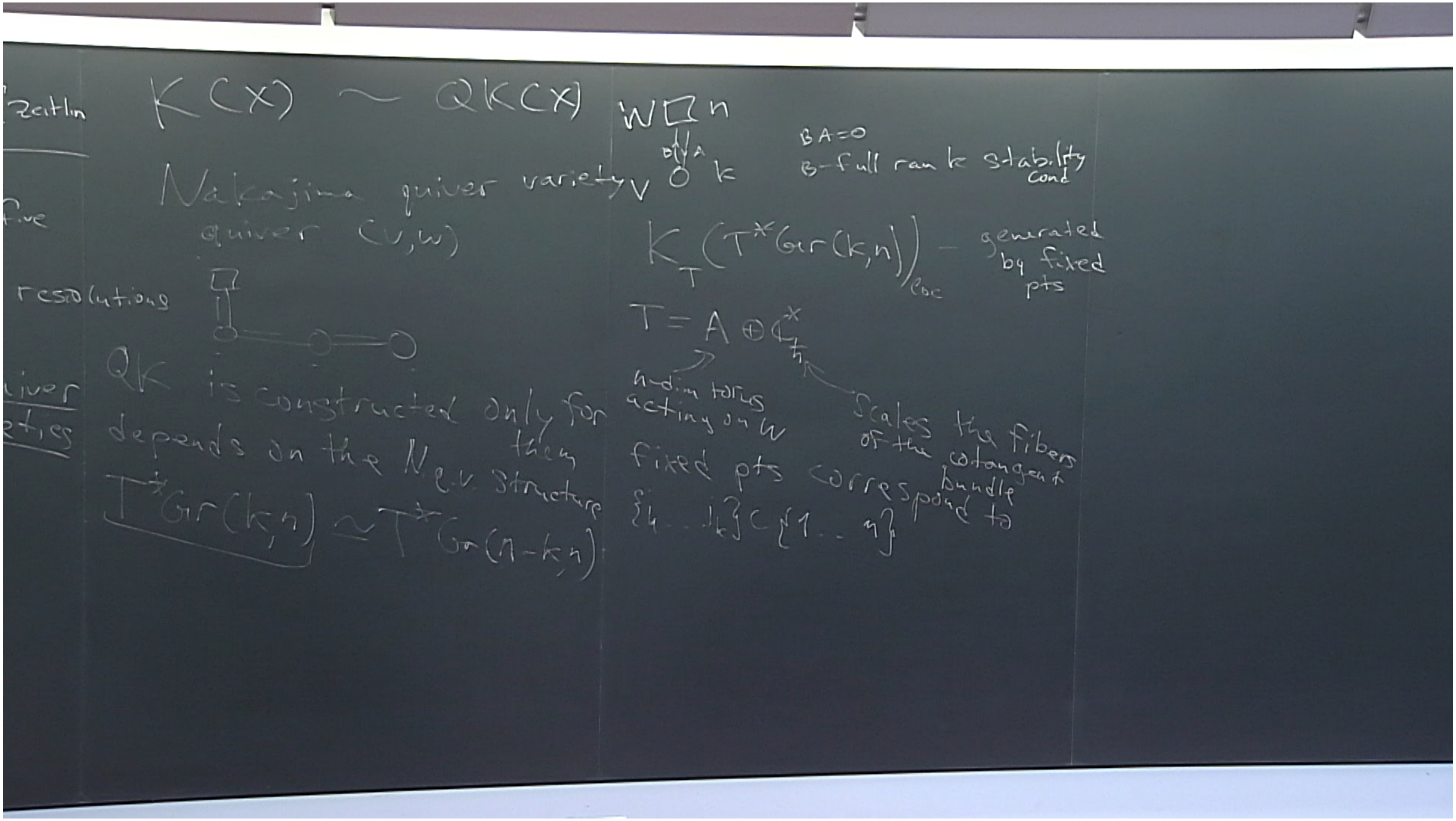

Zeitlin
K(X) ~ QK(X)
Nakajima quiver variety
quiver (v,w)
resolutions
QK is const
depends on th
T*Gr(k,n) ~
n
k
BA=0
B-full rank stability cond
K_T(T*Gr(k,n))


Zeitlin
K(X) ~ QK(X)
Nakajima quiver variety
quiver (v,w)
resolutions
QK is constructed only for them
depends on the N.e.v. structure
T*Gr(k,n) ≃ T*Gr(n-k,n)
W □ n
V ○ k
BA=0
B-full rank stability cond
K_T(T*Gr(k,n))_loc - generated by fixed pts
T = A ⊕ ℂ*_ħ
n-dim torus acting on W
scales the fibers of the cotangent bundle
fixed pts correspond to
{i_1 ... i_k} ⊂ {1 ... n}

Zeitlin

five

resolutions

quiver
...eties

$K(X) \sim QK(X)$

Nakajima quiver variety
quiver (v,w)

QK is constructed only for them
depends on the N e.v. structure

$T^*Gr(k,n) \simeq T^*Gr(n-k,n)$

$W \; \square \; n$
$B \downarrow \uparrow A$
$V \; \bigcirc \; k$

$BA=0$
B - full rank stability cond

$K_T(T^*Gr(k,n))_{loc}$ - generated by fixed pts — $\binom{n}{k}$ dim

$T = A \oplus \mathbb{C}^*_\hbar$

n-dim torus acting on W

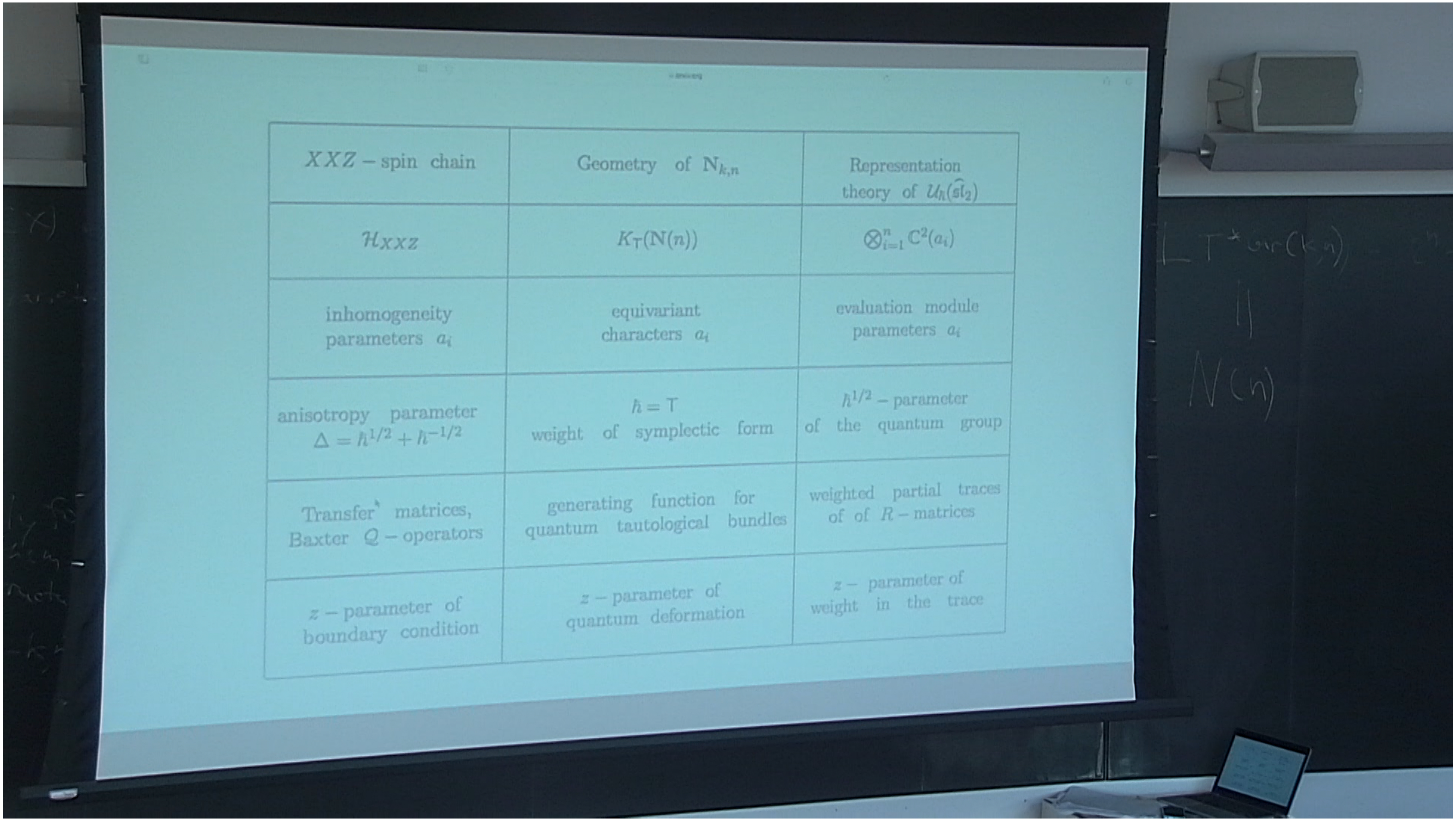
scales the fibers of the cotangent bundle

fixed pts correspond to
$\{i_1 \dots i_k\} \subset \{1 \dots n\}$


K_1(⊔ T*Gr(k,n)) - 2^n-dim

arxiv.org

$XXZ-$spin chain	Geometry of $\mathbf{N}_{k,n}$	Representation theory of $\mathcal{U}_\hbar(\widehat{\mathfrak{sl}}_2)$
$\mathcal{H}_{XXZ}$	$K_{\mathsf{T}}(\mathbf{N}(n))$	$\bigotimes_{i=1}^{n} \mathbb{C}^2(a_i)$
inhomogeneity parameters a_i	equivariant characters a_i	evaluation module parameters a_i
anisotropy parameter $\Delta = \hbar^{1/2} + \hbar^{-1/2}$	$\hbar = \mathsf{T}$ weight of symplectic form	$\hbar^{1/2}-$parameter of the quantum group
Transfer matrices, Baxter $\mathcal{Q}-$operators	generating function for quantum tautological bundles	weighted partial traces of of $R-$matrices
$z-$parameter of boundary condition	$z-$parameter of quantum deformation	$z-$ parameter of weight in the trace



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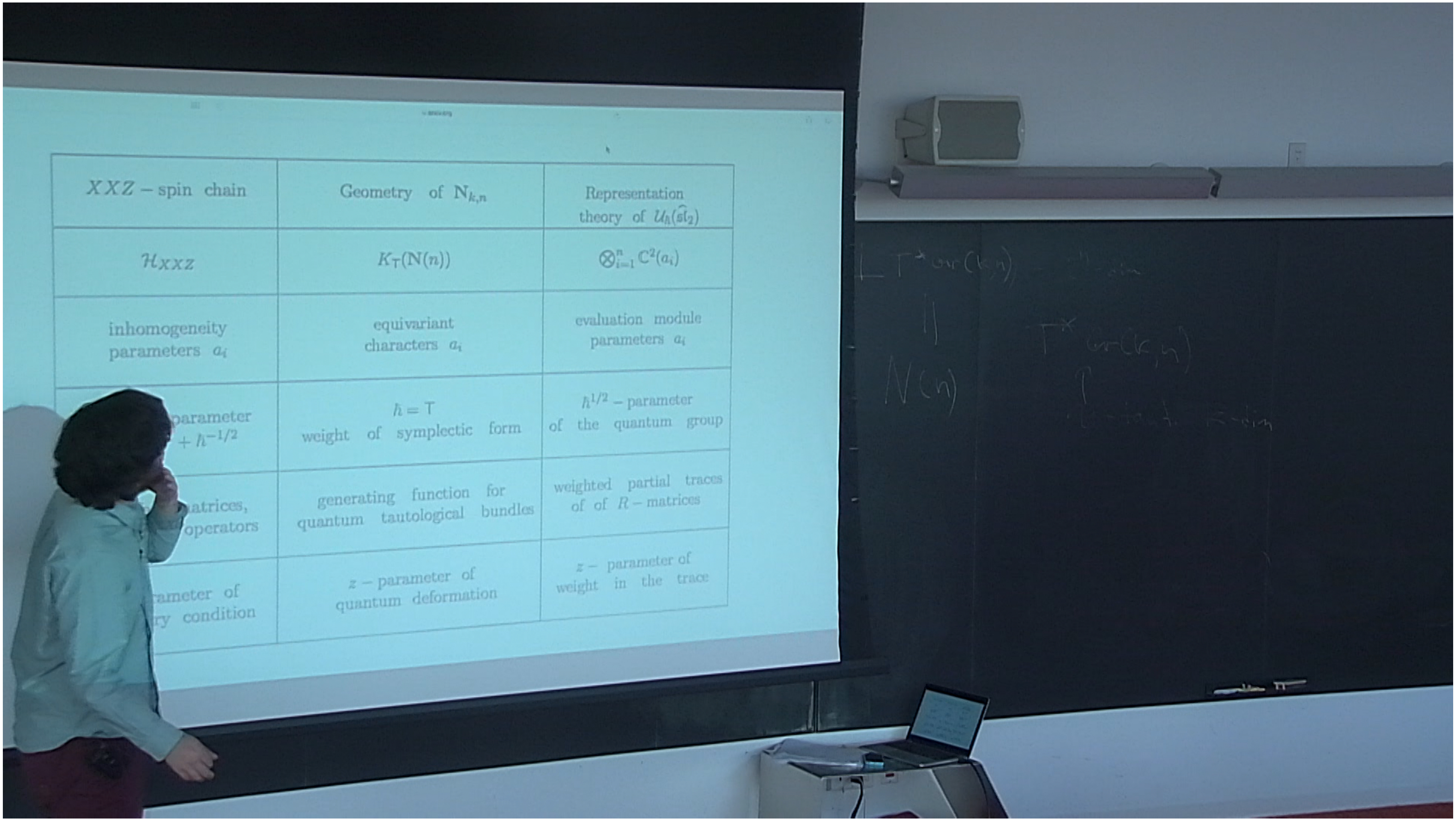
XXZ – spin chain
Geometry of N_{k,n}
Representation theory of U_ħ(ŝl_2)
H_XXZ
K_T(N(n))
⊗_{i=1}^n C^2(a_i)
inhomogeneity parameters a_i
equivariant characters a_i
evaluation module parameters a_i
parameter
+ ħ^{-1/2}
ħ = T
weight of symplectic form
ħ^{1/2} – parameter of the quantum group
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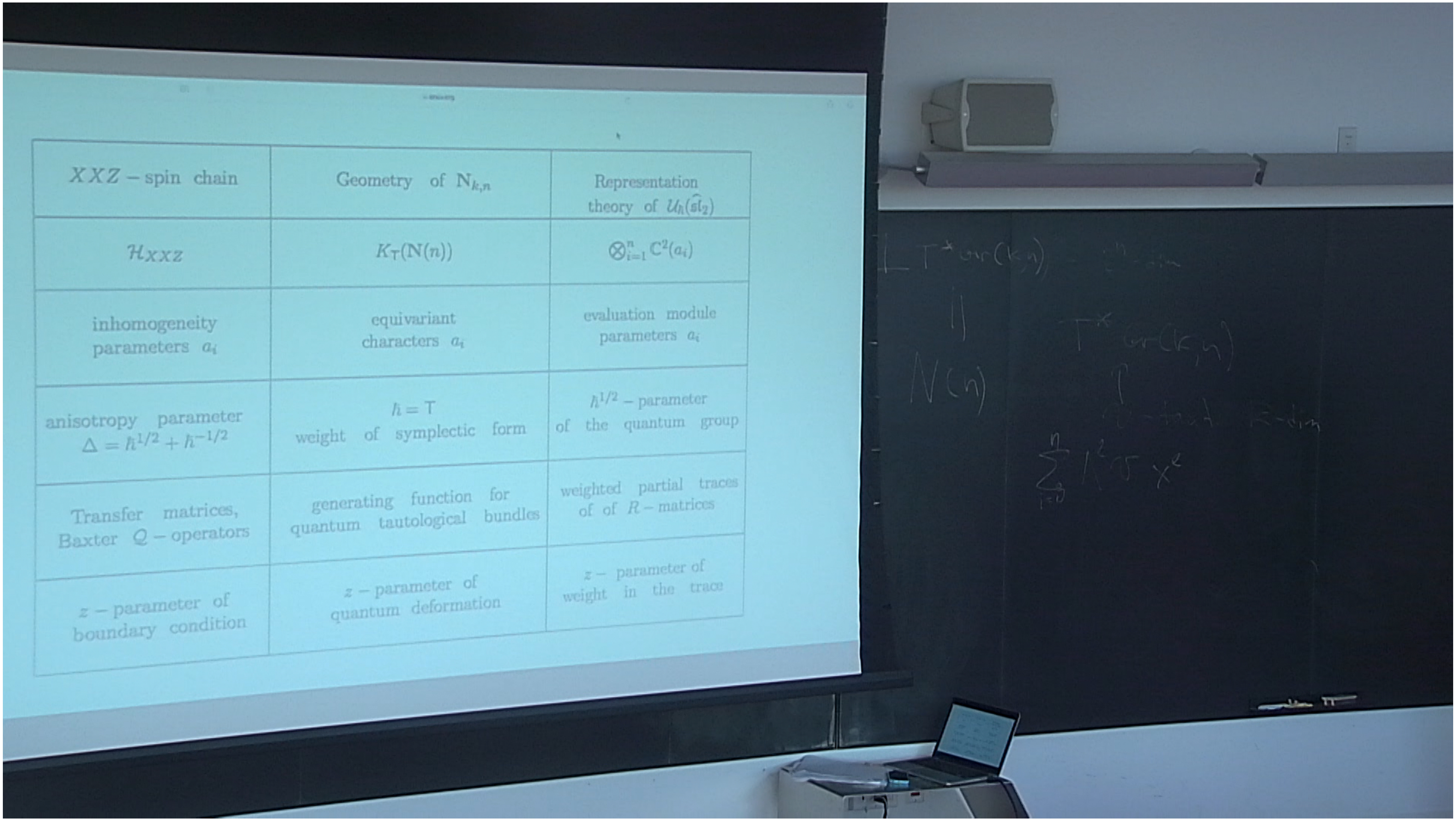
T* Gr(k,n)
N(n)



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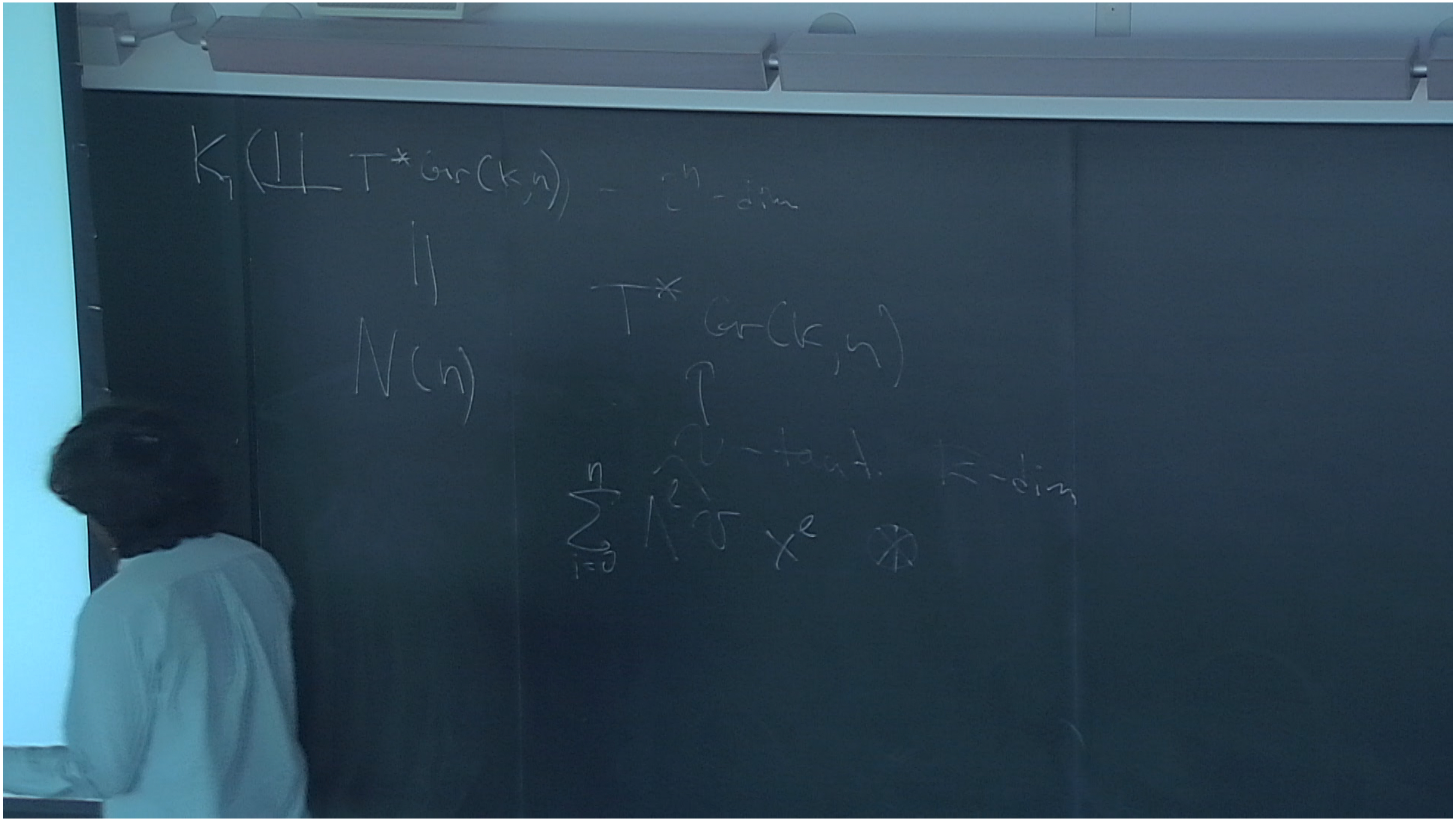
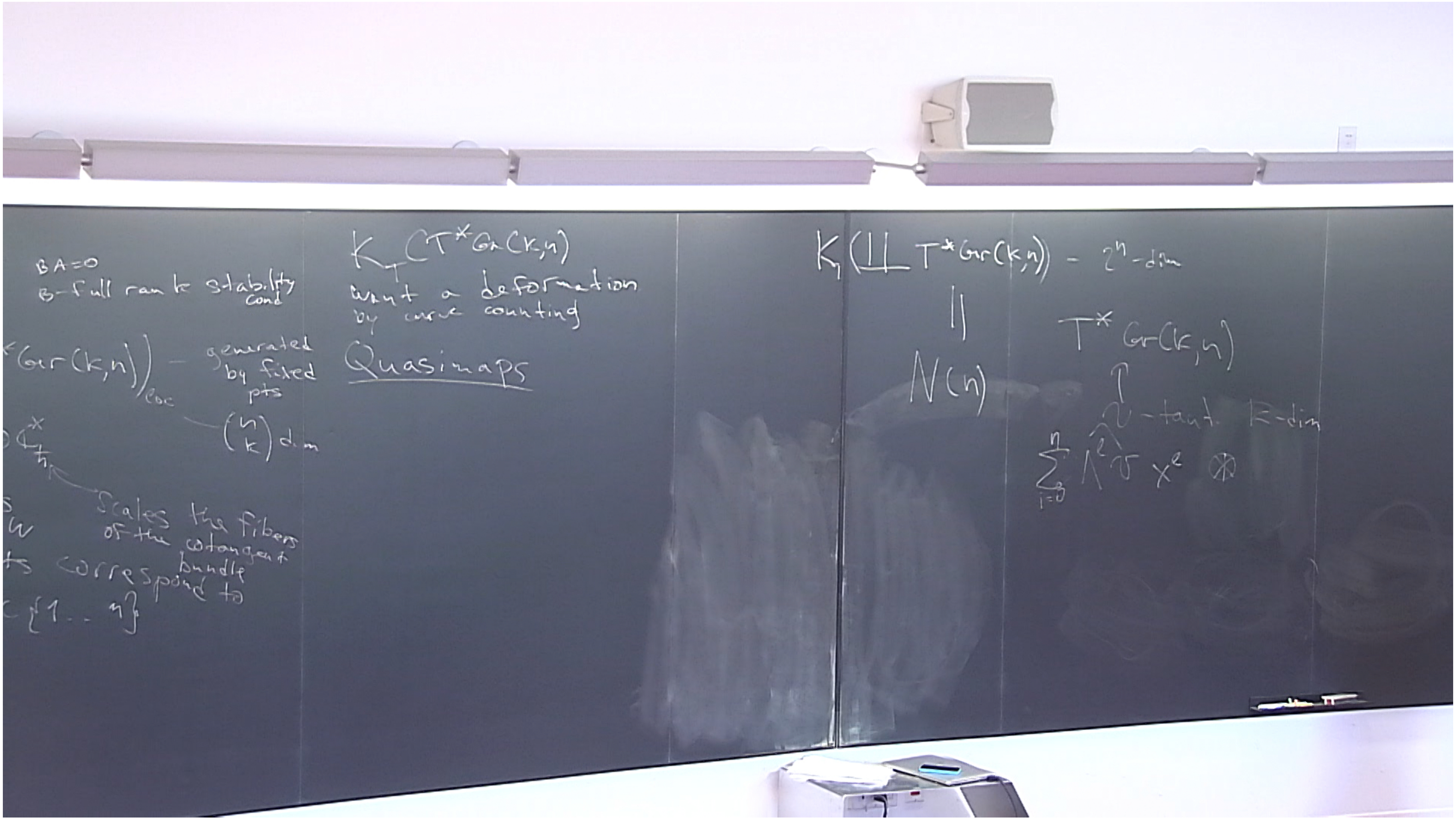
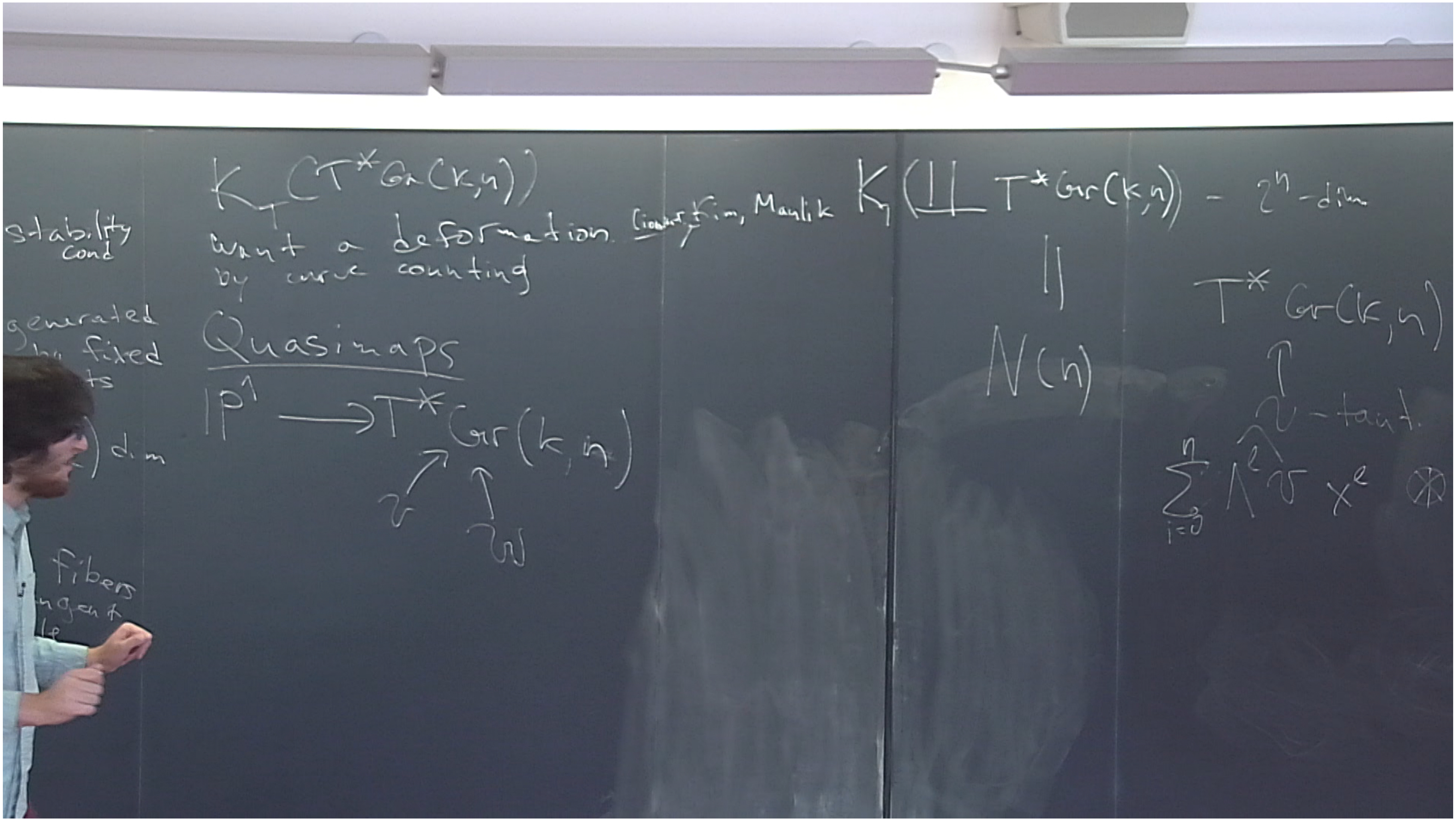
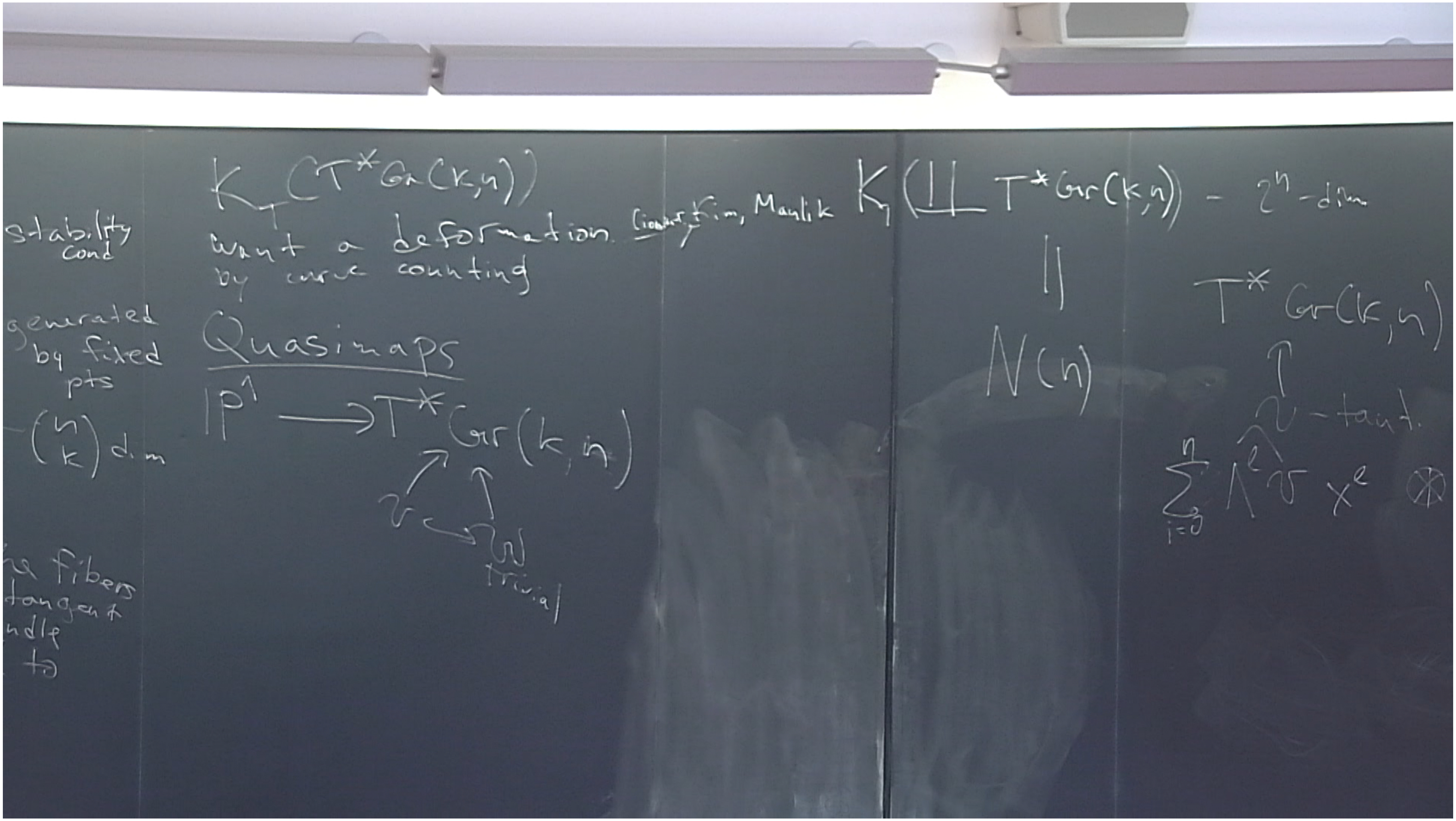
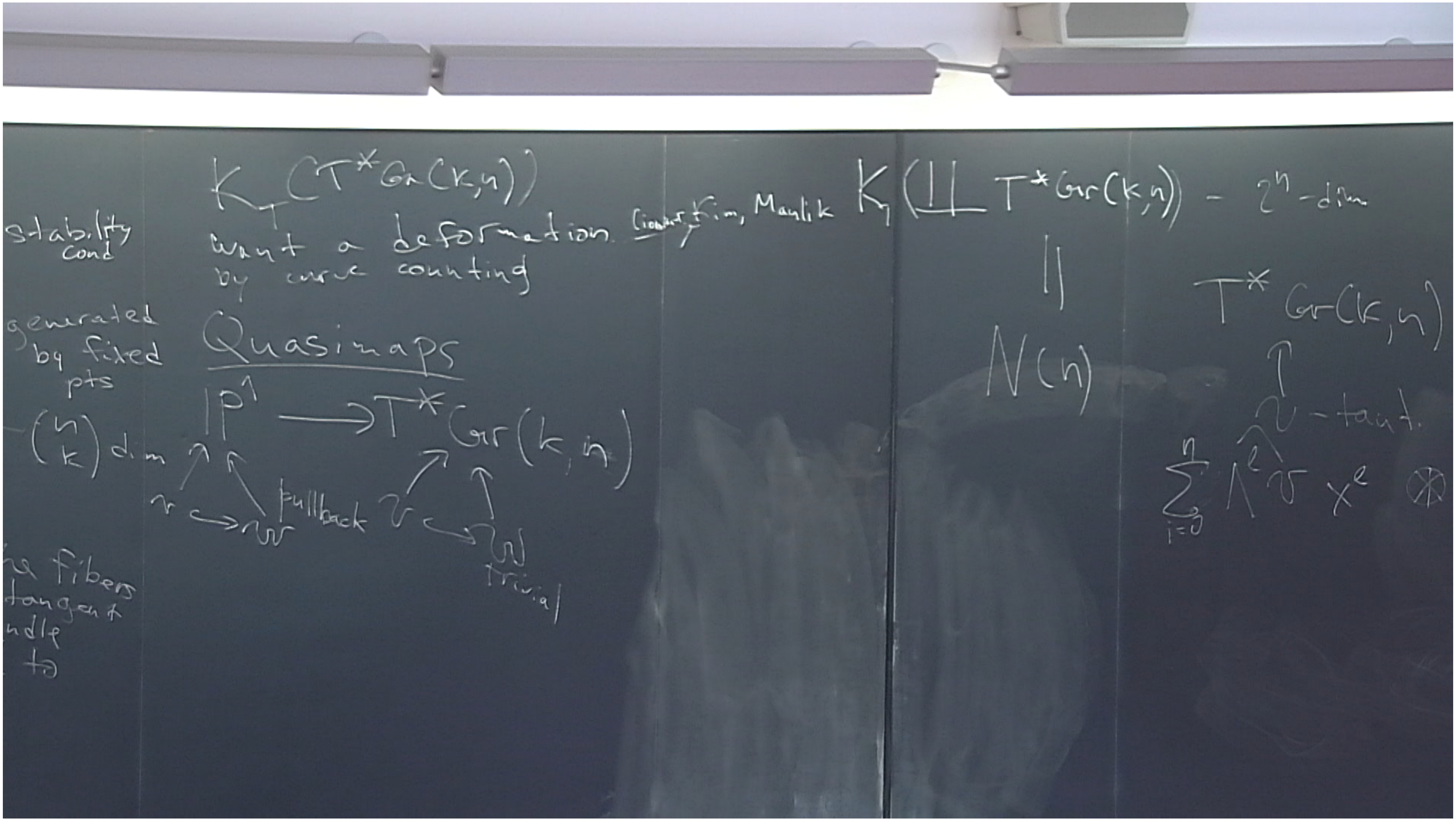

K_T(⊔ T* Gr(k,n)) – C^n-dim
N(n)
T* Gr(k,n)
V – taut. k-dim
Σ_{i=0}^n Λ^i V x^i

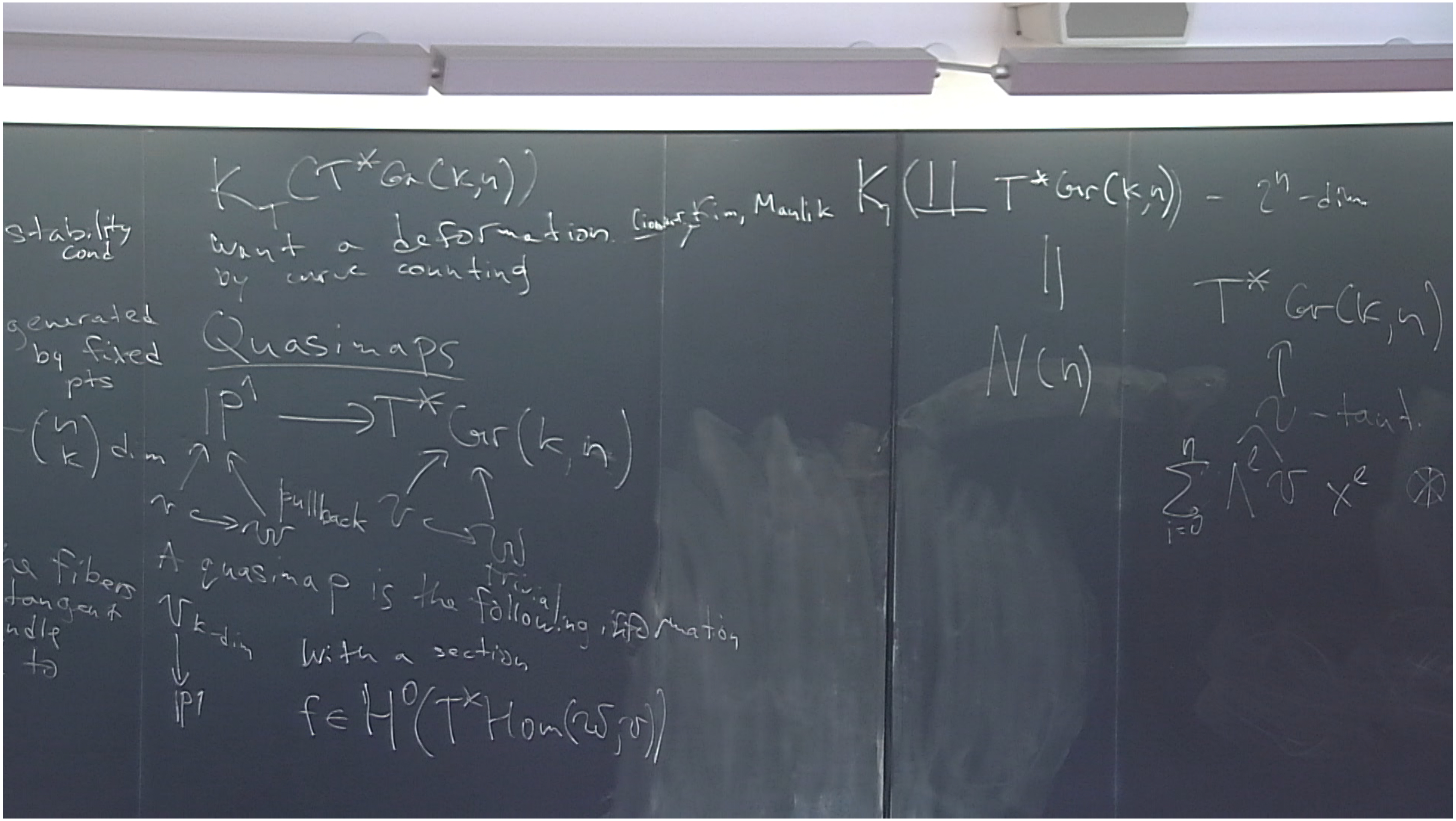

BA=0
B-full rank stability cond
*Gr(k,n))_loc - generated by fixed pts
(n k) dim
scales the fibers of the cotangent bundle
correspond to
{1 .. n}
K_T(T*Gr(k,n))
want a deformation by curve counting
Quasimaps
K_T(⊔ T*Gr(k,n)) - 2^n-dim
N(n)
T* Gr(k,n)
V - taut. k-dim


stability
cond
generated
fixed
dim
fibers
K_T(T*Gr(k,n))
want a deformation
by curve counting
Quasimaps
P^1 → T*Gr(k,n)
V
W
K_T(⊔ T*Gr(k,n)) - 2^n-dim
N(n)
T*Gr(k,n)
V - taut.
Σ_{i=0}^{n} Λ^i V x^i



stability cond
generated by fixed pts
(n k) dim
fibers
tangent
ndle
to
K_T(T*Gr(k,n))
Want a deformation
by curve counting
Quasimaps
P^1 → T*Gr(k,n)
V ↪ W
trivial
K_T(⨆ T*Gr(k,n)) - 2^n-dim
N(n)
T* Gr(k,n)
V - taut.
Σ_{i=0}^n Λ^l V x^l


stability cond
generated by fixed pts
dim
fibers
tangent
K_T(T*Gr(k,n))
want a deformation by curve counting
Maulik
Quasimaps
P^1 → T*Gr(k,n)
pullback
trivial
K_T(⊔ T*Gr(k,n)) - 2^n-dim
N(n)
T*Gr(k,n)
- taut.


K_T(T*Gr(k,n))
want a deformation
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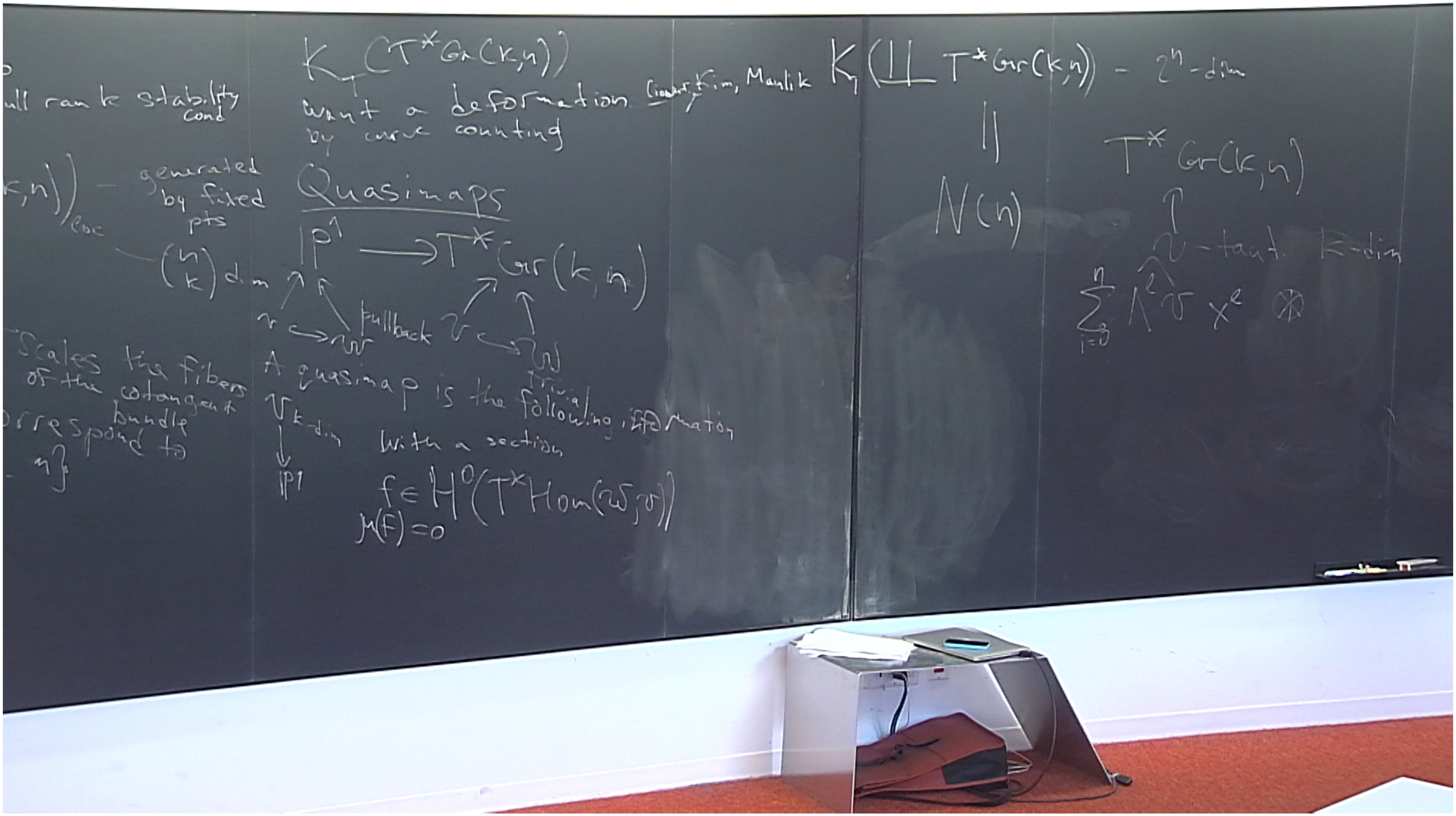
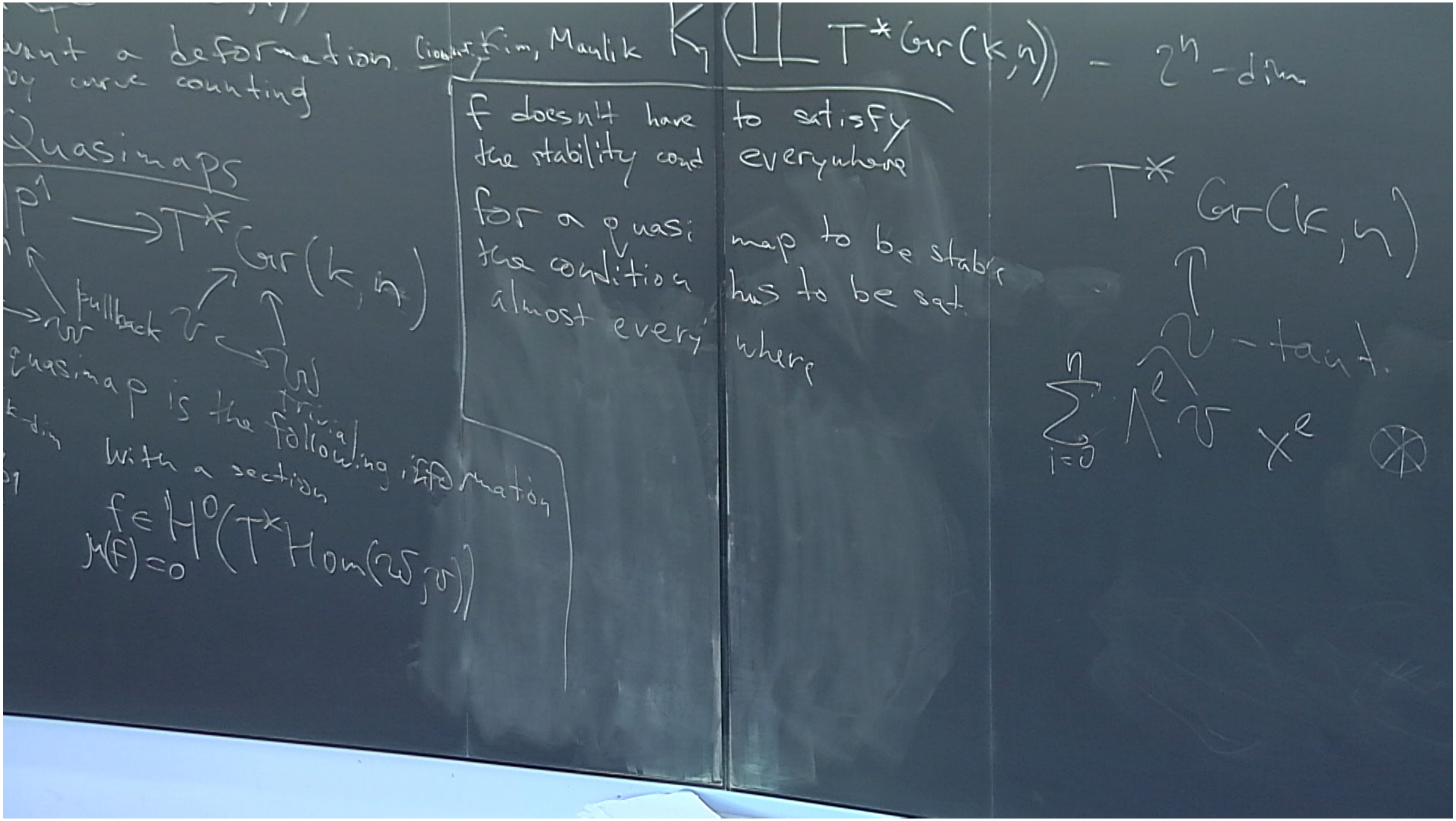
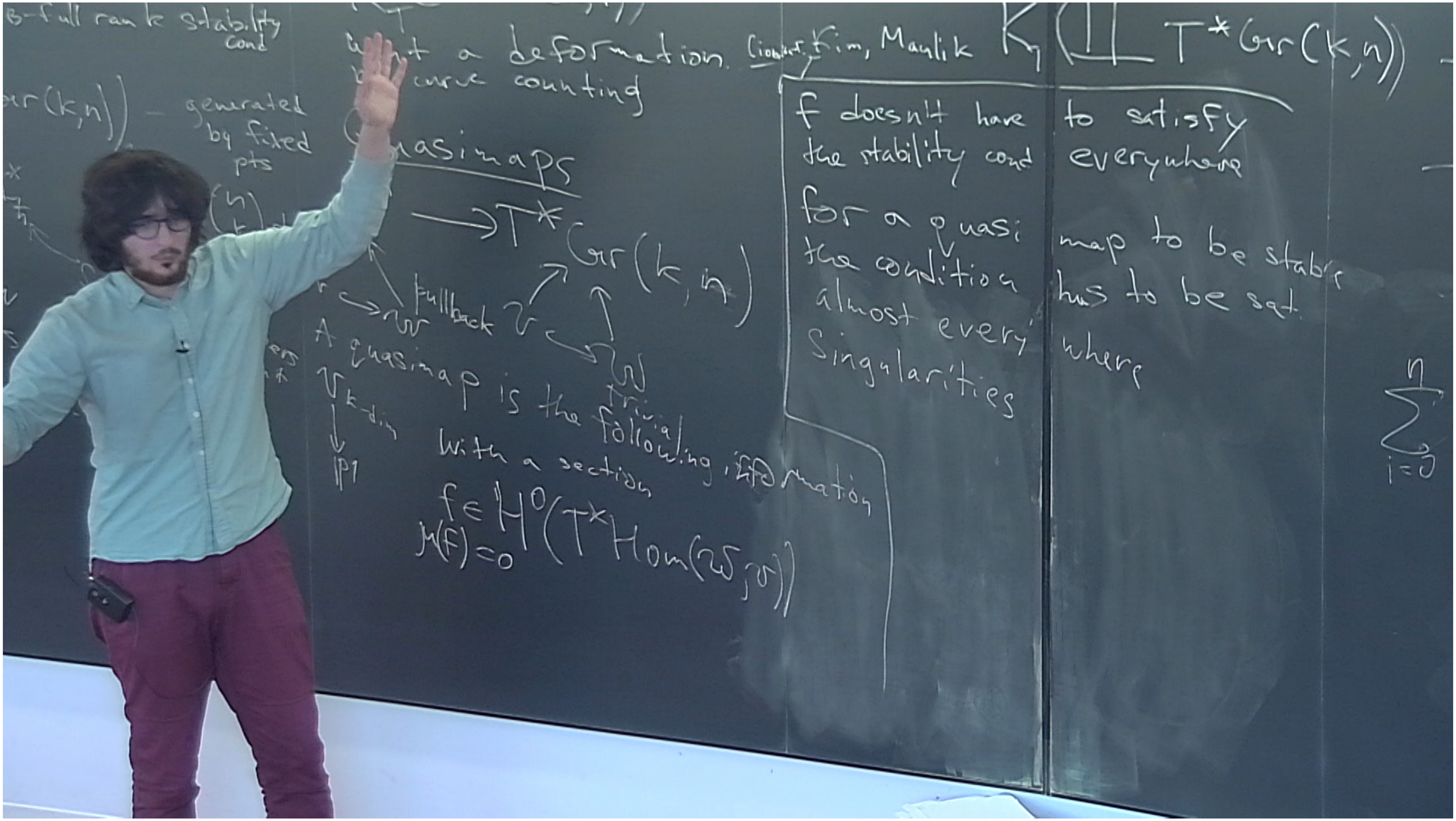
Maulik
K_T(∐ T*Gr(k,n)) - 2^n-dim
N(n)
T* Gr(k,n)
taut.
stability cond
generated by fixed pts
(n k) dim
Quasimaps
P1 → T*Gr(k,n)
pullback
trivial
A quasimap is the following information
V k-dim
P1
with a section
f ∈ H0(T*Hom(W,V))
fibers
tangent
bundle
to


full rank stability cond
generated by fixed pts
Scales the fibers of the cotangent bundle
correspond to
$K_T(T^*Gr(k,n))$
want a deformation
by curve counting
Quasimaps
$\mathbb{P}^1 \longrightarrow T^*Gr(k,n)$
pullback
trivial
A quasimap is the following information
$V_{k\text{-dim}}$
with a section
$f \in H^0(T^*Hom(W,V))$
$\mu(f)=0$
Kim, Maulik
$K_T(\coprod T^*Gr(k,n))$ - 2^n-dim
$N(n)$
$T^*Gr(k,n)$
taut. k-dim
$\sum_{i=0}^{n} \Lambda^i V x^i$


want a deformation
by curve counting
Quasimaps
pullback
trivial
Quasimap is the following information
With a section
Kim, Maulik
f doesn't have to satisfy
the stability cond everywhere
for a quasi map to be stable
the condition has to be sat.
almost everywhere
2^n - dim
taut.



full rank stability cond
a deformation
curve counting
generated by fixed pts
Quasimaps
Kim, Maulik
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for a quasi map to be stable the condition has to be sat. almost everywhere
Singularities
pullback
trivial
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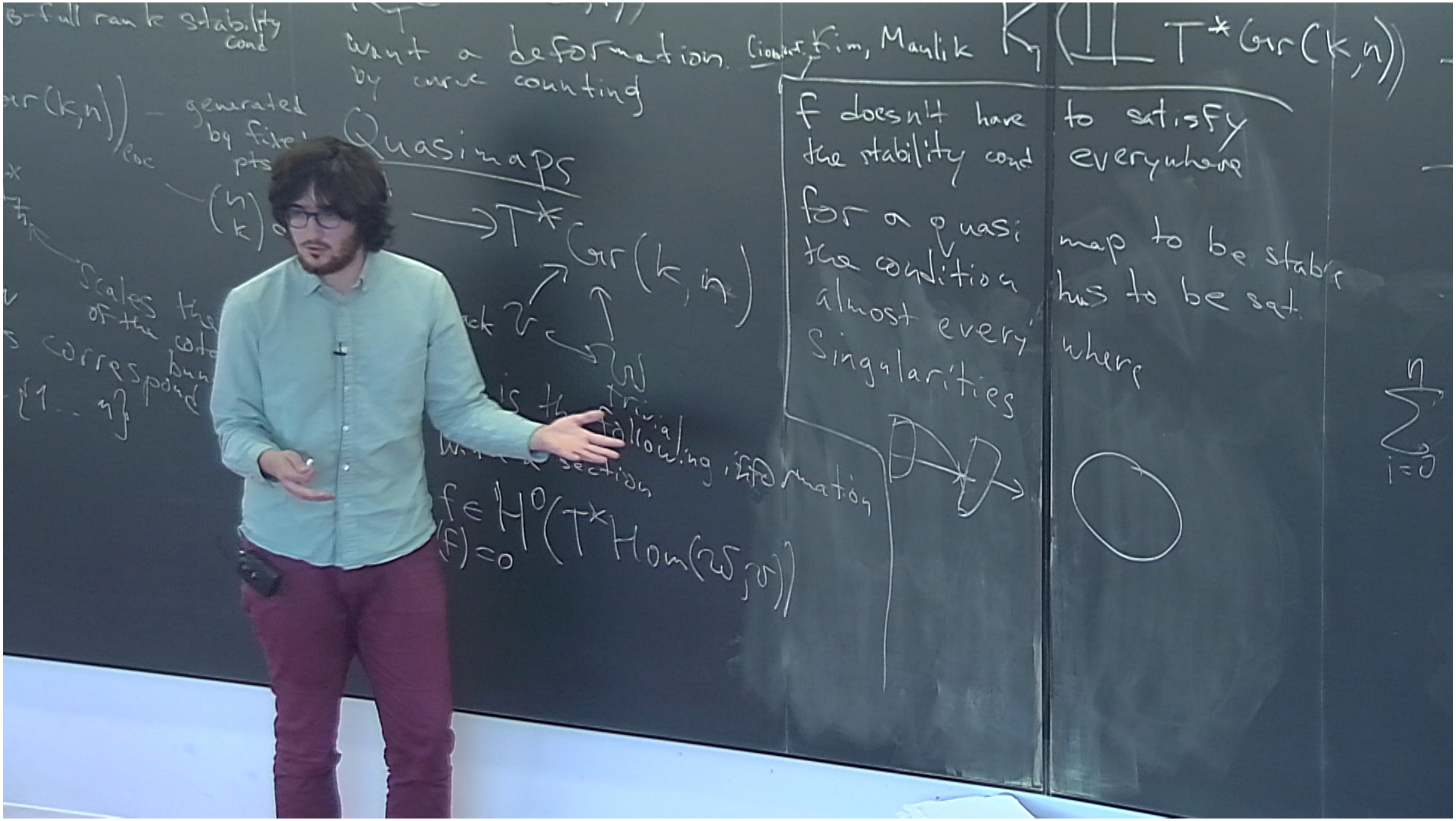
with a section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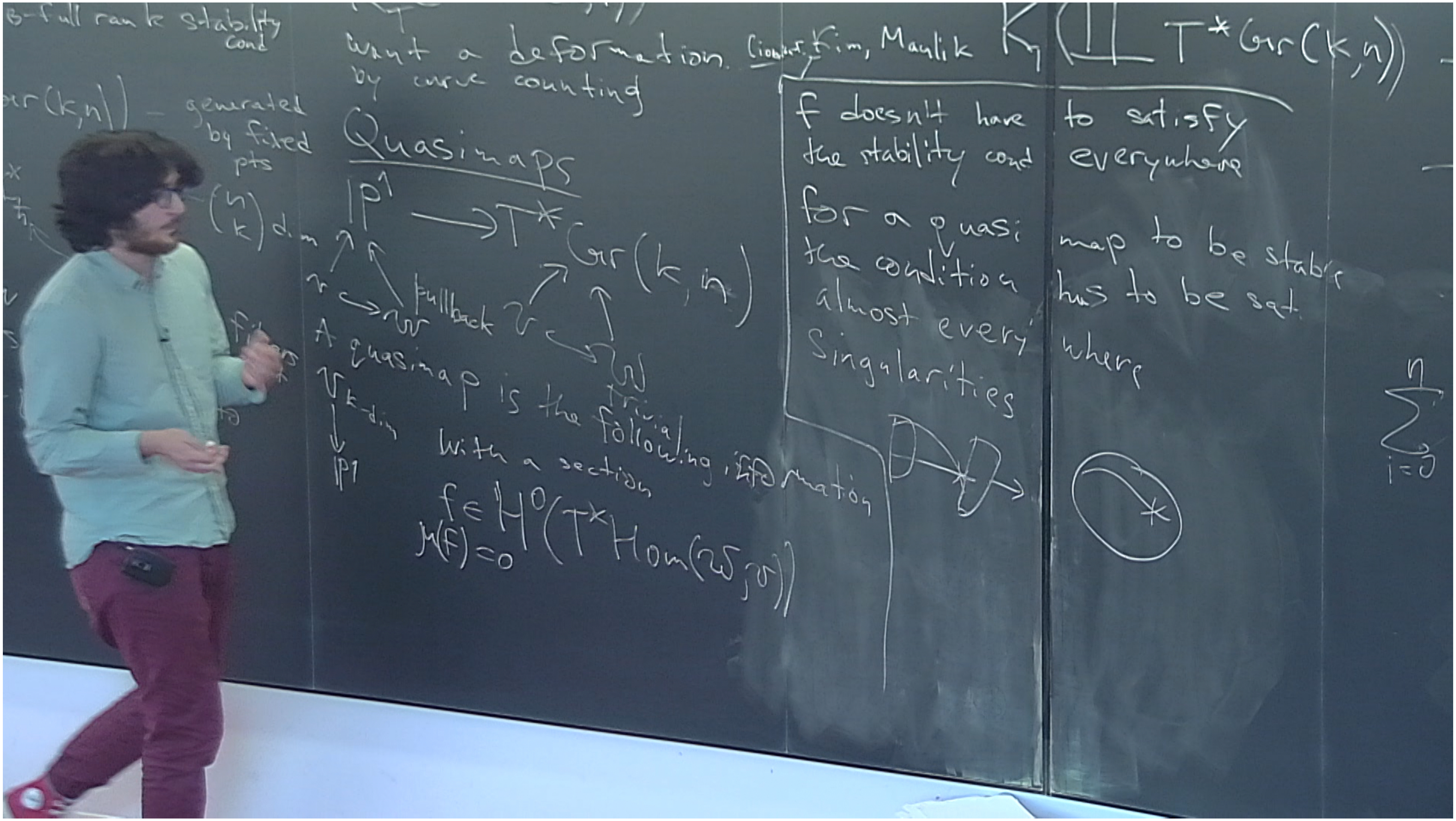
is the following information
trivial
section
Hom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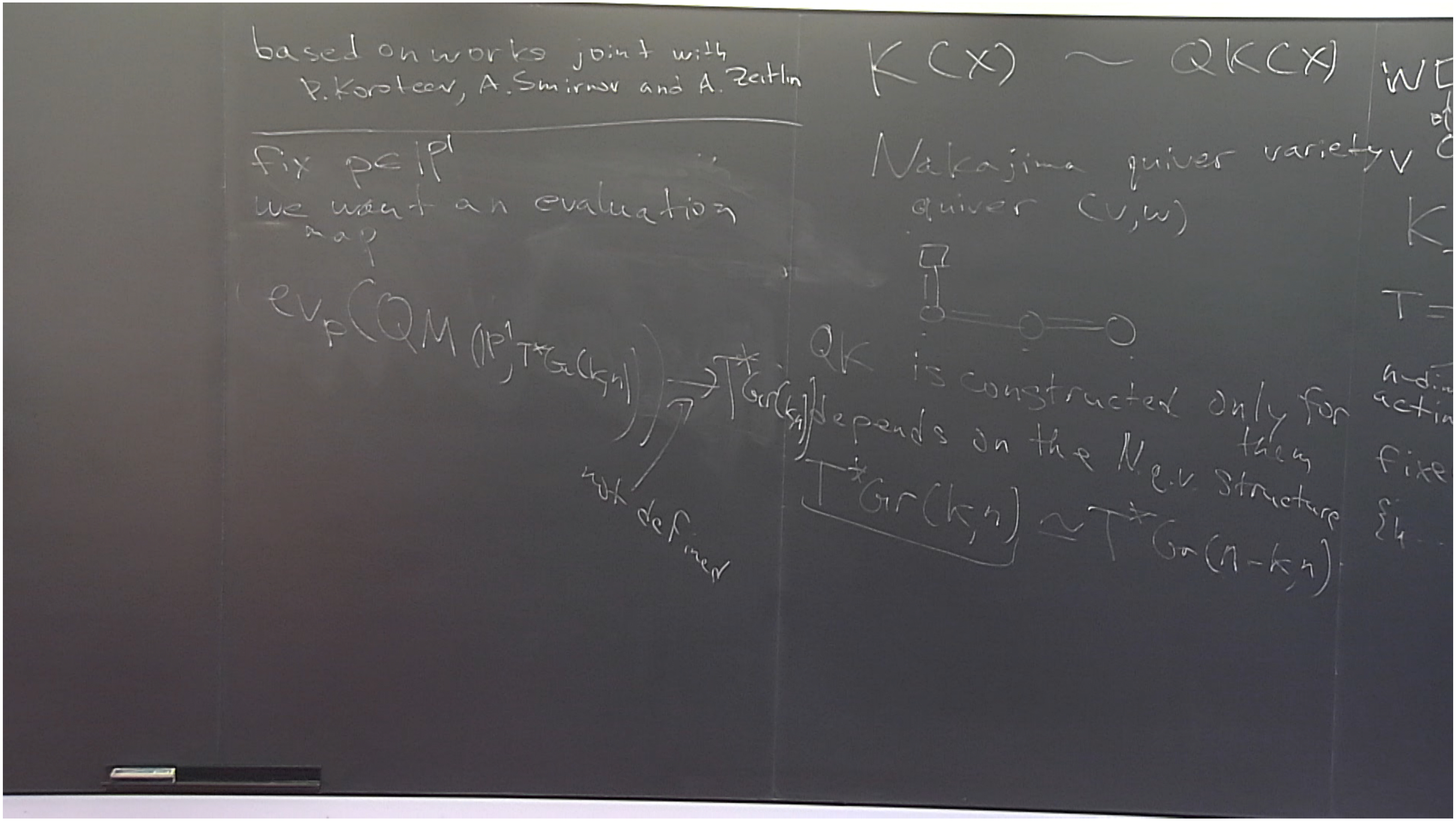
Quasimaps
pullback
trivial
A quasimap is the following information
with a section
f doesn't have to satisfy the stability cond everywhere
for a quasi map to be stable the condition has to be sat almost everywhere
Singularities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fix p ∈ P^1
we want an evaluation map
ev_p(QM(P^1, T*Gr(k,n))) → T*Gr(k,n)
not defined
K(X) ~ QK(X)
Nakajima quiver variety
quiver (v,w)
QK is constructed only for them
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T*Gr(k,n) ≃ T*Gr(n-k,n)
W
V
K
T=
n-dim
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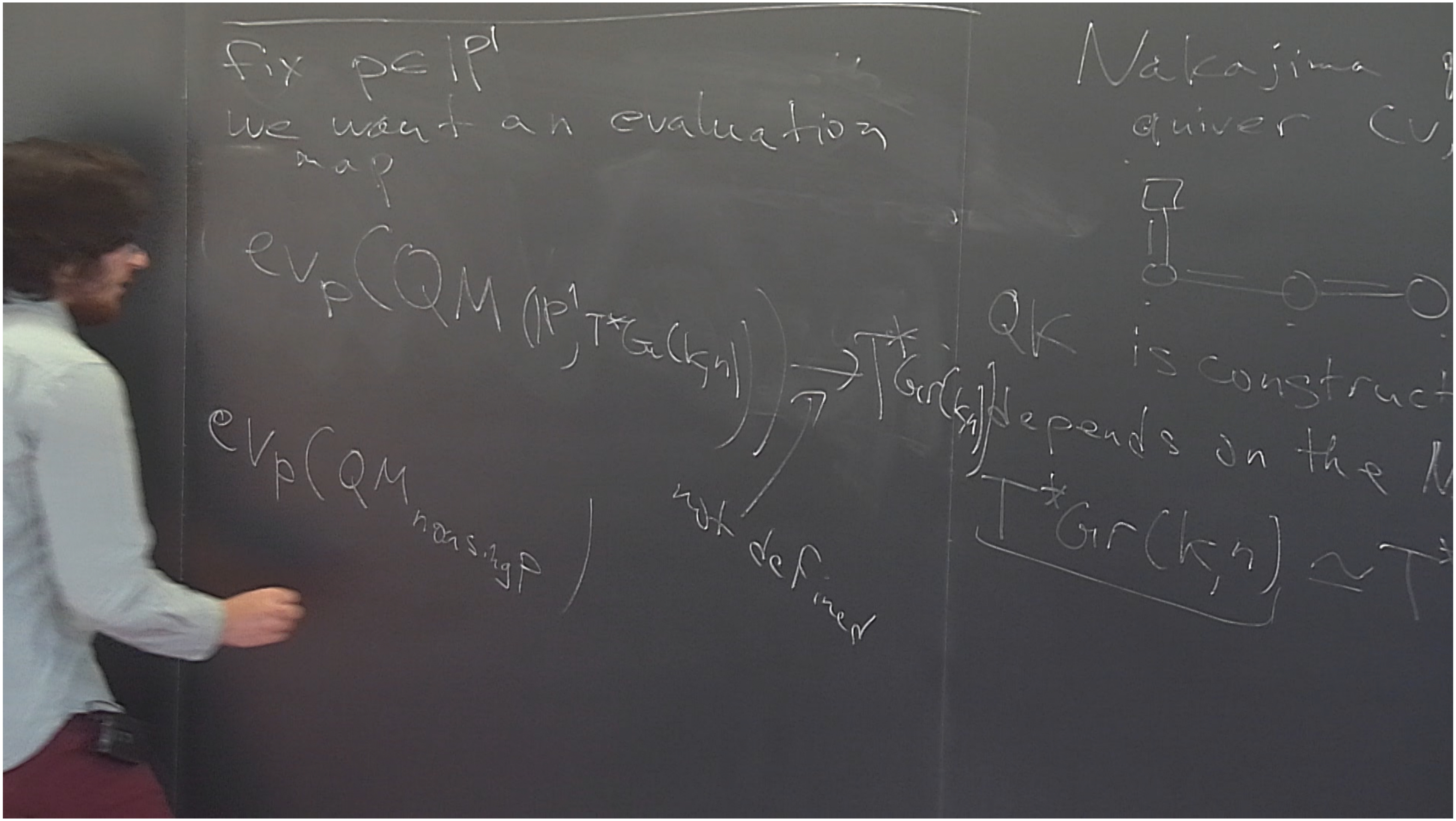
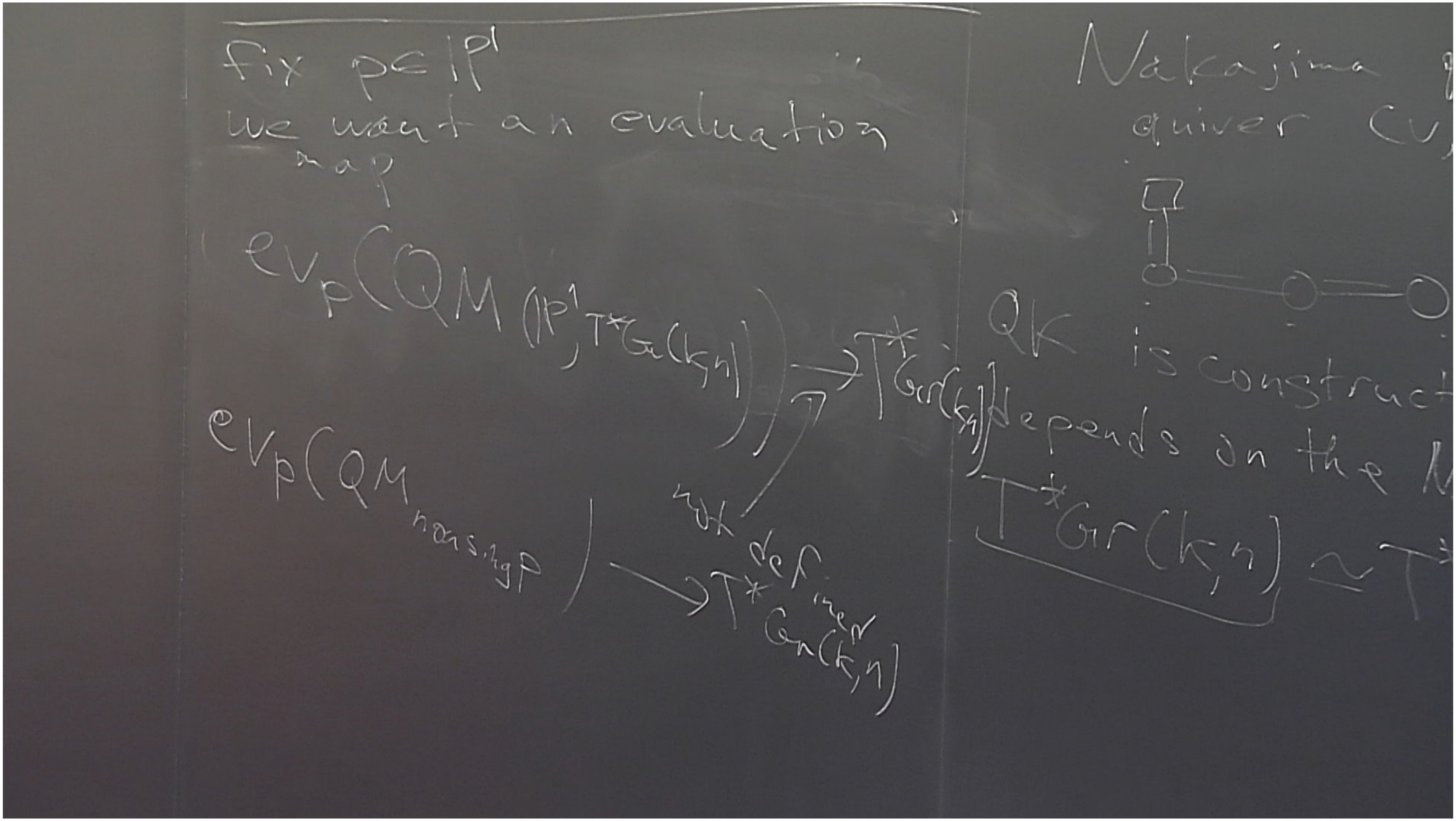
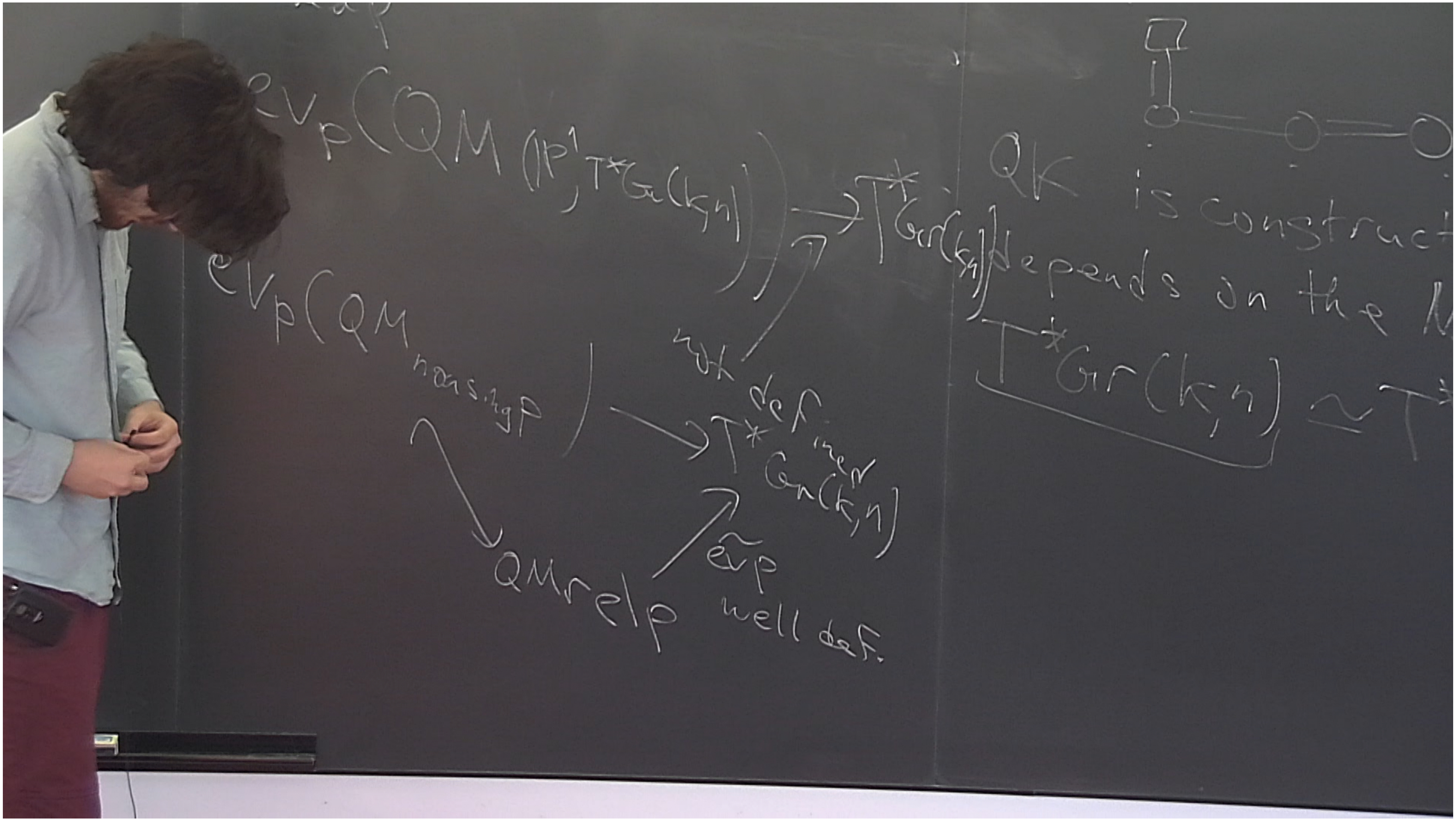

fix p∈P¹
we want an evaluation map
ev_p(QM(P¹, T*Gr(k,n))) → T*Gr(k,n)
ev_p(QM_nonsing)
not defined
Nakajima
quiver
QK is construct
depends on the
T*Gr(k,n) ≃ T*


fix p∈P¹
we want an evaluation map
ev_p(QM(P¹, T*Gr(k,n))) → T*Gr(k,n)
ev_p(QM_nonsing p) → T*Gr(k,n)
not defined
Nakajima
quiver
QK is construct
depends on the
T*Gr(k,n) ≃ T*


QK is construct
depends on the
not def.
well def.
QMrelp

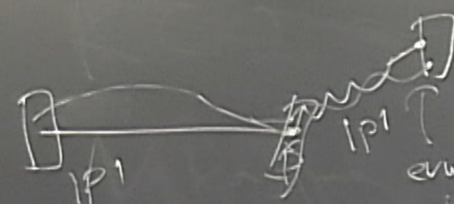
based on works joint with
P. Koroteev, A. Smirnov and A. Zeitlin

fix $p \in \mathbb{P}^1$
we want an evaluation map

$ev_p(QM(\mathbb{P}^1, T^*Gr(k,n))) \to T^*Gr(k,n)$

$ev_p(QM_{nonsing\,p}) \to T^*Gr(k,n)$
not defined

$QM_{rel\,p}$ $\widetilde{ev}_p$ well def.



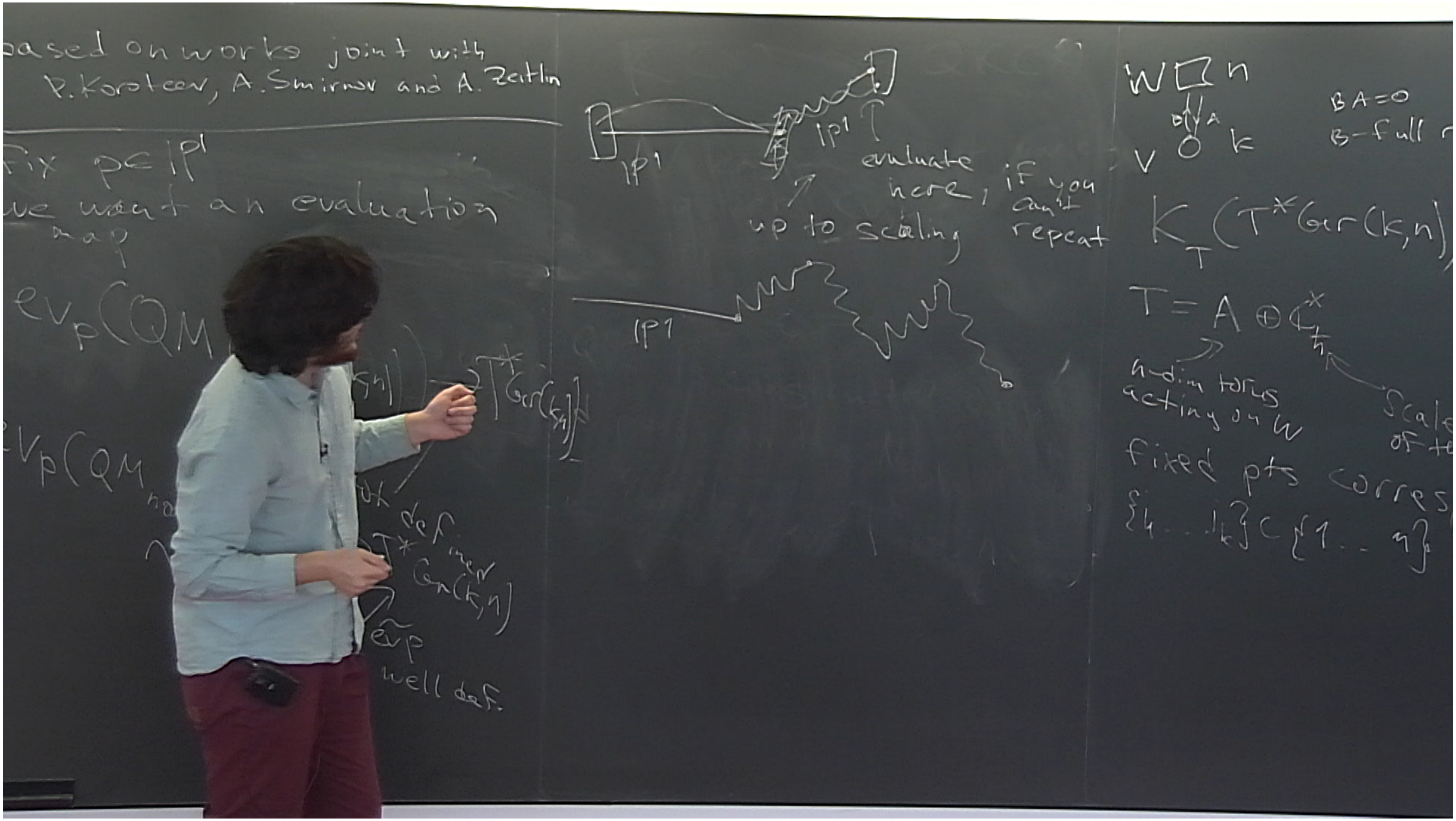
W □ n
B ↓↑ A
V ○ k

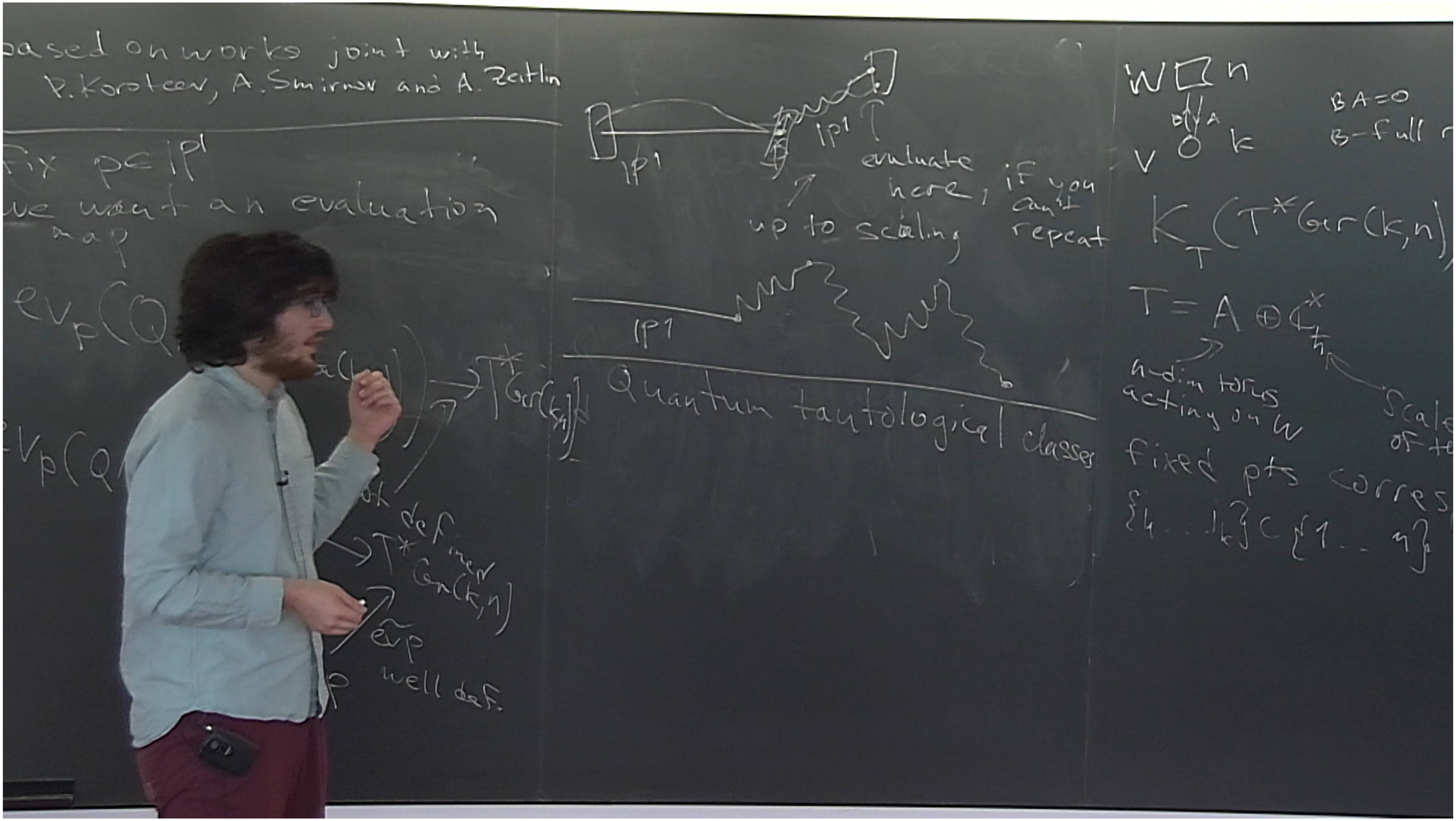
BA = 0
B - full r

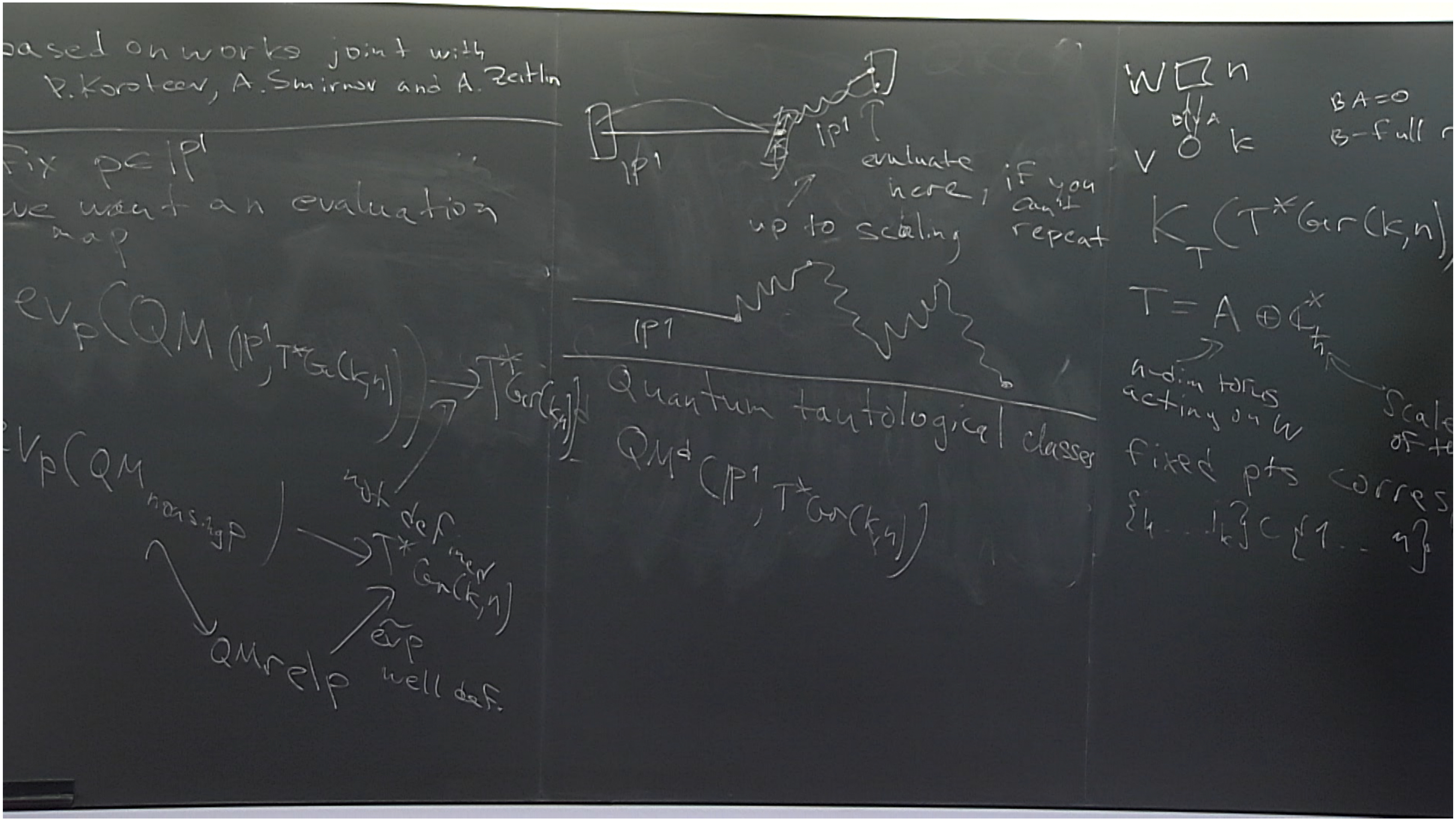
$K_T(T^*Gr(k,n))$

$T = A \oplus \mathbb{C}^*_\hbar$
n-dim torus acting on W
scale of t

fixed pts corres
$\{i_1 \ldots i_k\} \subset \{1 \ldots n\}$


based on works joint with
P. Koroteev, A. Smirnov and A. Zeitlin
fix p ∈ IP¹
we want an evaluation map
evaluate here, up to scaling
if you can't repeat
well def.
W □ n
V ○ k
BA=0
B-full
n-dim torus acting on W
fixed pts corres


based on works joint with
P. Koroteev, A. Smirnov and A. Zeitlin
fix p ∈ IP¹
we want an evaluation map
IP¹
evaluate here, if you can't repeat
up to scaling
Quantum tautological classes
def
well def.
W n
V k
BA = 0
B - full
T = A ⊕ ℂ*
n-dim torus acting on W
fixed pts corres
{i₁ ... iₖ} ⊂ {1 ... n}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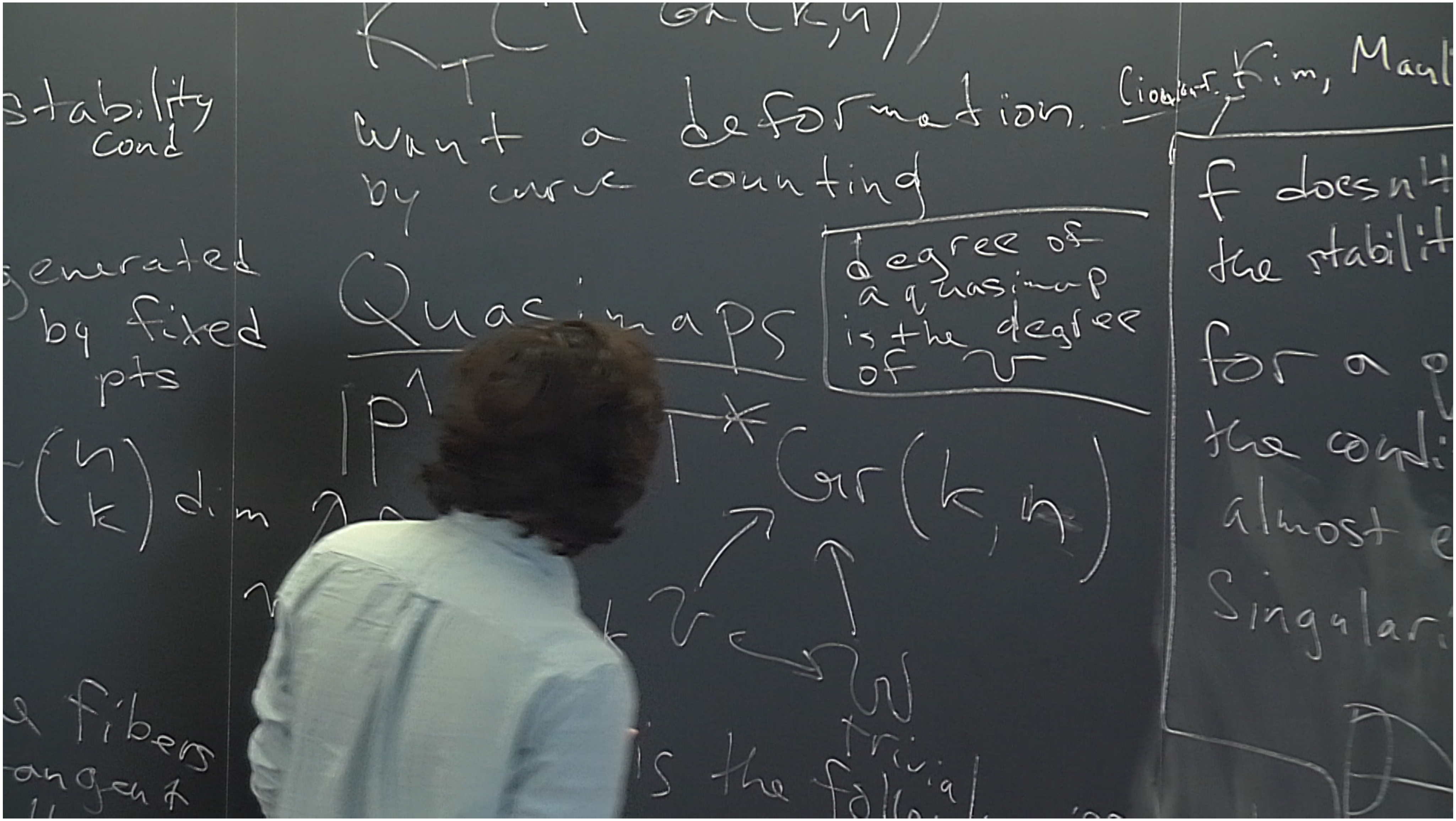
Fix p∈ℙ¹
we want an evaluation map
ev_p(QM(ℙ¹, T*Gr(k,n))) → T*Gr(k,n)
ev_p(QM_nonsing p) → T*Gr(k,n)
not defined
QM_rel p
well def.
ℙ¹
evaluate here, if you can't repeat
up to scaling
Quantum tautological classes
QM^d(ℙ¹, T*Gr(k,n))
W □ n
V ○ k
BA=0
B-full
K_T(T*Gr(k,n))
T = A ⊕ ℂ*_ħ
n-dim torus acting on W
fixed pts corres
{i_1 ... i_k} ⊂ {1 ... n}


stability
cond
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pts
dim
fibers
Want a deformation
by curve counting
Quasimaps
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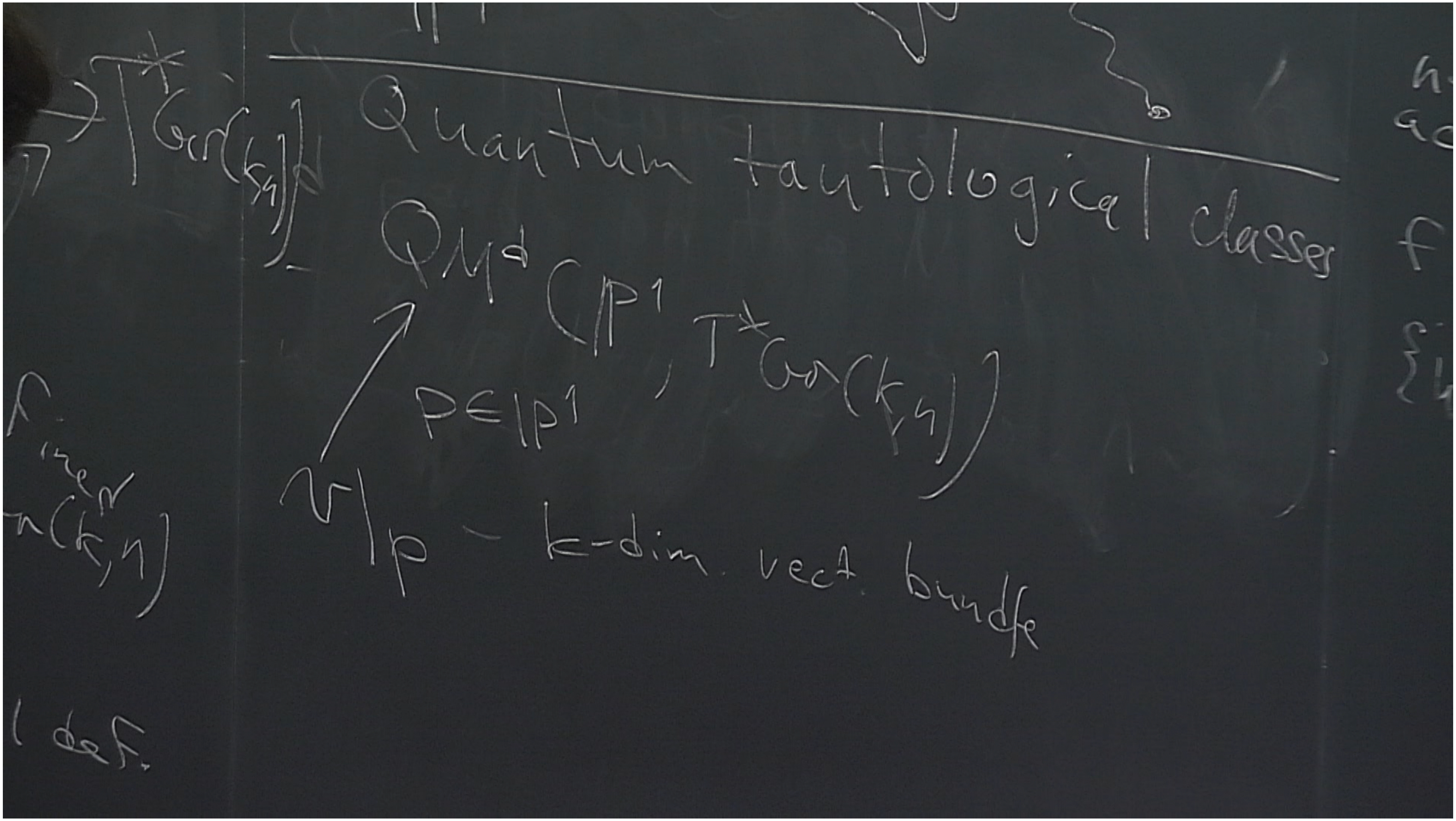
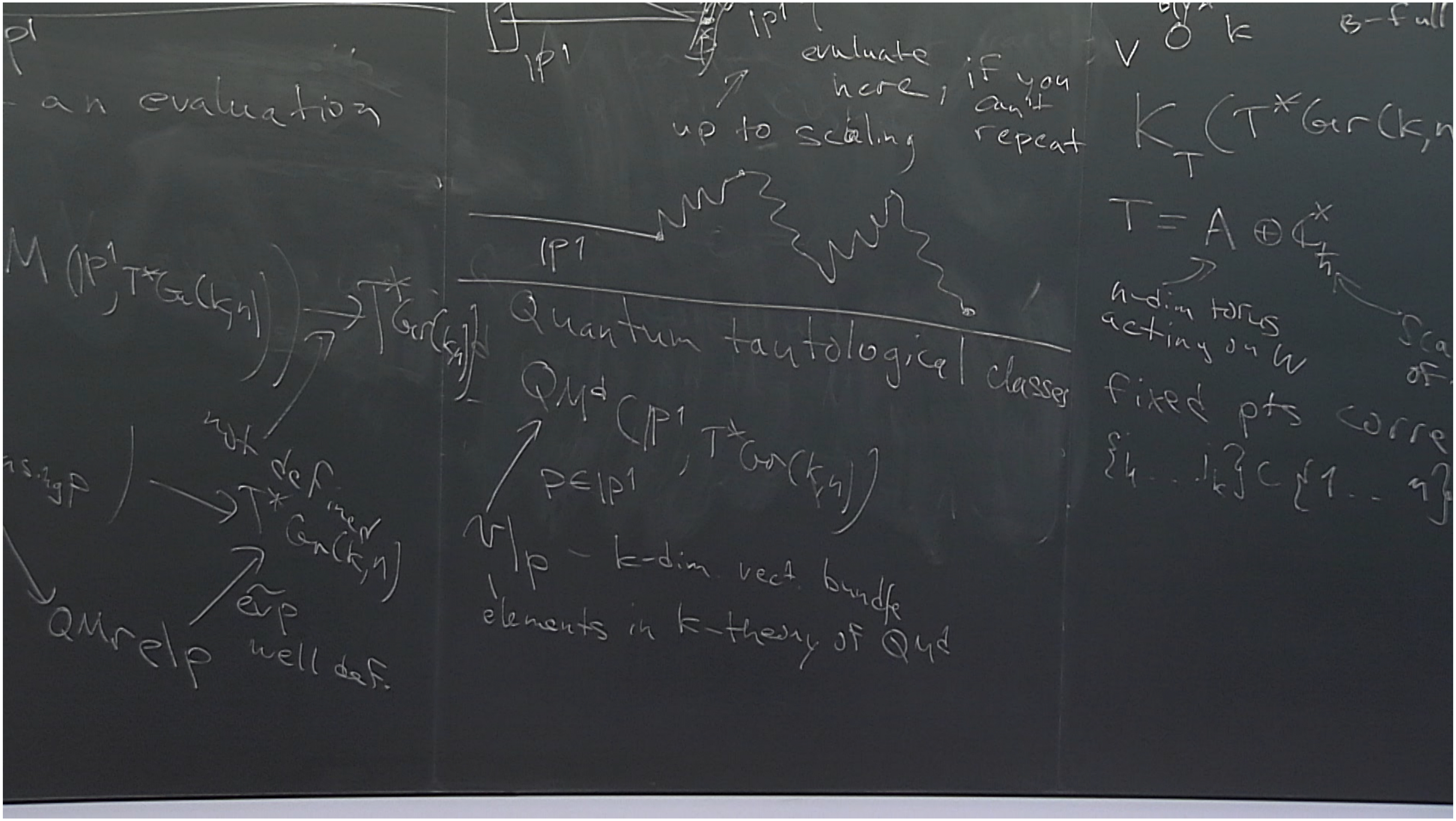
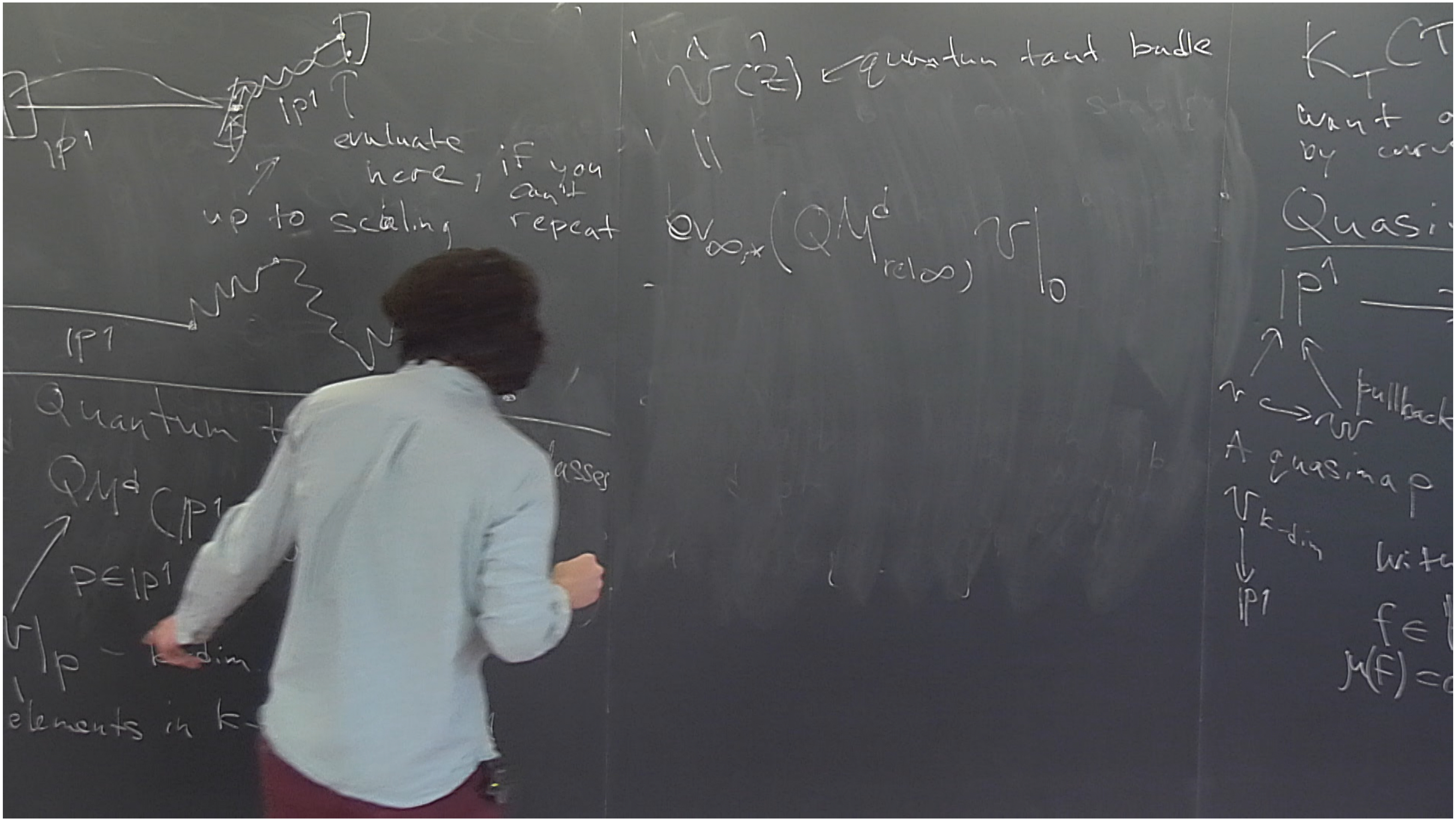
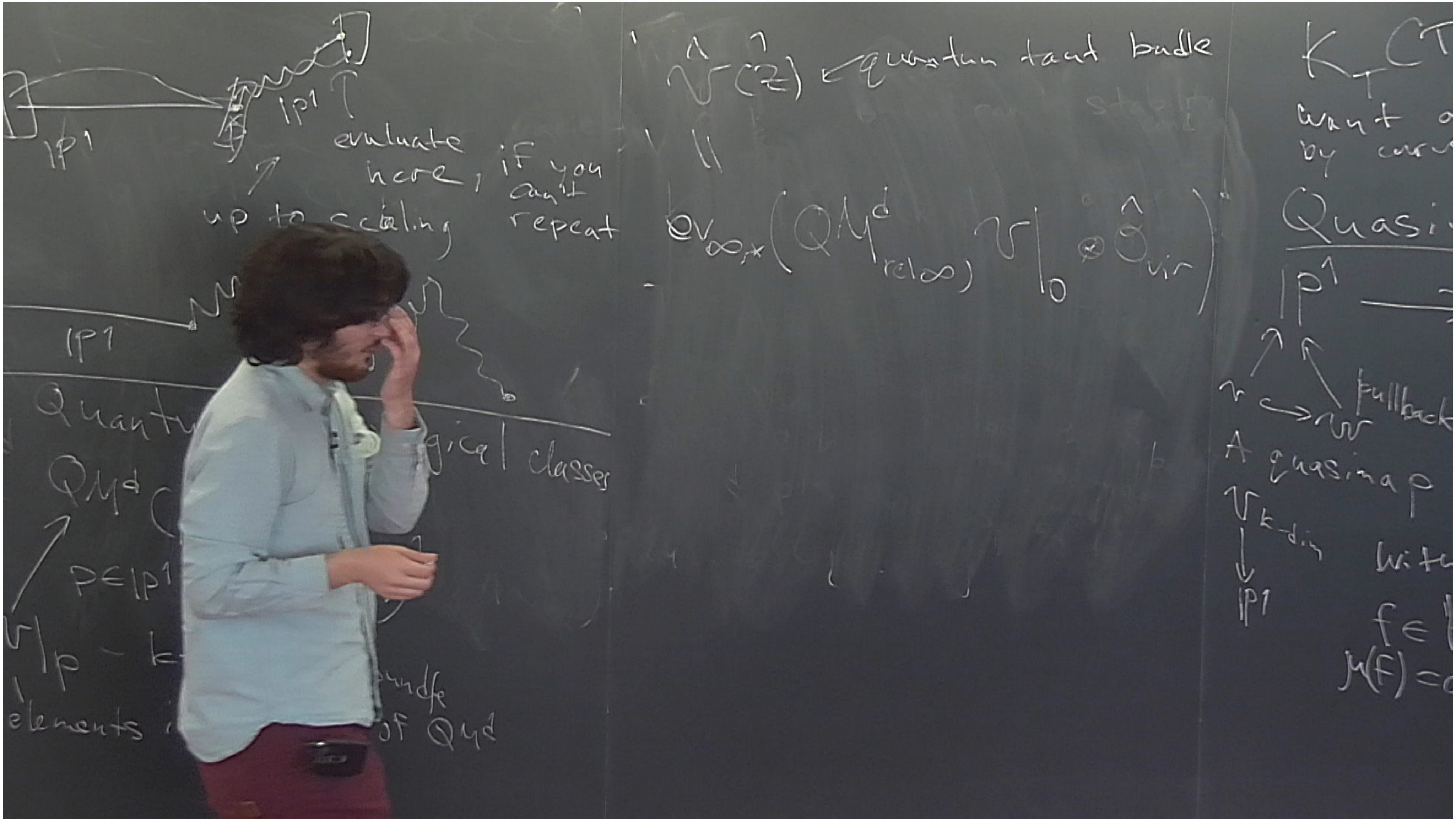
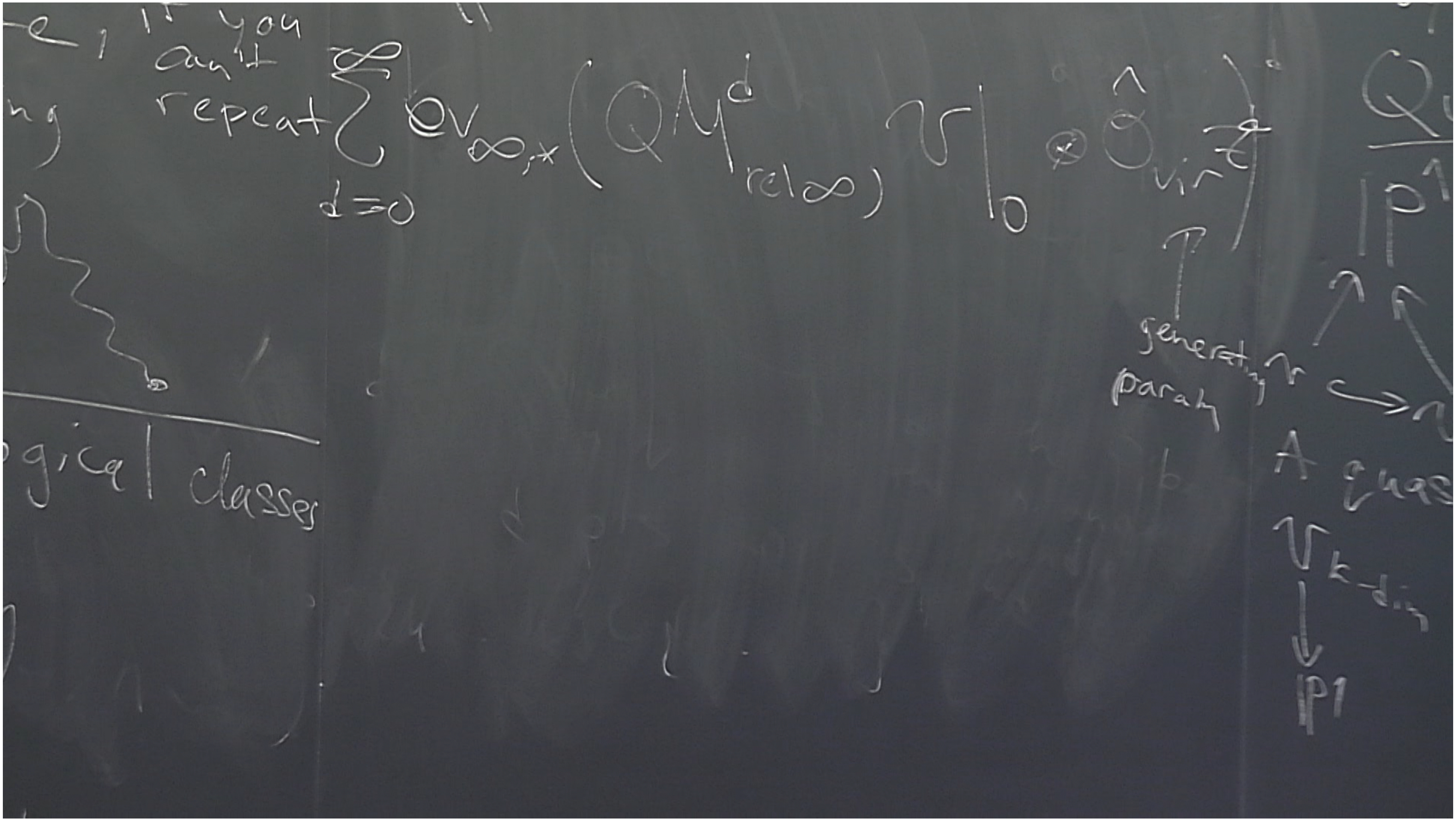

Quantum tautological classes
QM^d (P^1, T*Gr(k,n))
p∈P^1
V|p - k-dim. vect. bundle
def.

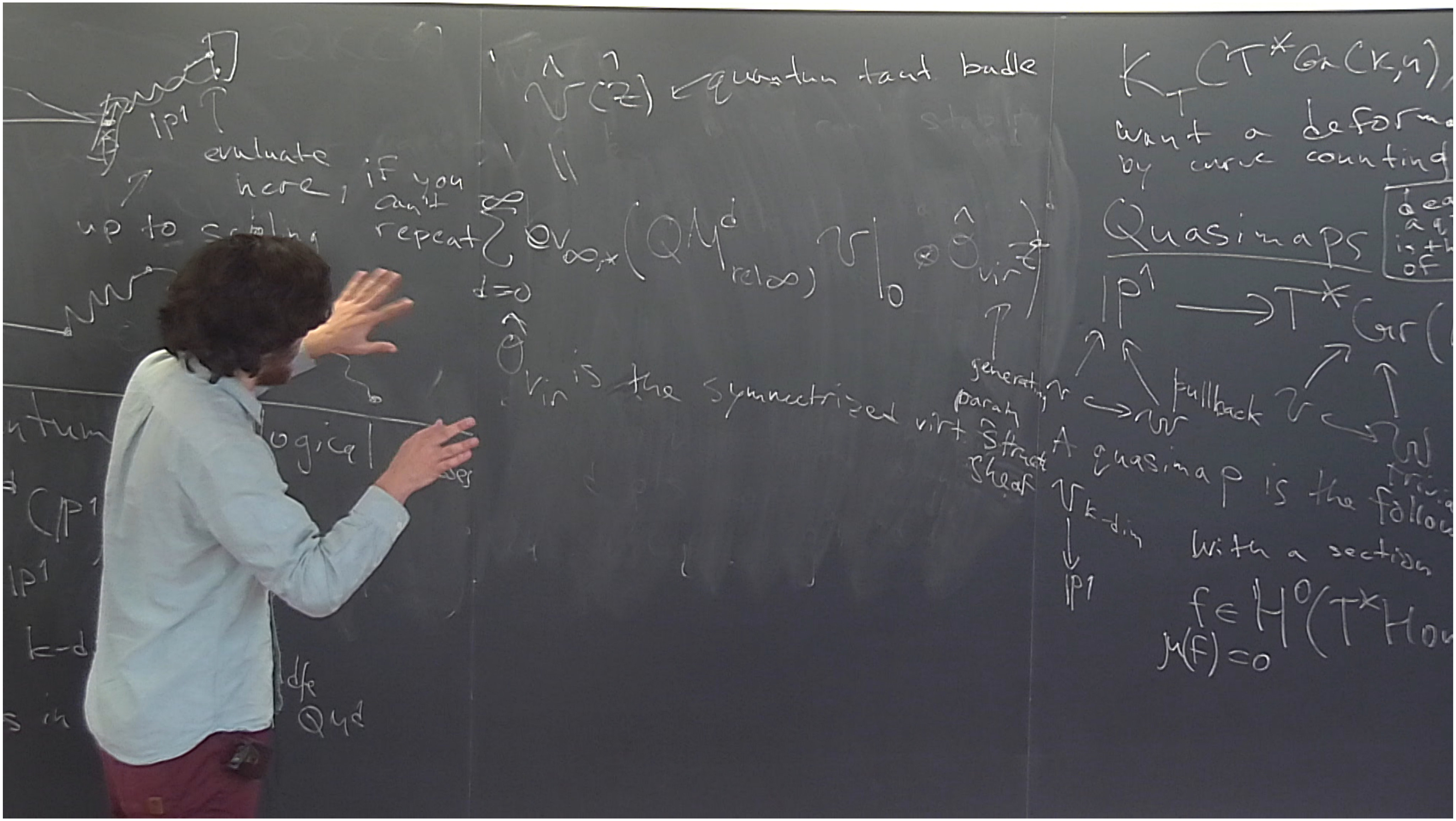

an evaluation
evaluate here, if you can't repeat
up to scaling
Quantum tautological classes
elements in K-theory of QM^d
p ∈ P^1
k-dim. vect. bundle
well def.
T = A ⊕ ℂ*
n-dim torus acting on W
fixed pts corre

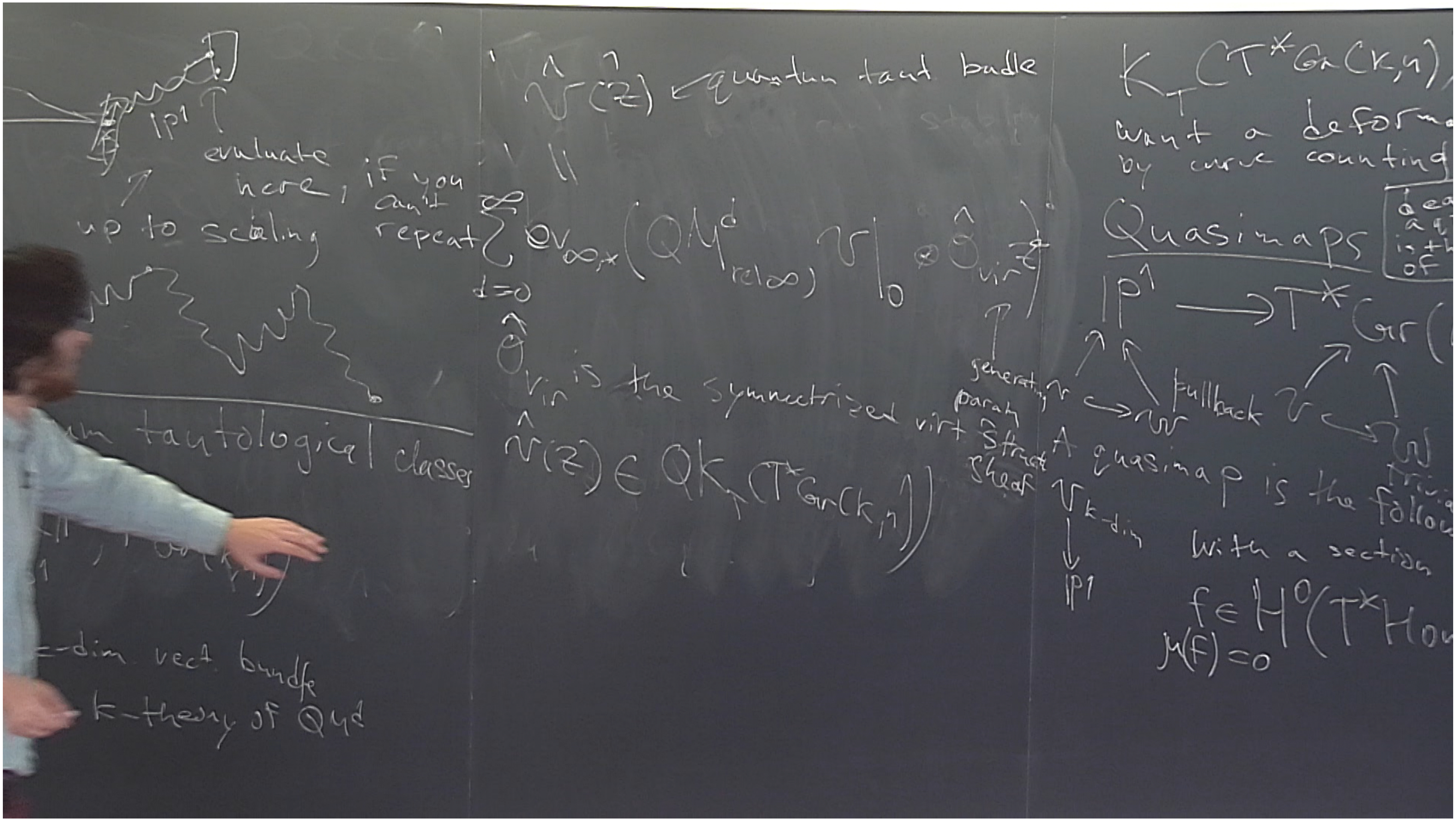


evaluate
here, if you
can't
up to scaling
repeat
Quantum t
QM^d (P^1
p∈P^1
elements in k-
quantum taut bundle
Quasi
pullback
A quasimap
want
by


quantum taut bundle
evaluate here, if you can't repeat
up to scaling
classes
elements
of QM^d
Quasi
A quasimap
pullback


if you
can't
repeat
d=0
generating
param
classes


quantum taut bundle
evaluate here, if you can't repeat
up to scaling
is the symmetrized virt struct sheaf
generating param
pullback
A quasimap is the following
with a section
Want a deformation by curve counting
Quasimaps


evaluate here, if you can't repeat
up to scaling
tautological classes
dim. vect. bundle
k-theory of QM^d
quantum taut bundle
is the symmetrized virt struct sheaf
generating param
K_T(T*Gr(k,n))
want a deform
by curve counting
Quasimaps
pullback
A quasimap is the follow
with a section
f ∈ H^0(T*Hom
μ(f) = 0
P^1

$\mathbb{P}^1$

evaluate here, up to scaling

if you can't repeat

$\sum_{d=0}^{\infty} ev_{\infty,*}\left(QM^d_{rel\,\infty}, \hat{\mathcal{V}}\big|_0 \otimes \hat{\mathcal{O}}_{vir}\, z^d\right)$

$\hat{\mathcal{V}}(z)$ ← quantum taut. bundle

$\hat{\mathcal{O}}_{vir}$ is the symmetrized virt. str. sheaf

$\hat{\mathcal{V}}(z) \in K_T(T^*Gr(k,n))[[z]]$

$\widehat{\Lambda^\ell \mathcal{V}}(z)$ is defined analogously

ntum tautological classes

$^d(\mathbb{P}^1, T^*Gr(k,n))$

$\mathbb{P}^1$

k-dim. vect. bundle

s in K-theory of QM^d

$K_T(T^*Gr(k,n))$

want a deform... by curve counting

Quasimaps

$\mathbb{P}^1 \longrightarrow T^*Gr(...$

generating param

$\mathcal{V} \hookrightarrow \mathcal{W}$ pullback $\mathcal{V} \hookrightarrow \mathcal{W}$ trivial

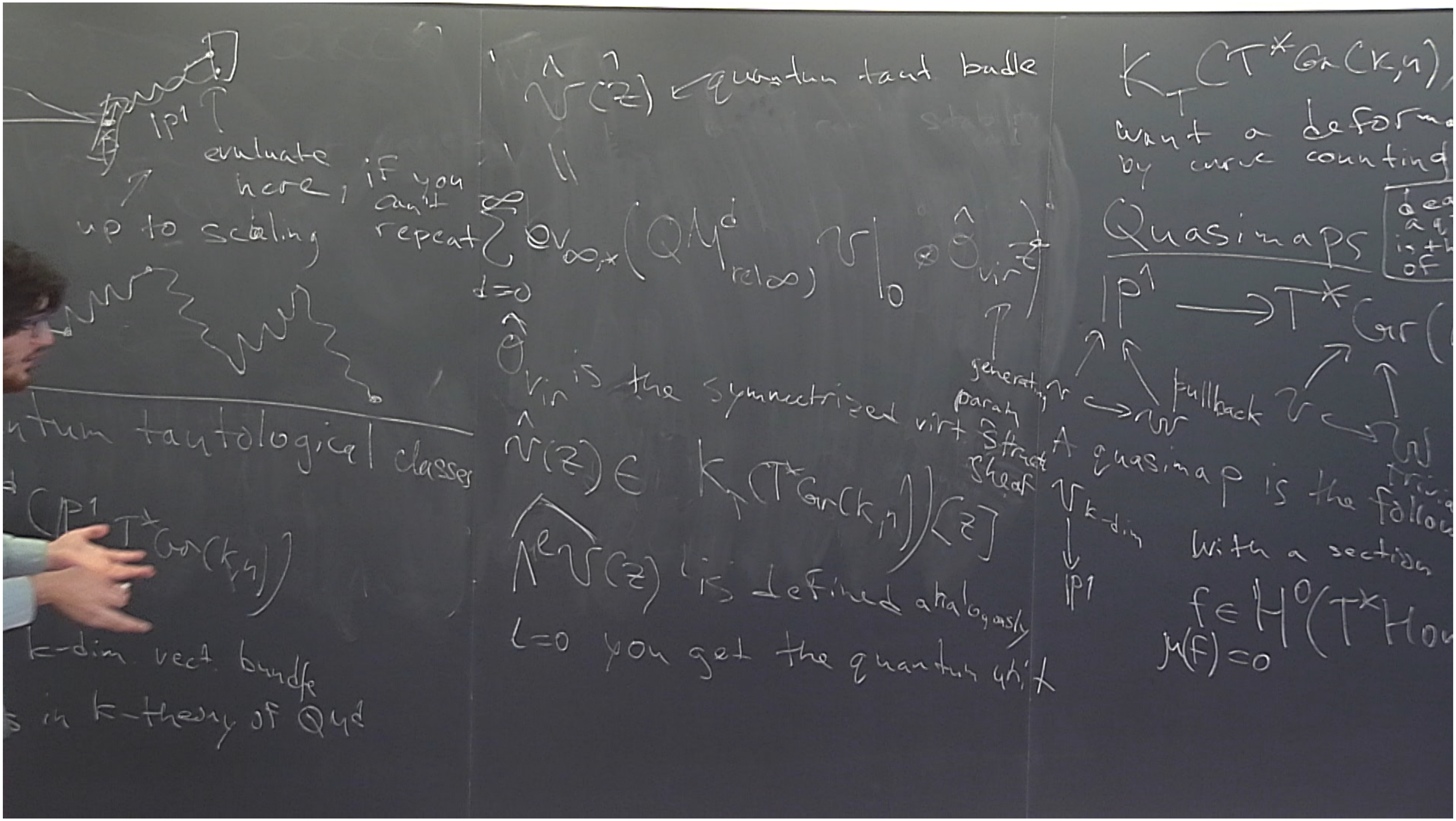
A quasimap is the follow...

$\mathcal{V}_{k\text{-dim}} \downarrow \mathbb{P}^1$

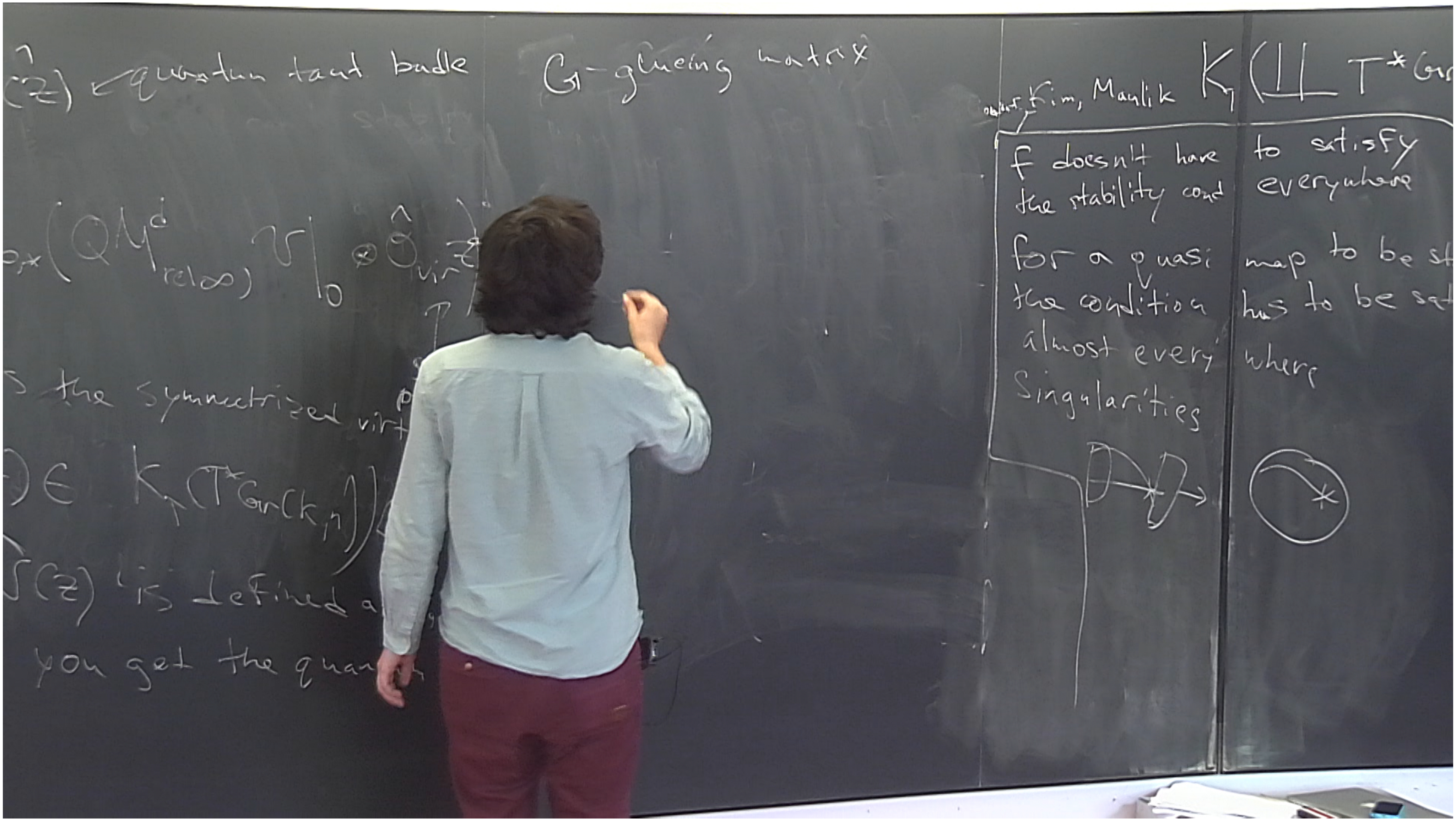
with a section

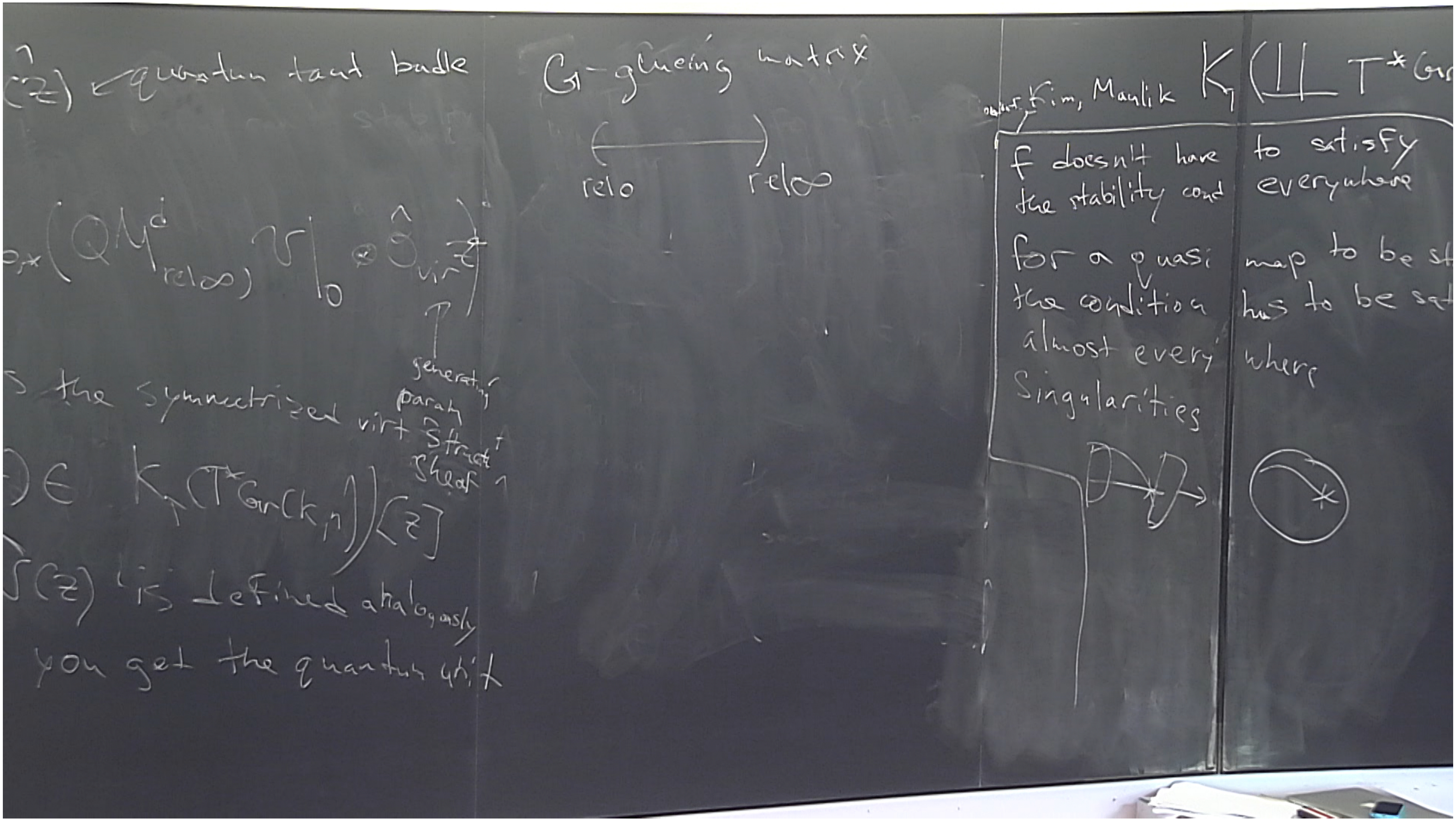
$f \in H^0(T^*Hom...$

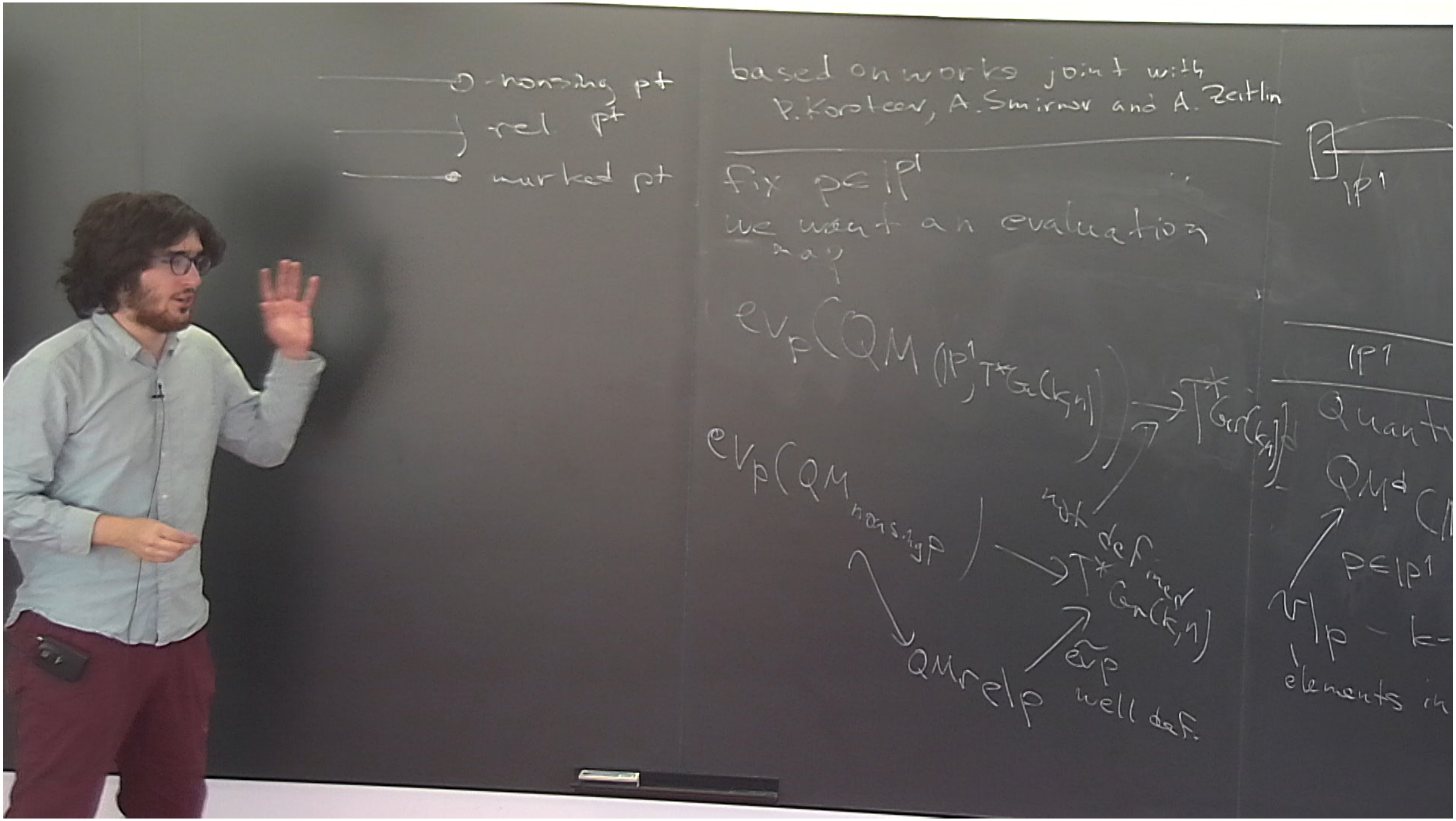
$\mu(f) = 0$



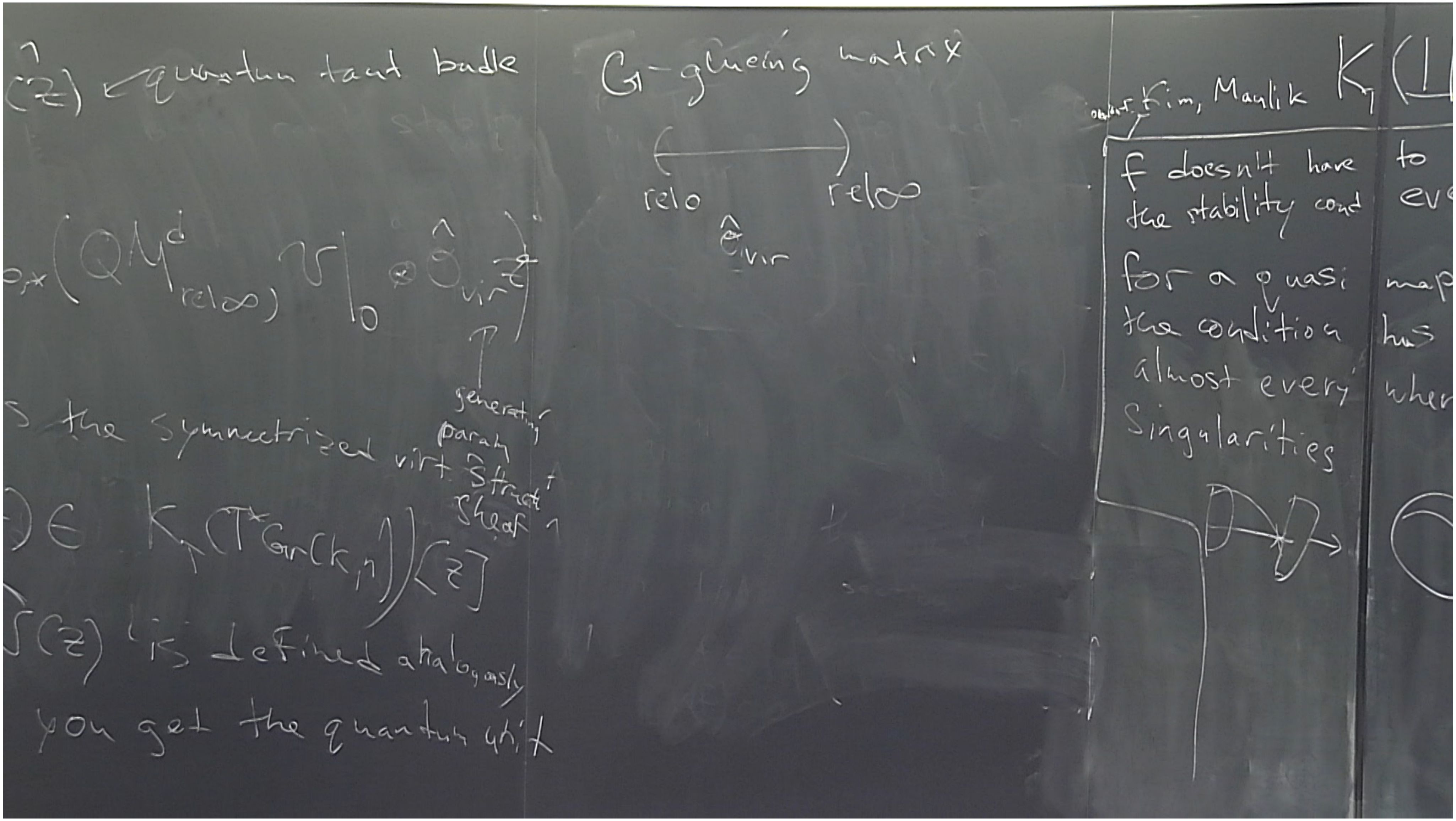
evaluate here, if you can't repeat
up to scaling
$\mathbb{P}^1$
ntum tautological classes
k-dim. vect. bundle
s in K-theory of QM^d
$\hat{\mathcal{V}}(z)$ ← quantum taut bundle
$\sum_{d=0}^{\infty} ev_{\infty,*}(QM^d_{rel\infty}, \mathcal{V}|_0 \otimes \hat{\mathcal{O}}_{vir} z^d)$
$\hat{\mathcal{O}}_{vir}$ is the symmetrized virt struct sheaf
generating param
$\hat{\mathcal{V}}(z) \in K_T(T^*Gr(k,n))[[z]]$
$\hat{\Lambda^\ell \mathcal{V}}(z)$ is defined analogously
$\ell=0$ you get the quantum unit
$K_T(T^*Gr(k,n))$
want a deform by curve counting
Quasimaps
$\mathbb{P}^1 \longrightarrow T^*Gr($
pullback
A quasimap is the follow
$\mathcal{V}$ k-dim
$\mathbb{P}^1$
With a section
$f \in H^0(T^*Hom$
$\mu(f)=0$

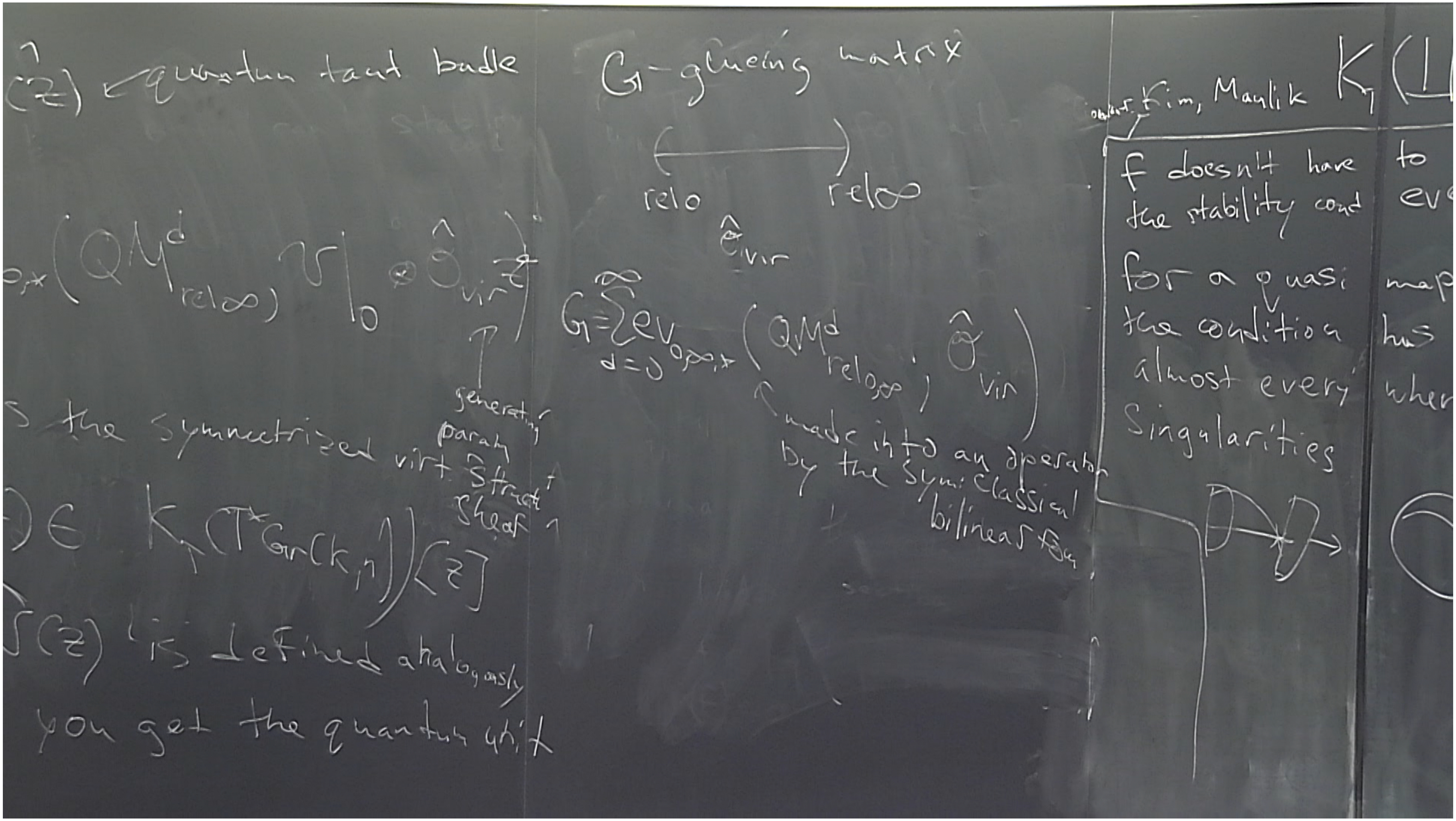

quantum taut bundle
G-glueing matrix
Kim, Maulik
f doesn't have the stability cond
to satisfy everywhere
for a quasi
map to be
the condition
has to be
almost every
where
Singularities
the symmetrized
is defined
you get the

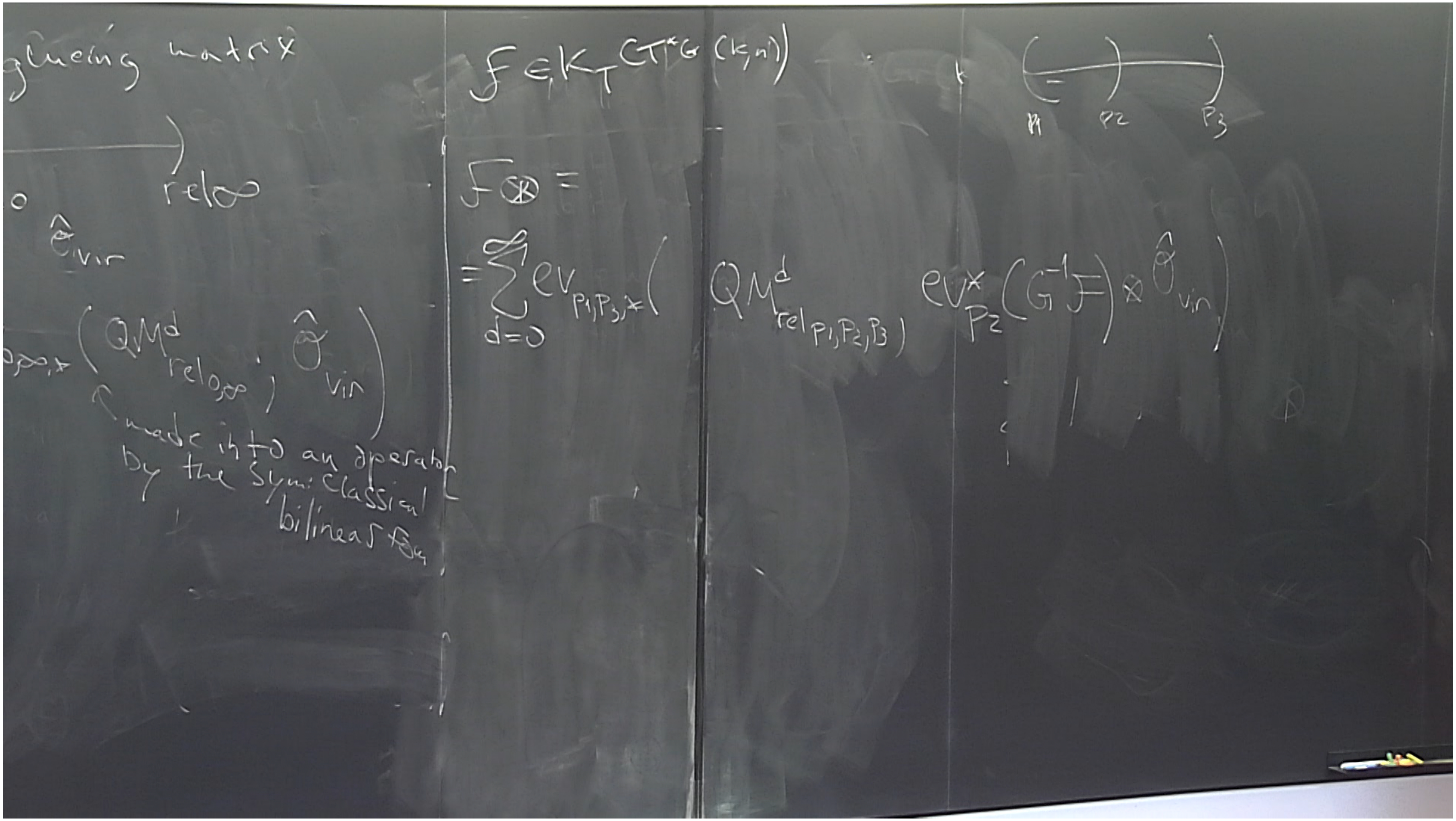

quantum tant bndle
G-glueing matrix
rel0
rel∞
$(QM^d_{rel\infty}, \mathcal{V}|_0 \otimes \hat{\mathcal{O}}_{vir}^{z})$
generating param
the symmetrized virt struct sheaf
$\in K_T(T^*Gr(k,n))[z]$
is defined analogously
you get the quantum virt
Kim, Maulik
$K_T(\coprod T^*Gr$
f doesn't have to satisfy the stability cond everywhere
for a quasi map to be st the condition has to be sat almost every where
Singularities

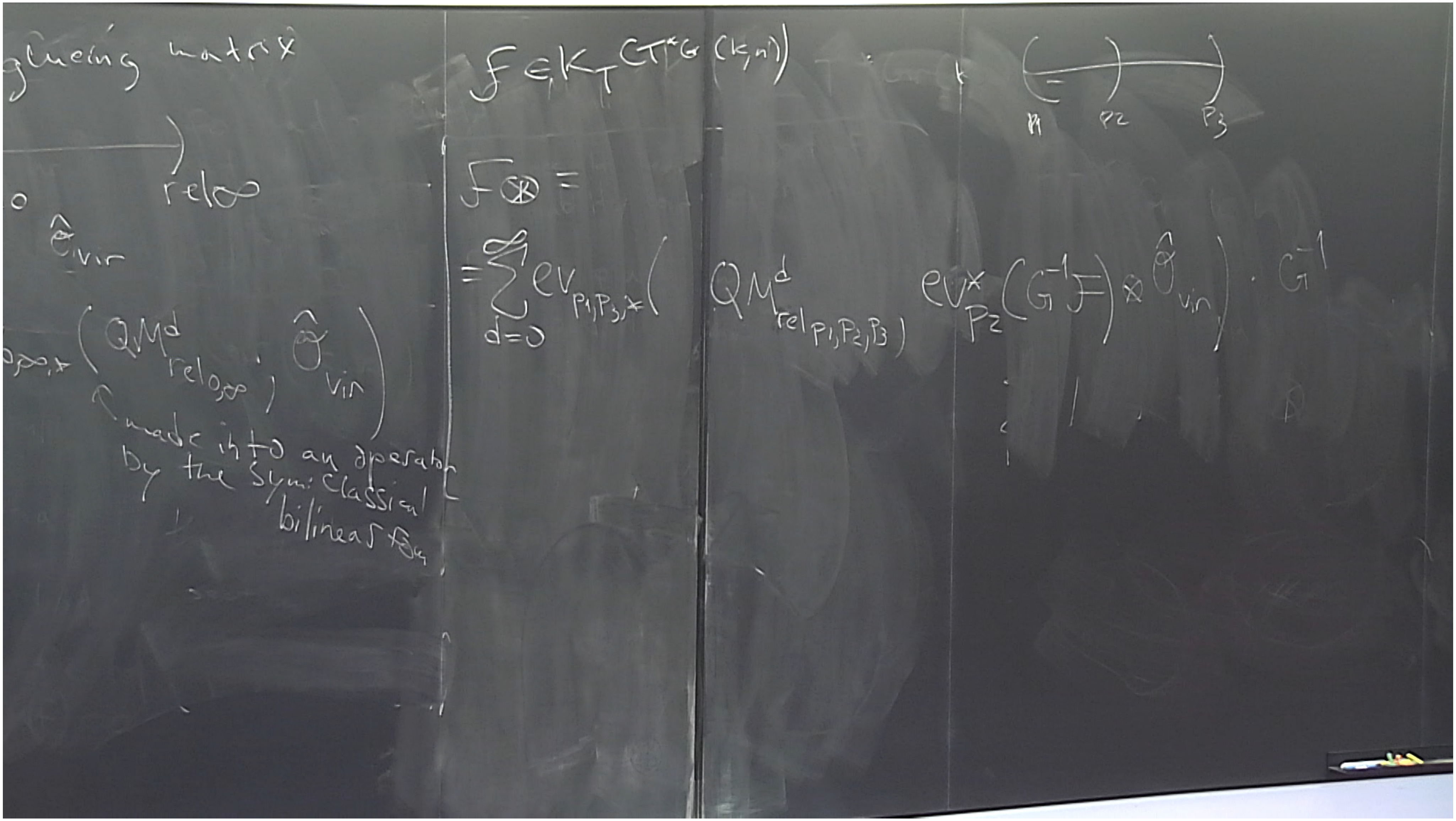


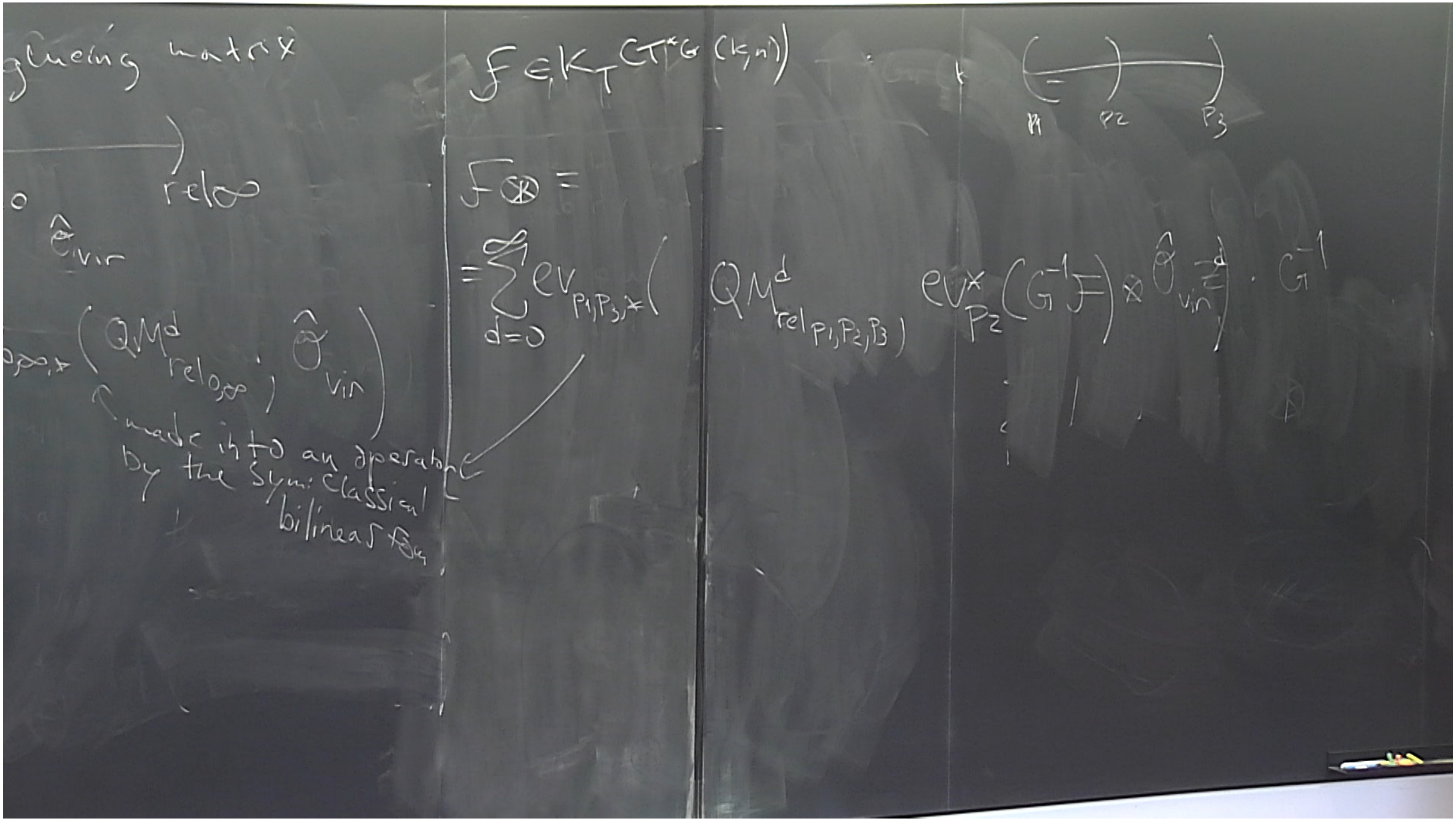
nonsing pt
rel pt
marked pt
based on works joint with
P. Koroteev, A. Smirnov and A. Zeitlin
fix p ∈ IP¹
we want an evaluation map
ev_p(QM(IP¹, T*Gr(k,n)))
T*Gr(k,n)
ev_p(QM nonsing p)
not def
def in gen
T*Gr(k,n)
QM rel p
ev_p
well def.
IP¹
p ∈ IP¹
elements in

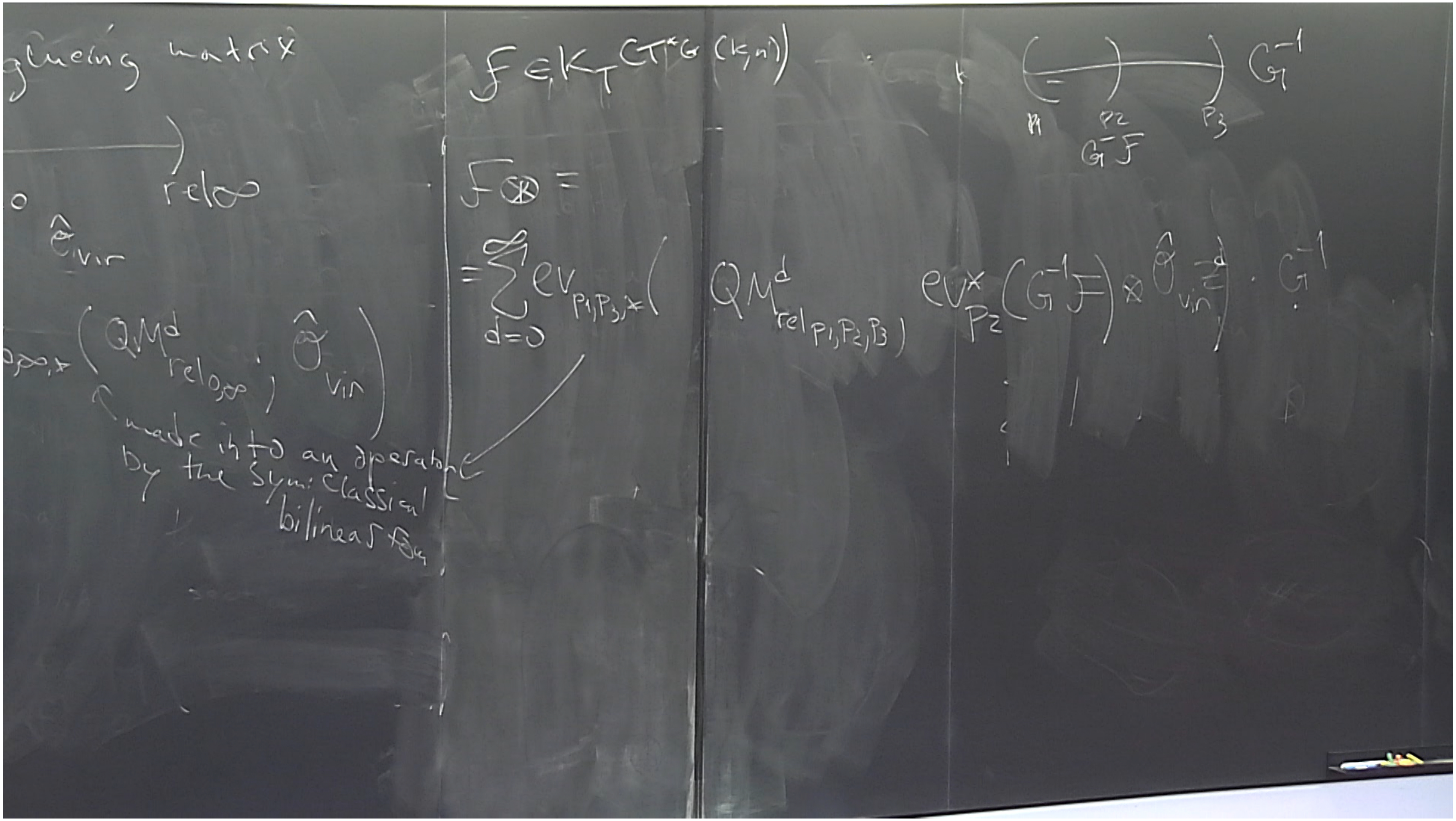

quantum taut bundle
G-glueing matrix
relo
rel∞
Kim, Maulik
f doesn't have the stability cond
for a quasi the condition almost every
Singularities
generating param
the symmetrized virt. struct sheaf
is defined analogously
you get the quantum unit

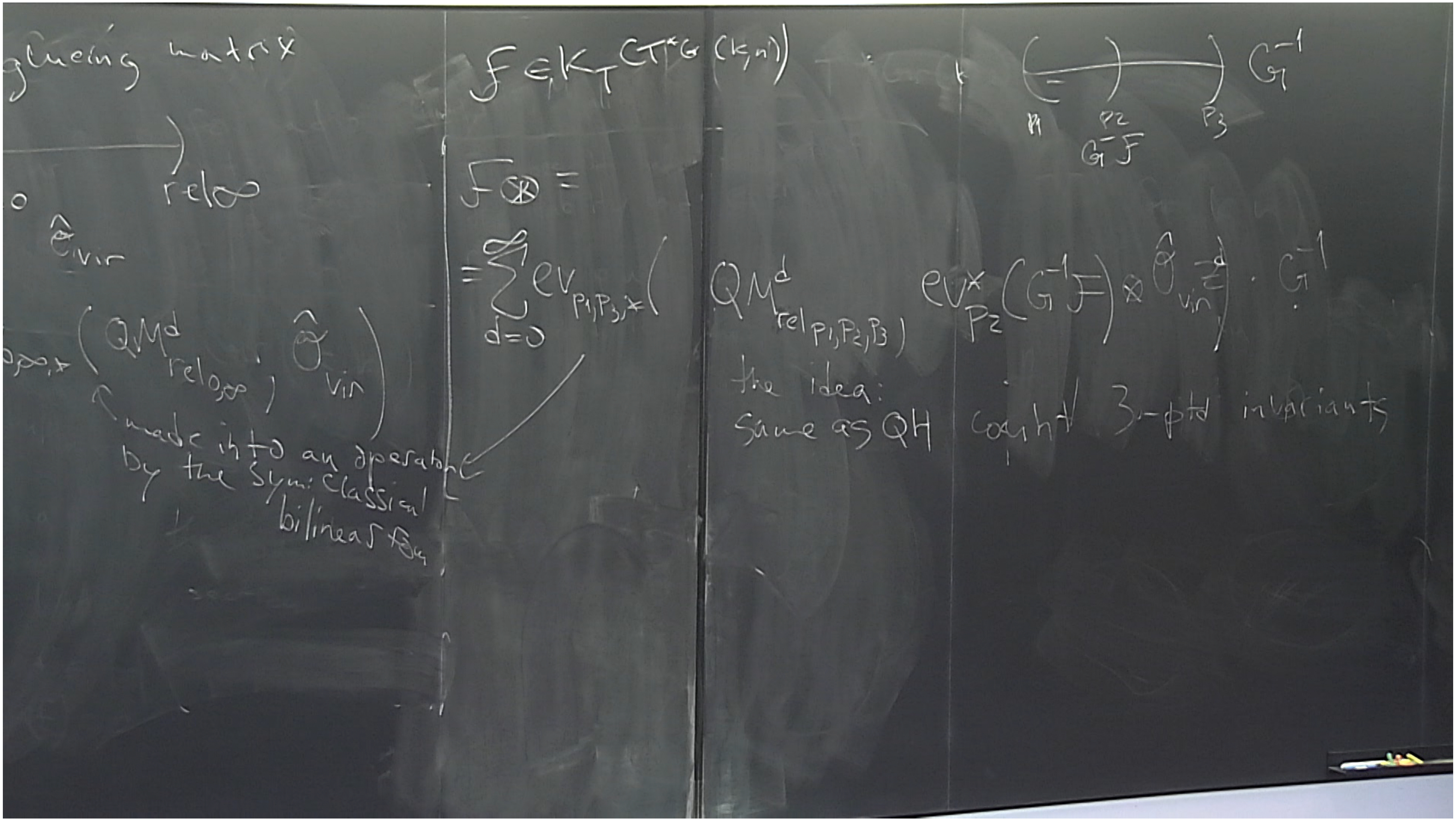

(z) ← quantum taut bundle
(QM^d_{rel∞}, v|_0 ⊗ Ô_vir z)
generating param
is the symmetrized virt struct sheaf
∈ K_T(T*Gr(k,n))[z]
(z) is defined analogously
you get the quantum unit
G - glueing matrix
rel0
rel∞
Ô_vir
G = Σ_{d=0}^∞ ev_{0,∞,*}(QM^d_{rel0,∞}, Ô_vir)
made into an operator by the sym. classical bilinear form
Kim, Maulik
f doesn't have the stability cond for a quasi
the condition almost every
Singularities
to
map
has
wher

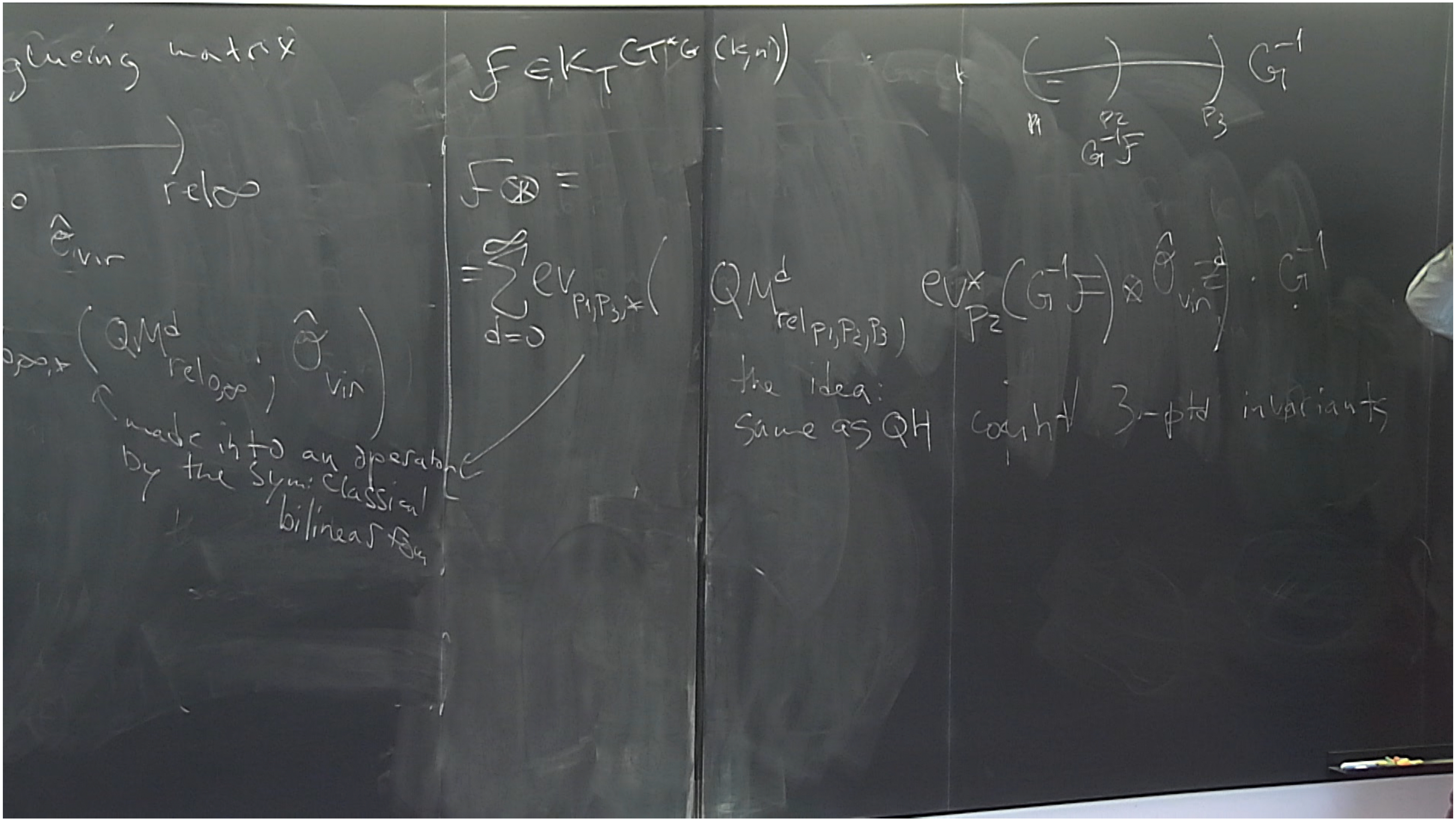

glueing matrix
rel∞
$\hat{\theta}_{vir}$
$(QM^d_{rel 0,\infty}, \hat{\theta}_{vir})$
made into an operator by the Sym. classical bilinear form
$F \in K_T(T^*G(k,n))$
p_1 p_2 p_3
$F \circledast =$
$= \sum_{d=0}^{\infty} ev_{p_1,p_3,*}(QM^d_{rel\, p_1,p_2,p_3}, ev^*_{p_2}(G^{-1}F) \otimes \hat{\theta}_{vir})$

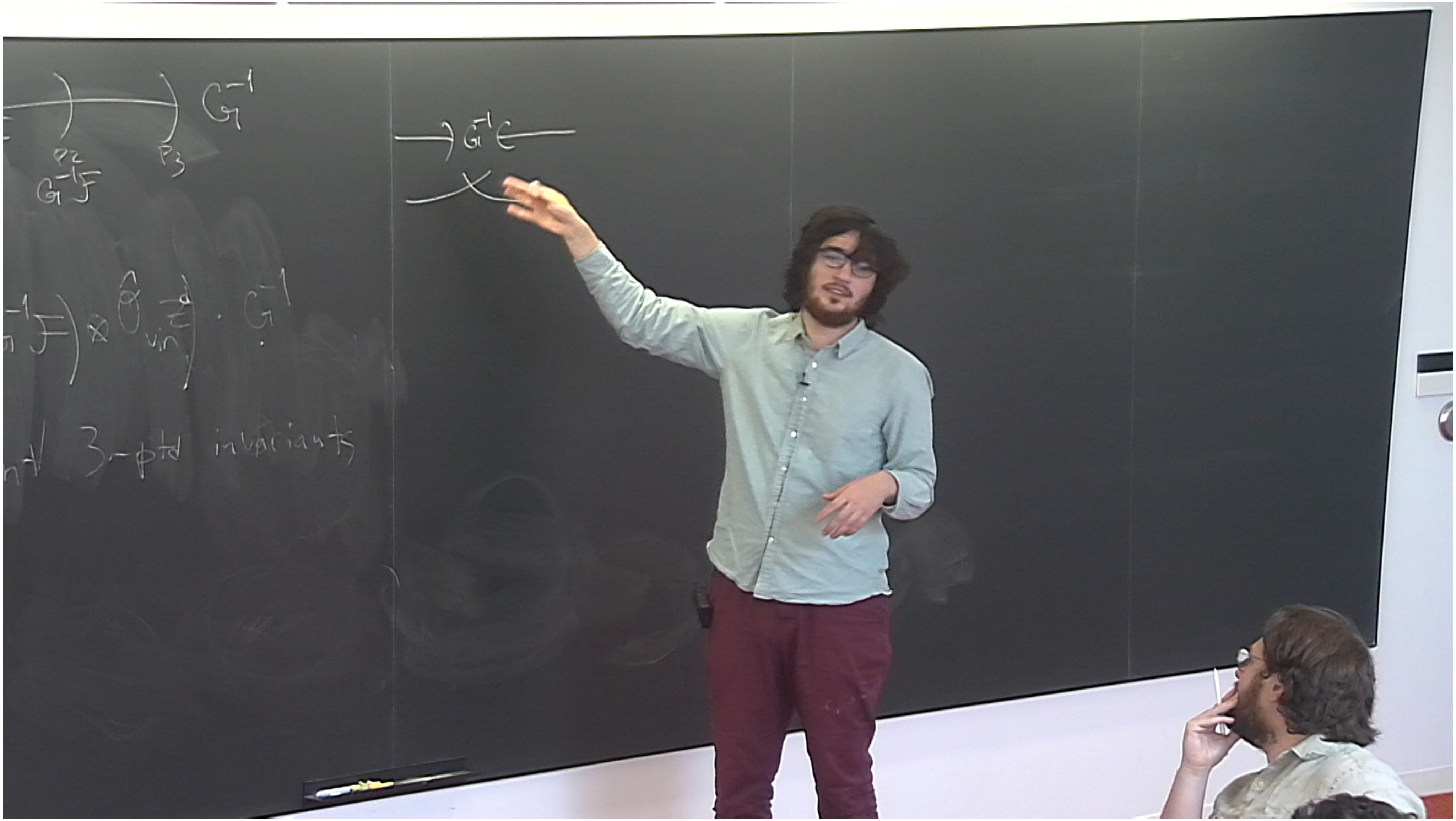

glueing matrix
rel∞
made into an operator
by the Sym: Classical
bilinear form
p1
p2
p3

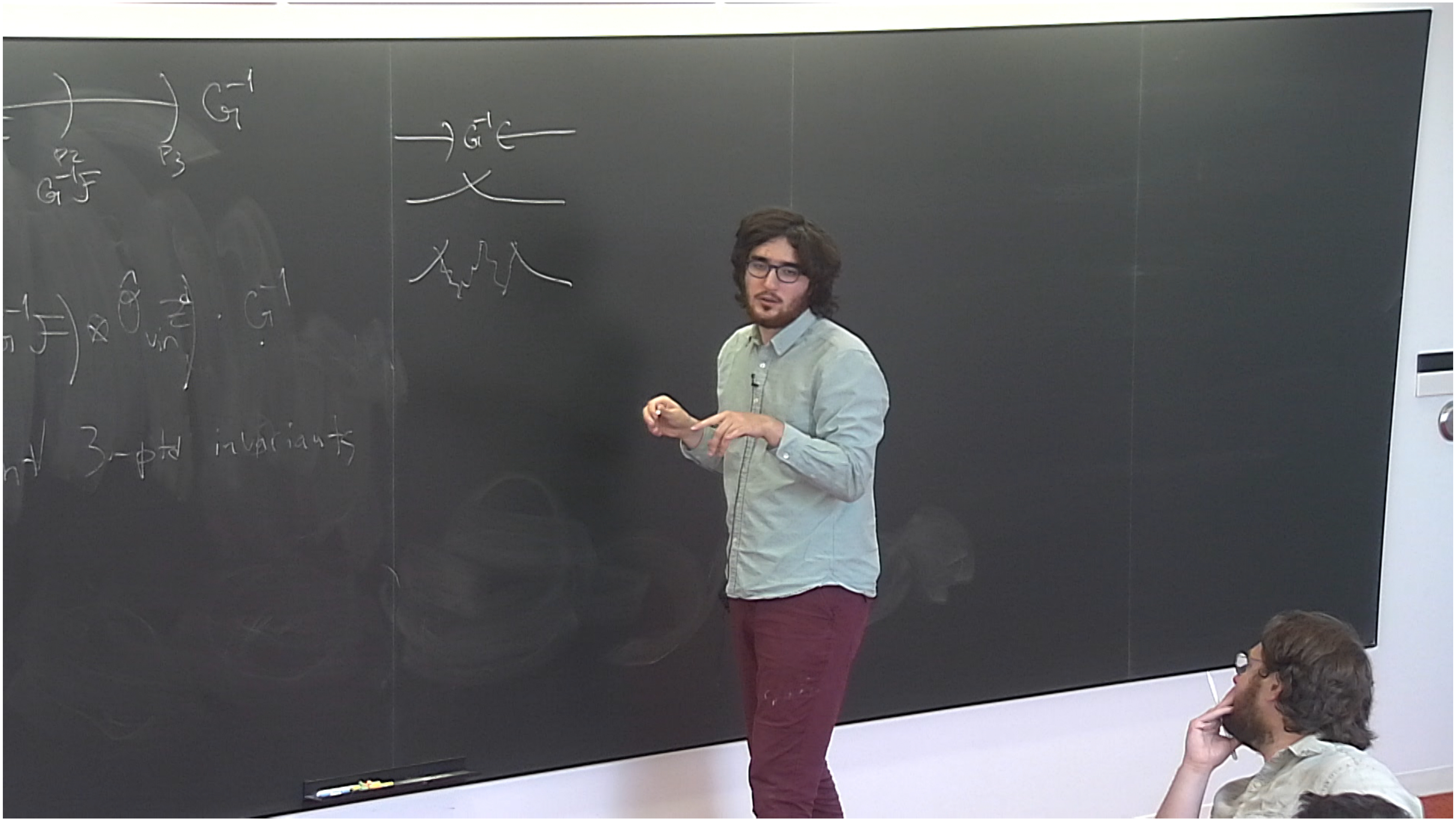

glueing matrix
rel∞
made into an operator
by the Sym: Classical
bilinear form
p1
p2
p3
d=0

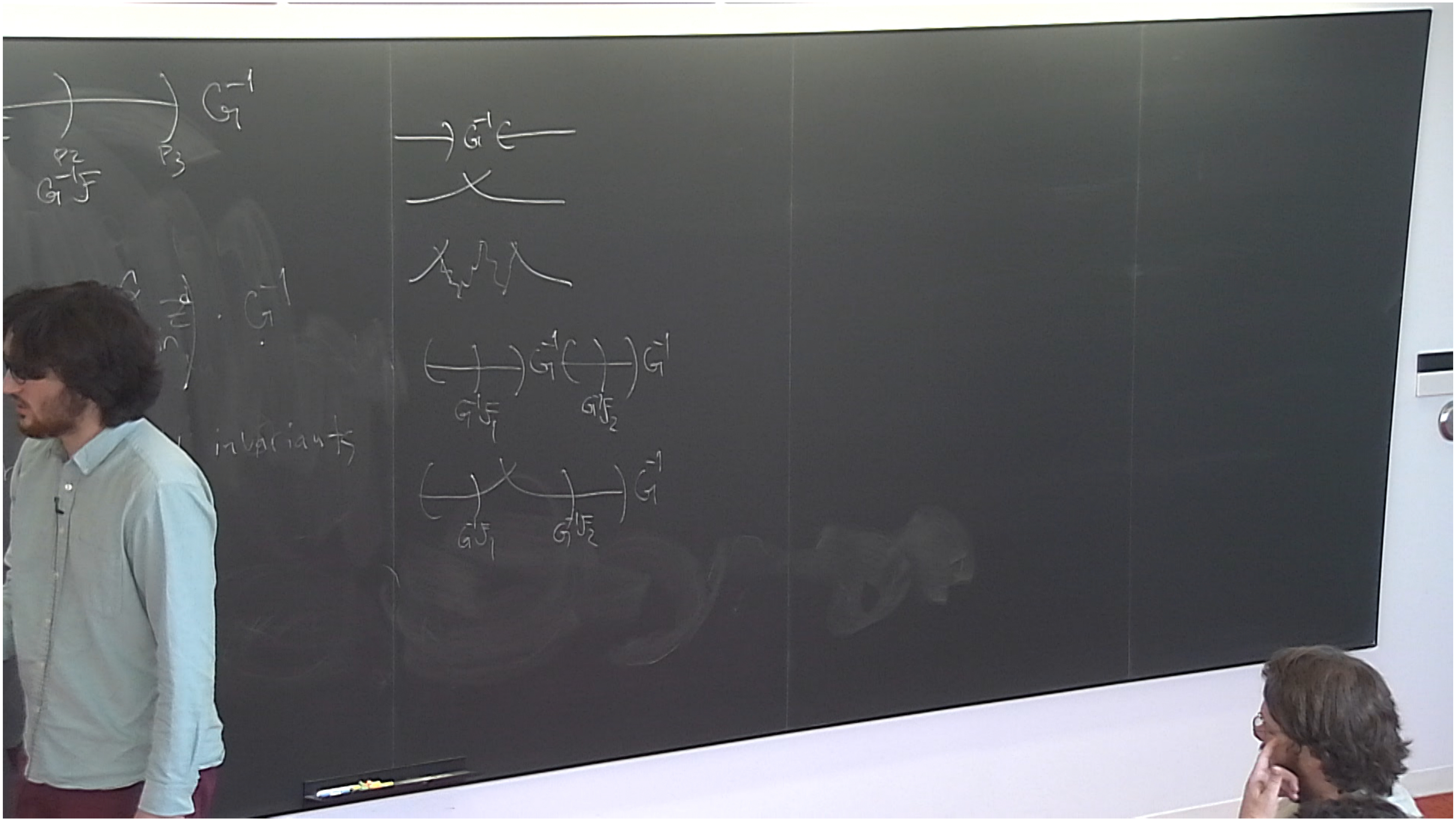

glueing matrix
rel∞
made into an operator by the Sym: classical bilinear form
$F \in K_T(T^*Gr(k,n))$
$F \circledast =$
$= \sum_{d=0}^{\infty} ev_{p_1,p_3,*}\left(QM^d_{rel\, p_1,p_2,p_3},\ ev^*_{p_2}(G^{-1}F) \otimes \hat{\mathcal{O}}_{vir}\, z^d \right) \cdot G^{-1}$
p1
p2
p3
$G^{-1}F$
G^{-1}

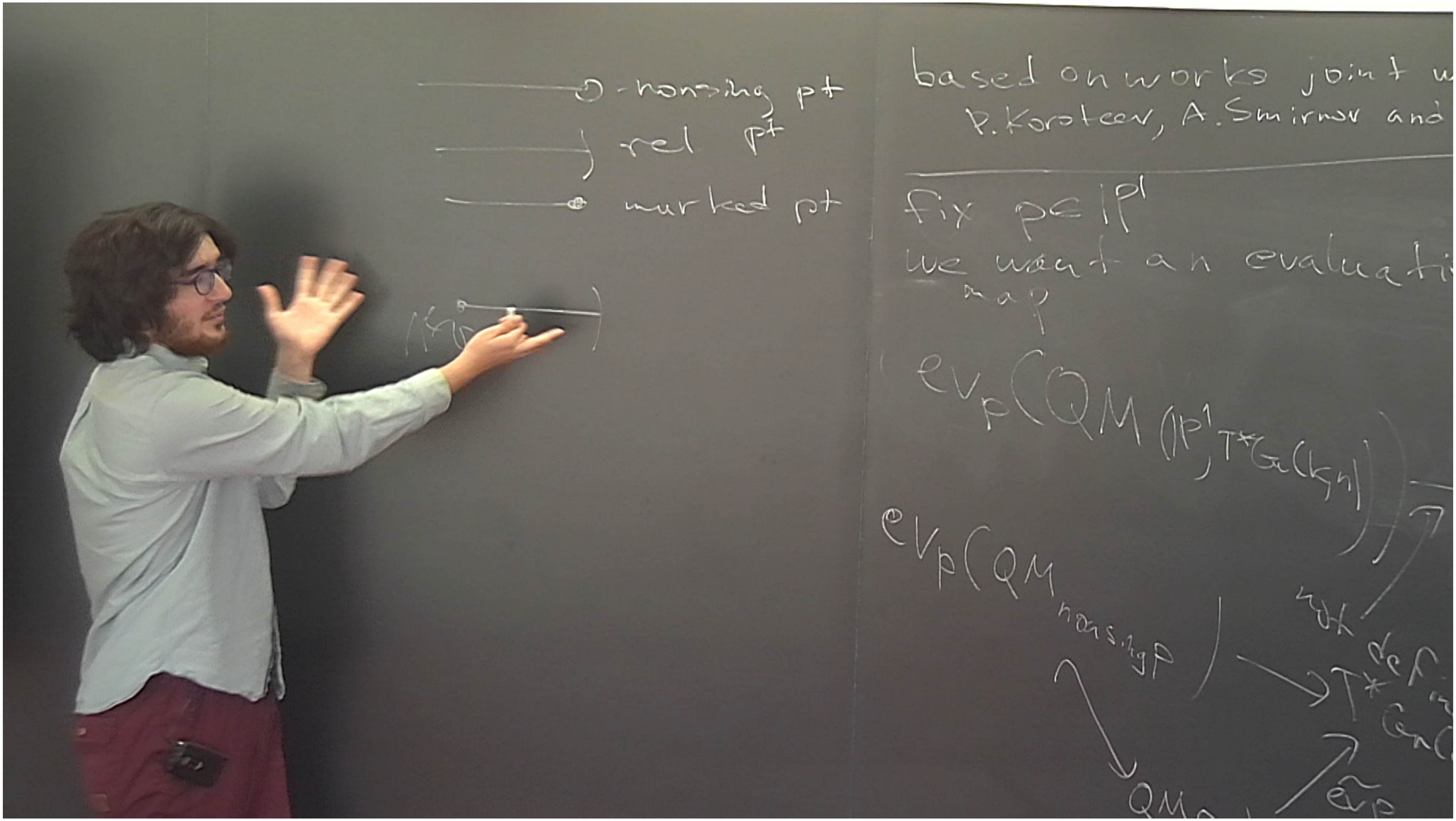

glueing matrix
rel∞
$\hat{\mathcal{O}}_{vir}$
$(QM^d_{rel\,0,\infty}, \hat{\mathcal{O}}_{vir})$
made into an operator by the sym. classical bilinear form
$\mathcal{F} \in K_T(T^*G(k,n))$
G^{-1}
p_1 p_2 p_3
$G^{-1}\mathcal{F}$
$\mathcal{F} \circledast =$
$= \sum_{d=0} ev_{p_1,p_3,*}(QM^d_{rel\,p_1,p_2,p_3}) \; ev^*_{p_2}(G^{-1}\mathcal{F}) \otimes \hat{\mathcal{O}}_{vir}\, z^d) \cdot G^{-1}$
the idea:
same as QH count 3-pted invariants


glueing matrix
rel∞
made into an operator by the sym. classical bilinear form
the idea:
same as QH corr. 3-pt'd invariants

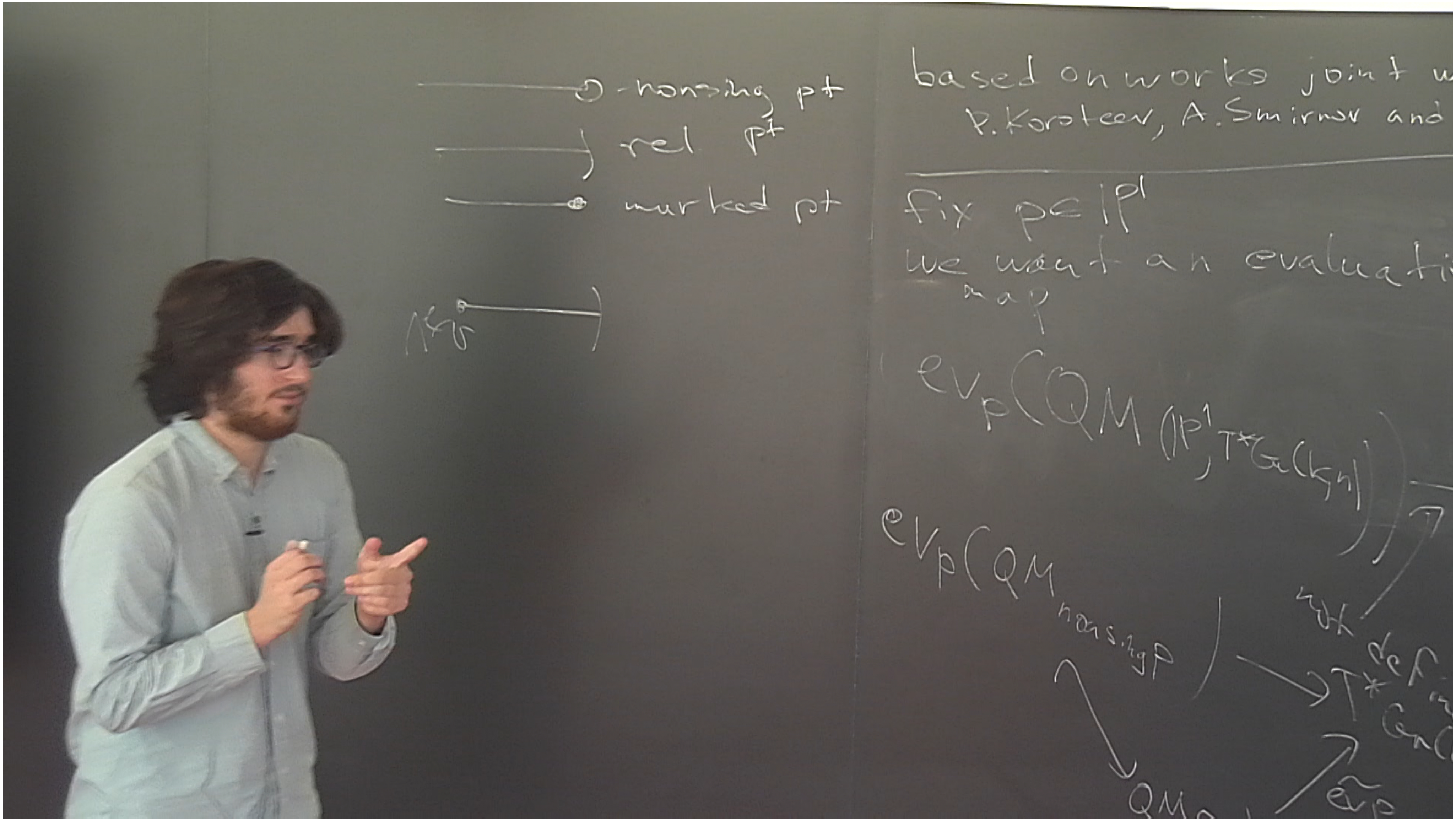

3-ptd invariants


3-pt'd invariants


invariants



- nonsing pt
rel pt
marked pt
based on works joint w
P. Koroteev, A. Smirnov and
fix p ∈ P1
we want an evaluati
map
evp(QM(P1, T*Gr(k,n)))
evp(QM nonsing p)
T*


nonsing pt
rel pt
marked pt
based on works joint w
P. Koroteev, A. Smirnov and
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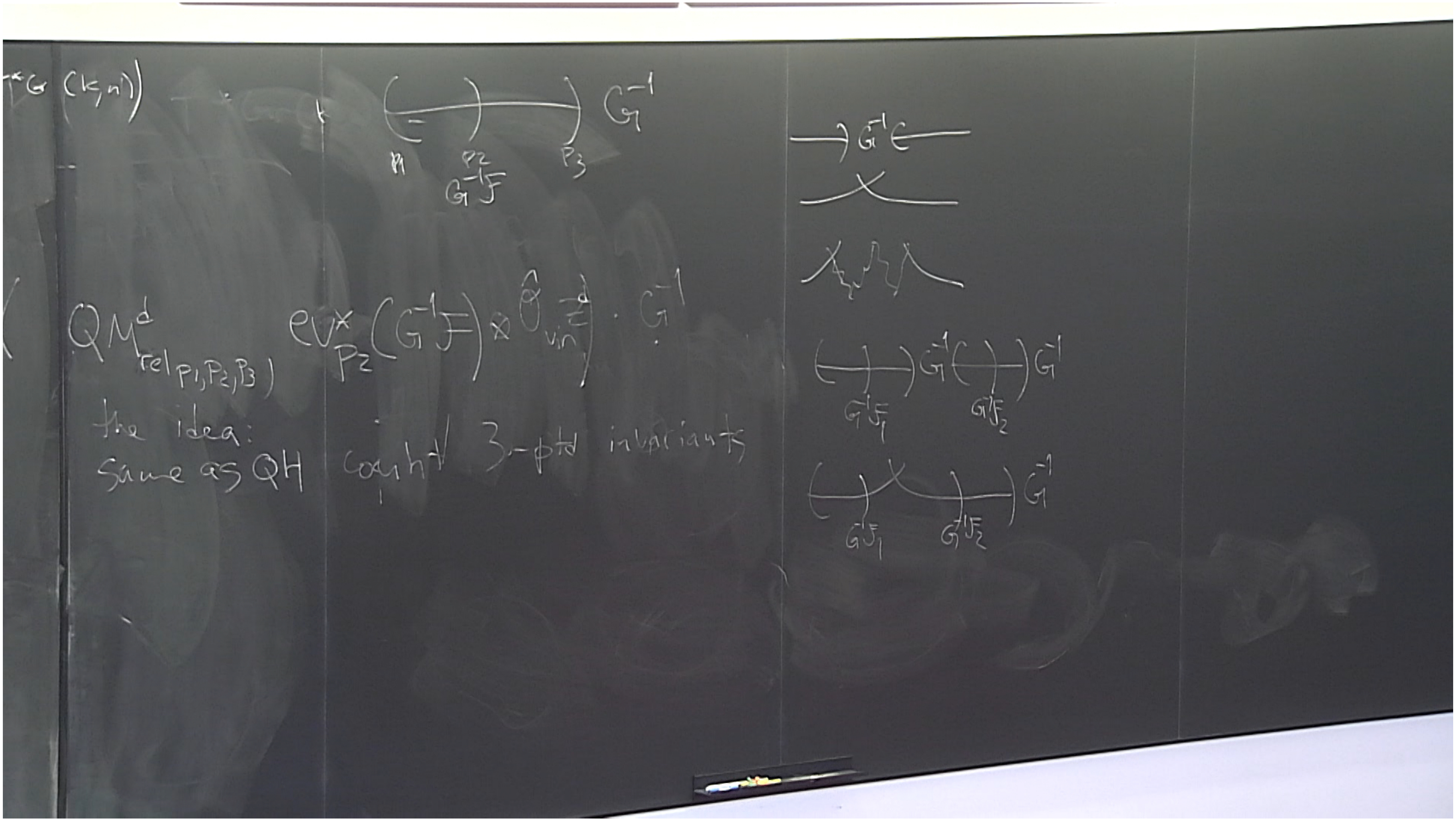
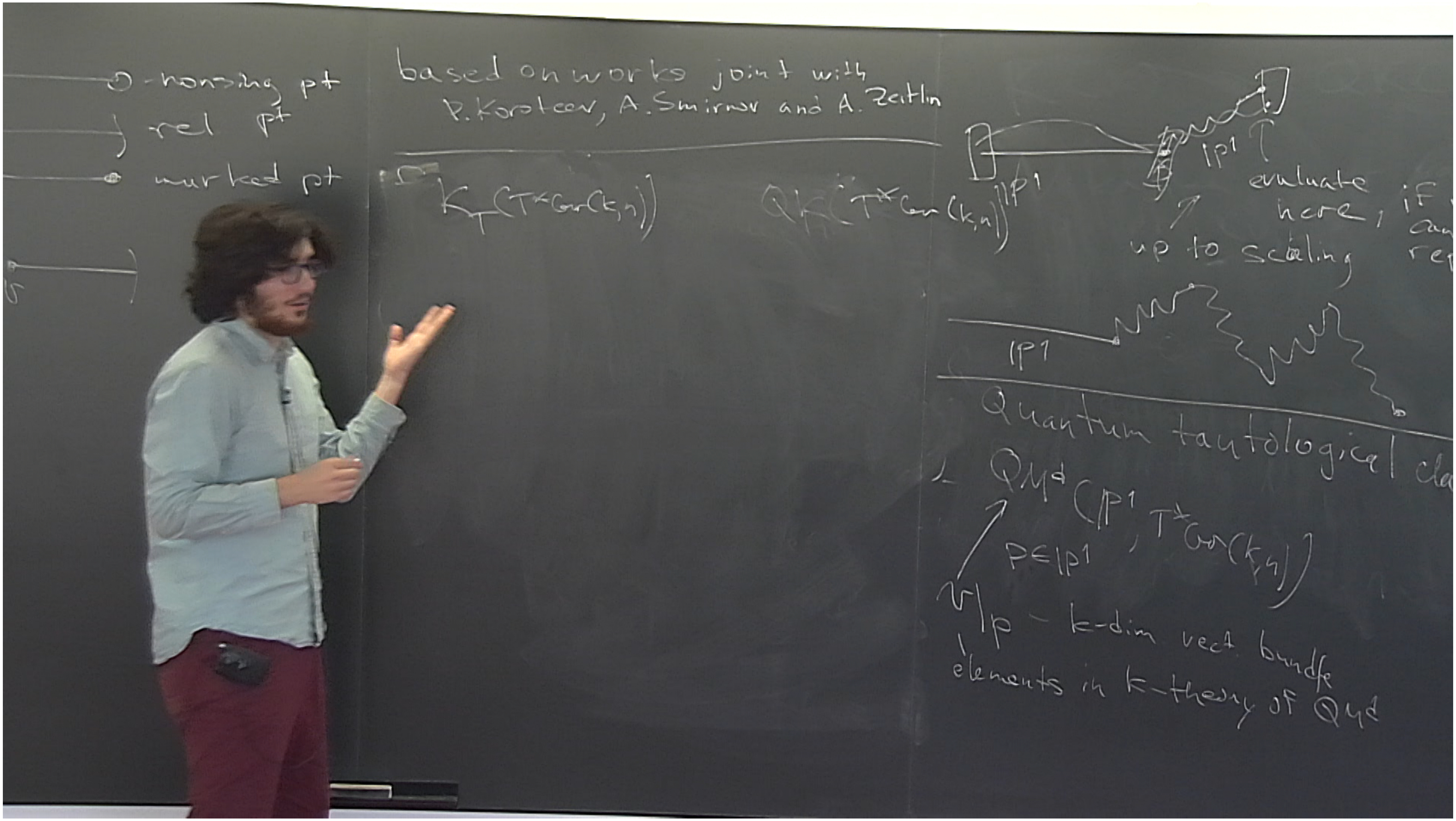
map
evp(QM(IP1, T*Gr(k,n)))
evp(QMnonsing p)


G^{-1}
p_1 p_2 p_3
$G^{-1}F$
$QM^d_{rel(p_1,p_2,p_3)}$ $ev^*_{p_2}(G^{-1}F) \otimes \hat{\theta}_{vir}$ $\cdot G^{-1}$
the idea:
same as QH
3-ptd invariants
$G^{-1}F_1$
$G^{-1}F_2$


rel pt
marked pt
based on works joint with
P. Koroteev, A. Smirnov and A. Zeitlin
K_T(T*Gr(k,n))
QK_T(T*Gr(k,n))
evaluate
here, if
up to scaling
Quantum tautological cla
QM^d(P^1, T*Gr(k,n))
p∈P^1
V|p - k-dim vect bundle
elements in K-theory of QM^d

- non-sing pt
- rel pt
- marked pt

based on works joint with
P. Koroteev, A. Smirnov and A. Zeitlin

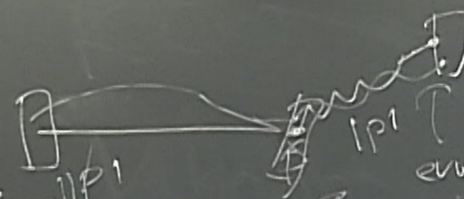
$K_T(T^*Gr(k,n))$

Fixed pts. (eigenbasis)

$\Lambda^i V$ – taut.

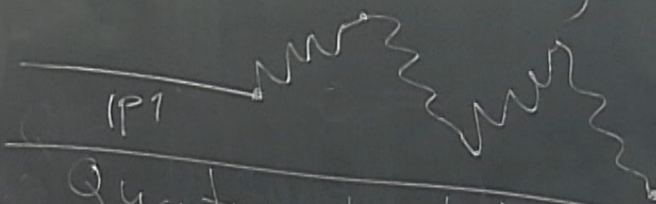
a_i – equiv characters

$QK_T(T^*Gr(k,n))$ $\mathbb{P}^1$



evaluate here, if ... can ... rep...

up to scaling



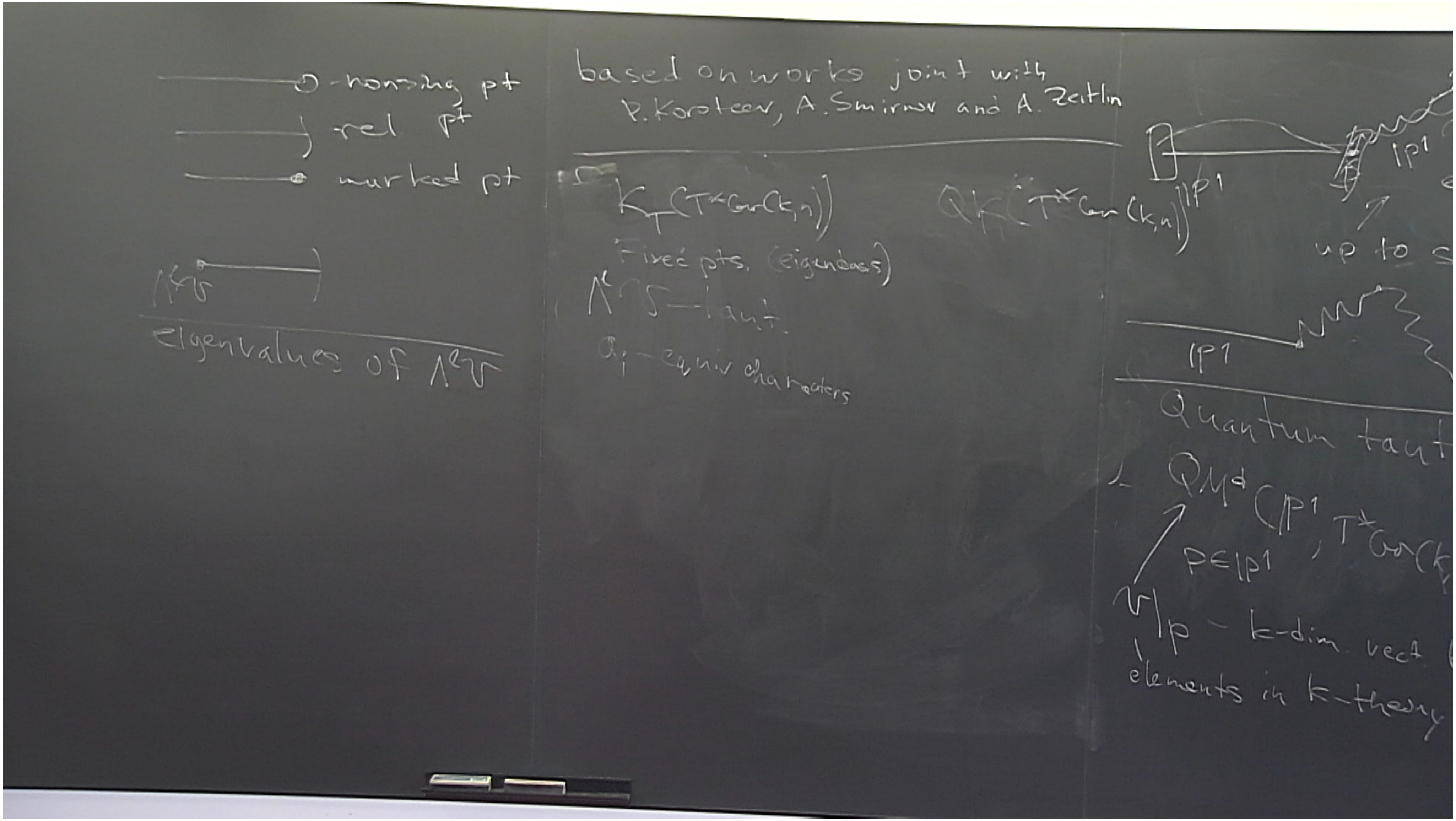
Quantum tautological cla...

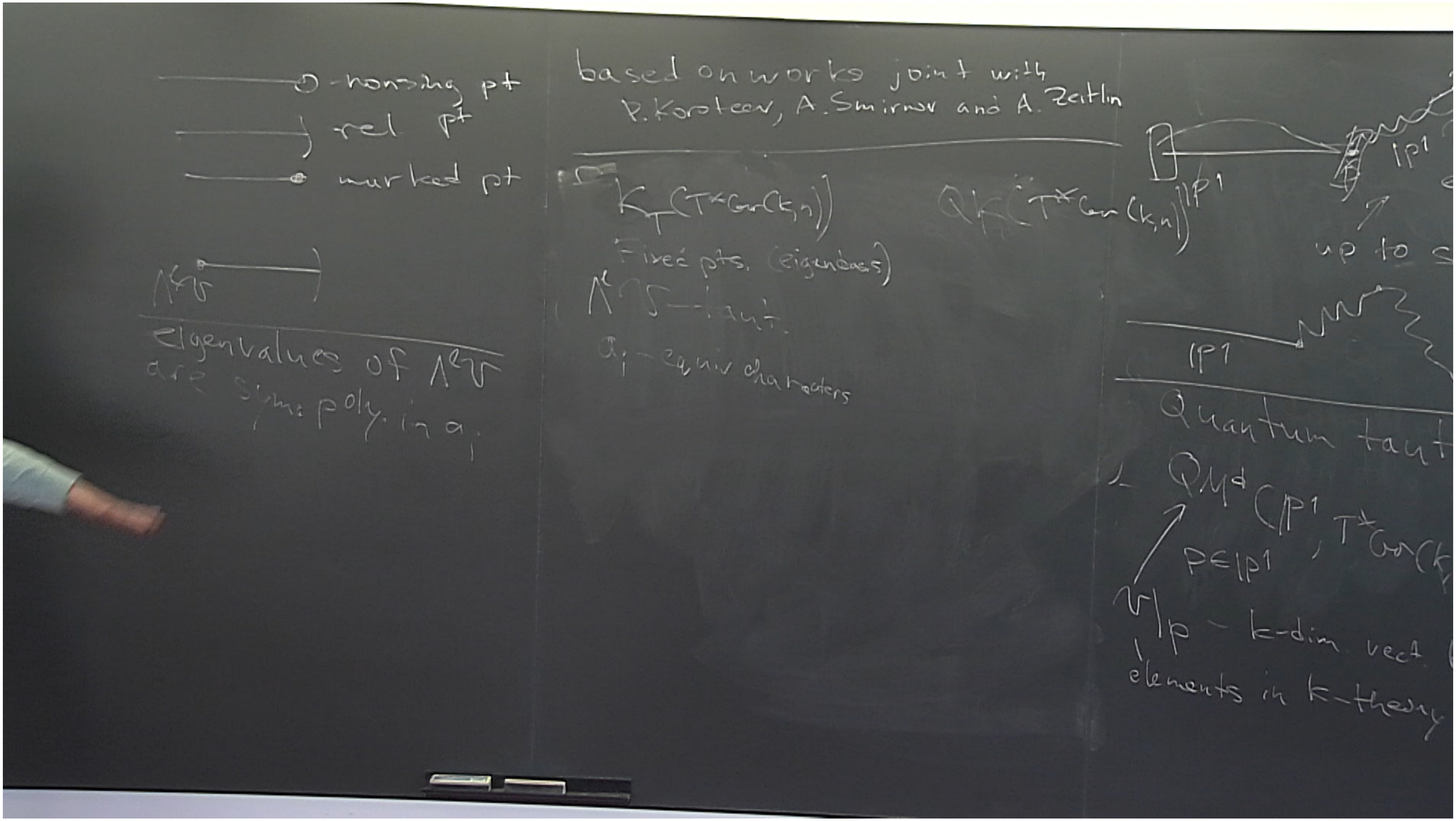
$QM^d(\mathbb{P}^1, T^*Gr(k,n))$

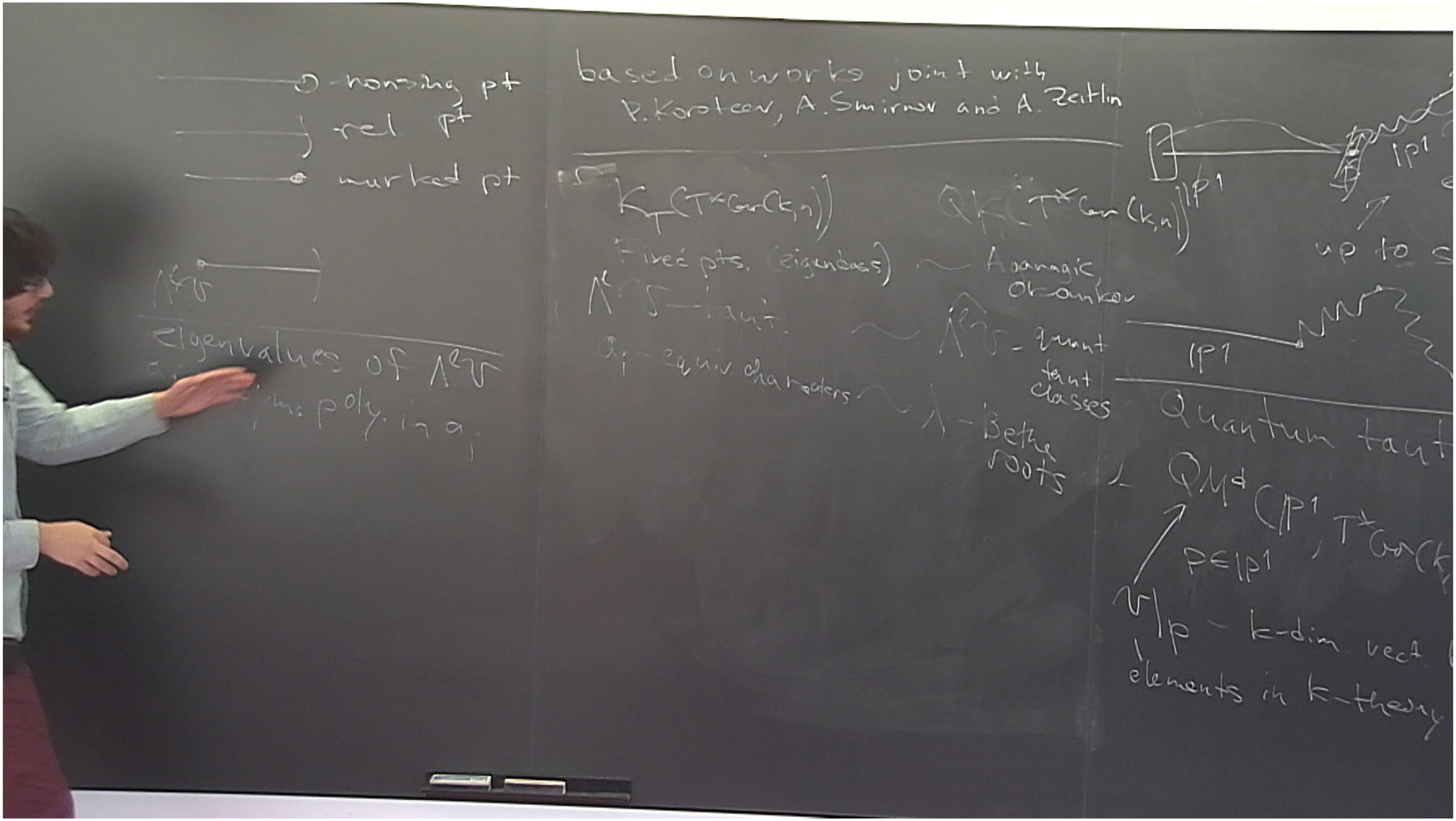
$p \in \mathbb{P}^1$

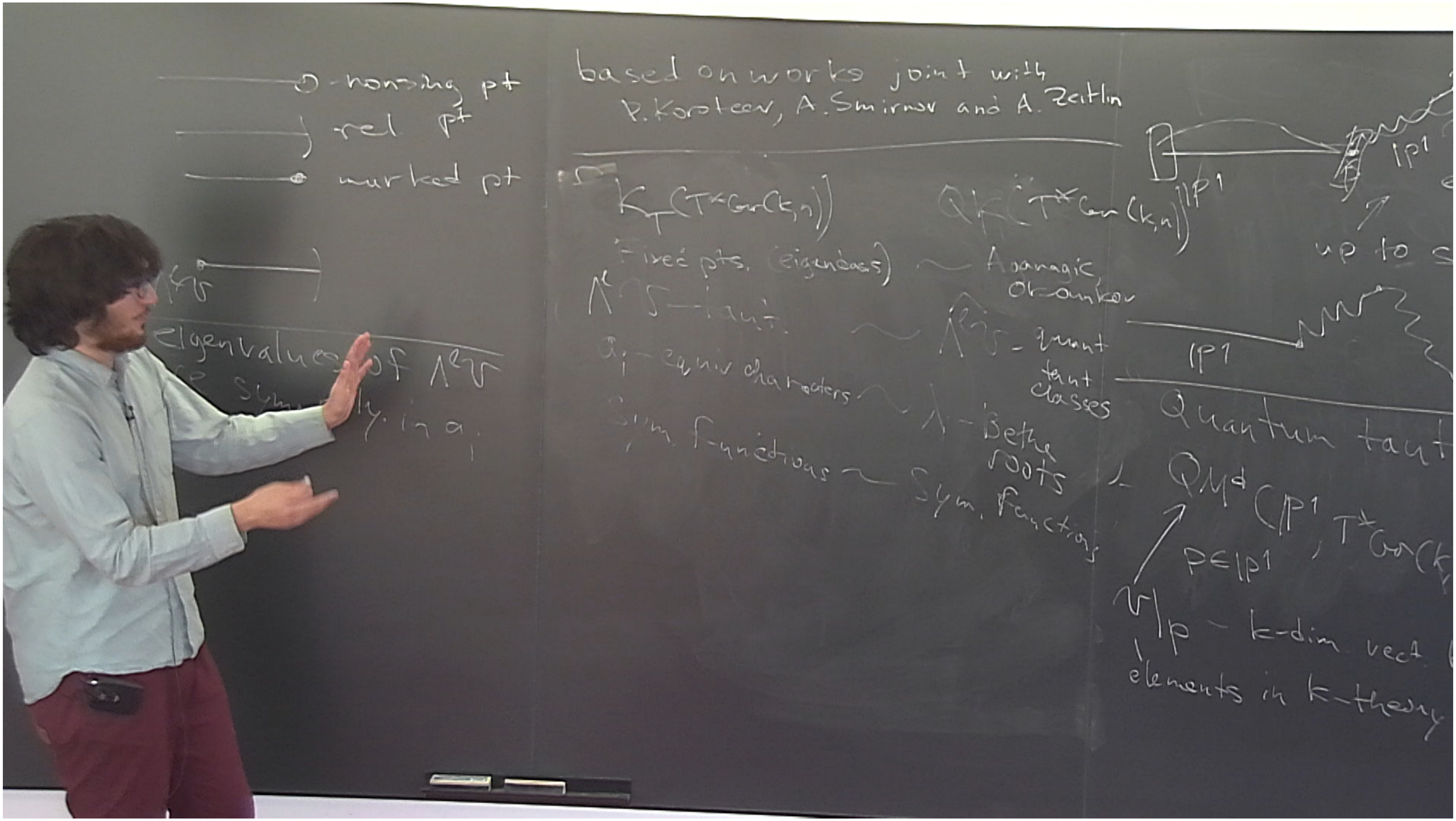
$V|_p$ – k-dim vect. bundle

elements in K-theory of QM^d

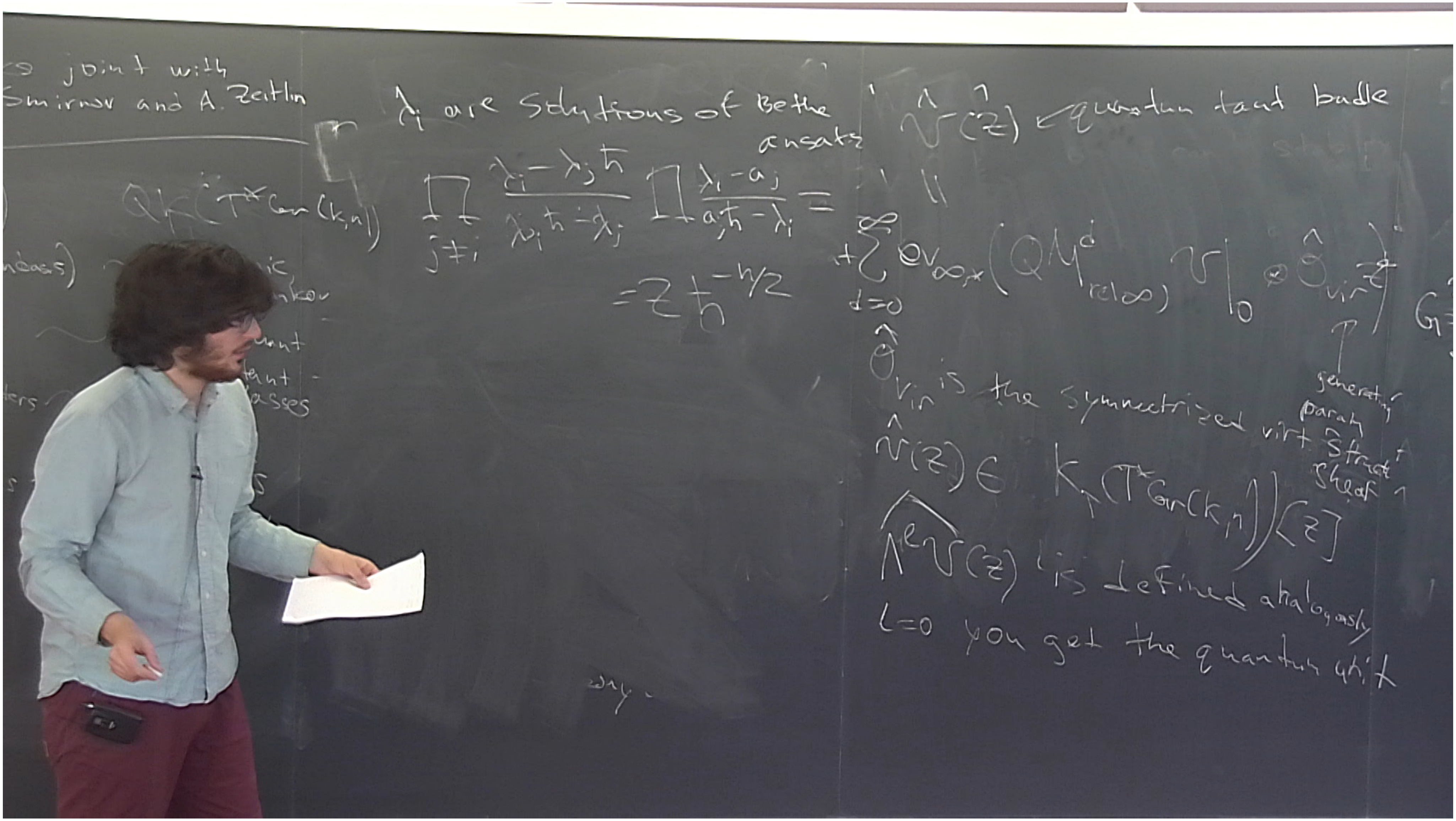

- nonsing pt
rel pt
marked pt
eigenvalues of Λ^k V
based on works joint with
P. Koroteev, A. Smirnov and A. Zeitlin
K_T(T*Gr(k,n))
QK_T(T*Gr(k,n))
fixed pts. (eigenbasis)
Λ^k V - taut.
a_i - equiv characters
up to s
Quantum taut
QM^d(P^1, T*Gr(k
p ∈ P^1
V|p - k-dim vect
elements in K-theory


nonsing pt
rel pt
marked pt
Eigenvalues of
are sym. poly. in a
based on works joint with
P. Koroteev, A. Smirnov and A. Zeitlin
Fixed pts. (eigenbasis)
equiv characters
up to s
Quantum taut
elements in K-theory


nonsing pt
rel pt
marked pt
eigenvalues of
poly. in
based on works joint with
P. Koroteev, A. Smirnov and A. Zeitlin
Fixed pts. (eigenbasis)
equiv characters
Aganagic, Okounkov
quant taut classes
Bethe roots
up to
Quantum taut
k-dim vect
elements in K-theory



based on works joint with
P. Koroteev, A. Smirnov and A. Zeitlin
vanishing pt
rel pt
marked pt
eigenvalues of
Fixed pts. (eigenbasis)
Aganagic, Okounkov
taut
quant taut classes
equiv characters
Bethe roots
Sym. functions
Sym. functions
Quantum taut
up to s
k-dim. vect
elements in K-theory


joint with
Smirnov and A. Zeitlin
λ_i are solutions of Bethe ansatz
= z ħ^{-n/2}
quantum taut bundle
is the symmetrized virt. struct. sheaf
generating param
is defined analogously
l=0 you get the quantum quot

...s joint with
Smirnov and A. Zeitlin

$QK(T^*Gr(k,n))$

...classes) ~ Aomagic Okounkov

~ $\hat{\Lambda^\ell \mathcal{V}}$ - quant taut classes

...ters ~ λ - Bethe roots

...s ~ Sym. functions

λ_i are solutions of Bethe ansatz

$$\prod_{j\neq i} \frac{\lambda_i - \lambda_j \hbar}{\lambda_j \hbar - \lambda_j} \prod \frac{\lambda_i - a_j}{a_j \hbar - \lambda_i} = z\,\hbar^{-n/2}$$

$i = 1 \ldots k$

$\hat{\mathcal{V}}(z)$ ← quantum taut bundle

$$= \sum_{d=0}^{\infty} ev_{\infty,*}\left(QM^d_{rel\,\infty}, \mathcal{V}|_0 \otimes \hat{\mathcal{O}}_{vir}\, z^d\right)$$

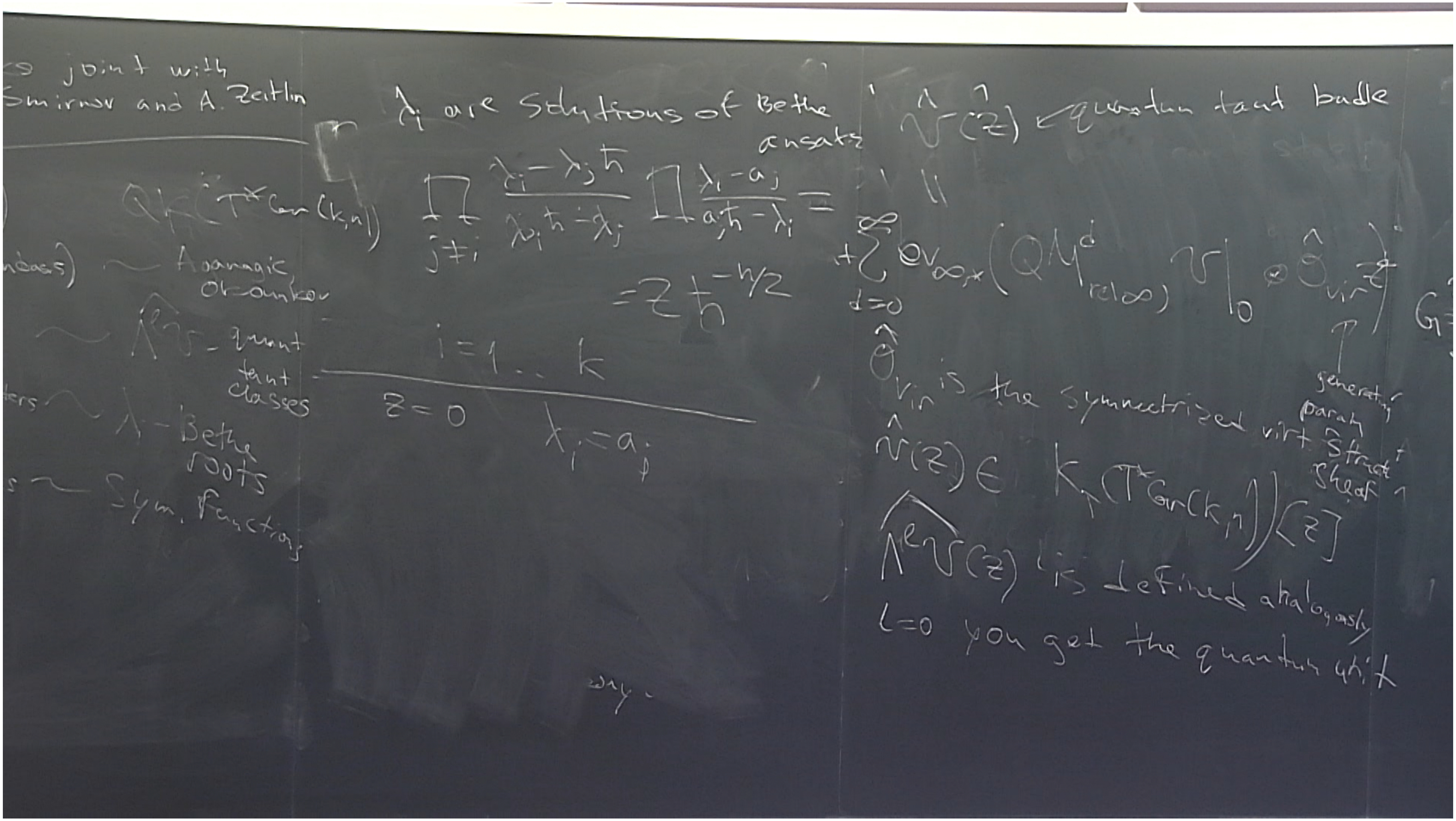
↑ generating param

$\hat{\mathcal{O}}_{vir}$ is the symmetrized virt struct sheaf

$\hat{\mathcal{V}}(z) \in K_T(T^*Gr(k,n))[[z]]$

$\widehat{\Lambda^\ell \mathcal{V}}(z)$ is defined analogously

$\ell = 0$ you get the quantum unit


joint with
Smirnov and A. Zeitlin
$QK(T^*Gr(k,n))$
Aganagic Okounkov
quant taut classes
Bethe roots
Sym. Functions
λ_i are solutions of Bethe ansatz
$\prod_{j\neq i} \frac{\lambda_i - \lambda_j \hbar}{\lambda_i \hbar - \lambda_j} \prod \frac{\lambda_i - a_j}{a_j \hbar - \lambda_i} = z\hbar^{-n/2}$
$i = 1 \ldots k$
$z = 0$
$\lambda_i = a_i$
$\hat{V}(z)$ ← quantum taut bundle
generating param
$\hat{V}(z) \in K_T(T^*Gr(k,n))[z]$
$\hat{\Lambda^l V}(z)$ is defined analogously
$l = 0$ you get the quantum unit

joint with Smirnov and A. Zeitlin

$QK_T(T^*Gr(k,n))$

(...classes) ~ Aomagic Okounkov

~ $\hat{\Lambda}^{\ell}\mathcal{V}$ - quant taut classes

...ers ~ λ - Bethe roots

...s ~ Sym. functions

λ_i are solutions of Bethe ansatz

$$\prod_{j\neq i} \frac{\lambda_i - \lambda_j \hbar}{\lambda_i \hbar - \lambda_j} \prod \frac{\lambda_i - a_j}{a_j \hbar - \lambda_i} = z\, \hbar^{-n/2}$$

$i = 1 \dots k$

$z = 0 \quad \lambda_i = a_i$

eigenvalues of quant taut classes are corresp. sym funct. of λ_i

$\hat{\mathcal{V}}(z)$ ← quantum taut bundle

$$\hat{\mathcal{V}}(z) = \sum_{d=0}^{\infty} ev_{\infty,*}\left(QM^d_{rel \infty}\right) \mathcal{V}|_0 \otimes \hat{\mathcal{O}}_{vir} z^d$$

↑ generating param.

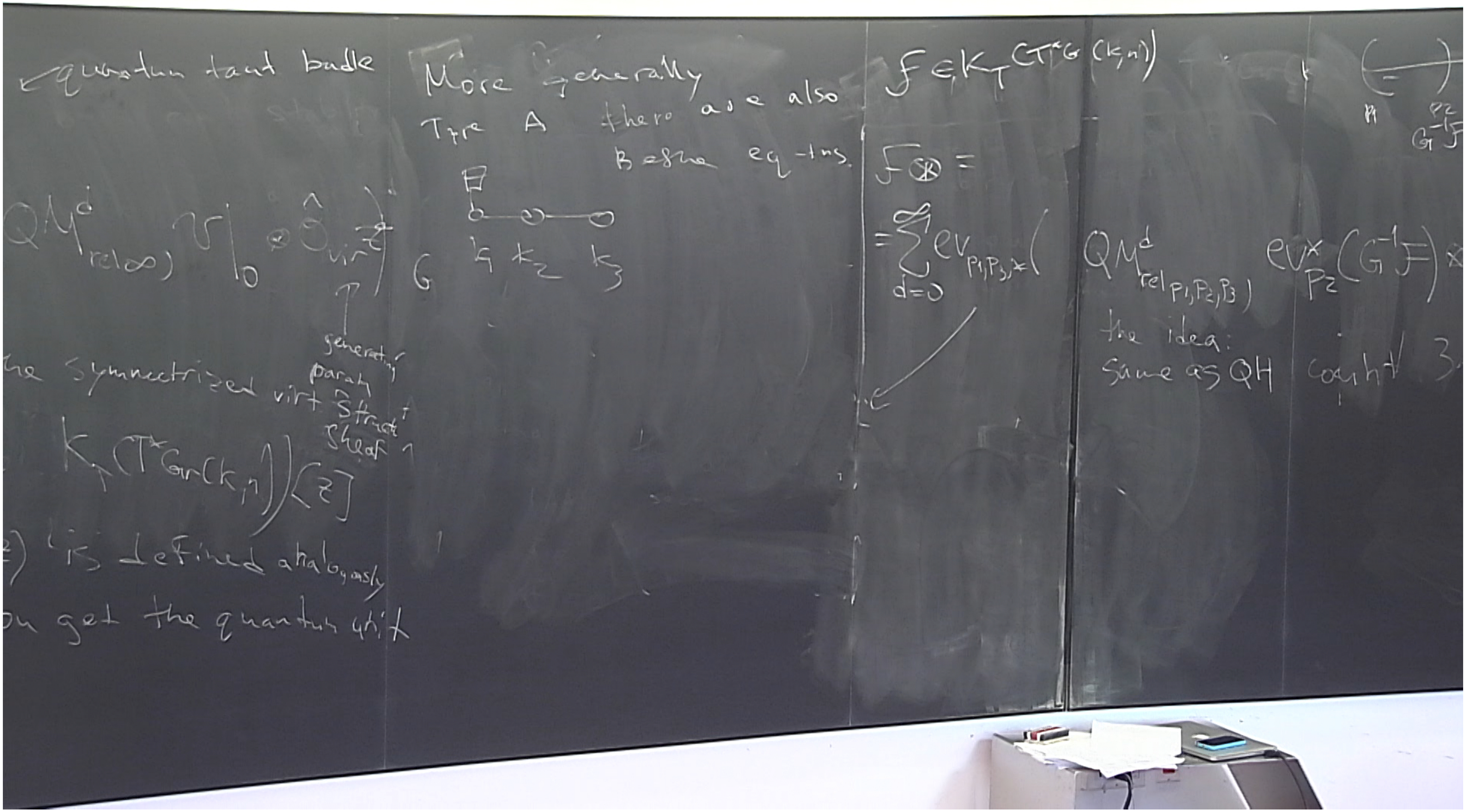
$\hat{\mathcal{O}}_{vir}$ is the symmetrized virt. struct. sheaf

$\hat{\mathcal{V}}(z) \in K_T(T^*Gr(k,n))[[z]]$

$\widehat{\Lambda^{\ell}\mathcal{V}(z)}$ is defined analogously

$\ell = 0$ you get the quantum unit


More generally
there are also
Bethe eq-ns
the symmetrized virt
the idea:
same as QH
you get the quantum


quantum taut bundle
More generally
Type A there are also
Bethe eq-tns.
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k_1 k_2 k_3
generating
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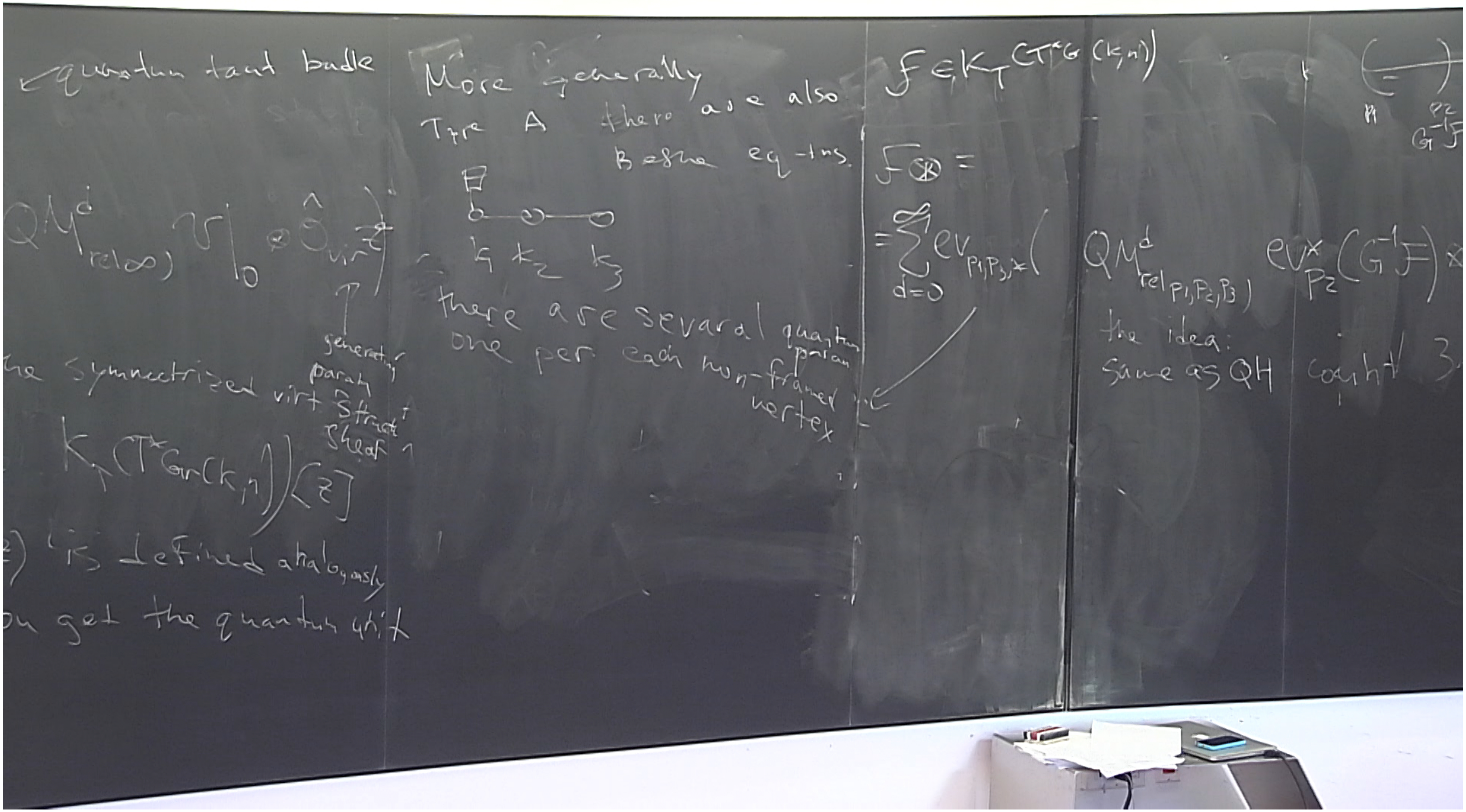
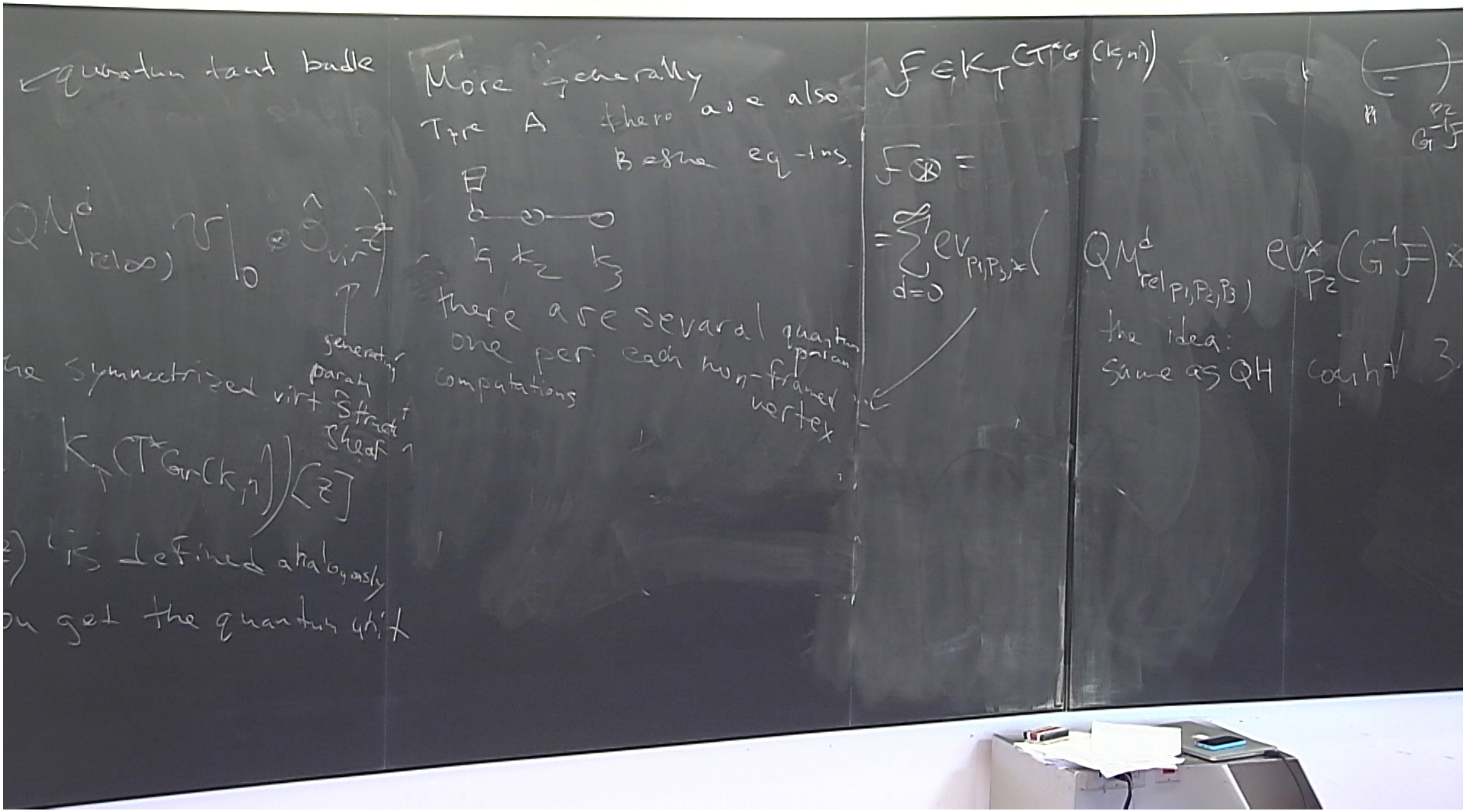
the symmetrized virt struct sheaf
is defined analogously
you get the quantum wrt
the idea:
same as QH
$\mathcal{F} \in K_T(T^*Gr(k,n))$


quantum tant bundle
the symmetrized virt struct sheaf
is defined analogously
you get the quantum wall
More generally
Type A there are also
Bethe eq-tns.
there are several quantum param
one per each non-framed vertex
the idea:
same as QH


quantum taut. bundle
the symmetrized virt. struct. sheaf
is defined analogously
you get the quantum wit
More generally
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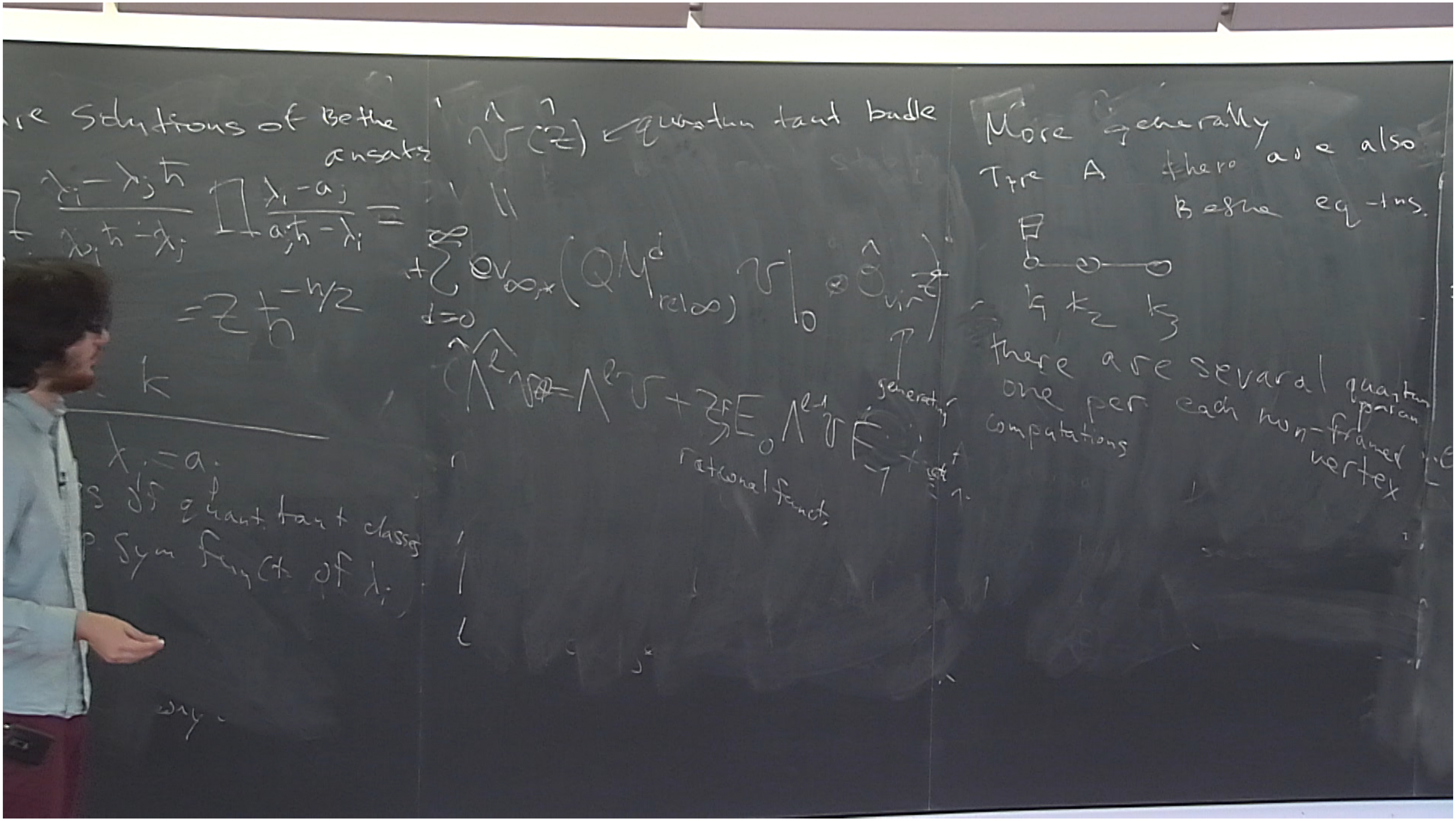
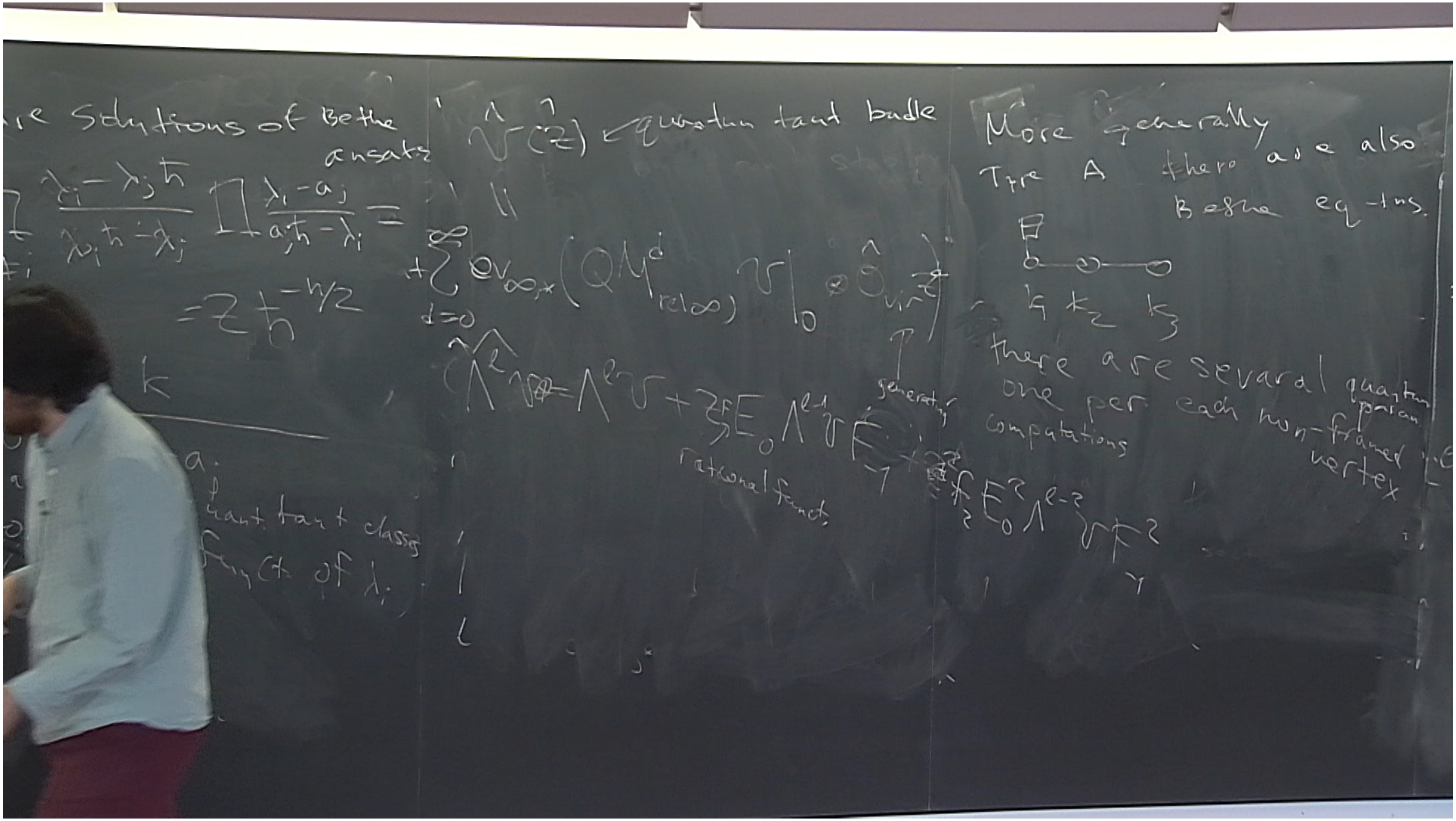
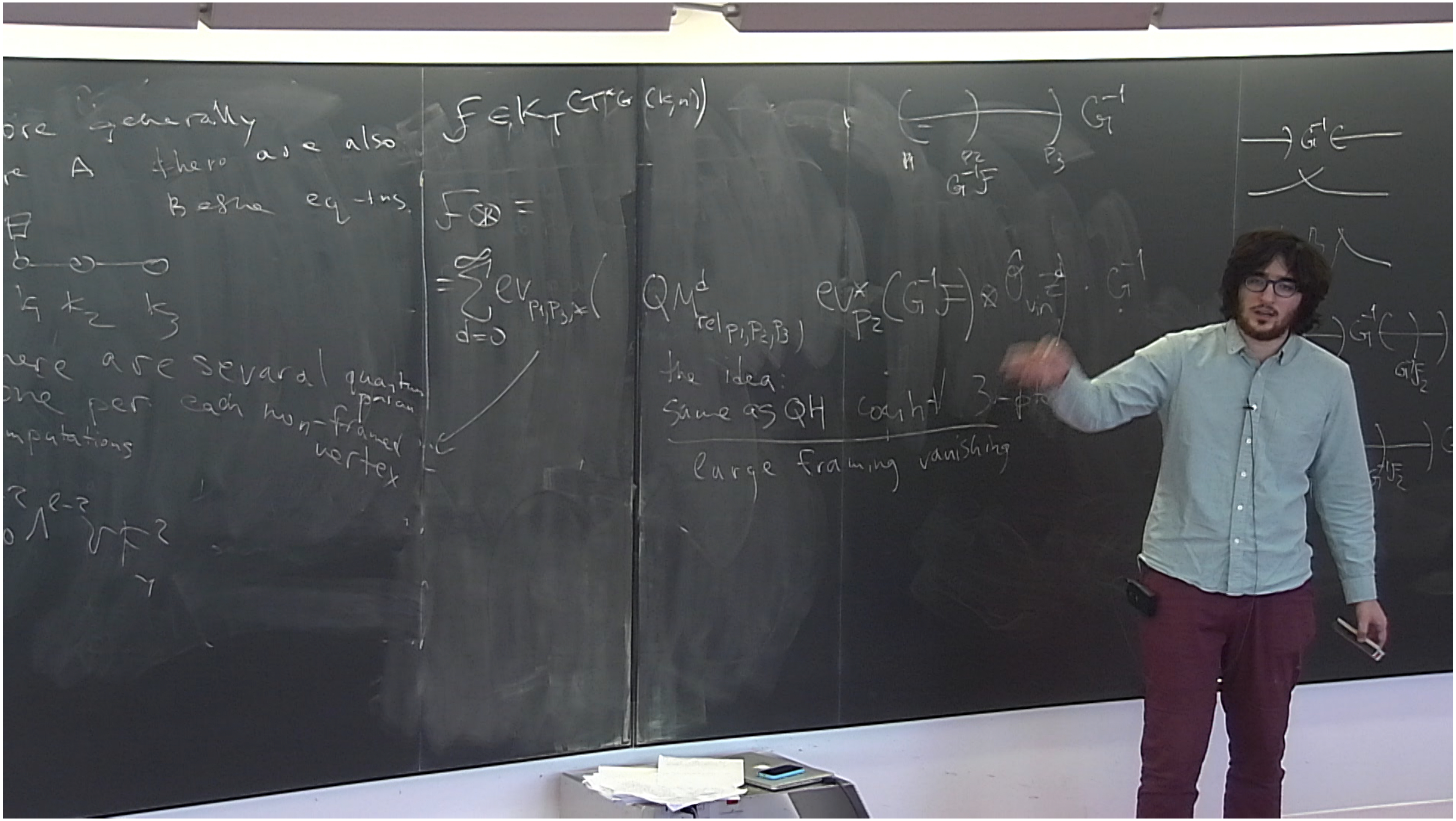
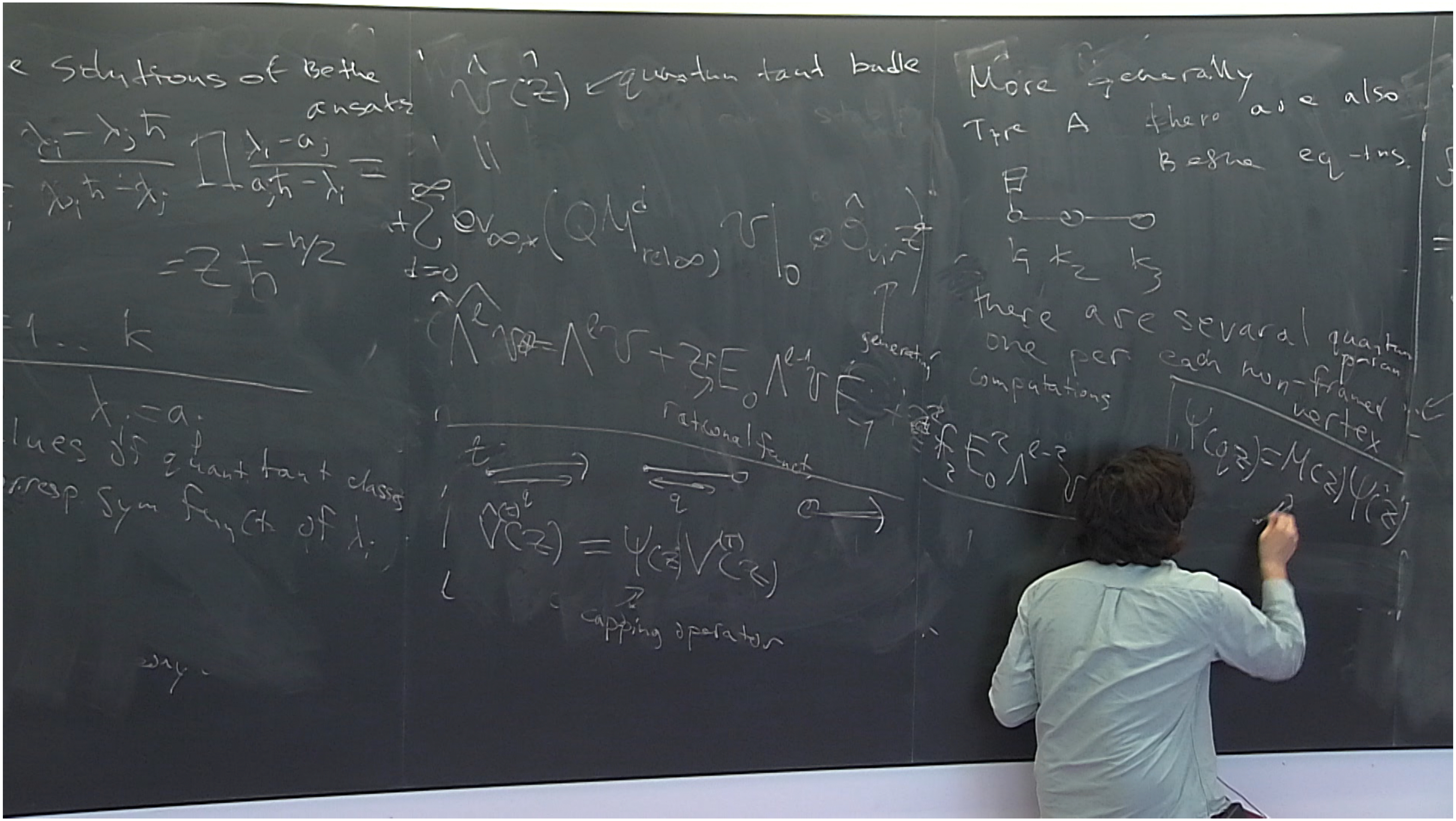
k1 k2 k3
there are several quantum param
one per each non-framed vertex
computations
the idea:
same as QH

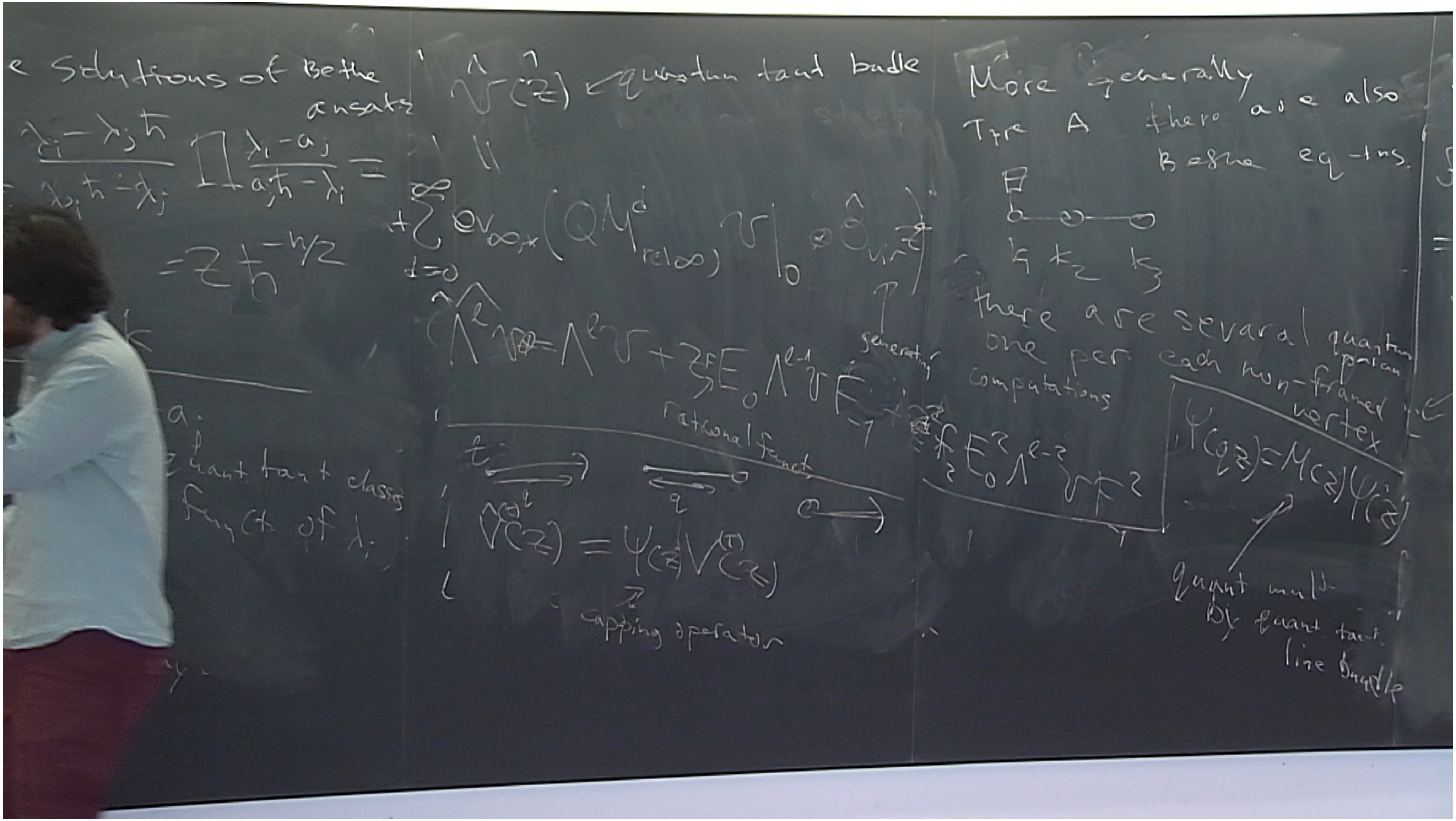


solutions of Bethe ansatz
quantum taut bundle
rational funct.
generating
More generally
Type A
there are also Bethe eq-tns.
there are several quantum param
one per each non-framed vertex
computations

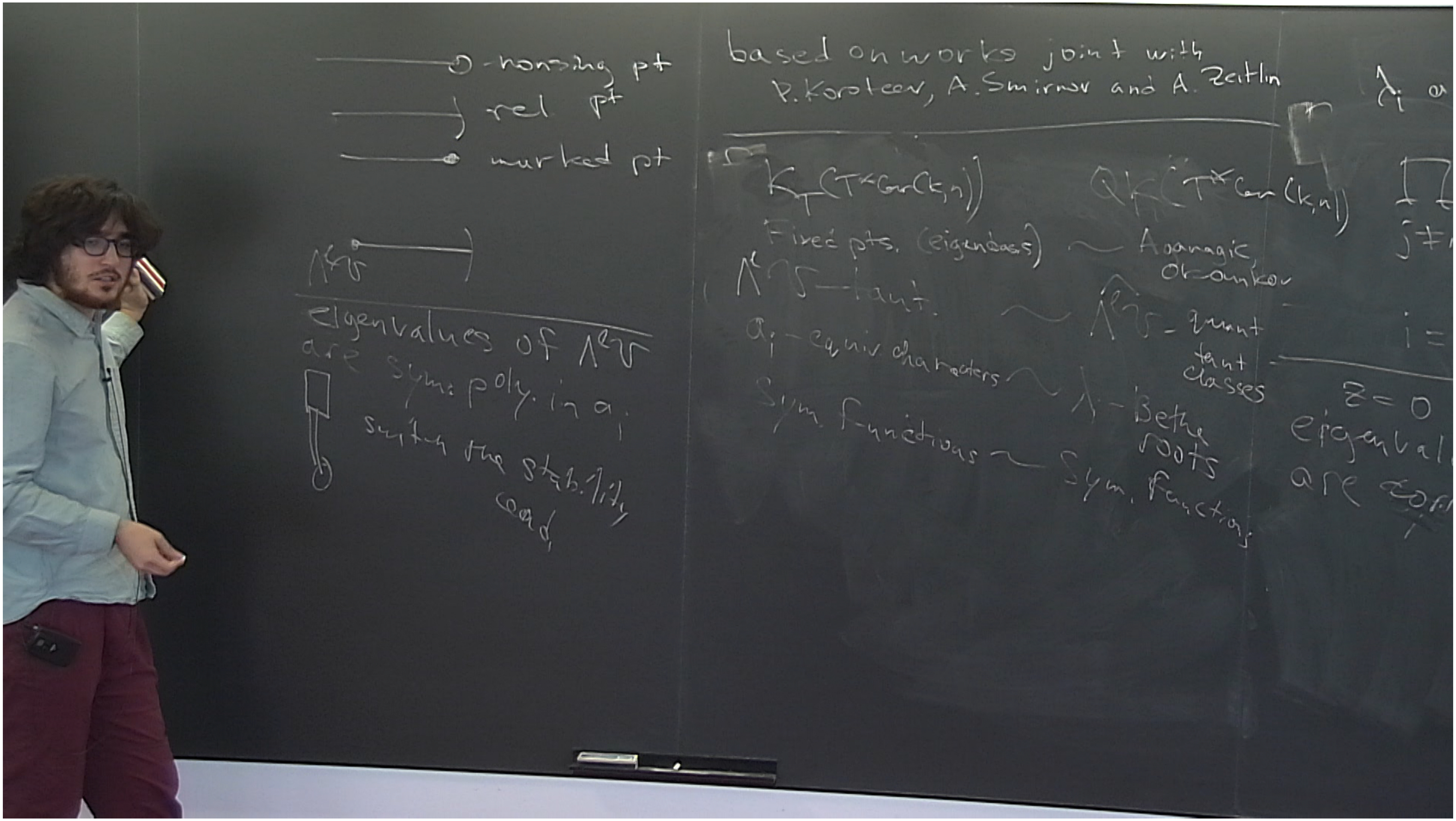

Solutions of Bethe ansatz
quantum taut bundle
More generally
Type A
there are also
Bethe eq-tns.
there are several quantum
one per each non-framed vertex
param
computations
generating
rational functs
quant taut class
fcts of


generally
there are also
Bethe eq-ns
are several quantum
one per each non-framed
vertex
the idea:
same as QH
large framing vanishing

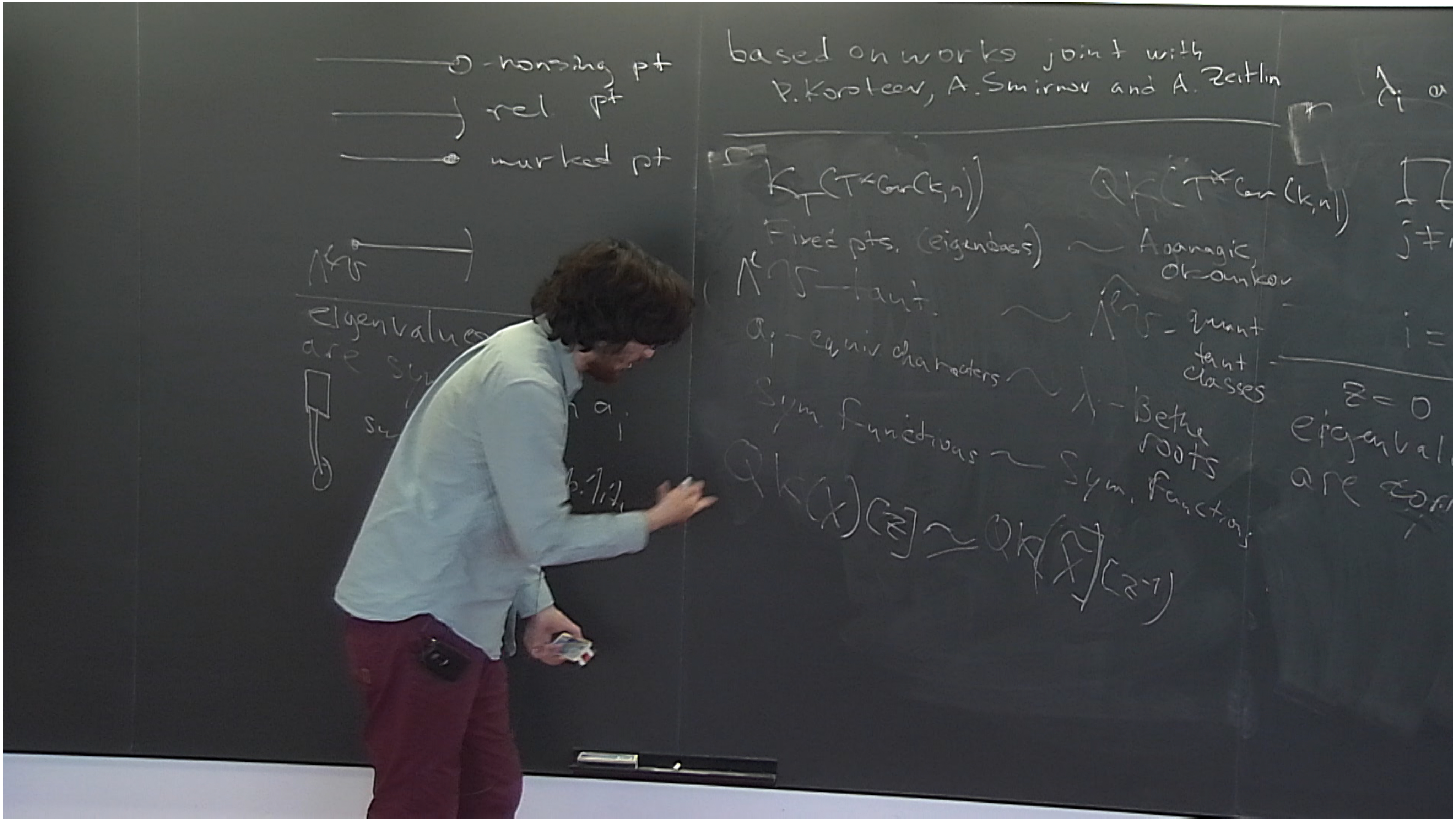

Solutions of Bethe ansatz
$= z\hbar^{-h/2}$
$\lambda_i = a_i$
values of quant taut classes
corresp sym func of λ_i
quantum taut bundle
generating
rational funct.
capping operator
$V^{(\tau)}(z) = \Psi(z) V^{(\tau)}(z)$
More generally
Type A there are also Bethe eq-tns.
$k_1 \; k_2 \; k_3$
there are several quantum param
one per each non-framed vertex
computations
$\Psi(q, z) = M(z) \Psi(z)$



Solutions of Bethe ansatz
quantum taut bundle
More generally
Type A
there are also
Bethe eq-tns.
there are several quantum param
one per each non-framed vertex
generating
computations
rational funct
capping operator
quant mult by quant taut line bundle
quant taut classes
funct of



nonsing pt
rel pt
marked pt
eigenvalues of $\Lambda^{\ell}\mathcal{V}$
are sym. poly. in a_i
based on works joint with
P. Koroteev, A. Smirnov and A. Zeitlin
$K_T(T^*Gr(k,n))$
$QK_T(T^*Gr(k,n))$
Fixed pts. (eigenbasis)
Aganagic, Okounkov
a_i - equiv characters
λ_i - Bethe roots
Sym. functions
Sym. functions
z = 0


nonsing pt
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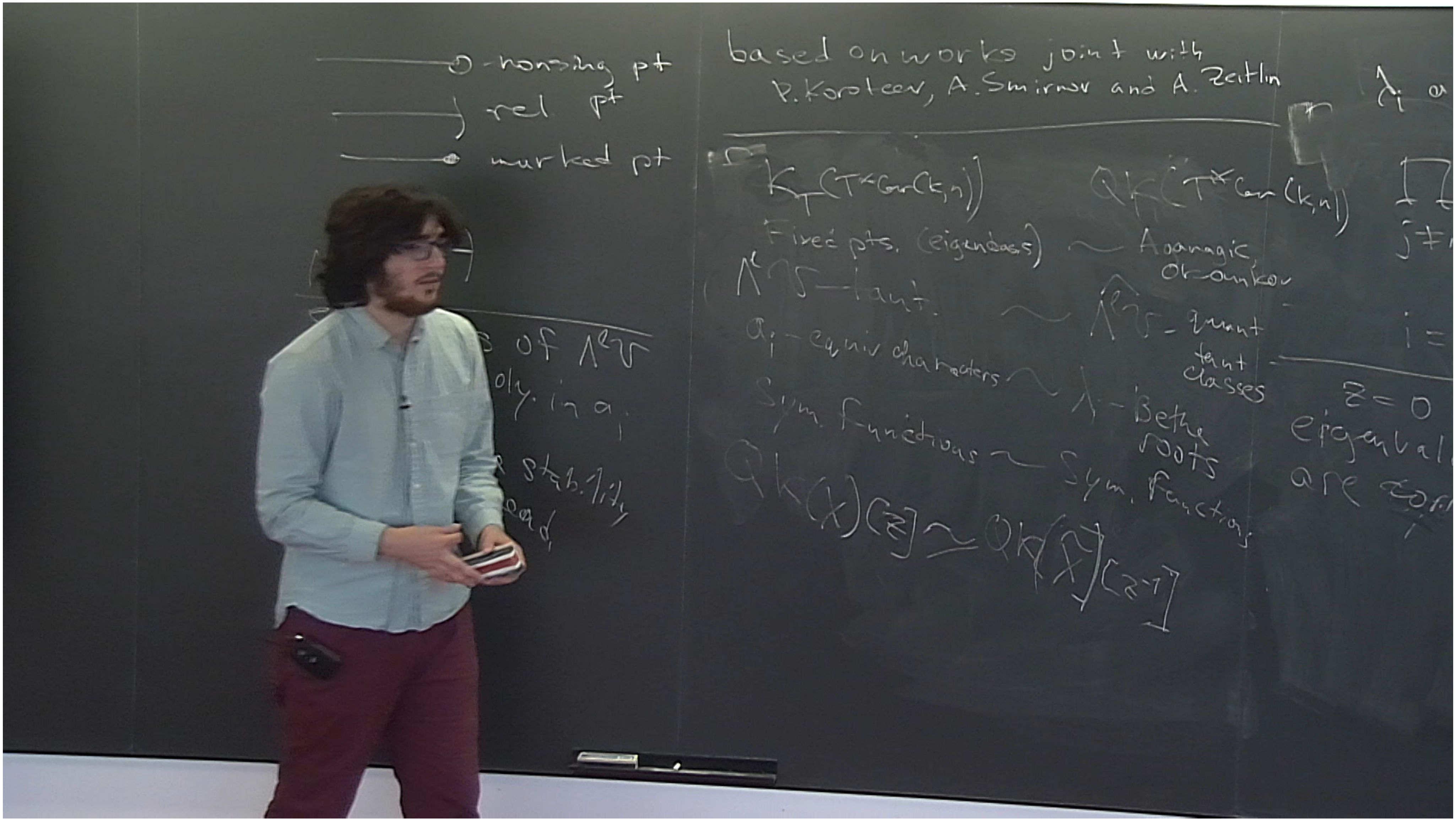
based on works joint with
P. Koroteev, A. Smirnov and A. Zeitlin
Fixed pts. (eigenbasis)
Aganagic, Okounkov
quant
taut
classes
Bethe
roots
Sym. functions
Sym. functions
z = 0


nonsing pt
rel pt
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based on works joint with
P. Koroteev, A. Smirnov and A. Zeitlin
Fixed pts. (eigenbasis)
Aganagic, Okounkov
quant taut classes
Bethe roots
Sym. functions
Sym. functions
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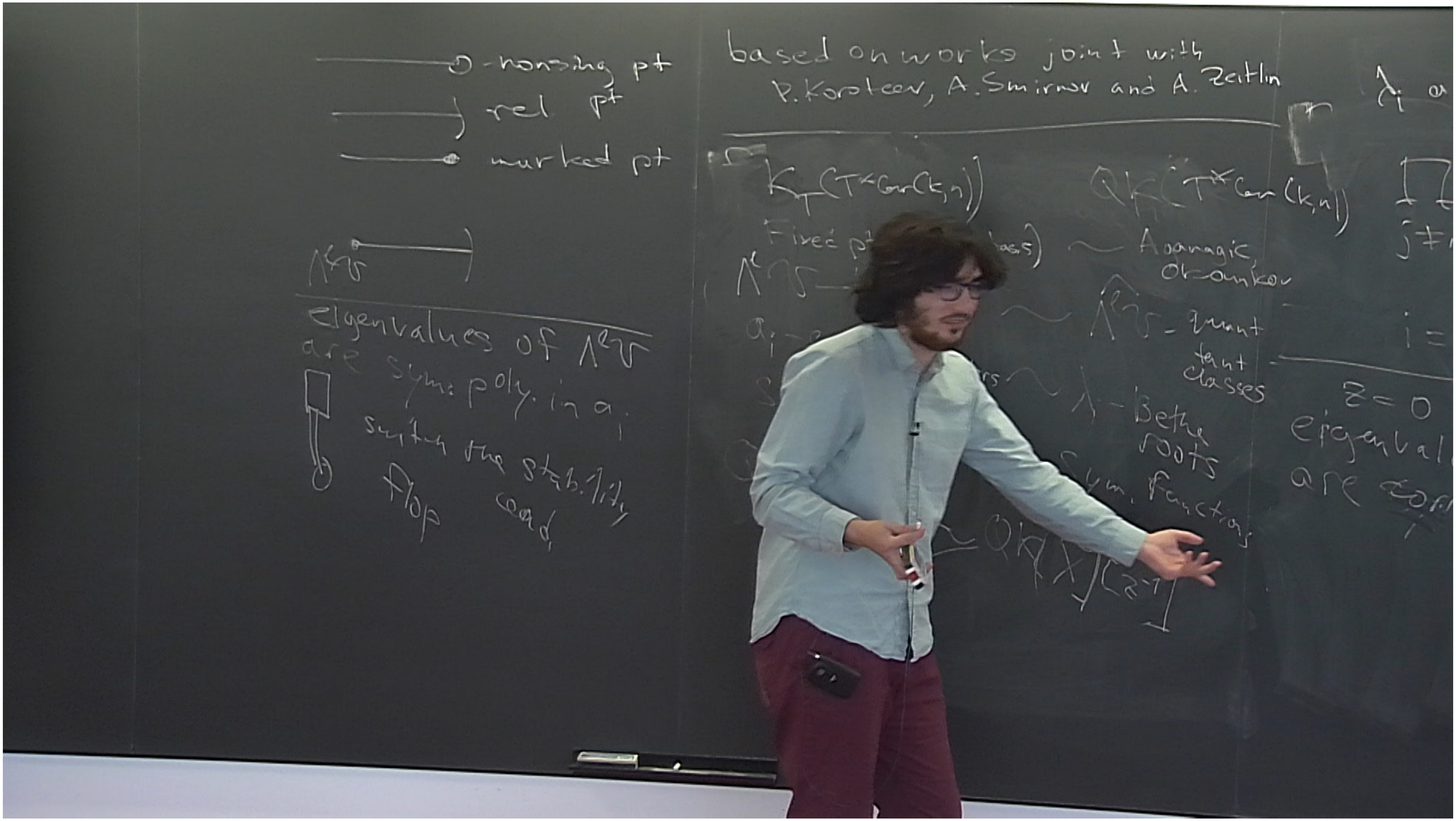
are


nonsing pt
rel pt
marked pt
eigenvalues of Λ^k V
are sym. poly. in a_i
flop
based on works joint with
P. Koroteev, A. Smirnov and A. Zeitlin
K_T(T*Gr(k,n))
QK_T(T*Gr(k,n))
Aganagic, Okounkov
quant taut classes
Bethe roots
sym. functions
z = 0