Title: Operational General Relativity - Lecture 4

Date: Nov 08, 2017 02:00 PM

URL: http://pirsa.org/17110059

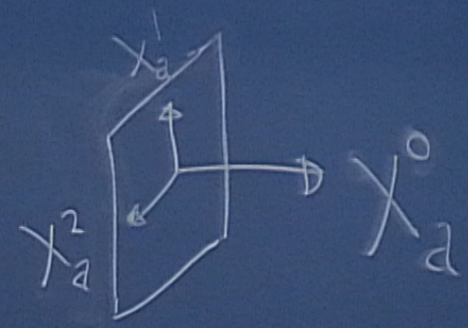
Abstract:

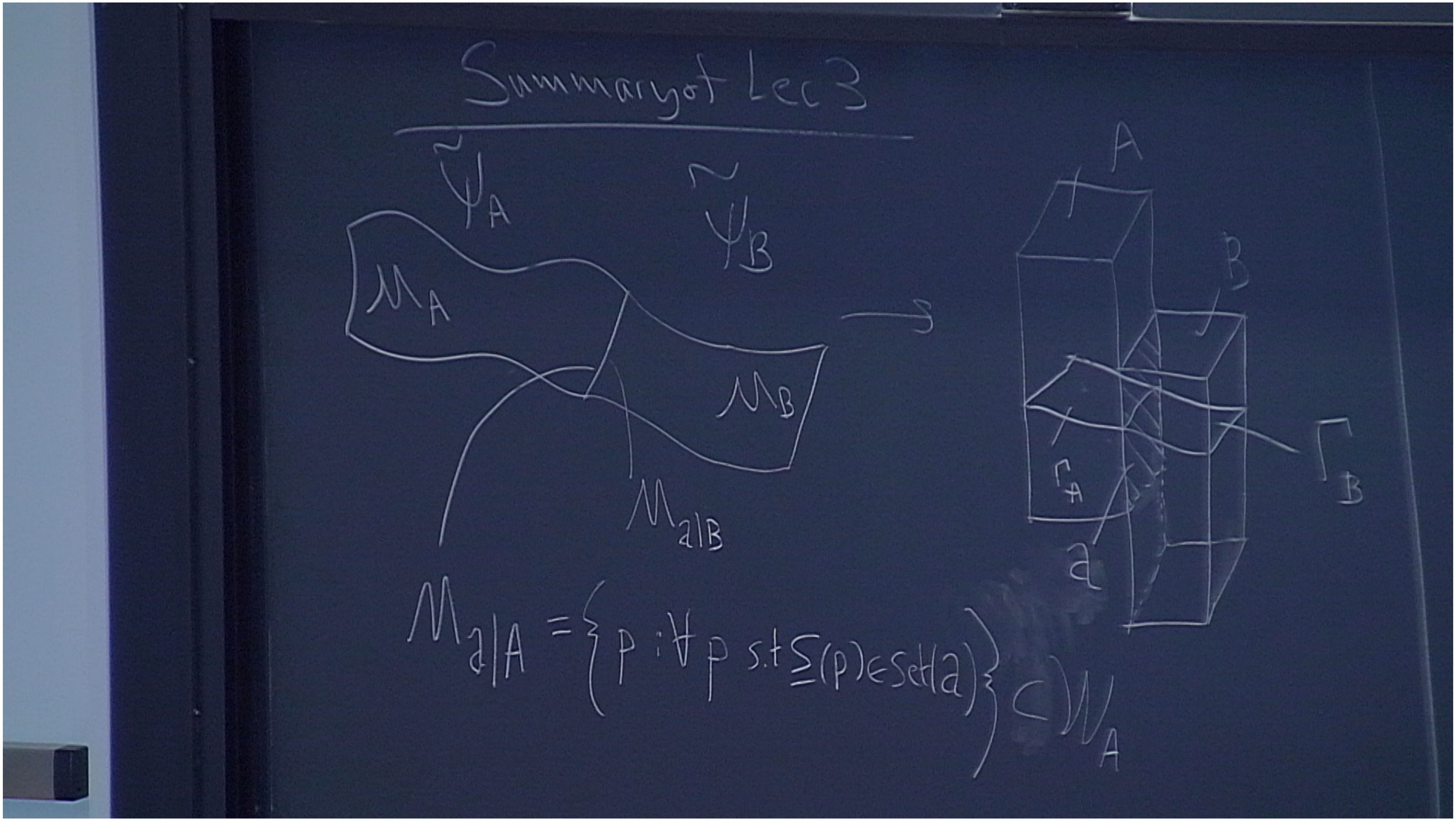
Γ_B

$$\psi_A \cup_a \psi_B = \{ \tilde{\psi}_A \cup_a \tilde{\psi}_B : \forall \tilde{\psi}_A \in \psi_A, \forall \tilde{\psi}_B \in \psi_B \}$$

$$a = (\text{set}(a), \text{coord}(a))$$

a^R has X_a^0 reversed.




Summary of Lec 3
$\tilde{\psi}_A$
$\tilde{\psi}_B$
M_A
M_B
$M_{a|B}$
A
B
Γ_A
Γ_B
a
$M_{a|A} = \{ p : \forall p \text{ s.t } \Sigma(p) \in \text{Set}(a) \} \subset M_A$

$\tilde{\psi}_A = \{ (p, \Phi) : \forall p \in \mathcal{M}_A \}$ and $\tilde{\psi}_B = \{ (p, \Phi) : \forall p \in \mathcal{M}_B \}$

chitecture

$\mathcal{M}_A \cup_a \mathcal{M}_{\underline{B}}$ must match

0. $\mathcal{M}_{a|A|a} = \mathcal{M}_{a|B|a}$ $\qquad \mathcal{W}_a$
1. Direction matching
2. smoothness matching.

wall paper

Φ and Π must match

$\uparrow$

some derivatives of Φ

join $\tilde{\psi}_A = \{(p, \Phi) : \forall p \in \mathcal{M}_A\}$ and $\tilde{\psi}_B = \{(p, \Phi) : \forall p \in$

Architecture

wall paper

$\mathcal{M}_A \cup_a \mathcal{M}_B$ must match

Φ a

⓪ $\mathcal{M}_{a|A|a} = \mathcal{M}_{a|B|a}$ $\mathcal{W}_a$

① Direction matching

② smoothness matching.

\Theta $\Theta = \left(\mathrm{dirn}(p), \mathrm{smoothness}(p);\ \Phi(p), \Pi(p) \right)$

Define

$$\tilde{\Theta}_a(\tilde{\psi}_A) = \left\{ (p, \Theta) : \forall p \in \mathcal{M}_{a|A|a} \right\}$$

$$\Theta_a(\psi_A) = \left\{ \tilde{\Theta}_a(\tilde{\psi}_A) : \forall \tilde{\psi}_A \in \psi_A \right\}$$

Can now define $\tilde{\psi}_A \cup_a \tilde{\psi}_B = \begin{cases} \{(p,\Phi) : \forall p \in \mathcal{M}_A \cup_a \mathcal{M}_B\} & \text{if} \\ - & \text{else.} \end{cases}$

$\text{set} \cup \{-\} = \text{set}.$

$$\psi_A \cup_a \psi_B \neq \psi_{A\cup B}[\underline{O}] \quad \text{iff} \quad \Theta_a(\psi_A) \cap \Theta_a(\psi_B) \neq \emptyset$$

$$\begin{cases} \{(p,\Phi) : \forall p \in \mathcal{M}_A \cup_a \mathcal{M}_B\} & \text{if } \tilde{\Theta}_a(\tilde{\psi}_A) = \tilde{\Theta}_a(\tilde{\psi}_B) \\ - & \text{else.} \end{cases}$$

set.

] iff $\Theta_a(\psi_A) \cap \Theta_a(\psi_B) \neq \emptyset$

Local matching assumption

For $\psi_A \in \Omega_A\left[\begin{smallmatrix}\text{actual}\\ \text{pure}\end{smallmatrix}\right]$, $\psi_B \in \Omega_B\left[\begin{smallmatrix}\text{actual}\\ \text{pure}\end{smallmatrix}\right]$

$$\psi_A \Cup_\partial \psi_B \in \Omega_{A\cup B}\left[\begin{matrix}\text{actual}\\ \text{mixed}\end{matrix}\right]$$

$\Omega_A[\text{spec}]$

for some choice of Π with a finite # elements $\forall A, B, \partial, \ldots$

Curious non-separability

It is possible that

$$\psi_A \uplus_a \psi_B \text{ is mixed}$$

when

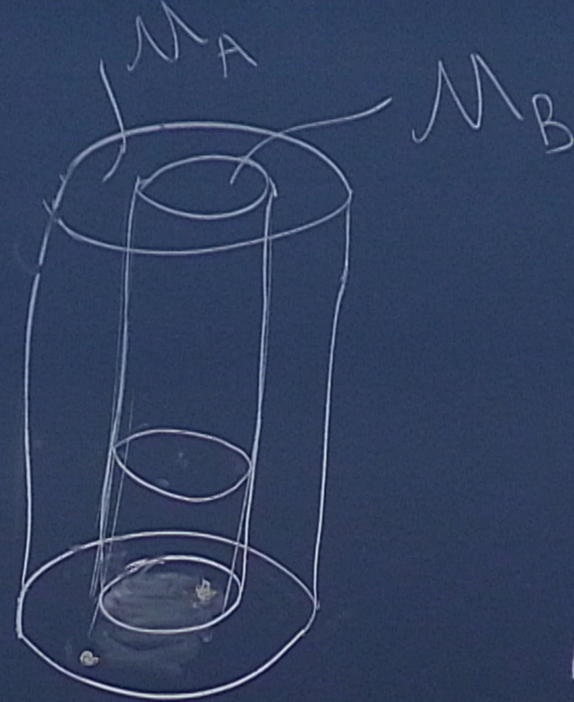
$$\psi_A \text{ and } \psi_B \text{ are pure.}$$

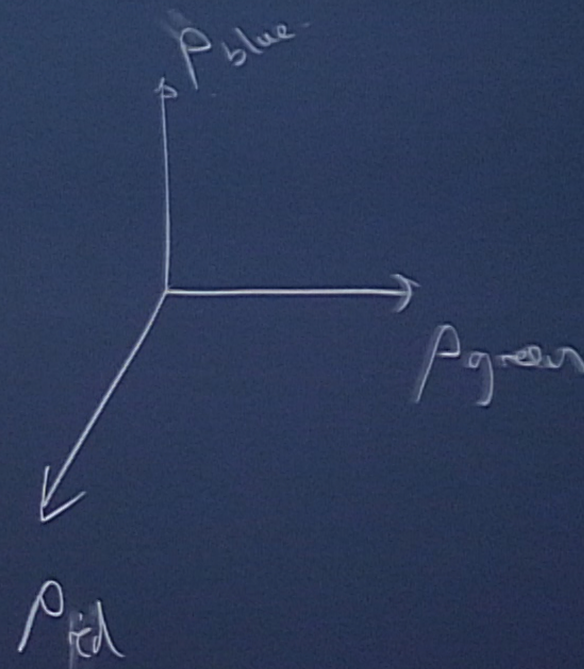
ility

nt

mixed

are pure.



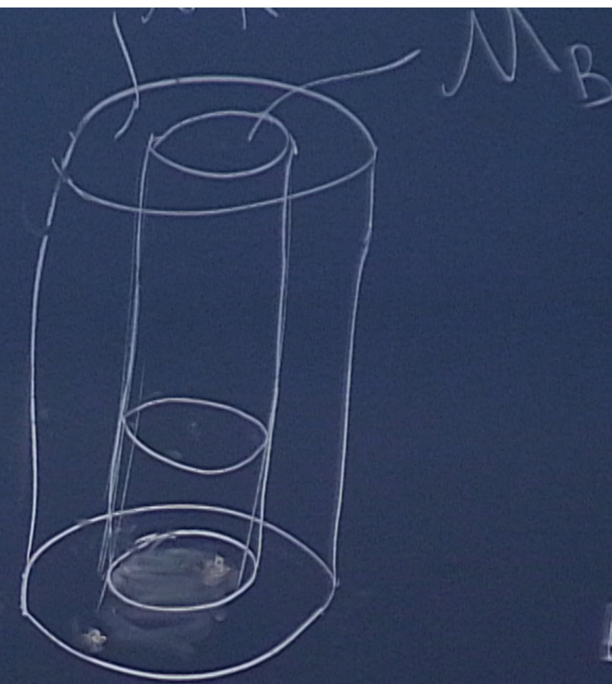


ossible that

A $\psi_a \psi_B$ in mixed

and ψ_B are pure.

$\psi_B(\Gamma_B)$ $\Gamma_A \cup \Gamma_B$



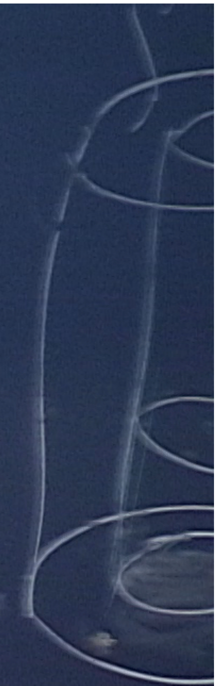
It is possible that

$$\psi_A \uplus_a \psi_B \text{ is mixed}$$

when

$$\psi_A \text{ and } \psi_B \text{ are pure.}$$

$$\psi_A[\Gamma_A] \uplus_a \psi_B(\Gamma_B) \qquad \Gamma_A \cup \Gamma_B$$



Set of boundary conditions

$$\Lambda_\partial[\text{spec}] = \Big\{ \Theta_\partial(\psi_A) : \forall \, \psi_A \in \Omega_A[\text{spec}],$$

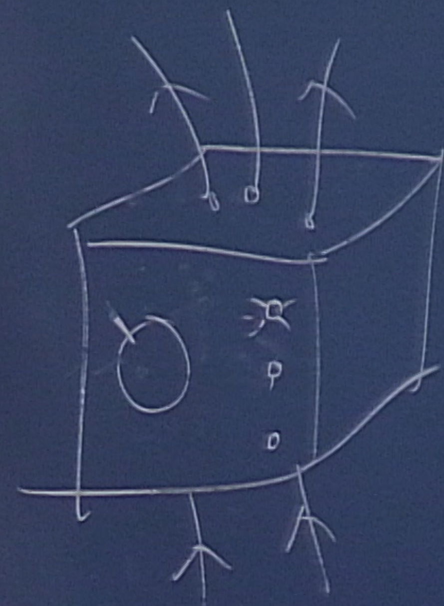
Actual pure case

$$\Lambda_\partial$$

nditions

$$(\psi_A) : \forall\ \psi_A \in \Omega_A[\text{spec}],\ \forall A \text{ s.t. } \text{set}(\partial) \subset \text{bound}(A) \}$$

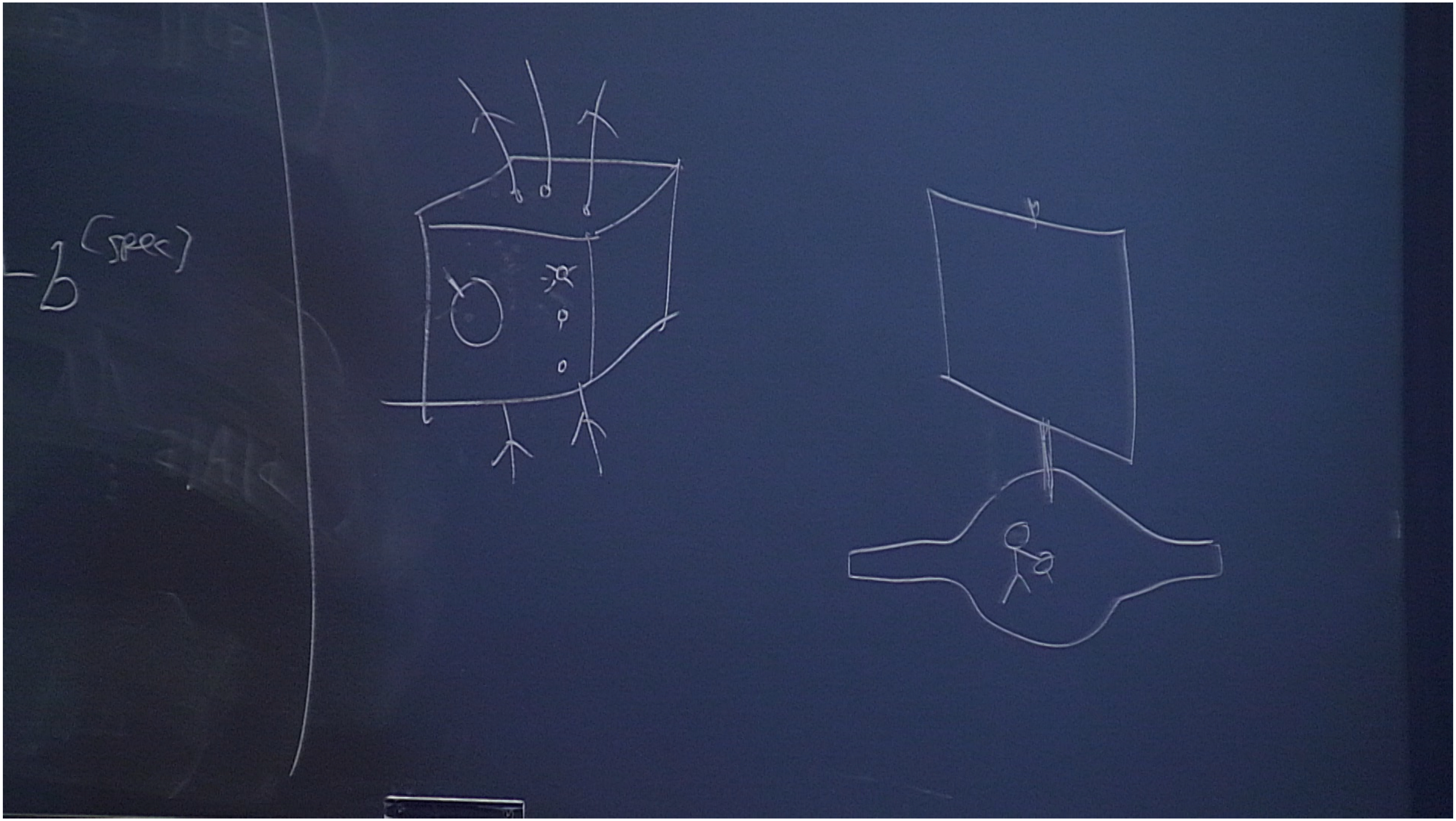
$a^{(\mathrm{spec})}, \quad b \in \Lambda_b^{(\mathrm{spec})}$



Notation.

write

$$a \in \Lambda_{a^{(\mathrm{spec})}}, \quad b \in \Lambda_{b^{(\mathrm{spec})}}$$


b^(spec)