Title: Light Cone Thermodynamics

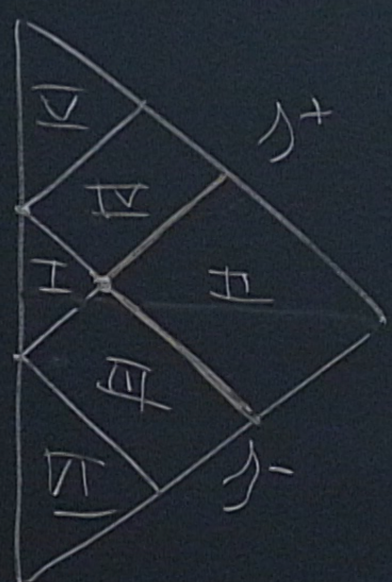
Date: Nov 30, 2017 02:30 PM

URL: http://pirsa.org/17110055

Abstract: <p>In this talk, I will show that light cones in MInkowski spacetime are a beautiful analoue of black hole horizons in curved spacetime. To do so, I will prove the analogue of the four laws of black hole thermodynamics in this setting. This is what we called light cone thermodynamics. More precisely, I will consider null surfaces defined by the out-going and in-falling wave fronts emanating from and arriving at a sphere in Minkowski spacetime. Such null surfaces, made of pieces of light cones, are bifurcate conformal Killing horizons for suitable conformally stationary observers. They can be extremal and non-extremal depending on the radius of the shining sphere. Such conformal Killing horizons have a constant light cone (conformal) temperature, given by the standard expression in terms of the surface gravity. Considering exchanges of conformally invariant energy across the conformal horizon, one can prove, in perturbation theory, a first law where entropy changes are given by 1/4 of the changes of the area of the bifurcate surface in Planck units. In the extremal case they become light cones associated with a single event; these have vanishing temperature as well as vanishing entropy. I will conclude with possible generalisations and applications of such results.</p>

LIGHT CONE THERMODYNAMICS [170

→ THE SETTING



0th law: $\kappa_{SG} = \text{const}$

1-st law: $\delta M = T\,\dfrac{\delta A}{4} + \delta M_\infty$

$$T = \frac{\kappa_{SG}}{2\pi}$$

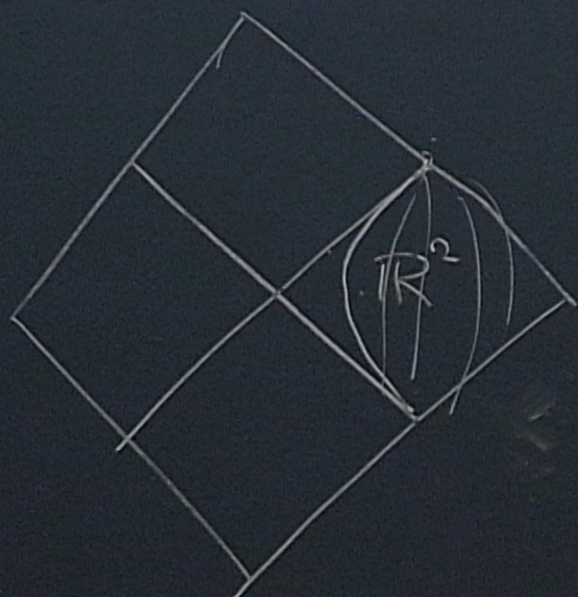
2nd law: $\delta S = \dfrac{\delta A}{4} \geq 0$

3rd law: $T \to 0,\ S \to 0$

[1707.00479]

WHY?

$+\ \delta M_\infty$

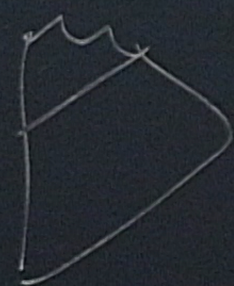


$\mathbb{R}^2 \times \mathbb{R}$

$\downarrow$

$A = \infty$

$S^2 \times \mathbb{R}$



[1707.00479]

M_∞

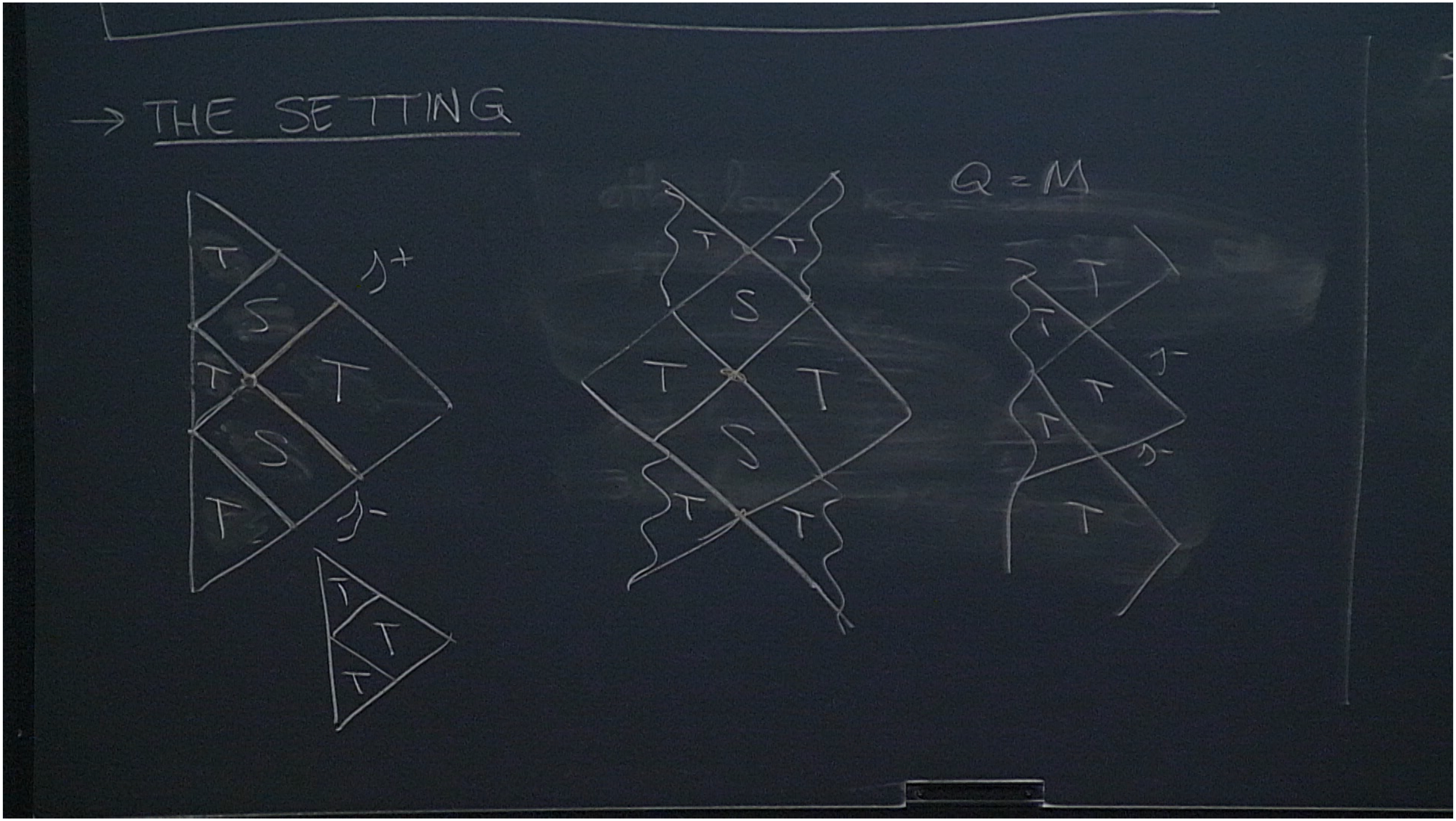
SO(5,1), 15 gen.

$$\mathcal{L}_\xi(g_{ab}) = \frac{\psi}{2} g_{ab} \qquad \psi = \nabla_a \xi^a$$

$$\xi^a \partial_a = (av^2 + c)\partial_v + (au^2 + c)\partial_u$$

$$\xi \xi = -(ar^2 + c)(au^2 + c)$$

$$v_\pm = u_\pm = -4ac$$


→ THE SETTING
Q = M
T
S
$\mathcal{J}^+$
$\mathcal{J}^-$

$$\mathcal{L}_\xi g_{ab} = \frac{\psi}{2} g_{ab} = \nabla_a \xi_b + \nabla_b \xi_a \qquad \psi = \nabla_a \xi^a$$

$$\xi \cdot \xi \hat{=} 0$$

$$\nabla_a(\xi \cdot \xi) \hat{=} -2 K_{SG} \xi_a$$

$$\xi^a \nabla_a \xi_b \hat{=} K \cdot \xi_b$$

$$K_{SG} = K - \frac{\psi}{2}$$

$$\mathcal{L}_\xi K_{SG} \hat{=} 0$$

$$M = \int_\Sigma J_a \, d\Sigma^a$$

$$J_a = T_{ab} \xi^b \implies \nabla_a J^a \propto T^a{}_a \implies$$

0th law

$\kappa_{SG} \hat{=} \text{const}$

1st law

$$\delta M = \underbrace{\int_H \delta T_{ab} \xi^b d\Sigma^b}_{\frac{\kappa_{SG}}{8\pi}\delta A} + \delta M_\infty$$

$\omega_{ab} \hat{=} 0$

$\sigma_{ab} \hat{=} 0$

$\theta = \theta_0 + \theta_1$

$\xi^a = \alpha l^a = \partial_v$

$$\theta = \frac{1}{dA}\frac{\partial}{\partial v} dA$$

$$\boxed{\kappa = \partial_v \alpha}$$

$$\frac{d\theta}{dv} = -\frac{1}{2}\theta^2 - \sigma_{ab}\sigma^{ab} + \omega_{ab}\omega^{ab} - \frac{1}{8\pi} T_{ab} l^a l^b$$

$$\mathcal{L}_\xi g_{ab} = \frac{\psi}{2} g_{ab} = \nabla_a \xi_b + \nabla_b \xi_a$$

$\xi \cdot \xi \hat{=} 0$

$\nabla_a(\xi \cdot \xi) \hat{=} -2\kappa_{SG}\xi_a$

$\xi^a \nabla_a \xi_b \hat{=} \kappa \xi_b$

$\mathcal{L}_\xi \kappa_{SG} \hat{=} 0$

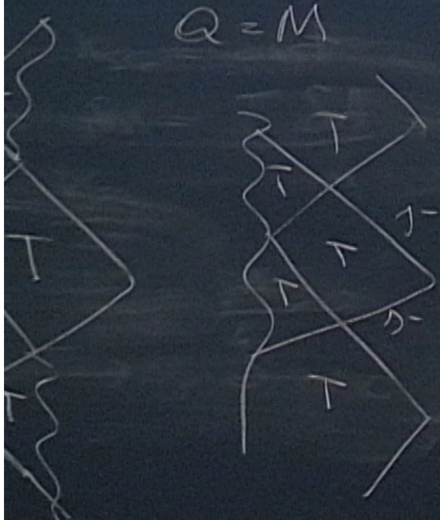
$\kappa_{SG} = \kappa$

$J_a = T_{ab}\xi^b \Rightarrow \nabla_a J^a \text{ or } T^a{}_a$

MODYNAMICS

[1707.00479]

Q = M



SO(5,1), 15 gen.

$$\mathcal{L}_\xi(g_{ab}) = \frac{\psi}{2} g_{ab} \qquad \psi = \nabla_a \xi^a$$

$$\xi^a \partial_a = (a v^2 + c)\partial_v + (a u^2 + c)\partial_u$$

$$\xi \xi = -(a v^2 + c)(a u^2 + c)$$

$$v_\pm = u_\pm = -4ac$$

$$\Delta = -4ac$$

dA

$$\frac{d\theta}{d\lambda} = -\frac{1}{2}\theta^2 - \sigma_{ab}\sigma^{ab} + \omega_{ab}\omega^{ab} - \frac{1}{8\pi}\delta T_{ab} l^a l^b$$

$$J_a = T_{ab}\xi^b \Rightarrow \nabla_a J^a \propto T^a{}_a =$$

$$\frac{d\cancel{\theta_0}}{d\lambda} + \frac{d\theta_1}{d\lambda} = -\cancel{\frac{1}{2}\theta_0^2} - \theta_0\theta_1 - \frac{1}{8\pi}\delta T_{ab} l^a l^b$$

$$\delta M = \int_H \alpha\, \delta T_{ab} l^a l^b \, dA\, d\lambda + \delta M_\infty =$$

$$= -\int_H \frac{\alpha}{8\pi}\left(\frac{d\theta_1}{d\lambda} + \theta_0\theta_1\right) dA\, d\lambda = -\oint \frac{\alpha}{8\pi}\theta_1\, dA \Big|_{\lambda_H}^{\infty} + \int_H (\partial_\lambda \alpha)\, \theta_1\, dA\, d\lambda + \delta M_\infty$$

$$\delta M = \frac{\kappa_{SG}}{8\pi} \underbrace{\int_H \frac{\kappa}{\kappa_{SG}} \theta_s \, dA \, dv}_{\equiv \delta A} + \delta M_\infty$$

$$\boxed{\delta M = \frac{\kappa_{SG}}{8\pi} \delta A + \delta M_\infty}$$

$$\delta T_{ab} \xrightarrow{\Omega^2 g_{ab}} \Omega^{-2} \delta T_{ab}$$

$$d\Sigma^a \xrightarrow{\Omega^2 g_{ab}} \Omega^2 \, d\Sigma^a$$

$$\delta M \longrightarrow \delta M$$

$$\kappa_{SG} \longrightarrow \kappa_{SG}$$

$$\mathcal{L}_\xi (\Omega$$

$$\Omega |$$

$(\tau, \rho, \theta, \varphi)$

$$= \Omega^2 \underbrace{\left[-d\tau^2 + d\rho^2 + \frac{\sinh^2 \rho\sqrt{\Delta}}{\Delta}\, dS^2\right]}_{\text{FRW}}$$

$|0\rangle_M = $ th. $\boxed{T = \frac{K_{SG}}{2\pi}}$

$|0\rangle_C \longrightarrow$ FOCK SPACE

$$\delta M = \frac{\kappa_{SG}}{8\pi}\underbrace{\int_H \frac{\kappa}{\kappa_{SG}}\,\theta_\Delta\, dA\, dv}_{\equiv\ \delta A} + \delta M_\infty$$

$$\boxed{\delta M = \frac{\kappa_{SG}}{8\pi}\,\delta A + \delta M_\infty}$$

$$\delta M = T\,\delta S + \delta M_\infty \qquad \frac{\delta A}{4} = \delta S$$

$$\delta T_{ab} \xrightarrow{\Omega^2 g_{ab}} \Omega^{-2}\,\delta T_{ab}$$

$$d\Sigma^a \xrightarrow{\Omega^2 g_{ab}} \Omega^2\, d\Sigma^a$$

$$\delta M \longrightarrow \delta M$$

$$\kappa_{SG} \longrightarrow \kappa_{SG}$$

$$\mathcal{L}_\xi\,(\Omega^2$$

$$\Omega|$$

$+ \delta M_\infty$

$\frac{\delta A}{4} = \delta S$

$$\delta T_{ab} \xrightarrow{\Omega^2 g_{ab}} \Omega^{-2} \delta T_{ab}$$

$$d\Sigma^a \xrightarrow{\Omega^2 g_{ab}} \Omega^2 d\Sigma^{(a)}$$

$$\delta M \longrightarrow \delta M$$

$$\kappa_{SG} \longrightarrow \kappa_{S\tilde{G}}$$

$$\mathcal{L}_\xi \left(\Omega^2 \eta_{ab}\right) = 0$$

$$\Omega\big|_{r_H} = 1$$

2ND : $\delta T_{ab} \ell^a \ell^b \geq 0 \Rightarrow \delta A \geq 0$

3RD : $T \to 0,\ S \to 0$

$- \delta M_\infty$

$$\delta T_{ab} \xrightarrow{\Omega^2 g_{ab}} \Omega^{-2} \delta T_{ab}$$

$$d\Sigma^a \xrightarrow{\Omega^2 g_{ab}} \Omega^2 d\Sigma^a$$

$$\delta M \longrightarrow \delta M$$

$$\kappa_{SG} \longrightarrow \kappa_{SG}$$

$$\frac{\delta A}{4} = \delta S$$

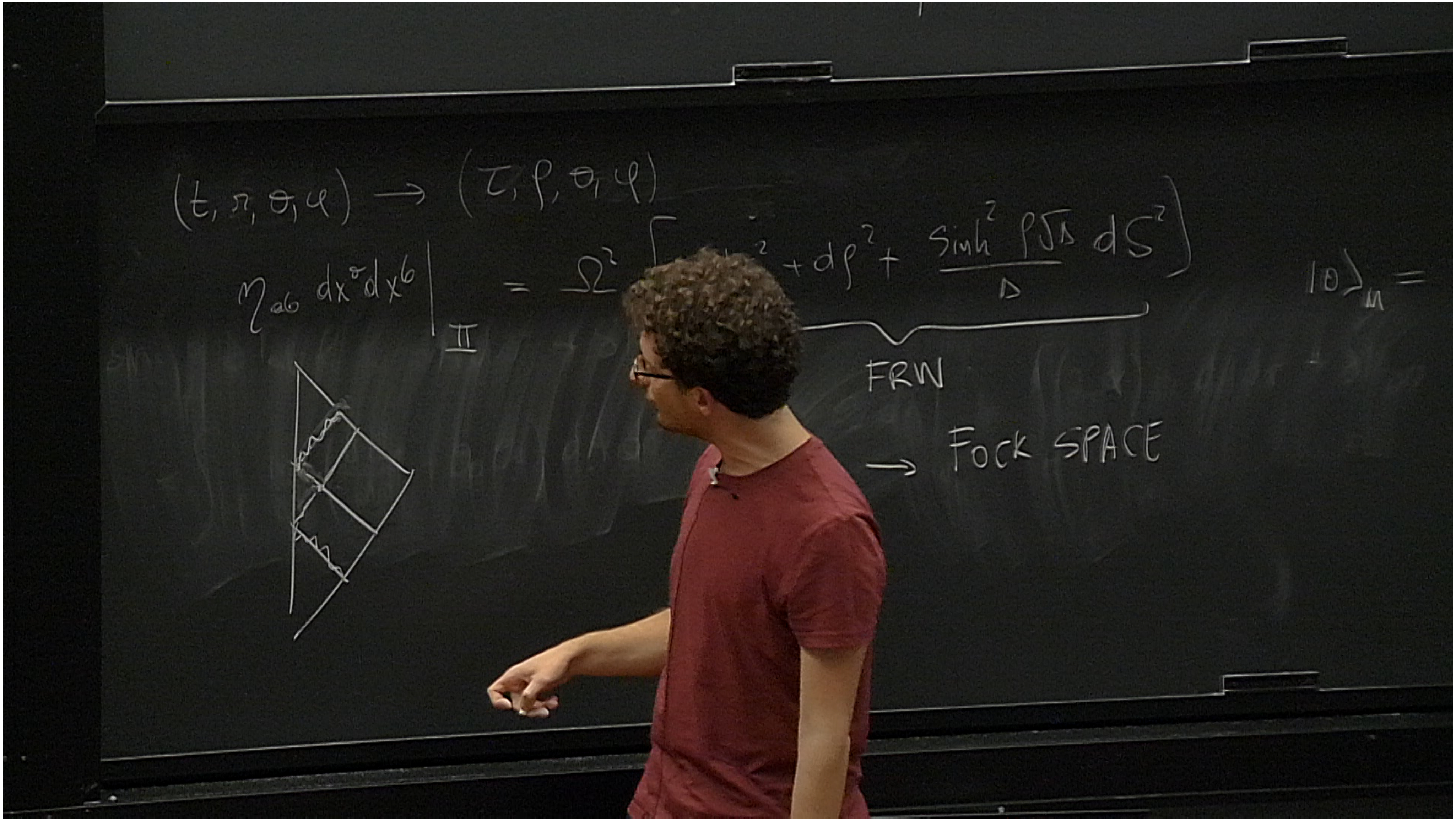
$$\mathcal{L}_\xi \left(\Omega^2 \eta_{ab} \right) = 0$$

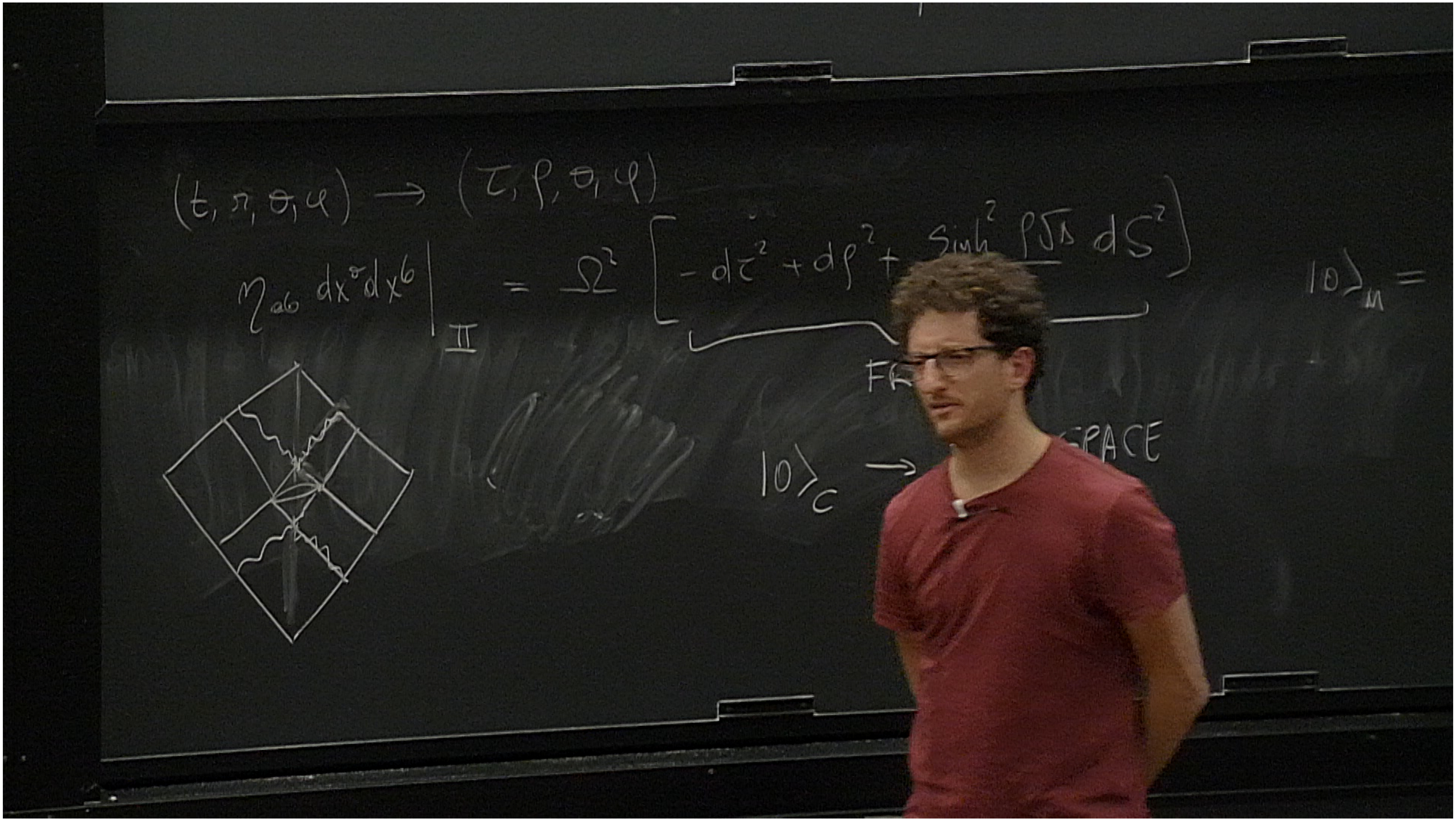
$$\Omega \Big|_{r_H} = 1$$

$$\Omega = \frac{4 r_H^2}{(r - r_H)(r + r_H)}$$

2ND: $\delta T_{ab} \ell^a \ell^b \geq 0 \Rightarrow \delta A \geq 0$

3RD: $T \to 0, \; S \to 0$


(t, r, θ, φ) → (τ, ρ, θ, φ)
FRW
→ FOCK SPACE
|0⟩_M =


(t, r, θ, φ) → (τ, ρ, θ, φ)
η_ab dx^a dx^b |_II = Ω² [-dτ² + dρ² + sinh² ρ dS²]
|0⟩_M =
FR
SPACE
|0⟩_C →