Title: Edge modes and entanglement in diffeomorphism-invariant theories.

Date: Nov 09, 2017 11:00 AM

URL: http://pirsa.org/17110054

Abstract: <p>The Hilbert space of a theory with diffeomorphism symmetry does not factorize into spatial subregions due to gauge constraints. This presents a challenge for defining a notion of entanglement entropy associated with a subregion in these theories. In this talk, I will describe the extended phase space method of Donnelly and Freidel for handling this nonfactorization. It involves introducing edge modes living at the boundary of the subregion, whose purpose is to restore the diffeomorphism invariance that was broken by the subregion's presence. These edge modes are then expected to contribute to the subregion's entanglement entropy. I will further discuss the relevance of the edge mode entanglement to the entropy of black holes, where it may provide a statistical interpretation of the Wald entropy within the low energy effective description.</p>

Edge modes and entanglement in diffeomorphism-invariant theories

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Quantum Gravity Seminar
Perimeter Institute
November 9, 2017

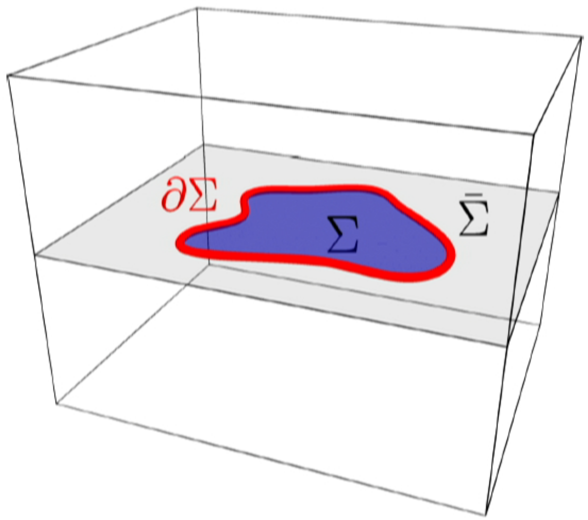
arXiv:1706.05061

Outline

1. Entanglement in gravitational theories
2. Diff(M) edge modes
3. Extended phase space
4. Surface symmetry algebra
5. Conclusion

Entanglement in gravitational theories

Goal: understand entanglement entropy in gravitational theories



Usual picture:

- Specify subregion Σ
- Reduced density matrix $\rho_\Sigma = \mathrm{Tr}_{\bar{\Sigma}}\, \rho$
- Entanglement entropy is $S = -\mathrm{Tr}\, \rho_\Sigma \log \rho_\Sigma$
- Describes spatial entanglement of field theory degrees of freedom

Explore what happens when theory has gauge symmetry Diff(M)

Entanglement in gravitational theories: motivations

Generalized entropy of black holes

Sum of Bekenstein-Hawking entropy and external matter entropy

$$S_{\text{gen}} = S_{\text{BH}} + S_{\text{out}} = \frac{A}{4G} + S_{\text{out}}$$

UV finite $\rightarrow$ divergences in G and S_{out} appear to cancel

[Susskind, Uglum 1994; Jacobson 1994; Cooperman, Luty 2014]

Raises intriguing questions:

- Is generalized entropy a von Neumman entropy in a finite UV completion of gravity?
- Statistical interpretation of S_{BH} within the IR effective description?
- Graviton contributions to S_{out}?

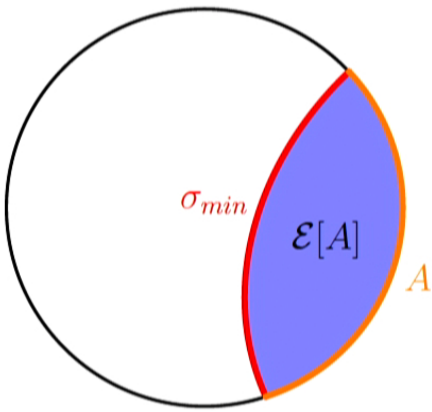
Entanglement in gravitational theories: motivations

Holography

- Entanglement wedge reconstruction: $\mathcal{E}[A]$ is a bulk subregion dual to boundary region A
- FLM quantum corrections to RT from gravitons:

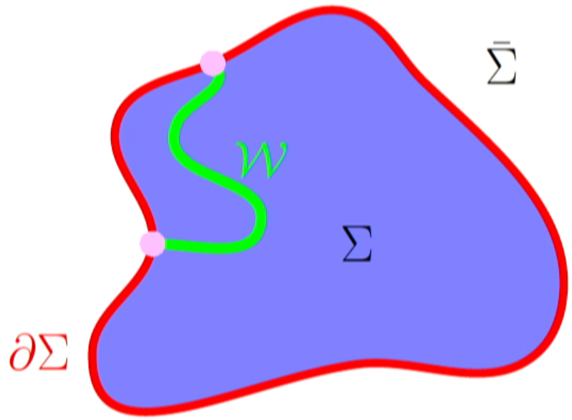
$$S_{\mathsf{bdy}} = \frac{A(\sigma_{\mathsf{min}})}{4G} + \underset{\substack{\uparrow \\ \text{gravitons contribute}}}{S_{\mathsf{bulk}}}$$

- Develop gravity side of the duality



Local subregions in gauge theories

Gauge invariance introduces complications



- Constraints (Gauss Law) relate data in Σ to $\bar{\Sigma}$
- $\mathcal{H} \neq \mathcal{H}_\Sigma \otimes \mathcal{H}_{\bar{\Sigma}}$
- $\mathcal{A}_\Sigma$ has nontrivial center

Wilson loops $\mathcal{W}$ cut by $\partial\Sigma$ not gauge-invariant within Σ

Boundary breaks gauge invariance!

Local subregions in gravitational theories

Two separate issues in diffeomorphism-invariant theories:

1. Diffeos move the subregion $\rightarrow$ nontrivial to even define Σ invariantly
2. Gravitational Wilson lines cut by $\partial\Sigma$

Several ways to handle 1, e.g. minimal surface anchored at boundary, relational constructions [Giddings, Marolf, Hartle 2006]

Edge modes arise in dealing with 2:

- Necessary even after invariant specification of Σ
- Parameterize ways in which boundary breaks diff-invariance
- Contribute to entanglement entropy

Entanglement in diffeomorphism-invariant theories

Extended Hilbert space

Edge modes on $\partial\Sigma$ restore gauge invariance.

Hilbert is a sum over edge mode superselection sectors, labeled by representations of the *surface symmetry algebra*,

$$\mathcal{H}_\Sigma^{\text{ext}} = \sum_i \mathcal{H}_\Sigma^i \otimes \mathcal{H}_{\text{edge}}^i$$

Density matrix is block diagonal

$$\rho_\Sigma^{\text{ext}} = \sum_i p_i \, \rho_\Sigma^i \otimes \rho_{\text{edge}}^i$$

ρ_{edge}^i is maximally mixed in each representation, $\rho_{\text{edge}}^i = \frac{\mathbb{1}}{\dim R_i}$

Entanglement in diffeomorphism-invariant theories

Entanglement entropy: [Donnelly 2014]

$$S = \sum_i \left(p_i S_i - p_i \log p_i + p_i \log \dim R_i \right)$$

1. entanglement within each sector
2. classical Shannon entropy
3. entanglement between edge modes

Similarity to $S_{\text{gen}} = S_{\text{out}} + S_{\text{Wald-like}}$ → edge modes give Wald entropy?

Related to entanglement within code subspace [Harlow 2016, Lin 2017]
$\mathcal{H}_{\text{code}} \subset \mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_{\bar{A}}$,

$$S(\rho_A) = S_{\text{alg}}(\rho_A) + \text{Tr}(\rho \hat{\mathcal{L}}_A)$$

$\hat{\mathcal{L}}_A$ term tracks entanglement of "UV modes" whose state is fixed in $\mathcal{H}_{\text{code}}$

Outline

Edge modes

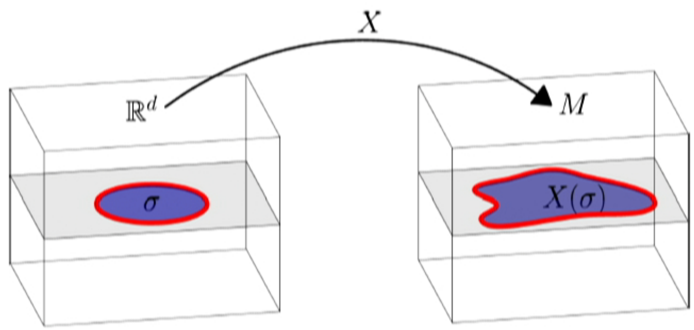
[Donnelly, Freidel 2016] construction

- Classical, continuum description of edge modes for vacuum GR
- Eventually quantize and regulate to compute entanglement
- Lattice regulator used for Yang-Mills, but breaks diff-invariance
- Edge modes introduce new symmetries, remnants of Diff(M)
- $\mathcal{H}_\Sigma^{\text{ext}}$ furnishes a representation of surface symmetry algebra

[AJS arXiv:1706.05061]–extend to arbitrary diff-invariant theories:

- Nonzero cosmological constant (AdS, dS)
- Matter
- Higher curvature theories
- Simpler diff-invariant theories, e.g. dilaton gravity

X fields



Edge mode fields X: [Donnelly, Freidel 2016]

- Elements of the group Diff(M) $\rightarrow$ Stueckelberg fields
- Coordinate system near the subregion
- Collection of d "scalars" X^μ, with transformation under diffeos

$$X^\mu \rightarrow X^\mu - \xi^\mu$$

(compare to usual scalar transformation $\varphi \rightarrow \varphi + \xi^\mu \partial_\mu \varphi$)

X fields

Add them to the theory through pullbacks: e.g. $g_{ab} \to X^* g_{ab}$,

$$(X^*g)_{\alpha\beta}(y) = \frac{\partial X^\mu}{\partial y^\alpha}\frac{\partial X^\nu}{\partial y^\beta} g_{\mu\nu}(X(y))$$

Any pulled-back field $X^*\phi$ is *diffeomorphism-invariant* (not covariant).

Variation $\delta X \leftrightarrow$ infinitesimal diffeomorphism $\leftrightarrow$ vector field χ^a,

$$\delta X^* \phi = X^*(\delta\phi + \pounds_\chi \phi)$$

Other appearances:

- parameterized theories [Arnowitt, Deser, Misner 1960; Kuchař 1976;...]
- bimetric/massive gravity [Arkani-Hamed, Georgi, Schwartz 2003; de Rham, Gabadadze, Tolley 2011]
- hydrodynamics [Friedman 1978; Green, Schiffrin, Wald 2014; Haehl, Loganayagam, Rangamani 2017,...]

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Solution space

Phase space constructed from $\mathcal{S}$, space of field equation solutions

Geometry of $\mathcal{S}$:

- Functions: fields at a point ϕ^x, functionals of fields, e.g. $S = \int_M L[\phi]$, evaluated on-shell
- Variations $\delta\phi^x$ define one-forms on $\mathcal{S}$
- Higher forms given by exterior product (keep $\wedge$ implicit)

$$\delta\phi^x \delta\psi^y \equiv \delta_1\phi^x \delta_2\psi^y - \delta_1\psi^y \delta_2\phi^x$$

- δ defines exterior derivative on arbitrary forms,

$$\delta(\alpha\beta) = (\delta\alpha)\beta + (-1)^p \alpha(\delta\beta)$$

Covariant phase space

Dynamics: Lagrangian $L = L[\phi]$, dynamical fields ϕ, variation

$$\delta L = E \cdot \delta\phi + d\theta[\phi; \delta\phi]$$

- Defines potential current θ, presymplectic potential $\Theta_0 = \int_\Sigma \theta$
- Symplectic current from exterior derivative (antisymmetric variation)

$$\omega = \delta\theta \equiv \delta_1\theta[\delta_2\phi] - \delta_2\theta[\delta_1\phi]$$

- Presymplectic form $\Omega_0 = \delta\Theta_0 = \int_\Sigma \omega$

Covariant phase space: examples

Scalar field

$$L = -\frac{1}{2}\nabla_a\varphi\nabla^a\varphi\,\epsilon$$

$$E\cdot\delta\phi = \nabla_a\nabla^a\varphi\,\delta\varphi\,\epsilon$$

$$\theta = -\,\delta\varphi\nabla^a\varphi\,\epsilon_a$$

$$\omega = \delta\varphi\nabla^a\delta\varphi\,\epsilon_a$$

General Relativity

$$L = \frac{1}{16\pi G}R\,\epsilon$$

$$E\cdot\delta\phi = -\frac{1}{16\pi G}G^{ab}\delta g_{ab}\,\epsilon$$

$$\theta = \frac{1}{16\pi G}\epsilon_a(g^{ac}g^{bd}-g^{ad}g^{bc})\nabla_d\delta g_{bc}$$

$$\omega = -\frac{1}{16\pi G}\delta(\epsilon_a g^{ac}\delta^b_d - \epsilon_d g^{bc})\delta\Gamma^d_{bc}$$

Covariant phase space

Diff(M) is a gauge symmetry $\leftrightarrow$ Ω_0 is degenerate,

$$\Omega_0[\delta\phi, \pounds_\xi \phi] = \int_{\partial\Sigma} (\delta Q_\xi - i_\xi \theta)$$

- Q_ξ is the Iyer-Wald Noether charge [Iyer, Wald 1994]

$$Q_\xi = -\epsilon_{ab} \, E^{abc}{}_d \nabla_c \xi^d + W_c \xi^c; \quad E^{abcd} \equiv \frac{\delta \mathcal{L}}{\delta R_{abcd}}$$

- Zero if $\xi^a \xrightarrow{\partial\Sigma} 0$, but should be degenerate for *all* ξ^a.

Introduce X fields to enforce degeneracy.

Edge mode symplectic form

Make replacement $L[\phi] \to L[X^*\phi]$, variations now include $(\delta\phi, \chi^a)$

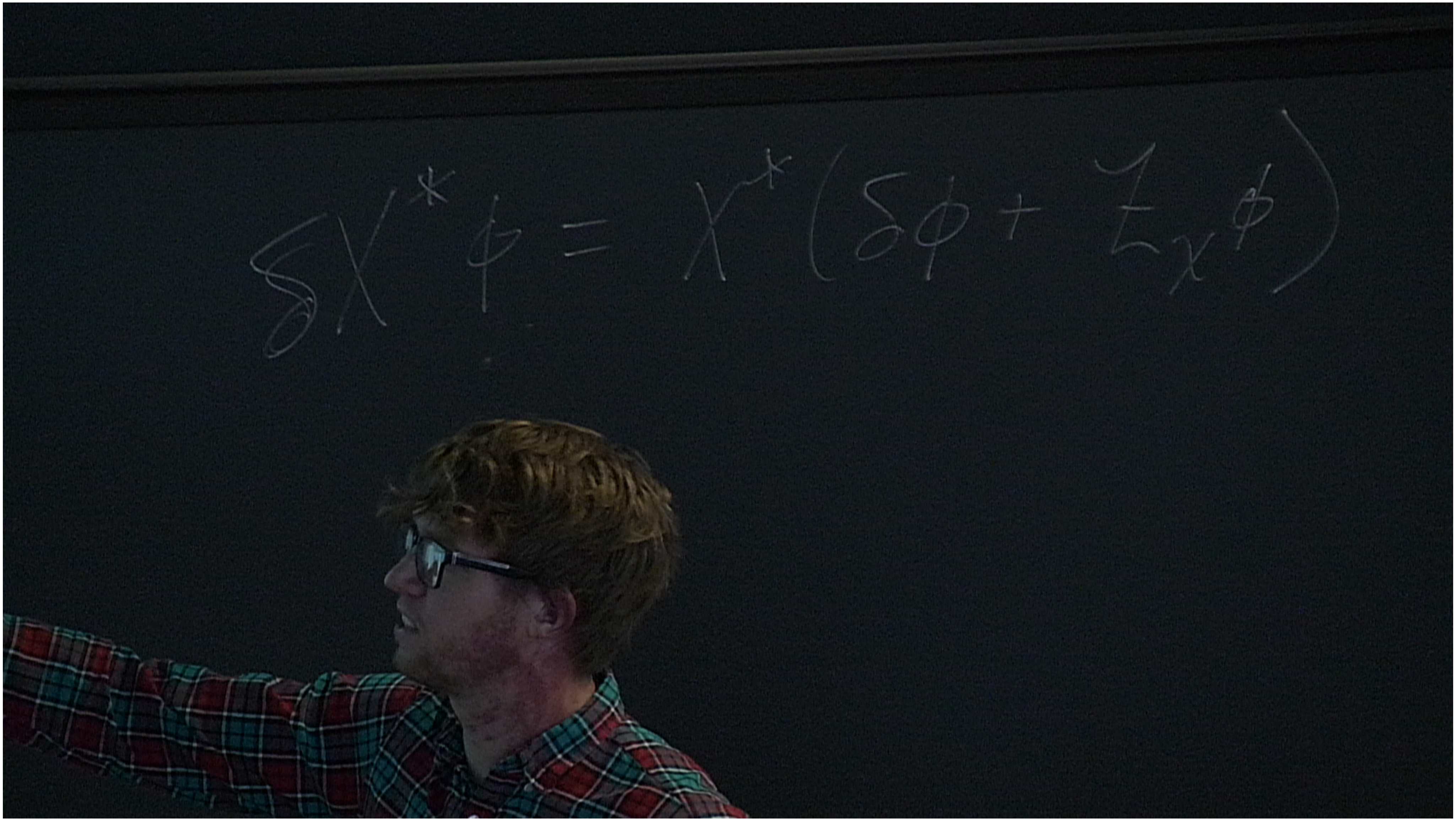
$$\delta L = X^*\Big(E \cdot (\delta\phi + \pounds_\chi \phi)\Big) + d\theta[X^*\phi; \delta X^*\phi]$$

ϕ satisfy same field equation $E[\phi] = 0$, X satisfy no dynamical equations.

New presymplectic potential:

$$\Theta = \int_\Sigma (\theta + i_\chi L) + \int_{\partial\Sigma} Q_\chi$$

- X appears implicitly through pullbacks: $\int_\Sigma = \int_{X(\sigma)} = \int_\sigma X^*$
- χ^a contributes everywhere on Σ if $L \neq 0$, not localized on $\partial\Sigma$?


δX*φ = X*(δφ + £_X φ)

Edge mode symplectic form

Make replacement $L[\phi] \to L[X^*\phi]$, variations now include $(\delta\phi, \chi^a)$

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Edge mode symplectic form

Variation gives presymplectic form:

$$\Omega = \int_\Sigma \omega + \int_{\partial\Sigma} \left(\delta Q_\chi + \pounds_\chi Q_\chi + i_\chi \theta + \frac{1}{2} i_\chi i_\chi L \right)$$

- Depends on χ^a, $\nabla_b \chi^a$ at $\partial\Sigma$, values elsewhere are pure gauge
- Annihilates all diffeos, $(\delta\phi, \chi^a) \to (\pounds_\xi \phi, -\xi^a)$

Since Θ depends on χ^a everywhere on Σ, it does not descend to a (single-valued) form on phase space $\to \Omega$ is closed but not exact.

JKM ambiguities

Various ambiguities Noether charge formalism [Jacobson, Kang, Myers 1994]:

$$Q_\xi \to Q_\xi + d\gamma_\xi, \qquad L \to L + d\alpha, \qquad \theta \to \theta + d\beta$$

γ ambiguity integrated over $\partial\Sigma \to$ no effect

α ambiguity sends $\Theta \to \Theta + \delta \int_{\partial\Sigma} \alpha$, so $\Omega = \delta\Theta$ unchanged

β ambiguity important: changes symplectic form

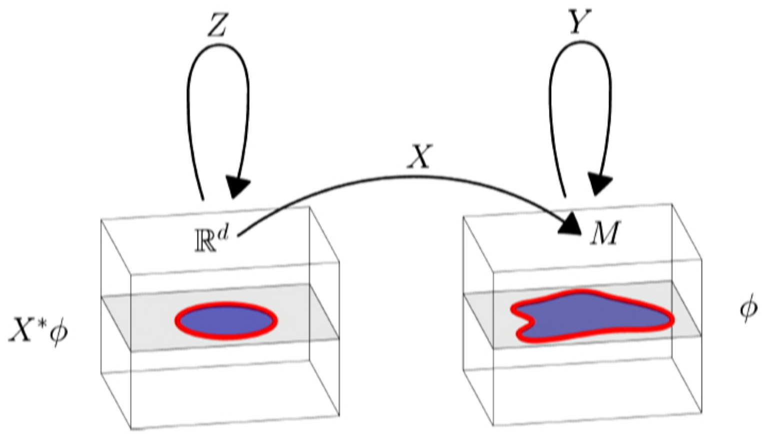
$$\Omega \to \Omega + \delta \int_{\partial\Sigma} \beta[\delta\phi + \pounds_\chi \phi]$$

- Can add higher derivatives of χ^a
- Affects surface symmetry algebra
- Extrinsic curvature terms $\to$ Wald vs. Dong entropy
- Resoltion requires extra input: replica trick, second law, surface translations,...

Outline

Surface symmetry generators

X fields introduce additional symmetries, Diff($\mathbb{R}^d$), most are gauge



$Z \in$ Diff($\mathbb{R}^d$), $\quad Y \in$ Diff(M)

Surface symmetries are non-gauge diffeos of $\mathbb{R}^d$, change X but not ϕ,

$$(\delta\phi, \chi^a) \to (0, V^a)$$

Surface symmetry generators

Generated by Hamiltonians H_V

$$\delta H_V = \Omega[\,\cdot\,, V^a]$$
$$= \delta \int_{\partial\Sigma} Q_V - \int_{\partial\Sigma} i_V \theta[\delta\phi + \pounds_\chi \phi]$$

Hamiltonian exists as long as second term is exact.

- Surface preserving: second term vanishes
- Surface translations: need $i_V\theta = i_V \delta B$

Surface-preserving transformations

V^a parallel or zero at $\partial\Sigma$:

$$\text{Generators: } H_V = \int_{\partial\Sigma} Q_V$$

$$\text{Poisson brackets: } \{H_V, H_W\} = H_{[V,W]}$$

$$[V, W]^a = V^b \partial_b W^a - W^b \partial_b V^a$$

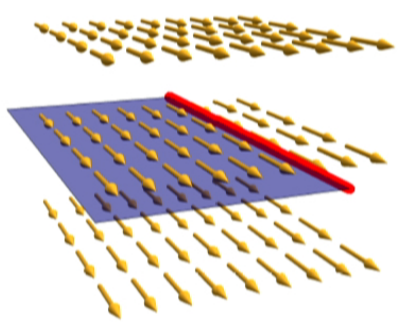
Lie algebra homomorphism from surface-preserving diffeos (vector field commutators) onto surface symmetry algebra, no central extension.

Not faithful: many diffeos map to zero

Surface-preserving transformations

Surface preserving symmetry algebra: Diff$(\partial\Sigma) \ltimes (SL(2,\mathbb{R}) \ltimes \mathbb{R}^{2\cdot(d-2)})^{\partial\Sigma}$

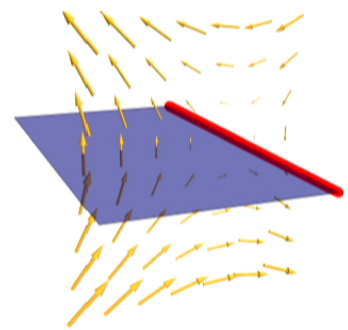
<u>Surface Diffeos</u>



Diff$(\partial\Sigma)$

Universal

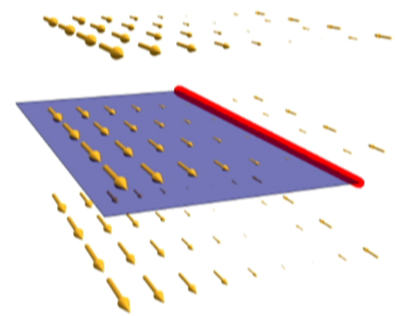
<u>Surface boosts</u>



$SL(2,\mathbb{R})$

Universal

<u>Normal shears</u>



$\mathbb{R}^{2\cdot(d-2)}$

Higher curvature only

β-ambiguity-dependent

Surface translations

Poisson brackets acquire a central extension:

$$\{H_V, H_W\} = H_{[V,W]} + K(V,W)$$
$$K(V,W) = \int_{\partial\Sigma} i_V i_W (L - dB)$$

$K(V,W)$ involves on-shell Lagrangian (different for AdS vs. flat space), boundary conditions imply it is central $\delta K(V,W) = 0$

TODO: Relate to Brown-Henneaux construction for AdS_3 [Brown, Henneaux 1983]

Surface translations

Derivation of B and b.c.'s [w.i.p. with Camps, Donnelly, Lewkowycz]
Take ξ^a to look like a boost at $\partial\Sigma$,

$$\xi^a \overset{\partial\Sigma}{=} 0, \quad \nabla_b \xi^a \overset{\partial\Sigma}{=} \kappa n^a{}_b \implies H_\xi = \frac{\kappa}{8\pi G} A$$

Poisson bracket with surface translation H_V gives change in area due to deformation

$$\{H_\xi, H_V\} = \frac{\kappa}{8\pi G} \int_{\partial\Sigma} \mu V^a K_a$$

But must also equal

$$\{H_\xi, H_V\} = H_{[\xi,V]} = \kappa \int_{\partial\Sigma} V^b n^a{}_b B_a$$

Fixes $B_a = \frac{1}{8\pi G} K_b n^b{}_a \wedge \mu$, b.c.'s follow from $i_V \theta = i_V \delta B$.

Outline

Summary

- General construction of edge modes for diff-invariant theories, canonical perspective on subregions and entanglement
- Surface symmetry algebra (nearly) universal for all diff-invariant theories; higher curvature $\not\Rightarrow$ more edge modes (up to ambiguities)
- Surface translations possible with b.c.'s, lead to a central extension of Poisson algebra

Future work

- Classify b.c.'s for surface translations, any relation to extremality?
- Understand the role of the central extension, related to central charge of a CFT dual? [Brown, Henneaux 1986; Carlip 1995; Strominger 1998]
- Procedure for fixing β ambiguity. Surface translations may play a role, affect B and b.c.'s
- Quantize edge modes and compute entanglement entropy [Donnelly, Wall 2015; Fliss et.al. 2017; Wieland 2017; Wong 2017]
- Extend to vielbeins [Geiller 2017]
- Black hole mechanics with X fields, understand gauge-dependence of Hollands-Wald canonical energy [Hollands, Wald 2013]
- Clarify fiber bundle structure of solution space $\mathcal{S}$, connection properties of χ^a [w.i.p. with Jacobson]