Title: Gravitational shockwaves on rotating black holes

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Abstract: <p>Gravitational shockwaves may signal the breakdown of effective field theory near black hole horizons. Motivated by this, I will revisit the Dray-â€˜t Hooft solution and explain how to generalize it to the Kerr-Newman background. In doing so I will emphasize the method of spin coefficients (the Newman-Penrose formalism) in its compacted form (the Geroch-Held-Penrose formalism).</p>

Gravitational shockwaves on rotating black holes

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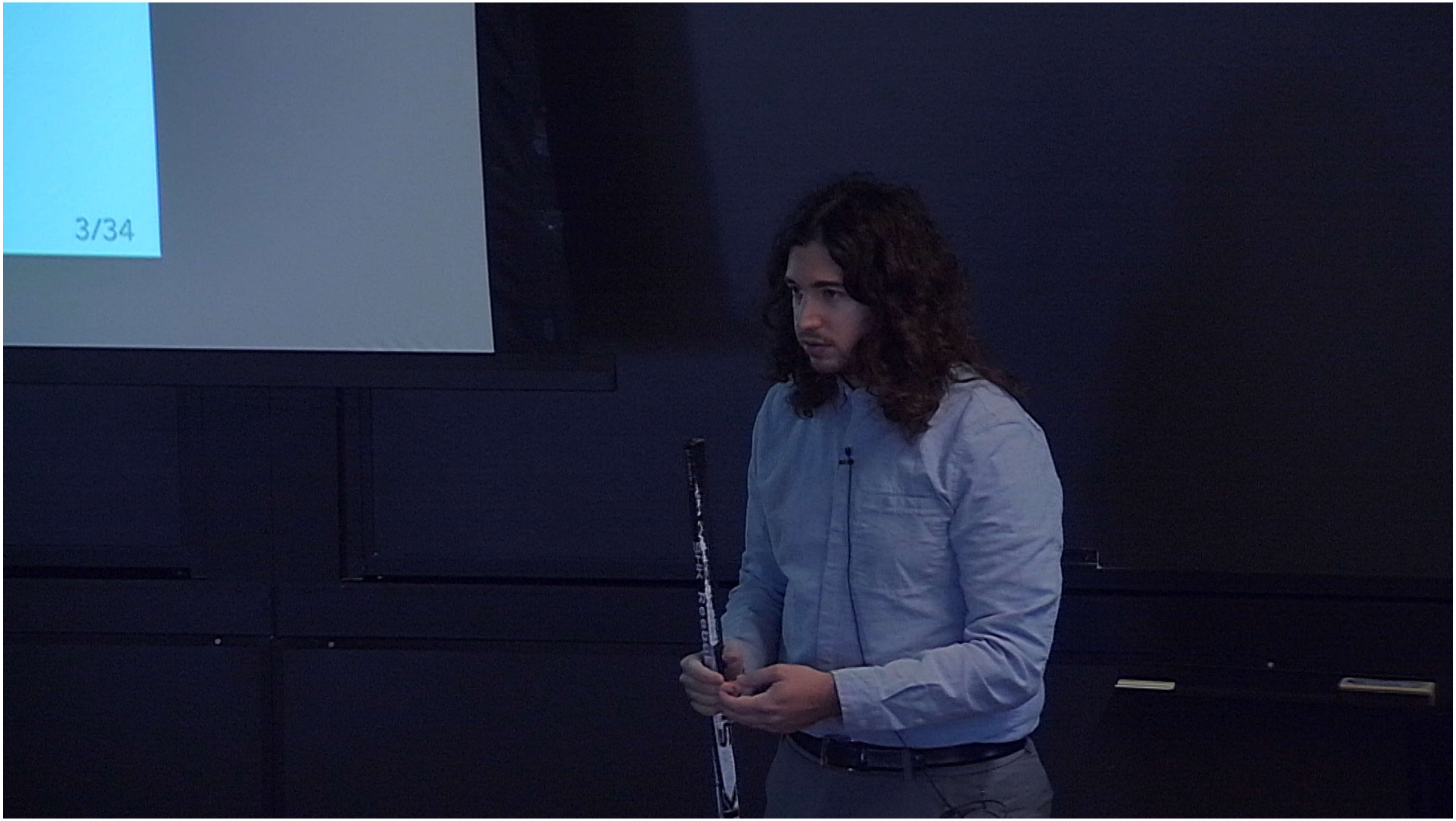
Black hole information problem

- "The approximation I shall use in this paper is that the matter fields [...] obey the usual wave equations with the Minkowski metric replaced by a classical space-time metric g_{ab}." (Hawking, 1975)
- "[...] the three postulates that are in conflict – purity of the Hawking radiation, absence of infalling drama, and semiclassical behavior outside the horizon – are widely held [...]" (Almheiri et al., 2013)

How do correlations leak out of an evaporating black hole?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Quantum butterfly effect

Kiem, Verlinde, and Verlinde, 1995:

- Near horizon: $\phi(u, v, \vec{x}) \approx \phi_{\text{out}}(u, \vec{x}) + \phi_{\text{in}}(v, \vec{x})$.
- But outgoing and ingoing modes do *not* commute:

$$[\phi_{\text{out}}(u', \vec{x}\,'), \phi_{\text{in}}(v, \vec{x})] \approx$$
$$16\pi i f(\vec{x}, \vec{x}\,')\, e^{\alpha(u'-v)} \partial_{u'} \phi_{\text{out}}(u', \vec{x}\,') \partial_v \phi_{\text{in}}(v, \vec{x}) \ .$$

- "...this will result in large, super-Planckian stress-energy fluctuations near the horizon."
- "...these modes interact violently with all infalling particles, via the stress-energy fluctuations..."
- "This will in turn result in a fast growing quantum uncertainty in the geometry near the horizon..."

Revisited by Stanford and Shenker, 2014; Kitaev, 2015; Polchinski, 2015.

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Conventions and coordinates

Signature: $(-+++)$

Spherical coordinates: (t, r, θ, φ)

Tortoise coordinate: $r_* = r + 2M \ln\left|\frac{r}{2M} - 1\right|$

Eddington coordinates: $u = t - r_*$, $v = t + r_*$

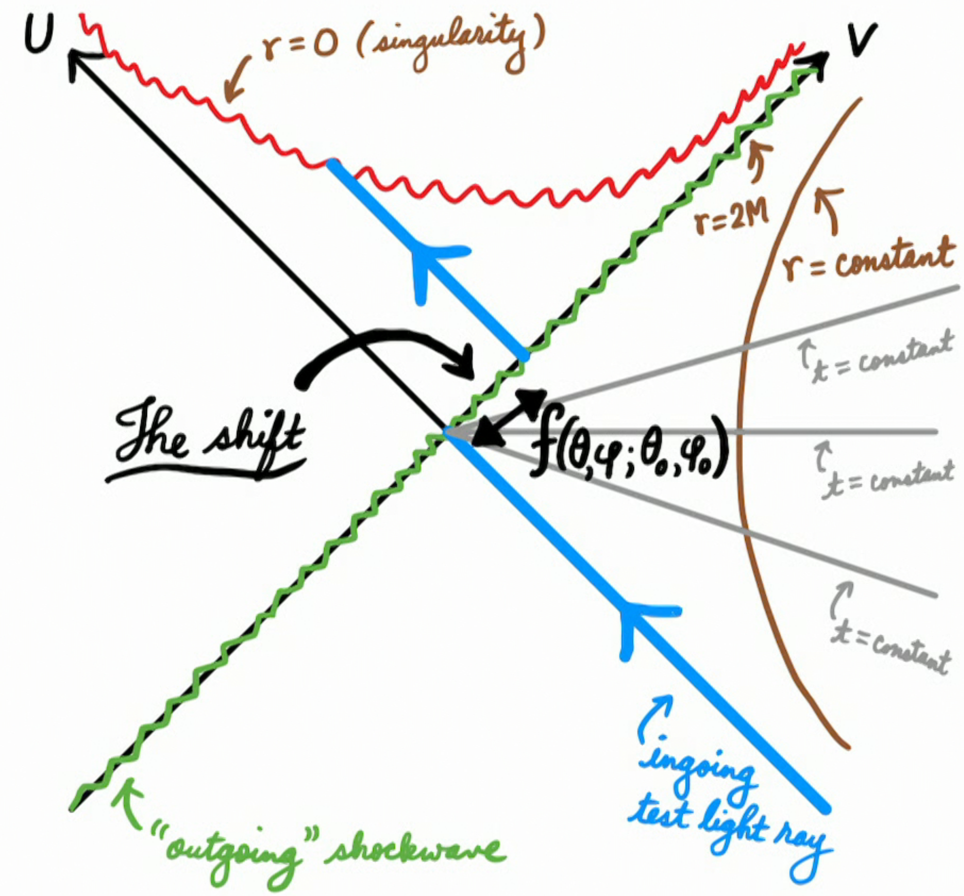
Kruskal coordinates: $U = -e^{-\frac{u}{4M}}$, $V = +e^{+\frac{v}{4M}}$

Schwarzschild metric (exterior):

$$ds^2 = -\left(1 - \tfrac{2M}{r}\right) dt^2 + \frac{1}{1 - \frac{2M}{r}} dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

$$= -\frac{32M^3}{r(UV)}\, e^{-\frac{r(UV)}{2M}}\, dU\, dV + r(UV)^2(d\theta^2 + \sin^2\theta\, d\varphi^2)\ .$$

Gravitational shockwave (Dray and 't Hooft, 1985; Matzner, 1986)



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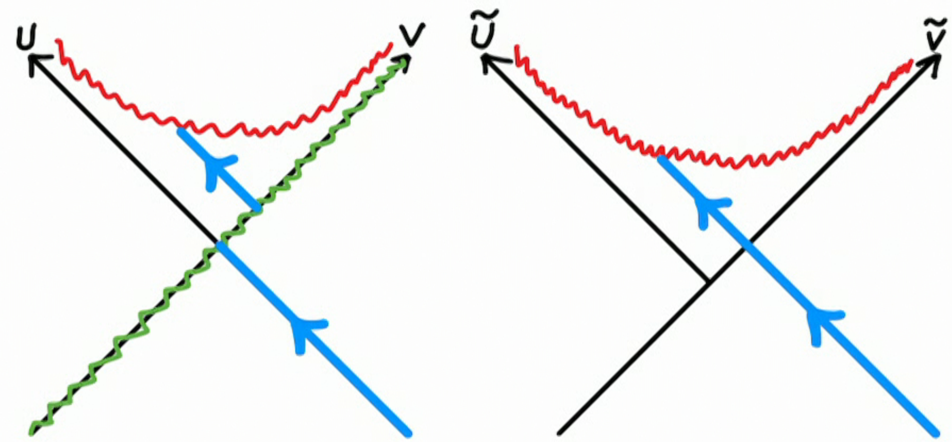
Equivalent depictions of outgoing shockwave



$$(\tilde{U}, \tilde{V}) = (U, V + \Theta(U) f(\theta, \varphi))$$

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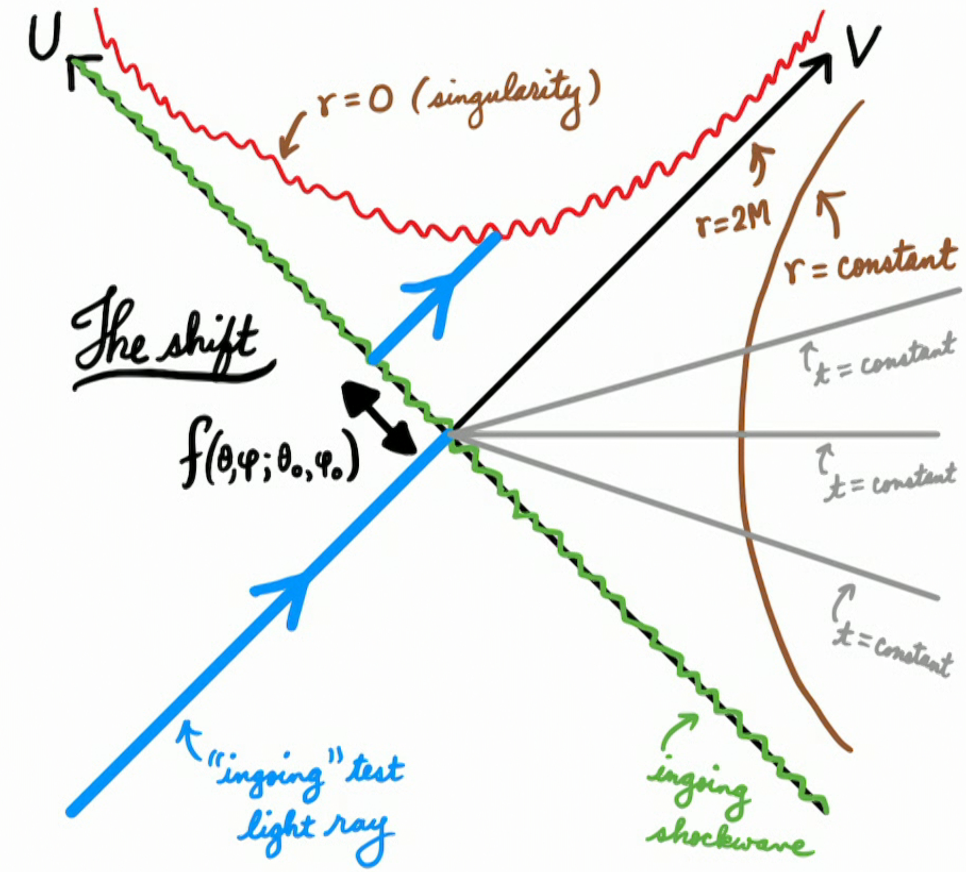
Gravitational shockwave (see Eardley, 1974)



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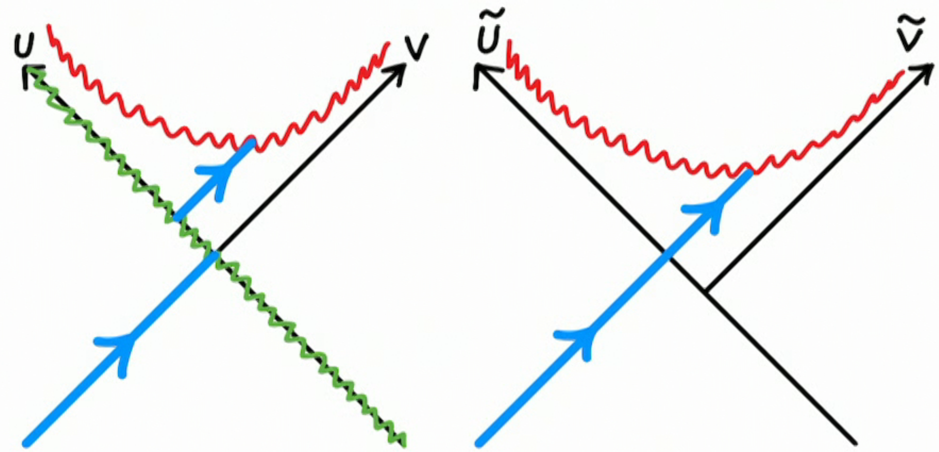
Equivalent depictions of ingoing shockwave



$$(\tilde{U}, \tilde{V}) = (U + \Theta(V) f(\theta, \varphi), V)$$

Goal: Generalize Dray-'t Hooft to Kerr-Newman.

Null tetrad

Perform a 2+2 decomposition (Newman and Penrose, 1962):

$$g_{\mu\nu} \equiv -(l_\mu l'_\nu + l'_\mu l_\nu) + (m_\mu m'_\nu + m'_\mu m_\nu) \ .$$

Conditions:

- Null: $l^\mu l_\mu = l'^\mu l'_\mu = m^\mu m_\mu = m'^\mu m'_\mu = 0$.
- Normalized: $l^\mu l'_\mu = -1$, $m^\mu m'_\mu = +1$.

Useful transformations:

- Conjugation: $(l_\mu, l'_\mu, m_\mu, m'_\mu) \to (l_\mu, l'_\mu, m'_\mu, m_\mu)$.
- Priming: $(l_\mu, l'_\mu, m_\mu, m'_\mu) \to (l'_\mu, l_\mu, m'_\mu, m_\mu)$.
- Sachs: $(l_\mu, l'_\mu, m_\mu, m'_\mu) \to (m_\mu, -m'_\mu, -l_\mu, l'_\mu)$.

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Kerr-Newman black hole

Forms:[1]

$$l_\mu dx^\mu = -dt + \frac{|R|^2}{\Delta} dr + a\sin^2\theta\, d\varphi,$$

$$l'_\mu dx^\mu = \frac{\Delta}{2|R|^2}\left(-dt - \frac{|R|^2}{\Delta} dr + a\sin^2\theta\, d\varphi\right),$$

$$m_\mu dx^\mu = \frac{1}{\sqrt{2}\,R}\left[|R|^2 d\theta + i|R_0|^2 \sin\theta\left(d\varphi - \frac{a}{|R_0|^2} dt\right)\right].$$

Vectors:

$$l^\mu \partial_\mu = \frac{|R_0|^2}{\Delta}\left(\partial_t + \frac{a}{|R_0|^2}\partial_\varphi\right) + \partial_r,$$

$$l'^\mu \partial_\mu = \frac{\Delta}{2|R|^2}\left[\frac{|R_0|^2}{\Delta}\left(\partial_t + \frac{a}{|R_0|^2}\partial_\varphi\right) - \partial_r\right],$$

$$m^\mu \partial_\mu = \frac{1}{\sqrt{2}\,R}\left(\partial_\theta + \frac{i}{\sin\theta}\partial_\varphi + ia\sin\theta\,\partial_t\right).$$

[1] Notation: $\Delta \equiv r^2 - 2Mr + a^2 + Q^2$, $R \equiv r + ia\cos\theta$, $R_0 \equiv r + ia$.

Gravity as a gauge theory

- Ambiguity in frame:

$$e^a_\mu(x) \sim O^a{}_b(x)\, e^b_\mu(x) \ \text{ if } \ O^a{}_b(x) \in SO(3,1)\ .$$

- Subgroup (Geroch, Held, and Penrose, 1973):

$$l_\mu \to \lambda\lambda^* l_\mu\ ,\ \ l'_\mu \to \lambda^{-1}\lambda^{*\,-1} l'_\mu\ ,$$
$$m_\mu \to \lambda\lambda^{*\,-1} m_\mu\ ,\ \ m'_\mu \to \lambda^{-1}\lambda^*\, m'_\mu\ .$$

- Weighted objects ("matter fields"):

$$f_{h,\bar h} \sim (h,\bar h) \quad \text{if} \quad f_{h,\bar h} \to \lambda^{2h}\lambda^{*\,2\bar h} f_{h,\bar h}\ .$$

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Spin connection

- Covariant derivative for $SO(3,1)$:

$$(\mathcal{D}_\mu)^a{}_b \equiv \delta^a{}_b \partial_\mu + (\omega_\mu)^a{}_b \ .$$

- Spin coefficients (Newman and Penrose, 1962):

$$\gamma_{abc} \equiv (\omega_\mu)_{ab}\, e^\mu_c \ .$$

- Torsion-free:

$$de_a = \gamma_{abc}\, e^b \wedge e^c \ .$$

Under the GHP subgroup, 8 spin coefficients transform as matter fields, while the remaining 4 transform as gauge fields.

Covariant derivatives for GHP

- NP derivatives (1962):

$$D \equiv l^\mu \nabla_\mu \,, \quad \delta \equiv m^\mu \nabla_\mu \,.$$

- Gauge fields:

$$\varepsilon \equiv \tfrac{1}{2}(-\gamma_{121} + \gamma_{341}) \,, \quad \beta \equiv \tfrac{1}{2}(-\gamma_{123} + \gamma_{343}) \,.$$

- GHP derivatives (1973):

$$\text{þ} \equiv D + 2h\,\varepsilon + 2\bar{h}\,\varepsilon^* \;\sim (\tfrac{1}{2}, \tfrac{1}{2}) \,,$$
$$\eth \equiv \delta + 2h\,\beta - 2\bar{h}\,\beta'^* \sim (\tfrac{1}{2}, -\tfrac{1}{2}) \,.$$

Priming flips weights, conjugation exchanges them.

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Weighted spin coefficients

Geometrical properties of null congruences:

- Refraction: $\kappa \equiv \gamma_{311} \sim (\frac{3}{2}, \frac{1}{2})$
- Expansion + i Twist : $\rho \sim \gamma_{314} \sim (\frac{1}{2}, \frac{1}{2})$
- Complex shear: $\sigma \equiv \gamma_{313} \sim (\frac{3}{2}, -\frac{1}{2})$
- Angular velocity: $\tau \equiv \gamma_{312} \sim (\frac{1}{2}, -\frac{1}{2})$

Horizons: $\rho\rho' = 0$.

Poles: $\tau\tau' = 0$.

Stationary limit surfaces: $\rho\rho' = \tau\tau'$.

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Dynamical equations for light rays

Given 1-form A, integrate along curve C with embedding coordinates $X^\mu(s)$:

$$I[X] \equiv \int_C A(X) = \int_{s_1}^{s_2} ds\, \frac{dX^\mu(s)}{ds}\, A_\mu(X(s)) \; .$$

Vary the curve ($X^\mu \to X^\mu + \blacktriangle^\mu$), bring the endpoints close ($s_2 = s_1 + \epsilon$), set the linear terms to zero:

$$\dot{X}^\nu \nabla_\nu (\blacktriangle^\mu A_\mu) = \dot{X}^\mu \blacktriangle^\nu (\partial_\mu A_\nu - \partial_\nu A_\mu) \; .$$

- Choose $A_\mu = l_\mu$ to get:

  $\text{þ}(\blacktriangle^\mu l_\mu) = \kappa\, \blacktriangle^\mu m'_\mu + c.c.$ $\leftarrow$ **refraction**

- Choose $A_\mu = m_\mu$ to get (let $z \equiv \blacktriangle^\mu m_\mu$):

  $\text{þ} z = \rho\, z + \sigma\, z^* - (\tau - \tau'^*) \blacktriangle^\mu l_\mu$. $\leftarrow$ **expansion, twist, shear**

- Choose $A_\mu = l'_\mu$ to get:

  $\text{þ}(\blacktriangle^\mu l'_\mu) = \tau'\, \blacktriangle^\mu m_\mu + c.c.$ $\leftarrow$ **angular velocity**

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Spin coefficients for Kerr-Newman

Outgoing and ingoing congruences are:

- Geodesic (i.e., $\kappa = \kappa' = 0$);
- Shear-free (i.e., $\sigma = \sigma' = 0$);
- Expanding *and twisting* (i.e., $\rho = \frac{1}{R^*}$ and $\rho' = \frac{\Delta}{2|R|^2}\left(-\frac{1}{R^*}\right)$); and
- Rotating (i.e., $\tau = \frac{ia\sin\theta}{\sqrt{2}|R|^2}$ and $\tau' = \frac{ia\sin\theta}{\sqrt{2}(R^*)^2}$).

Horizons: $\Delta = 0$.

Stationary limit surfaces: $\Delta = a^2\sin^2\theta$.

Spin coefficients for Kerr-Newman (cont.)

- Affinity parameters:

$$\varepsilon = 0\ , \quad \varepsilon' = \frac{\partial_r \Delta}{4|R|^2} + \rho'\ .$$

Surface gravity:

$$\alpha \equiv 2\,\mathrm{Re}(\varepsilon')\big|_{\rho\rho' = \tau\tau' = 0} = \frac{r_+ - r_-}{2(r_+^2 + a^2)}\ .$$

- Transverse gauge fields:

$$\beta = -\frac{\cot\theta}{2\sqrt{2}\,R}\ , \quad \beta' = \beta^* + \tau'\ .$$

Angular velocity at horizon:

$$\Omega_H = \frac{a}{r_+^2 + a^2}\ .$$

Gravitational compass (Szekeres, 1965)

$\Psi_\perp \equiv \Psi_4 \equiv -C_{\mu\nu\rho\sigma}\, l'^\mu m'^\nu l'^\rho m'^\sigma \leftarrow$ outgoing transverse

$\Psi_\parallel \equiv \Psi_3 \equiv -C_{\mu\nu\rho\sigma}\, l'^\mu l^\nu l'^\rho m'^\sigma \leftarrow$ outgoing longitudinal

$\Psi_C \equiv \Psi_2 \equiv -C_{\mu\nu\rho\sigma}\, l^\mu m^\nu m'^\rho l'^\sigma \leftarrow$ Coulomb

$\Psi'_\parallel \equiv \Psi_1 \equiv -C_{\mu\nu\rho\sigma}\, l^\mu l'^\nu l^\rho m^\sigma \leftarrow$ ingoing longitudinal

$\Psi'_\perp \equiv \Psi_0 \equiv -C_{\mu\nu\rho\sigma}\, l^\mu m^\nu l^\rho m^\sigma \leftarrow$ ingoing transverse

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Examples of curvature in GHP notation

Ingoing transverse gravitational wave:

$$\Psi_0 = -\left[\text{þ} + (\rho + \rho^*)\right]\sigma + \left[\eth + (\tau + \tau'^*)\right]\kappa \ .$$

Response to ingoing energy-momentum (i.e., to $T_{\mu\nu}\, l^\mu l^\nu$):

$$\Phi_{00} = -\left(\text{þ}\rho + \rho^2 + |\sigma|^2\right) + \eth'\kappa + \tau'\kappa + \tau\,\kappa^* \ .$$

Curvature scalars for Kerr-Newman

The only components of the curvature are:

- Weyl:

$$\Psi_2 = \frac{-M}{(R^*)^3} + \frac{Q^2}{R(R^*)^3} \, .$$

- Ricci:

$$\Phi_{11} \equiv \tfrac{1}{2} R_{\mu\nu}(l^\mu l'^\nu + m^\mu m'^\nu) = \frac{Q^2}{2|R|^4} \, .$$

Note that Schwarzschild to Kerr is basically $r \to R$.
(Newman and Janis, 1965.)

Geometrize the shockwave

The Dray-'t Hooft metric is

$$g_{\mu\nu} = g_{\mu\nu}^{\mathsf{Sch}} - 2S l_\mu l_\nu$$

with[2]

$$S = -\frac{\Delta}{2r^2}\, e^{-t/2M}\, \delta(U)\, f(\theta, \varphi)\ .$$

This is called "generalized Kerr-Schild" (Taub, 1981).

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[2] $U = -e^{-\alpha u}$, $V = e^{\alpha v}$, $u = t - r_*$, $v = t + r_*$, $r_* = \int dr\, \frac{r^2}{\Delta}$.

Shifted tetrad

- Define the *shifted tetrad 1-forms*:

$$\tilde{l} \equiv l \ , \ \ \tilde{l}' \equiv l' + S\, l \ , \ \ \tilde{m} \equiv m \ .$$

- If background is shear-free geodesic, then:

$$\tilde{\kappa}' = [\eth' + (\tau^* - \tau')]S \ , \qquad \leftarrow \textbf{refraction}$$

$$\tilde{\rho}' = \rho' - \rho S \ , \qquad \leftarrow \textbf{shift}$$

$$\tilde{\text{þ}}' f_{h,\bar{h}} = [\text{þ}' - S\text{þ} - (h+\bar{h})(\text{þ}S) + 2i(h-\bar{h})\text{Im}(\rho)S] f_{h,\bar{h}} \ .$$

Absence of nonlinearity

The only term of $O(S^2)$ is in:[4]

$$\tilde{\Phi}_{22} = \Phi_{22} + \mathrm{Re}\left[(\rho \text{þ}' - \rho' \text{þ}) S\right] + \Phi_{00} S^2$$
$$+ \tfrac{1}{2}\left[\Box_\perp + (\tau - \tau'^*)\eth' + (\tau^* - \tau')\eth\right] S$$
$$+ \left\{-4\,\mathrm{Im}(\rho)\,\mathrm{Im}(\rho') + (\eth'\tau + \eth\tau^*)\right.$$
$$\left. + 2|\tau|^2 + 2[\mathrm{Re}(\Psi_2) + 2\Pi]\right\} S \;.$$

If $\Phi_{00} = 0$, then $\tilde{\Phi}_{22} = O(S)$.

Linearization is exact.

[4] Transverse box: $\Box_\perp \equiv \eth\eth' + (\tau + \tau'^*)\eth' + \;'$.

Transverse "outgoing" gravitational wave

Weights: $\Psi_4 \sim (-2, 0)$, $f \sim (-1, -1)$. Look for $(-1, +1)$:

$$\tilde{\Psi}_4 = \frac{-c}{2|R|^2}\,\delta(U)\left[\eth'\eth' + (k_1\,\tau^* + k_2\,\tau')\eth' + (k_3\,\tau^{*2} + k_4\,\tau'^2 + k_5\,\tau^*\tau')\right] f(\theta, \chi)\ .$$

Results:

$$k_1 = \frac{\alpha|R|^2}{r}\ ,\quad k_2 = -2\left(1 + \frac{\alpha|R|^2}{2r}\right)\ ,\quad k_3 = \left(\frac{\alpha|R|^2}{2r}\right)^2\ ,$$
$$k_4 = 1 + \left(1 + \frac{\alpha|R|^2}{2r}\right)^2\ ,\quad k_5 = -\frac{\alpha|R|^2}{r}\left(1 + \frac{\alpha|R|^2}{2r}\right)\ .$$

$$c \equiv -\left.\frac{\Delta}{UV}\right|_{r=r_+} = r_+ r_- \left(\frac{r_+}{r_-} - 1\right)^{\frac{r_-^2+a^2}{r_+^2+a^2}+1} e^{-2\alpha r_+}\ .$$

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Gravitational field of "outgoing" particle

Weights: $\Phi_{22} \sim (-1,-1)$, $f \sim (-1,-1)$. Look for $(0,0)$.

Result:

$$\Phi_{22} = \frac{-c}{4|R|^2}\delta(U)\left\{ ðð' + \left[-(\tau+\tau'^*) + \left(1+\frac{\alpha|R|^2}{r}\right)(\tau-\tau'^*)\right]ð' + c.c. \right.$$
$$\left. +2\,\mathrm{Re}(\Psi_2) - (ð'\tau + ð\tau^*) + \tfrac{1}{2}|\tau+\tau'^*|^2 + \tfrac{1}{2}\left(1+\frac{\alpha|R|^2}{r}\right)^2|\tau-\tau'^*|^2 \right\} f(\theta,\chi).$$

This is the gravitational backreaction from a massless particle on the horizon of a Kerr-Newman black hole.

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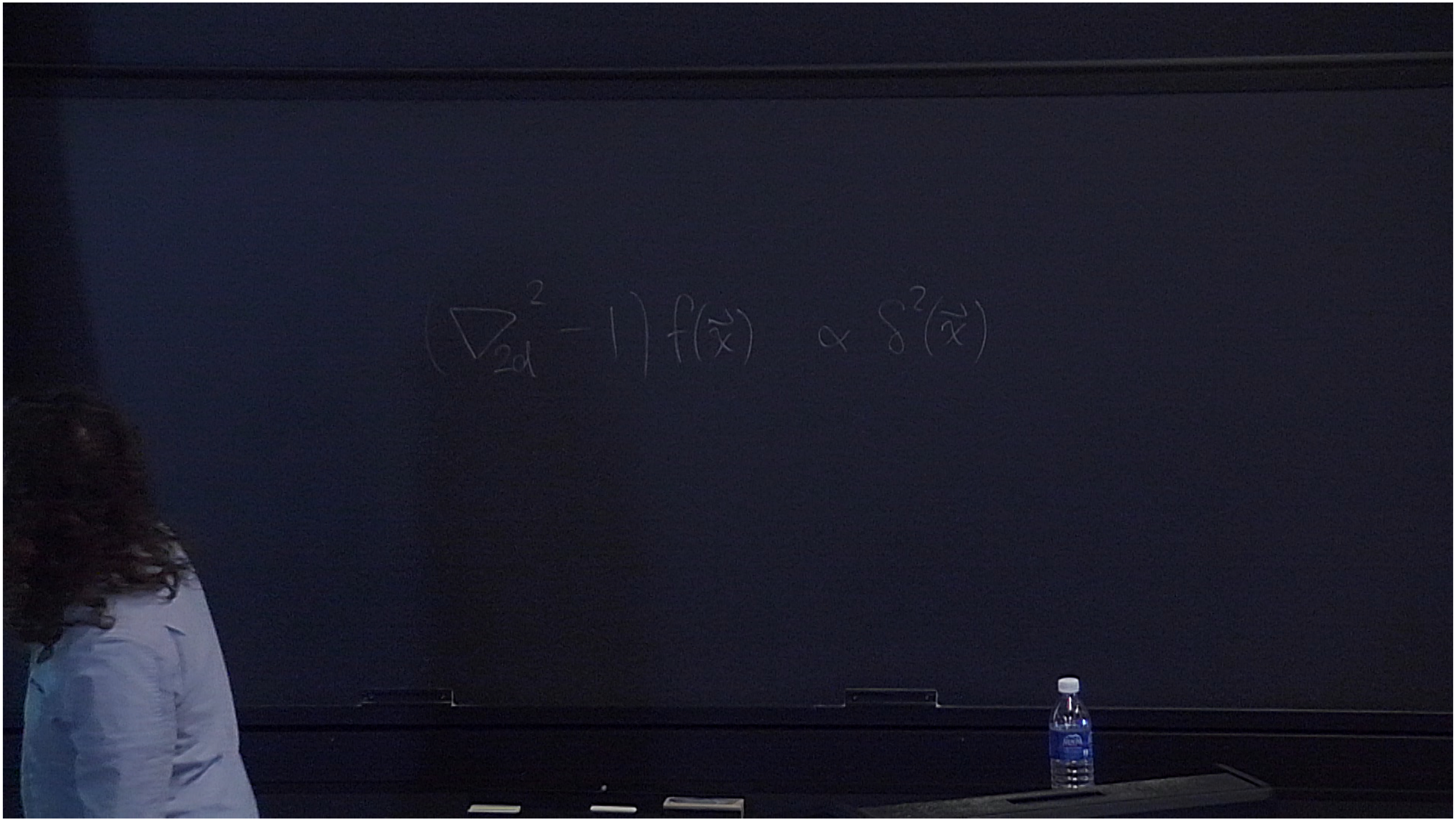
$(\nabla_{2d}^2 - 1) f(\vec{x}) \propto \delta^2(\vec{x})$