Title: All point correlation functions in SYK

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Abstract: <p>The SYK model and its variants are a new class of large N conformal field theories. In this talk, we solve SYK, computing all connected correlation functions. Our techniques and results for summing all leading large N Feynman diagrams are applicable to a significantly broader class of theories. </p>

<p> </p>

All point correlation functions in SYK

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SYK is 0+1 dimensional theory of N fermions with 4 body (or, more generally, q body) random all-to-all interactions.
It flows to a CFT in the infrared, at strong coupling.

SYK

$$L = \sum_i \frac{1}{2}\chi_i \frac{d}{d\tau}\chi_i - \sum_{i,j,k,l} J_{ijkl}\, \chi_i \chi_j \chi_k \chi_l$$

fermion dimension in infrared (large J):

$$\Delta = 1/q \ , \qquad q = 4$$

Kitaev introduced SYK as a solvable model of holography

In this talk we will solve SYK

We will compute all correlation functions, at strong coupling, to leading nontrivial order in 1/N

Why do we want to solve SYK?

Let us recall the two canonical examples of AdS/CFT

$\mathcal{N} = 4$, maximally supersymmetric SU(N) Yang-Mills, is a CFT, for any value of the ’t Hooft coupling

$\mathcal{N} = 4$, at large N, is dual to string theory in AdS. At large ’t Hooft coupling, there is a gap, leading to Einstein gravity and black holes.

$\mathcal{N} = 4$ is incredibly rich, and is integrable. Over the last decade, major progress has been made in solving $\mathcal{N} = 4$, enriching our understanding of AdS/CFT.

The free/critical vector O(N) model is another solvable, large N, CFT

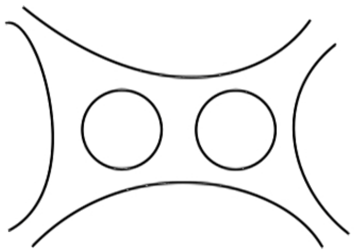
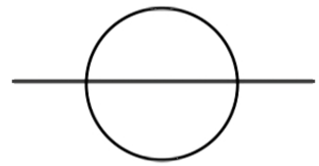
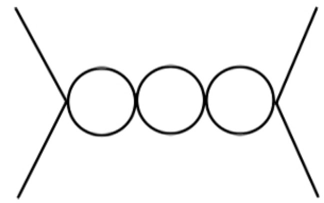
The O(N) model is dual to Vasiliev higher spin theory

$\mathcal{N}$ = 4 and the vector O(N) model are very different theories

Correspondingly, the bulk duals, string theory and Vasiliev theory, are very different.

SYK is a new class of solvable, strongly coupled, large N, CFTs.

SYK is harder than the O(N) model, but easier than $\mathcal{N} = 4$.

	Matrix model	SYK	Vector model
			
	planar diagrams	melon diagrams	bubble diagrams
	$\mathcal{N} = 4$	SYK	O(N)
Bulk:	Large gap	Tower massive particles	Tower massless particles
	String theory	?	Vasiliev

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- It was just something Kitaev made up
- Yet, it had the remarkable properties of being a CFT in the infrared, solvable at large N, and maximally chaotic (like a black hole)

A number of generalizations and variations of SYK started being found. These also had the salient features of SYK.

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One could, for instance, add flavor Gross, VR ‘16
or supersymmetry Fu, Gaiotto, Maldacena, Sachdev, ‘16

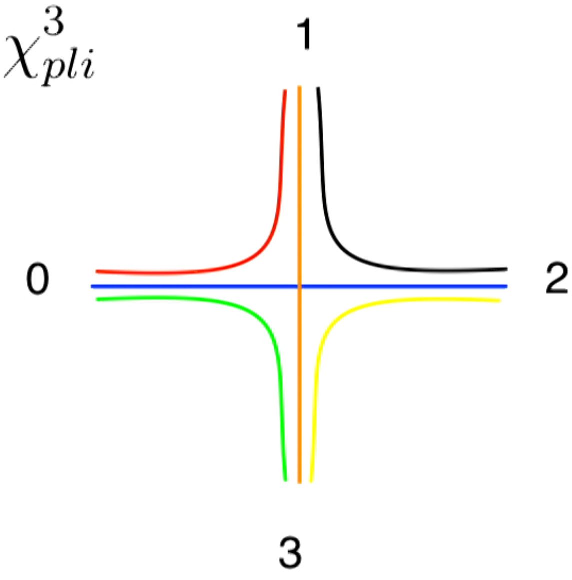
The two canonical examples of AdS/CFT that I mentioned were maximally supersymmetric SU(N) Yang-Mills (a matrix model), and the O(N) vector model (a vector model)

We generally don't discuss tensor models.
One might assume they would be too difficult to study.

Melonic Tensor Models

Bonzom, Gurau, Riello, Rivasseau
Gurau
Witten

$$H = \sum \chi^0_{ijk} \chi^1_{klm} \chi^2_{mjp} \chi^3_{pli}$$



Since we have studied vector models and matrix models, it is only natural to study tensor models.

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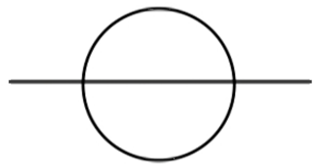
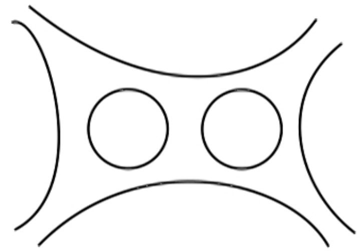
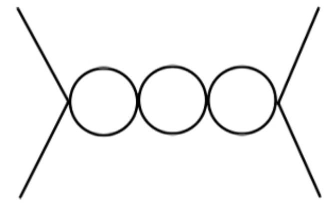
In fact, there are arguments that the description of M2 branes stretching between M5 branes should be described by tensors

Beccaria, Tseytlin, '17
Klebanov Tsyeltin, '96

Tensor model	Matrix model	Vector model
		
melon diagrams	planar diagrams	bubble diagrams
Gurau-Witten Klebanov-Tarnopolsky	$\mathcal{N} = 4$	O(N)
Many towers massive particles	Large gap	Tower massless particles
?	String theory	Vasiliev

SYK and the melonic tensor models have the same large N fermion correlation functions.

For the rest of the talk we will focus on SYK, but the results trivially extend to tensor models and all variations and generalizations of SYK.

1) To better understand holography and black holes

2) To find the AdS dual of SYK

3) SYK is also a model of a strange metal, and of a maximally chaotic quantum system

4) Our solution applies to all melonic tensor models

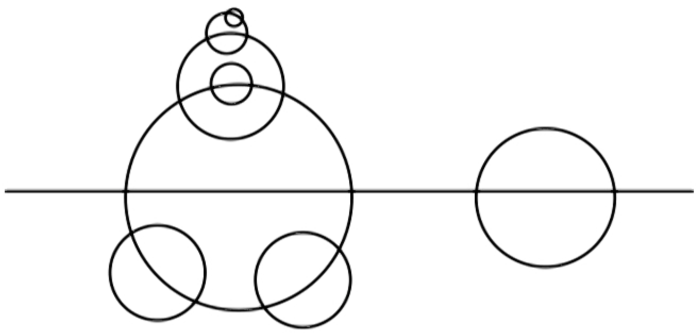
The computation of SYK correlation functions will be direct. We sum all Feynman diagrams.

The result will be interesting.

All higher-point functions will follow from the six-point function through simple rules.

2-point function

Melons



IR dimension: Δ

Sachdev Ye '93

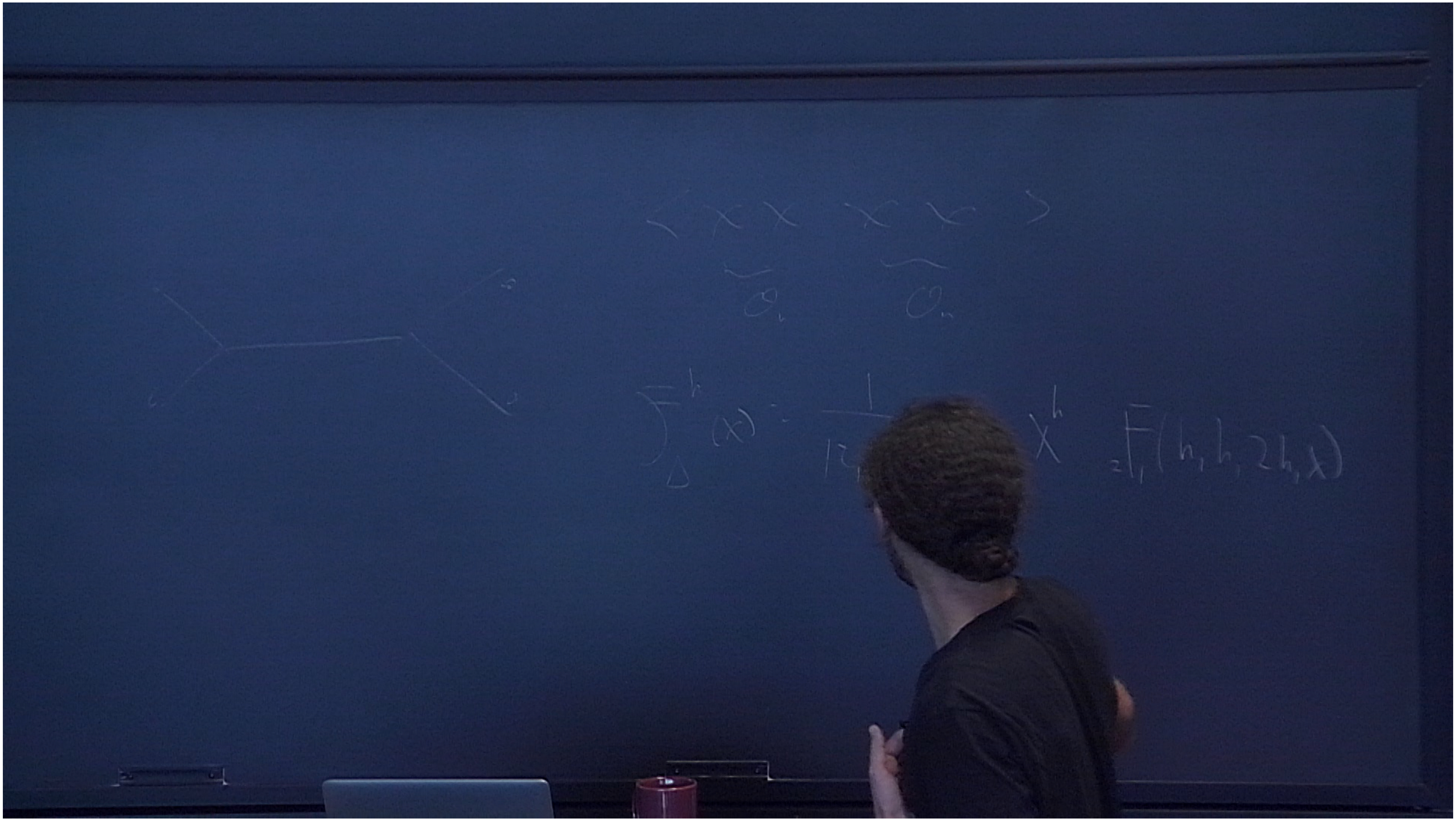
4-point function

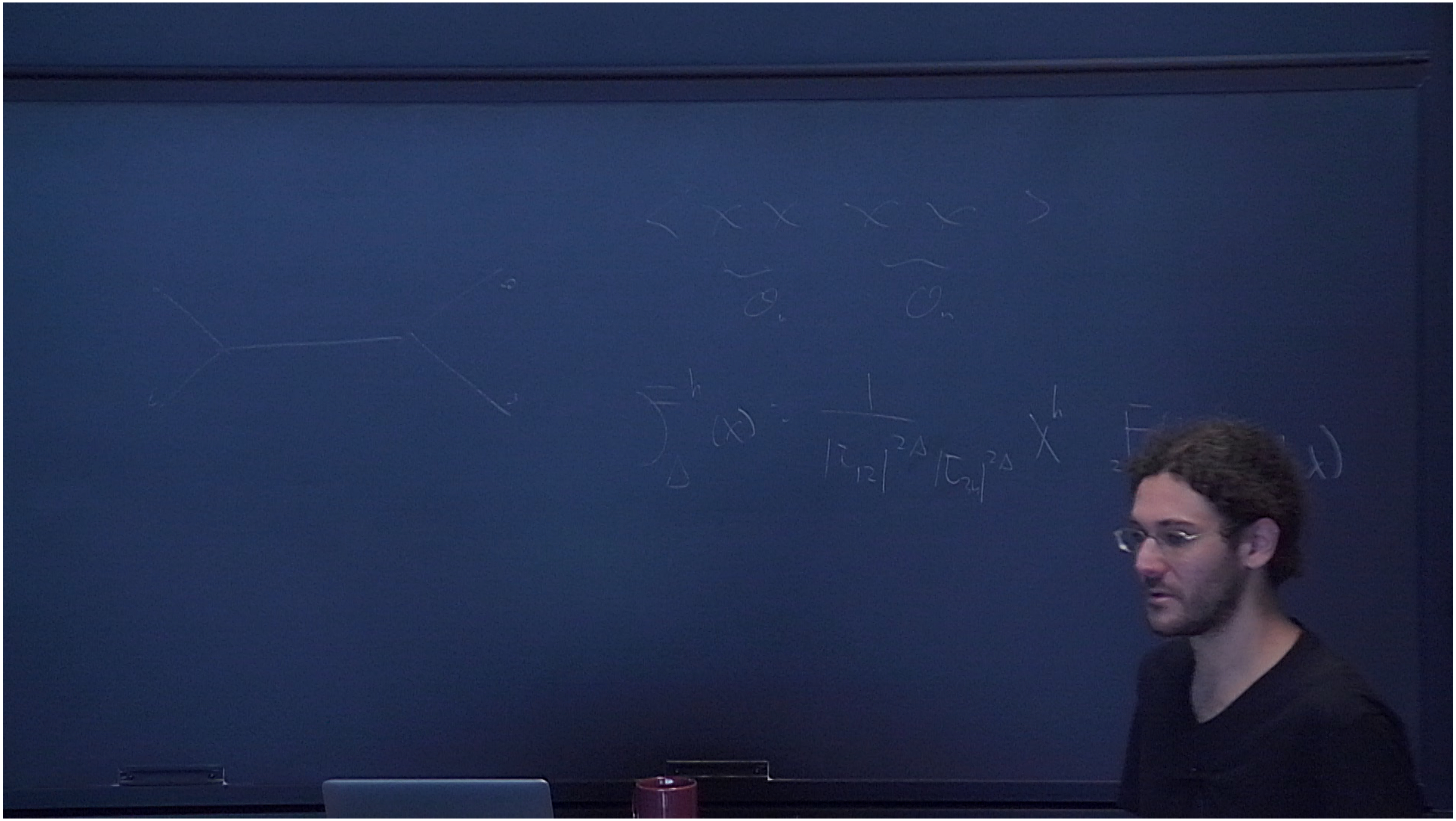
Fermion four-point function is a sum of conformal blocks,

$$\mathcal{F}(\tau_1, \ldots, \tau_4) = \frac{b^2}{J^{4\Delta}} \sum_{h_n} c_n^2 \, \mathcal{F}_{\Delta}^{h_n}(x) \qquad x = \frac{\tau_{12}\tau_{34}}{\tau_{13}\tau_{24}}$$

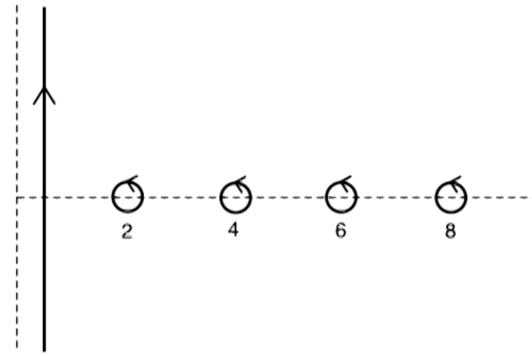
These are the conformal blocks of the primary, fermion bilinear, O(N) invariant singlets, $\mathcal{O}_n$ with dimension h_n

$$\mathcal{O}_n \sim \frac{1}{N} \sum_{i=1}^{N} \chi_i \partial_\tau^{1+2n} \chi_i$$





We may also write this as,



$$\mathcal{F}(\tau_1,\ldots,\tau_4) \;=\; G(\tau_{12})G(\tau_{34})\int_{\mathcal{C}} \frac{dh}{2\pi i}\,\rho(h)\Psi_h(x)$$

$$\frac{2}{|\tau_{12}|^{2\Delta}|\tau_{34}|^{2\Delta}}\Psi_h(x) = \beta(h,0)\mathcal{F}^h_\Delta(x) + \beta(1-h,0)\mathcal{F}^{1-h}_\Delta(x)$$

Fermion four-point function is a sum of conformal blocks,

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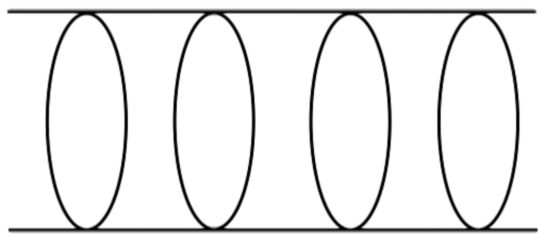
$$\mathcal{O}_n \sim \frac{1}{N} \sum_{i=1}^{N} \chi_i \partial_\tau^{1+2n} \chi_i$$

This was general

For SYK, the four-point function is a sum of ladder diagrams



So we know $\rho(h)$

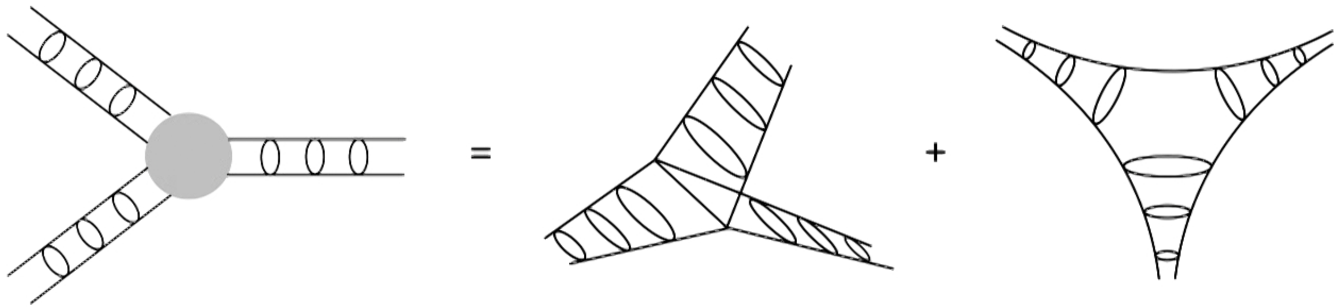
Maldacena, Stanford '16
Polchinski, V.R., '16
Kitaev, '16

Summary

2-point function: Δ

4-point function: $\rho(h)$

6-point function



The fermion 6-point function has the same information as the bilinear 3-point function

$$\langle \mathcal{O}_1(\tau_1)\mathcal{O}_2(\tau_2)\mathcal{O}_3(\tau_3)\rangle = \frac{1}{\sqrt{N}} \frac{c_{123}}{|\tau_{12}|^{h_1+h_2-h_3}|\tau_{23}|^{h_2+h_3-h_1}|\tau_{13}|^{h_1+h_3-h_2}}$$

$$c_{123} = c_1 c_2 c_3 (\mathcal{I}_{123}^{(1)} + \mathcal{I}_{123}^{(2)})$$

$$\mathcal{I}_{123}^{(1)} = \frac{\sqrt{\pi}\, 2^{h_1+h_2+h_3-1}\, \Gamma(1-h_1)\Gamma(1-h_2)\Gamma(1-h_3)}{\Gamma\left(\frac{3-h_1-h_2-h_2}{2}\right)} [\rho(h_1,h_2,h_3)+\rho(h_2,h_3,h_1)+\rho(h_3,h_1,h_2)]$$

$$\rho(h_1,h_2,h_3) = \frac{\Gamma(\frac{h_2+h_3-h_1}{2})}{\Gamma(\frac{2-h_1-h_2+h_3}{2})\Gamma(\frac{2-h_1-h_3+h_2}{2})} \left(1 + \frac{\sin(\pi h_2)}{\sin(\pi h_3) - \sin(\pi h_1 + \pi h_2)}\right)$$

$$
\begin{aligned}
\mathcal{I}_{123}^{(2)} &= \alpha_1\ {}_4F_3\left[\begin{matrix} 1-h_1 & h_1 & 2\Delta-h_3 & 1-h_3 \\ 1+h_2-h_3 & 2\Delta & 2-h_2-h_3 & \end{matrix};1\right] \\
&+ \alpha_2\ {}_4F_3\left[\begin{matrix} 1-h_1-h_2+h_3 & h_1-h_2+h_3 & 2\Delta-h_2 & 1-h_2 \\ 2-2h_2 & 1-h_2+h_3 & 2\Delta-h_2+h_3 & \end{matrix};1\right] \\
&+ \alpha_3\ {}_4F_3\left[\begin{matrix} 2-h_1-2\Delta & 1+h_1-2\Delta & 1-h_3 & 2-h_3-2\Delta \\ 2+h_2-h_3-2\Delta & 3-h_2-h_3-2\Delta & 2-2\Delta & \end{matrix};1\right] \\
&+ \alpha_4\ {}_4F_3\left[\begin{matrix} h_2+h_3-h_1 & h_1+h_2+h_3-1 & h_2-1+2\Delta & h_2 \\ 2h_2 & h_2+h_3-1+2\Delta & h_2+h_3 & \end{matrix};1\right] .
\end{aligned}
$$

$$
\alpha_1 = -\frac{\Gamma(\frac{2\Delta+1}{2})^2}{\Gamma(1-\Delta)^2}\prod_{i=1}^{3}\frac{\Gamma(\frac{1-h_i}{2})}{\Gamma(\frac{h_i}{2})}\ \frac{\Gamma(\frac{3-h_2-2\Delta}{2})\Gamma(\frac{2+h_2-2\Delta}{2})}{\Gamma(\frac{h_2+2\Delta}{2})\Gamma(\frac{1-h_2+2\Delta}{2})}\frac{\Gamma(\frac{h_3-h_2}{2})\Gamma(\frac{h_2+h_3-1}{2})}{\Gamma(\frac{2-h_2-h_3}{2})\Gamma(\frac{1+h_2-h_3}{2})}\frac{\Gamma(\frac{h_1+h_2-h_3}{2})}{\Gamma(\frac{1-h_1-h_2+h_3}{2})}\ ,
$$

$$
\begin{aligned}
\alpha_2 &= -\frac{\Gamma(\frac{2\Delta+1}{2})^3}{\Gamma(1-\Delta)^3}\frac{\Gamma(\frac{1-h_1}{2})}{\Gamma(\frac{h_1}{2})}\frac{\Gamma(\frac{1-h_2}{2})^2\,\Gamma(\frac{2h_2-1}{2})}{\Gamma(\frac{h_2}{2})^2\,\Gamma(\frac{2-2h_2}{2})}\frac{\Gamma(\frac{3-h_2-2\Delta}{2})}{\Gamma(\frac{h_2+2\Delta}{2})}\frac{\Gamma(\frac{2+h_3-2\Delta}{2})}{\Gamma(\frac{1-h_3+2\Delta}{2})} \\
&\quad\cdot\frac{\Gamma(\frac{h_2-h_3}{2})\Gamma(\frac{h_2-h_3+2-2\Delta}{2})}{\Gamma(\frac{1-h_2+h_3}{2})\Gamma(\frac{h_3-h_2+1+2\Delta}{2})}\frac{\Gamma(\frac{h_1-h_2+h_3}{2})}{\Gamma(\frac{1-h_1+h_2-h_3}{2})}\ ,
\end{aligned}
$$

$$
\begin{aligned}
\alpha_3 &= -\frac{\Gamma(\frac{2\Delta+1}{2})^3\,\Gamma(\Delta)}{\Gamma(1-\Delta)^3\,\Gamma(\frac{3-2\Delta}{2})}\prod_{i=1}^{3}\frac{\Gamma(\frac{1-h_i}{2})\Gamma(\frac{2+h_i-2\Delta}{2})\Gamma(\frac{3-h_i-2\Delta}{2})}{\Gamma(\frac{h_i}{2})\Gamma(\frac{1-h_i+2\Delta}{2})\Gamma(\frac{h_i+2\Delta}{2})} \\
&\quad\cdot\frac{\Gamma(\frac{h_3-h_2+2\Delta}{2})\Gamma(\frac{h_2+h_3-1+2\Delta}{2})}{\Gamma(\frac{3+h_2-h_3-2\Delta}{2})\Gamma(\frac{4-h_2-h_3-2\Delta}{2})}\frac{\Gamma(\frac{h_1+h_2-h_3}{2})}{\Gamma(\frac{1-h_1-h_2+h_3}{2})}\ ,
\end{aligned}
$$

$$
\begin{aligned}
\alpha_4 &= -\frac{\Gamma(\frac{2\Delta+1}{2})^3}{\Gamma(1-\Delta)^3}\frac{\Gamma(\frac{1-h_1}{2})}{\Gamma(\frac{h_1}{2})}\frac{\Gamma(\frac{1-2h_2}{2})}{\Gamma(h_2)}\frac{\Gamma(\frac{2+h_2-2\Delta}{2})}{\Gamma(\frac{1-h_2+2\Delta}{2})}\frac{\Gamma(\frac{2+h_3-2\Delta}{2})}{\Gamma(\frac{1-h_3+2\Delta}{2})} \\
&\quad\cdot\frac{\Gamma(\frac{1-h_2-h_3}{2})}{\Gamma(\frac{h_2+h_3}{2})}\frac{\Gamma(\frac{3-h_2-h_3-2\Delta}{2})}{\Gamma(\frac{h_2+h_3+2\Delta}{2})}\frac{\Gamma(\frac{h_1+h_2-h_3}{2})\Gamma(\frac{-h_1+h_2+h_3}{2})\Gamma(\frac{h_1+h_2+h_3-1}{2})}{\Gamma(\frac{1-h_1-h_2+h_3}{2})\Gamma(\frac{1+h_1-h_2-h_3}{2})\Gamma(\frac{2-h_1-h_2-h_3}{2})}\ .
\end{aligned}
$$

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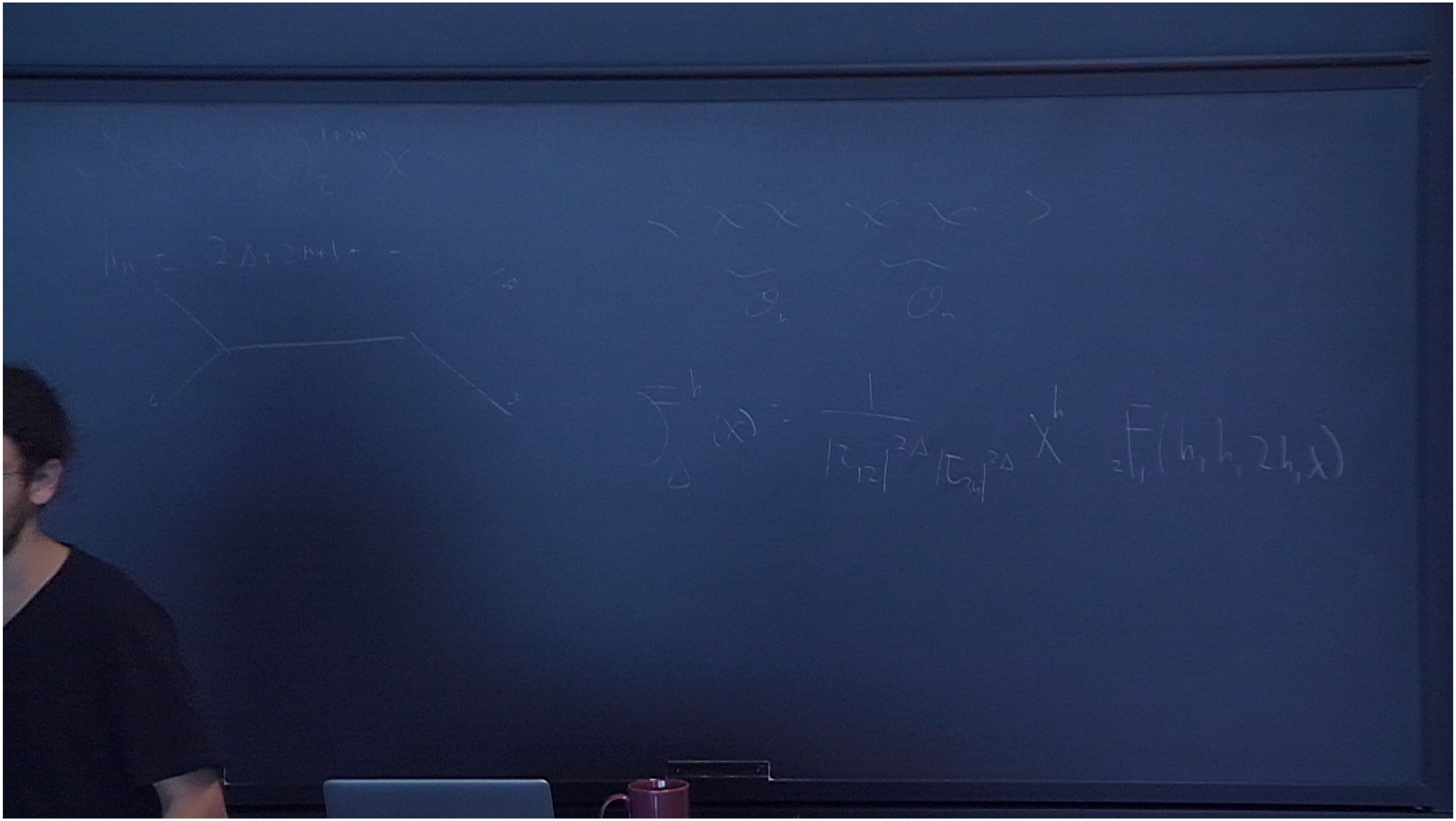
\mathcal{I}_{123}^{(2)} &= \alpha_1 \; {}_4F_3\left[\begin{matrix} 1-h_1 & h_1 & 2\Delta-h_3 & 1-h_3 \\ 1+h_2-h_3 & 2\Delta & 2-h_2-h_3 & \end{matrix} ;1\right] \\
&+ \alpha_2 \; {}_4F_3\left[\begin{matrix} 1-h_1-h_2+h_3 & h_1-h_2+h_3 & 2\Delta-h_2 & 1-h_2 \\ 2-2h_2 & 1-h_2+h_3 & 2\Delta-h_2+h_3 & \end{matrix} ;1\right] \\
&+ \alpha_3 \; {}_4F_3\left[\begin{matrix} 2-h_1-2\Delta & 1+h_1-2\Delta & 1-h_3 & 2-h_3-2\Delta \\ 2+h_2-h_3-2\Delta & 3-h_2-h_3-2\Delta & 2-2\Delta & \end{matrix} ;1\right] \\
&+ \alpha_4 \; {}_4F_3\left[\begin{matrix} h_2+h_3-h_1 & h_1+h_2+h_3-1 & h_2-1+2\Delta & h_2 \\ 2h_2 & h_2+h_3-1+2\Delta & h_2+h_3 & \end{matrix} ;1\right] .
\end{aligned}
$$

$$
\begin{aligned}
\alpha_1 &= -\frac{\Gamma(\frac{2\Delta+1}{2})^2}{\Gamma(1-\Delta)^2}\prod_{i=1}^{3}\frac{\Gamma(\frac{1-h_i}{2})}{\Gamma(\frac{h_i}{2})}\;\frac{\Gamma(\frac{3-h_2-2\Delta}{2})\Gamma(\frac{2+h_2-2\Delta}{2})}{\Gamma(\frac{h_2+2\Delta}{2})\Gamma(\frac{1-h_2+2\Delta}{2})}\,\frac{\Gamma(\frac{h_3-h_2}{2})\Gamma(\frac{h_2+h_3-1}{2})}{\Gamma(\frac{2-h_2-h_3}{2})\Gamma(\frac{1+h_2-h_3}{2})}\,\frac{\Gamma(\frac{h_1+h_2-h_3}{2})}{\Gamma(\frac{1-h_1-h_2+h_3}{2})}\;, \\
\alpha_2 &= -\frac{\Gamma(\frac{2\Delta+1}{2})^3}{\Gamma(1-\Delta)^3}\,\frac{\Gamma(\frac{1-h_1}{2})}{\Gamma(\frac{h_1}{2})}\,\frac{\Gamma(\frac{1-h_2}{2})^2\,\Gamma(\frac{2h_2-1}{2})}{\Gamma(\frac{h_2}{2})^2\,\Gamma(\frac{2-2h_2}{2})}\,\frac{\Gamma(\frac{3-h_2-2\Delta}{2})}{\Gamma(\frac{h_2+2\Delta}{2})}\,\frac{\Gamma(\frac{2+h_3-2\Delta}{2})}{\Gamma(\frac{1-h_3+2\Delta}{2})} \\
&\quad\cdot\frac{\Gamma(\frac{h_2-h_3}{2})\Gamma(\frac{h_2-h_3+2-2\Delta}{2})}{\Gamma(\frac{1-h_2+h_3}{2})\Gamma(\frac{h_3-h_2+1+2\Delta}{2})}\,\frac{\Gamma(\frac{h_1-h_2+h_3}{2})}{\Gamma(\frac{1-h_1+h_2-h_3}{2})}\;, \\
\alpha_3 &= -\frac{\Gamma(\frac{2\Delta+1}{2})^3\,\Gamma(\Delta)}{\Gamma(1-\Delta)^3\,\Gamma(\frac{3-2\Delta}{2})}\prod_{i=1}^{3}\frac{\Gamma(\frac{1-h_i}{2})\Gamma(\frac{2+h_i-2\Delta}{2})\Gamma(\frac{3-h_i-2\Delta}{2})}{\Gamma(\frac{h_i}{2})\Gamma(\frac{1-h_i+2\Delta}{2})\Gamma(\frac{h_i+2\Delta}{2})} \\
&\quad\cdot\frac{\Gamma(\frac{h_3-h_2+2\Delta}{2})\Gamma(\frac{h_2+h_3-1+2\Delta}{2})}{\Gamma(\frac{3+h_2-h_3-2\Delta}{2})\Gamma(\frac{4-h_2-h_3-2\Delta}{2})}\,\frac{\Gamma(\frac{h_1+h_2-h_3}{2})}{\Gamma(\frac{1-h_1-h_2+h_3}{2})}\;, \\
\alpha_4 &= -\frac{\Gamma(\frac{2\Delta+1}{2})^3}{\Gamma(1-\Delta)^3}\,\frac{\Gamma(\frac{1-h_1}{2})}{\Gamma(\frac{h_1}{2})}\,\frac{\Gamma(\frac{1-2h_2}{2})}{\Gamma(h_2)}\,\frac{\Gamma(\frac{2+h_2-2\Delta}{2})}{\Gamma(\frac{1-h_2+2\Delta}{2})}\,\frac{\Gamma(\frac{2+h_3-2\Delta}{2})}{\Gamma(\frac{1-h_3+2\Delta}{2})} \\
&\quad\cdot\frac{\Gamma(\frac{1-h_2-h_3}{2})}{\Gamma(\frac{h_2+h_3}{2})}\,\frac{\Gamma(\frac{3-h_2-h_3-2\Delta}{2})}{\Gamma(\frac{h_2+h_3+2\Delta}{2})}\,\frac{\Gamma(\frac{h_1+h_2-h_3}{2})\Gamma(\frac{-h_1+h_2+h_3}{2})\Gamma(\frac{h_1+h_2+h_3-1}{2})}{\Gamma(\frac{1-h_1-h_2+h_3}{2})\Gamma(\frac{1+h_1-h_2-h_3}{2})\Gamma(\frac{2-h_1-h_2-h_3}{2})}\;.
\end{aligned}
$$

- We have computed this for general h_1, h_2, h_3. The physical 3-point function is for h_1, h_2, h_3 at the physical dimensions.



- We have computed this for general h_1, h_2, h_3. The physical 3-point function is for h_1, h_2, h_3 at the physical dimensions.

- Viewed as a function of h_1, there are poles at,

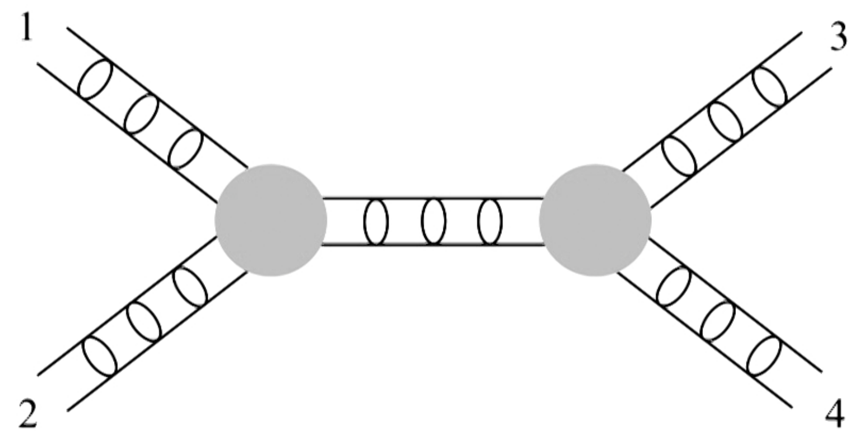
$$h_1 = h_2 + h_3 + 2n$$

Summary

2-point function: Δ

4-point function: $\rho(h)$

6-point function: c_{123}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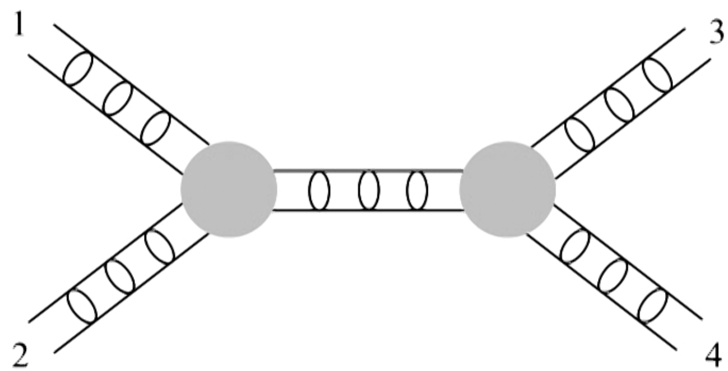
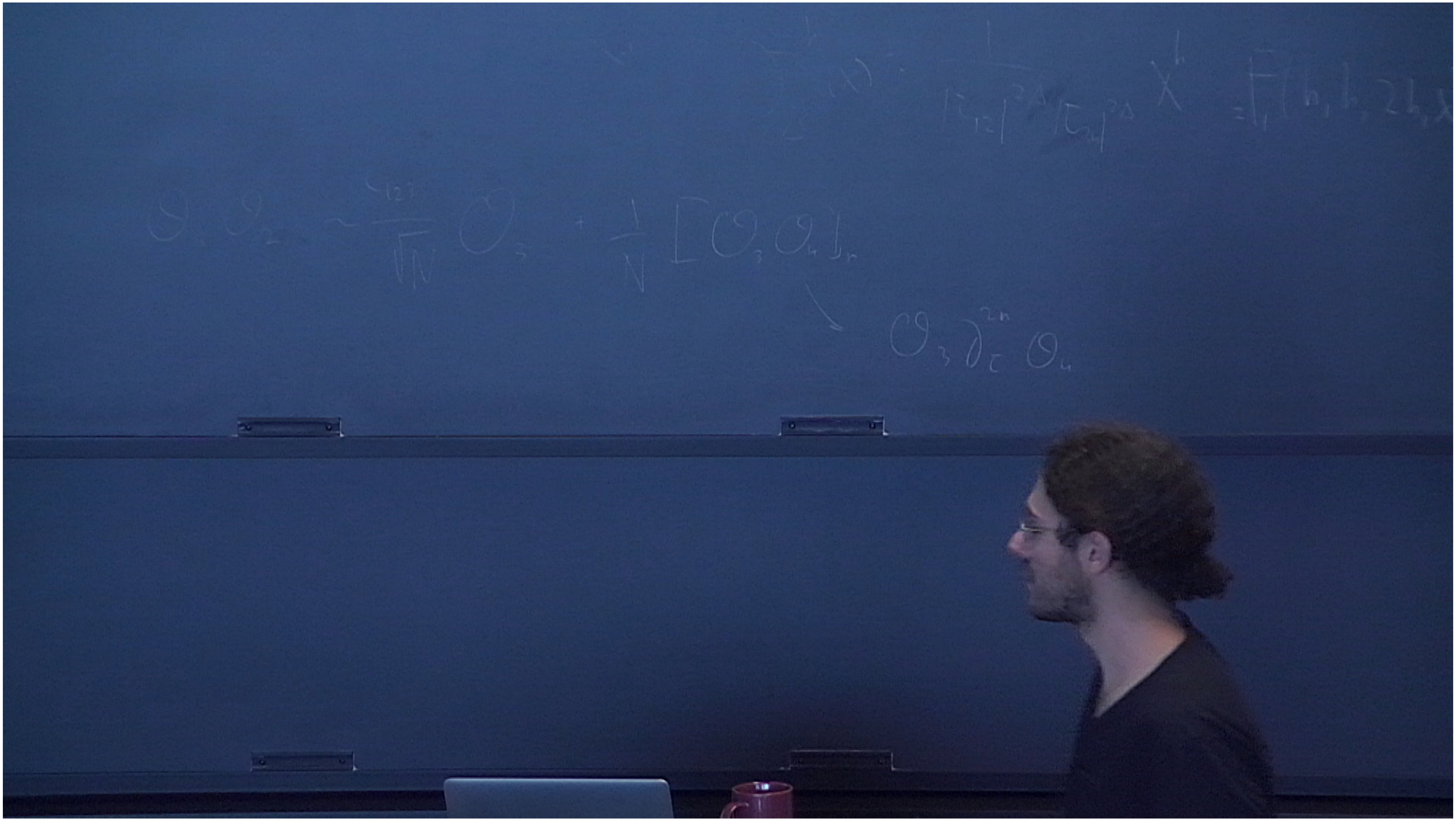


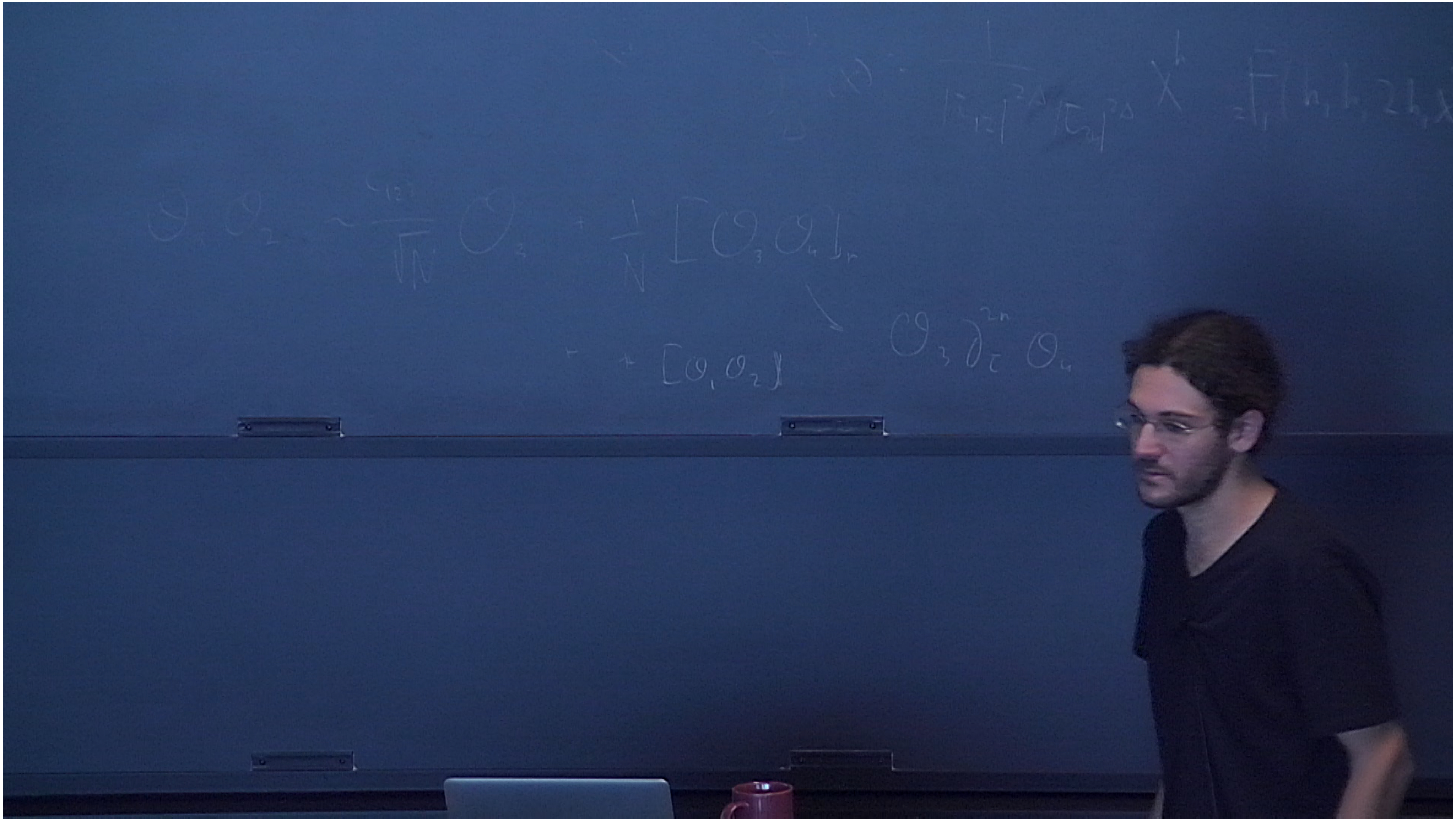
$$\langle \mathcal{O}_1(\tau_1) \cdots \mathcal{O}_4(\tau_4) \rangle_s = \int_{\mathcal{C}} \frac{dh}{2\pi i} \frac{\rho(h)}{c_h^2} \frac{\Gamma(h)^2}{\Gamma(2h)} \, c_{12h} c_{34h} \, \mathcal{F}_{1234}^h(x)$$

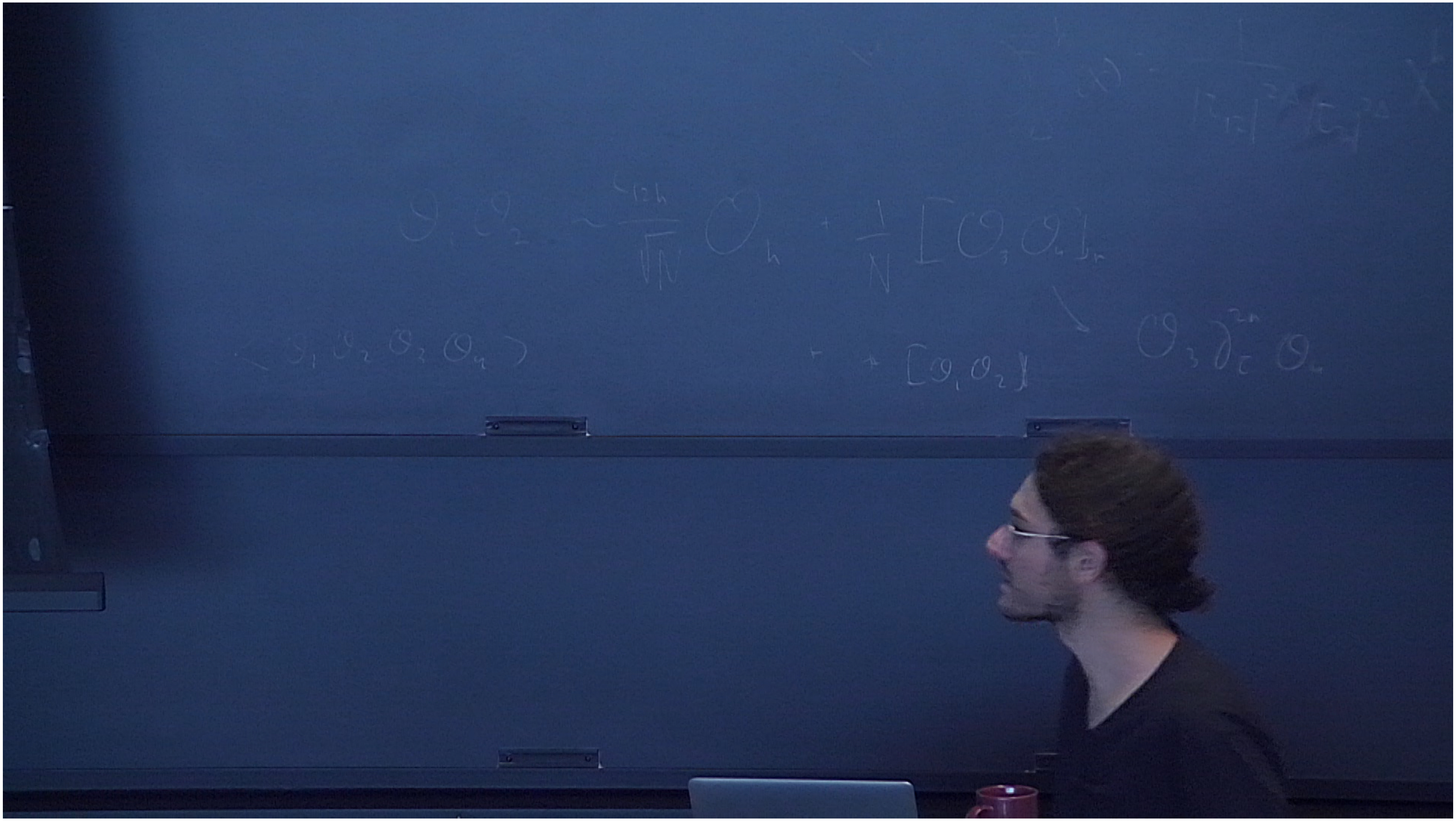
$$\langle \mathcal{O}_1(\tau_1)\cdots\mathcal{O}_4(\tau_4)\rangle_s = \int_{\mathcal{C}} \frac{dh}{2\pi i}\,\frac{\rho(h)}{c_h^2}\,\frac{\Gamma(h)^2}{\Gamma(2h)}\,c_{12h}c_{34h}\,\mathcal{F}^h_{1234}(x)$$

Closing the contour,

$$\begin{aligned}\langle \mathcal{O}_1(\tau_1)\cdots\mathcal{O}_4(\tau_4)\rangle_s &= \sum_{h=h_n} c_{12h}c_{34h}\,\mathcal{F}^h_{1234}(x)\\ &+ \sum_{n=0}^{\infty} -\mathrm{Res}\,\left[\frac{c_{12h}}{c_h}\right]_{h=h_1+h_2+2n}\left[\rho(h)\frac{\Gamma(h)^2}{\Gamma(2h)}\frac{c_{34h}}{c_h}\mathcal{F}^h_{1234}(x)\right]_{h=h_1+h_2+2n}\\ &+ \sum_{n=0}^{\infty} -\mathrm{Res}\,\left[\frac{c_{34h}}{c_h}\right]_{h=h_3+h_4+2n}\left[\rho(h)\frac{\Gamma(h)^2}{\Gamma(2h)}\frac{c_{12h}}{c_h}\mathcal{F}^h_{1234}(x)\right]_{h=h_3+h_4+2n}\end{aligned}$$


$\mathcal{O}_1 \mathcal{O}_2 \sim \frac{c_{123}}{\sqrt{N}} \mathcal{O}_3 + \frac{1}{N} [\mathcal{O}_3 \mathcal{O}_4]_n$
$\mathcal{O}_3 \partial_\tau^{2n} \mathcal{O}_4$





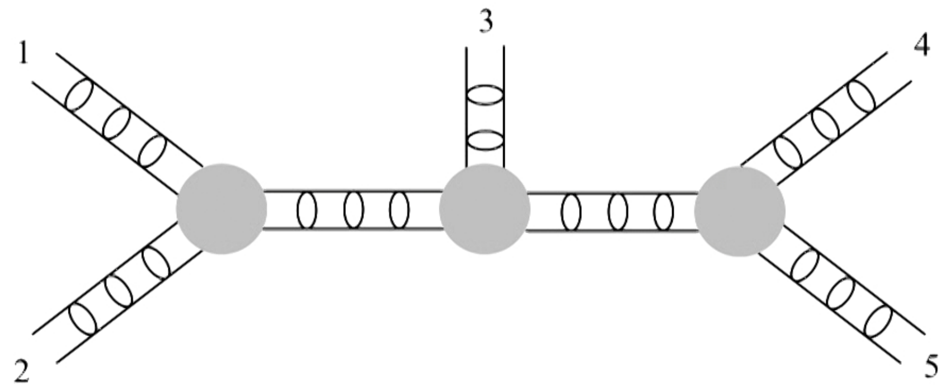
O_1 O_2 ~ C_{12h}/√N O_h + 1/N [O_3 O_4]_n
⟨O_1 O_2 O_3 O_4⟩
+ [O_1 O_2]
O_3 ∂_τ^{2n} O_4

$$\langle \mathcal{O}_1(\tau_1)\cdots\mathcal{O}_4(\tau_4)\rangle_s = \int_{\mathcal{C}} \frac{dh}{2\pi i}\frac{\rho(h)}{c_h^2}\frac{\Gamma(h)^2}{\Gamma(2h)}\, c_{12h}c_{34h}\,\mathcal{F}^h_{1234}(x)$$

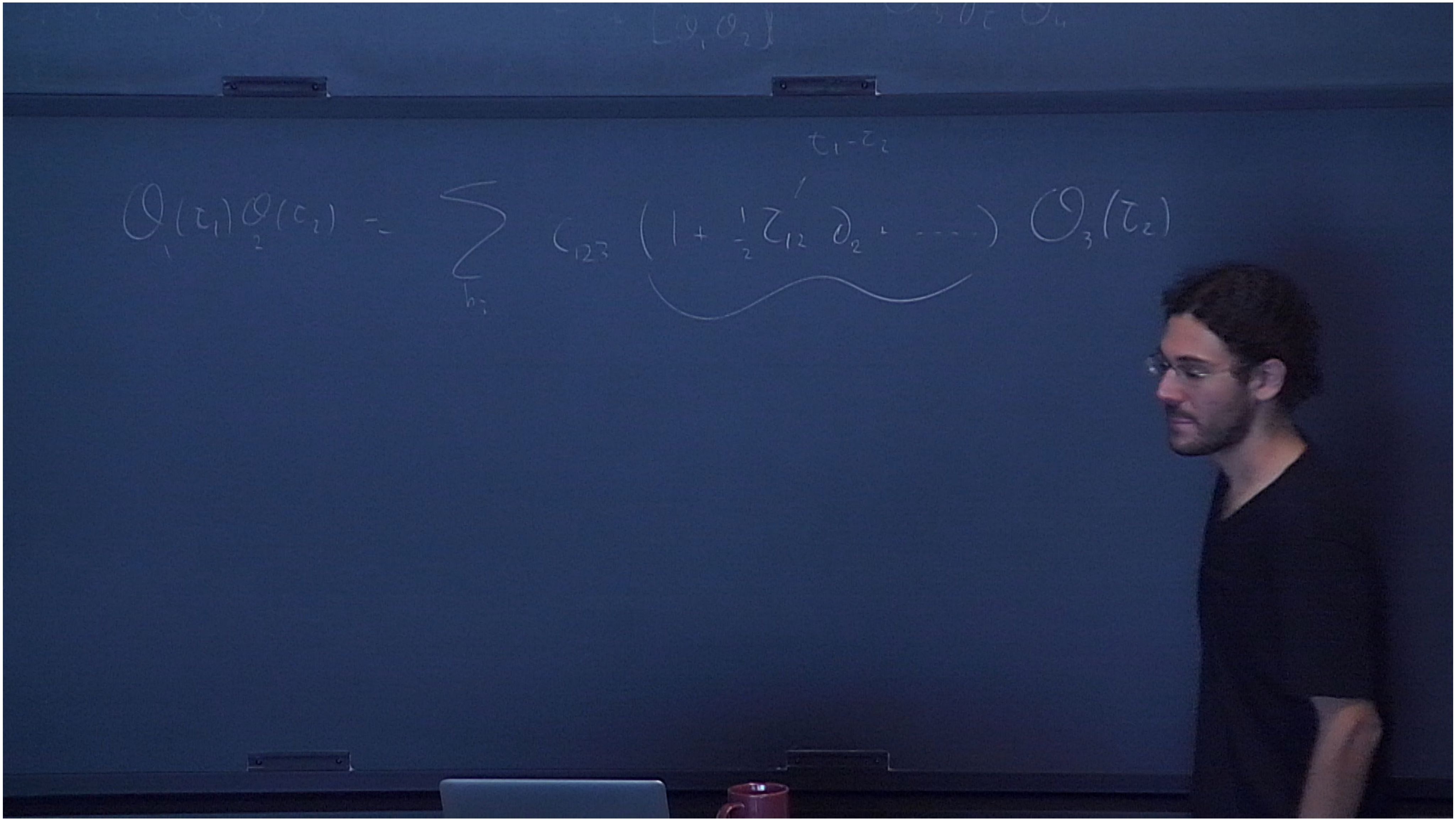
Closing the contour,

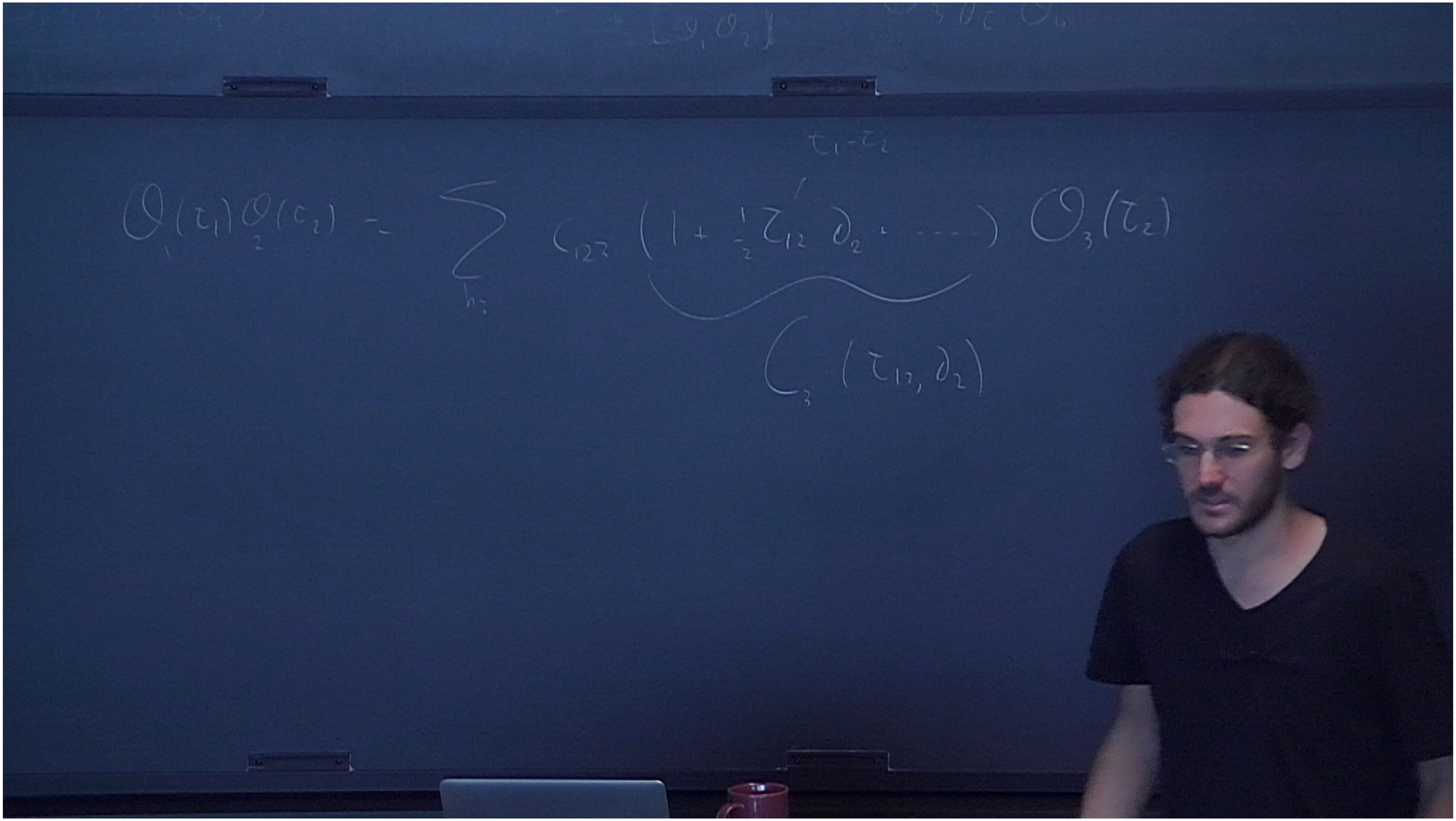
$$\begin{aligned}\langle \mathcal{O}_1(\tau_1)\cdots\mathcal{O}_4(\tau_4)\rangle_s \;&=\; \sum_{h=h_n} c_{12h}c_{34h}\,\mathcal{F}^h_{1234}(x)\\ &+\; \sum_{n=0}^{\infty} -\mathrm{Res}\,\left[\frac{c_{12h}}{c_h}\right]_{h=h_1+h_2+2n}\left[\rho(h)\frac{\Gamma(h)^2}{\Gamma(2h)}\frac{c_{34h}}{c_h}\mathcal{F}^h_{1234}(x)\right]_{h=h_1+h_2+2n}\\ &+\; \sum_{n=0}^{\infty} -\mathrm{Res}\,\left[\frac{c_{34h}}{c_h}\right]_{h=h_3+h_4+2n}\left[\rho(h)\frac{\Gamma(h)^2}{\Gamma(2h)}\frac{c_{12h}}{c_h}\mathcal{F}^h_{1234}(x)\right]_{h=h_3+h_4+2n}\end{aligned}$$

10-point function



$$\langle \mathcal{O}_1(\tau_1)\cdots\mathcal{O}_5(\tau_5)\rangle_s = \int_{\mathcal{C}} \frac{dh_a}{2\pi i}\frac{\rho(h_a)}{c_{h_a}^2}\frac{\Gamma(h_a)^2}{\Gamma(2h_a)} \int_{\mathcal{C}} \frac{dh_b}{2\pi i}\frac{\rho(h_b)}{c_{h_b}^2}\frac{\Gamma(h_b)^2}{\Gamma(2h_b)}\, c_{12\,h_a} c_{h_a\,3\,h_b} c_{h_b\,45}\, \mathcal{F}_{12345}^{h_a,h_b}(x_1,x_2)$$


[O₁, O₂]
τ₁ - τ₂
O₁(τ₁) O₂(τ₂) = Σ_{h₃} C₁₂₃ (1 + ½ τ₁₂ ∂₂ + ...) O₃(τ₂)


$\tau_1 - \tau_2$
$\mathcal{O}_1(\tau_1)\mathcal{O}_2(\tau_2) = \sum_{h_i} C_{123} \left(1 + \frac{1}{2}\tau_{12}\partial_2 + \cdots\right) \mathcal{O}_3(\tau_2)$
$C_3(\tau_{12}, \partial_2)$

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- This analytically continued three-point function had singularities in just the right places.
  There should be a general argument why this occurred.

- Our results apply to all theories in which higher-point correlators are built out of four-point functions joined together.

- Our results apply to all theories in which higher-point correlators are built out of four-point functions joined together.

- Actually, in SYK this is not completely true. There is an additional diagram, 4 ladders joined to a single melon, which we must also be included.

- One point I skipped over is that SYK is not fully a CFT in the infrared, but only ``nearly'' a CFT

The Bulk Dual

We have solved SYK at large N. So we have, in principle, determined the tree level bulk Lagrangian.

The bulk has a tower of fields dual to the primary fermion bilinear O(N) singlets

$$\mathcal{O}_n \sim \frac{1}{N}\sum_{i=1}^{N}\chi_i\partial_\tau^{1+2n}\chi_i \quad \leftrightarrow \quad \phi_n$$

On general grounds,

$$S_{bulk} = \int d^2x\sqrt{g}\left[\frac{1}{2}(\partial\phi_n)^2 + \frac{1}{2}m_n^2\phi_n^2 + \frac{1}{\sqrt{N}}\lambda_{nmk}\,\phi_n\phi_m\phi_k + \frac{1}{N}\left(\lambda^0_{nmkl}\,\phi_n\phi_m\phi_k\phi_l + \lambda^1_{nmkl}\,\partial\phi_n\partial\phi_m\phi_k\phi_l + \ldots\right) + \ldots\right]$$

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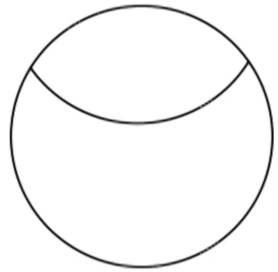
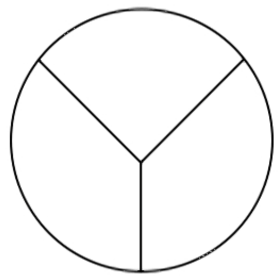
$$\left. + \quad \frac{1}{N}\left(\lambda^0_{nmkl}\,\phi_n\phi_m\phi_k\phi_l + \lambda^1_{nmkl}\,\partial\phi_n\partial\phi_m\phi_k\phi_l + \ldots\right) + \ldots\right]$$

Using this Lagrangian, we compute Witten diagrams to obtain CFT correlators, and fix the coefficients of the Lagrangian so as to match the SYK answers.

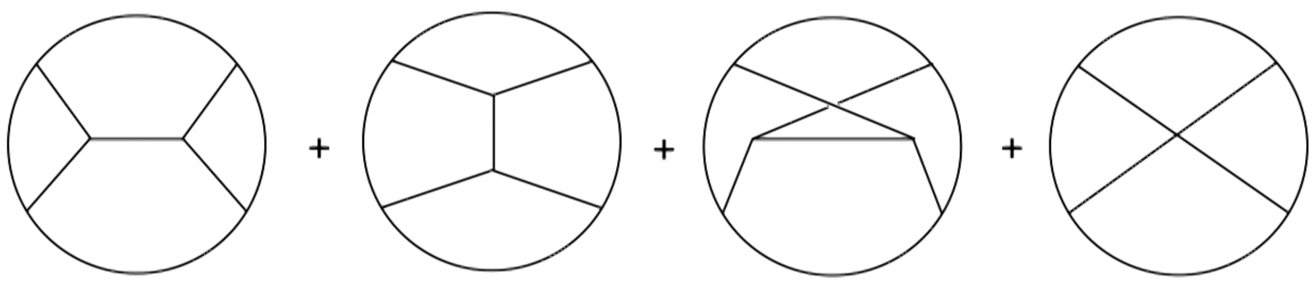
2-pt



3-pt



4-pt



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The bilinear three-point functions for SYK, which determined the cubic couplings, were complicated.

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Universality: the c_{123} are analytic functions of Δ and h_i. Therefore, to the extent that two theories in the SYK family have similar Δ and h_i, they will have similar three-point functions, and by extension, all-point functions.

The high dimension bilinears have small anomalous dimensions,

$$\mathcal{O}_n \sim \frac{1}{N}\sum_{i=1}^{N} \chi_i \partial_\tau^{1+2n} \chi_i$$

$$h_n \approx 2\Delta + 2n + 1 \,, \quad n \gg 1$$

These are the same as the dimensions of the bilinears for cSYK at weak coupling (generalized free field theory of fermions of dimension Δ, in the singlet sector).

For the bilinear 3-point function, and hence the cubic couplings,

$$\lambda_{n_1 n_2 n_3} \approx \frac{N!}{\Gamma(N-2n_1+\frac{1}{2})\Gamma(N-2n_2+\frac{1}{2})\Gamma(N-2n_3+\frac{1}{2})}$$

$$N = n_1 + n_2 + n_3$$

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We have written it in a form that is suggestive of a string bit like interpretation, but we have no concrete statement.

4-point function

$$\langle \mathcal{O}_{n_1}(\tau_1)\dots\mathcal{O}_{n_1}(\tau_4)\rangle \approx \frac{-1}{(\tau_{12}^2\tau_{34}^2)^{n_1}}\left(\frac{(\sqrt{x}+\sqrt{1-x}+1)^4}{(1-x)^2}\right)^{n_1}\frac{1}{(n_1!)^4}$$

$$n_1 \gg 1$$

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$$n_1 \gg 1$$

From preliminary studies of the Mellin transform, this gives bulk quartic couplings that can not be obtained from a local theory. This is not surprising.

The dual of large N $\mathcal{N} = 4$, at large 't Hooft coupling, has:

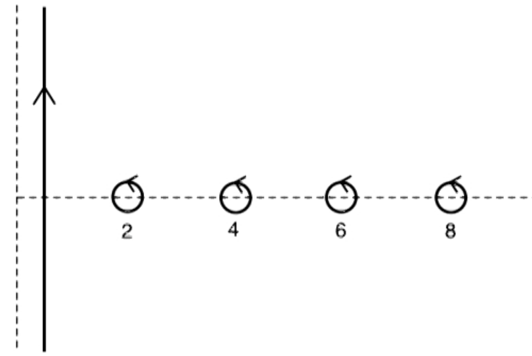
a) Einstein gravity

b) Stringy modes

- The dual of SYK contains dilaton gravity. This is related to the breaking of conformal invariance in the four-point function.

- The dual of cSYK does not contain dilaton gravity. The lowest dimension operator is just dual to the lightest bulk scalar. cSYK is quantum field theory on a fixed AdS background

We wrote the fermion four-point function as:

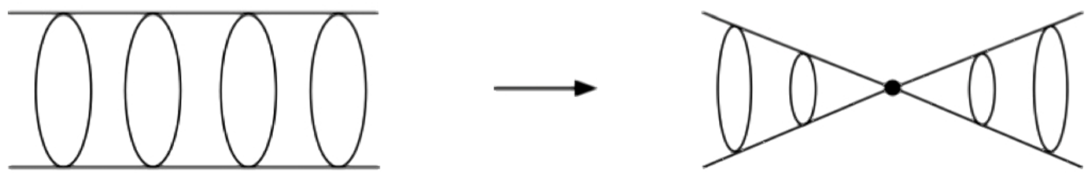


$$\mathcal{F}(\tau_1, \ldots, \tau_4) \;=\; G(\tau_{12})G(\tau_{34}) \int_{\mathcal{C}} \frac{dh}{2\pi i}\, \rho(h)\Psi_h(x)$$

$$\frac{2}{|\tau_{12}|^{2\Delta}|\tau_{34}|^{2\Delta}}\Psi_h(x) = \beta(h,0)\mathcal{F}^h_\Delta(x) + \beta(1-h,0)\mathcal{F}^{1-h}_\Delta(x)$$

The shadow formalism allows us to write,

$$2\, c_h c_{1-h} \frac{b\,\mathrm{sgn}(\tau_{12})\, b\,\mathrm{sgn}(\tau_{34})}{|J\tau_{12}|^{2\Delta}|J\tau_{34}|^{2\Delta}} \Psi_h(x) = \int d\tau_0\, \langle \chi(\tau_1)\chi(\tau_2)\mathcal{O}_h(\tau_0)\rangle \langle \chi(\tau_3)\chi(\tau_4)\mathcal{O}_{1-h}(\tau_0)\rangle$$



Summary

- The SYK fermion two-point, four-point, six-point correlation functions are encoded in $\Delta, \rho(h), \text{and } c_{123}$ These determine all higher-point correlators through simple rules: Feynman like rules for gluing four-point functions.

Summary

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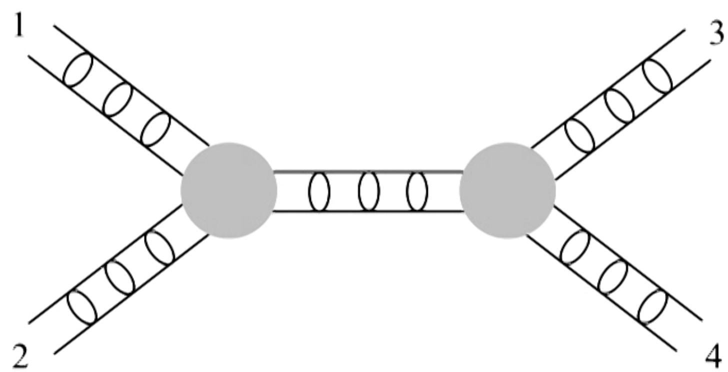

$$\langle \mathcal{O}_1(\tau_1)\cdots\mathcal{O}_4(\tau_4)\rangle_s = \int_{\mathcal{C}} \frac{dh}{2\pi i}\frac{\rho(h)}{c_h^2}\frac{\Gamma(h)^2}{\Gamma(2h)}\, c_{12h}c_{34h}\,\mathcal{F}^h_{1234}(x)$$

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- Details of strongly coupled SYK are in the correlation functions of low dimension operators. Correlation functions of high dimension operators are the same as at weak coupling of cSYK, generalized free field theory of fermions in the singlet sector.

- Bulk: We have given simple expressions for the cubic couplings of the very massive fields, and the four-point functions of the very heavy operators.

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- The next step is to find a theory of extended objects which naturally gives these.