Title: Wonders of viscous electronics in graphene: anomalies, super-ballistic conductance, etc

Date: Nov 16, 2017 02:00 PM

URL: http://pirsa.org/17110044

Abstract: <p>Quantum-critical strongly correlated systems feature universal collision-dominated collective transport. Viscous electronics is an emerging field dealing with systems in which strongly interacting electrons flow like a fluid. I shall describe recent theoretical and experimental results: negative resistance, current vortices, expulsion of electric field, conductance exceeding the fundamental quantum-ballistic limit and other wonders of viscous electronics.</p>

<p>Nature Physics 2016, 2017,  PNAS 2017,  PhysRevLet 2017</p>

# Wonders of viscous electronics

Falkovich Levitov
WIS MIT

Levitov, Falkovich, **Electron viscosity, current vortices and negative nonlocal resistance in graphene.** *Nature Physics* **12** : 672-676 (2016)

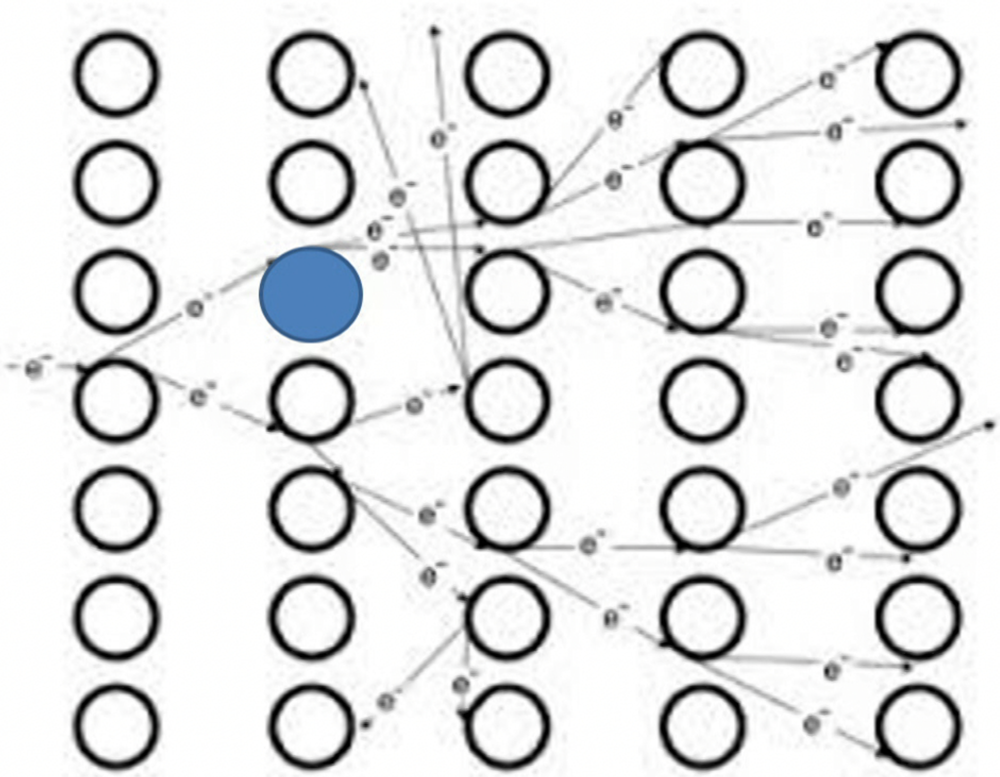
Falkovich, Levitov, **Linking spatial distributions of potential and current in viscous electronics.** *Phys Rev Lett 2017 119, 066601,*

Guo, Ilseven, Falkovich, Levitov, **Higher-than-ballistic conduction of viscous electron flows.** *PNAS 2017* **1607.07269** :

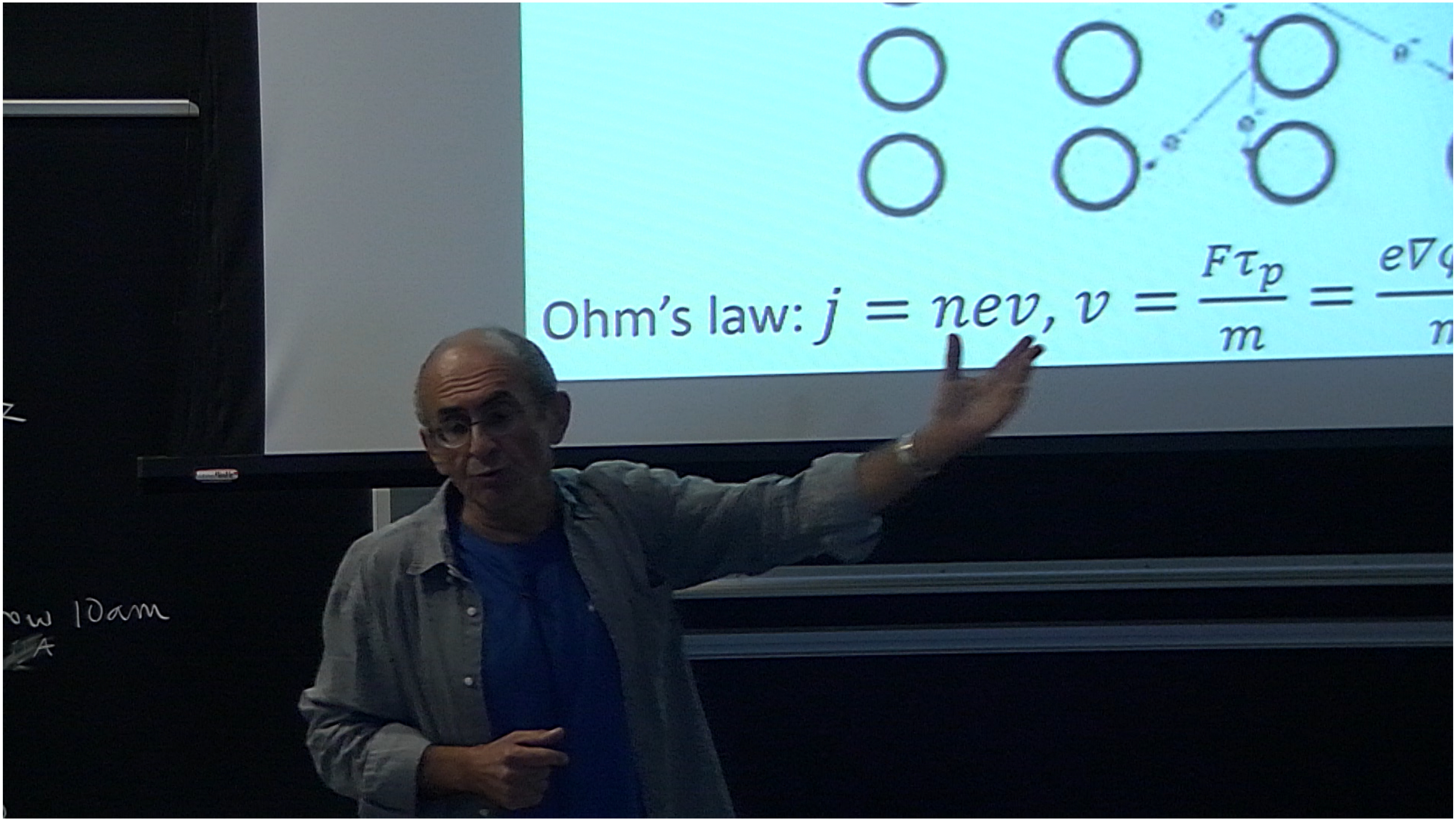
Kumar, Bandurin, Pellegrino, Cao, Principi, Guo, Auton, Ben Shalom, Ponomarenko, Falkovich, Grigorieva, Levitov, Polini, Geim, **Super-ballistic flow of viscous electron fluid through graphene constrictions** *Nature Physics doi:10.1038/nphys4240 August 21, 2017*

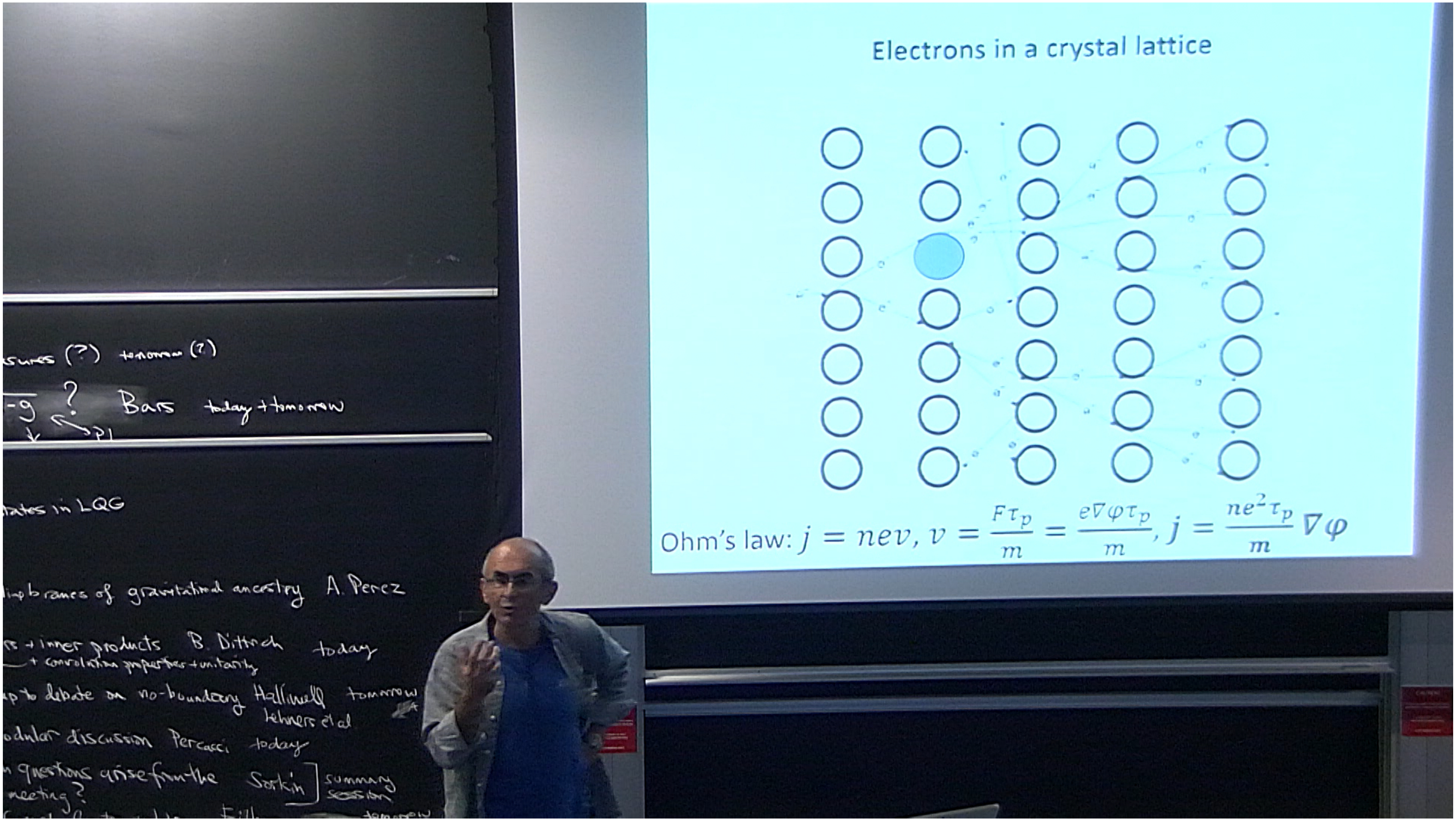
Nov 16, 2017, Perimeter Institute

# Electrons in a crystal lattice



Ohm's law: $j = nev,\ v = \frac{F\tau_p}{m} = \frac{e\nabla\varphi\tau_p}{m},\ \boldsymbol{j} = \frac{\boldsymbol{ne^2\tau_p}}{\boldsymbol{m}}\boldsymbol{\nabla\varphi}$


Ohm's law: $j = nev,\ v = \frac{F\tau_p}{m} =$


Electrons in a crystal lattice
Ohm's law: $j = nev,\ v = \frac{F\tau_p}{m} = \frac{e\nabla\varphi\tau_p}{m},\ j = \frac{ne^2\tau_p}{m}\nabla\varphi$

# Is hydrodynamics ever relevant in metals?

In one-component fluid or gas a hydrodynamic approach works because one has local conservation of energy and momentum. Macroscopic hydrodynamic equations describe propagation of conserved quantities in space.

Electron fluid in a solid can exchange energy and momentum with the lattice. Hydrodynamics not relevant?

If the disorder scattering time $\tau_p$ exceeds the electron-electron scattering time $\tau_{ee} = l/v_F$, then electrons behave as viscous liquid.

High-mobility electron systems (GaAs 2DES, graphene). Non-Fermi liquids, high-Tc superconductors, strange metals.

$\gamma_{\mathrm{ee}}^{-1} \approx 80\,\mathrm{fs}$ $\gamma_{\mathrm{p}}^{-1} \sim 0.5\,\mathrm{ps}$

Fritz, L., Schmalian, J., Müller, M. & Sachdev, S. Quantum critical transport in clean graphene. Phys. Rev. B **78**, 085416 (2008).

Kashuba, A. B. Conductivity of defectless graphene. Phys. Rev. B 78, 085415 (2008).

# Dimensionless coupling constant

In graphene (or any other Dirac material), the strength of electron-electron interactions is controlled by the dimensionless parameter, called "fine structure constant" (because of its analogy with the QED fine structure constant):

$$\alpha_{\rm ee} = \frac{e^2/(\epsilon L)}{\hbar v_{\rm F}/L} = \frac{e^2}{\epsilon \hbar v_{\rm F}}$$

This dimensionless number is:
1) **not** small (of order unity)
2) **not** gate tunable (Fermi wave number drops out)
3) **sensitive** to dielectric environment (the "epsilon factor")

V. N. Kotov *et al.*, Rev. Mod. Phys. **84**, 1067 (2012)

# Kinematic viscosity - diffusivity of momentum

$$\nu \approx v_F^2 \gamma_{ee}^{-1} \approx v_F l_{ee}$$

$$\nu \approx v l$$

typical velocity — mean free path

| | |
|---|---|
| water | $\nu = 0.01\ cm^2 sec^{-1}$ |
| air | $\nu = 0.15\ cm^2 sec^{-1}$ |
| electrons in graphene | $\nu = 1000\ cm^2 sec^{-1}$ |

The main problem:

# WHAT IS THE MOST FUNDAMENTAL MANIFESTATION OF STRONGLY INTERACTING ELECTRON FLOW?

# Hydrodynamic description of transport

$$\gamma_{\mathrm{p}} \ll \gamma_{\mathrm{ee}}$$

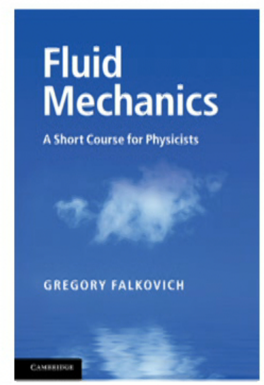
## Linearized Navier-Stokes equation

$$\partial_t v - \nu \nabla^2 v = -\nabla P / mn$$

$$\nu \approx (1/2) v_{\mathrm{F}}^2 \gamma_{\mathrm{ee}}^{-1}$$

$$P = e \int_{n_0}^{n} \Phi(n') \mathrm{d}n'$$

$E_{\mathrm{F}} \gg k_{\mathrm{B}} T$ $\qquad P \approx e(n - n_0)\Phi$

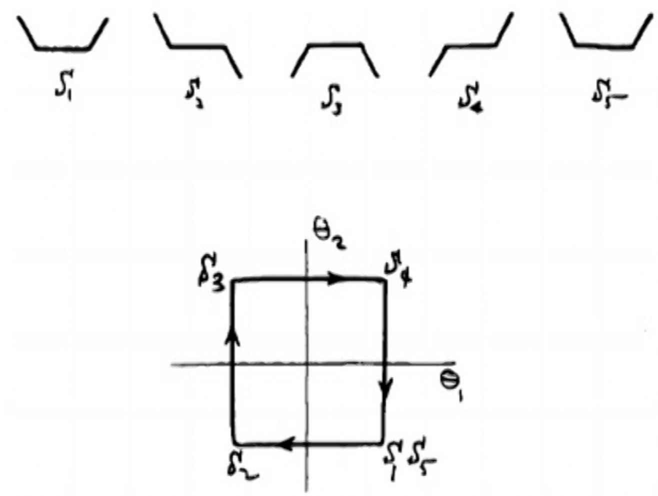


# Life at low Reynolds numbers

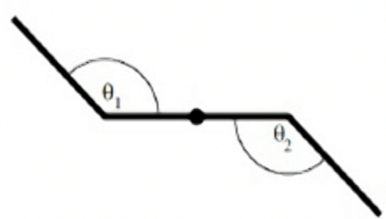
## Aristotelean world

$$Re = vL/\nu << 1 \qquad \eta \Delta v = \nabla P$$

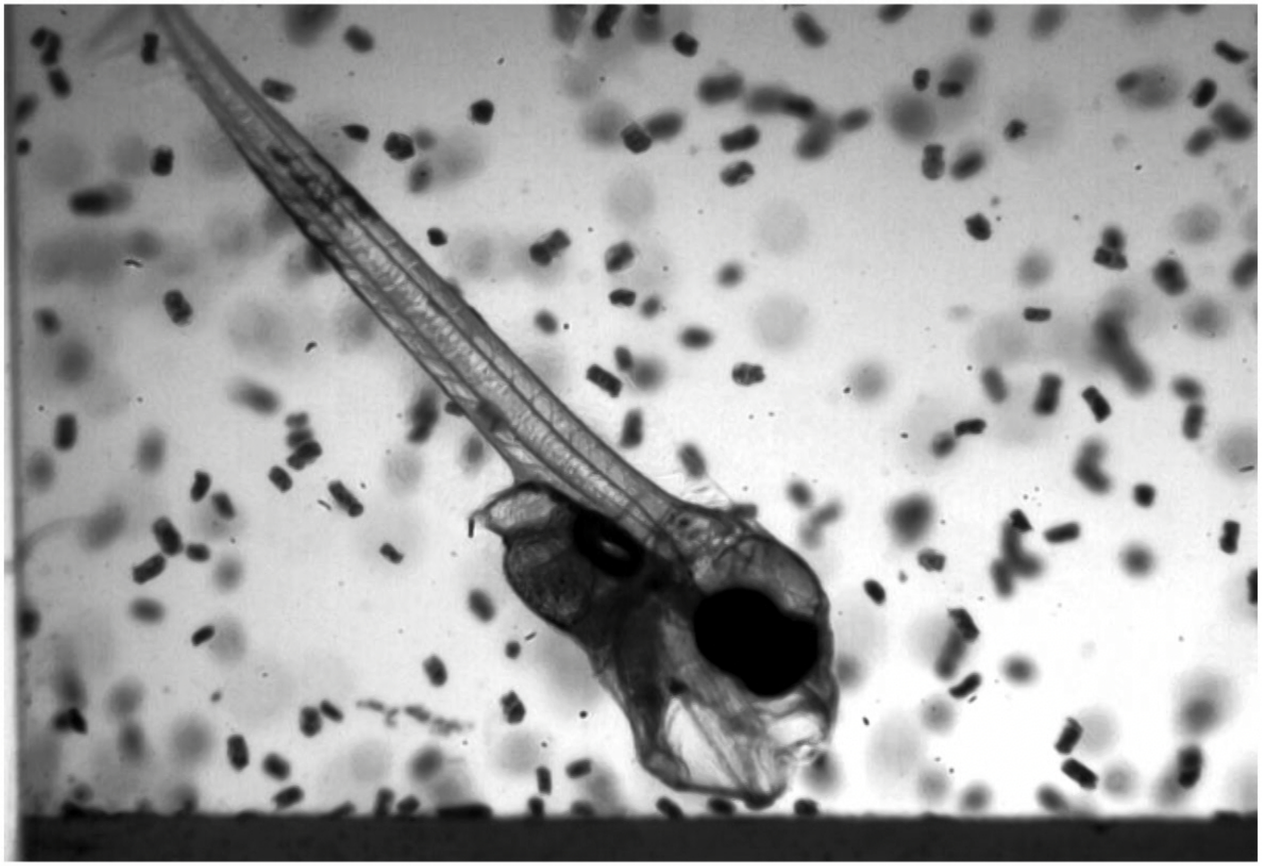
- **Scallop theorem** (Purcell 1977) to achieve pumping or propulsion at low Reynolds number one must deform in a way that is not invariant under time-reversal.
- **Berry phase & non-abelian gauge theory:** Wilczek, Shapere (1989), Geometry of self-propulsion at low *Re*. Avron, Kenneth, Gat (2004).




Swimming (pumping) consists in periodically changing shape to move relative to the fluid



# Life and death at low Reynolds number



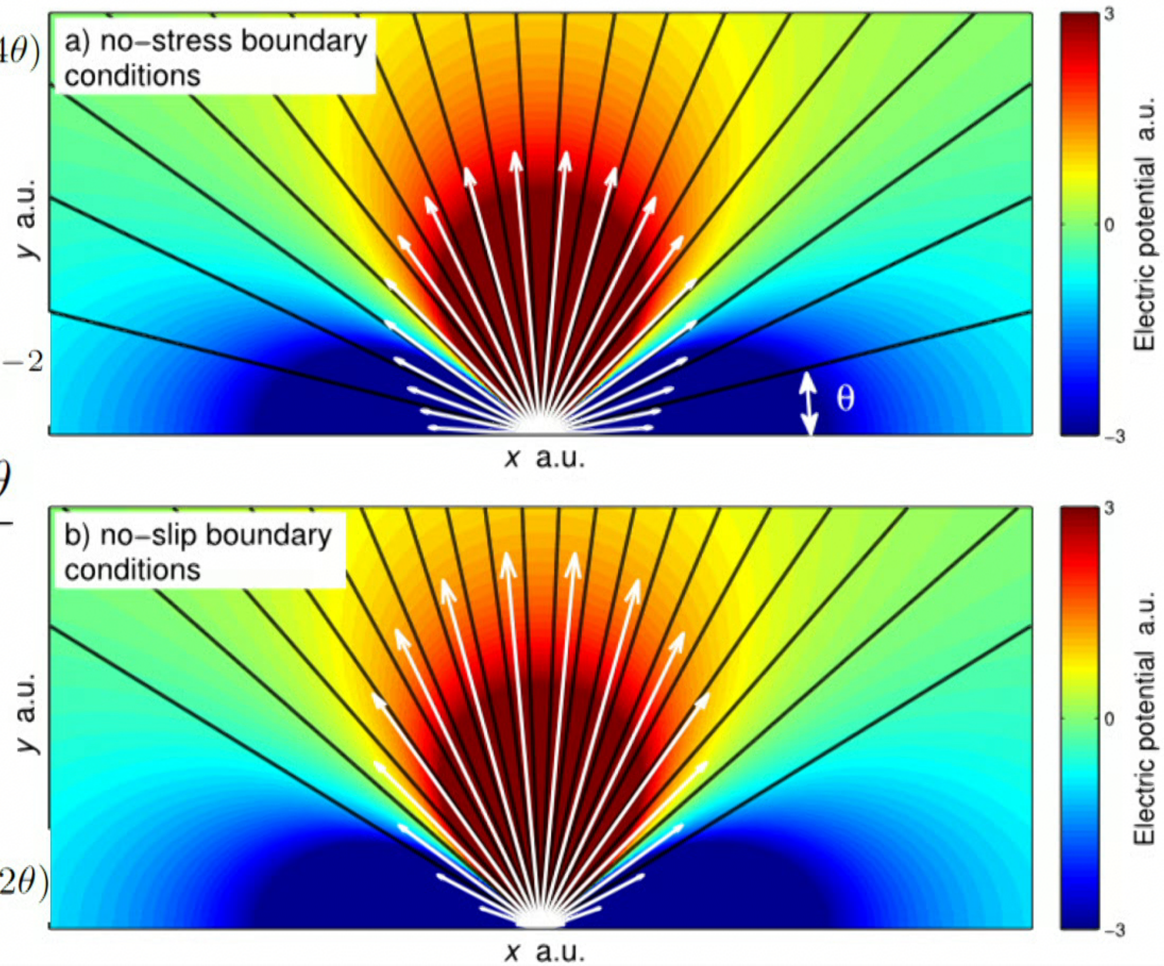
# The simplest non-trivial viscous current flow
## (non-metallic boundary)

$$\psi_1(\theta) = \frac{\tilde{I}}{4\pi}(\sin 2\theta - 4\theta)$$

$$\phi(x,y) = \frac{\tilde{I}\eta}{2ne}\,\mathrm{Re}\, z^{-2}$$

$$= -\frac{\tilde{I}\eta}{2ne}\frac{\cos 2\theta}{r^2}$$

$$\psi_2(\theta) = \frac{\tilde{I}}{2\pi}(\sin 2\theta - 2\theta)$$



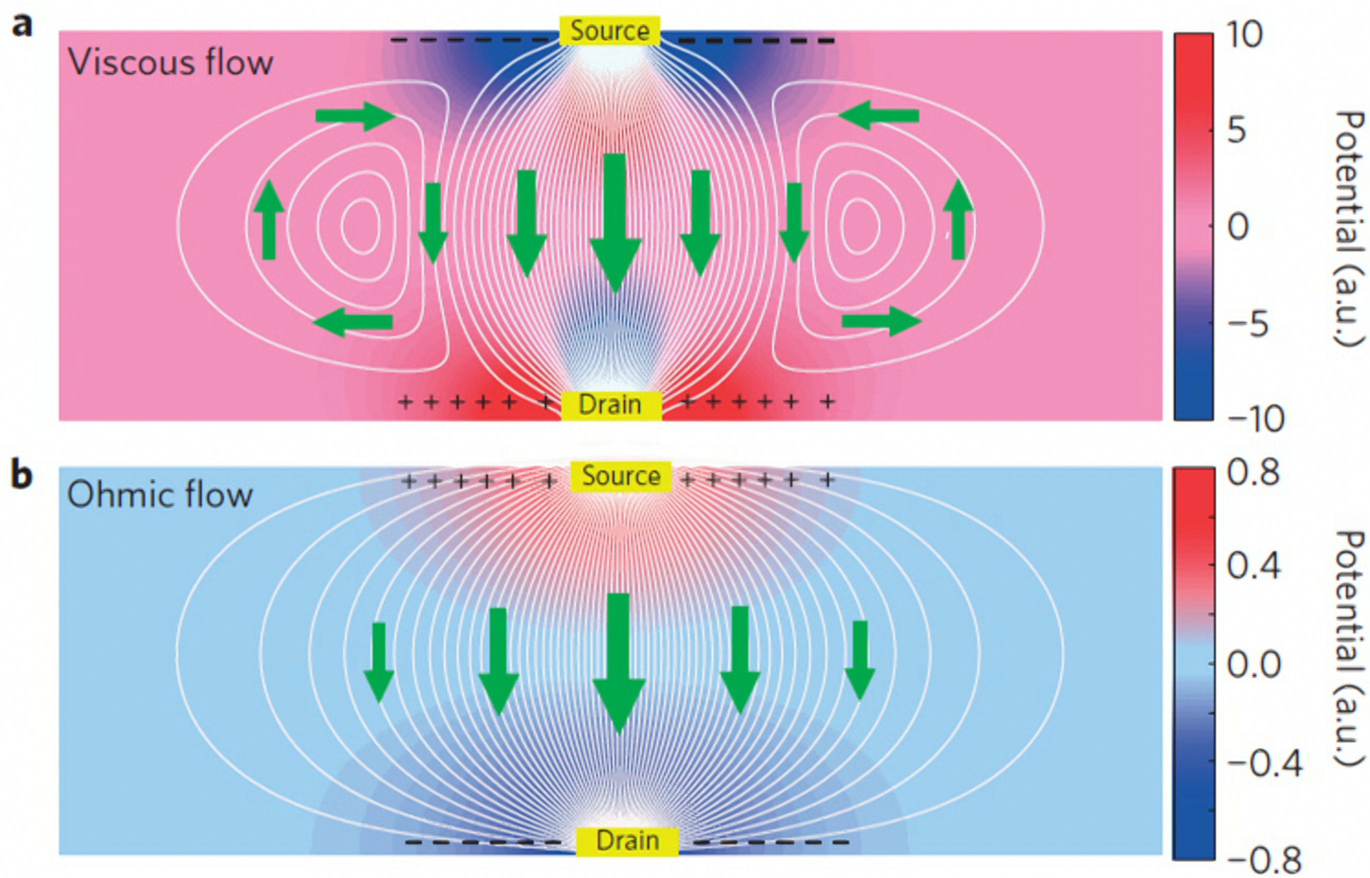



**Figure 1 | Current streamlines and potential map for viscous and ohmic flows.** White lines show current streamlines, colours show electrical potential

# Microfluidic flows and Ohmic-viscous currents

Consider plane viscous flow between two plates separated by *h*.

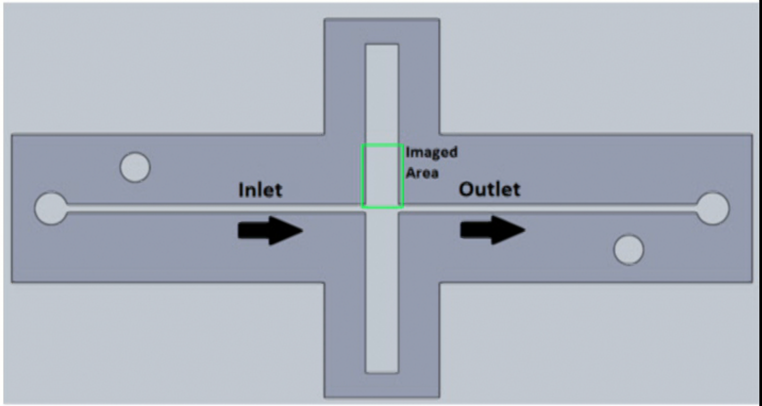
$$\boldsymbol{v}(x,y,z) = 6z(h-z)\boldsymbol{v}(x,y)$$

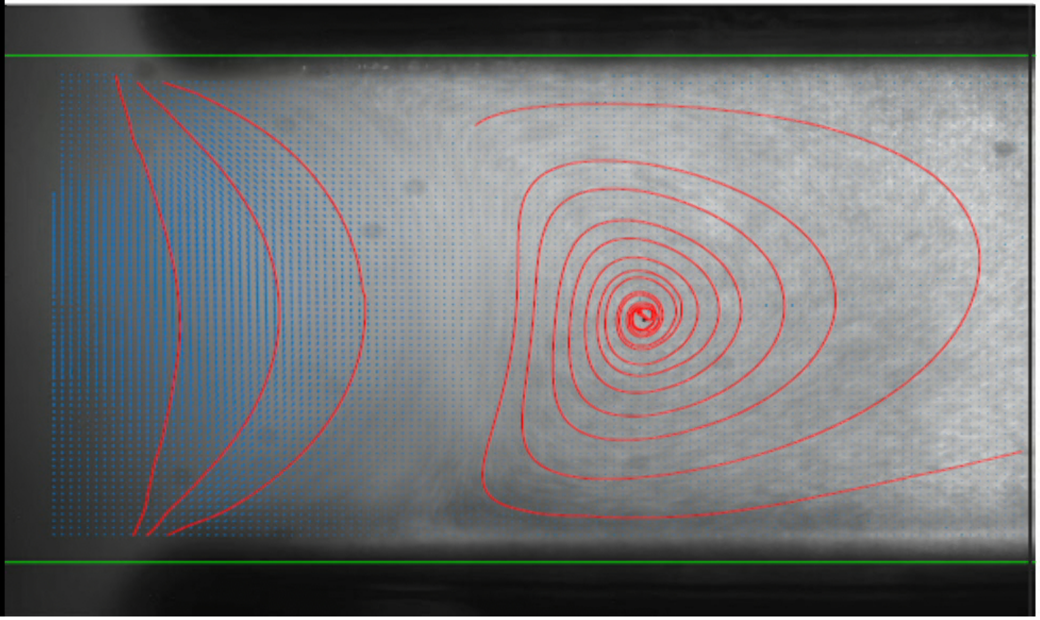

$$\frac{1}{h}\int_0^h dz(\eta\Delta v - \nabla P) = \eta\Delta_\perp v - \frac{12\eta}{h^2}v - \nabla P = 0$$

$$\eta\Delta_\perp v - \rho(ne)^2 v - ne\nabla\varphi = 0$$

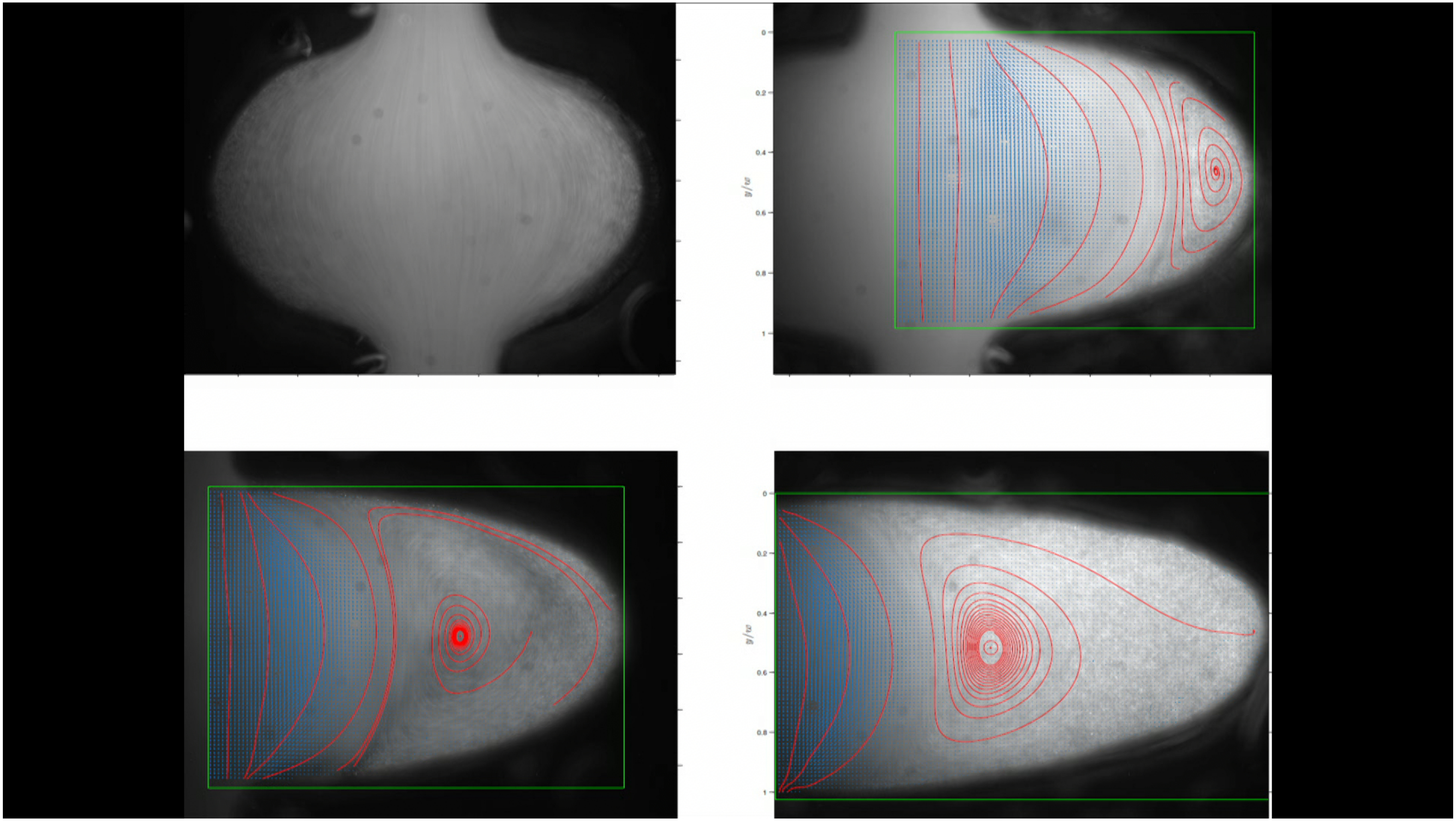
Ohmic resistance

# Microfluidic experiment



Y. Mayzel, MSc Thesis, 2017



y/w
0
0.2
0.4
0.6
0.8
1

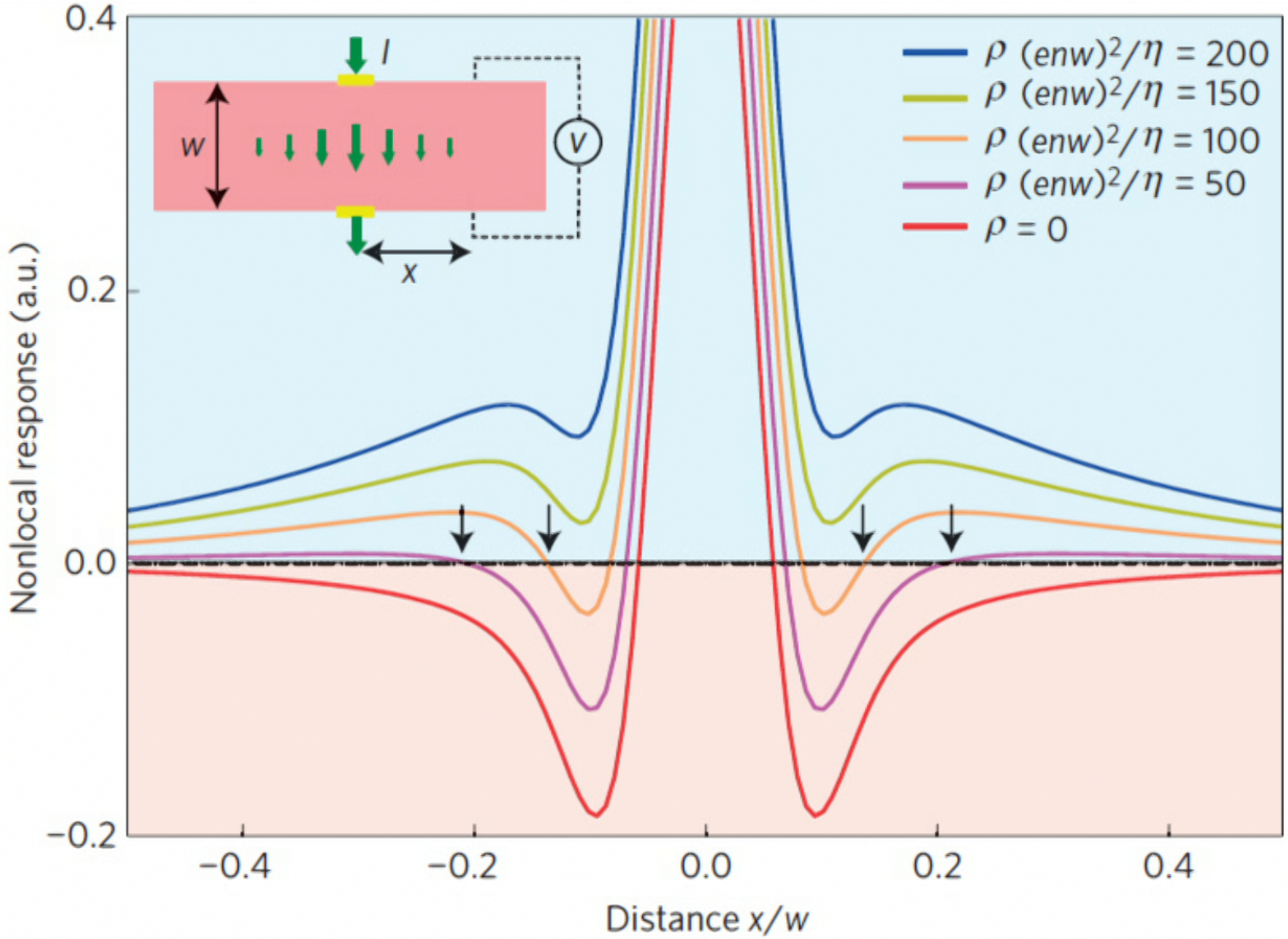


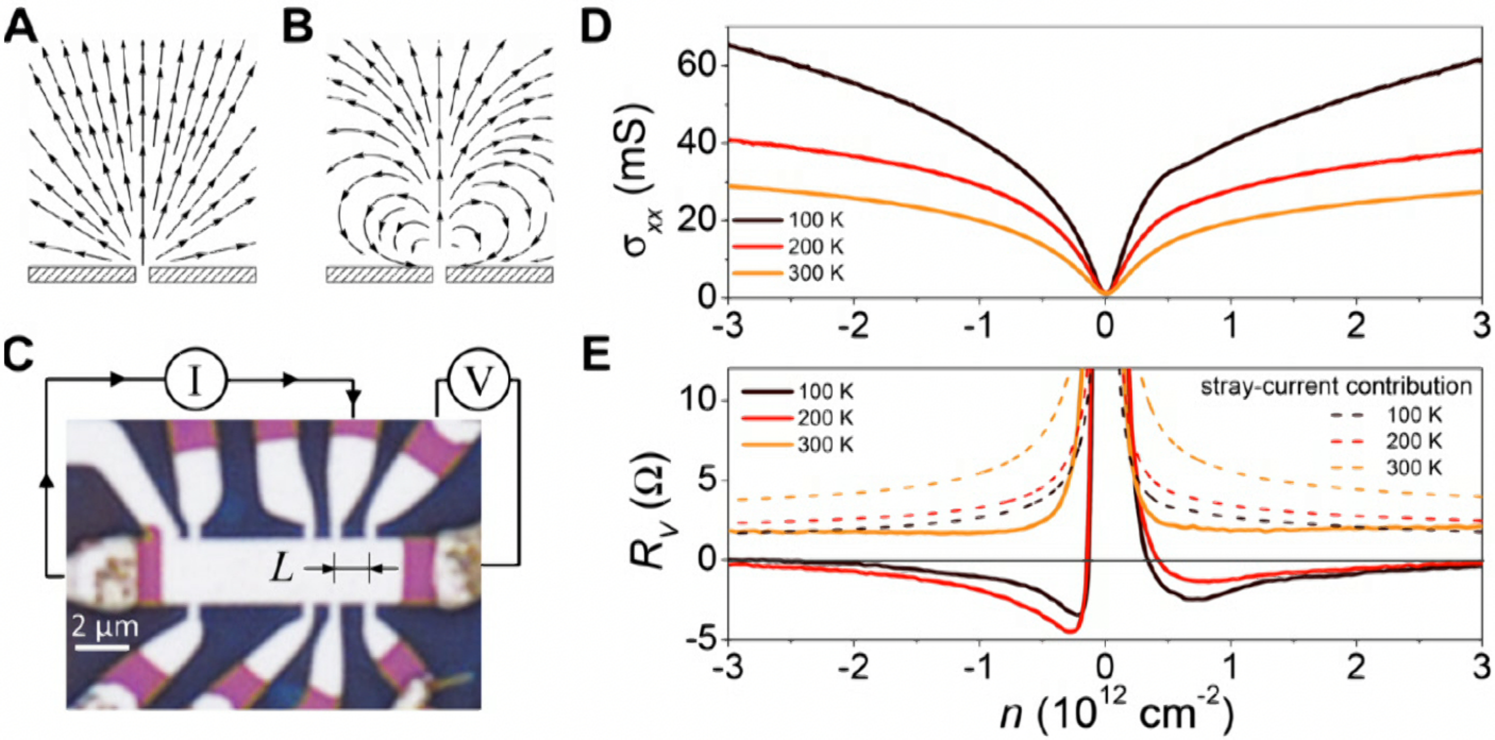

**Figure 2 | Nonlocal response for different resistivity-to-viscosity ratios** $\rho/\eta$

$$\epsilon = \rho(enw)^2/\eta \approx 2\gamma_{ee}\gamma_p(w/v_F)^2$$

# Negative local resistance caused by viscous electron backflow in graphene

Science
11 February 2016

D. A. Bandurin,[1] I. Torre,[2] R. Krishna Kumar,[1,3] M. Ben Shalom,[1,4] A. Tomadin,[5] A. Principi,[6] G. H. Auton,[4] E. Khestanova,[1,4] K. S. Novoselov,[4] I. V. Grigorieva,[1] L. A. Ponomarenko,[1,3] A. K. Geim,[1*] M. Polini[7*]



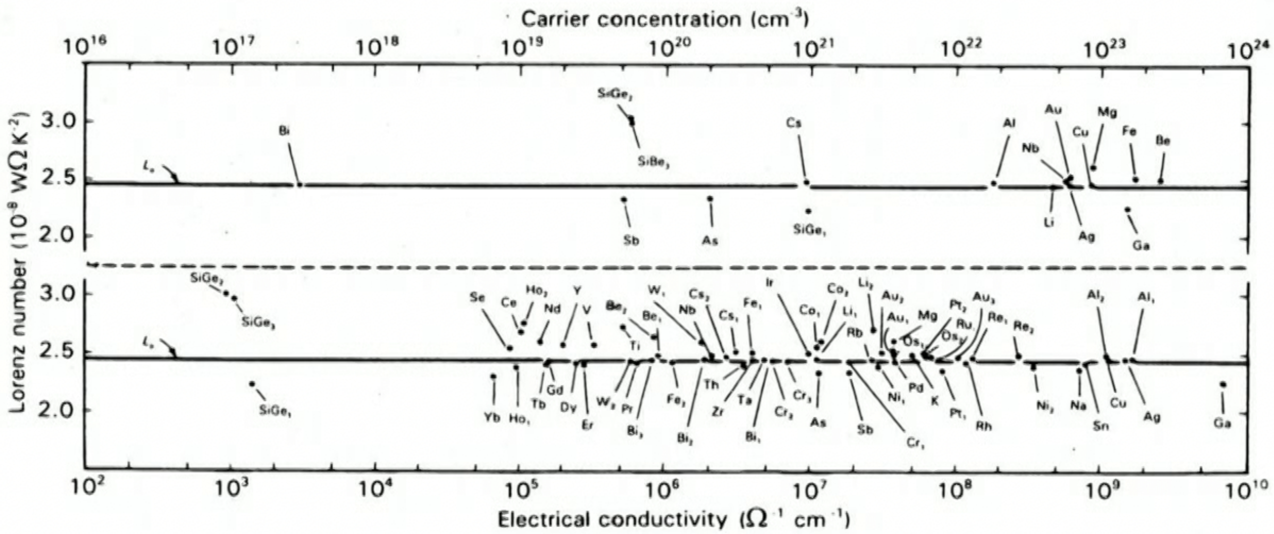

$I$ = 0.3 μA; $L$ = 1 μm; $W$ = 2.5 μm

**Measured viscosity** ≈ 0.1 m² s⁻¹

# Wiedemann-Franz law

$$\frac{\kappa_{WF}}{\sigma_{elec} T} = L_0 = \frac{\pi^2}{3}\left(\frac{k_B}{e}\right)^2$$

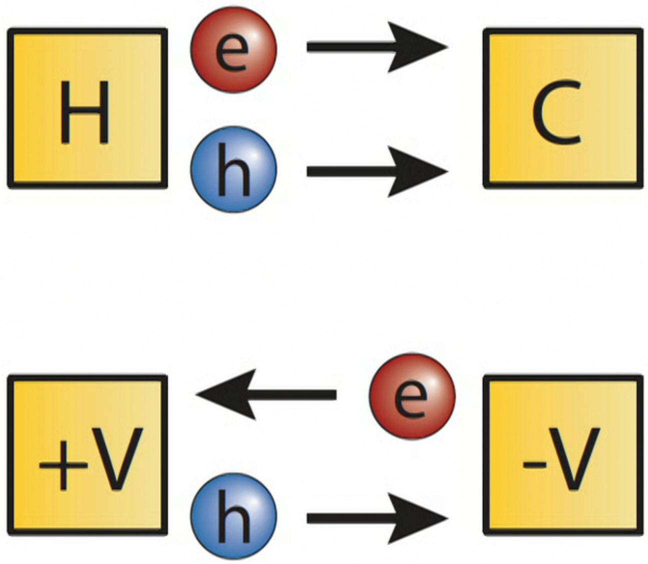
- Independent of density, mass, mean-free-path, scattering time




Kumar, Prasad, Pohl, J. of Materials Sci. 28, 4261 (1993)

kc.fong@bbn.com

# Experimental signature of Dirac fluid



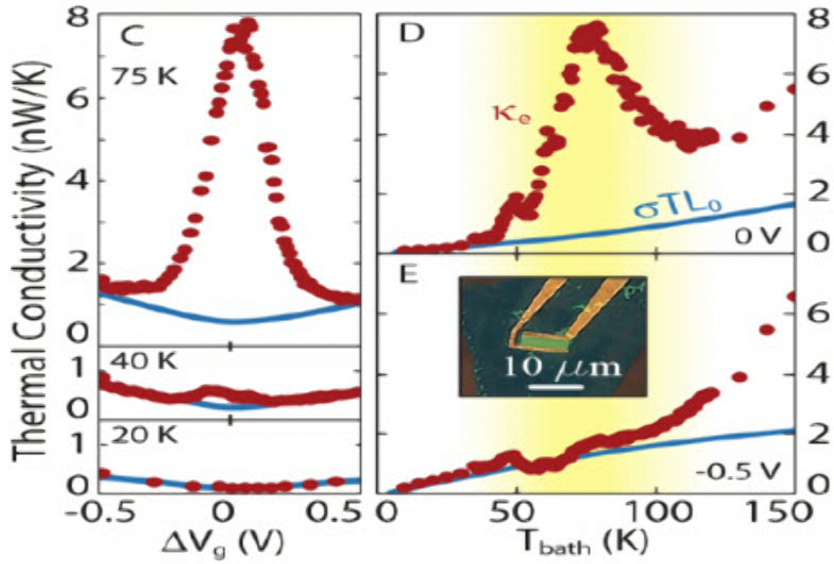

kc.fong@bbn.com

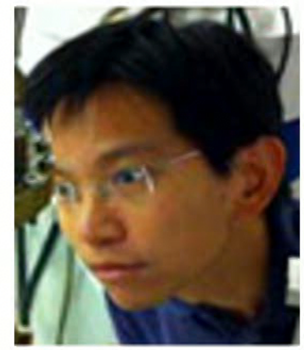
# Observation of the Dirac fluid and the breakdown of the Wiedemann-Franz law in graphene

Science
10.1126/science.aad0343 (2016)

Jesse Crossno,[1,2] Jing K. Shi,[1] Ke Wang,[1] Xiaomeng Liu,[1] Achim Harzheim,[1] Andrew Lucas,[1] Subir Sachdev,[1,3] Philip Kim,[1,2*] Takashi Taniguchi,[4] Kenji Watanabe,[4] Thomas A. Ohki,[5] Kin Chung Fong[5*]

Interactions between particles in quantum many-body systems can lead to collective behavior described by hydrodynamics. One such system is the electron-hole plasma in graphene near the charge neutrality point, which can form a strongly coupled Dirac fluid. This charge neutral plasma of quasi-relativistic fermions is expected to exhibit a substantial enhancement of the thermal conductivity, thanks to decoupling of charge and heat currents within hydrodynamics. Employing high sensitivity Johnson noise thermometry, we report an order of magnitude increase in the thermal conductivity and the breakdown of the Wiedemann-Franz law in the thermally populated charge neutral plasma in graphene. This result is a signature of the Dirac fluid, and constitutes direct evidence of collective motion in a quantum electronic fluid.

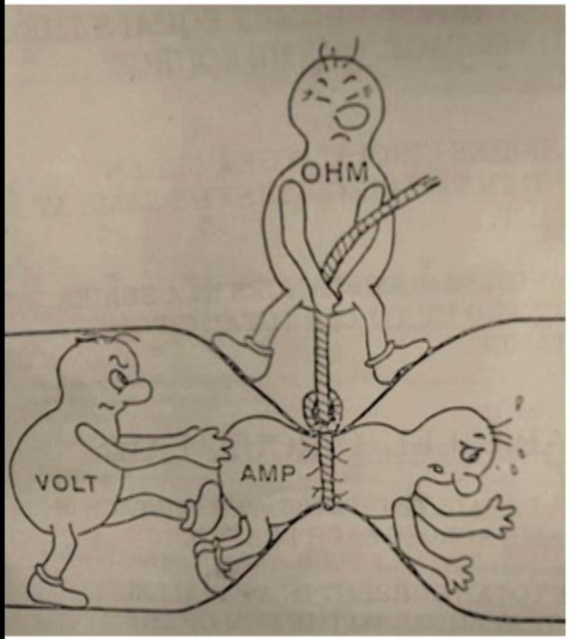


How interaction between carriers of the same sign changes resistance? Does the resistance in the viscous regime always exceed that in the Ohmic regime?

One must distinguish between Ohmic resistance due to scattering on
a) phonons, b) impurities or boundaries.
We consider b).

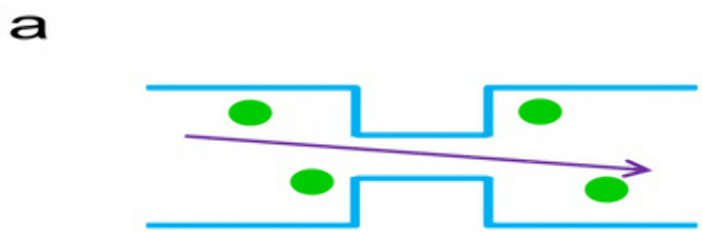
# Exceeding ballistic conductance in viscous flows




ballistic free-fermion

$$R_{\text{ball}} = \frac{h}{2e^2} N^{-1}$$

$$N \approx 2w/\lambda_F$$



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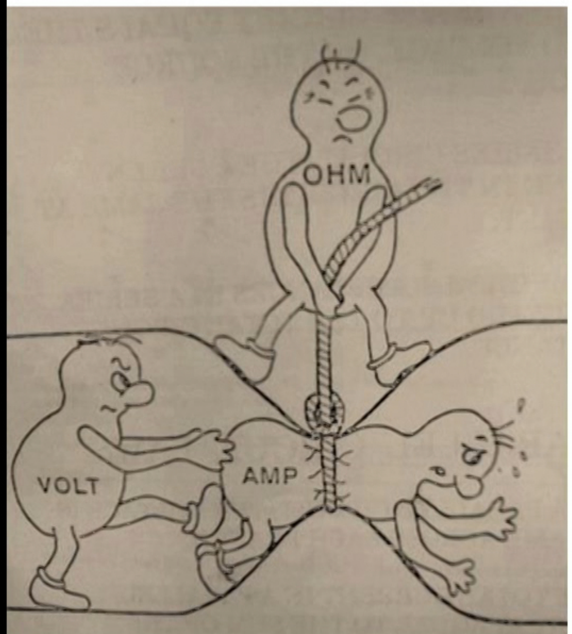


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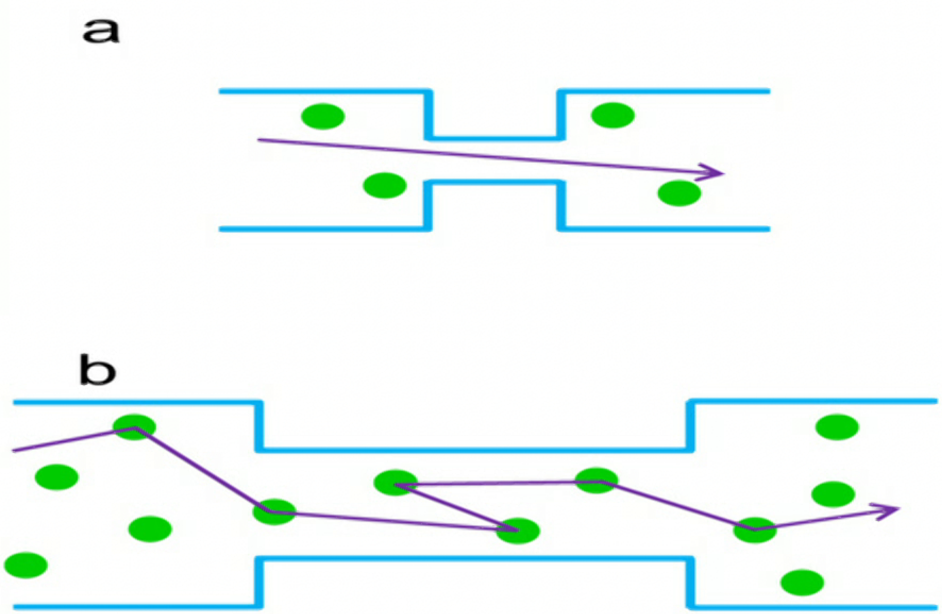


viscous resistance

$$R_{\text{vis}}(w) = \frac{32\eta}{\pi(ne)^2 w^2}$$

# Exceeding ballistic conductance in viscous flows


ballistic free-fermion

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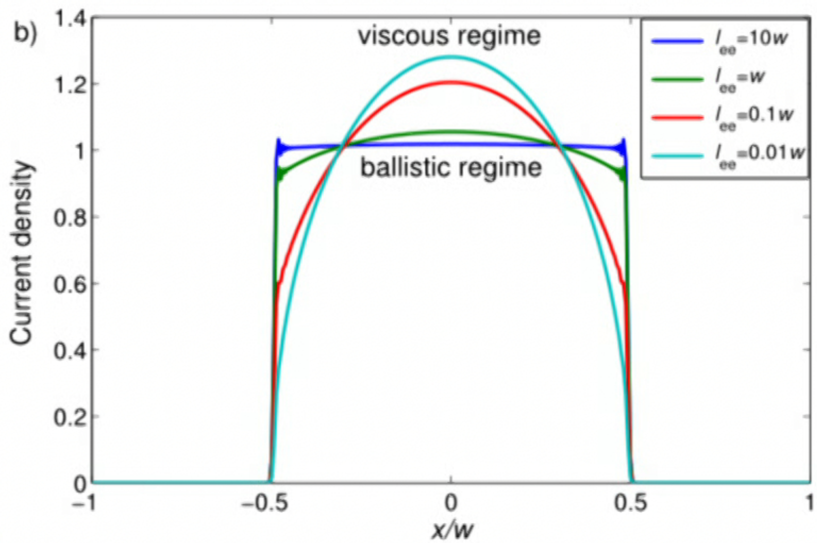
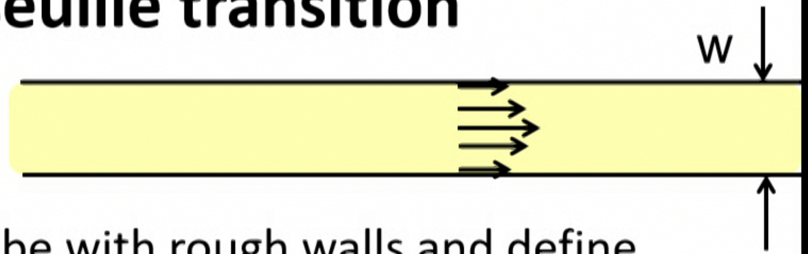



$$R_{\mathrm{vis}}(w) = \frac{32\eta}{\pi (ne)^2 w^2}$$

it is easier for **interacting** electrons to go through the eye of a needle...

# Knudsen-Poiseuille transition

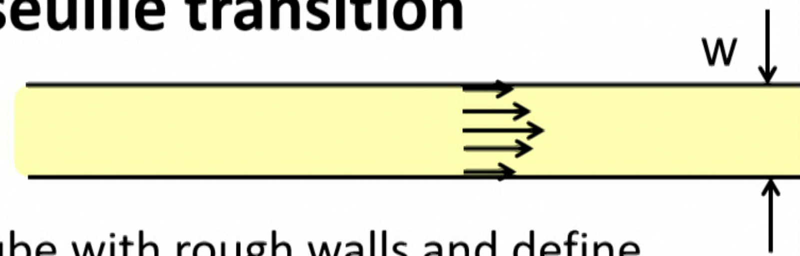



Consider gas flowing through a tube with rough walls and define resistance as force divided by momentum: $R \equiv \frac{1}{\tau} \equiv \frac{\nabla P}{mnU}$, analog $R \equiv \frac{V}{I}$

Knudsen regime w $< l, \tau \sim w/v_T$.

Poiseuille regime w $> l, \tau \sim w^2/\nu \sim w^2/v_T l = \frac{w}{v_T}\frac{w}{l}$

# Knudsen-Poiseuille transition

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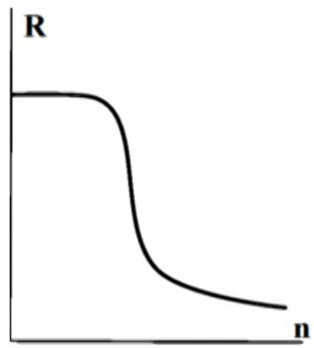

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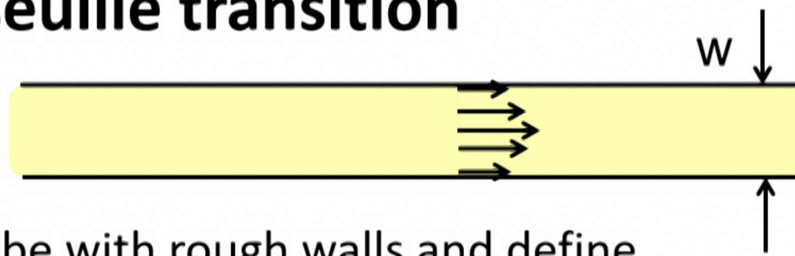


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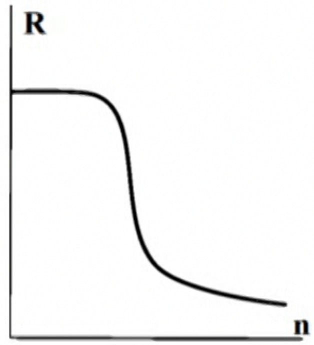
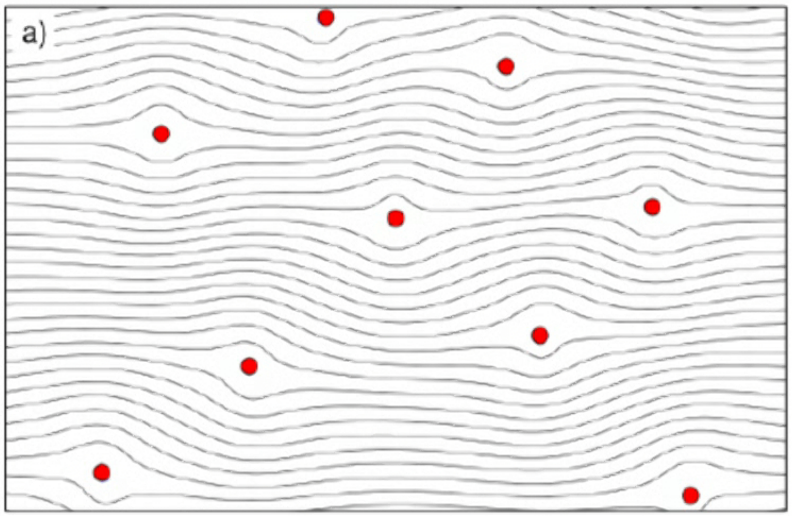
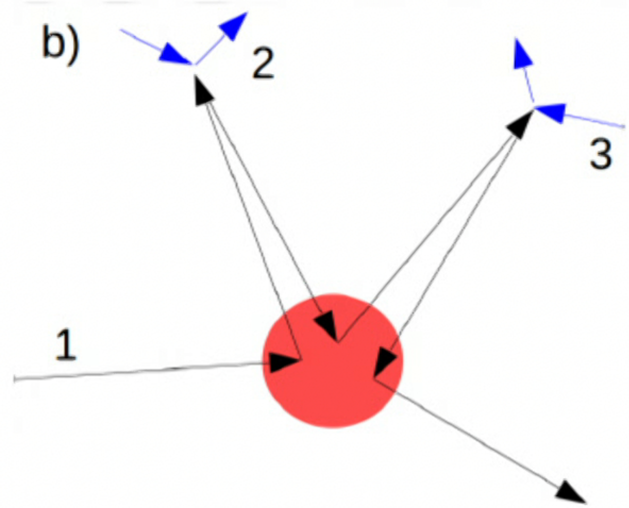


# Stokes Paradox, Back Reflections and Interaction-Enhanced Conduction



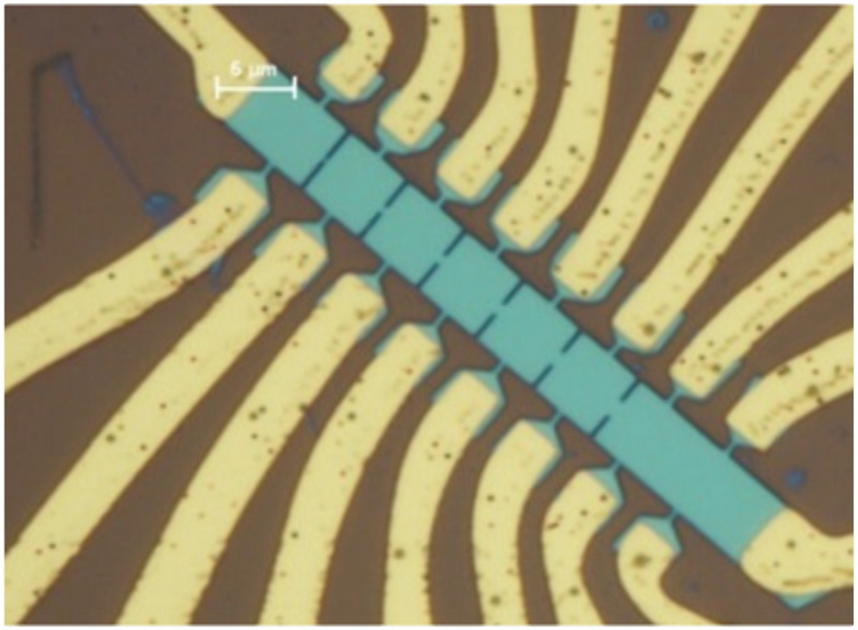

$$\sigma = \frac{Ne^2 v_F^2}{2n_s}\left(\frac{1}{U_0} + \frac{1}{4\pi\nu}\ln\frac{L}{a_*}\right)$$

$$a_* = (a\ell_{\mathrm{ee}})^{1/2}$$

H Guo, E Ilseven, G Falkovich and L Levitov, *PNAS 2017* **1607.07269**

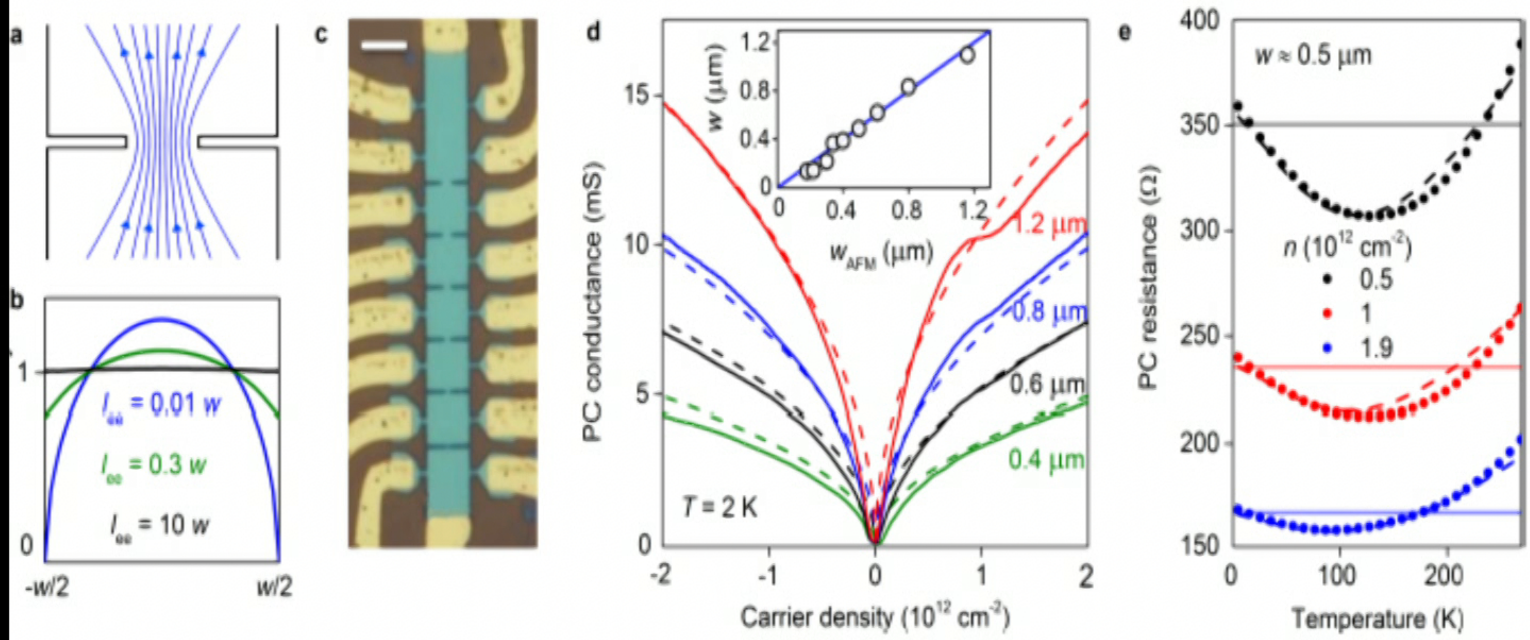
*Arxiv:1607.07269*

# Experiment




R.K. Kumar, D.A. Bandurin, F.M.D. Pellegrino, Y Cao, A Principi, H Guo, G Auton, M Ben Shalom, L.A. Ponomarenko, G. Falkovich, I.V. Grigorieva, L.S. Levitov, M. Polini, and A.K. Geim,
*Nature Physics doi:10.1038/nphys4240 August 21, 2017*

# Experiment




R.K. Kumar, D.A. Bandurin, F.M.D. Pellegrino, Y Cao, A Principi, H Guo, G Auton, M Ben Shalom, L.A. Ponomarenko, G. Falkovich, I.V. Grigorieva, L.S. Levitov, M. Polini, and A.K. Geim,
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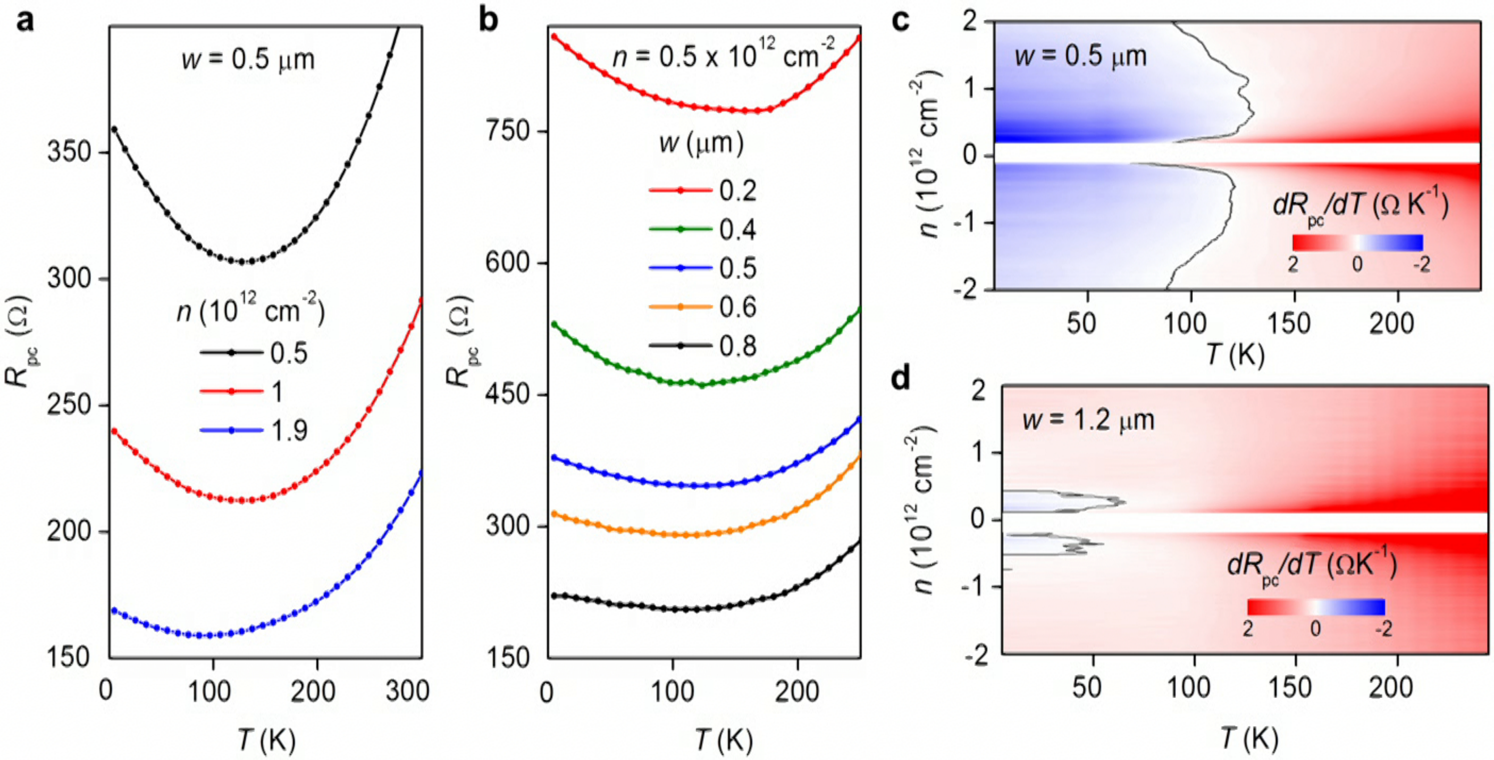
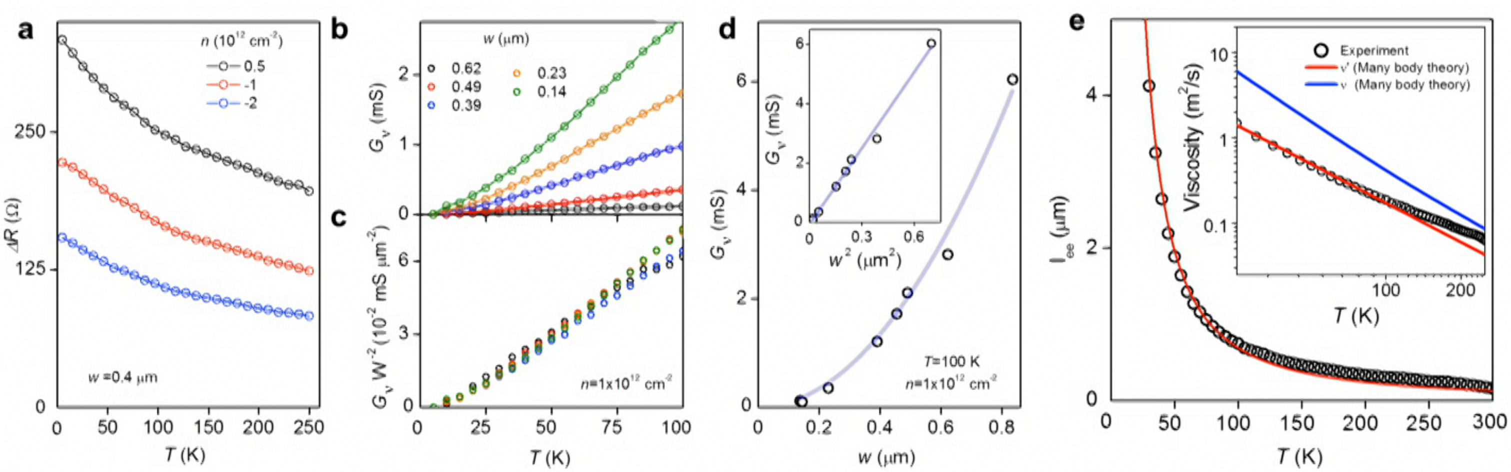
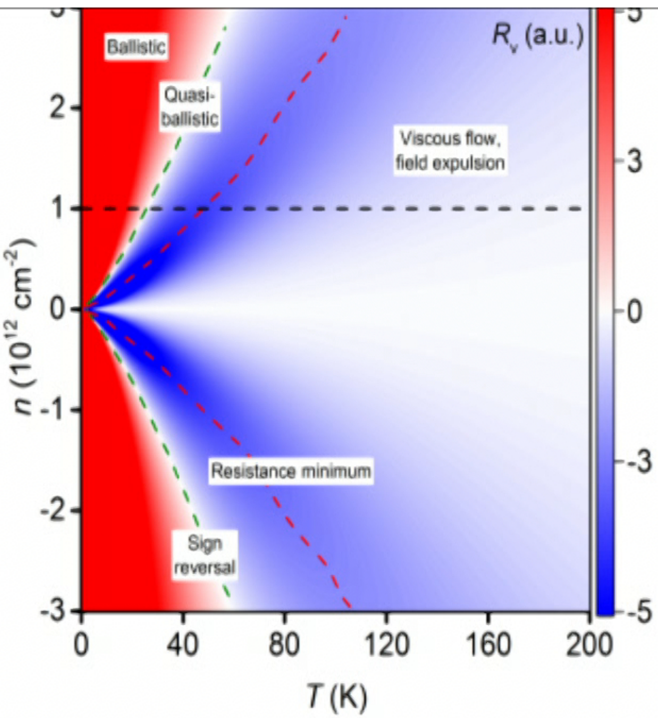
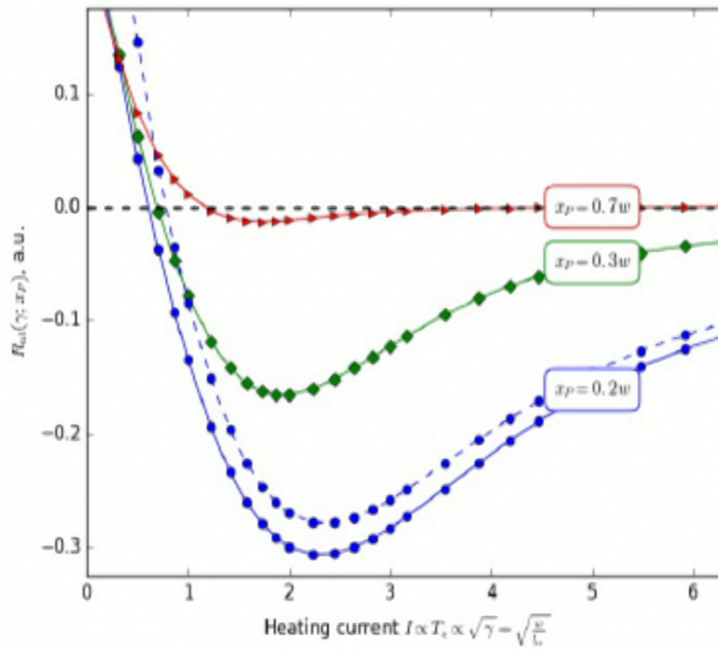


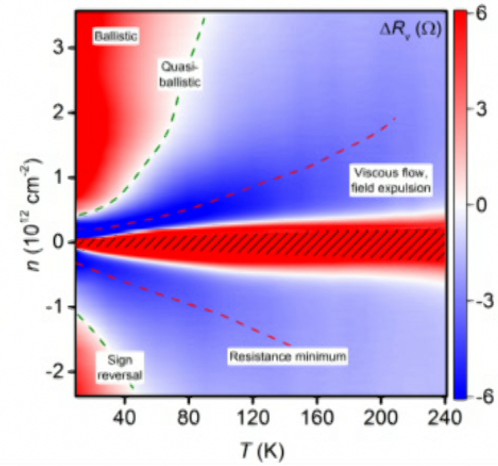

Kumar, Bandurin, Pellegrino, Cao, Principi, Guo, Auton, Ben Shalom, Ponomarenko, Falkovich, Grigorieva, Levitov, Polini, Geim, *Nature Physics* *doi:10.1038/nphys4240 August 21, 2017*

$$R_{\rm pc} = \frac{1}{G_{\rm b} + G_{\nu}} + R_{\rm C}$$






Shytov, Levitov, Falkovich, Kumar, Bandurin, Geim,
*Anomalies in viscous electronics at the onset of fluidity*
*in preparation*

# Purely Ohmic case

Consider incompressible flow $\nabla \cdot \mathbf{j} = ne\nabla \cdot \mathbf{v} = 0$

$$\mathbf{v} = \mathbf{z} \times \nabla\psi = (-\partial_y \psi, \partial_x \psi)$$

$$\mathbf{j} = en\mathbf{v} = -\sigma\nabla\phi$$

$$en\mathbf{z} \times \nabla\psi = -\sigma\nabla\phi$$

Cauchy-Riemann conditions

$$\Delta\psi = \Delta\varphi = 0$$

# Purely viscous case
# Current and potential in viscous electronics

$$\eta \nabla^2 v_i = ne \nabla_i \phi, \quad \nabla_i v_i = 0$$

vorticity $\omega = \nabla \times \mathbf{v} = \mathbf{z} \nabla^2 \psi$ is non-zero

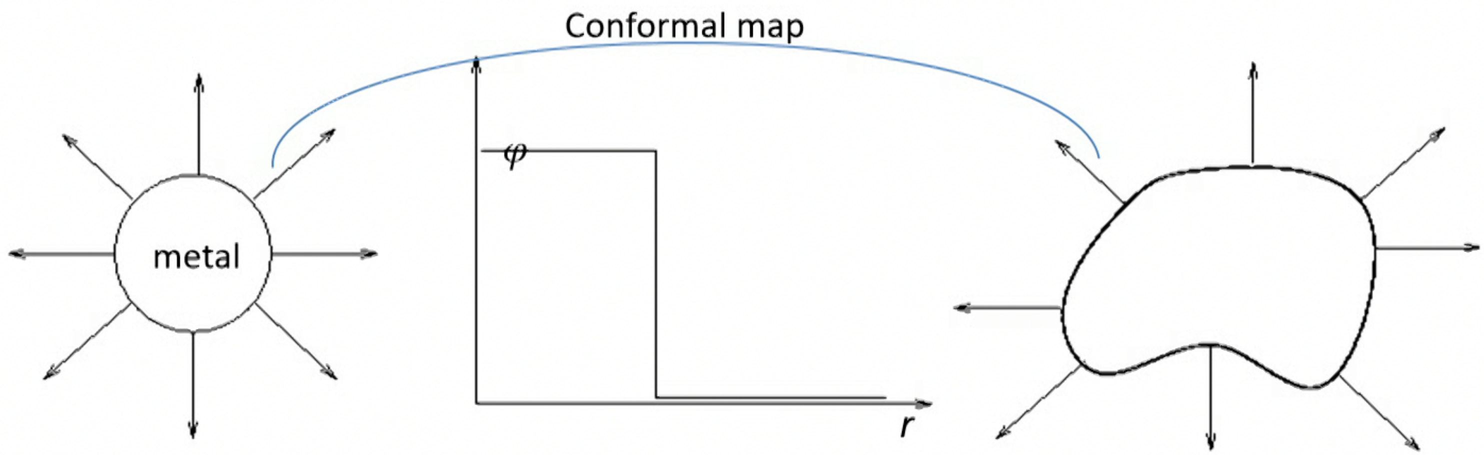
$$\partial_x \omega = (en/\eta) \partial_y \phi, \quad \partial_y \omega = -(en/\eta) \partial_x \phi$$

$$\Delta\omega = \Delta^2\psi = \Delta\varphi = 0$$

# Field expulsion from viscous current flow

$$\omega + i\varphi \ - analytic\ function$$

$$v \propto \frac{1}{r} \rightarrow \Delta v = 0 \rightarrow \omega = 0, \varphi = const$$




Levitov, Shytov, Falkovich, Geim, Bandurin, Kumar… *in preparation*.

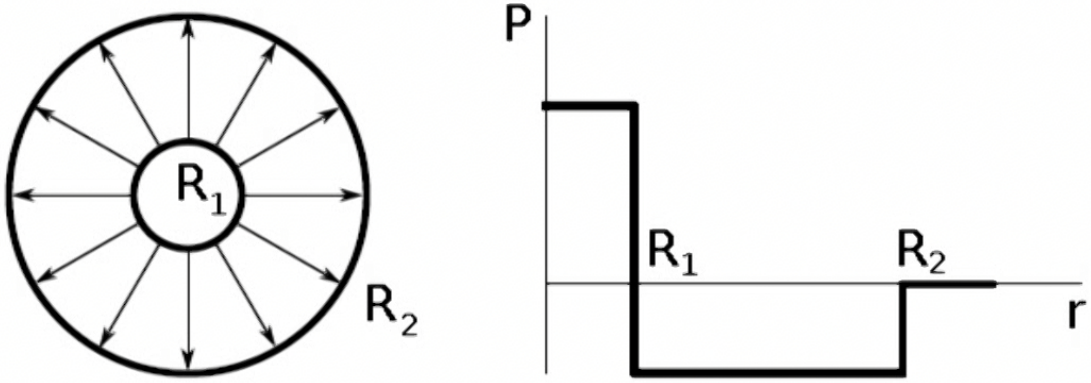
# Freely flowing dissipating viscous flows

$$v \propto \frac{1}{r} \rightarrow \Delta v = 0 \rightarrow \omega = 0, P = const$$

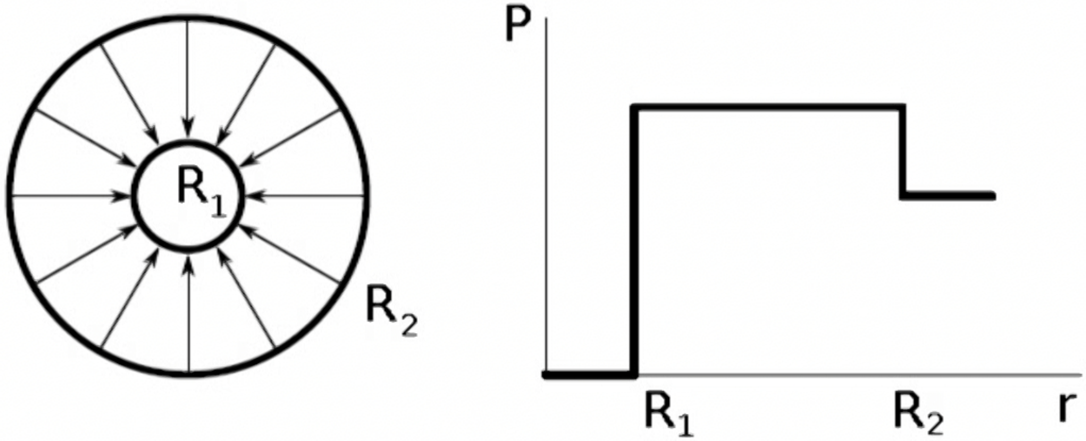
$$\sigma_{rr} = 2\eta dv/dr$$

$$\mathrm{dE/dt} = -\int \sigma_{rr}{}^{2} / 2\eta$$

We need a voltage to do the work compensating this dissipation




$$t \to -t, v \to -v, \nabla P \to -\nabla P$$



# So where was the cheating?

# So where was the cheating?

**If there is no cheating, then it is not theoretical physics but mathematics.**

viscosity makes current-voltage relation nonlocal opening new possibilities; Strongly interacting electrons can flow like a laminar viscous flow and demonstrate negative resistance,

super-ballistic conductance

and other wonders.

Future "viscous electronics" needs *fluid mechanics*