Title: New understandings of unconventional quantum critical points

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Abstract: <p>Quantum critical points (QCP) beyond the Landau-Ginzburg paradigm are often called unconventional QCPs. There are in general two types of unconventional QCP: type I are QCPs between ordered phases that spontaneously break very different symmetries, and type II involve topological (or quasi-topological) phases on at least one side of the QCP. Recently significant progress has been made in understanding (2+1)-dimensional unconventional QCPs, using the recently developed (2+1)d dualities, i.e., seemingly different theories may actually be identical in the infrared limit. One group of dualities between unconventional QCPs have attracted particular interests in the field of condensed matter theory. This group of dualities include the so called deconfined QCP between the Neel and valence bond solid phases, and the topological transition between a bosonic topological insulator and a trivial Mott insulator. Each of the transitions mentioned above is also "self-dual". This group of dualities make extremely powerful predictions for numerical test. We will review the theoretical aspects and most recent numerical evidences for these new results.</p>

New Understandings of unconventional Quantum Critical Points

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New Understandings of unconventional Quantum Critical Points

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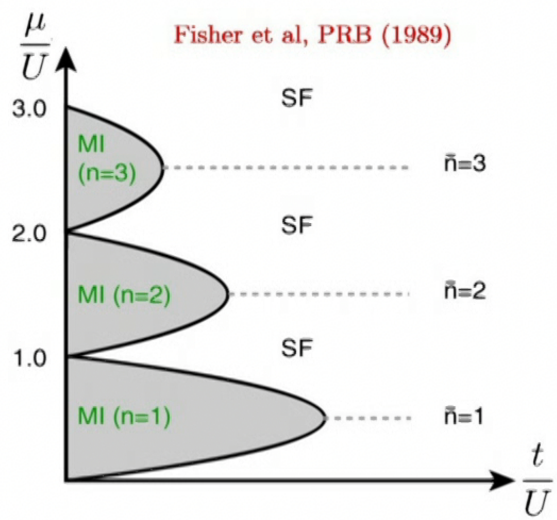
Main references:
PRB, 92, 220416(R) Xu, You
PRB, 95, 205137, Metlitski, Vishwanath, Xu
PRX, 7, 031051, Wang, Nahum, Metlitski, Xu, Senthil
PRX, 7, 031052, Qin, He, You, Lu, Sen, Sandvik, Xu, Meng
PRL, 118, 126801, Bi, Zhang, You, Young, Balents, Liu, Xu

Introduction

Quantum critical point/Continuous quantum phase transition: sandwiched between two ground states of a quantum many body system; gapless, power-law correlation between local operators, etc.

Classic example of QCP: quantum phase transition between superfluid and Bose Mott insulator, realized in cold atom trapped in optical lattice:



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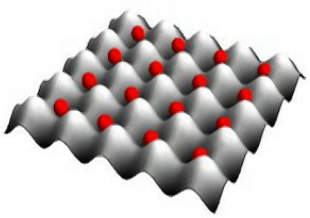
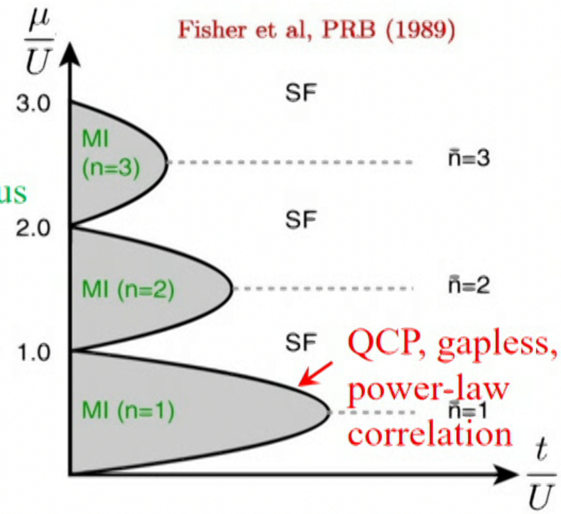
MI: Quantum disordered phase, gapped, no spontaneous symmetry breaking





SF: Quantum ordered phase, gapless (Goldstone), spontaneous U(1) symmetry breaking Long range correlation

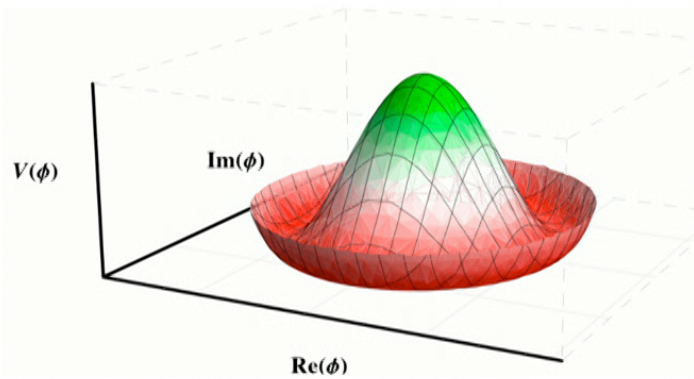
Introduction

What are we interested in about a quantum critical point?

A set of universal numbers called "critical exponents".

Classical phase transition theory: Landau-Ginzburg-Wilson-Fisher (LGWF) paradigm.

Central idea: order parameter and spontaneous symmetry breaking



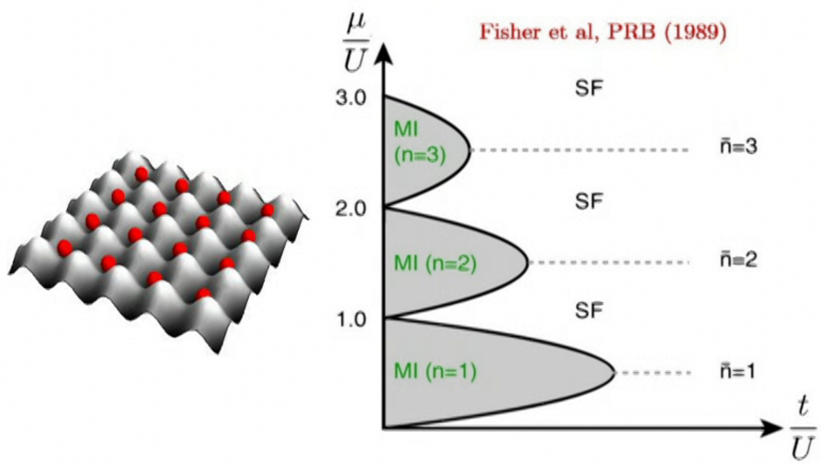
$$f = |\nabla\phi|^2 + r|\phi|^2 + g|\phi|^4$$

$$\xi \sim |r|^{-\nu} \sim |T - T_c|^{-\nu}$$

$$\langle \phi(0)\phi(x)^* \rangle \sim \frac{1}{|x|^{d-2+\eta}}$$

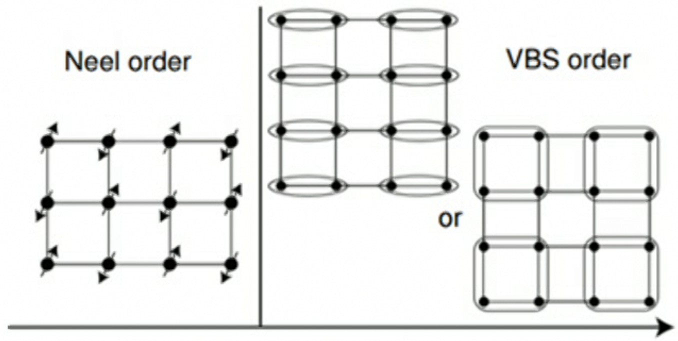
Conventional v.s. Unconventional QCP

Conventional QCP: between a "direct product disordered state" (quantum analogue of classical disordered state) and a spontaneous symmetry breaking state, example: the MI to SF transition. Although the system is quantum, we can use the classical LGWF paradigm in d+1-dimensional space, to describe the d-dimensional conventional QCP.



Conventional v.s. Unconventional QCP

Unconventional QCP:
Type 1, direct 2nd order transition between two ordered phases that spontaneously break two totally different symmetries. (deconfined QCP, Senthil, Vishwanath, Balents, Sachdev, Fisher, 2004)



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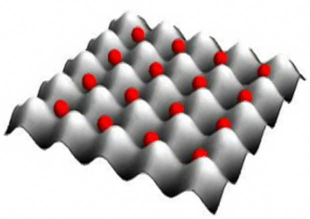

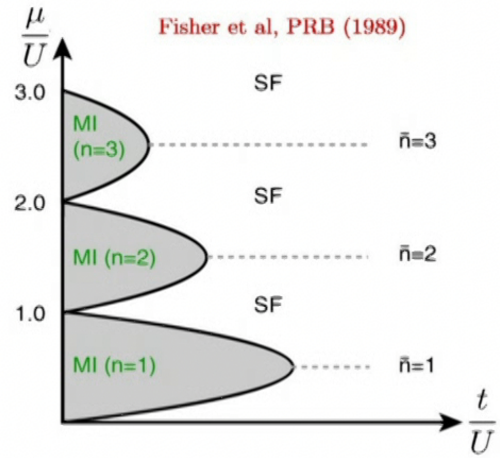

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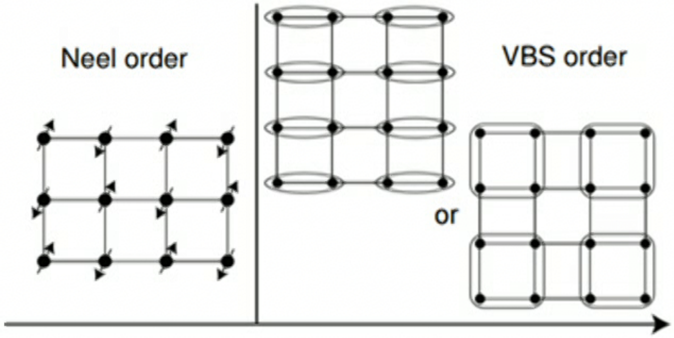

Type 2: transition that involves (quasi) topological phase (quantum disordered non-direct product state) on at least one side of the transitions.

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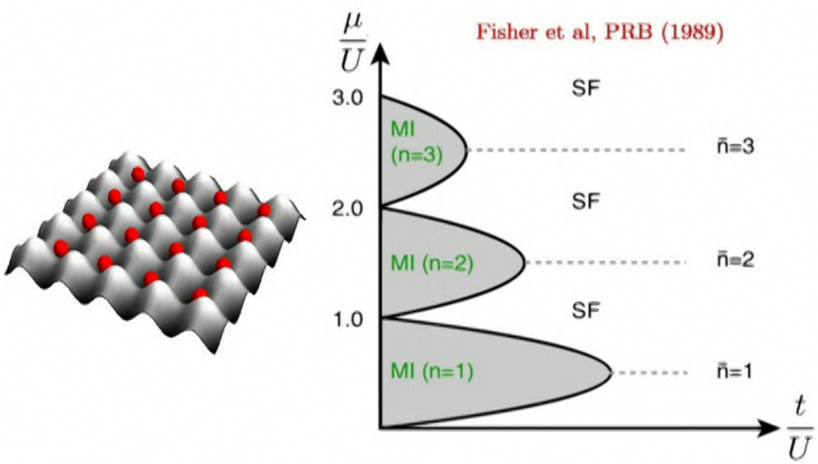

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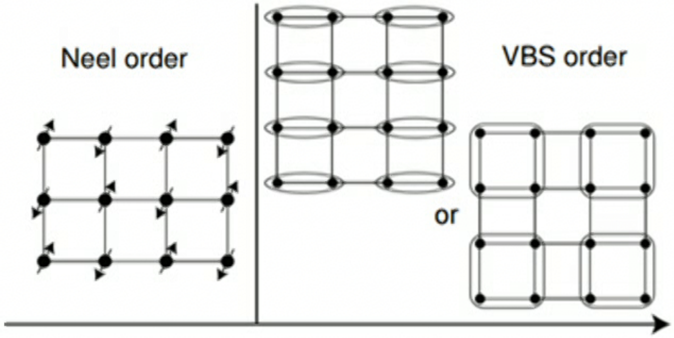

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Elementary Dualities

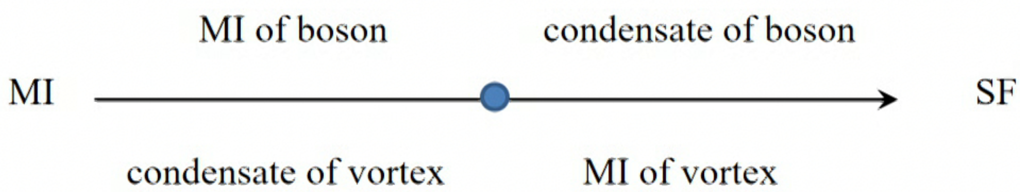
Challenge of studying 2+1d QCPs:

Standard methods of studying 2+1d QCPs are, $1/N$ expansion, epsilon expansion, both have difficulty of convergence for 2+1d QCPs.

A powerful nonperturbative method: duality

Duality maps an unknown problem to a (hopefully) known problem.

Classic example: Particle-Vortex duality (Peskin, 1978, Dastupta, Halperin, 1981, Fisher, Lee, 1989):



Elementary Dualities

Classic example: Boson-Vortex duality (Peskin, 1978, Dastupta, Halperin, 1981, Fisher, Lee, 1989):

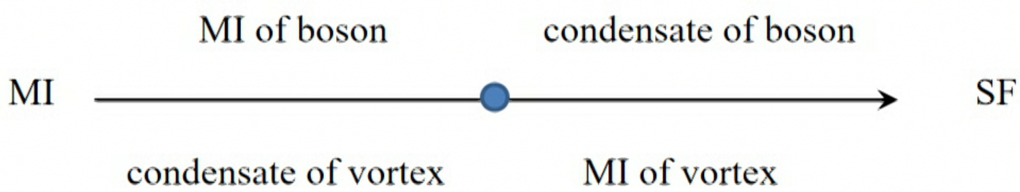
A 2d (2+1d) superfluid phase is dual to a photon phase, gapless Goldstone mode is dual to the photon excitation.

How do we describe a vortex in a superfluid? It is surrounded with nonlocal superfluid current pattern, but in 2d (2+1d), it can be mapped to a point charge coupled to an electric field (gauge field).

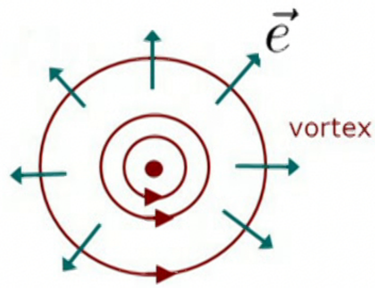
Elementary Dualities

Classic example: Boson-Vortex duality (Peskin, 1978, Dastupta, Halperin, 1981, Fisher, Lee, 1989):

$$\mathcal{L} = |\partial_\mu \Phi|^2 + r|\Phi|^2 + g|\Phi|^4$$



$$\mathcal{L} = |(\partial_\mu - i a_\mu)\tilde{\Phi}|^2 - r|\tilde{\Phi}|^2 + g|\tilde{\Phi}|^4$$



Elementary Dualities

Example 2: "bosonization of" 2+1d Dirac fermion (Chen, Wu, Fisher 1993, Hsin, et.al. 2016; connection with SUSY duality, Kachru, et.al. 2016; Lattice duality: Chen, et.al. 2017):

$$\mathcal{L} = |(\partial_\mu - i a_\mu)\Phi|^2 + r|\Phi|^2 + g|\Phi|^4 + \frac{i}{4\pi} a \wedge da - \frac{i}{2\pi} a \wedge dA$$

$r < 0$ Trivial insu. Integer QH $r > 0$

$m < 0$ $m > 0$

$$\mathcal{L} = \bar{\psi}\gamma_\mu(\partial_\mu - iA_\mu)\psi + m\bar{\psi}\psi + \bar{\Psi}\gamma_\mu(\partial_\mu - iA_\mu)\Psi + M\bar{\Psi}\Psi$$

Spiritually analogous to bosonization/fermionization in 1+1d (such as 1d quantum Ising model): the bound state between a boson/spin and a "kink" is a fermion.

Elementary Dualities

Example 3: fermionic particle-vortex duality: (Son, 2015, Metlitski, Vishwanath, 2015, Wang, Senthil, 2015, Seiberg, et.al. 2016)

$$\mathcal{L} = \bar{\chi}\gamma_\mu(\partial_\mu - iA_\mu)\chi + m\bar{\chi}\chi$$

$m < 0$ +1/2 QH -1/2 QH $m > 0$

$\tilde{m} > 0$ $\tilde{m} < 0$

$$\mathcal{L} = \bar{\psi}\gamma_\mu(\partial_\mu - ia_\mu)\psi + \frac{i}{4\pi} a \wedge dA + \tilde{m}\bar{\psi}\psi$$

These elementary dualities can lead to a large web of dualities, some of these dualities involve unconventional QCPs that are of great interest to CMT, and much easier to test numerically.

Self-dual N=2 QED3

Restating the conjecture: the N=1 QED3 flows to an IR fixed point, which is equivalent to a noninteracting Dirac fermion.

$$\mathcal{L} = \bar{\chi}\gamma_\mu(\partial_\mu - iA_\mu)\chi$$

$$\leftrightarrow \ \mathcal{L} = \bar{\psi}\gamma_\mu(\partial_\mu - ia_\mu)\psi + \frac{i}{4\pi} a \wedge dA$$

Assuming this is true, we can derive the following descendant duality: The N=2 QED3, if it is a CFT, is self-dual, Xu, You, arXiv:1510.06032

$$\mathcal{L} = \sum_{j=1}^{2} \bar{\chi}_j\gamma_\mu(\partial_\mu - ia_\mu)\chi_j + iA_\mu\bar{\chi}\gamma_\mu\tau^z\chi + \frac{i}{2\pi} a \wedge dB$$

$$\leftrightarrow \ \mathcal{L} = \sum_{j=1}^{2} \bar{\psi}_j\gamma_\mu(\partial_\mu - ib_\mu)\psi_j + iB_\mu\bar{\psi}\gamma_\mu\tau^z\psi + \frac{i}{2\pi} b \wedge dA$$

Self-dual N=2 QED3

Deriving the self-duality (Xu, You, arXiv:1510.06032):

$$\mathcal{L} = \sum_{j=1}^{2} \bar{\chi}_j \gamma_\mu (\partial_\mu - i a_\mu) \chi_j + i A_\mu \bar{\chi} \gamma_\mu \tau^z \chi + \frac{i}{2\pi} a \wedge dB$$

Step 1: run the fermion-fermion duality for each flavor:

$$\begin{aligned} \leftrightarrow\ \mathcal{L} &= \bar{\psi}_1 \gamma_\mu (\partial_\mu - i b_\mu) \psi_1 + \bar{\psi}_2 \gamma_\mu (\partial_\mu - i c_\mu) \psi_2 \\ &- \frac{i}{4\pi} a \wedge d(b + c - 2B) - \frac{i}{4\pi} A \wedge d(b - c) \end{aligned}$$

Step 2: Integrating out dynamical gauge field a:

$$\leftrightarrow\ \mathcal{L} = \sum_{j=1}^{2} \bar{\psi}_j \gamma_\mu (\partial_\mu - i b_\mu) \psi_j + i B_\mu \bar{\psi} \gamma_\mu \tau^z \psi + \frac{i}{2\pi} b \wedge dA$$

Other Derivation of this duality: Mross, et.al. arXiv:1510.08455
Karch, Tong, arXiv:1606.01893, Hsin, Seiberg, arXiv:1607.07457

Self-dual N=2 QED3

$$\mathcal{L} = \sum_{j=1}^{2} \bar{\chi}_j \gamma_\mu (\partial_\mu - i a_\mu) \chi_j + i A_\mu \bar{\chi} \gamma_\mu \tau^z \chi + \frac{i}{2\pi} a \wedge dB$$

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Implications of this duality:

1, the N=2 QED3 can at most have O(4) symmetry. The SO(4) ~ SU(2) x SU(2) is the flavor symmetry of both ψ and χ. The Z_2 subgroup of O(4) is the self-duality transformation.
Another way to see the O(4) symmetry, by mapping this model to a low-energy effective field theory in terms of gauge invariant O(4) vector boson (Senthil, Fisher 2006).

Self-dual N=2 QED3

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Implications of this duality:

2, there is an O(4) breaking but SO(4) invariant relevant perturbation:

$$m\bar{\psi}\psi \sim -m\bar{\chi}\chi$$

Tuning *m* drives a bosonic topological transition (type II unconventional QCP), between a bosonic "topological insulator", i.e. bosonic symmetry protected topological (BSPT) state, and a trivial state (Grover, Vishwanath, 2012, Lu, Lee, 2012):

Self-dual N=2 QED3

$$\mathcal{L} = \sum_{j=1}^{2} \bar{\chi}_j \gamma_\mu (\partial_\mu - i a_\mu) \chi_j + i A_\mu \bar{\chi} \gamma_\mu \tau^z \chi + \frac{i}{2\pi} a \wedge dB$$

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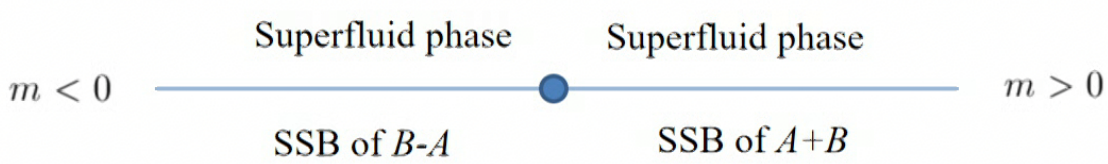
Implications of this duality:

3, there is also a SO(4) breaking fermion mass term, that drives the system into two different superfluid phases (type I):

$$m \bar{\psi} \sigma^z \psi \sim -m \bar{\chi} \sigma^z \chi$$



Self-dual N=2 QED3

4, evidences for the N=2 QED3 to be a conformal field theory:
4.1 Direct numerical evidence for CFT and emergent O(4) symmetry: Karthik, Narayanan, 2016, 2017

4.2 the tuning parameter $m\bar{\psi}\psi \sim -m\bar{\chi}\chi$ drives a bosonic topological phase transition (Type II), and the Chern-Simons level of the back ground field A and B change by +2 and -2 respectively. If we enhance A and B to SU(2) background gauge fields, their levels change by +1 and -1 respectively.

Simulation on a lattice model with the same transition (Slagle, You, Xu, 2014, He, etc. 2015),

More duality of N=2 QED3

More duality of N=2 QED3 (Karch et.al. 2016, Potter, et.al. 2016, Wang et.al. 2017)

$$\mathcal{L} = \sum_{j=1}^{2} \bar{\chi}_j \gamma_\mu (\partial_\mu - i a_\mu) \chi_j + i A_\mu \bar{\chi} \gamma_\mu \tau^z \chi + \frac{i}{2\pi} a \wedge dB$$

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$$\leftrightarrow \ \mathcal{L} = |(\partial_\mu - i b_\mu) z_1|^2 + g|z_1|^4 + |(\partial_\mu - i c_\mu) z_2|^2 + g|z_2|^4 + \frac{i}{2\pi} a \wedge (b - c) + \frac{i}{4\pi} b \wedge db - \frac{i}{4\pi} c \wedge dc$$

Step 2: Integrating out dynamical gauge field a:

$$\mathcal{L} = \sum_j |(\partial_\mu - i b_\mu) z_j|^2 + g|z_j|^4$$

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$r < 0$ SF ordered SF (VBS) order $r > 0$

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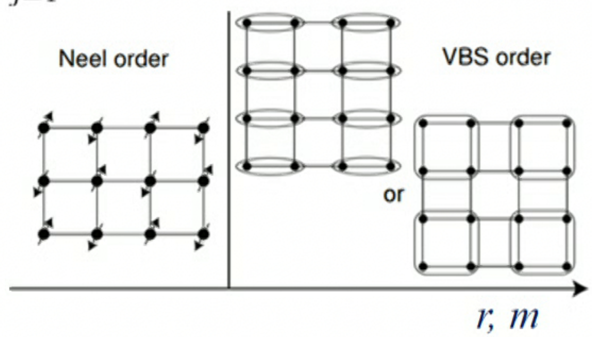

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$m < 0$ — $m > 0$

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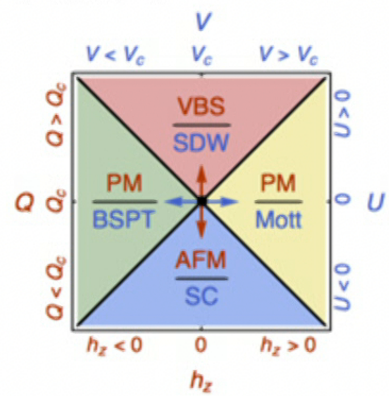
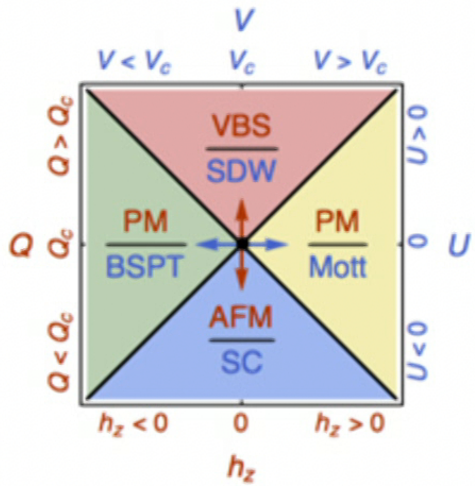
"Miniweb" of duality: each of the Lagrangians above are self-dual, and they are dual to each other: duality between (self-dual) type I and (self-dual) type II unconventional QCPs!



Implications of the proposed duality



$$3 - \frac{1}{\nu_{\mathrm{BTT}}} = \frac{1+\eta^{z}_{\mathrm{dQCP}}}{2},$$

$$3 - \frac{1}{\nu^{xy}_{\mathrm{dQCP}}} = \frac{1+\eta_{\mathrm{qed}}}{2} = \frac{1+\eta_{\mathrm{BTT}}}{2},$$

$$\eta^{O(4)}_{\mathrm{BTT}} = \eta^{xy}_{\mathrm{dQCP}} = \eta^{vbs}_{\mathrm{dQCP}}$$

Duality predicts:
The easy-plane dQCP has an emergent O(4) symmetry, and a set of exact relations between the critical exponents of the bosonic topological transition and deconfined QCP:
(Wang, Nahum, Metlitski, Xu, Senthil, 2017)

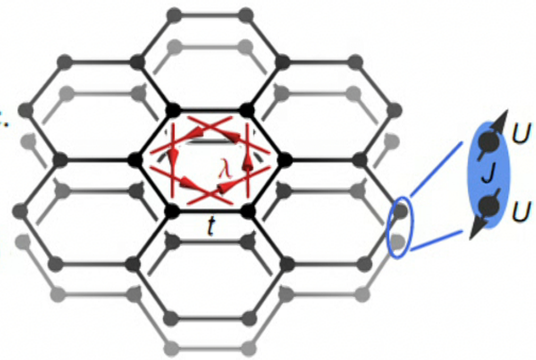
Numerical Simulation for Bosonic topological transition

We want to design a similar lattice model with all the key physics, and "easy" to study numerically:

$$H = H_{\text{band}} + H_{\text{int}},$$

$$H_{\text{band}} = -t \sum_{\langle ij \rangle, \ell} c^{\dagger}_{i\ell} c_{j\ell} + \sum_{\langle\langle ij \rangle\rangle, \ell} \mathrm{i}\lambda_{ij} c^{\dagger}_{i\ell} \sigma^{z} c_{j\ell} + H.c.$$

$$H_{\text{int}} = +J \sum_{i} \left[\boldsymbol{S}_{i1} \cdot \boldsymbol{S}_{i2} + \frac{1}{4}(n_{i1} - 1)(n_{i2} - 1) \right.$$



Simple limits of this model:
(1) Noninteracting: bilayer quantum spin Hall, boundary has two channels of gapless fermion modes

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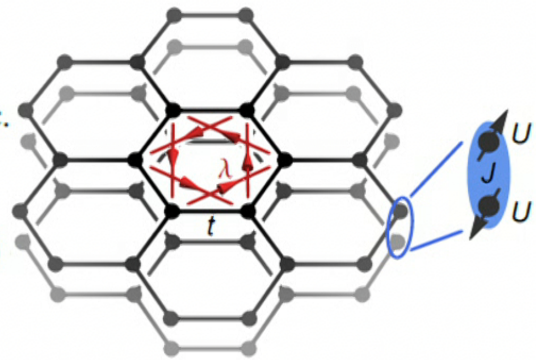


Simple limits of this model:

(2) Strong J-interacting limit: trivial Mott insulator, with inter-layer spin singlet one every site. $|\Psi\rangle = \prod |\text{Singlet}\rangle$

What happens at intermediate J ?

Numerical Simulation for Bosonic topological transition

Apparently, this model has at least $U(1)_{spin}$ x $U(1)_{charge}$ symmetry. At relatively weak interaction J, we can directly bosonize the edge states:

$$H_0 = \int dx \sum_{l=1}^{2} \psi_{l,L} iv\partial_x \psi_{l,L} - \psi_{l,R} iv\partial_x \psi_{l,R}$$

$$\Downarrow$$

$$H_0 = \int dx \sum_{l=1}^{2} \frac{v}{2K}(\partial_x \theta_l)^2 + \frac{vK}{2}(\partial_x \phi_l)^2$$

$\Downarrow$ interaction $H_v \sim \alpha \cos(2\pi\phi_1 - 2\pi\phi_2)$

$$\tilde{H} = \int dx \frac{\tilde{v}}{2\tilde{K}}(\partial_x \theta)^2 + \frac{\tilde{v}\tilde{K}}{2}(\partial_x \phi)^2$$

When H_v is relevant, all the fermion modes are gapped at the boundary, but bosonic modes are gapless, and protected by symmetry. Thus the system becomes effectively a "bosonic topological insulator"

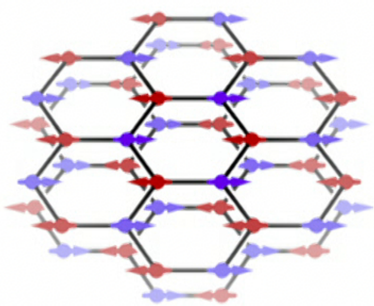
Numerical Simulation for Bosonic topological transition

This model actually has an exact SO(4) symmetry. Spin-up and spin-down fermions have their individual SU(2) symmetry.
SO(4) vector:

$$\boldsymbol{n}_i = (N_i^x, \mathrm{Im}\,\Delta_i, \mathrm{Re}\,\Delta_i, N_i^y). \\ = f_{i\downarrow}^\dagger(\tau^0, \mathrm{i}\tau^1, \mathrm{i}\tau^2, \mathrm{i}\tau^3) f_{i\uparrow} + h.c.,$$

$$f_{i\uparrow} = \begin{pmatrix} c_{1i\uparrow} \\ (-1)^i c_{2i\uparrow}^\dagger \end{pmatrix}, f_{i\downarrow} = \begin{pmatrix} (-1)^i c_{1i\downarrow} \\ c_{2i\downarrow}^\dagger \end{pmatrix}$$

(N^x, N^y)



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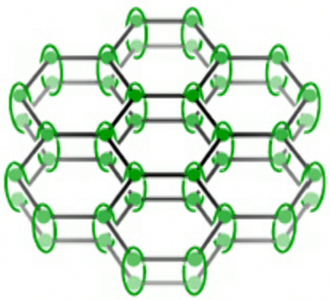
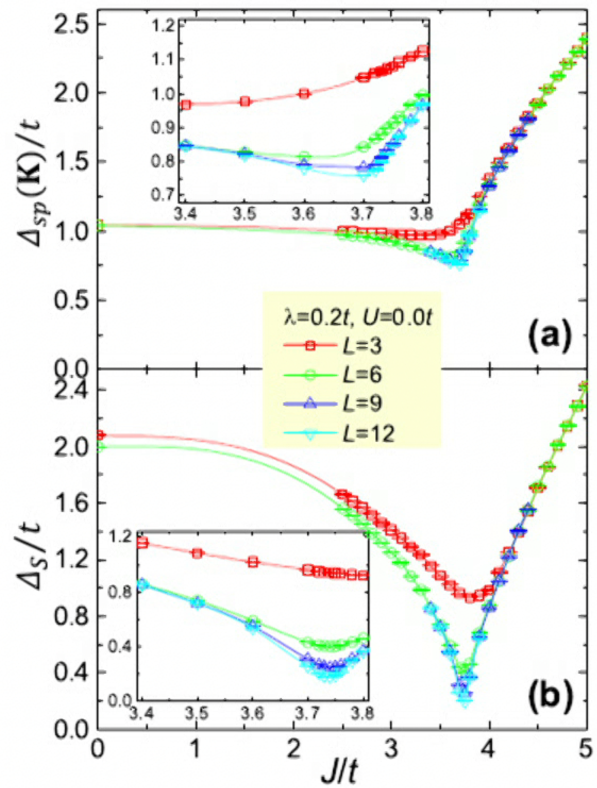


$$H = \sum_{i,j,\sigma} (-)^\sigma f_{i\sigma}^\dagger(-t_{ij} + i\lambda_{ij}) f_{j\sigma} + h.c. - \frac{J}{16} \sum_i \left(D_i D_i^\dagger + D_i^\dagger D_i\right) \qquad D_i = \sum_\sigma f_{i\sigma} i\tau^2 f_{i\sigma}$$

Numerical Simulation for Bosonic topological transition

Determinant QMC for the bosonic topological phase transition: (Slagle, You, Xu, 2014, He, etc. 2015), this model has an exact SO(4) symmetry on the lattice, consistent with the emergent N=2 QED in the infrared.

In terms of the N=2 QED language, this transition corresponds to changing the sign of the SU(2) invariant fermion mass term $m\bar{\psi}\psi$



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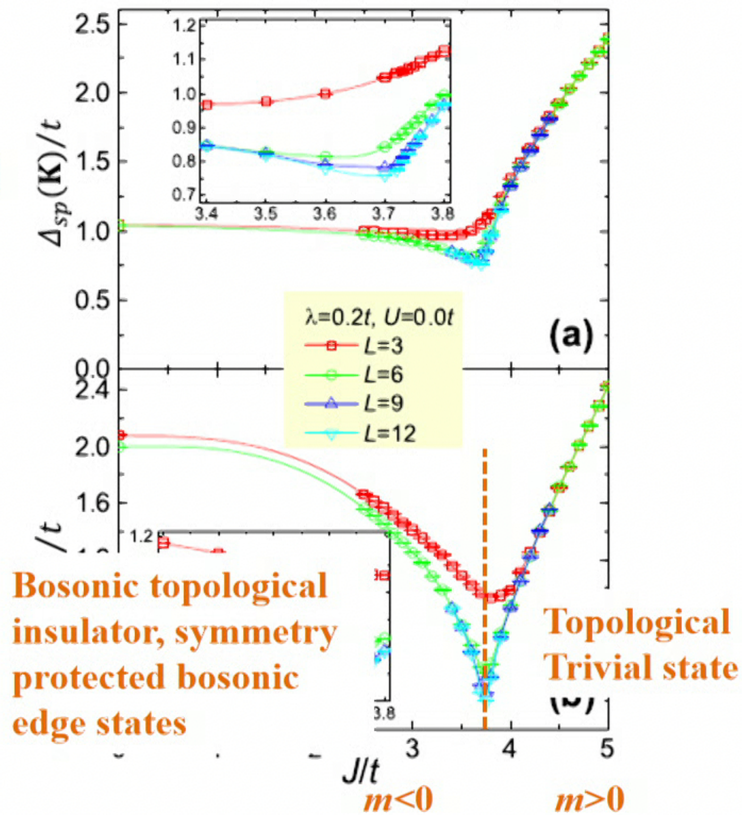
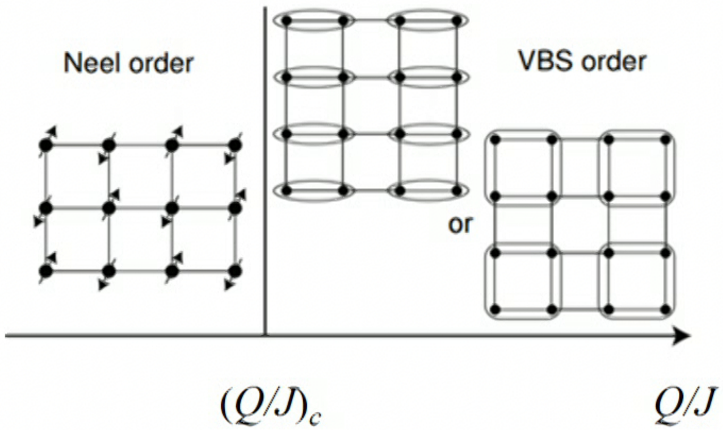


The easy-plane J-Q model with a continuous AF-VBS transition

$$H_{\mathrm{JQ}} = -J \sum_{\langle ij \rangle} (P_{ij} + \Delta S_i^z S_j^z) - Q \sum_{\langle ijklmn \rangle} P_{ij} P_{kl} P_{mn}$$

$$\boldsymbol{P}_{ij} = \tfrac{1}{4} - \boldsymbol{S}_i \cdot \boldsymbol{S}_j,$$



Qin, He, You, Lu, Sen, Sandvik, Xu, Meng, 2017

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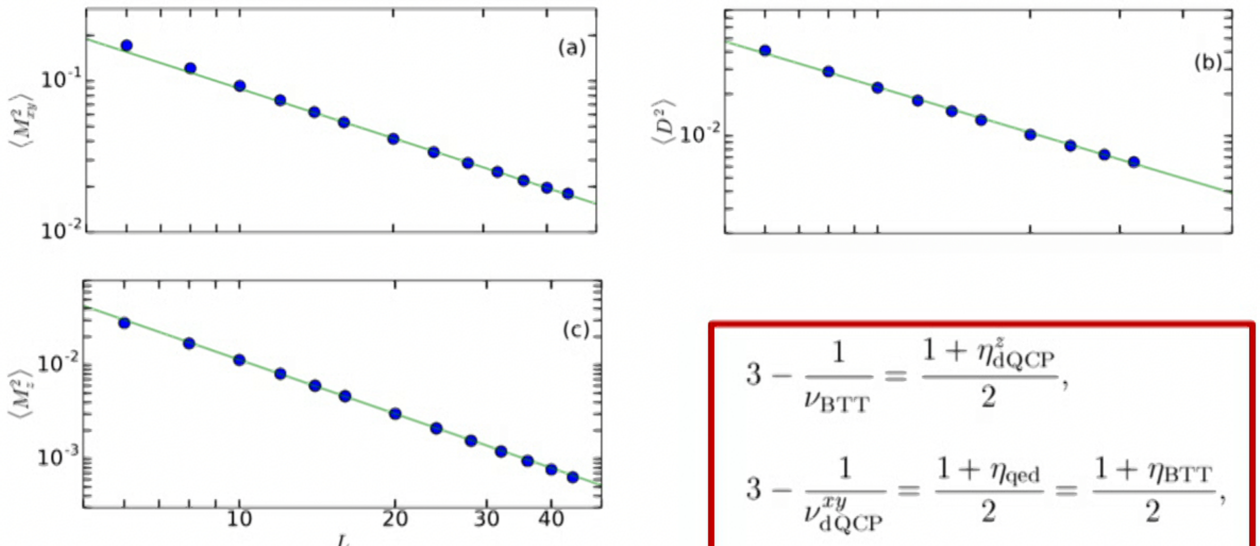


Within error bar, the numerical data are consistent with predictions!

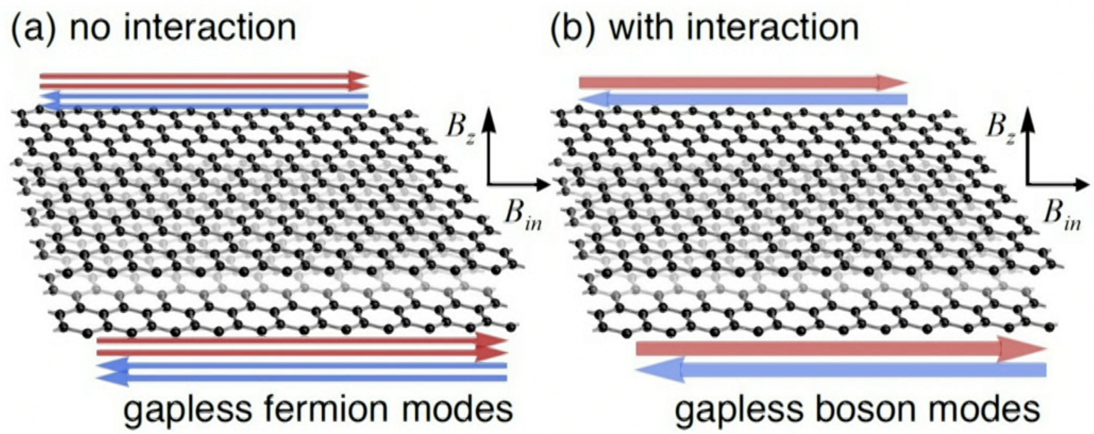
$$3 - \frac{1}{\nu_{\mathrm{BTT}}} = \frac{1 + \eta^z_{\mathrm{dQCP}}}{2},$$

$$3 - \frac{1}{\nu^{xy}_{\mathrm{dQCP}}} = \frac{1 + \eta_{\mathrm{qed}}}{2} = \frac{1 + \eta_{\mathrm{BTT}}}{2},$$

$$\eta^{O(4)}_{\mathrm{BTT}} = \eta^{xy}_{\mathrm{dQCP}} = \eta^{vbs}_{\mathrm{dQCP}}$$

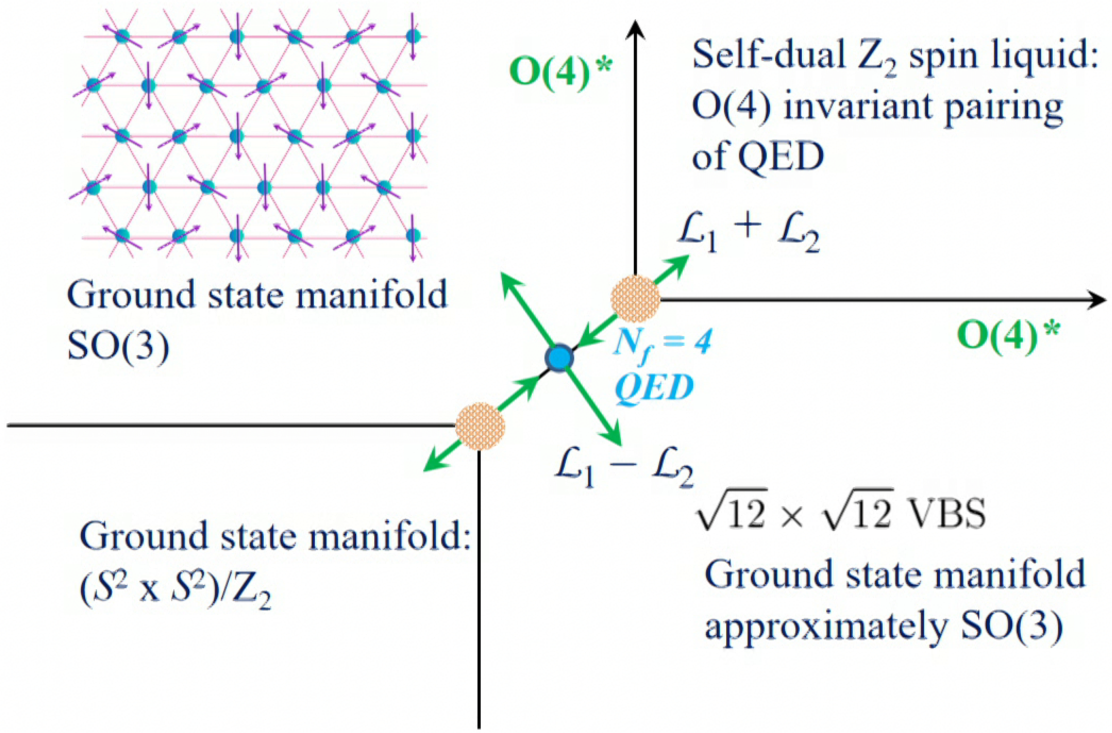
Possible Experimental Platform

bosonic topological insulator, and potentially bosonic topological transition, have been proposed to be realized in bilayer graphene. Interaction plays the crucial role.
Bi, Zhang, You, Young, Balents, Liu, Xu, 2016.



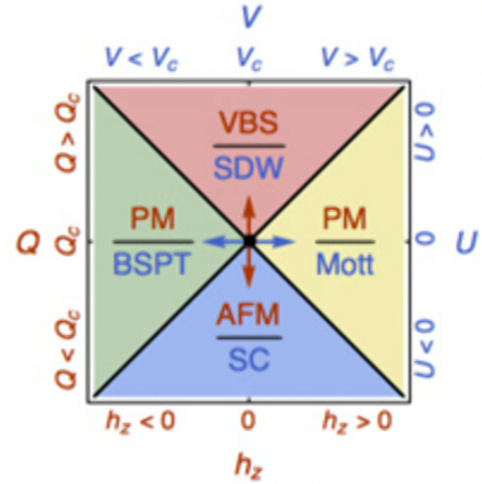
Deconfined Quantum Critical Point on the Triangular Lattice

Thomson, Rasmussen, Bi, Xu, 2017



Summary

Proposed duality between two (self-dual) unconventional quantum critical points: bosonic topological transition (type II) and the easy-plane deconfined quantum critical point (type I);



Numerical simulation does support the theoretical predictions!

dQCP and bosonic topological insulator have both been proposed to be realized in bilayer graphene!
Lee, Sachdev 2014,
Bi, Zhang, You, Young, Balents, Liu, Xu, 2016.