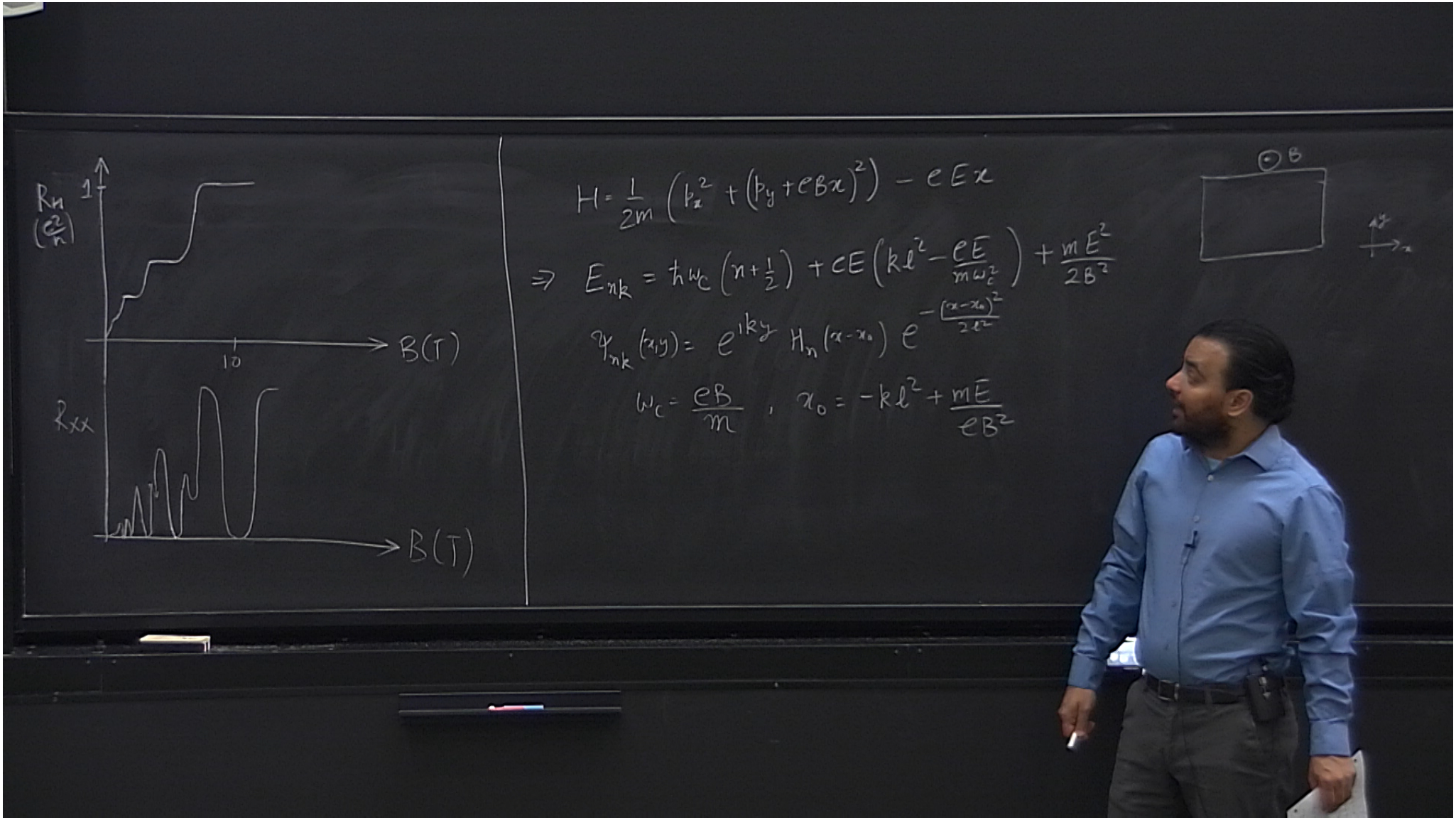
Title: PSI 17/18 - Condensed Matter - Lecture 15

Date: Nov 24, 2017 10:45 AM

URL: http://pirsa.org/17110040

Abstract:


R_H $(\frac{e^2}{h})$
1
10
B(T)
R_{xx}
B(T)
$H = \frac{1}{2m}\left(p_x^2 + (p_y + eBx)^2\right) - eEx$
$\Rightarrow E_{nk} = \hbar\omega_c\left(n+\frac{1}{2}\right) + eE\left(k\ell^2 - \frac{eE}{m\omega_c^2}\right) + \frac{mE^2}{2B^2}$
$\psi_{nk}(x,y) = e^{iky} H_n(x-x_0)\, e^{-\frac{(x-x_0)^2}{2\ell^2}}$
$\omega_c = \frac{eB}{m}$, $x_0 = -k\ell^2 + \frac{mE}{eB^2}$
B
y
x

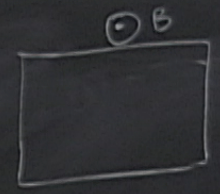
$$H = \frac{1}{2m}\left(p_x^2 + (p_y + eBx)^2\right) - eEx$$

$$\Rightarrow E_{nk} = \hbar\omega_c\left(n+\tfrac{1}{2}\right) + eE\left(k\ell^2 - \frac{eE}{m\omega_c^2}\right) + \frac{mE^2}{2B^2}$$

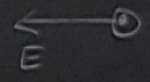
$$\psi_{nk}(x,y) = e^{iky}\, H_n(x-x_0)\, e^{-\frac{(x-x_0)^2}{2\ell^2}}$$

$$\omega_c = \frac{eB}{m}\,, \quad x_0 = -k\ell^2 + \frac{mE}{eB^2}\,, \quad \ell = \sqrt{\frac{\hbar}{eB}}$$

$$v_y = \frac{1}{\hbar}\frac{\partial E_{nk}}{\partial k} = \frac{1}{\hbar}eE\ell^2 = \frac{E}{B}$$



$E = 0$



$$\vec{v}_{drift} = \vec{E} \times \vec{B}$$

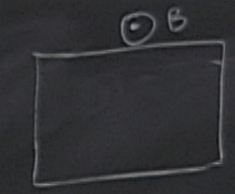
$$H = \frac{1}{2m}\left(p_x^2 + (p_y + eBx)^2\right) - eEx$$

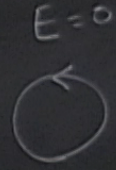
$$\Rightarrow E_{nk} = \hbar\omega_c\left(n+\tfrac{1}{2}\right) + eE\left(k\ell^2 - \frac{eE}{m\omega_c^2}\right) + \frac{mE^2}{2B^2}$$

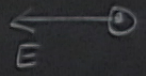
$$\psi_{nk}(x,y) = e^{iky}\, H_n(x-x_0)\, e^{-\frac{(x-x_0)^2}{2\ell^2}}$$

$$\omega_c = \frac{eB}{m}\,, \quad x_0 = -k\ell^2 + \frac{mE}{eB^2}\,,$$

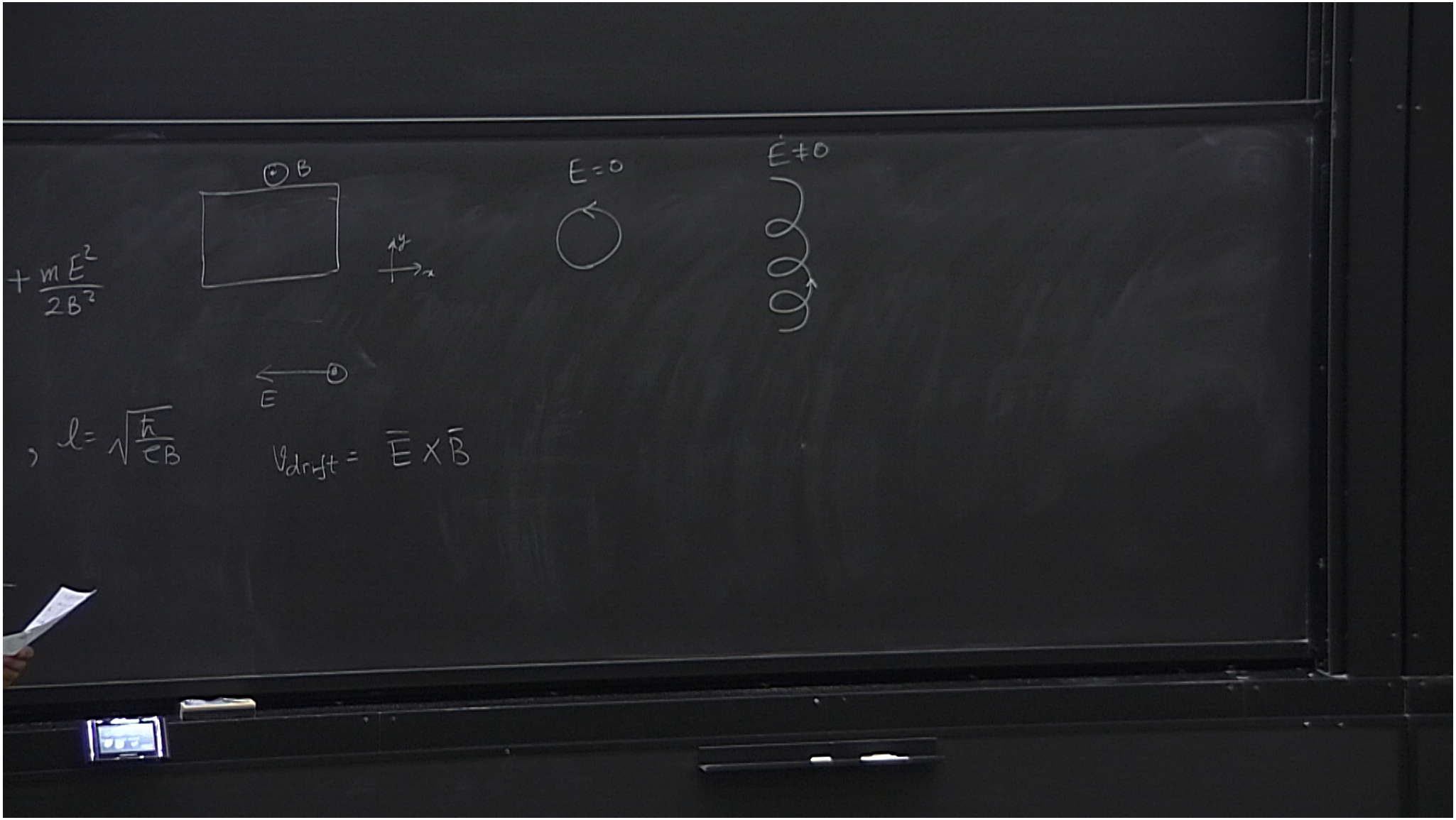
$$v_y = \frac{1}{\hbar}\frac{\partial E_{nk}}{\partial k} = \frac{1}{\hbar}eE\ell^2 = \frac{E}{B}$$

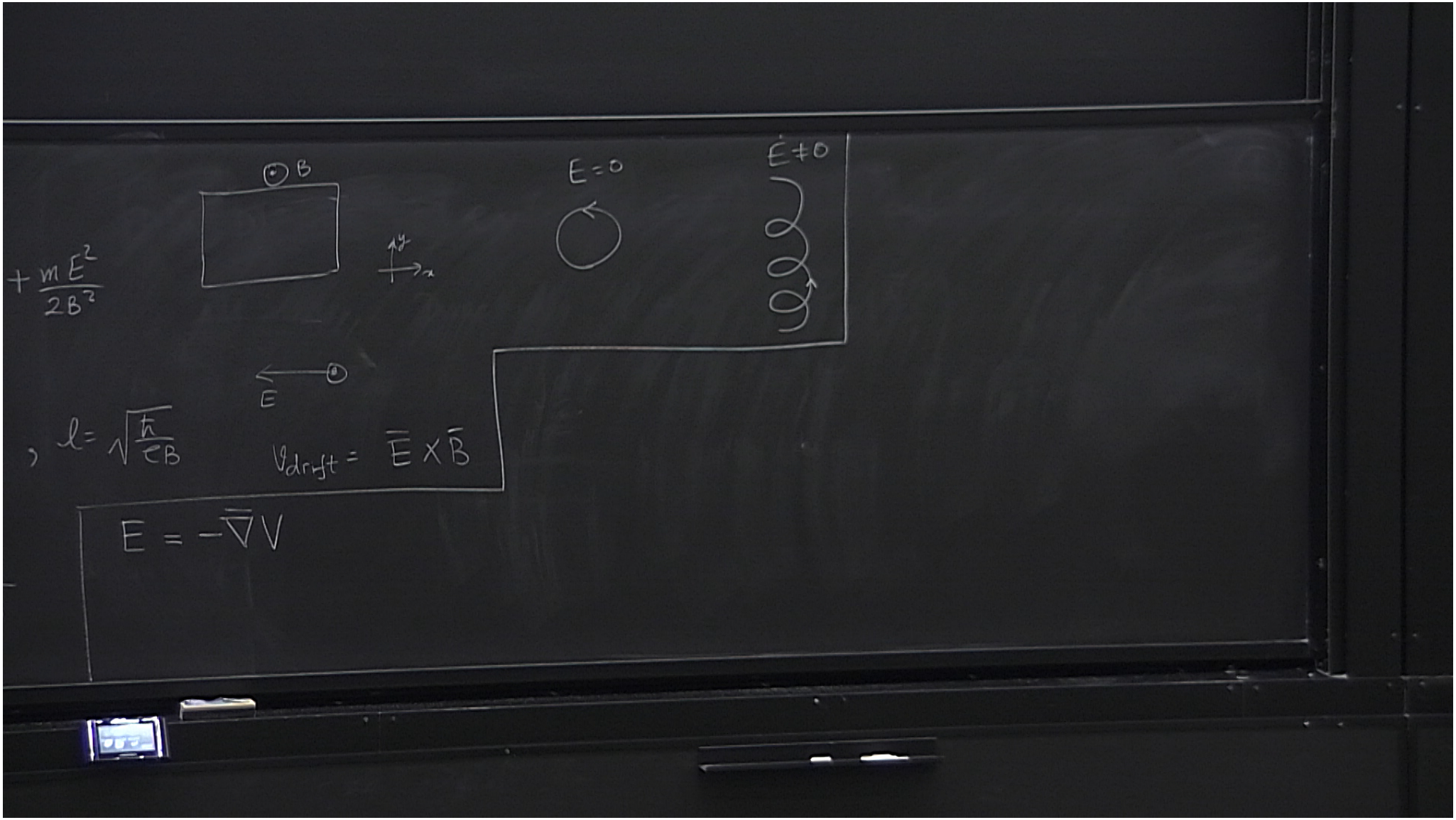


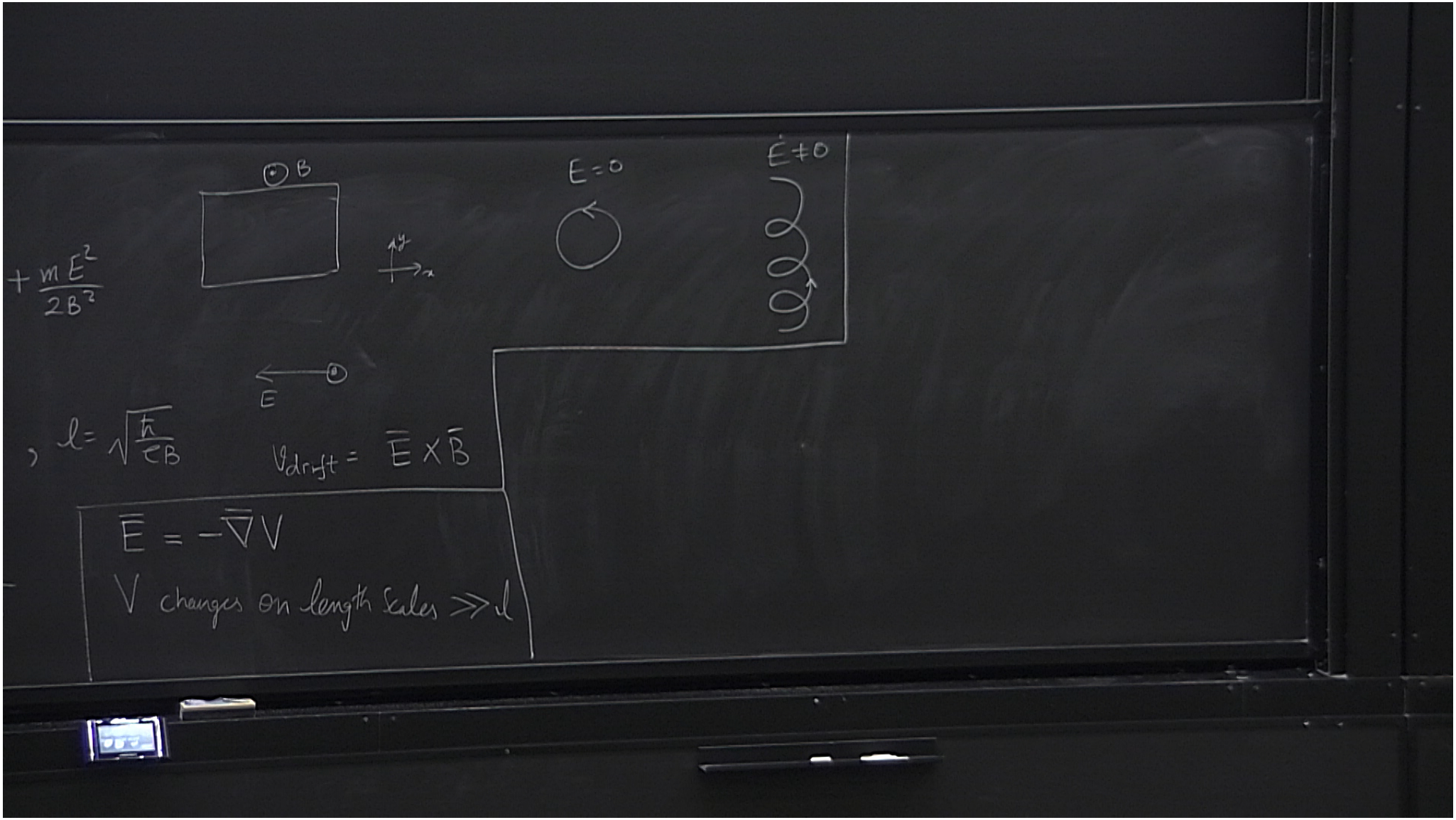


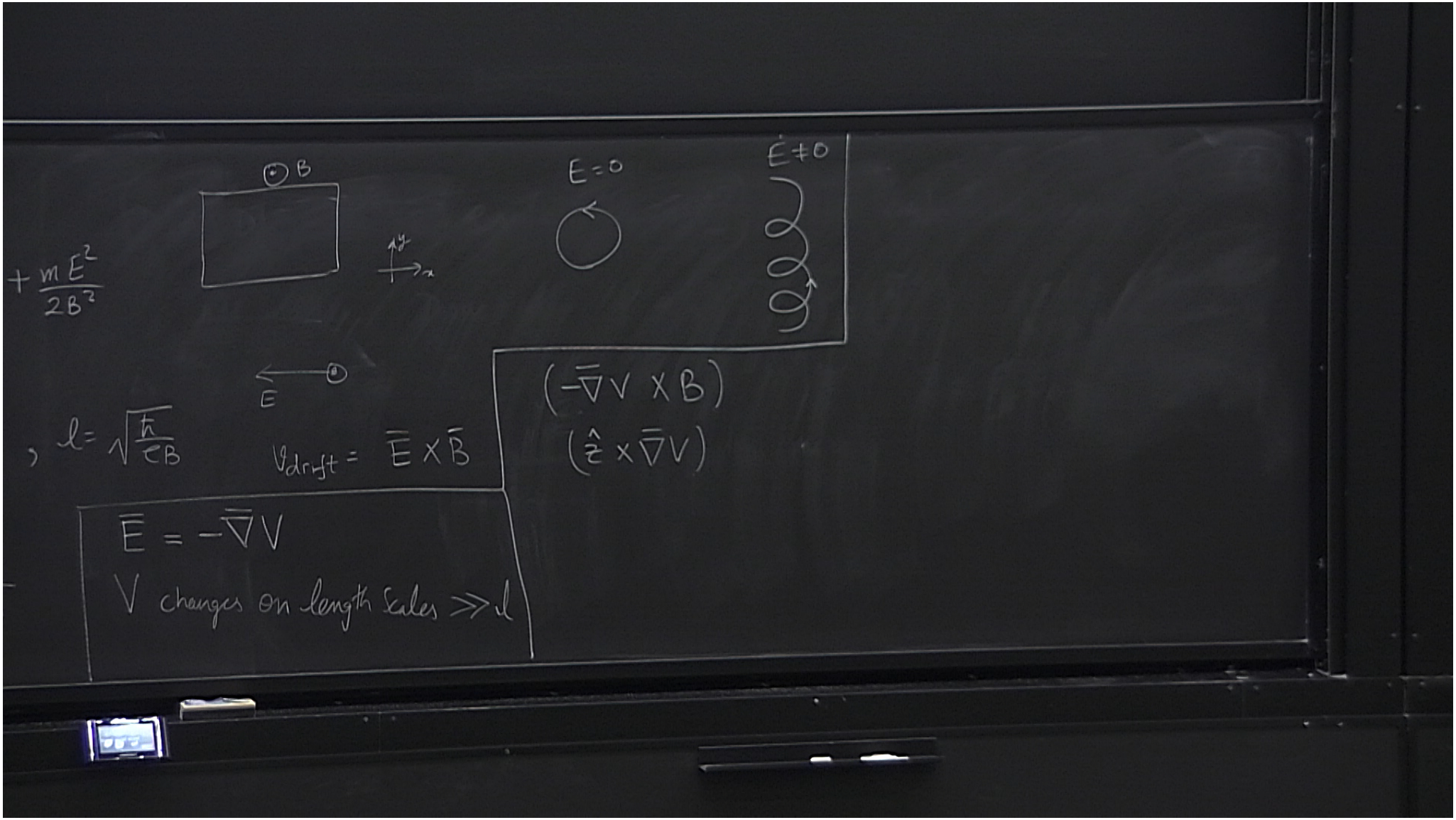


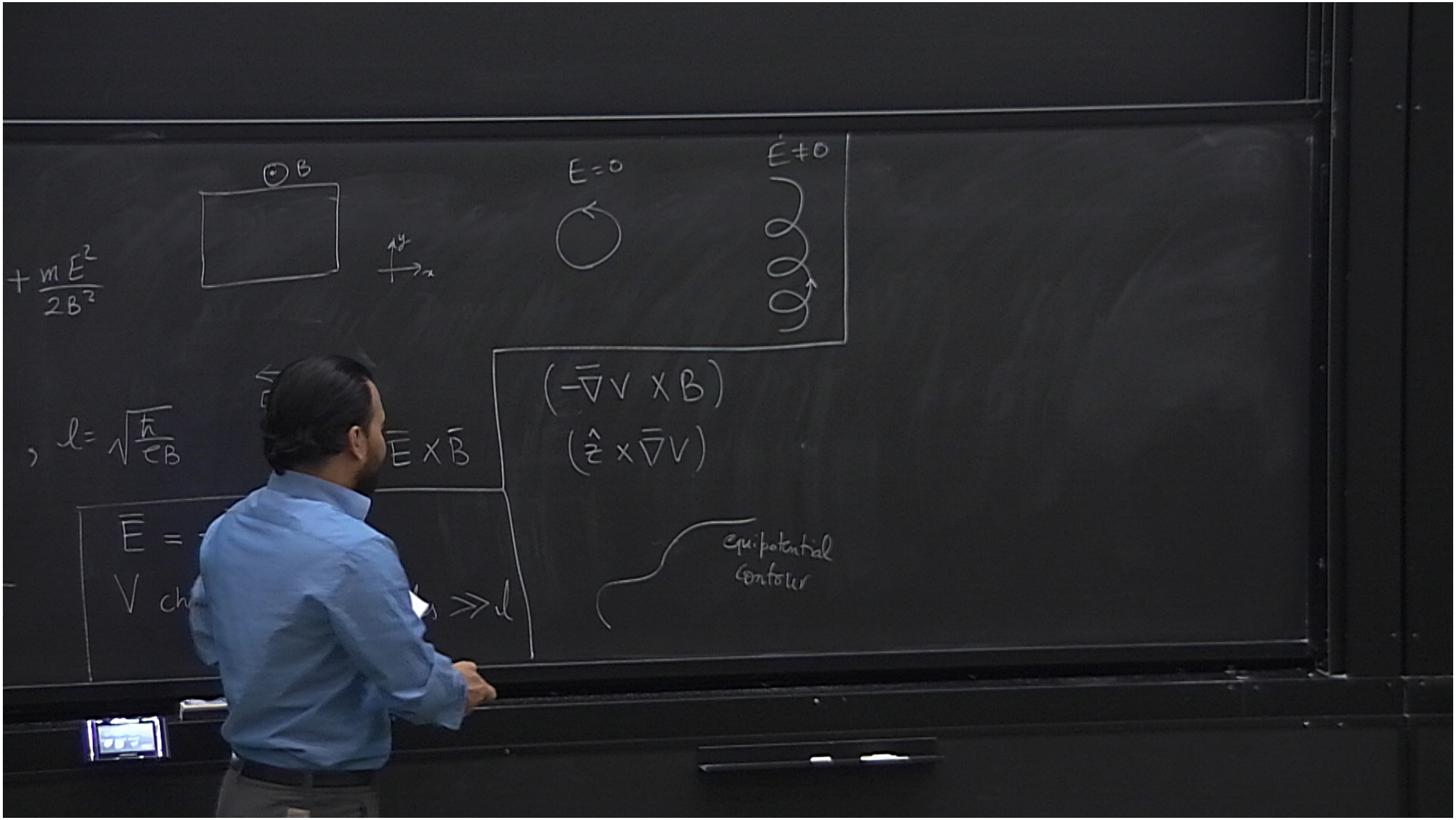
$$\vec{v}_{drift} = \vec{E} \times \vec{B}$$

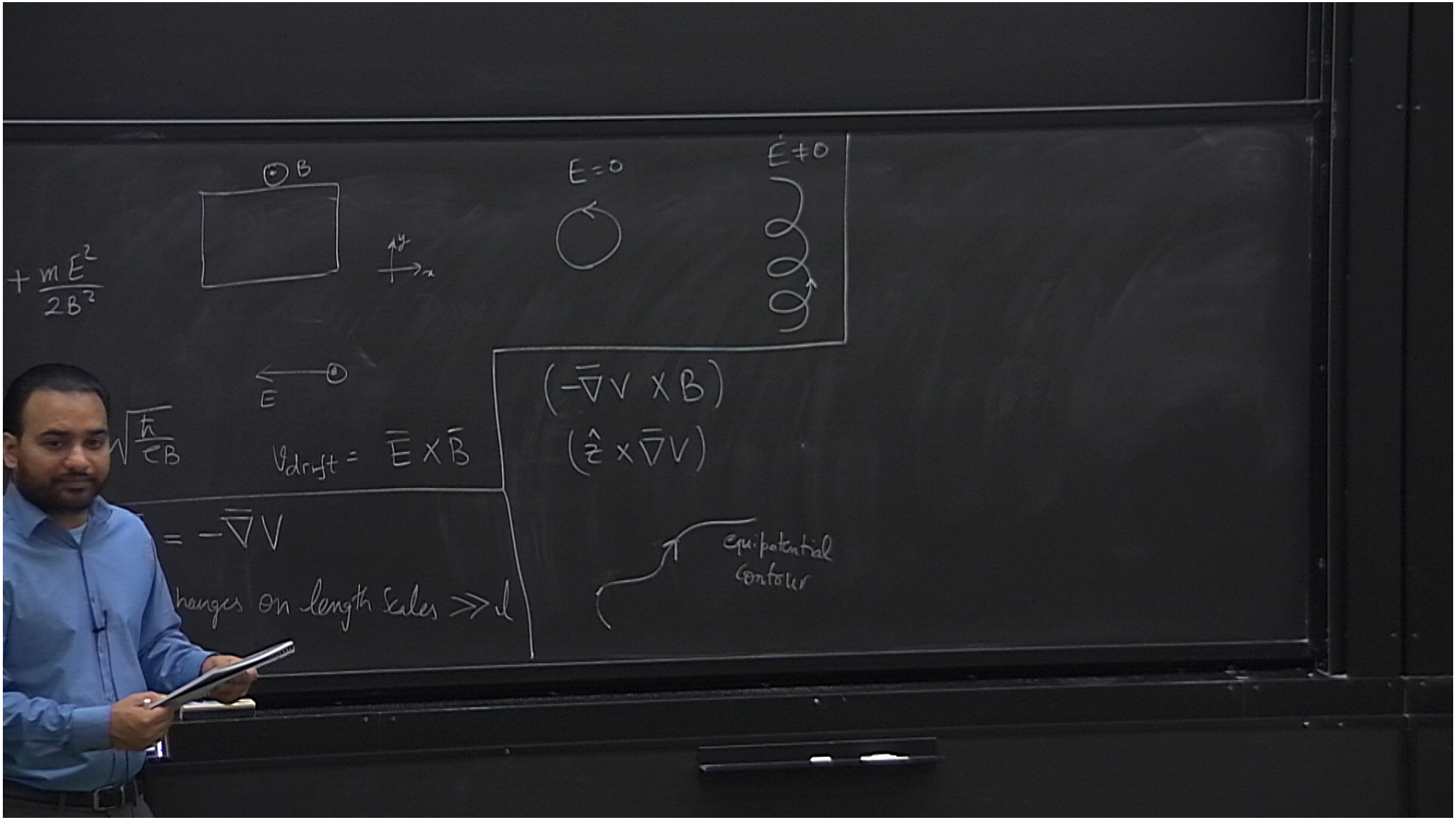

B
E=0
Ė≠0
y
x
E
$+\frac{mE^2}{2B^2}$
$\ell = \sqrt{\frac{\hbar}{eB}}$
$v_{drift} = \vec{E} \times \vec{B}$

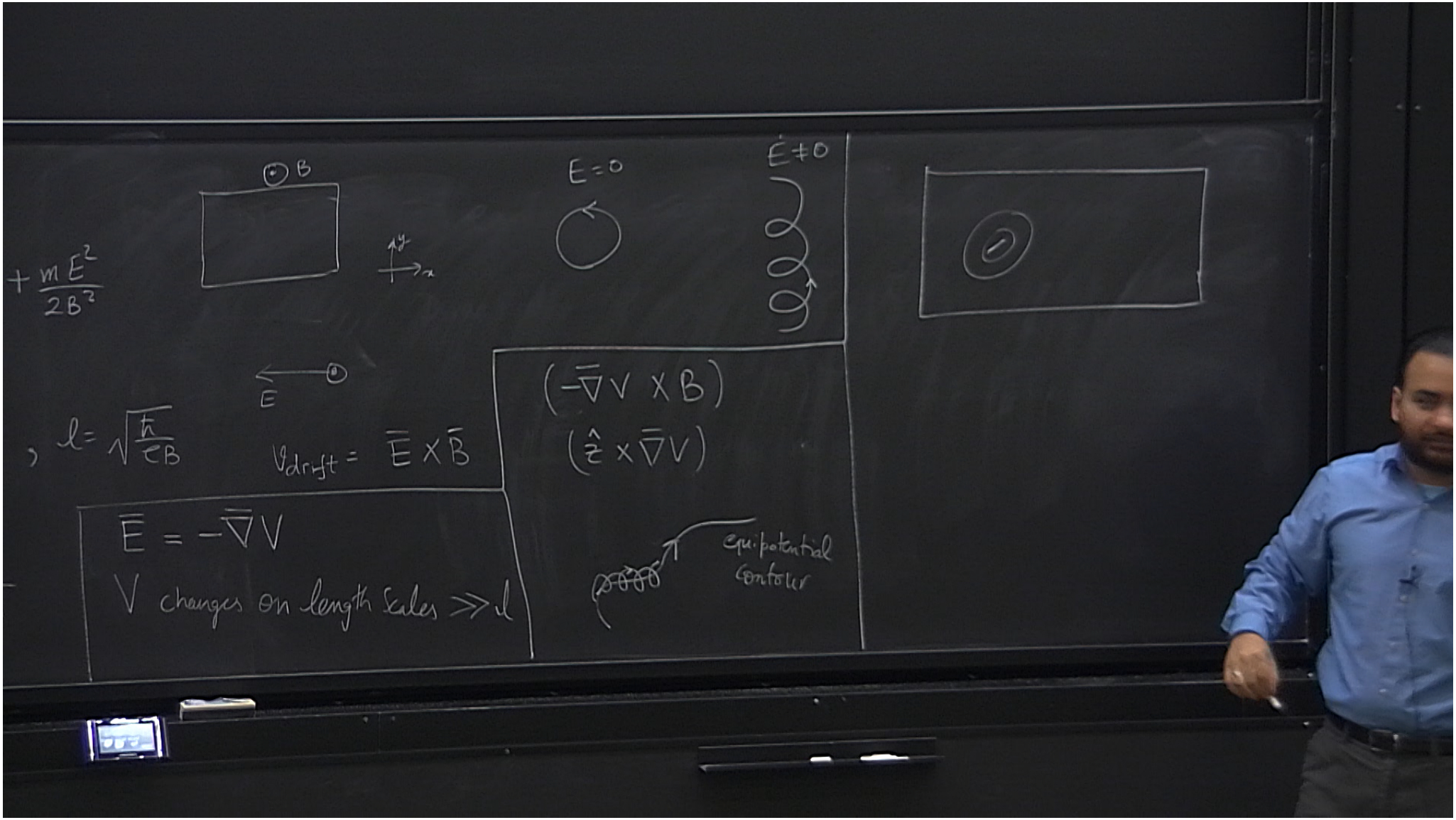

B
E=0
Ė≠0
y
x
E
$+\frac{mE^2}{2B^2}$
$\ell = \sqrt{\frac{\hbar}{eB}}$
$v_{drift} = \vec{E} \times \vec{B}$
$E = -\vec{\nabla} V$

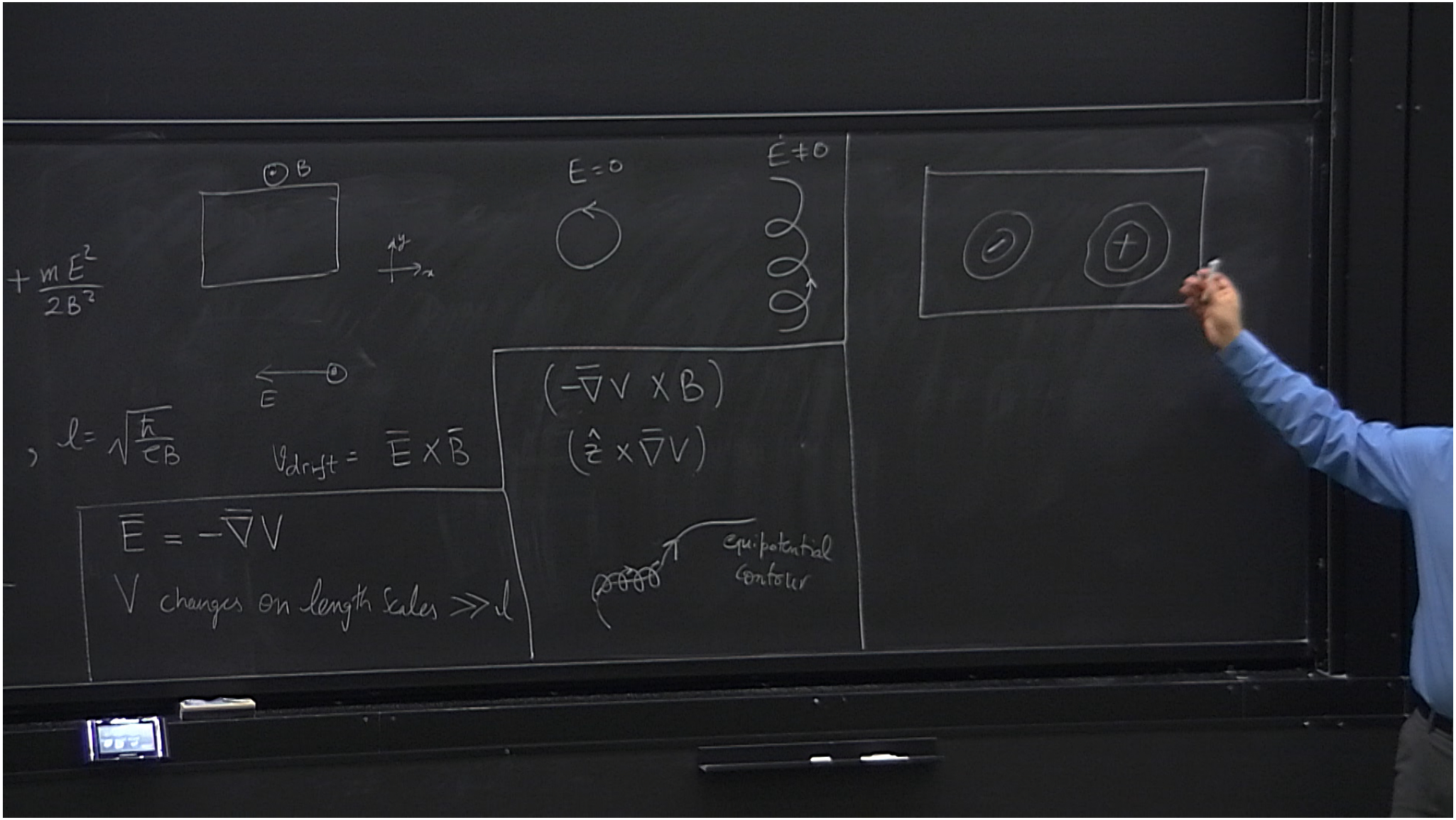

B
E=0
Ė≠0
E
$+\frac{mE^2}{2B^2}$
$\ell = \sqrt{\frac{\hbar}{eB}}$
$v_{drift} = \vec{E} \times \vec{B}$
$\vec{E} = -\vec{\nabla} V$
V changes on length scales $\gg \ell$

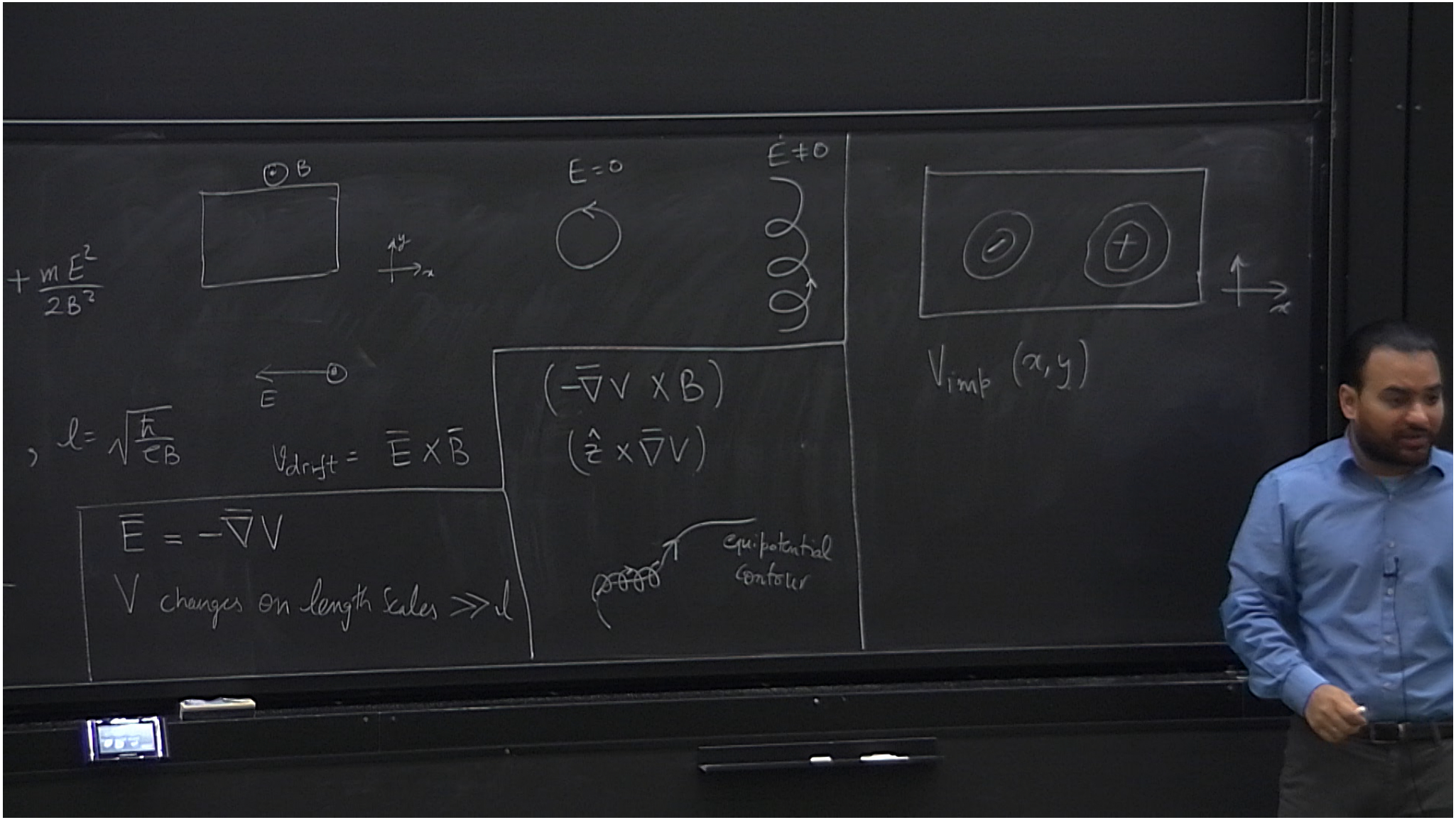

B
E=0
Ė≠0
E
$+\frac{mE^2}{2B^2}$
$\ell = \sqrt{\frac{\hbar}{eB}}$
$v_{drift} = \bar{E} \times \bar{B}$
$(-\bar{\nabla} V \times B)$
$(\hat{z} \times \bar{\nabla} V)$
$\bar{E} = -\bar{\nabla} V$
V changes on length scales $\gg \ell$

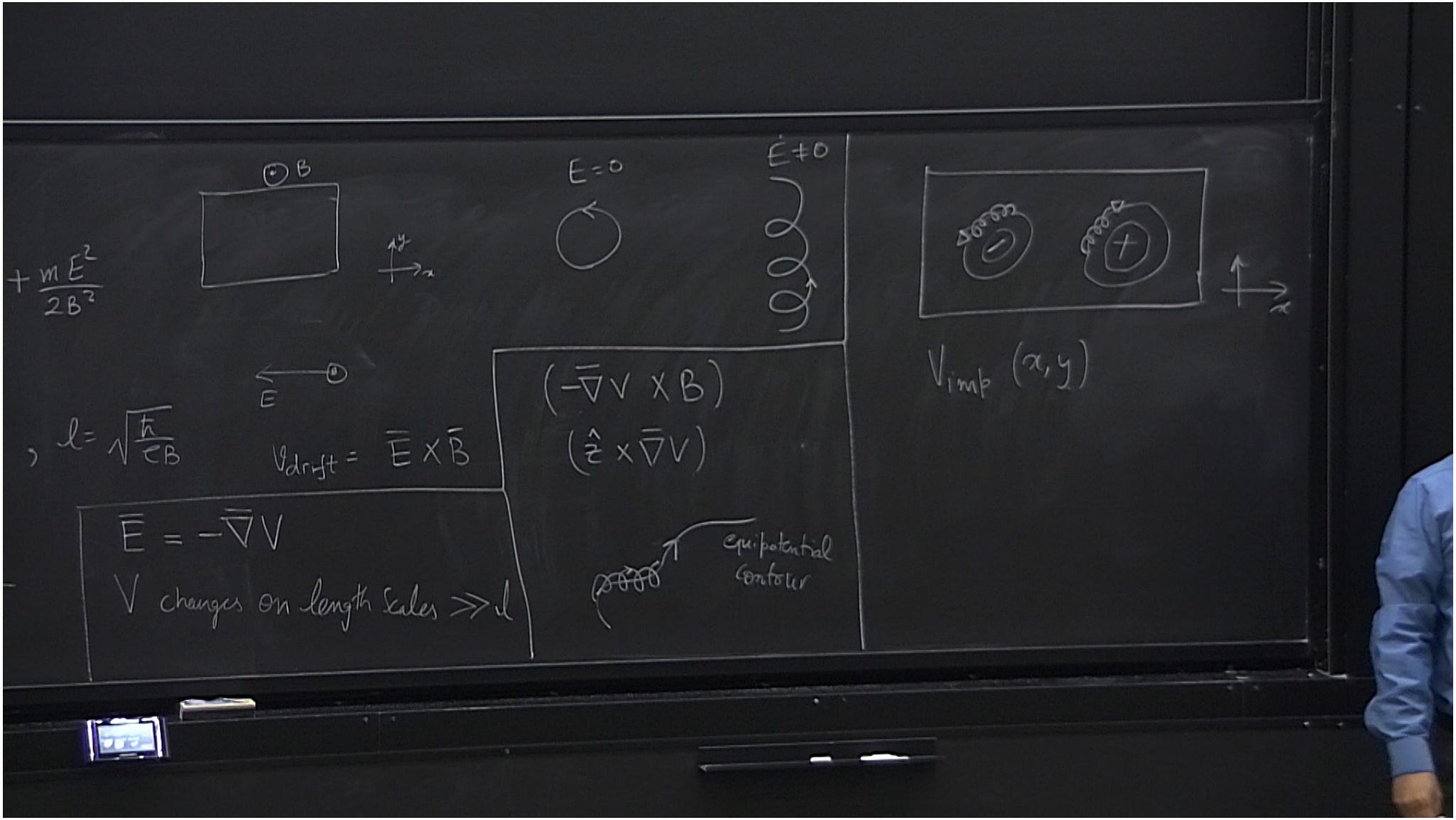

B
E=0
Ė≠0
$+\frac{mE^2}{2B^2}$
$\ell = \sqrt{\frac{\hbar}{eB}}$
$\bar{E} \times \bar{B}$
$(-\bar{\nabla} V \times B)$
$(\hat{z} \times \bar{\nabla} V)$
equipotential
contour

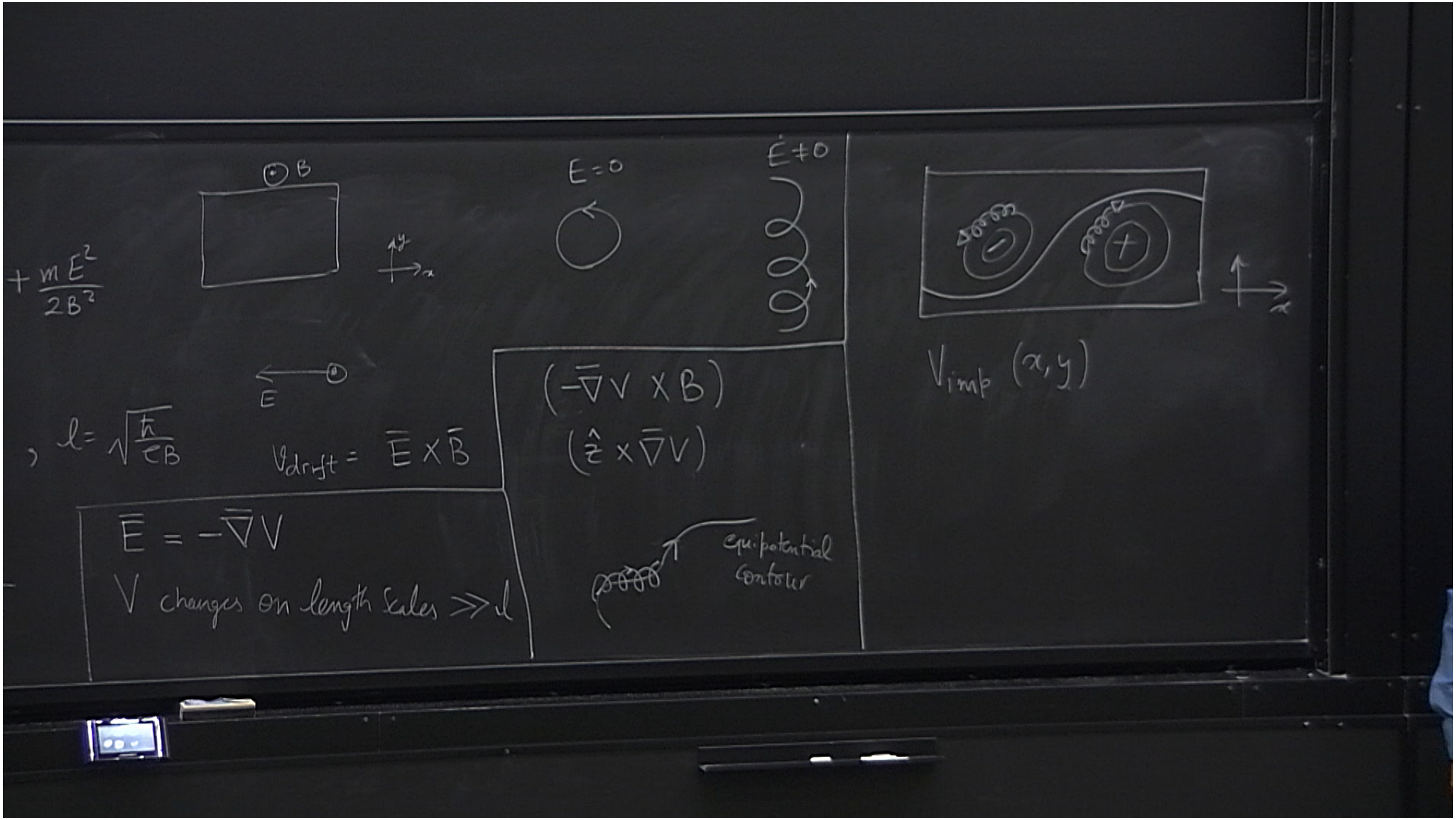

B
E=0
Ė≠0
y
x
E
$+\frac{mE^2}{2B^2}$
$\sqrt{\frac{\hbar}{eB}}$
$v_{drift} = \bar{E} \times \bar{B}$
$= -\bar{\nabla} V$
changes on length scales $\gg d$
$(-\bar{\nabla} V \times B)$
$(\hat{z} \times \bar{\nabla} V)$
equipotential contour

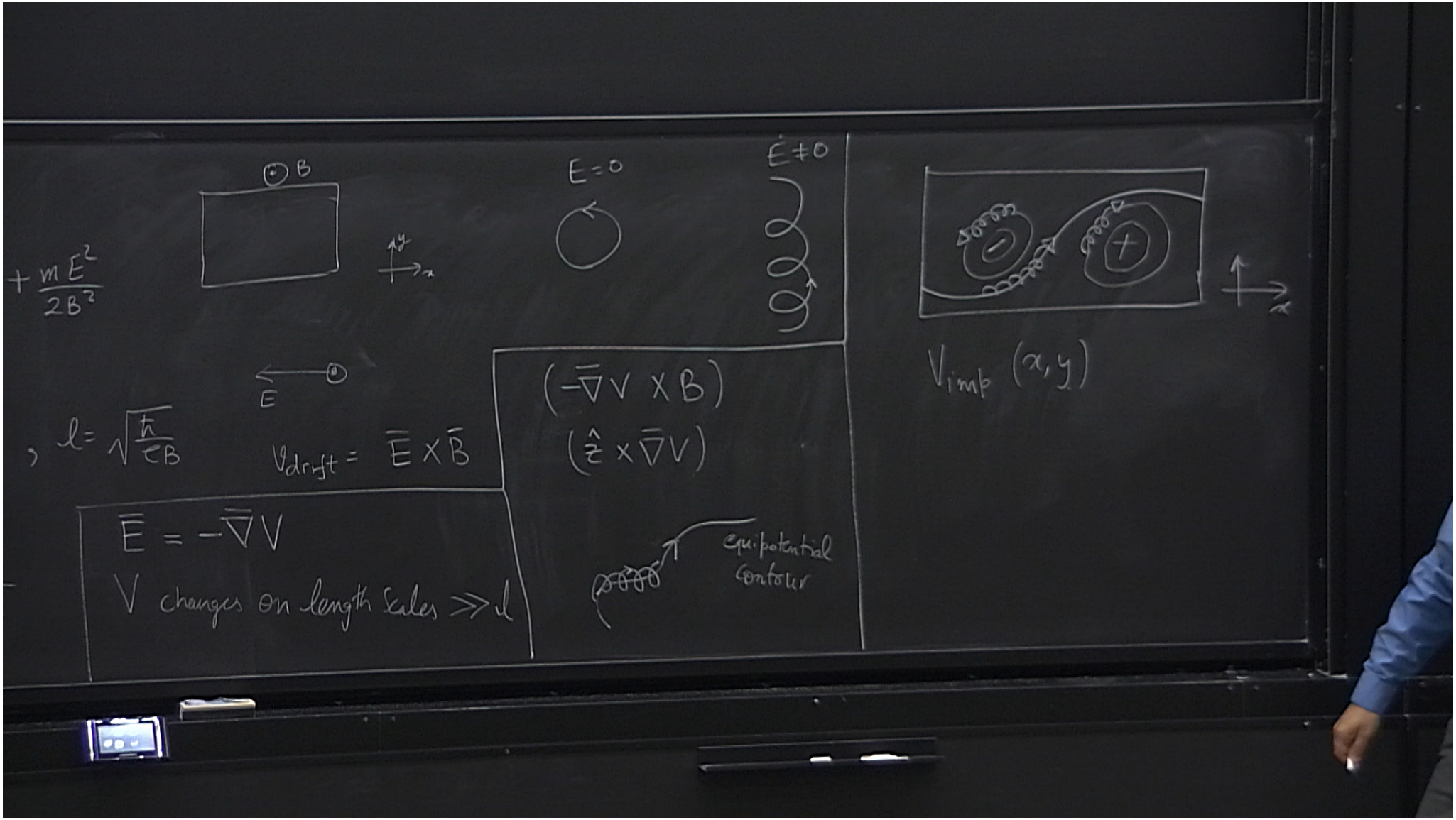

B
E=0
Ė≠0
+ mE²/2B²
E
ℓ = √(ħ/eB)
v_drift = E × B
(−∇V × B)
(ẑ × ∇V)
E = −∇V
V changes on length scales ≫ ℓ
equipotential contour

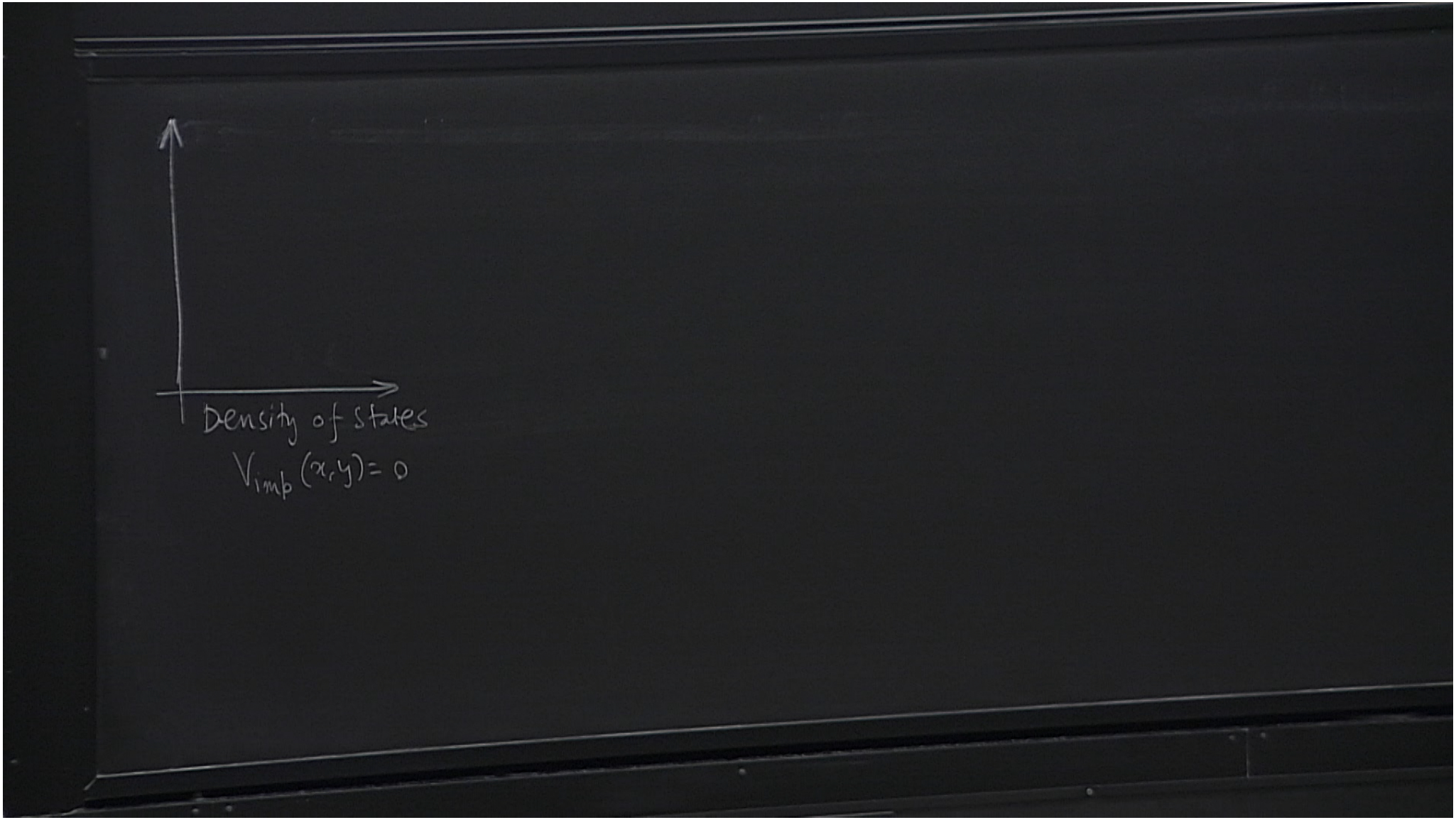

B
E = 0
Ė ≠ 0
+ mE²/2B²
y
x
E
ℓ = √(ħ/eB)
v_drift = E × B
(−∇V × B)
(ẑ × ∇V)
E = −∇V
V changes on length scales ≫ ℓ
equipotential contour
−
+

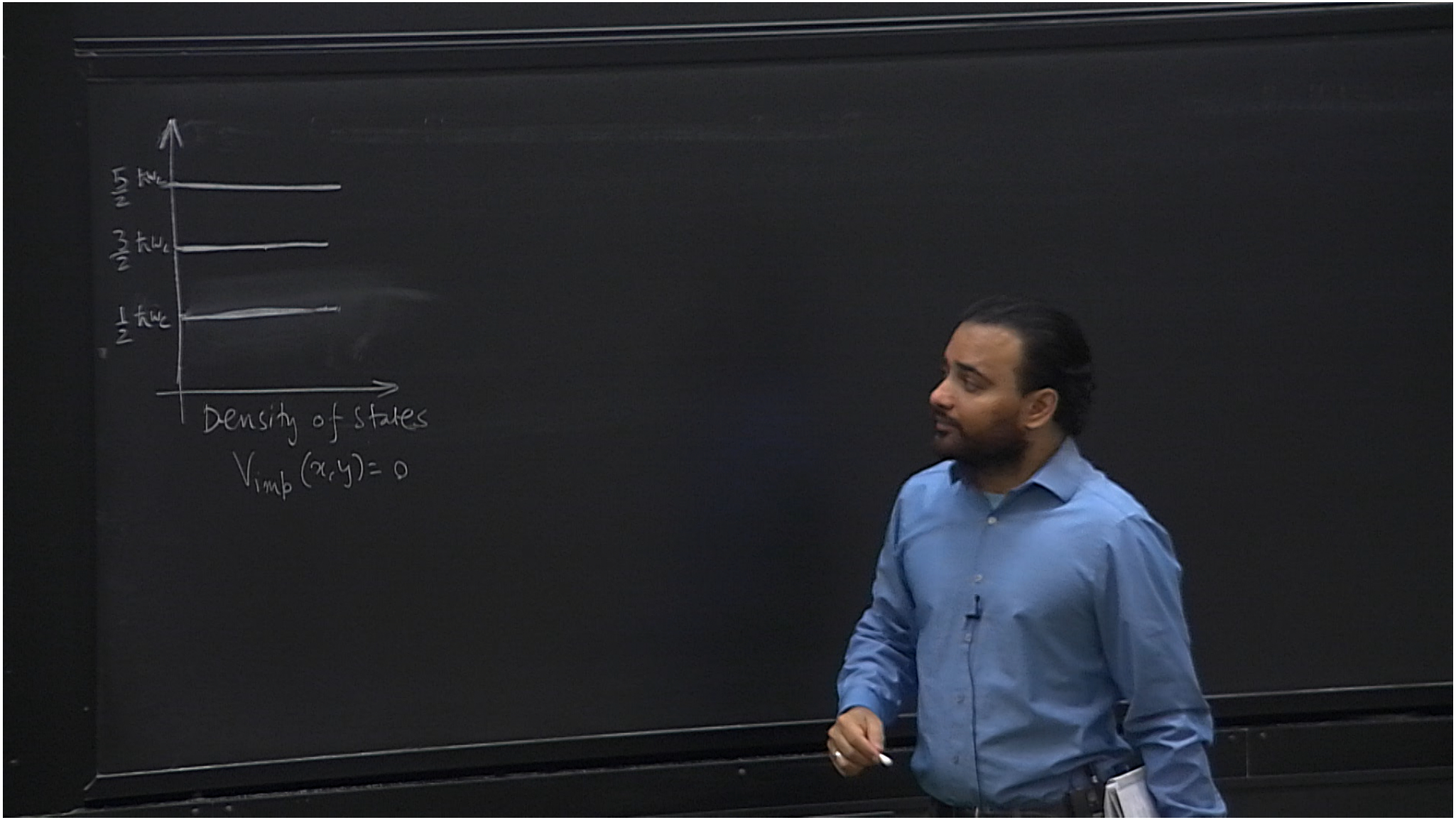

B
E=0
Ė≠0
$+\frac{mE^2}{2B^2}$
E
$\ell = \sqrt{\frac{\hbar}{eB}}$
$v_{drift} = \bar{E} \times \bar{B}$
$(-\bar{\nabla} V \times B)$
$(\hat{z} \times \bar{\nabla} V)$
$V_{imp}(x, y)$
$\bar{E} = -\bar{\nabla} V$
V changes on length scales $\gg \ell$
equipotential contour

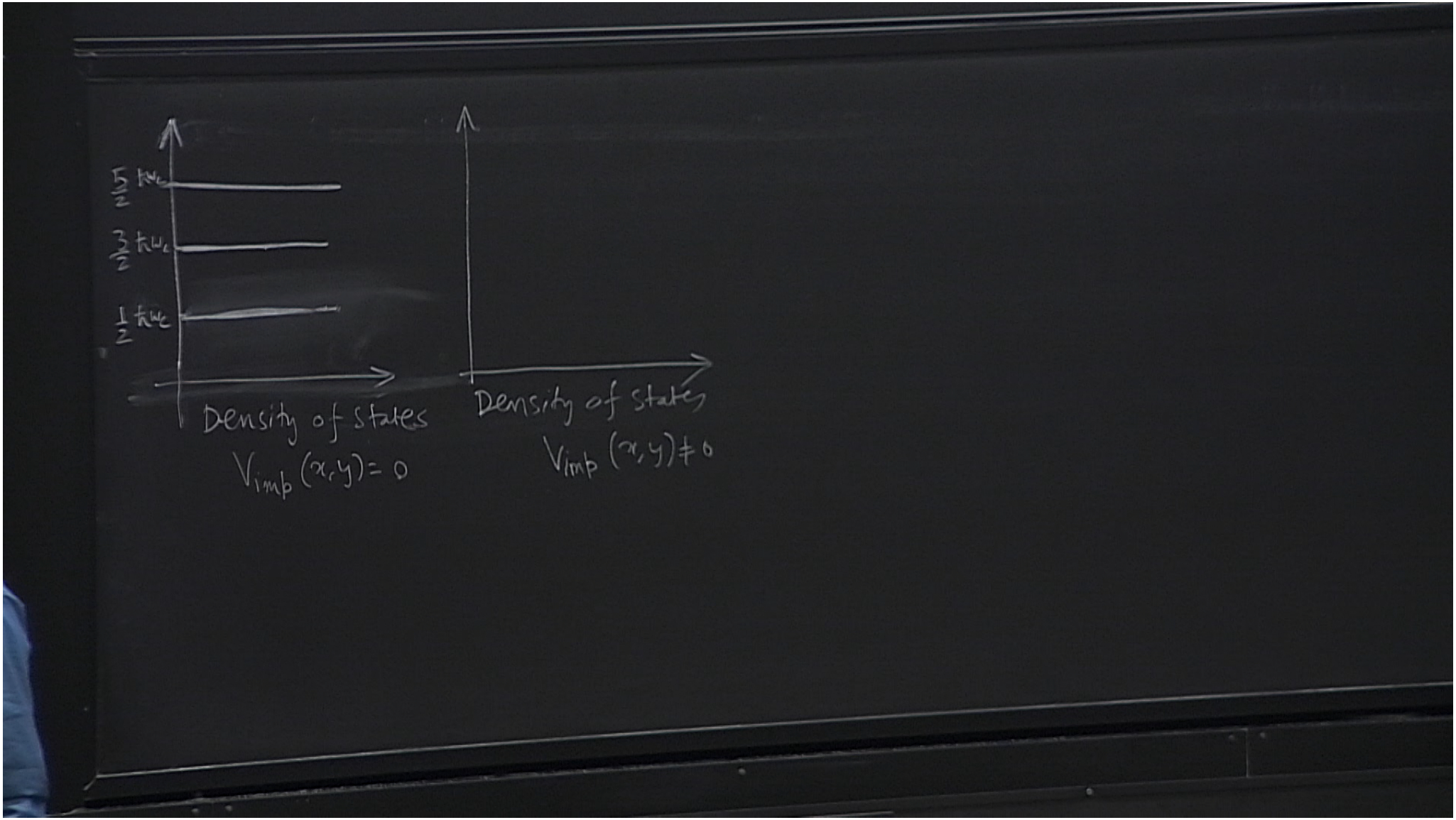

B
E=0
Ė≠0
+ mE²/2B²
E
ℓ = √(ħ/eB)
v_drift = E × B
(−∇V × B)
(ẑ × ∇V)
E = −∇V
V changes on length scales ≫ ℓ
equipotential contour
V_imp (x, y)

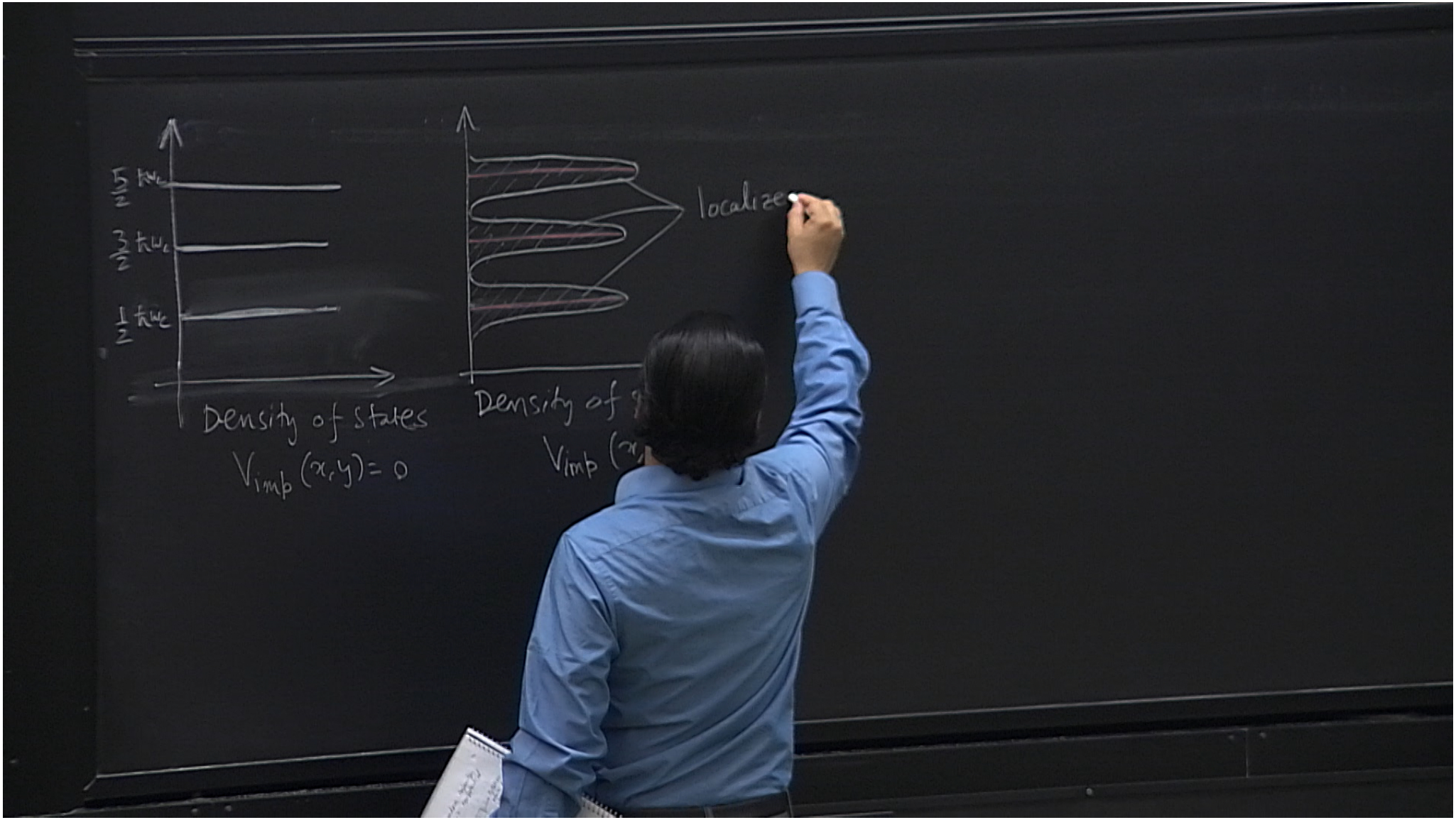

B
E=0
Ė≠0
+ mE²/2B²
E
ℓ = √(ħ/eB)
v_drift = E × B
(−∇V × B)
(ẑ × ∇V)
E = −∇V
V changes on length scales ≫ ℓ
equipotential contour
V_imp (x, y)


B
E=0
Ė≠0
$+\frac{mE^2}{2B^2}$
E
$\ell = \sqrt{\frac{\hbar}{eB}}$
$v_{drift} = \vec{E} \times \vec{B}$
$(-\vec{\nabla} V \times B)$
$(\hat{z} \times \vec{\nabla} V)$
$V_{imp}(x, y)$
$\vec{E} = -\vec{\nabla} V$
V changes on length scales $\gg \ell$
equipotential contour

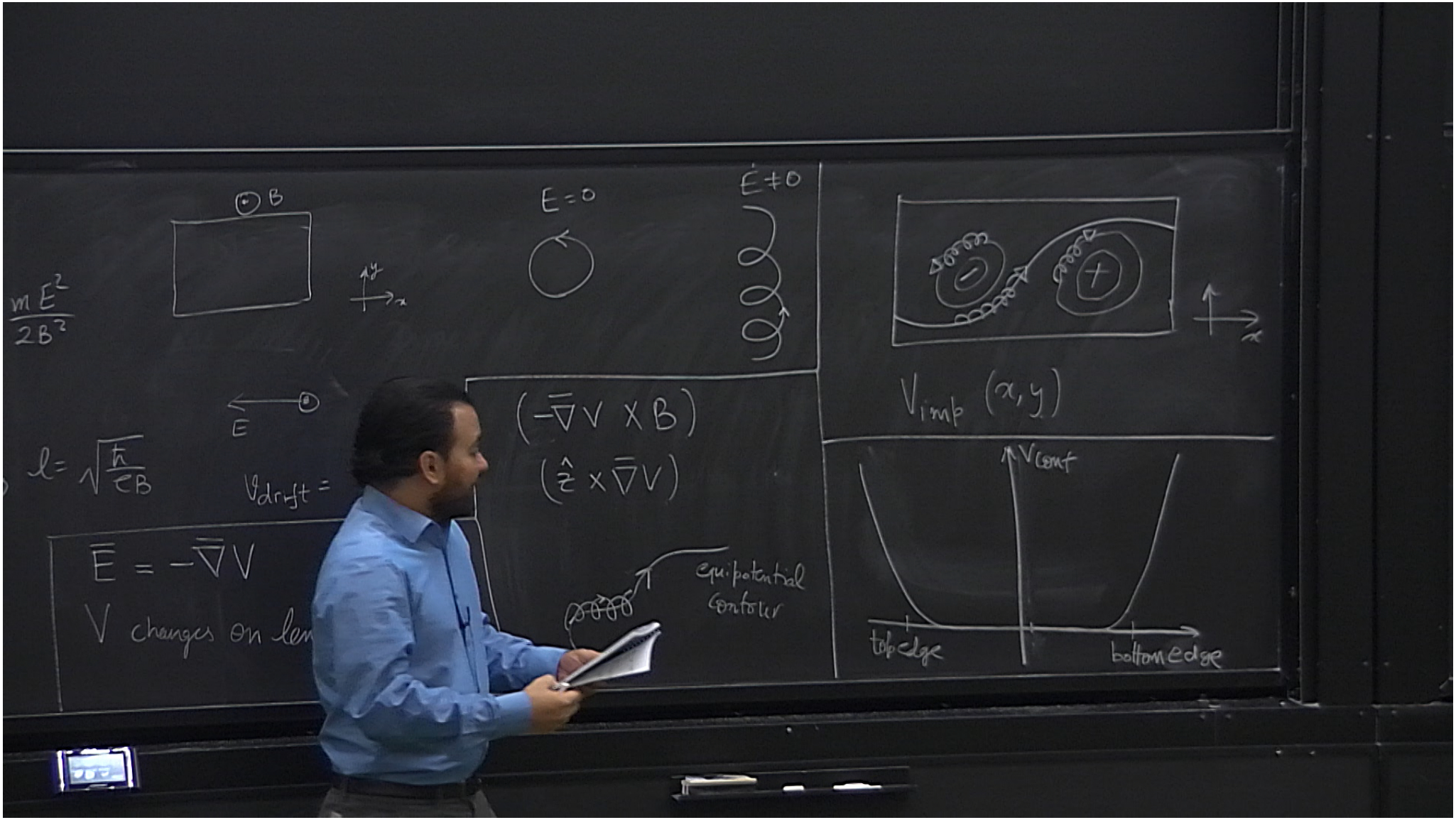

Density of states
$V_{imp}(x,y) = 0$

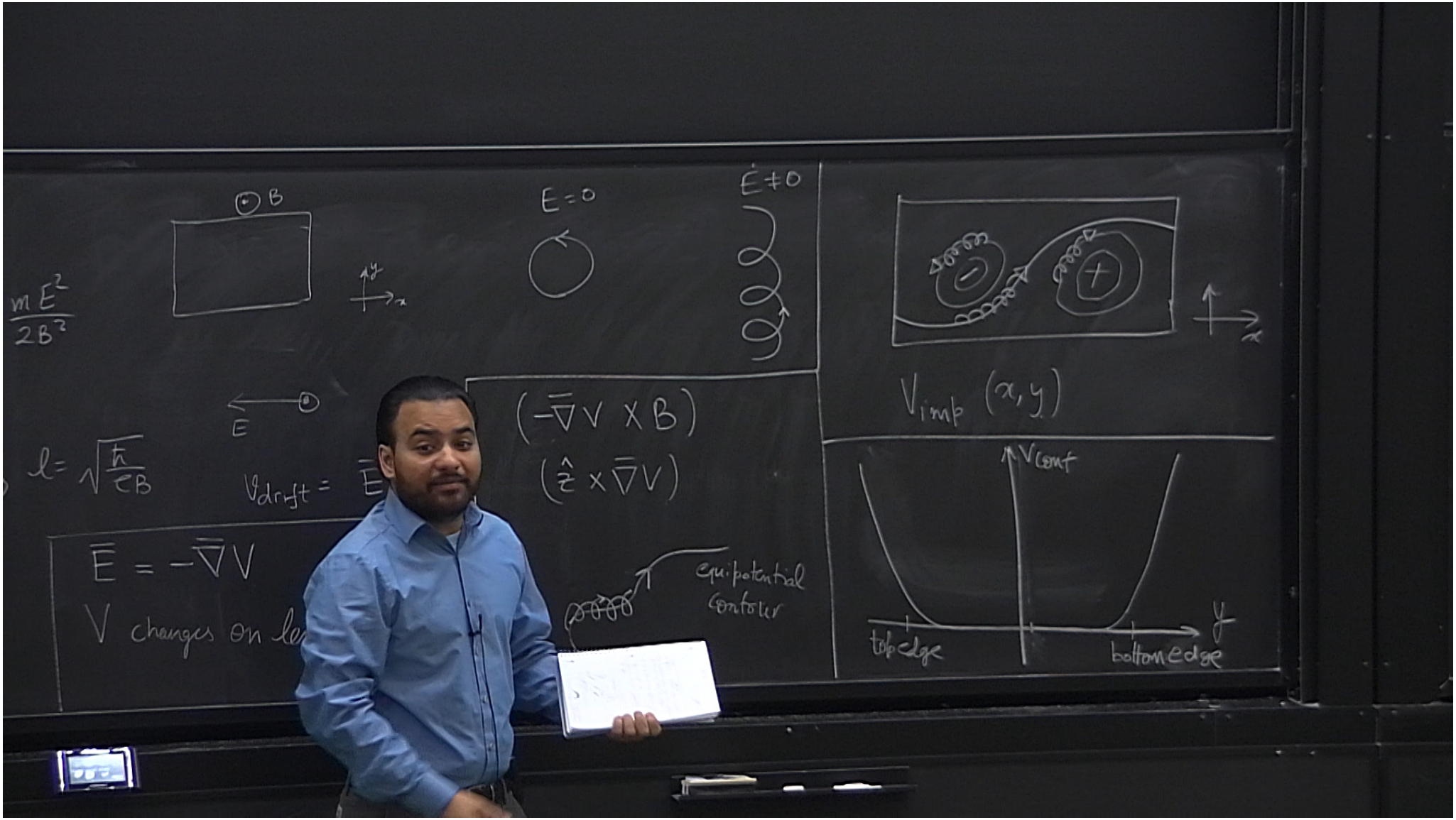

$\frac{5}{2}\hbar\omega_c$
$\frac{3}{2}\hbar\omega_c$
$\frac{1}{2}\hbar\omega_c$
Density of states
$V_{imp}(x,y) = 0$

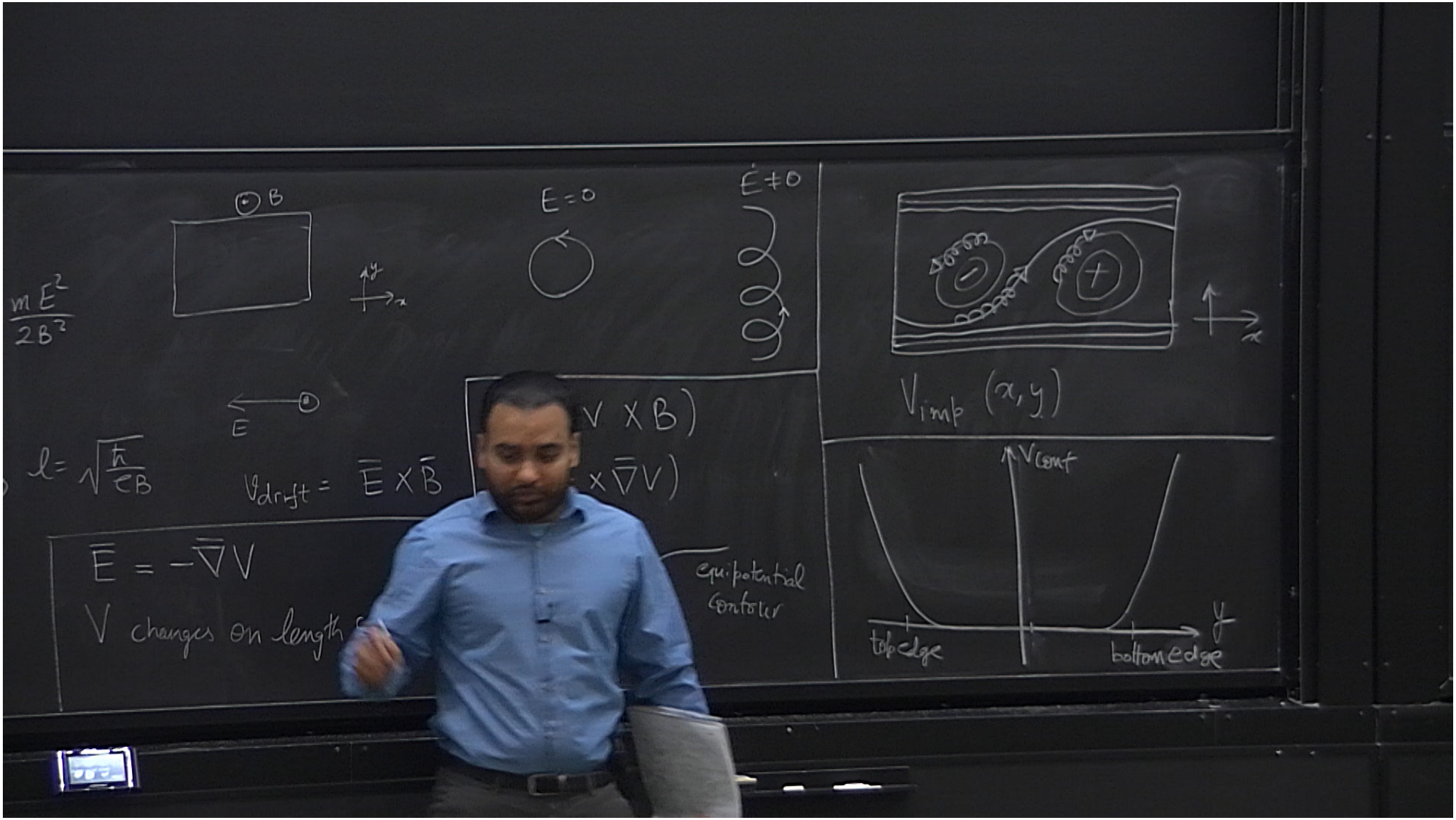

$\frac{5}{2}\hbar\omega_c$
$\frac{3}{2}\hbar\omega_c$
$\frac{1}{2}\hbar\omega_c$
Density of states
$V_{imp}(x,y) = 0$
Density of states
$V_{imp}(x,y) \neq 0$

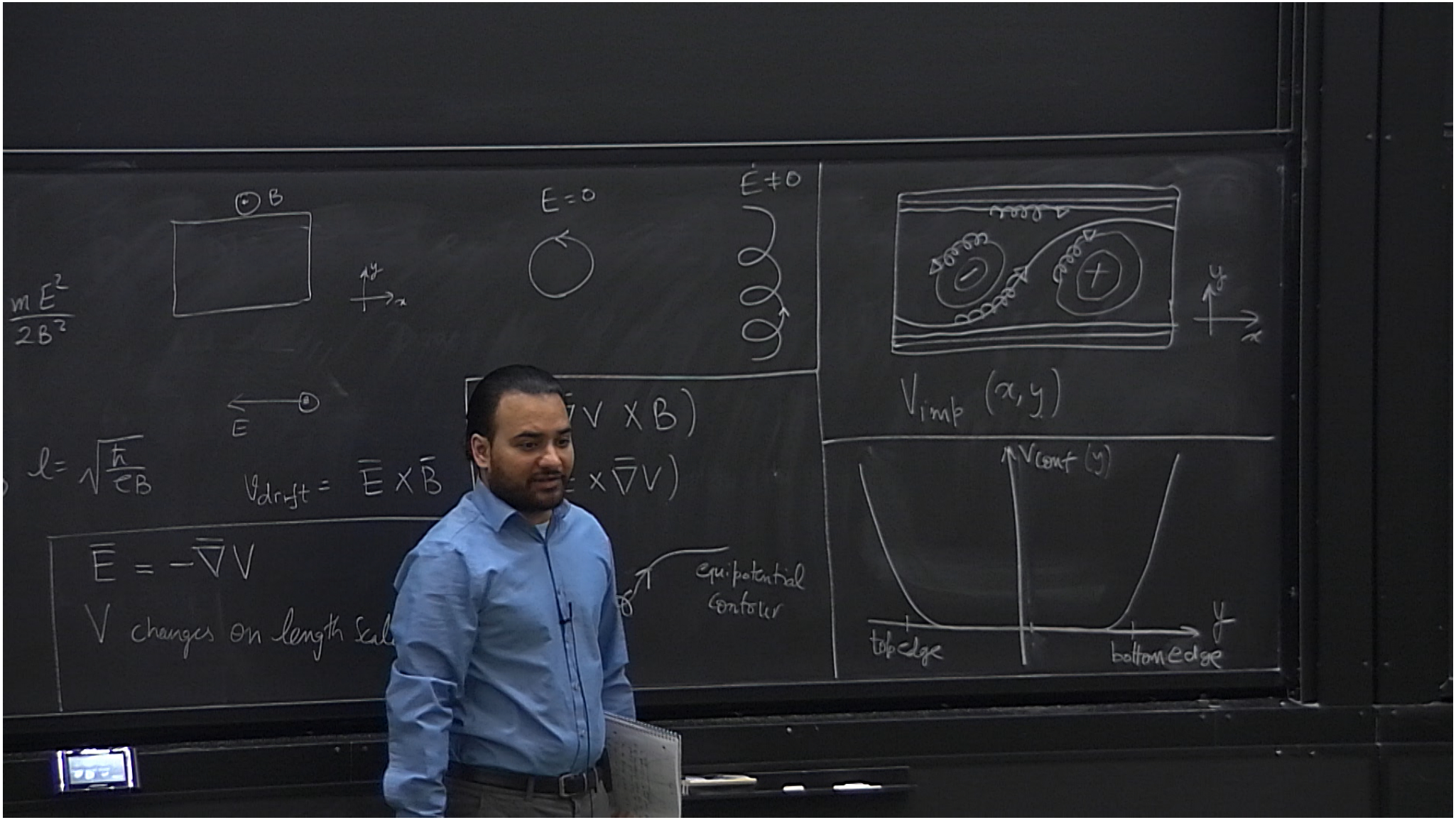

$\frac{5}{2}\hbar\omega_c$
$\frac{3}{2}\hbar\omega_c$
$\frac{1}{2}\hbar\omega_c$
Density of states
$V_{imp}(x,y) = 0$
localize
Density of
V_{imp}

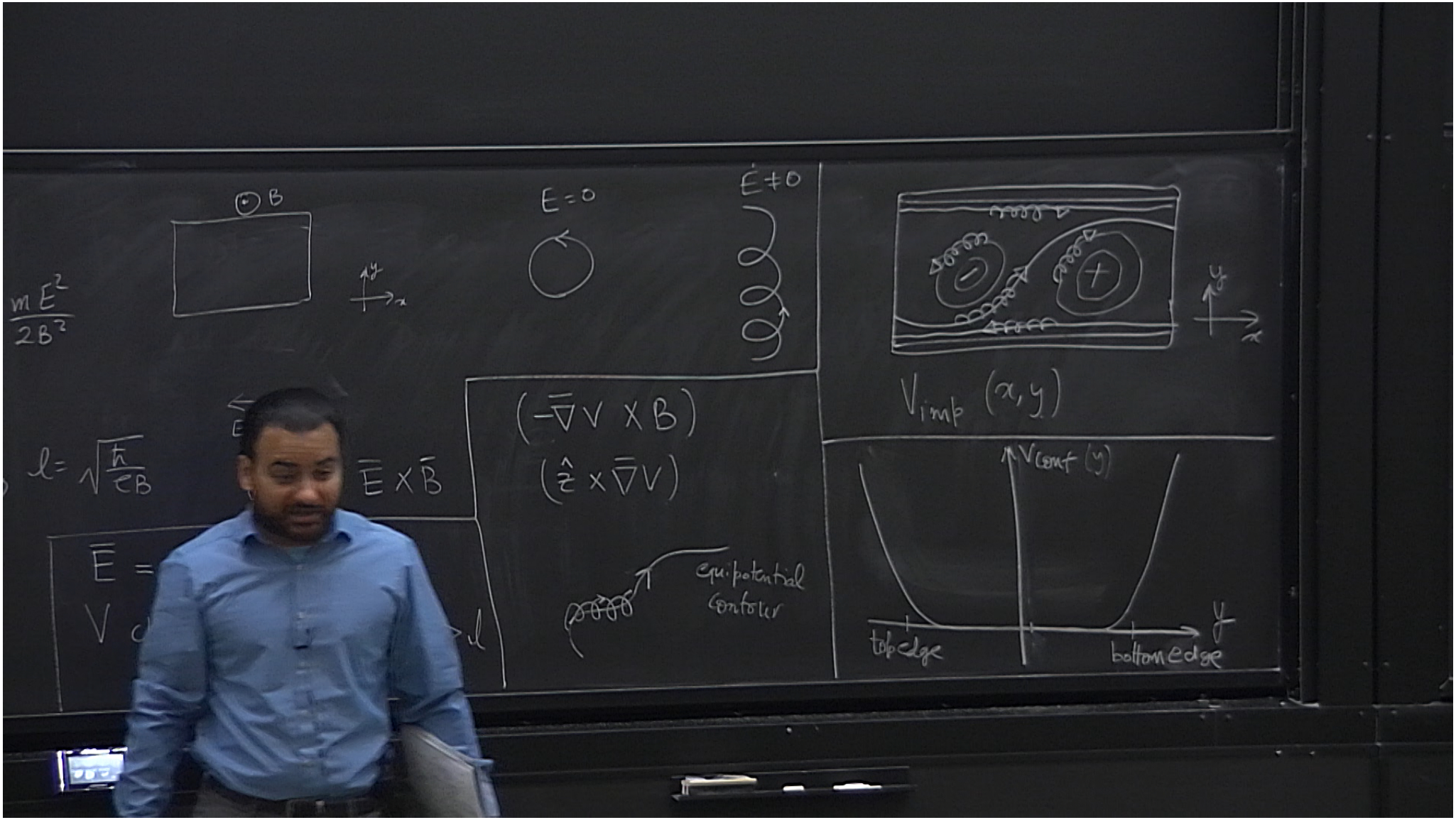

$\frac{5}{2}\hbar\omega_c$
$\frac{3}{2}\hbar\omega_c$
$\frac{1}{2}\hbar\omega_c$
Density of states
$V_{imp}(x,y) = 0$
localized
Extended
Density of states
$V_{imp}(x,y) \neq 0$

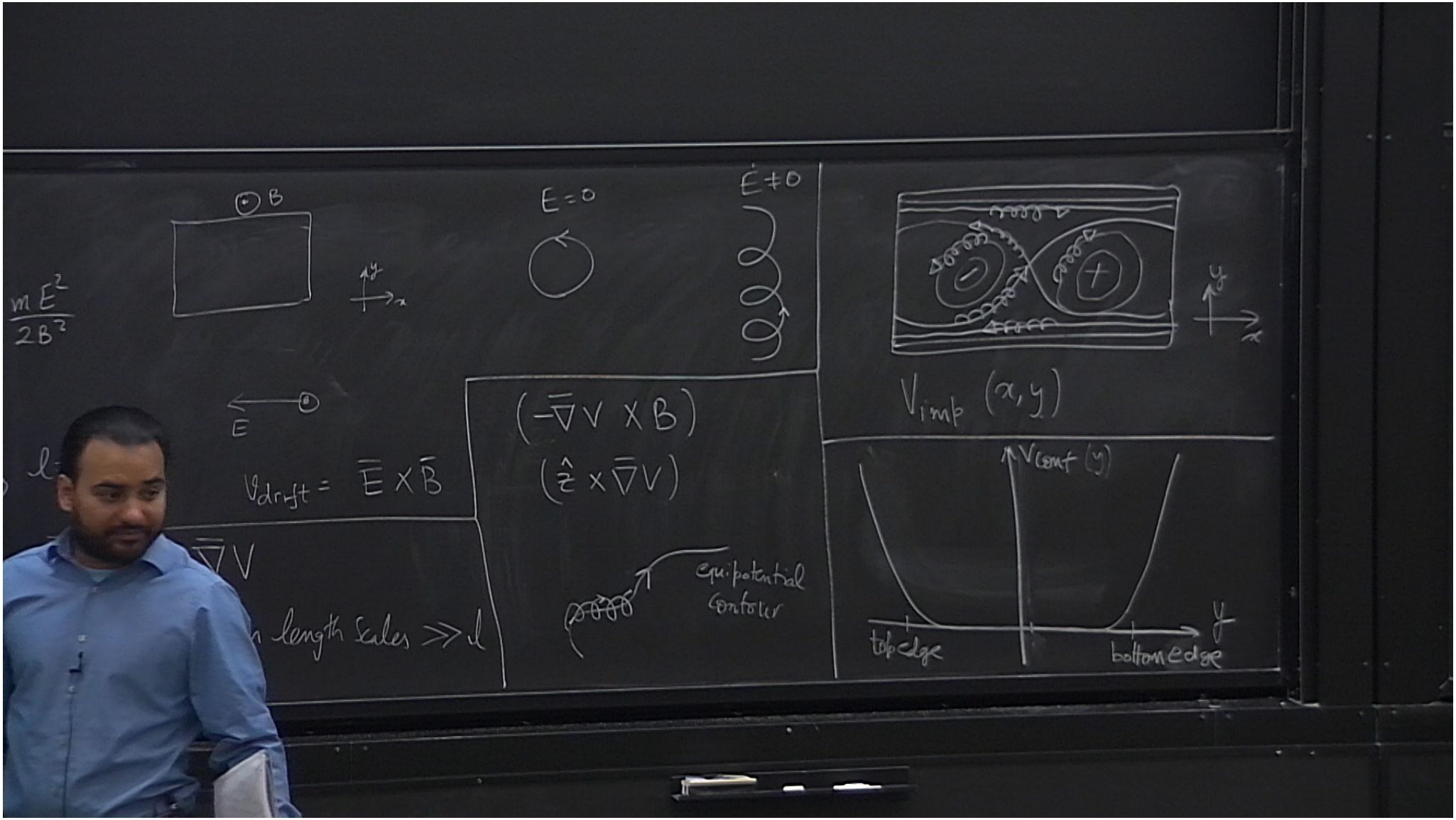

B
E = 0
Ė ≠ 0
E
$\frac{mE^2}{2B^2}$
$\ell = \sqrt{\frac{\hbar}{eB}}$
$v_{drift} =$
$\vec{E} = -\vec{\nabla}V$
V changes on len
$(-\vec{\nabla}V \times B)$
$(\hat{z} \times \vec{\nabla}V)$
equipotential contour
$V_{imp}(x,y)$
V_{cent}
top edge
bottom edge

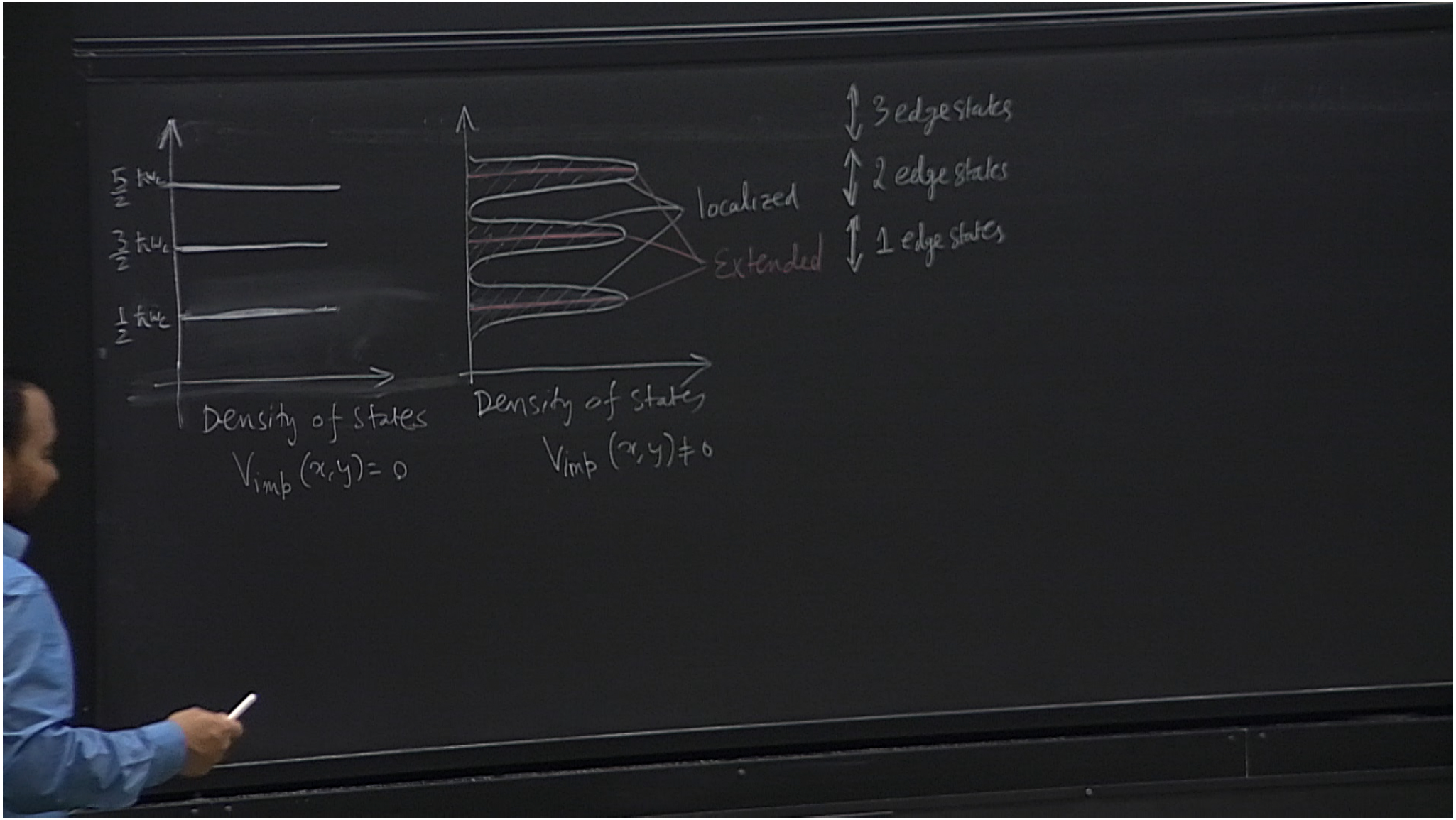

B
E = 0
Ė ≠ 0
E
$V_{imp}(x,y)$
$\bar{E} = -\bar{\nabla}V$
equipotential contour
top edge
bottom edge

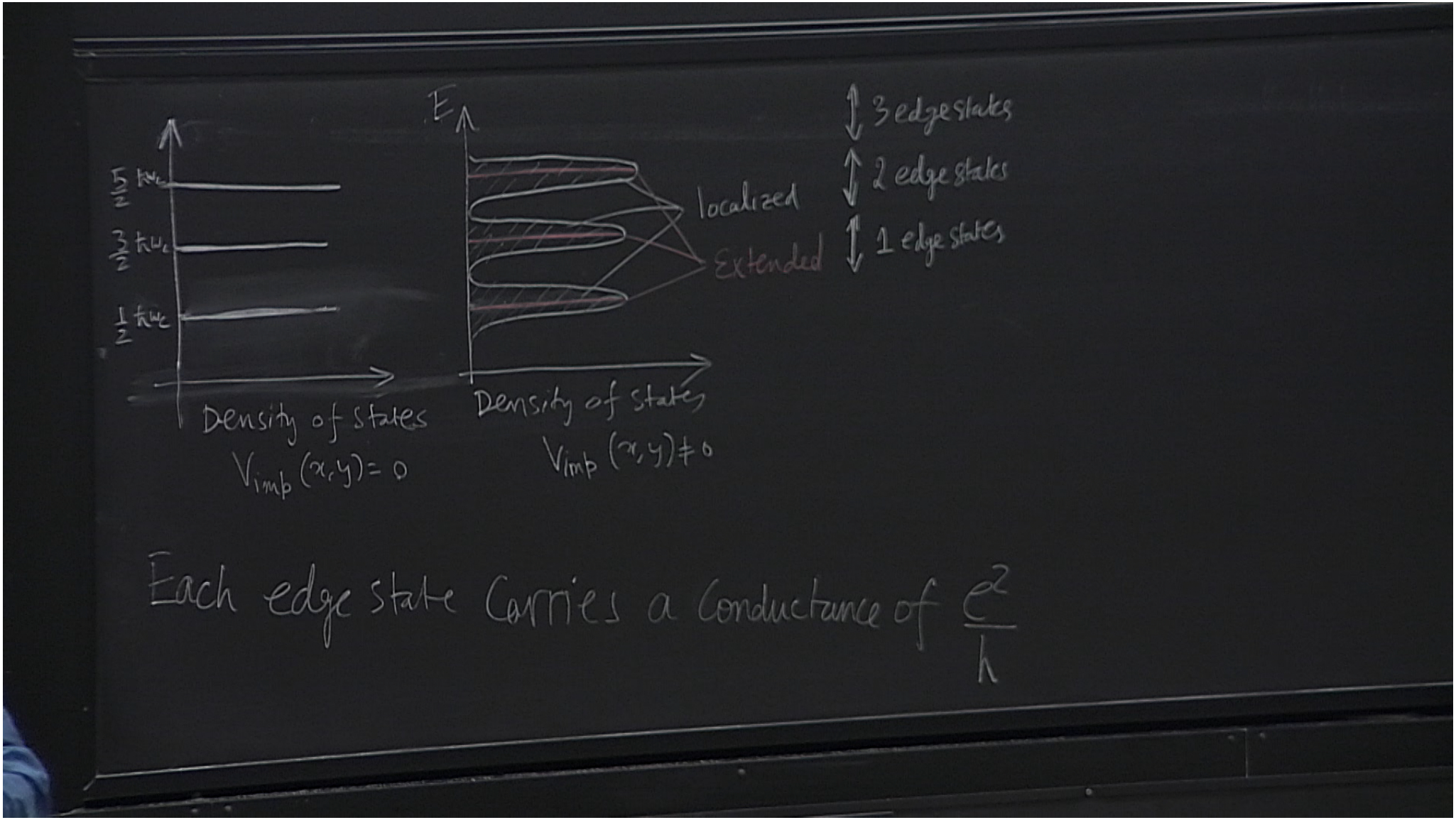

B
E = 0
Ė ≠ 0
E
$\frac{mE^2}{2B^2}$
$\ell = \sqrt{\frac{\hbar}{eB}}$
$v_{drift} = \vec{E} \times \vec{B}$
$\vec{E} = -\vec{\nabla} V$
V changes on length
equipotential contour
$V_{imp}(x, y)$
V_{cent}
top edge
bottom edge
y

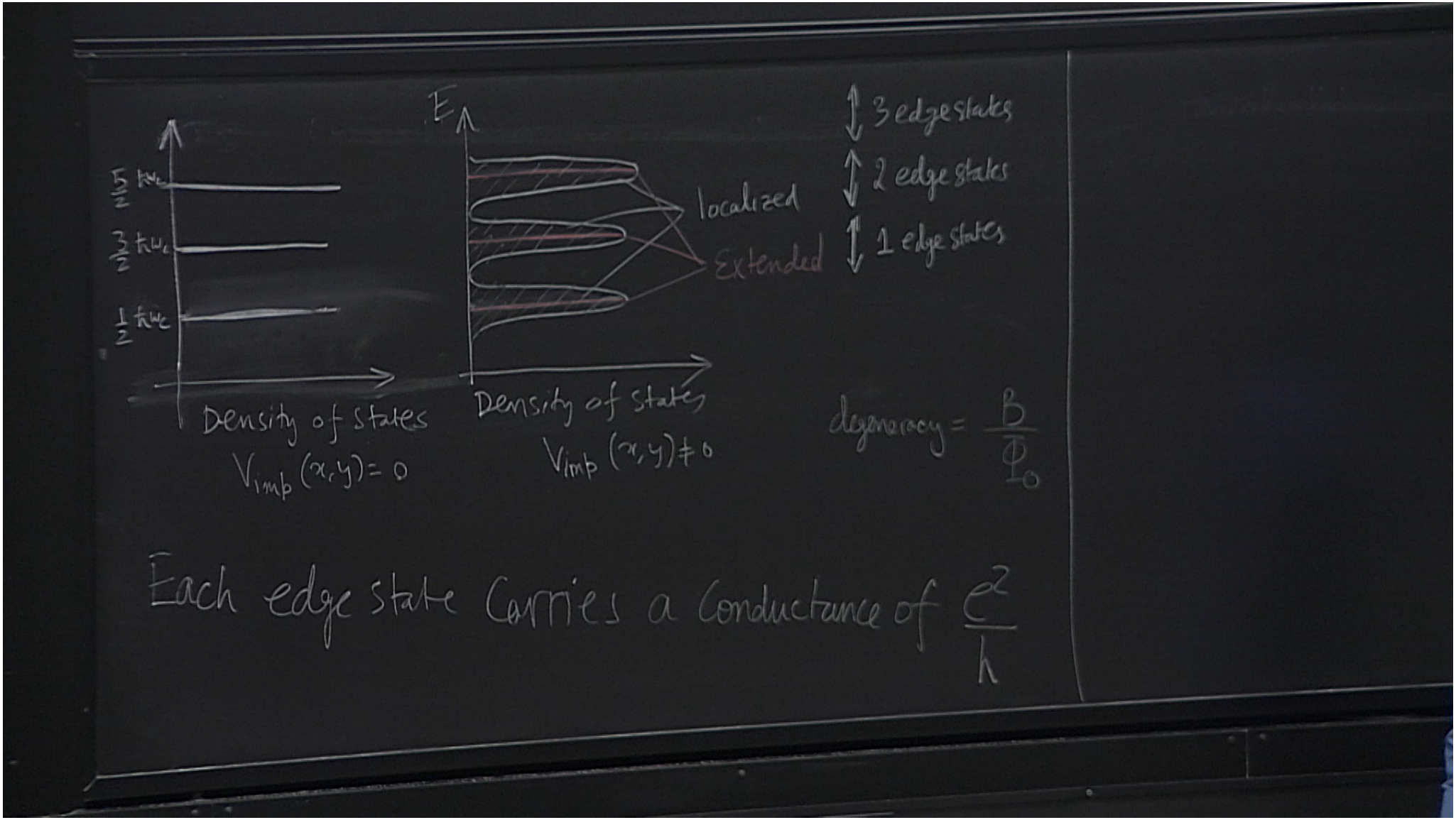

B
E = 0
Ė ≠ 0
$\frac{mE^2}{2B^2}$
E
$\ell = \sqrt{\frac{\hbar}{eB}}$
$v_{drift} = \vec{E} \times \vec{B}$
$\vec{E} = -\vec{\nabla} V$
V changes on length scal
equipotential contour
$V_{imp}(x, y)$
$V_{cent}(y)$
top edge
bottom edge
y

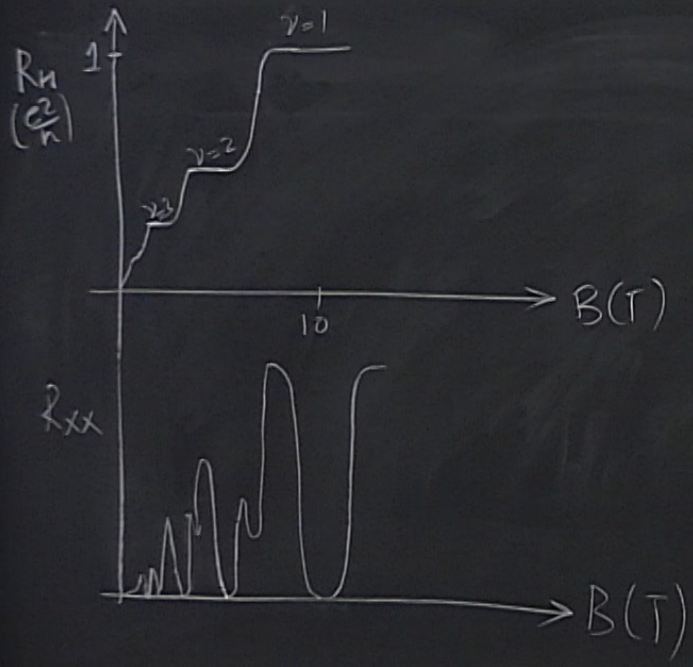

B
E = 0
Ė ≠ 0
$\frac{mE^2}{2B^2}$
$V_{imp}(x,y)$
$(-\bar{\nabla}V \times B)$
$(\hat{z} \times \bar{\nabla}V)$
$\bar{E} \times \bar{B}$
$\ell = \sqrt{\frac{\hbar}{eB}}$
equipotential contour
$V_{cent}(y)$
top edge
bottom edge
y


B
E = 0
Ė ≠ 0
mE²/2B²
E
v_drift = E × B
(−∇V × B)
(ẑ × ∇V)
equipotential contour
length scales ≫ d
V_imp (x, y)
V_cent (y)
top edge
bottom edge
y


$\frac{5}{2}\hbar\omega_c$
$\frac{3}{2}\hbar\omega_c$
$\frac{1}{2}\hbar\omega_c$
Density of states
$V_{imp}(x,y) = 0$
localized
Extended
Density of states
$V_{imp}(x,y) \neq 0$
3 edge states
2 edge states
1 edge states


E
$\frac{5}{2}\hbar\omega_c$
$\frac{3}{2}\hbar\omega_c$
$\frac{1}{2}\hbar\omega_c$
Density of states
$V_{imp}(x,y) = 0$
localized
Extended
Density of states
$V_{imp}(x,y) \neq 0$
3 edge states
2 edge states
1 edge states
Each edge state Carries a Conductance of $\frac{e^2}{h}$


E
$\frac{5}{2}\hbar\omega_c$
$\frac{3}{2}\hbar\omega_c$
$\frac{1}{2}\hbar\omega_c$
Density of states
$V_{imp}(x,y) = 0$
localized
Extended
Density of states
$V_{imp}(x,y) \neq 0$
3 edge states
2 edge states
1 edge states
degeneracy $= \frac{B}{\Phi_0}$
Each edge state carries a Conductance of $\frac{e^2}{h}$



$$H = \frac{1}{2m}\left(p_x^2 + (p_y + eBx)^2\right) - eEx$$

$$\Rightarrow E_{nk} = \hbar\omega_c\left(n+\tfrac{1}{2}\right) + eE\left(k\ell^2 - \frac{eE}{m\omega_c^2}\right) + \frac{mE^2}{2B^2}$$

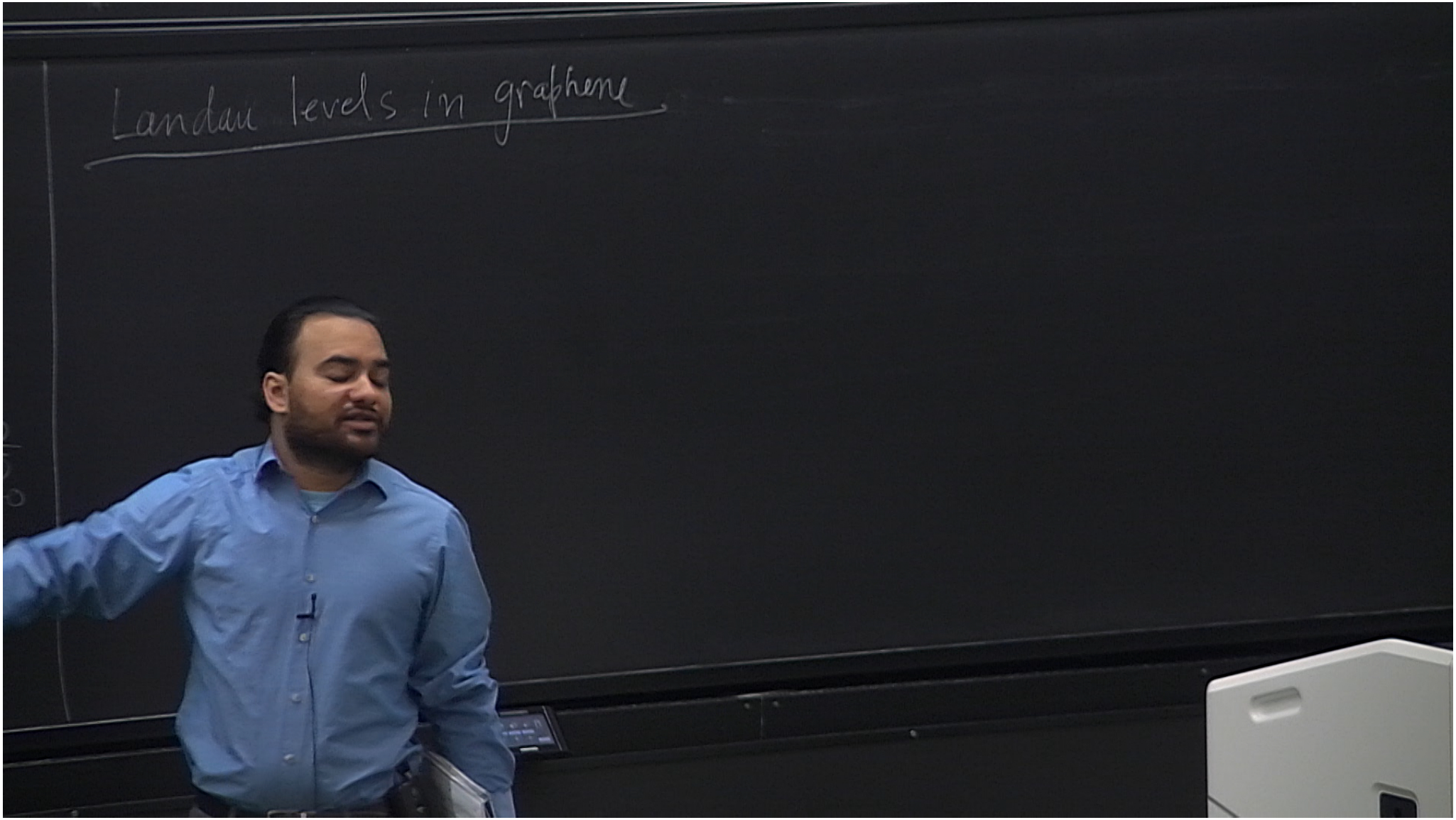
$$\psi_{nk}(x,y) = e^{iky}\, H_n(x-x_0)\, e^{-\frac{(x-x_0)^2}{2\ell^2}}$$

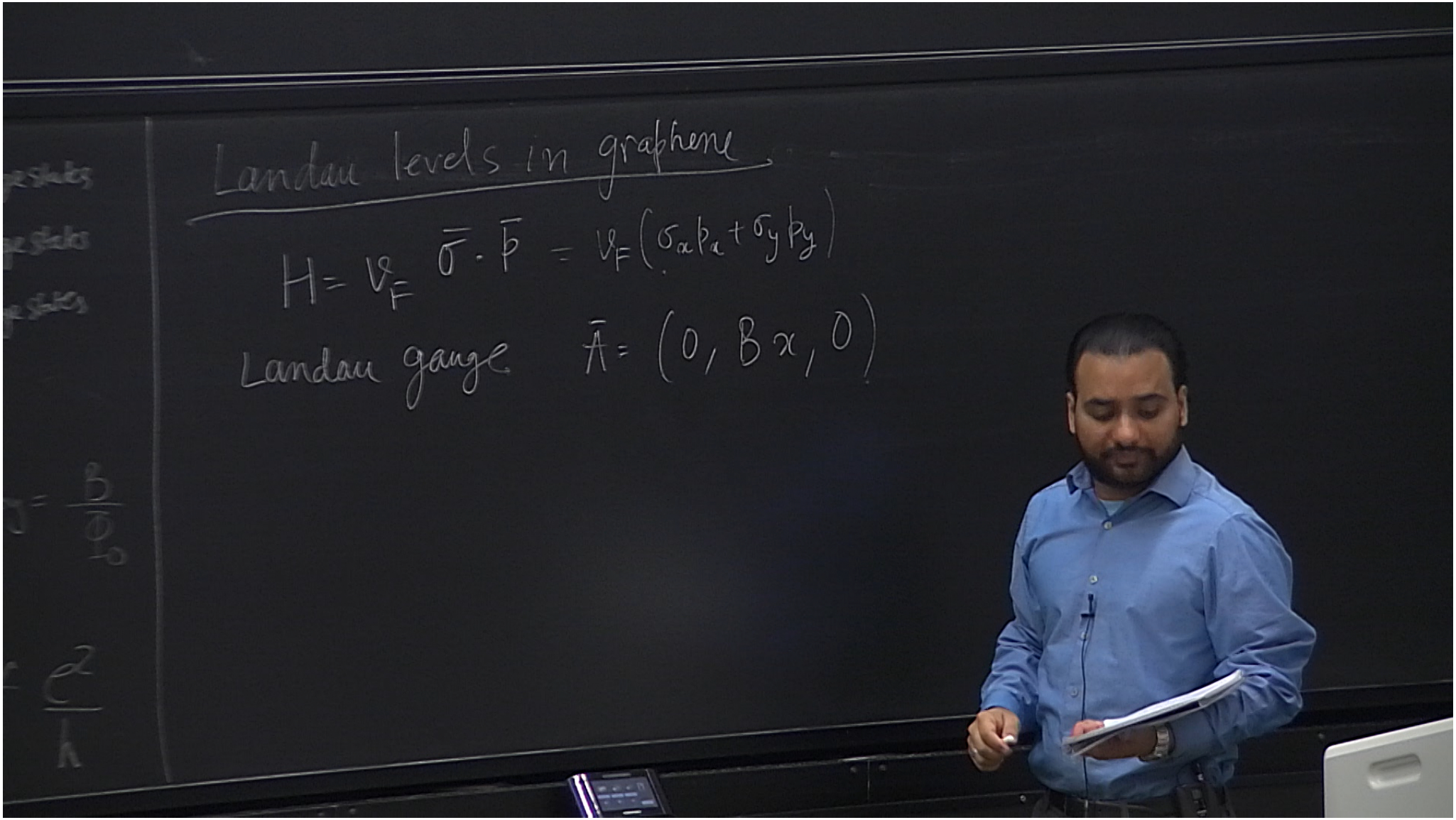
$$\omega_c = \frac{eB}{m}\,, \quad x_0 = -k\ell^2 + \frac{mE}{eB^2}\,, \quad \ell = \sqrt{\frac{\hbar}{eB}}$$

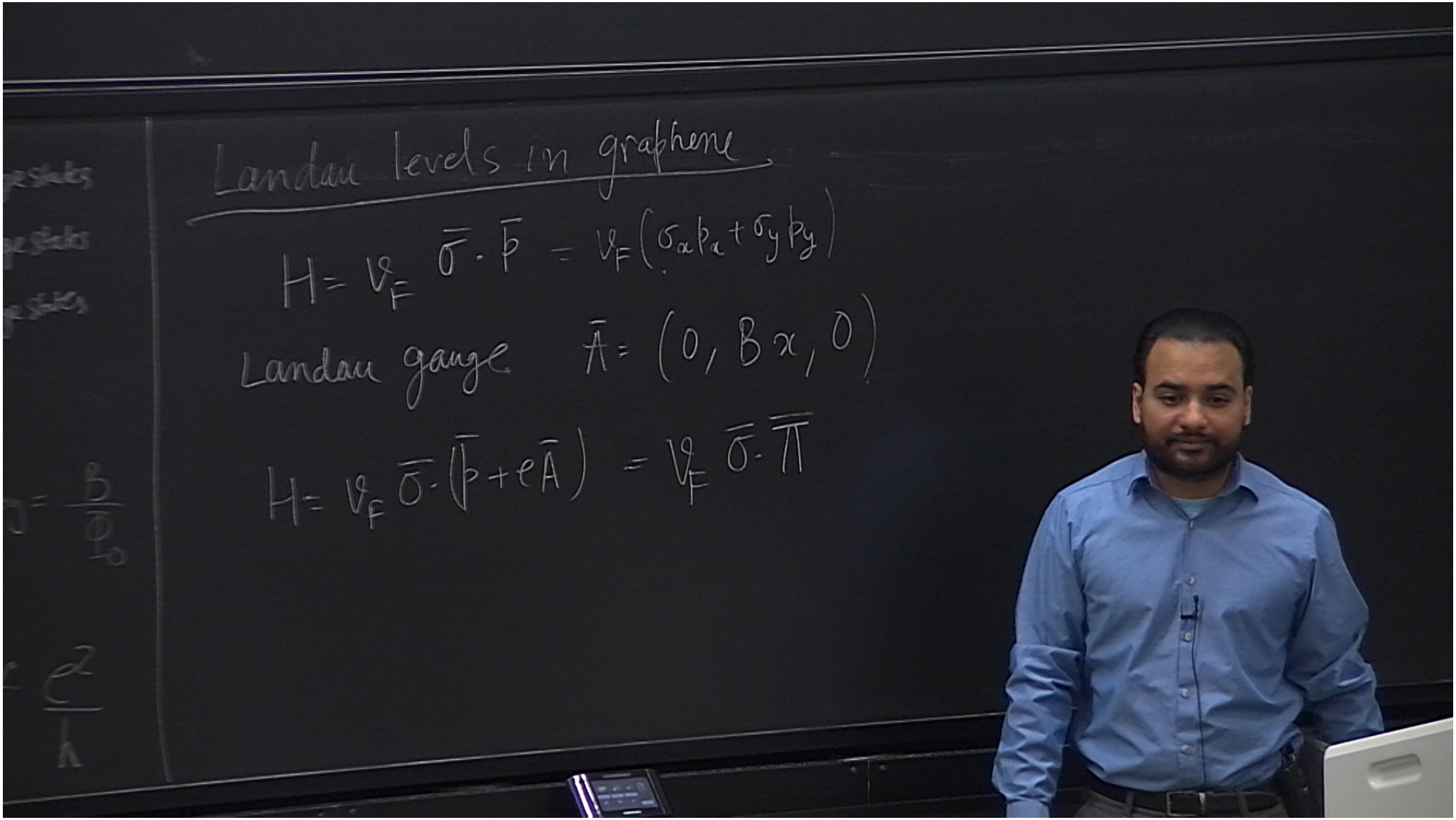
$$v_y = \frac{1}{\hbar}\frac{\partial E_{nk}}{\partial k} = \frac{1}{\hbar}eE\ell^2 = \frac{E}{B}$$

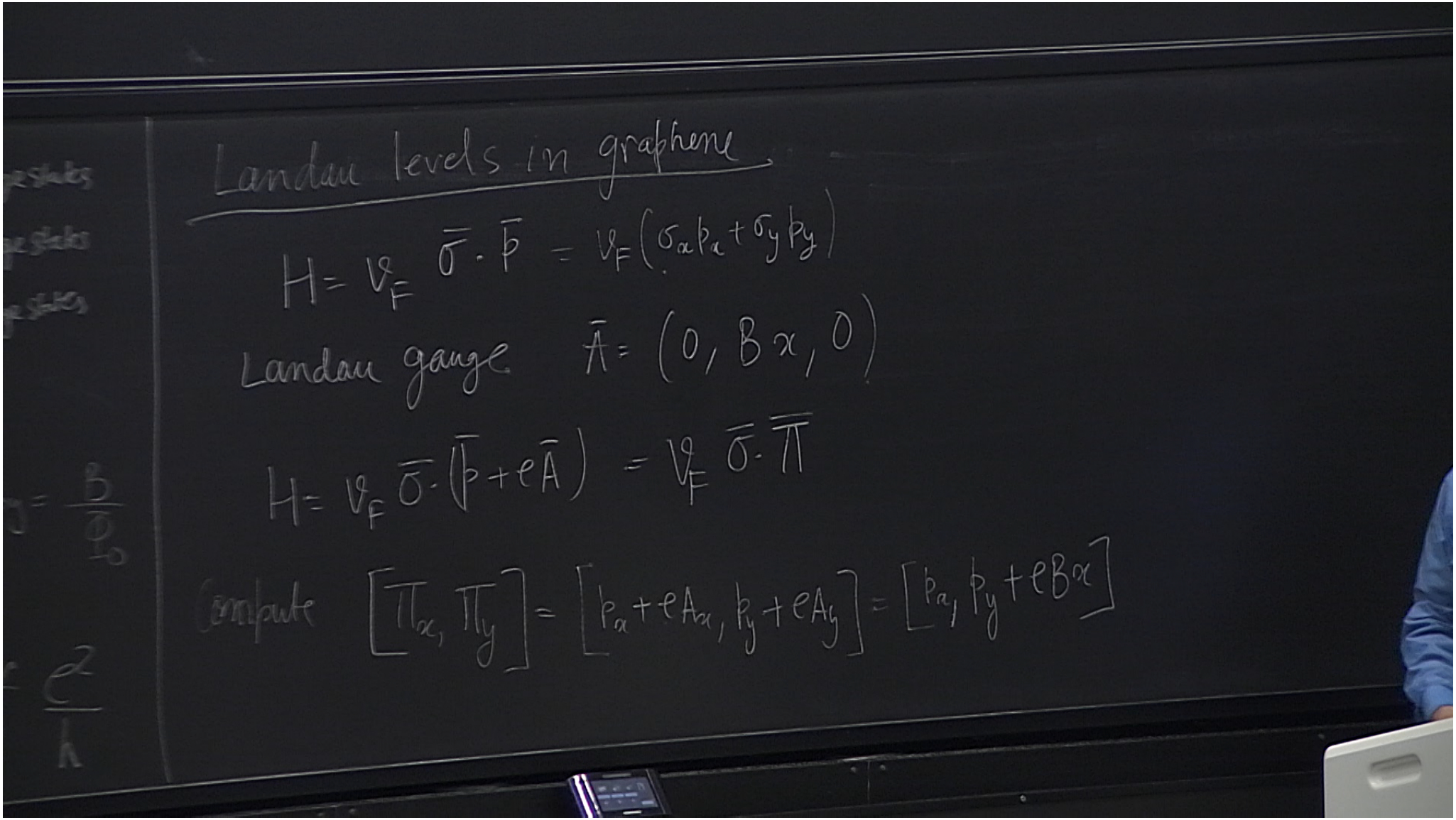
$\vec{E} = -\vec{\nabla}V$

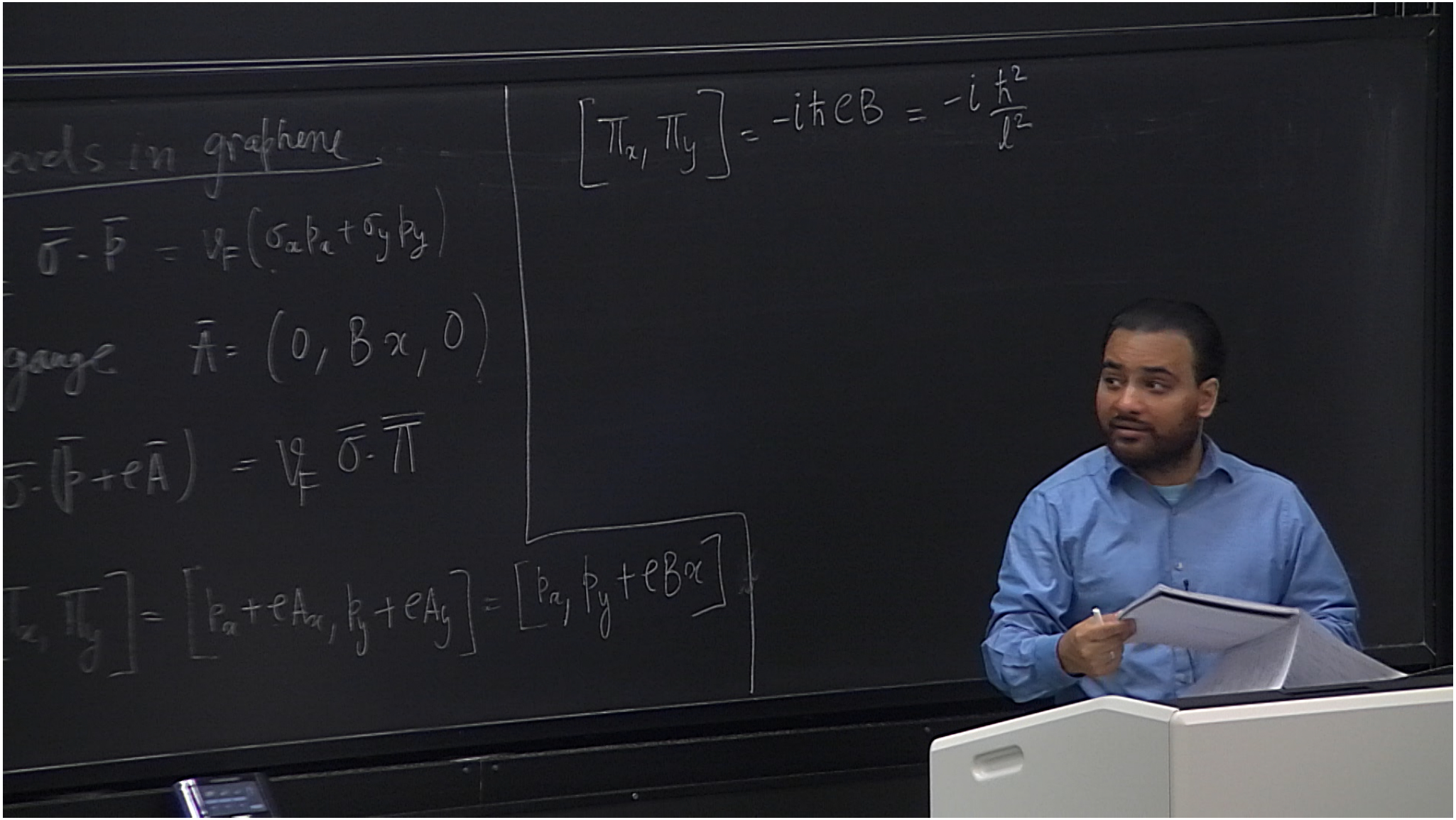
V changes on

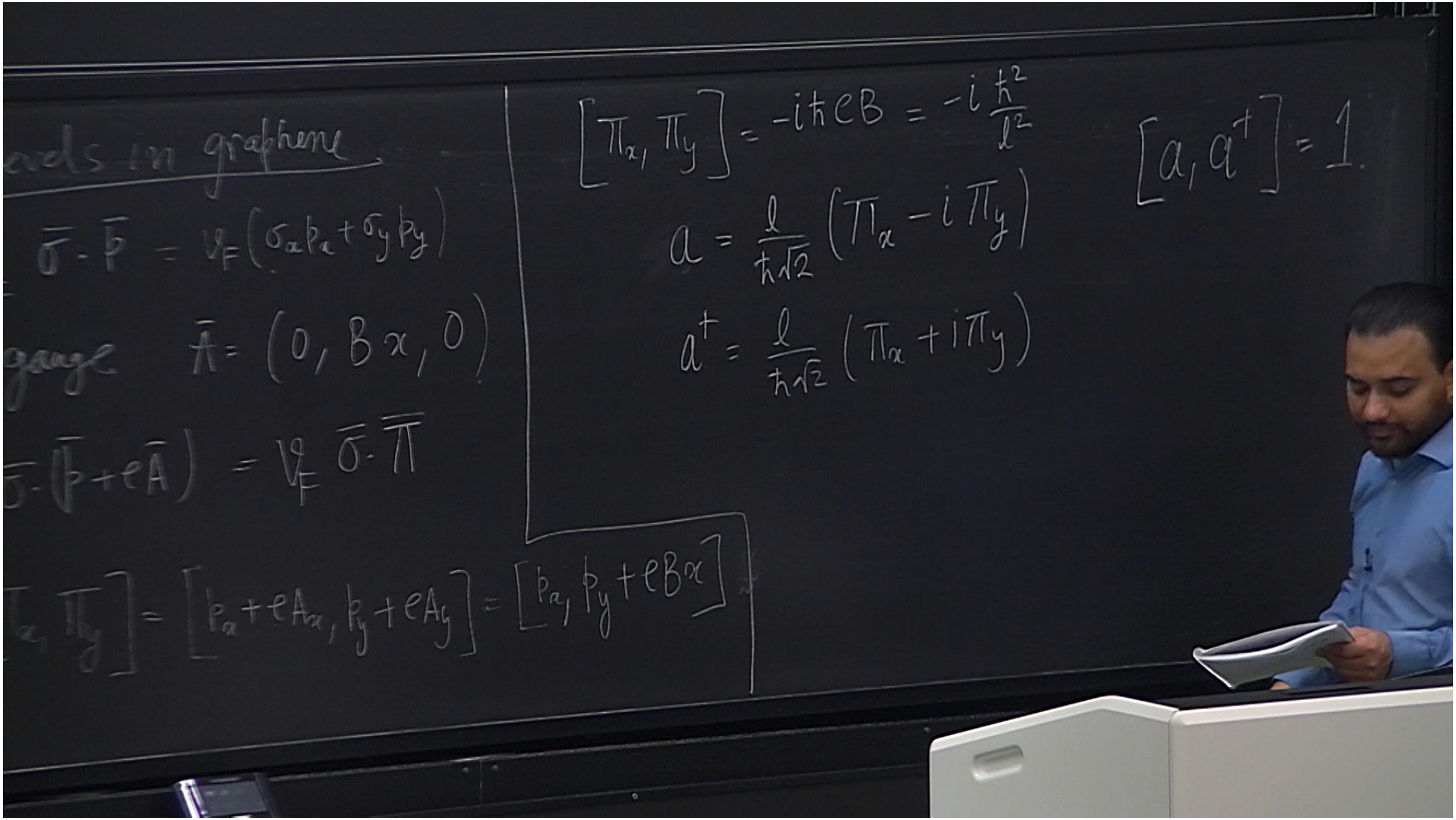

Landau levels in graphene

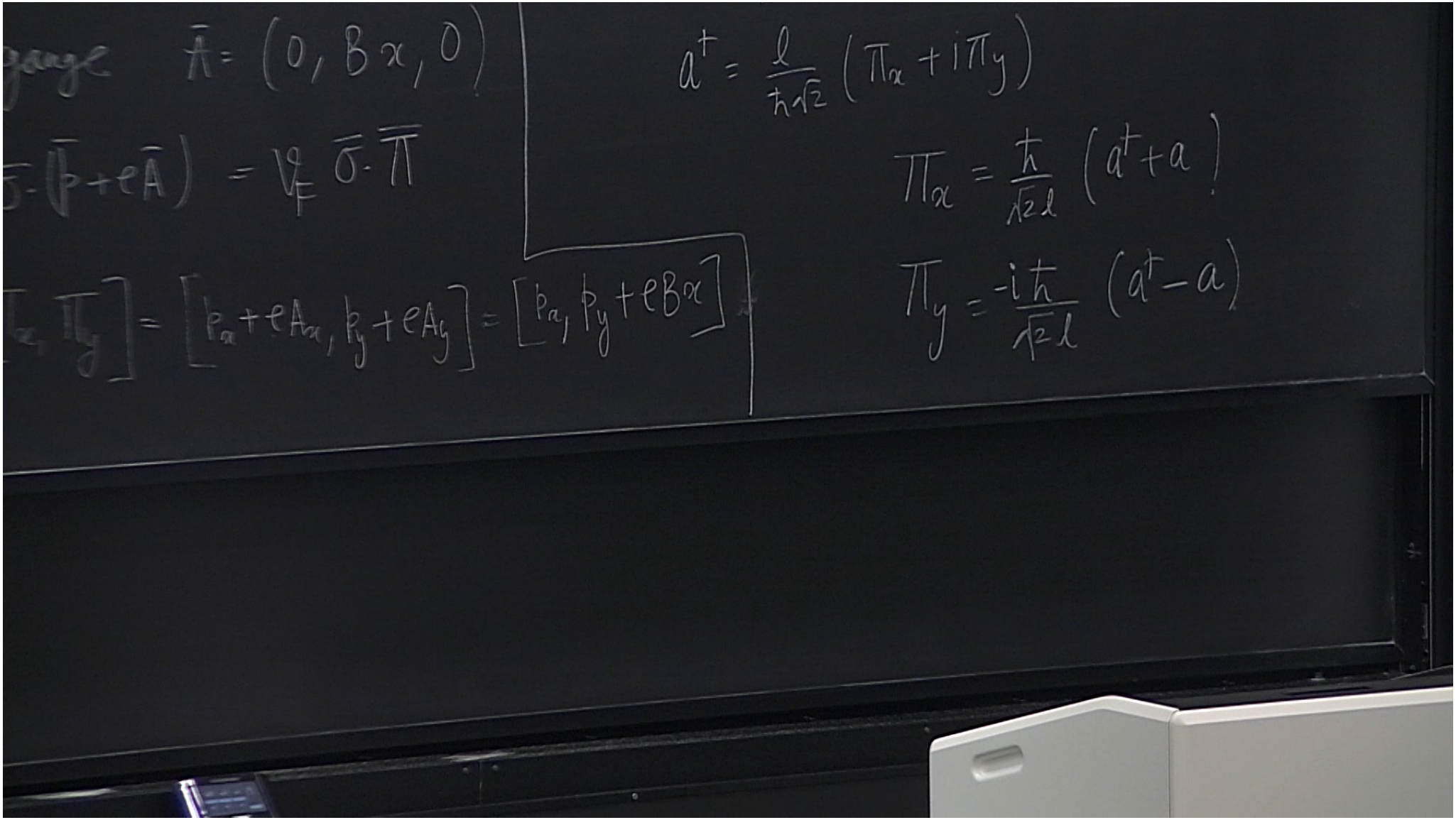

Landau levels in graphene
$H = v_F\, \bar{\sigma} \cdot \bar{p} = v_F(\sigma_x p_x + \sigma_y p_y)$
Landau gauge $\bar{A} = (0, Bx, 0)$

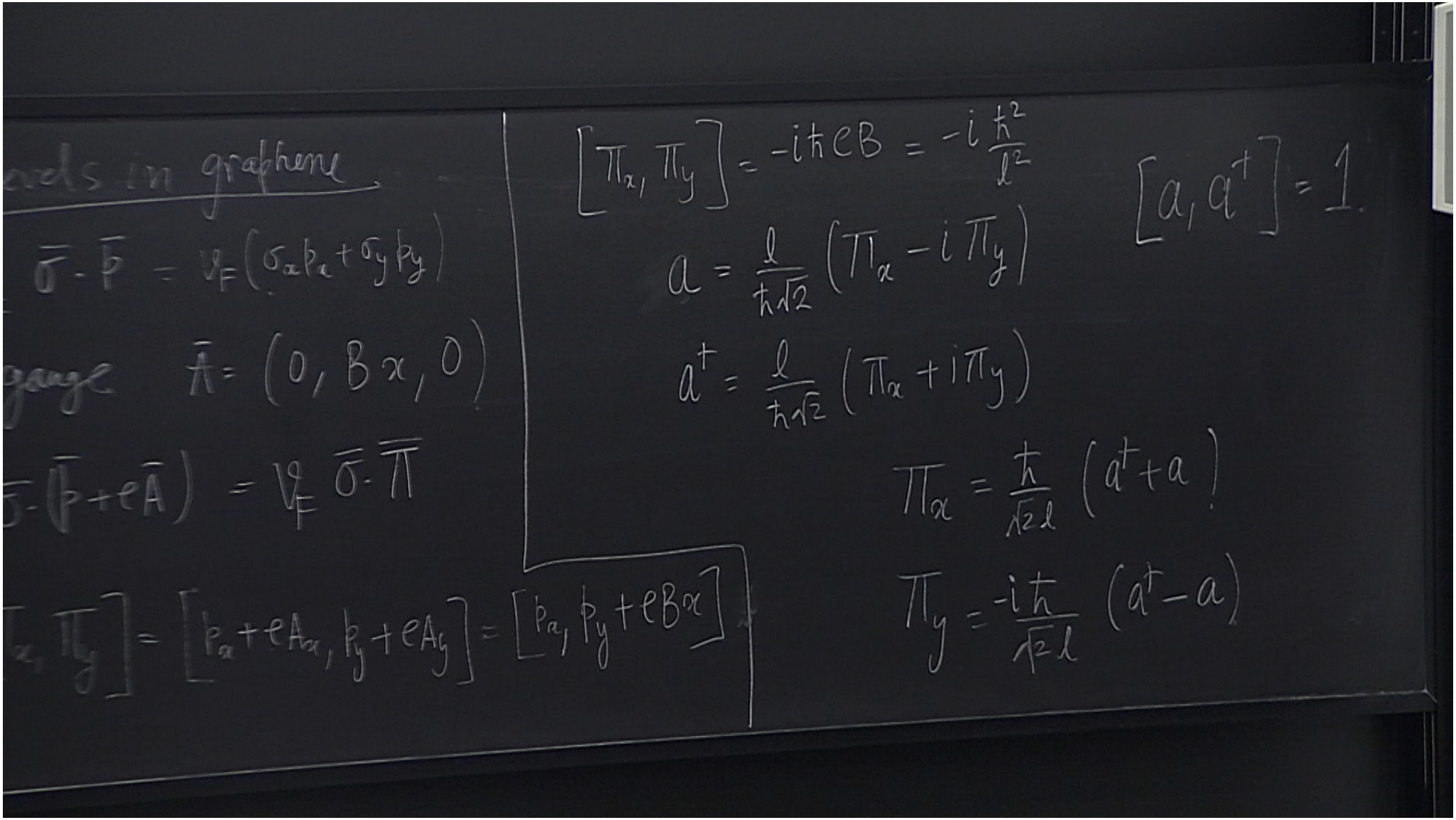

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$H = v_F\, \bar{\sigma}\cdot(\bar{p} + e\bar{A}) = v_F\, \bar{\sigma}\cdot\bar{\Pi}$

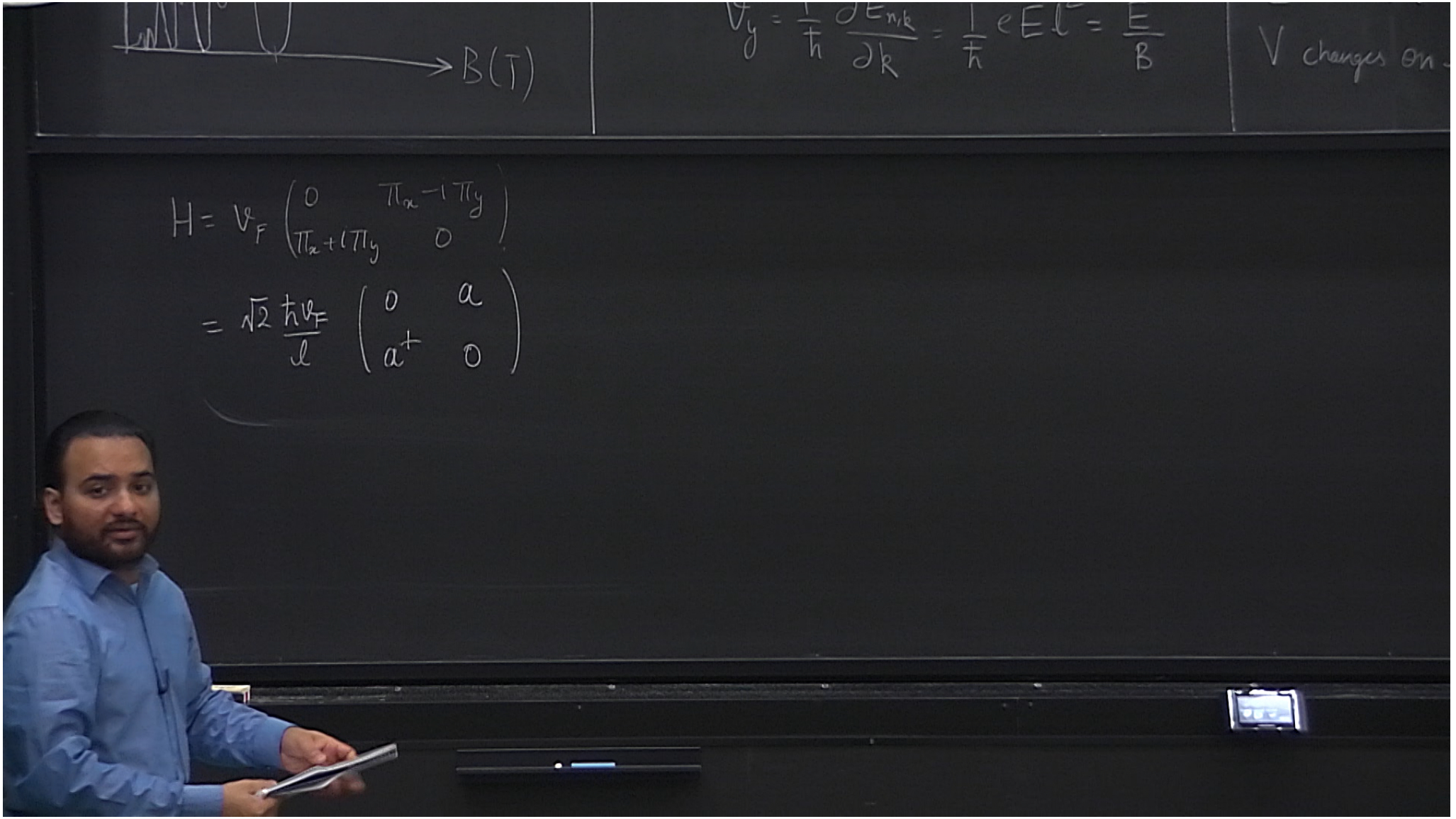

Landau levels in graphene
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$H = v_F\, \bar{\sigma}\cdot(\bar{p} + e\bar{A}) = v_F\, \bar{\sigma}\cdot\bar{\Pi}$
Compute $[\Pi_x, \Pi_y] = [p_x + eA_x, p_y + eA_y] = [p_x, p_y + eBx]$

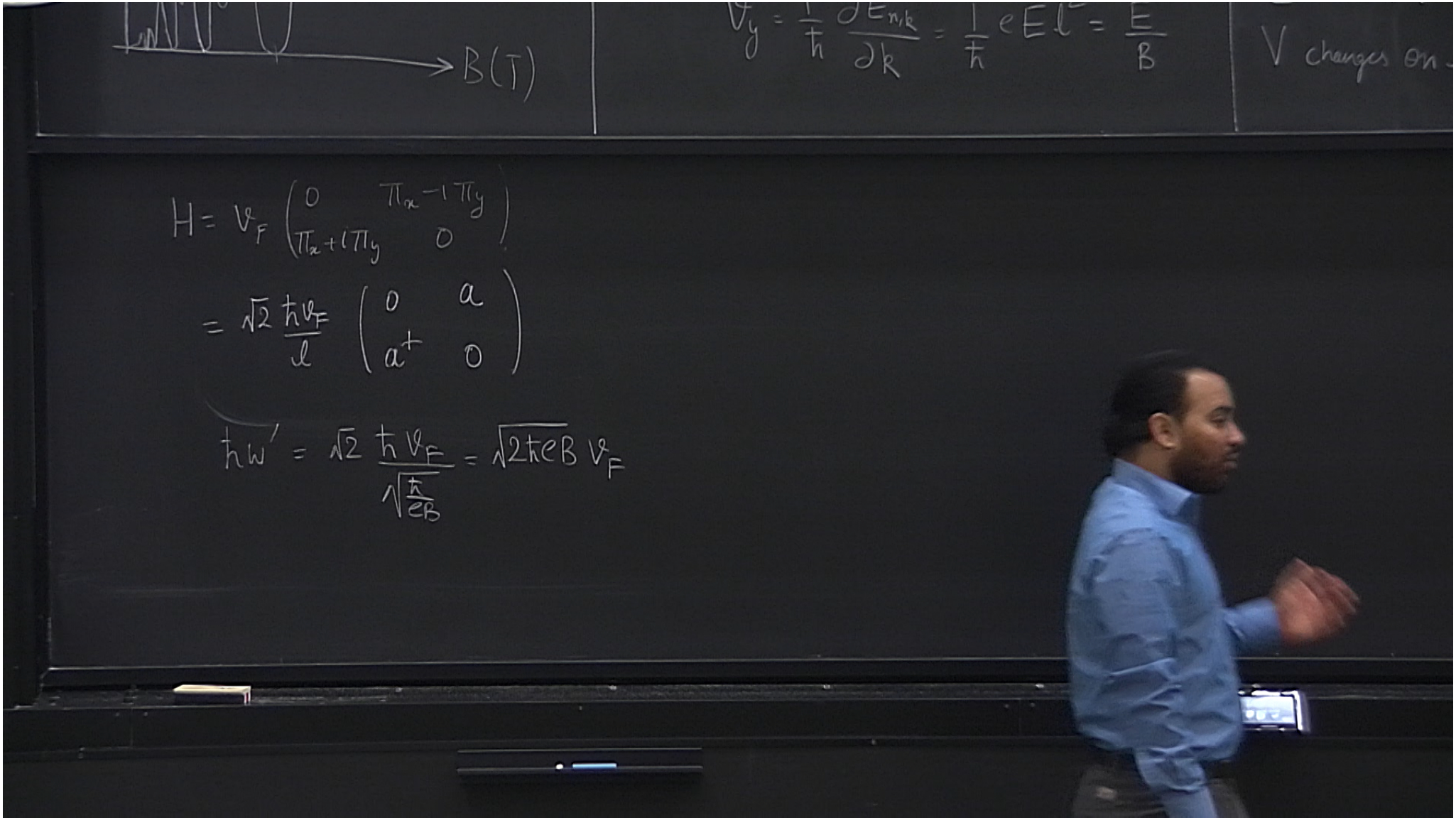

ards in graphene
$\bar{\sigma}\cdot\bar{p} = v_F(\sigma_x p_x + \sigma_y p_y)$
gauge $\bar{A} = (0, Bx, 0)$
$\bar{\sigma}\cdot(\bar{p}+e\bar{A}) = v_F\,\bar{\sigma}\cdot\bar{\pi}$
$[\pi_x, \pi_y] = [p_x + eA_x, p_y + eA_y] = [p_x, p_y + eBx]$
$[\pi_x, \pi_y] = -i\hbar eB = -i\frac{\hbar^2}{l^2}$

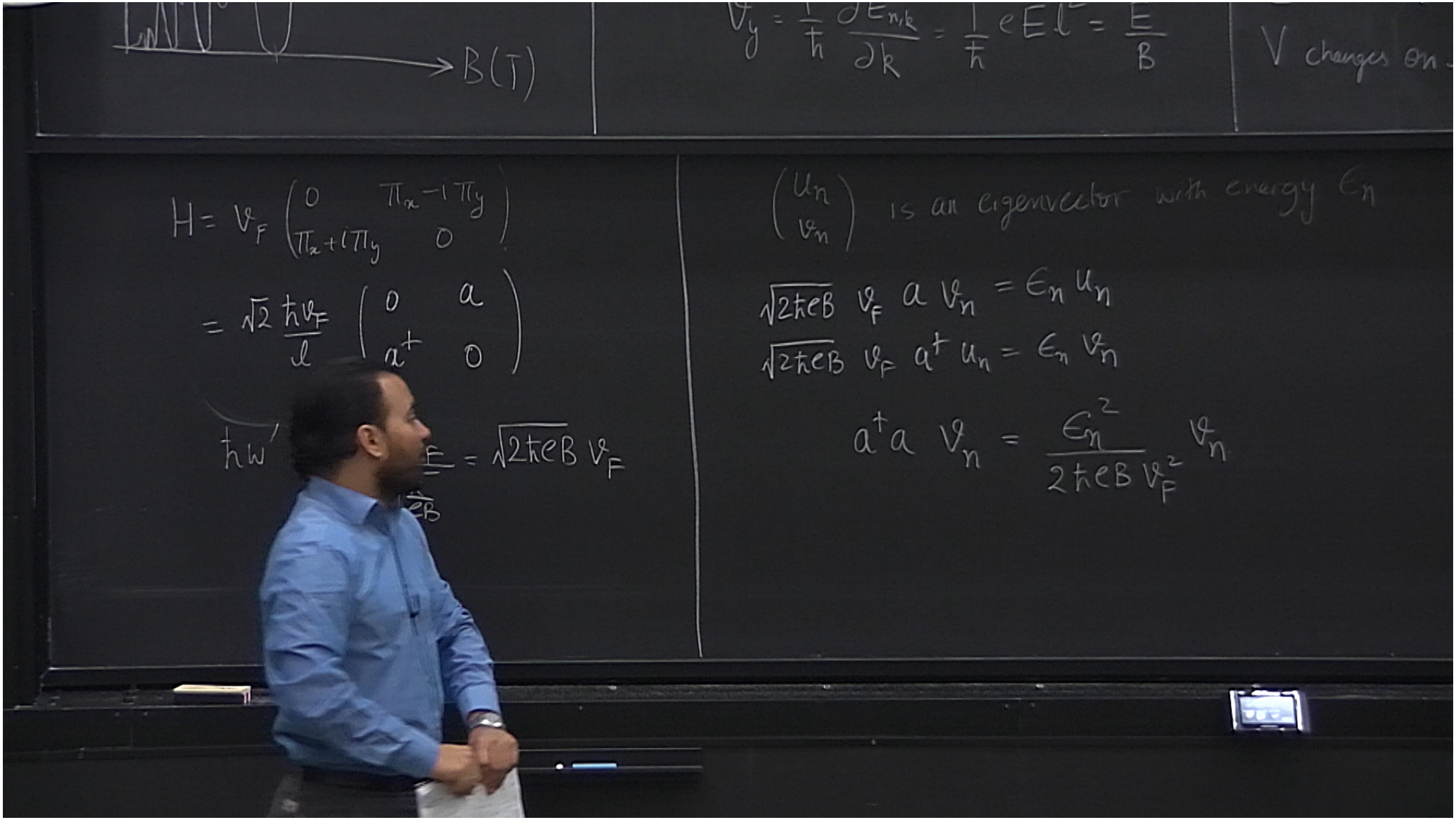

eads in graphene
$\bar{\sigma}\cdot\bar{p} = v_F(\sigma_x p_x + \sigma_y p_y)$
gauge $\bar{A} = (0, Bx, 0)$
$\bar{\sigma}\cdot(\bar{p}+e\bar{A}) = v_F\,\bar{\sigma}\cdot\bar{\pi}$
$[\pi_x, \pi_y] = [p_x + eA_x, p_y + eA_y] = [p_x, p_y + eBx]$
$[\pi_x, \pi_y] = -i\hbar eB = -i\frac{\hbar^2}{l^2}$
$a = \frac{l}{\hbar\sqrt{2}}(\pi_x - i\pi_y)$
$a^\dagger = \frac{l}{\hbar\sqrt{2}}(\pi_x + i\pi_y)$
$[a, a^\dagger] = 1$


gauge $\bar{A} = (0, Bx, 0)$
$\bar{\sigma}\cdot(\bar{p}+e\bar{A}) = v_F\,\bar{\sigma}\cdot\bar{\pi}$
$[\pi_x, \pi_y] = [p_x + eA_x, p_y + eA_y] = [p_x, p_y + eBx]$
$a^\dagger = \frac{l}{\hbar\sqrt{2}}(\pi_x + i\pi_y)$
$\pi_x = \frac{\hbar}{\sqrt{2}l}(a^\dagger + a)$
$\pi_y = \frac{-i\hbar}{\sqrt{2}l}(a^\dagger - a)$


levels in graphene
$\bar{\sigma}\cdot\bar{p} = v_F(\sigma_x p_x + \sigma_y p_y)$
gauge $\bar{A} = (0, Bx, 0)$
$\bar{\sigma}\cdot(\bar{p}+e\bar{A}) = v_F\,\bar{\sigma}\cdot\bar{\pi}$
$[\pi_x, \pi_y] = [p_x + eA_x, p_y + eA_y] = [p_x, p_y + eBx]$
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$a = \frac{l}{\hbar\sqrt{2}}(\pi_x - i\pi_y)$
$a^\dagger = \frac{l}{\hbar\sqrt{2}}(\pi_x + i\pi_y)$
$[a, a^\dagger] = 1$
$\pi_x = \frac{\hbar}{\sqrt{2}l}(a^\dagger + a)$
$\pi_y = \frac{-i\hbar}{\sqrt{2}l}(a^\dagger - a)$


B(T)
$v_y = \frac{1}{\hbar}\frac{\partial E_{n,k}}{\partial k} = \frac{1}{\hbar} eEl = \frac{E}{B}$
V changes on
$H = v_F \begin{pmatrix} 0 & \pi_x - i\pi_y \\ \pi_x + i\pi_y & 0 \end{pmatrix}$
$= \sqrt{2}\frac{\hbar v_F}{l} \begin{pmatrix} 0 & a \\ a^+ & 0 \end{pmatrix}$


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$= \sqrt{2}\frac{\hbar v_F}{l} \begin{pmatrix} 0 & a \\ a^+ & 0 \end{pmatrix}$
$\hbar\omega' = \sqrt{2}\frac{\hbar v_F}{\sqrt{\frac{\hbar}{eB}}} = \sqrt{2\hbar eB}\, v_F$

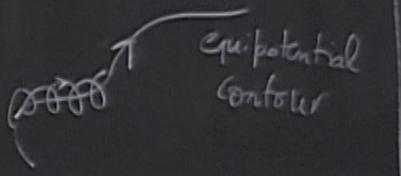

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$= \sqrt{2}\frac{\hbar v_F}{l}\begin{pmatrix} 0 & a \\ a^\dagger & 0 \end{pmatrix}$
$\hbar\omega$
$E = \sqrt{2\hbar eB}\, v_F$
eB
$\begin{pmatrix} u_n \\ v_n \end{pmatrix}$ is an eigenvector with energy ϵ_n
$\sqrt{2\hbar eB}\, v_F\, a\, v_n = \epsilon_n u_n$
$\sqrt{2\hbar eB}\, v_F\, a^\dagger u_n = \epsilon_n v_n$
$a^\dagger a\, v_n = \frac{\epsilon_n^2}{2\hbar eB v_F^2} v_n$


B(T)
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$\hbar w' = \sqrt{2}\frac{\hbar v_F}{\sqrt{\frac{\hbar}{eB}}}$ B v_F
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$\sqrt{2\hbar eB}\, v_F\, a^\dagger u_n = \epsilon_n v_n$
$a^\dagger a\, v_n = \frac{\epsilon_n^2}{2\hbar eB v_F^2} v_n$
$\Rightarrow \epsilon_n = \pm\sqrt{2neB\hbar}\, v_F$

$$v_y = \frac{1}{\hbar}\frac{\partial E_{n,k}}{\partial k} = \frac{1}{\hbar} e E l^2 = \frac{E}{B}$$

$$\vec{E} = -\vec{\nabla} V$$

V changes on length scales $\gg l$





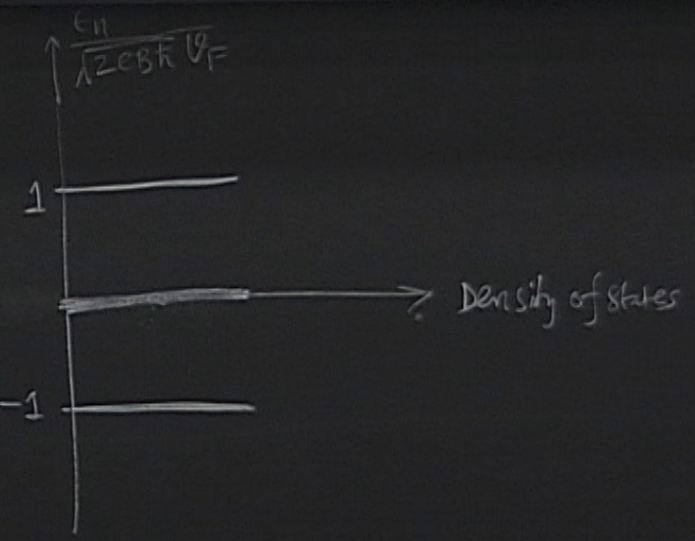
$\begin{pmatrix} u_n \\ v_n \end{pmatrix}$ is an eigenvector with energy ϵ_n

$$\sqrt{2\hbar e B}\, v_F\, a\, v_n = \epsilon_n u_n$$

$$\sqrt{2\hbar e B}\, v_F\, a^\dagger u_n = \epsilon_n v_n$$

$$a^\dagger a\, v_n = \frac{\epsilon_n^2}{2\hbar e B v_F^2} v_n$$

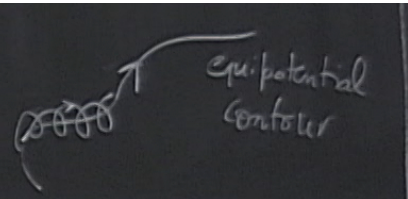
$$\Rightarrow \epsilon_n = \pm\sqrt{2 n e B \hbar}\, v_F$$



$$v_y = \frac{1}{\hbar}\frac{\partial E_{n,k}}{\partial k} = \frac{1}{\hbar} e E l^2 = \frac{E}{B}$$

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V changes on length scales $\gg l$





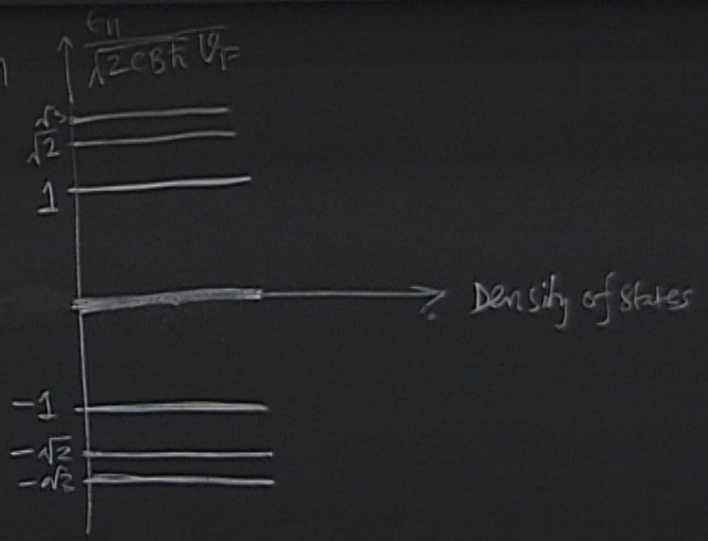
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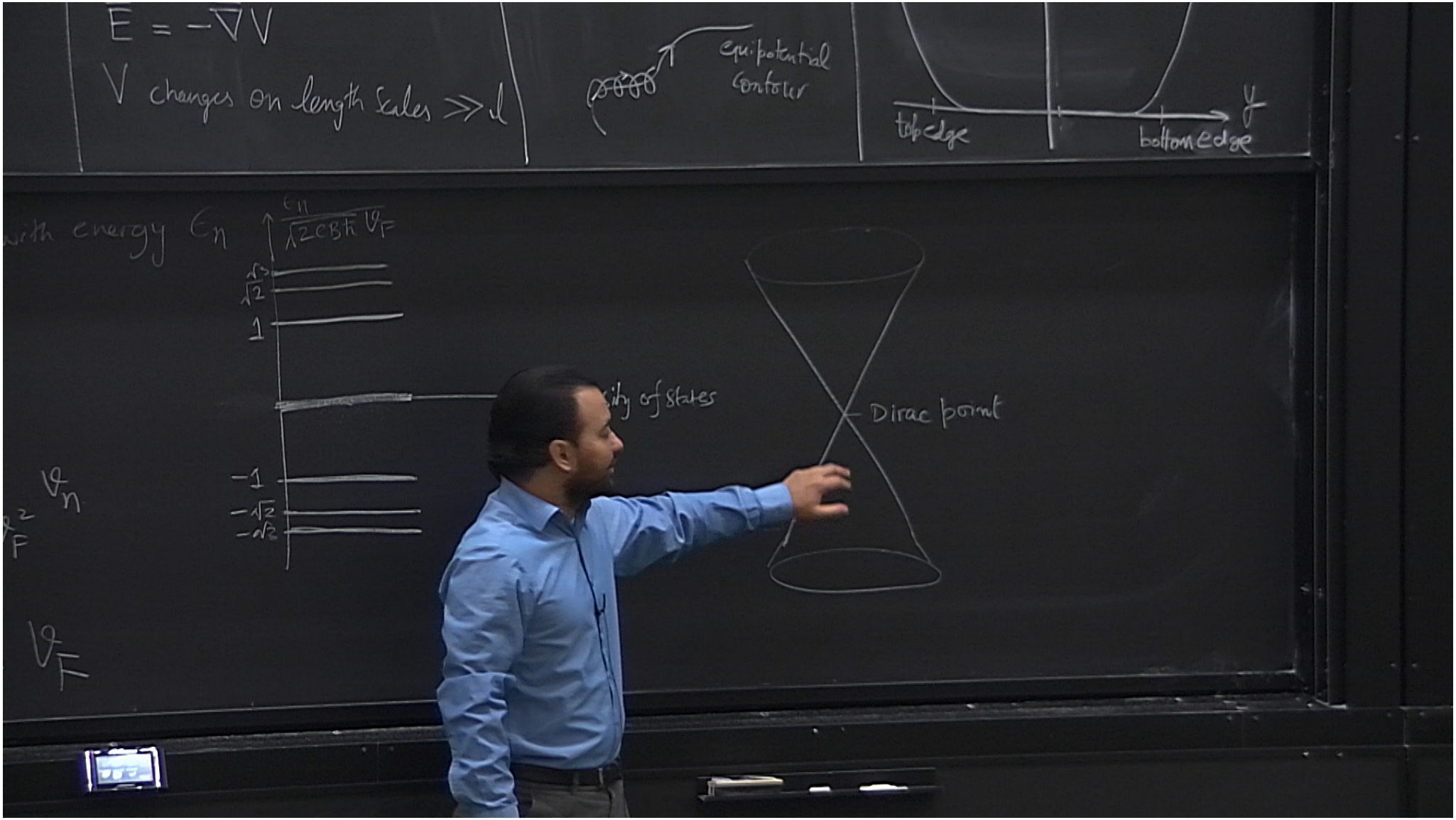
$$\sqrt{2\hbar e B}\, v_F\, a\, v_n = \epsilon_n u_n$$

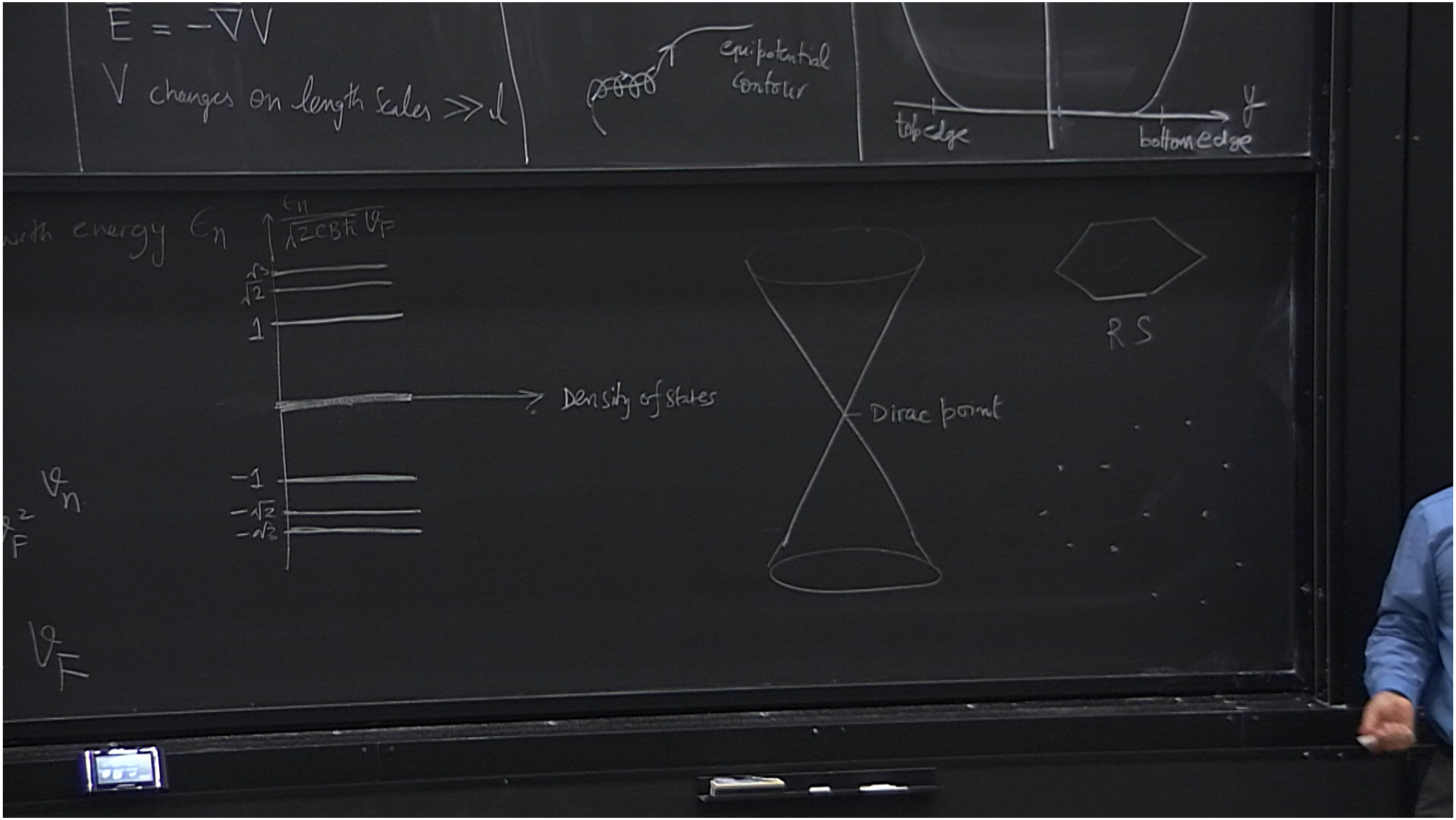
$$\sqrt{2\hbar e B}\, v_F\, a^\dagger u_n = \epsilon_n v_n$$

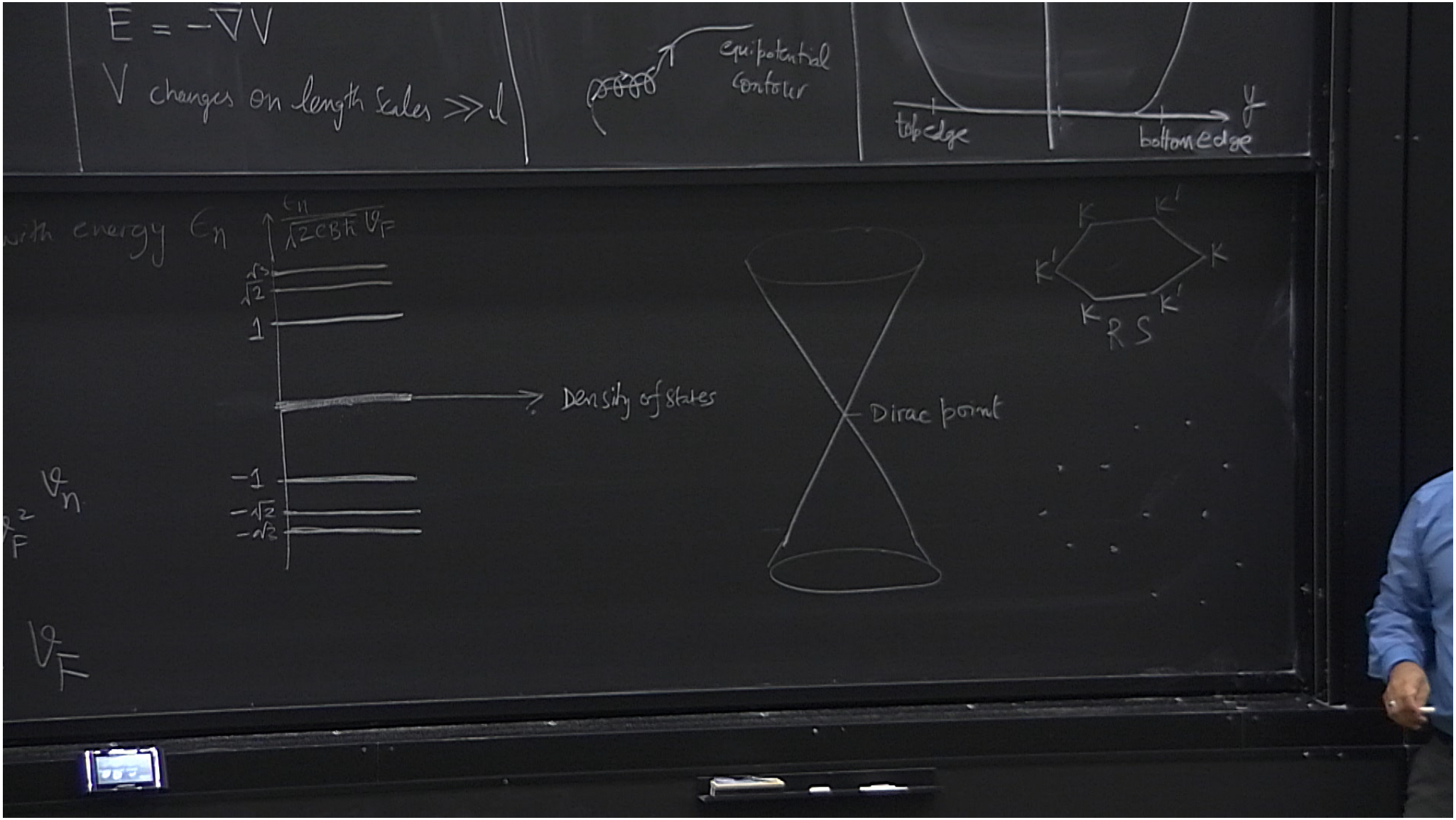
$$a^\dagger a\, v_n = \frac{\epsilon_n^2}{2\hbar e B v_F^2} v_n$$

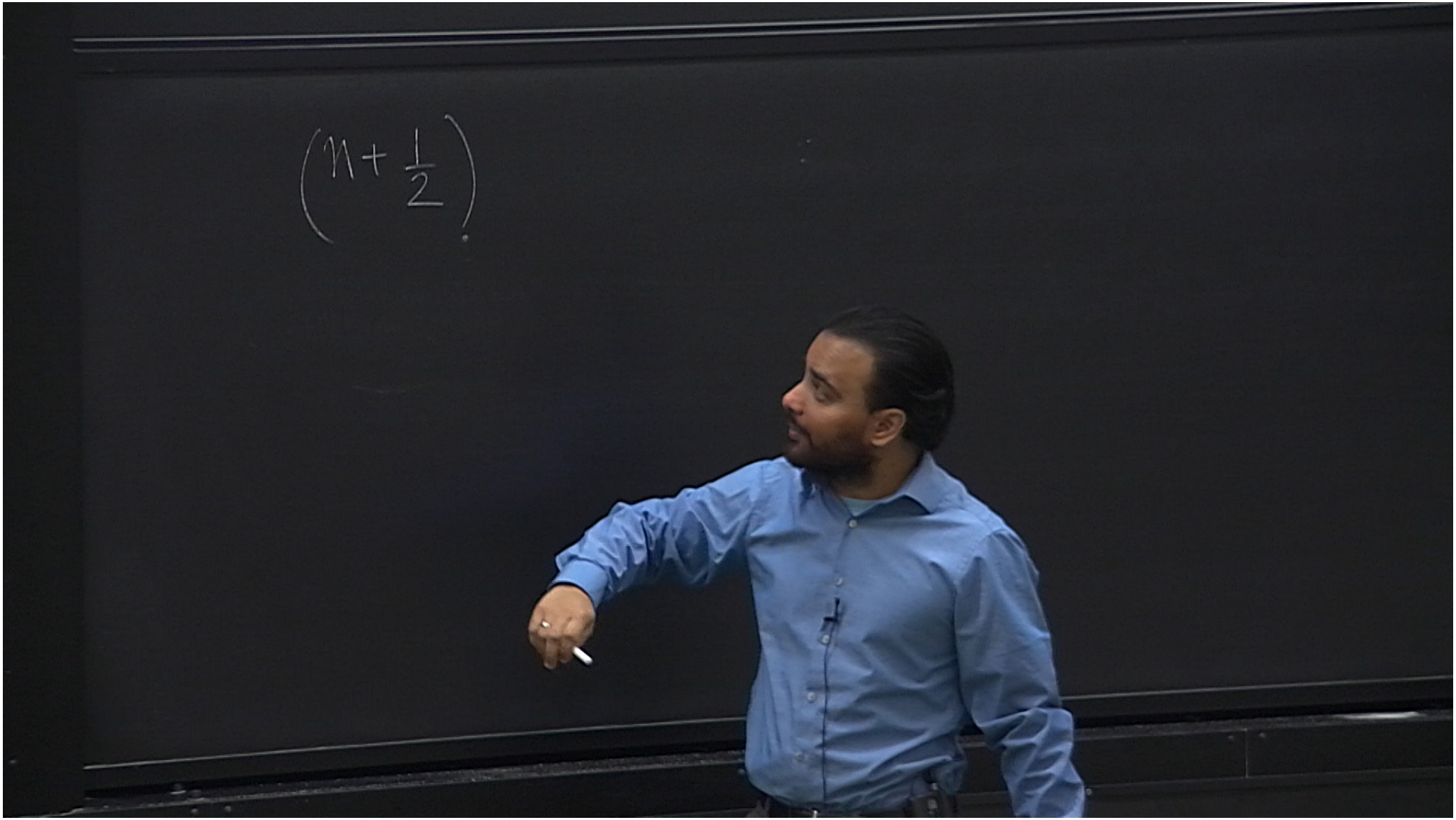
$$\epsilon_n = \pm\sqrt{2ne\hbar B}\, v_F$$

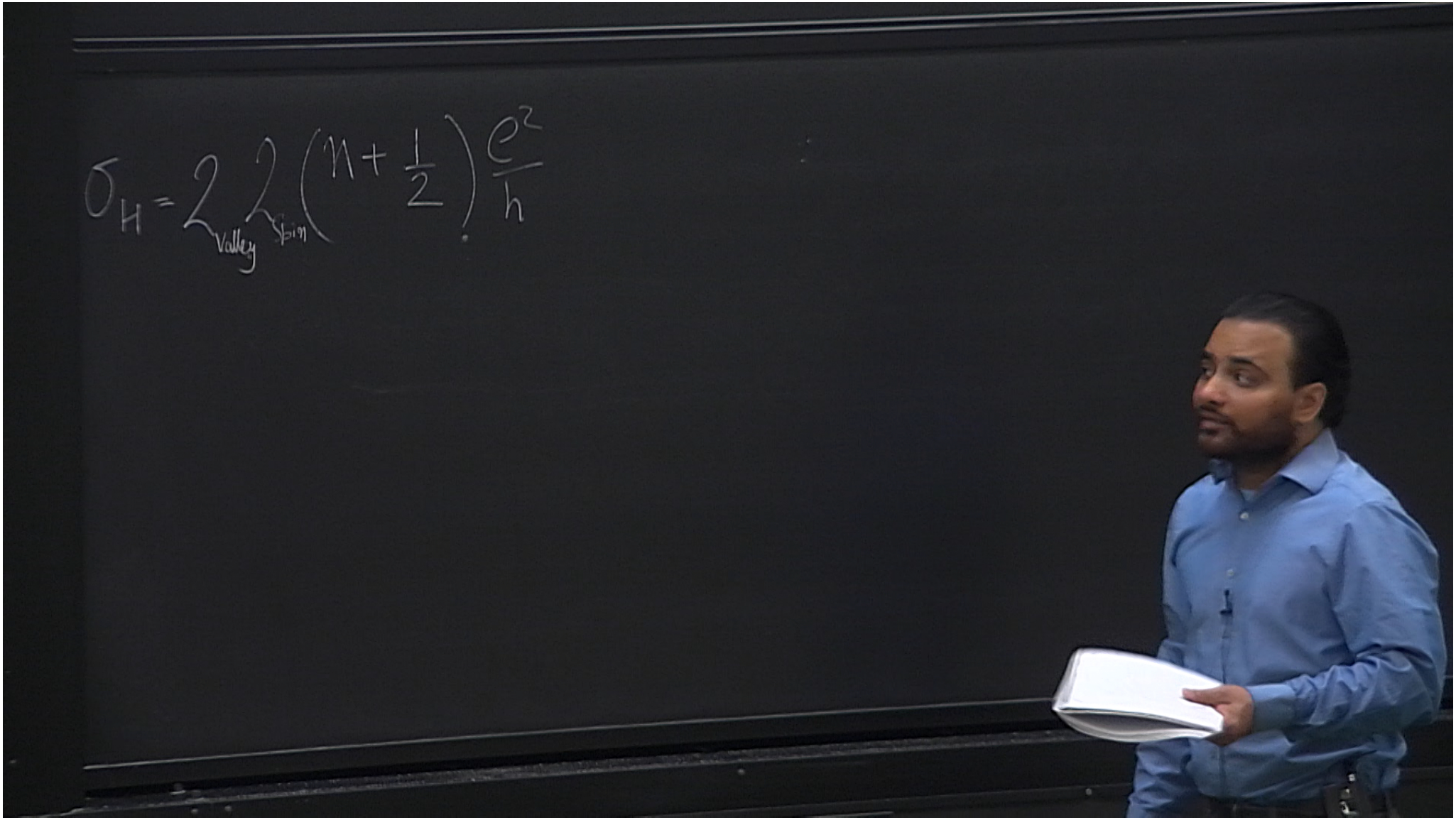



E = −∇V
V changes on length scales ≫ d
equipotential contour
top edge
bottom edge
y
with energy ϵn
ϵn / √2CBħ vF
√3
√2
1
−1
−√2
−√3
Dirac point


E = −∇V
V changes on length scales ≫ d
equipotential contour
top edge
bottom edge
y
with energy ε_n
ε_n / √2 ħ v_F
√3
√2
1
−1
−√2
−√3
Density of states
Dirac point
RS
v_n
v_F


E = −∇V
V changes on length scales ≫ d
equipotential contour
top edge
bottom edge
y
with energy ε_n
ε_n
√2 CBħ v_F
√3
√2
1
−1
−√2
−√3
Density of states
Dirac point
K
K'
K
K'
K
K'
R
S


$\left(n+\frac{1}{2}\right)!$


$\sigma_H = 2 \underset{\text{Valley}}{2} \underset{\text{Spin}}{} \left(n + \frac{1}{2}\right) \frac{e^2}{h}$

$$\sigma_H = 2 \underbrace{2}_{\text{Valley}} \underbrace{}_{\text{Spin}} \left(n + \frac{1}{2}\right) \frac{e^2}{h}$$

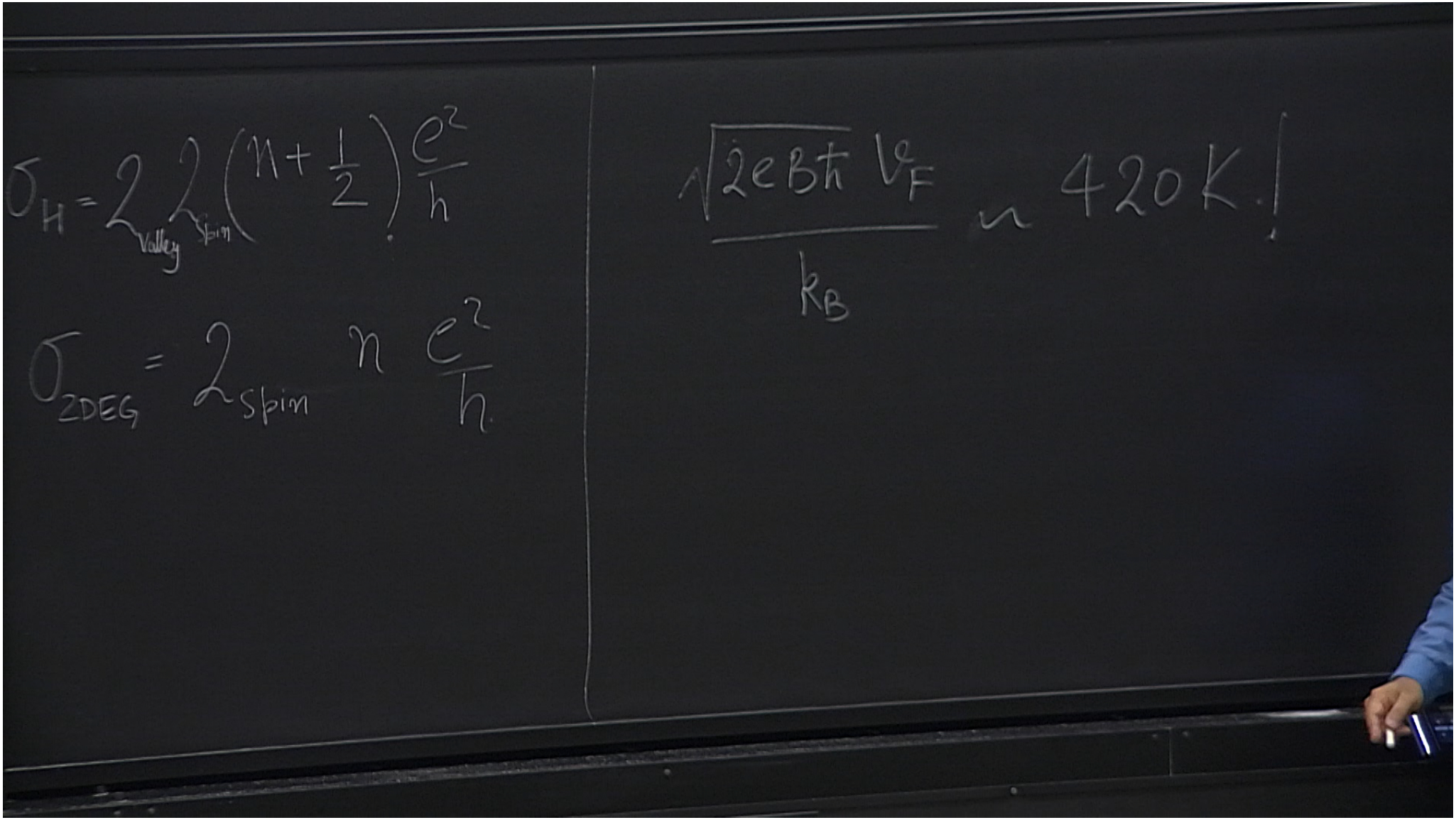
$$\sigma_{2DEG} = 2_{\text{Spin}} \; n \; \frac{e^2}{h}$$

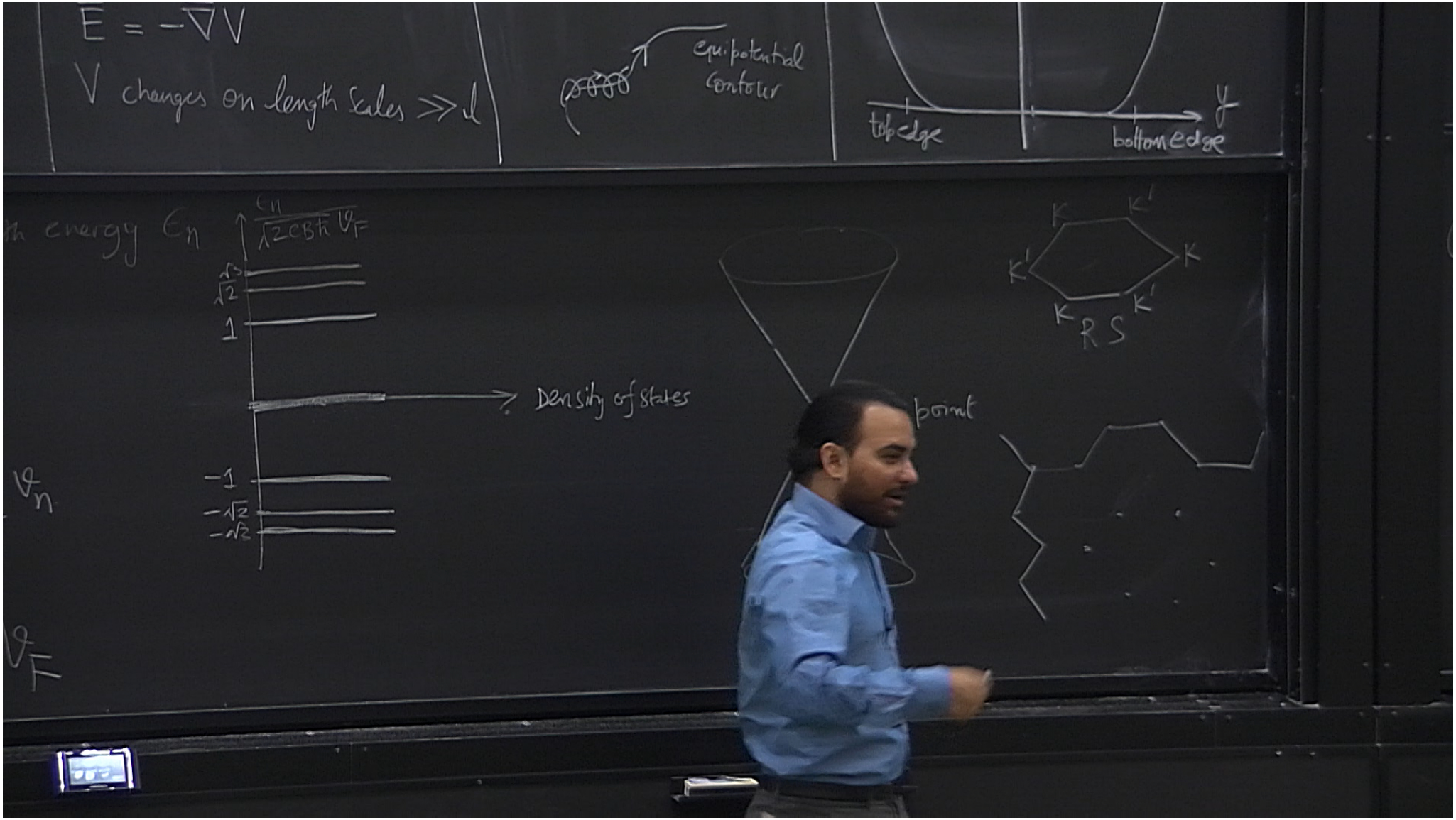


$$\sigma_H = 2_{\text{Valley}} 2_{\text{Spin}} \left(n + \frac{1}{2}\right) \frac{e^2}{h}$$

$$\sigma_{2DEG} = 2_{\text{Spin}}\, n\, \frac{e^2}{h}$$

$$\frac{\sqrt{2eB\hbar}\, v_F^2}{k_B}$$


$\sigma_H = 2_{Valley} 2_{Spin} \left(n + \frac{1}{2}\right) \frac{e^2}{h}$
$\sigma_{2DEG} = 2_{Spin}\, n\, \frac{e^2}{h}$
$\frac{\sqrt{2eBt}\, v_F}{k_B} \sim 420\,K.$


E = −∇V
V changes on length scales ≫ a
equipotential contour
top edge
bottom edge
y
energy
Density of states
point
K
K'
R
S