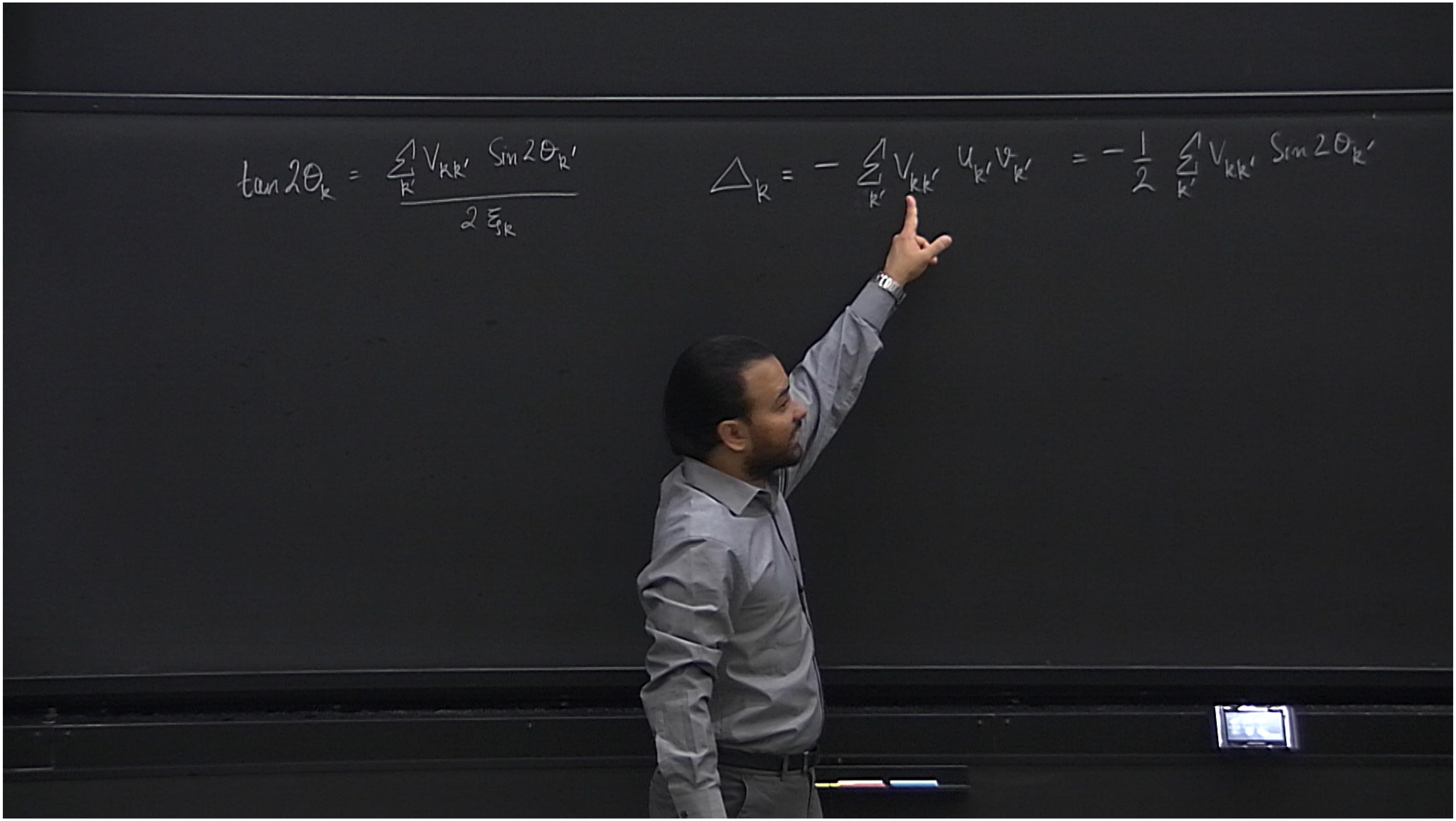
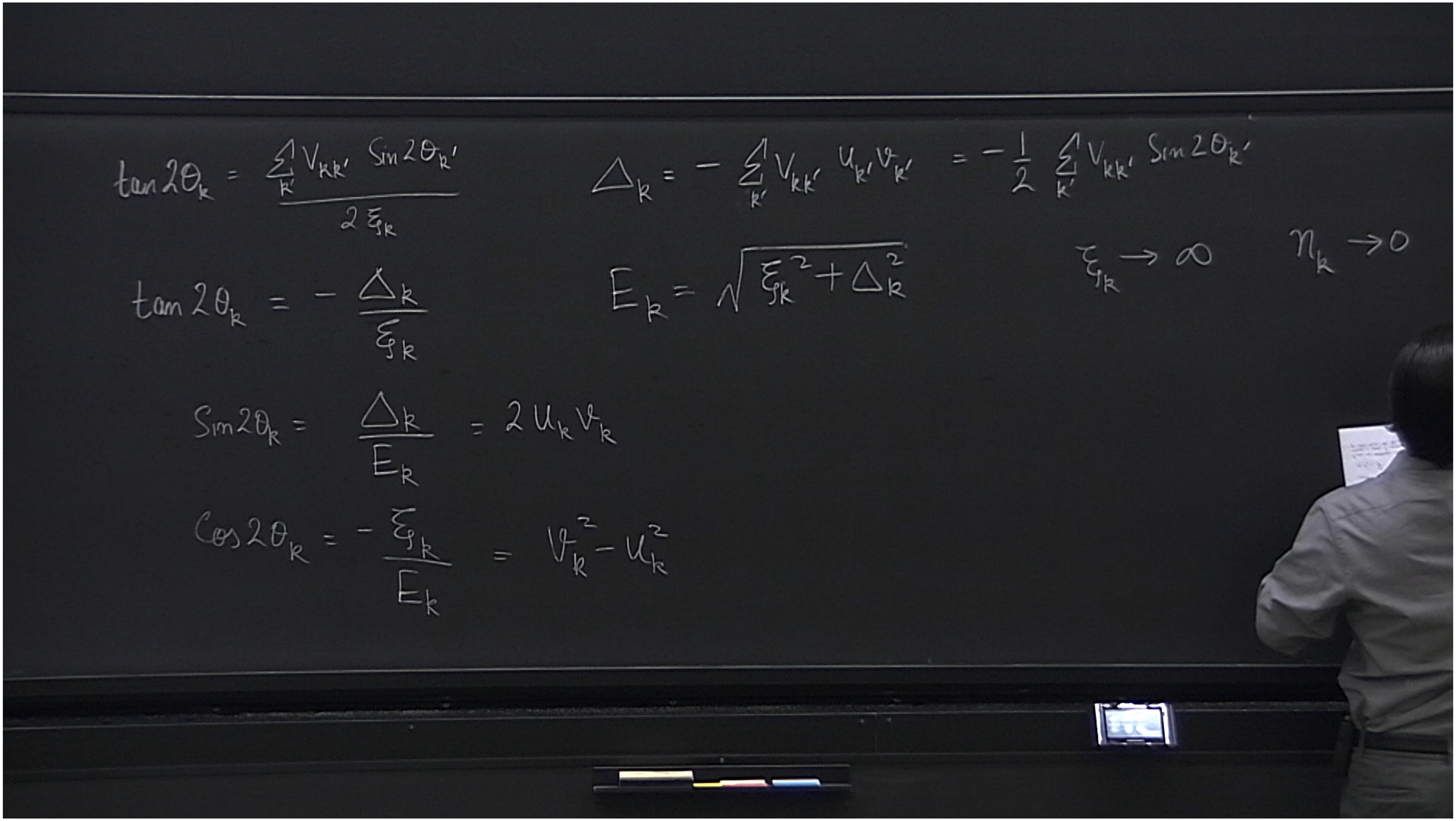
Title: PSI 17/18 - Condensed Matter - Lecture 12

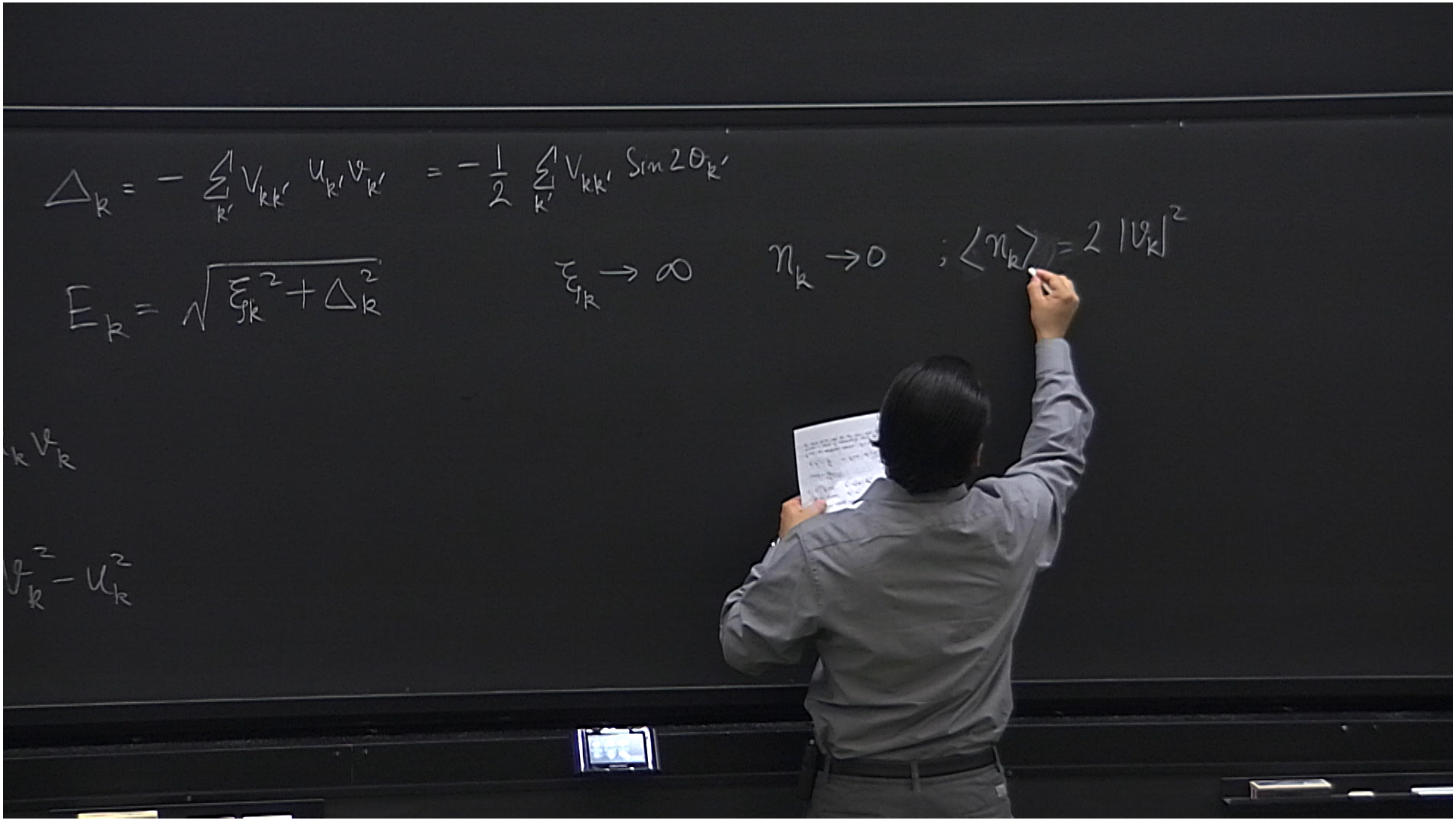
Date: Nov 21, 2017 10:45 AM

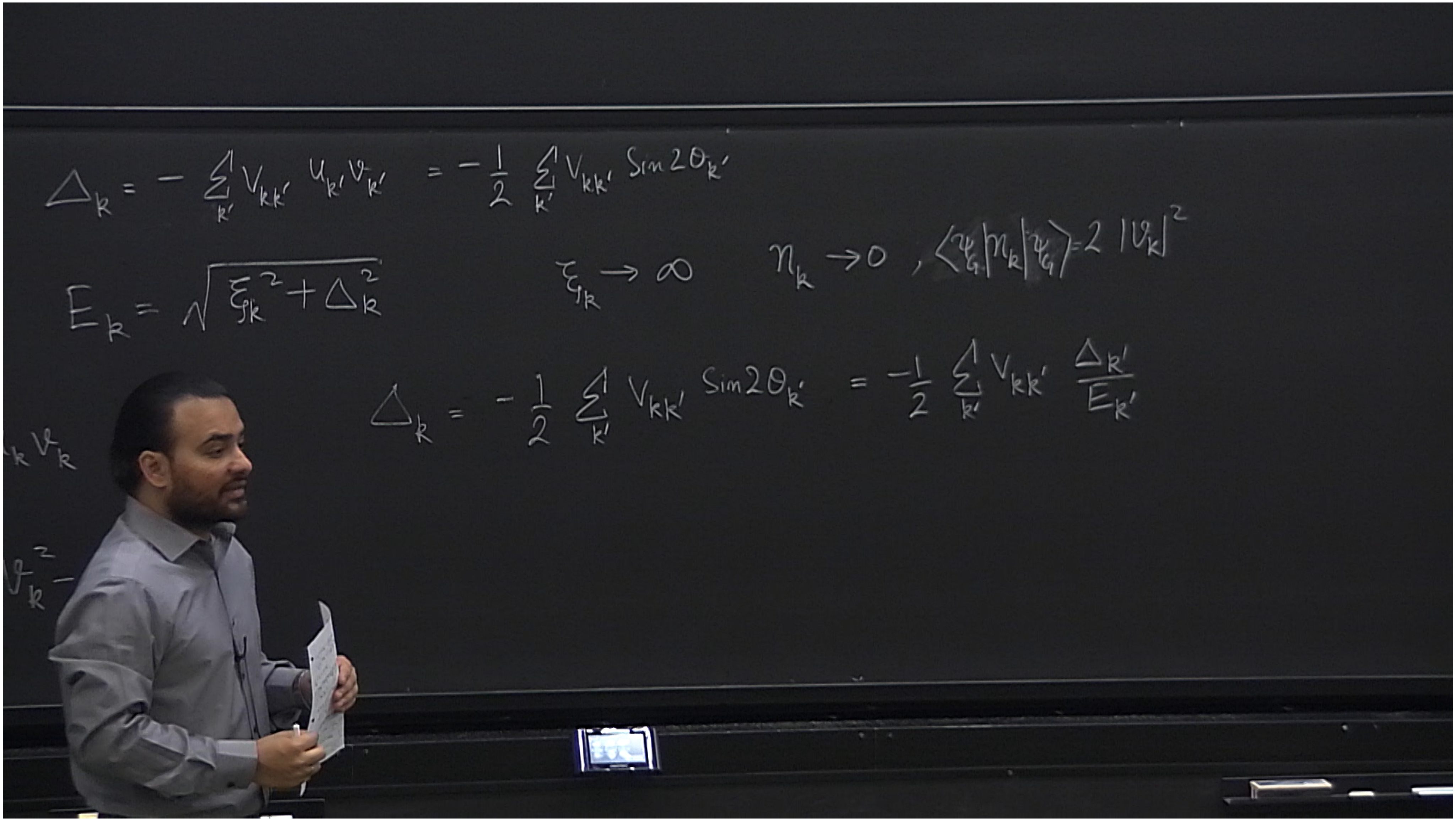
URL: http://pirsa.org/17110037

Abstract:


$\tan 2\theta_k = \frac{\sum_{k'} V_{kk'} \sin 2\theta_{k'}}{2\xi_k}$
$\Delta_k = -\sum_{k'} V_{kk'} u_{k'} v_{k'} = -\frac{1}{2} \sum_{k'} V_{kk'} \sin 2\theta_{k'}$


$\tan 2\theta_k = \frac{\sum_{k'} V_{kk'} \sin 2\theta_{k'}}{2\xi_k}$
$\Delta_k = -\sum_{k'} V_{kk'} u_{k'} v_{k'} = -\frac{1}{2} \sum_{k'} V_{kk'} \sin 2\theta_{k'}$
$\tan 2\theta_k = -\frac{\Delta_k}{\xi_k}$
$E_k = \sqrt{\xi_k^2 + \Delta_k^2}$
$\xi_k \to \infty$
$n_k \to 0$
$\sin 2\theta_k = \frac{\Delta_k}{E_k} = 2 u_k v_k$
$\cos 2\theta_k = -\frac{\xi_k}{E_k} = v_k^2 - u_k^2$


$\Delta_k = -\sum'_{k'} V_{kk'} u_{k'} v_{k'} = -\frac{1}{2} \sum'_{k'} V_{kk'} \sin 2\theta_{k'}$
$E_k = \sqrt{\xi_k^2 + \Delta_k^2}$
$\xi_k \to \infty$
$n_k \to 0 \; ; \langle n_k \rangle = 2|v_k|^2$
$_k v_k$
$v_k^2 - u_k^2$


$\Delta_k = -\sum_{k'}' V_{kk'} u_{k'} v_{k'} = -\frac{1}{2} \sum_{k'}' V_{kk'} \sin 2\theta_{k'}$
$E_k = \sqrt{\xi_k^2 + \Delta_k^2}$
$\xi_k \to \infty \quad n_k \to 0 , \langle \psi_G | n_k | \psi_G \rangle = 2 |v_k|^2$
$\Delta_k = -\frac{1}{2} \sum_{k'}' V_{kk'} \sin 2\theta_{k'} = -\frac{1}{2} \sum_{k'}' V_{kk'} \frac{\Delta_{k'}}{E_{k'}}$

$$\tan 2\theta_k = \frac{\sum_{k'} V_{kk'} \sin 2\theta_{k'}}{2\xi_k}$$

$$\tan 2\theta_k = -\frac{\Delta_k}{\xi_k}$$

$$\sin 2\theta_k = \frac{\Delta_k}{E_k} = 2 u_k v_k$$

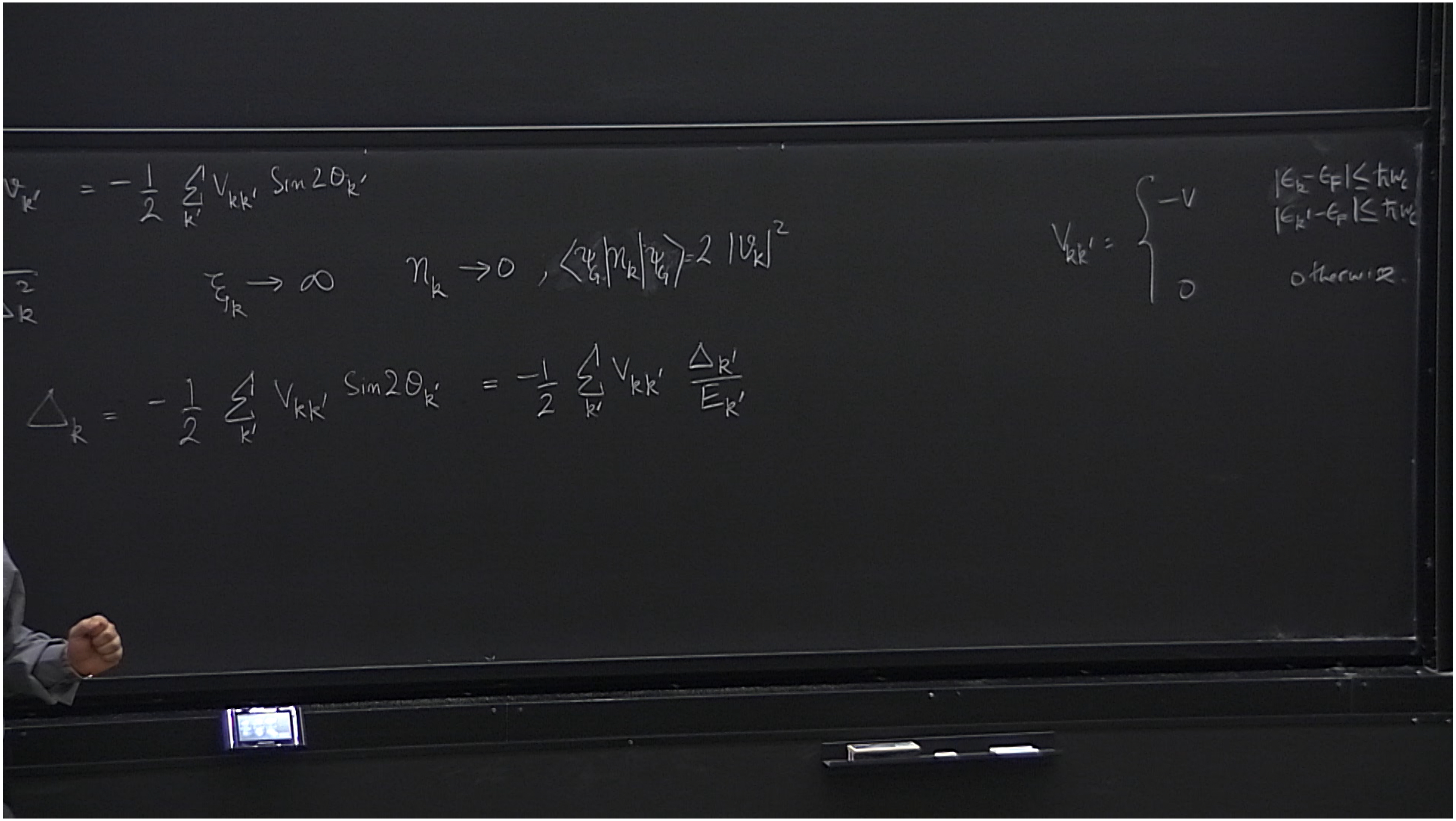
$$\cos 2\theta_k = -\frac{\xi_k}{E_k} = v_k^2 - u_k^2$$

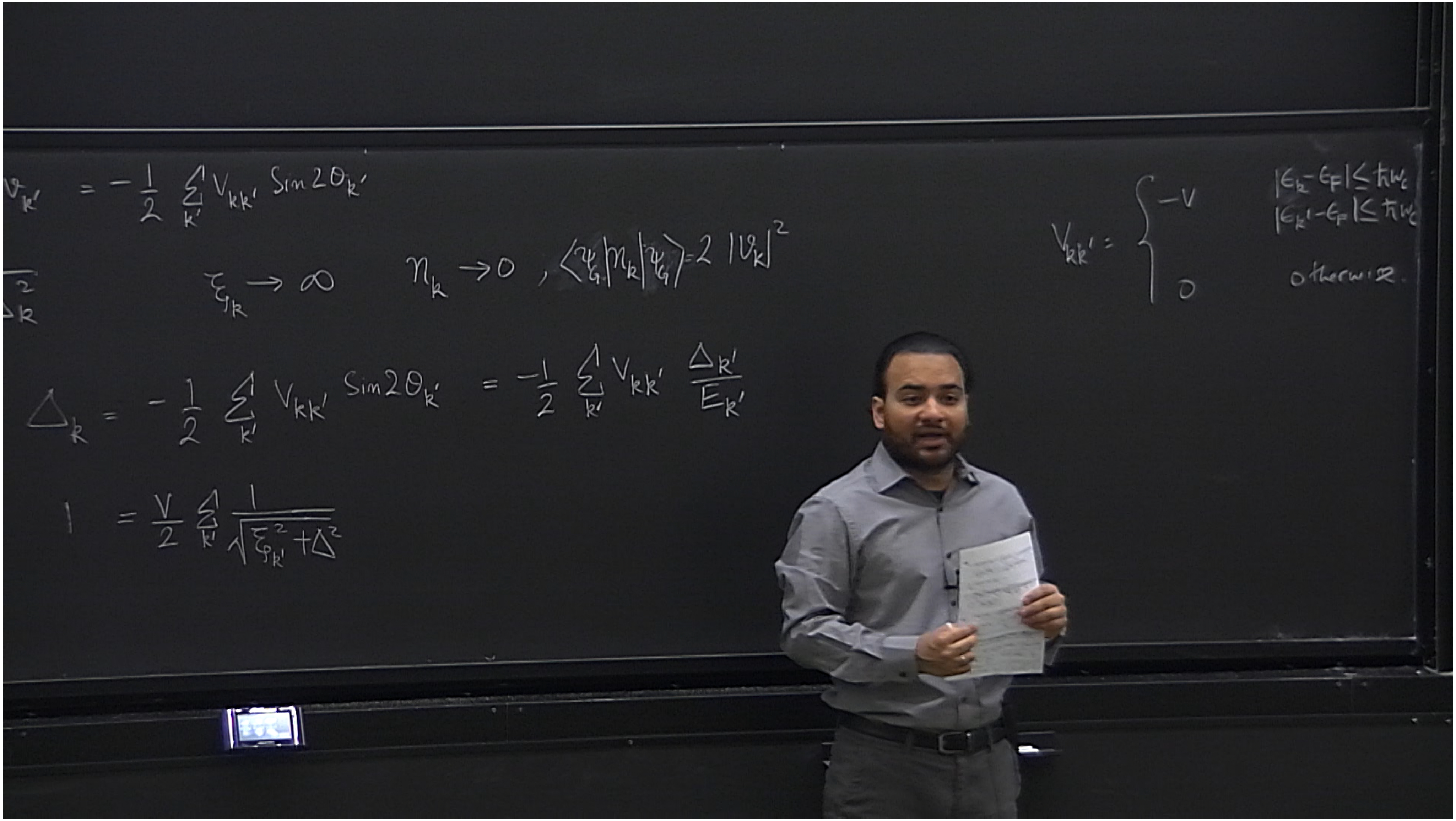
$$\Delta_k = -\sum_{k'} V_{kk'} u_{k'} v_{k'} = -\frac{1}{2} \sum_{k'} V_{kk'} \sin 2\theta_{k'}$$

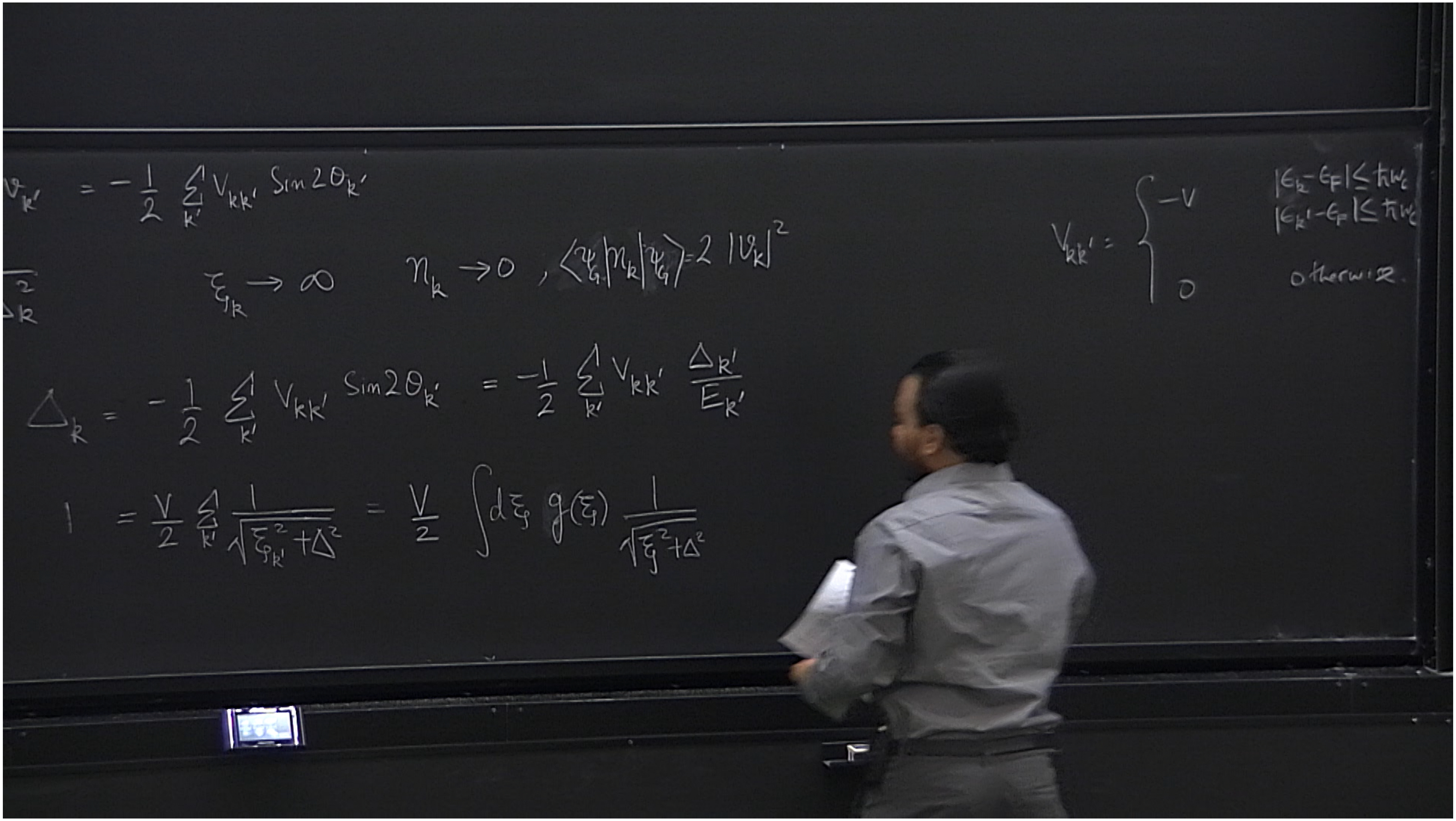
$$E_k = \sqrt{\xi_k^2 + \Delta_k^2}$$

$$\xi_k \to \infty \qquad n_k \to 0, \quad \langle \psi_G | n_k | \psi_G \rangle = 2 |v_k|^2$$

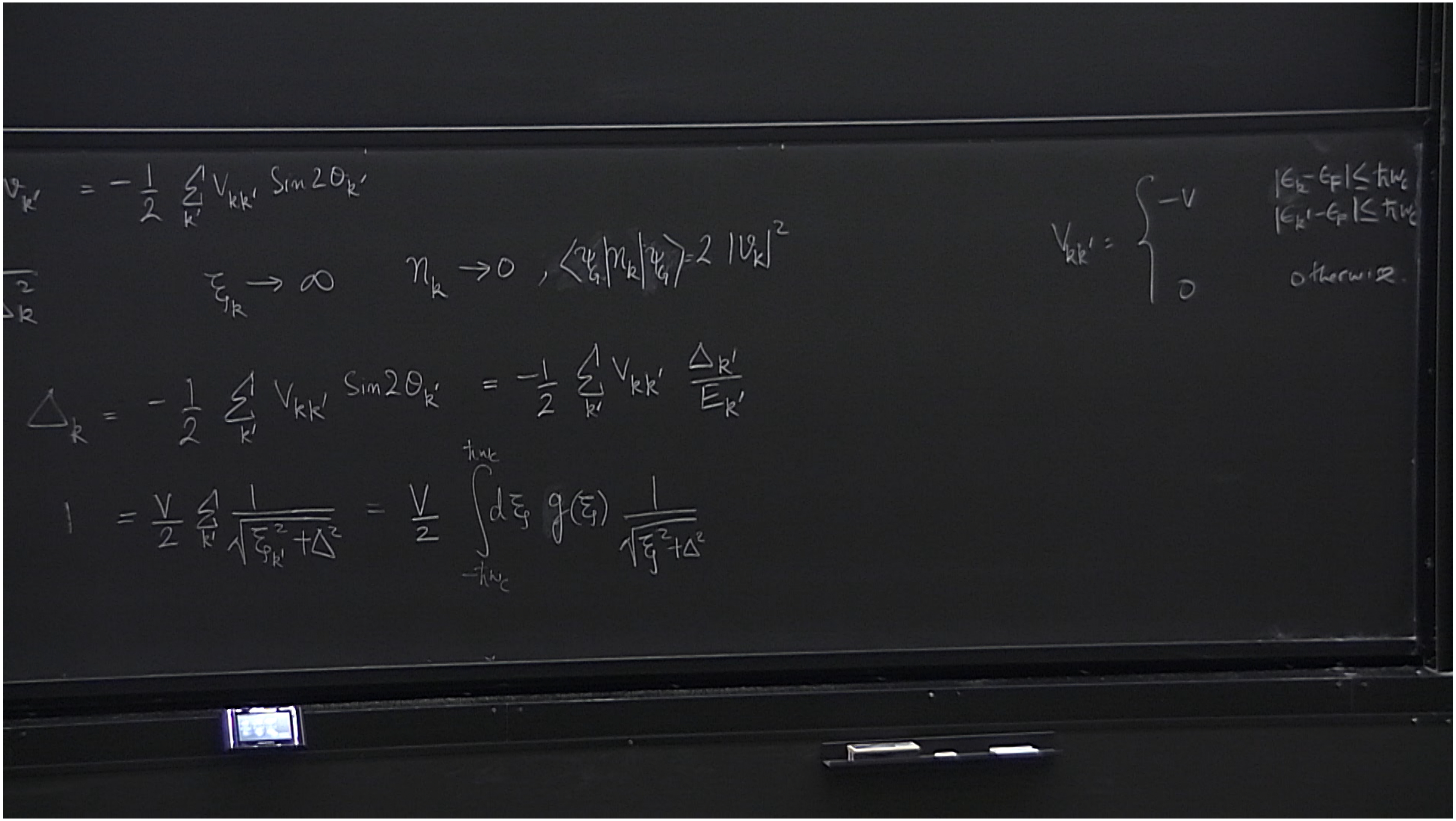
$$\Delta_k = -\frac{1}{2} \sum_{k'} V_{kk'} \sin 2\theta_{k'} = -\frac{1}{2} \sum_{k'} V_{kk'} \frac{\Delta_{k'}}{E_{k'}}$$

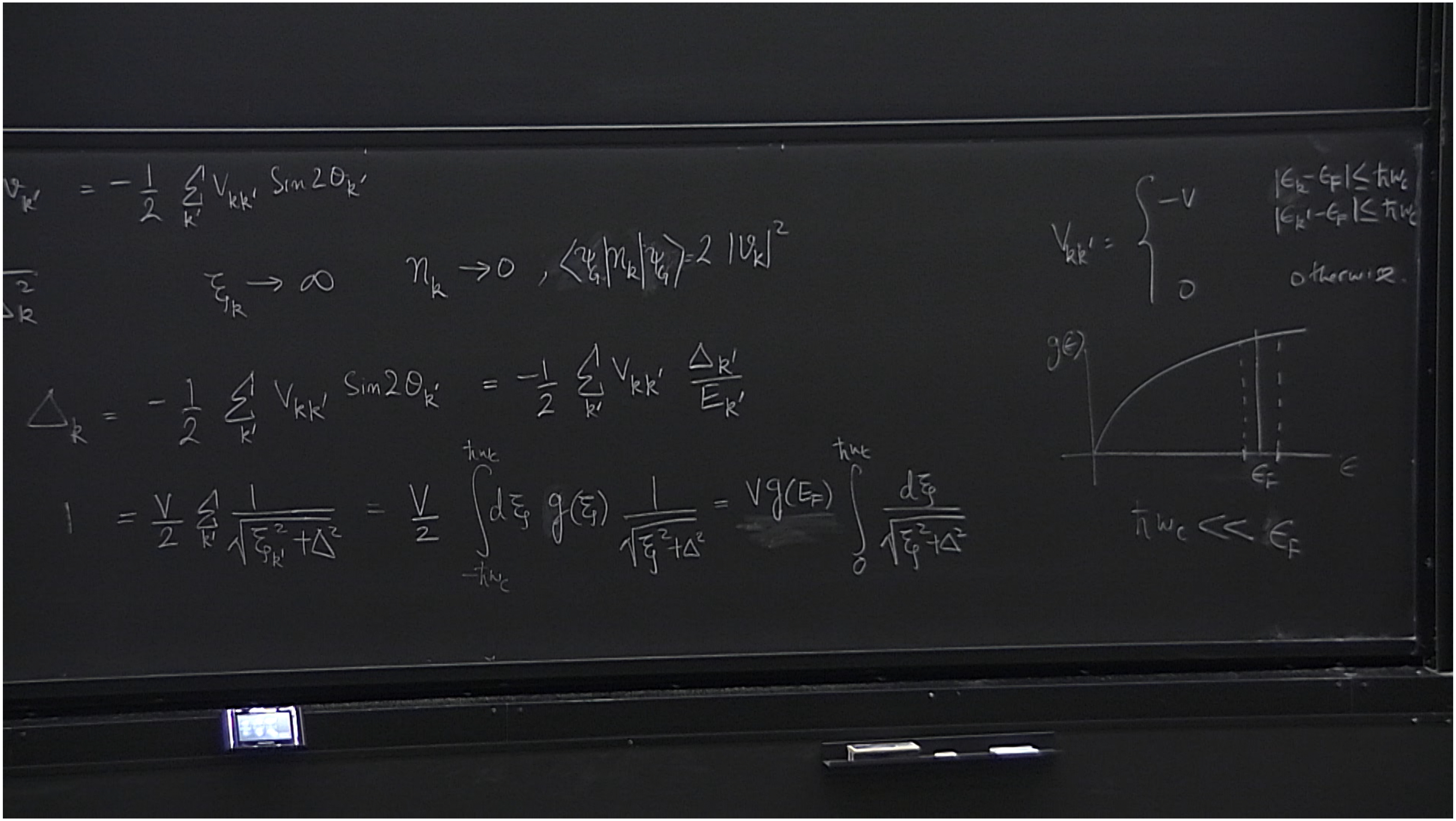

$v_{k'} = -\frac{1}{2} \sum_{k'} V_{kk'} \sin 2\theta_{k'}$
$\xi_k \to \infty \quad n_k \to 0 , \langle \psi_G | n_k | \psi_G \rangle = 2 |v_k|^2$
$\Delta_k = -\frac{1}{2} \sum_{k'} V_{kk'} \sin 2\theta_{k'} = -\frac{1}{2} \sum_{k'} V_{kk'} \frac{\Delta_{k'}}{E_{k'}}$
$V_{kk'} = \begin{cases} -V & |\epsilon_k - \epsilon_F| \leq \hbar\omega_c,\ |\epsilon_{k'} - \epsilon_F| \leq \hbar\omega_c \\ 0 & \text{otherwise.} \end{cases}$


$v_{k'} = -\frac{1}{2} \sum_{k'} V_{kk'} \sin 2\theta_{k'}$
$\xi_k \to \infty$
$n_k \to 0$, $\langle \psi_G | n_k | \psi_G \rangle = 2 |v_k|^2$
$V_{kk'} = \begin{cases} -V & |\epsilon_k - \epsilon_F| \le \hbar\omega_c \\ & |\epsilon_{k'} - \epsilon_F| \le \hbar\omega_c \\ 0 & \text{otherwise} \end{cases}$
$\Delta_k = -\frac{1}{2} \sum_{k'} V_{kk'} \sin 2\theta_{k'} = -\frac{1}{2} \sum_{k'} V_{kk'} \frac{\Delta_{k'}}{E_{k'}}$
$1 = \frac{V}{2} \sum_{k'} \frac{1}{\sqrt{\xi_{k'}^2 + \Delta^2}}$



$v_{k'} = -\frac{1}{2}\sum_{k'} V_{kk'} \sin 2\theta_{k'}$
$\xi_k \to \infty$
$n_k \to 0$, $\langle \psi_G | n_k | \psi_G \rangle = 2\,|v_k|^2$
$V_{kk'} = \begin{cases} -V & |\epsilon_k - \epsilon_F| \le \hbar\omega_c,\ |\epsilon_{k'} - \epsilon_F| \le \hbar\omega_c \\ 0 & \text{otherwise.} \end{cases}$
$\Delta_k = -\frac{1}{2}\sum_{k'} V_{kk'} \sin 2\theta_{k'} = -\frac{1}{2}\sum_{k'} V_{kk'} \frac{\Delta_{k'}}{E_{k'}}$
$1 = \frac{V}{2}\sum_{k'} \frac{1}{\sqrt{\xi_{k'}^2 + \Delta^2}} = \frac{V}{2}\int d\xi\, g(\xi)\, \frac{1}{\sqrt{\xi^2 + \Delta^2}}$


$v_{k'} = -\frac{1}{2} \sum_{k'} V_{kk'} \sin 2\theta_{k'}$
$\xi_k \to \infty \quad n_k \to 0, \ \langle \psi_G | n_k | \psi_G \rangle = 2 |v_k|^2$
$V_{kk'} = \begin{cases} -V & |\epsilon_k - \epsilon_F| \le \hbar\omega_c, \ |\epsilon_{k'} - \epsilon_F| \le \hbar\omega_c \\ 0 & \text{otherwise.} \end{cases}$
$\Delta_k = -\frac{1}{2} \sum_{k'} V_{kk'} \sin 2\theta_{k'} = -\frac{1}{2} \sum_{k'} V_{kk'} \frac{\Delta_{k'}}{E_{k'}}$
$1 = \frac{V}{2} \sum_{k'} \frac{1}{\sqrt{\xi_{k'}^2 + \Delta^2}} = \frac{V}{2} \int_{-\hbar\omega_c}^{\hbar\omega_c} d\xi \, g(\xi) \frac{1}{\sqrt{\xi^2 + \Delta^2}}$


$v_{k'} = -\frac{1}{2} \sum_{k'} V_{kk'} \sin 2\theta_{k'}$
$\xi_k \to \infty \quad n_k \to 0 , \langle \psi_G | n_k | \psi_G \rangle = 2 |v_k|^2$
$\Delta_k = -\frac{1}{2} \sum_{k'} V_{kk'} \sin 2\theta_{k'} = -\frac{1}{2} \sum_{k'} V_{kk'} \frac{\Delta_{k'}}{E_{k'}}$
$1 = \frac{V}{2} \sum_{k'} \frac{1}{\sqrt{\xi_{k'}^2 + \Delta^2}} = \frac{V}{2} \int_{-\hbar\omega_c}^{\hbar\omega_c} d\xi \, g(\xi) \frac{1}{\sqrt{\xi^2+\Delta^2}} = V g(\epsilon_F) \int_0^{\hbar\omega_c} \frac{d\xi}{\sqrt{\xi^2+\Delta^2}}$
$V_{kk'} = \begin{cases} -V & |\epsilon_k - \epsilon_F| \le \hbar\omega_c, \ |\epsilon_{k'} - \epsilon_F| \le \hbar\omega_c \\ 0 & \text{otherwise} \end{cases}$
$g(\epsilon)$
ϵ_F
ϵ
$\hbar\omega_c \ll \epsilon_F$

$$\frac{1}{V g(E_F)} = \sinh^{-1} \frac{\hbar\omega_c}{\Delta}$$

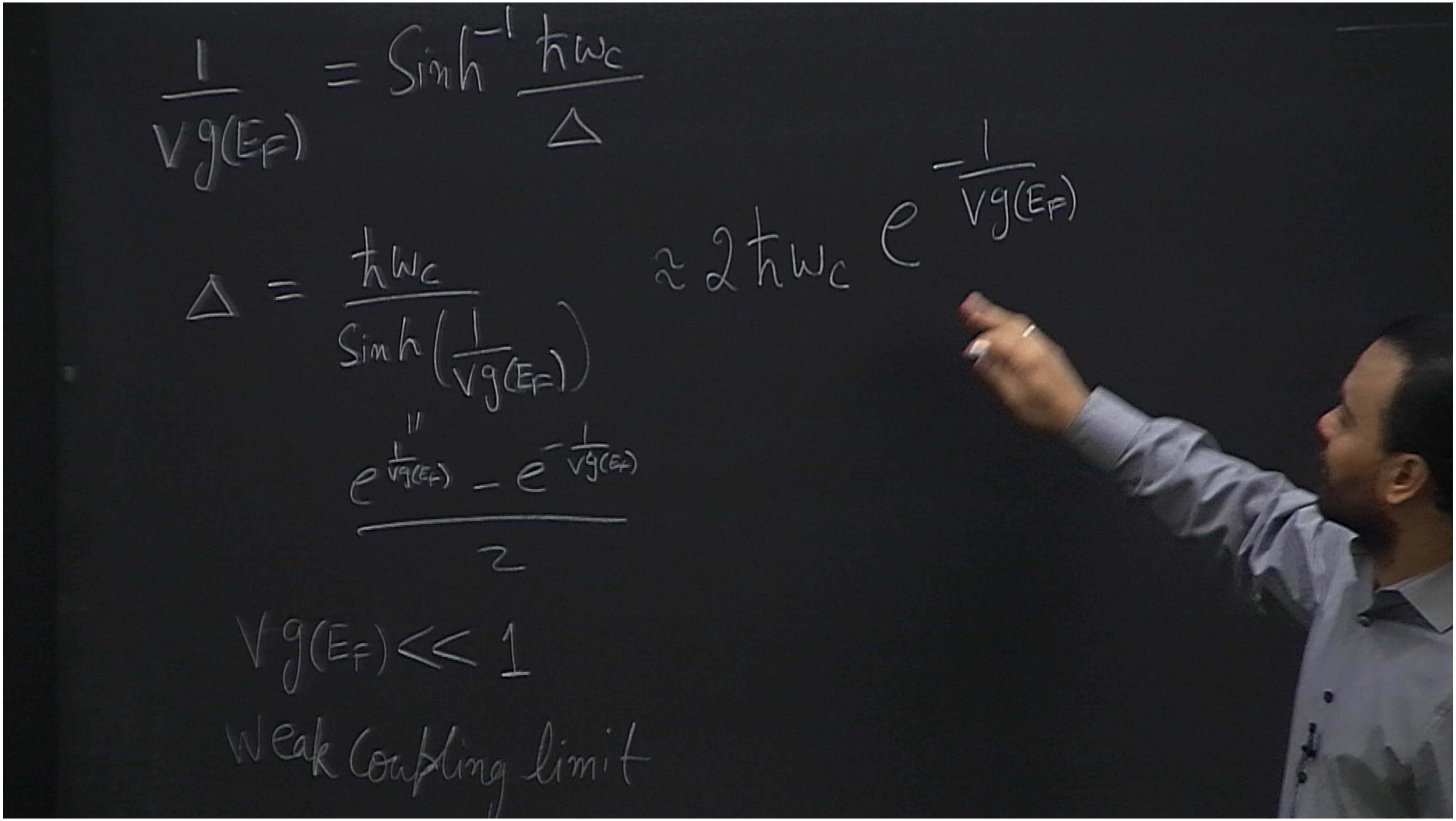
$$\Delta = \frac{\hbar\omega_c}{\sinh\left(\frac{1}{V g(E_F)}\right)}$$

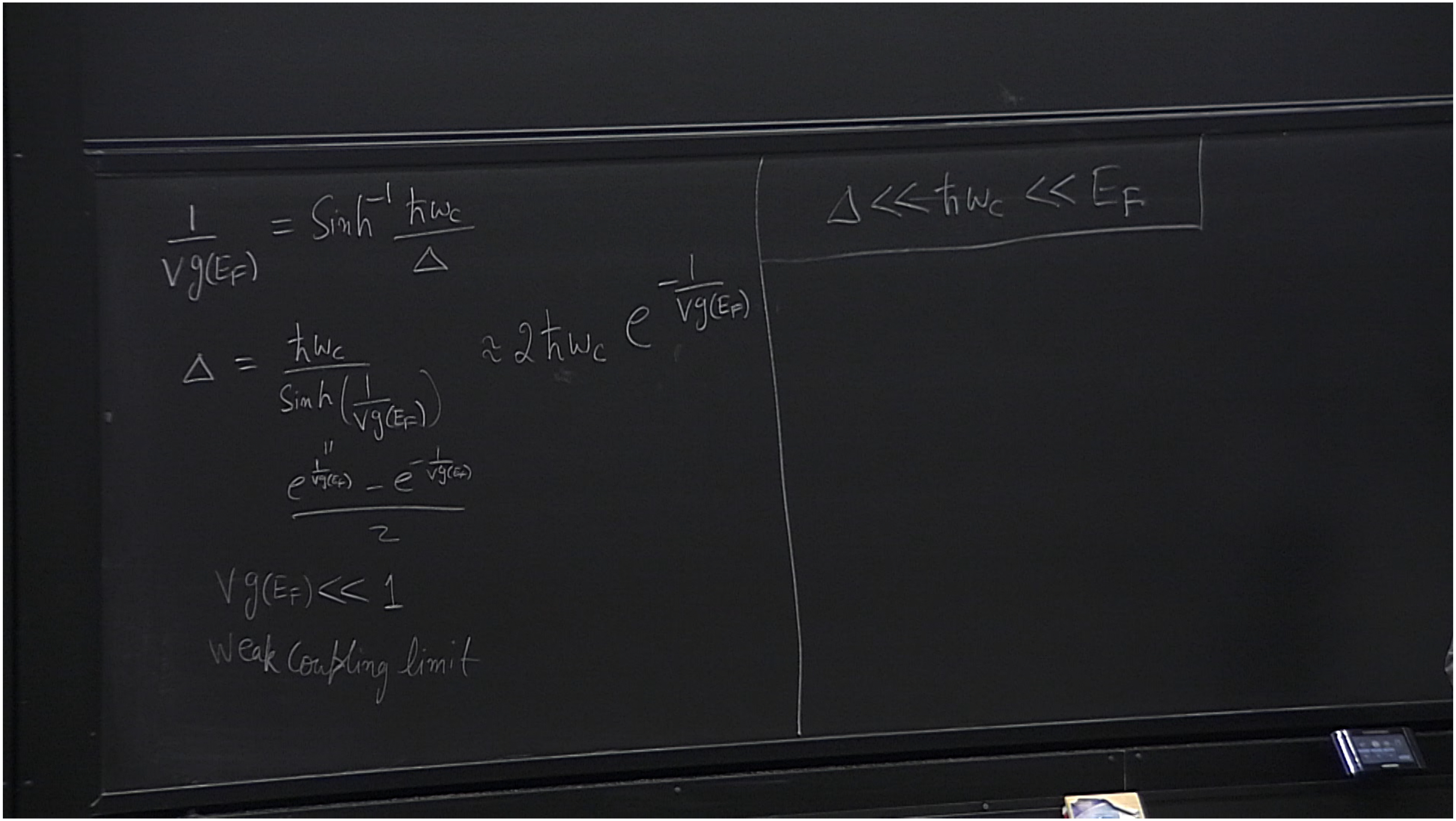
$$\frac{1}{V g(E_F)} = \sinh^{-1} \frac{\hbar \omega_c}{\Delta}$$

$$\Delta = \frac{\hbar \omega_c}{\sinh\left(\frac{1}{V g(E_F)}\right)}$$

$$\sinh\left(\frac{1}{V g(E_F)}\right) = \frac{e^{\frac{1}{V g(E_F)}} - e^{-\frac{1}{V g(E_F)}}}{2}$$

$$V g(E_F) \ll 1$$


$\frac{1}{Vg(E_F)} = \sinh^{-1}\frac{\hbar\omega_c}{\Delta}$
$\Delta = \frac{\hbar\omega_c}{\sinh\left(\frac{1}{Vg(E_F)}\right)} \simeq 2\hbar\omega_c\, e^{-\frac{1}{Vg(E_F)}}$
$\frac{e^{\frac{1}{Vg(E_F)}} - e^{-\frac{1}{Vg(E_F)}}}{2}$
$Vg(E_F) \ll 1$
weak coupling limit


$\frac{1}{Vg(E_F)} = \sinh^{-1} \frac{\hbar\omega_c}{\Delta}$
$\Delta = \frac{\hbar\omega_c}{\sinh\left(\frac{1}{Vg(E_F)}\right)} \simeq 2\hbar\omega_c e^{-\frac{1}{Vg(E_F)}}$
$\frac{e^{\frac{1}{Vg(E_F)}} - e^{-\frac{1}{Vg(E_F)}}}{2}$
$Vg(E_F) \ll 1$
Weak coupling limit
$\Delta \ll \hbar\omega_c \ll E_F$

$$\frac{1}{V g(E_F)} = \sinh^{-1} \frac{\hbar\omega_c}{\Delta}$$

$$\Delta = \frac{\hbar\omega_c}{\sinh\left(\frac{1}{V g(E_F)}\right)} \simeq 2\hbar\omega_c \, e^{-\frac{1}{V g(E_F)}}$$

$$\sinh\left(\frac{1}{V g(E_F)}\right) = \frac{e^{\frac{1}{V g(E_F)}} - e^{-\frac{1}{V g(E_F)}}}{2}$$

$$V g(E_F) \ll 1$$

Weak coupling limit

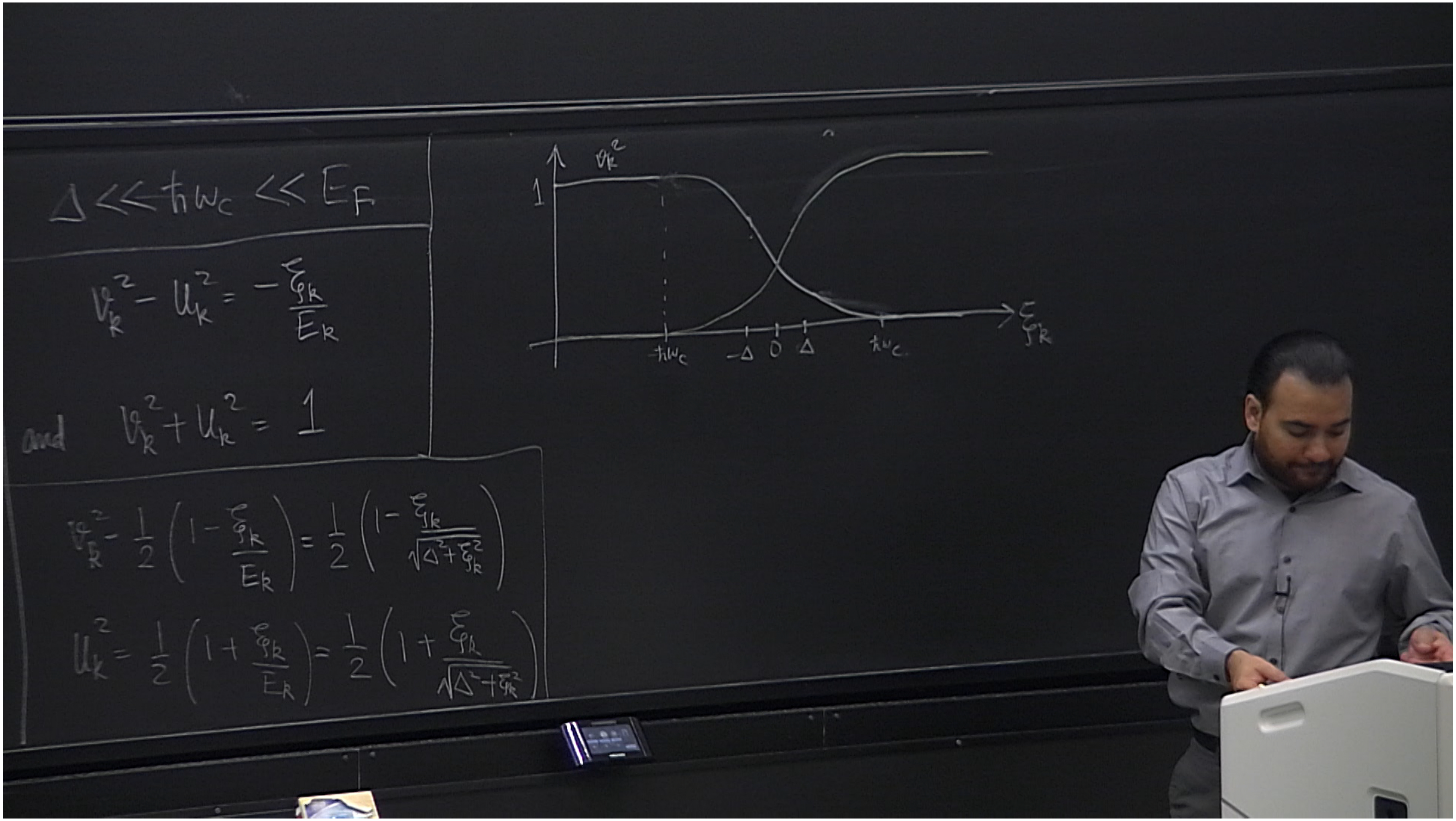
$$\Delta \ll \hbar\omega_c \ll E_F$$

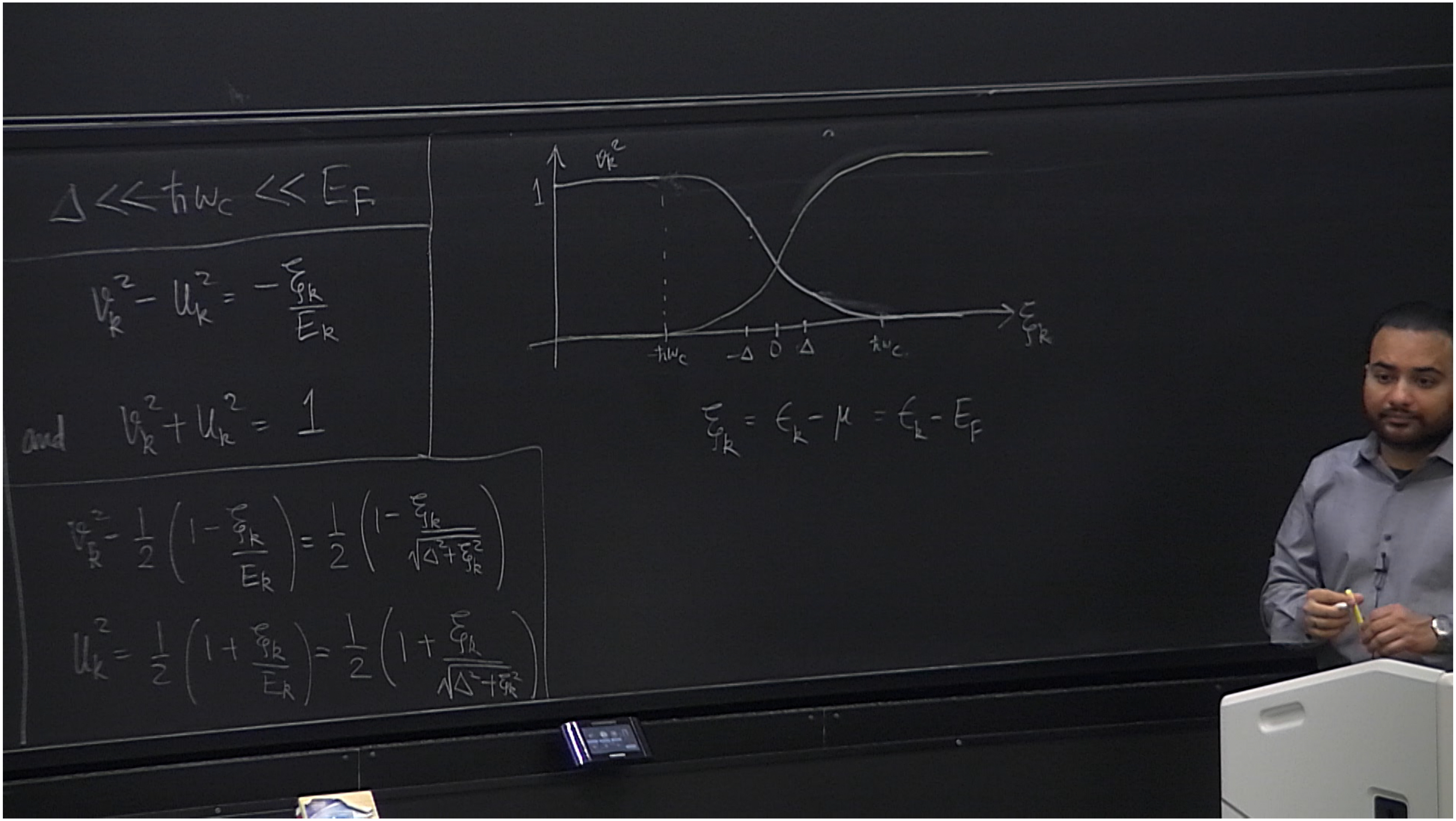
$$v_k^2 - u_k^2 = -\frac{\xi_k}{E_k}$$

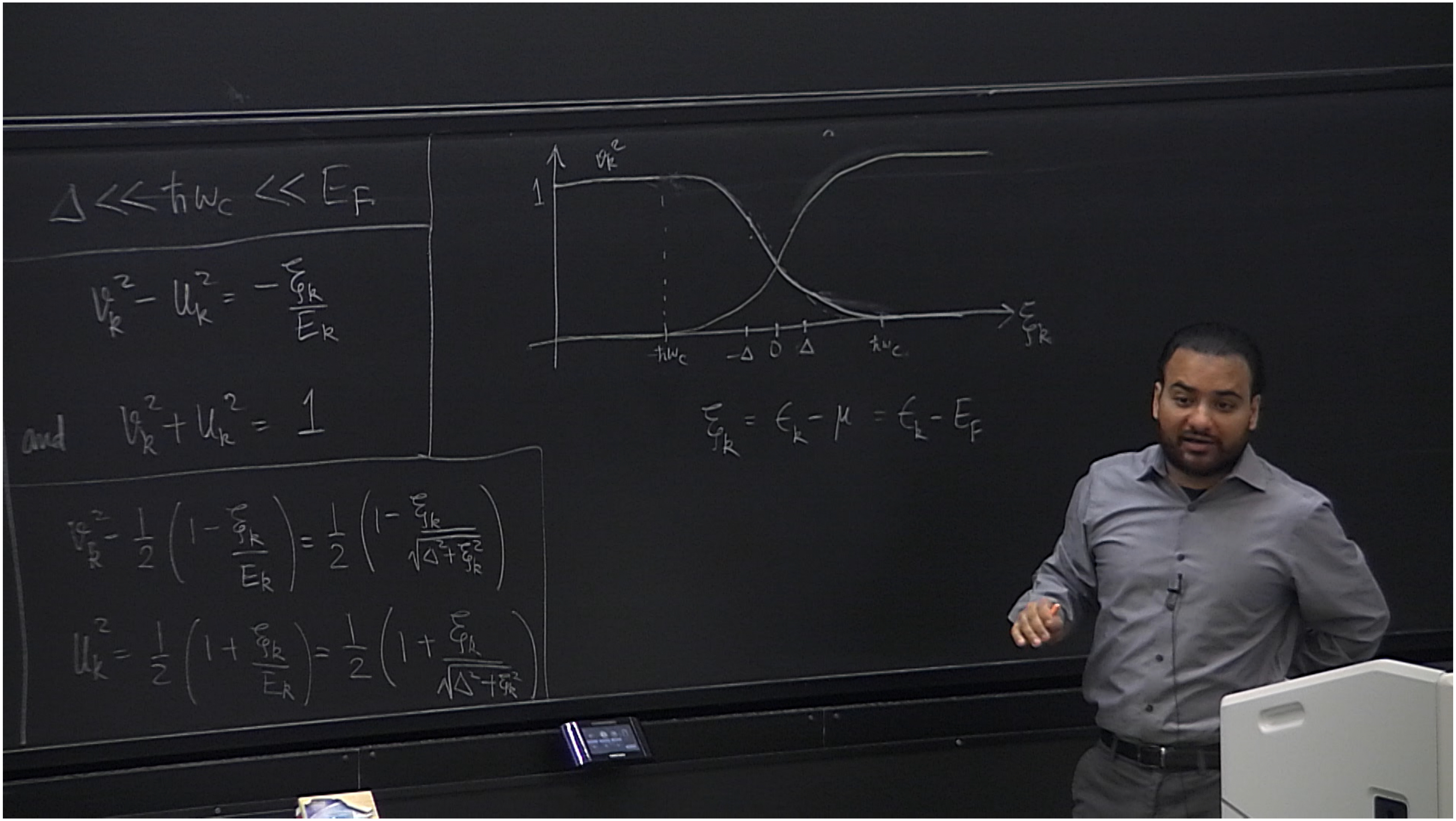
and $$v_k^2 + u_k^2 = 1$$

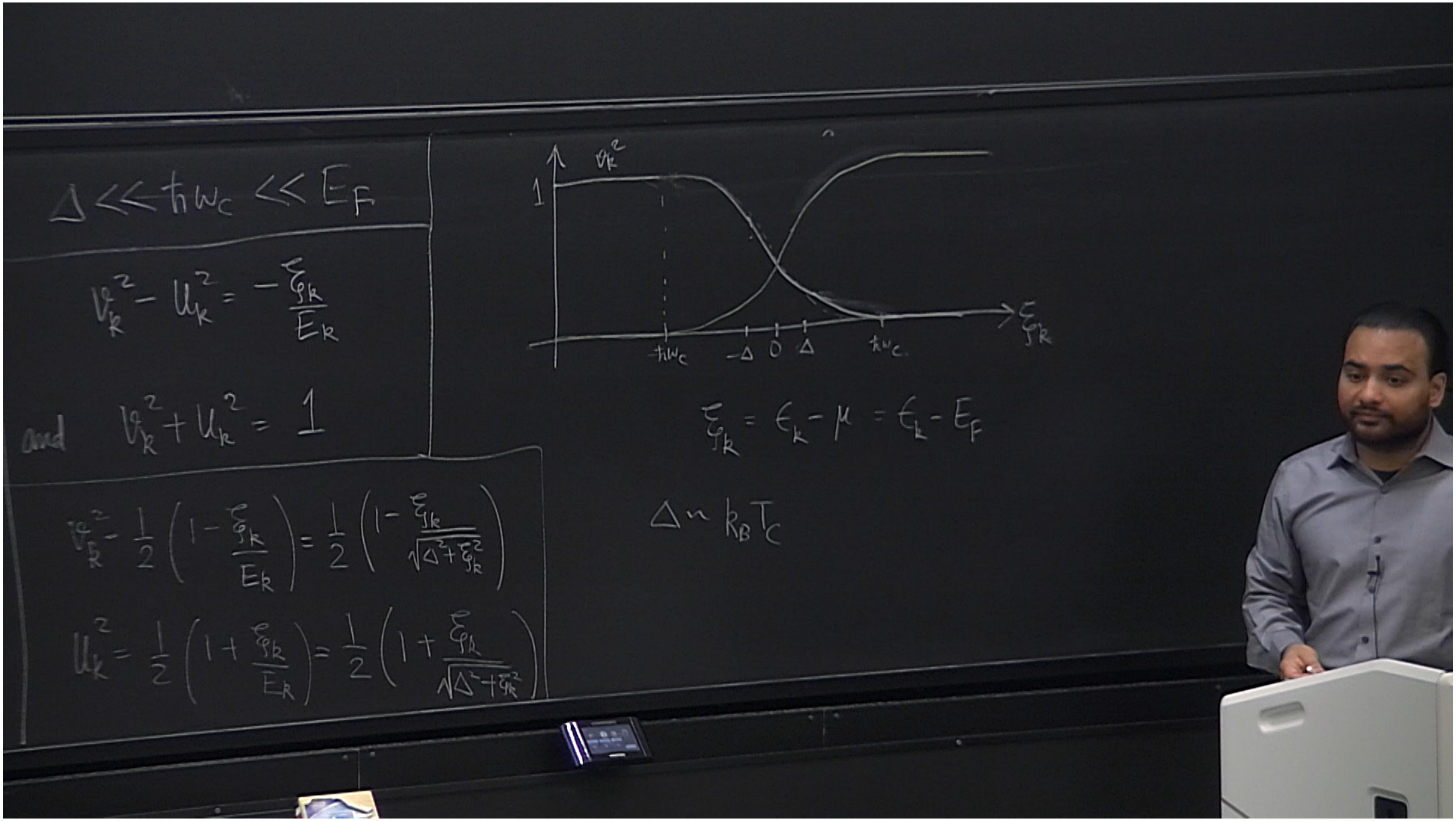
$$v_k^2 = \frac{1}{2}\left(1 - \frac{\xi_k}{E_k}\right) = \frac{1}{2}\left(1 - \frac{\xi_k}{\sqrt{\Delta^2 + \xi_k^2}}\right)$$

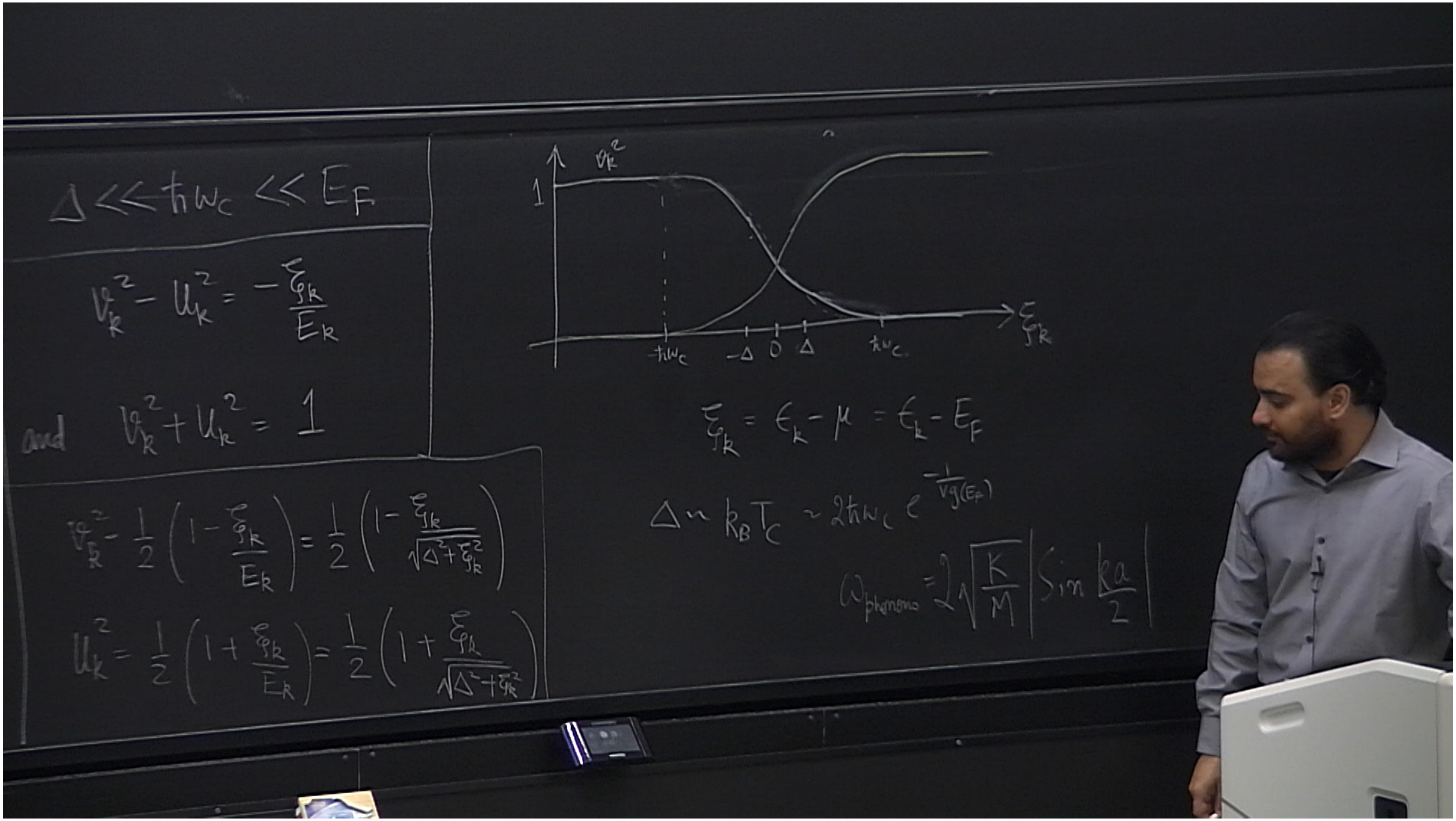
$$u_k^2 = \frac{1}{2}\left(1 + \frac{\xi_k}{E_k}\right) = \frac{1}{2}\left(1 + \frac{\xi_k}{\sqrt{\Delta^2 + \xi_k^2}}\right)$$

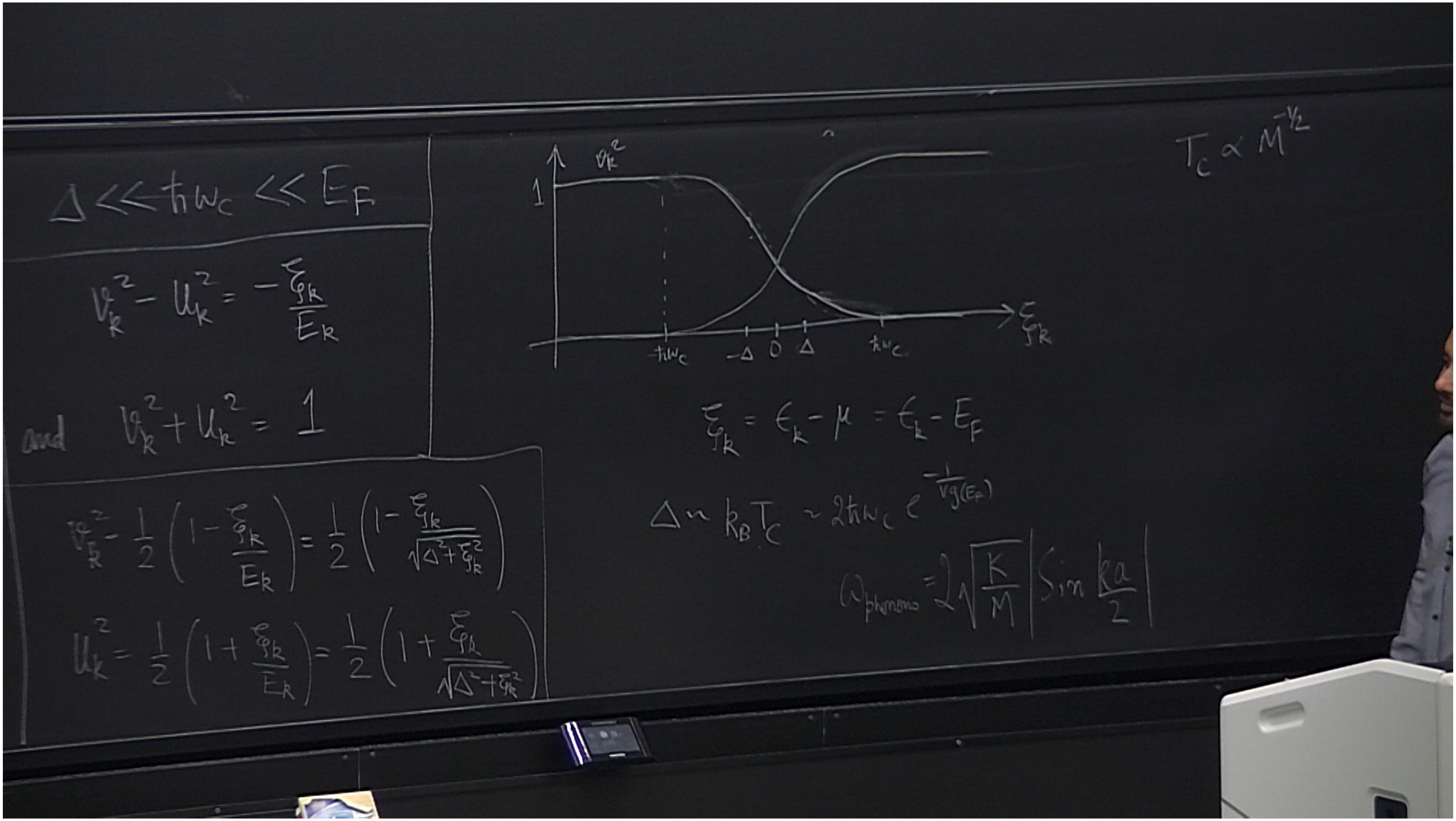

$\Delta \ll \hbar\omega_c \ll E_F$
$v_k^2 - u_k^2 = -\frac{\xi_k}{E_k}$
and $v_k^2 + u_k^2 = 1$
$v_k^2 = \frac{1}{2}\left(1 - \frac{\xi_k}{E_k}\right) = \frac{1}{2}\left(1 - \frac{\xi_k}{\sqrt{\Delta^2 + \xi_k^2}}\right)$
$u_k^2 = \frac{1}{2}\left(1 + \frac{\xi_k}{E_k}\right) = \frac{1}{2}\left(1 + \frac{\xi_k}{\sqrt{\Delta^2 + \xi_k^2}}\right)$
v_k^2
1
$-\hbar\omega_c$
$-\Delta$
0
Δ
$\hbar\omega_c$
ξ_k


Δ << ħω_c << E_F
v_k^2 − u_k^2 = − ξ_k / E_k
and v_k^2 + u_k^2 = 1
v_k^2 = 1/2 (1 − ξ_k / E_k) = 1/2 (1 − ξ_k / √(Δ^2 + ξ_k^2))
u_k^2 = 1/2 (1 + ξ_k / E_k) = 1/2 (1 + ξ_k / √(Δ^2 + ξ_k^2))
v_k^2
1
−ħω_c
−Δ
0
Δ
ħω_c
ξ_k
ξ_k = ε_k − μ = ε_k − E_F

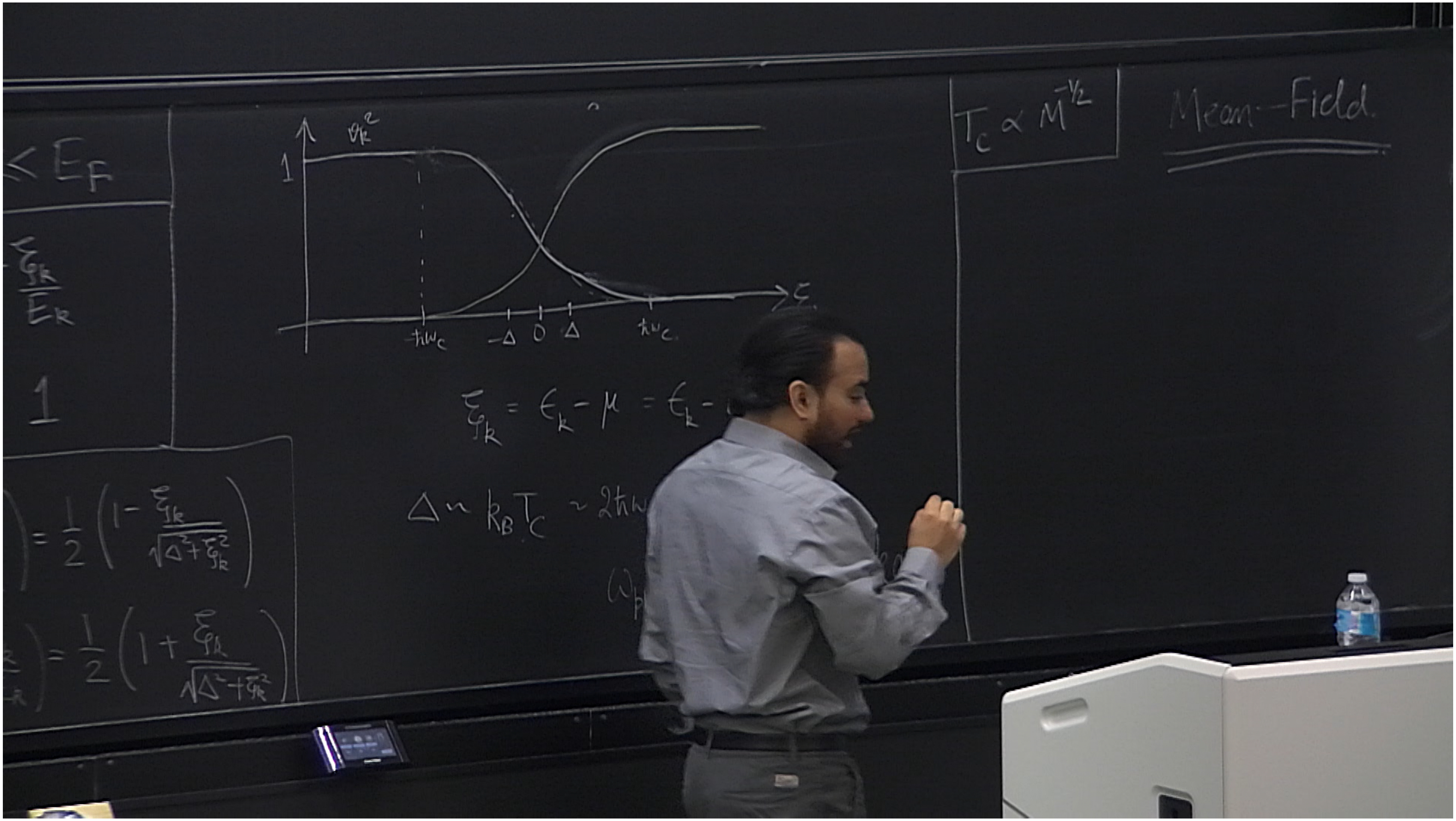

Δ << ħω_c << E_F
v_k² − u_k² = − ξ_k / E_k
and v_k² + u_k² = 1
v_k² = 1/2 (1 − ξ_k/E_k) = 1/2 (1 − ξ_k/√(Δ² + ξ_k²))
u_k² = 1/2 (1 + ξ_k/E_k) = 1/2 (1 + ξ_k/√(Δ² + ξ_k²))
v_k²
1
−ħω_c
−Δ
0
Δ
ħω_c
ξ_k
ξ_k = ε_k − μ = ε_k − E_F


$\Delta \ll \hbar\omega_c \ll E_F$
$v_k^2 - u_k^2 = -\frac{\xi_k}{E_k}$
and $v_k^2 + u_k^2 = 1$
$v_k^2 = \frac{1}{2}\left(1 - \frac{\xi_k}{E_k}\right) = \frac{1}{2}\left(1 - \frac{\xi_k}{\sqrt{\Delta^2+\xi_k^2}}\right)$
$u_k^2 = \frac{1}{2}\left(1 + \frac{\xi_k}{E_k}\right) = \frac{1}{2}\left(1 + \frac{\xi_k}{\sqrt{\Delta^2+\xi_k^2}}\right)$
v_k^2
1
$-\hbar\omega_c$ $-\Delta$ 0 Δ $\hbar\omega_c$
ξ_k
$\xi_k = \epsilon_k - \mu = \epsilon_k - E_F$
$\Delta \sim k_B T_c$

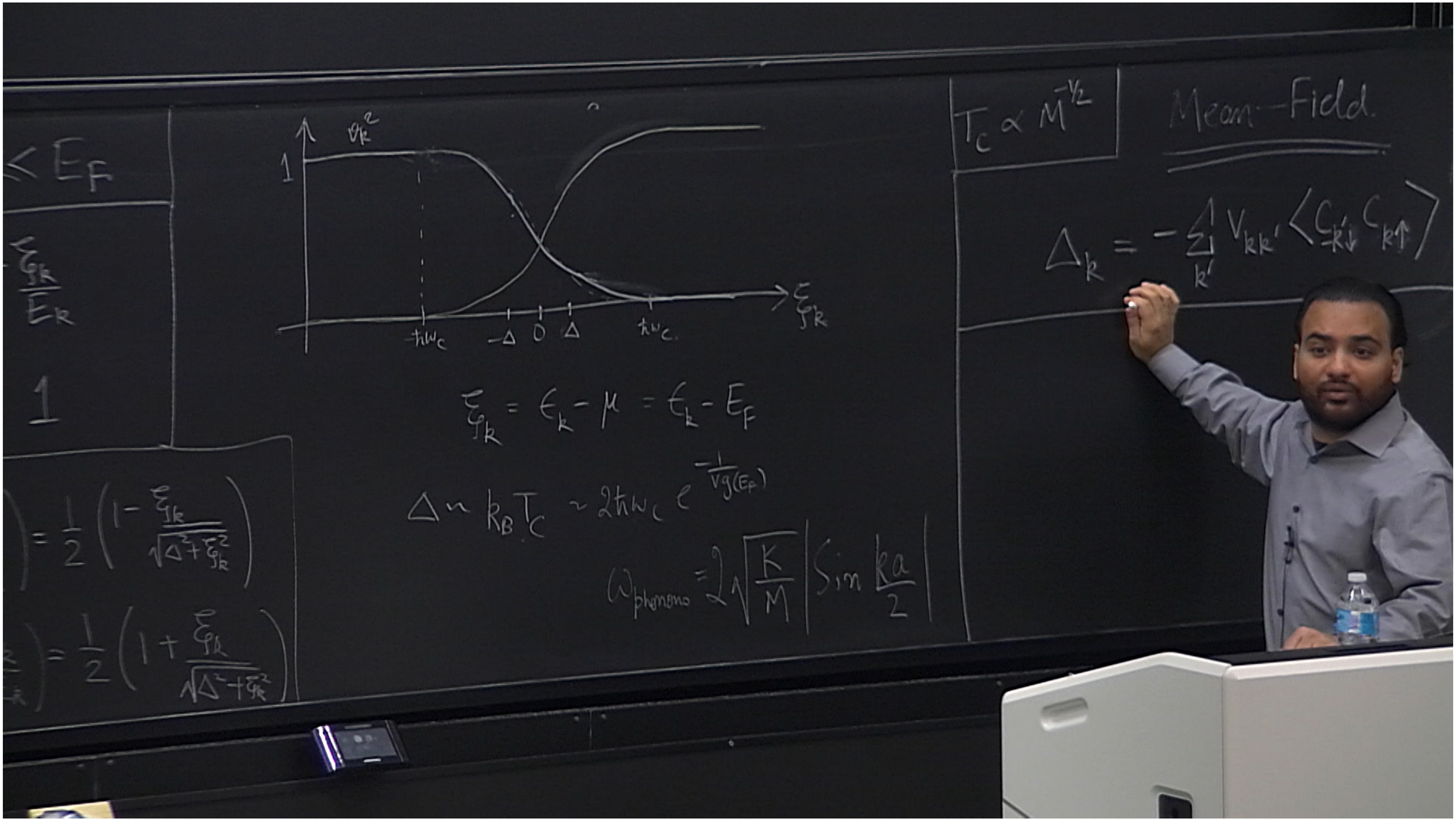

$\Delta \ll \hbar\omega_c \ll E_F$
$v_k^2 - u_k^2 = -\frac{\xi_k}{E_k}$
and $v_k^2 + u_k^2 = 1$
$v_k^2 = \frac{1}{2}\left(1 - \frac{\xi_k}{E_k}\right) = \frac{1}{2}\left(1 - \frac{\xi_k}{\sqrt{\Delta^2+\xi_k^2}}\right)$
$u_k^2 = \frac{1}{2}\left(1 + \frac{\xi_k}{E_k}\right) = \frac{1}{2}\left(1 + \frac{\xi_k}{\sqrt{\Delta^2+\xi_k^2}}\right)$
v_k^2
1
$-\hbar\omega_c$
$-\Delta$
0
Δ
$\hbar\omega_c$
ξ_k
$\xi_k = \epsilon_k - \mu = \epsilon_k - E_F$
$\Delta \sim k_B T_c \sim 2\hbar\omega_c e^{-\frac{1}{Vg(E_F)}}$
$\omega_{phonons} = 2\sqrt{\frac{K}{M}}\left|\sin\frac{ka}{2}\right|$



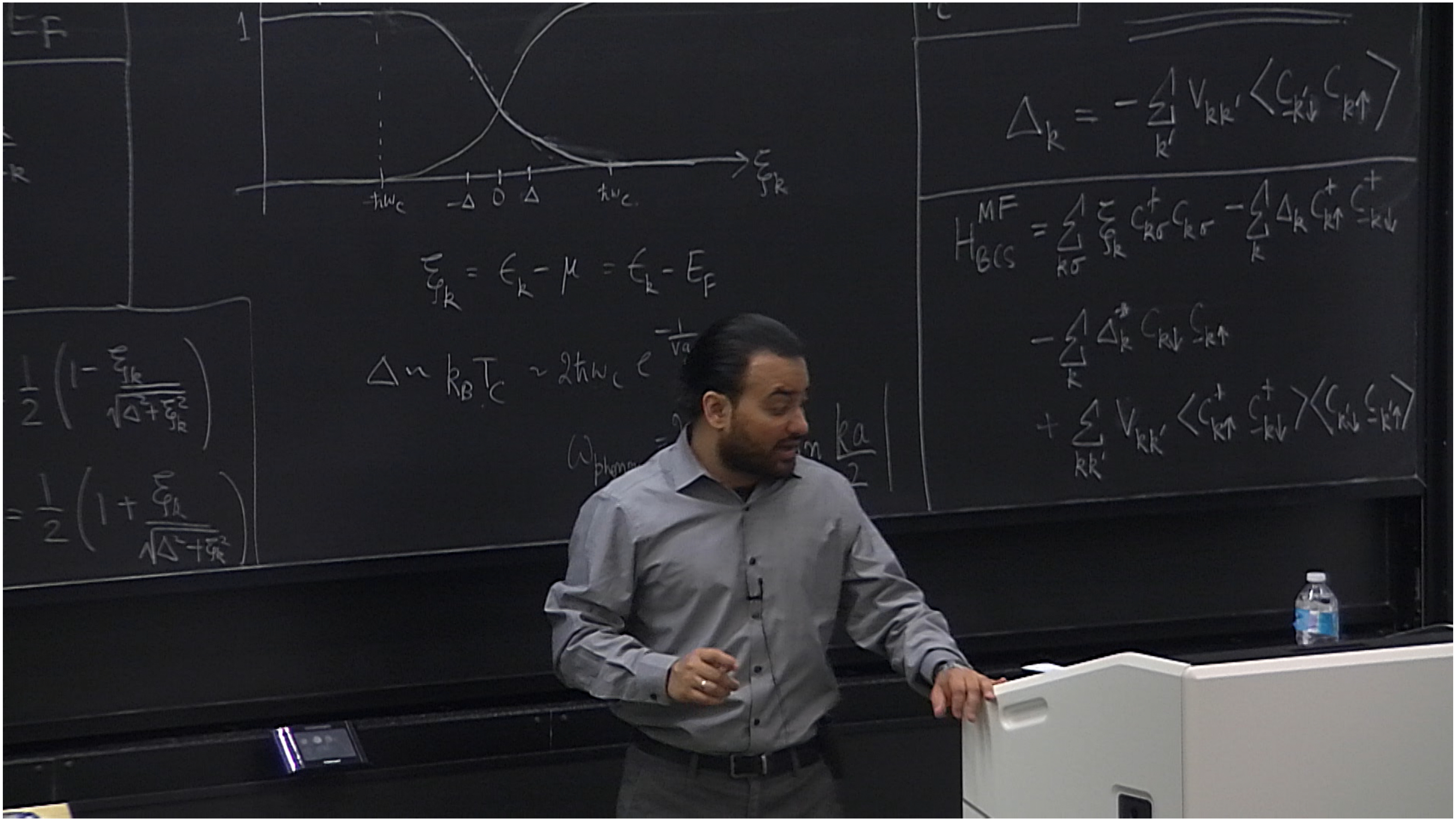
Δ << ħω_c << E_F
v_k^2 − u_k^2 = − ξ_k / E_k
and v_k^2 + u_k^2 = 1
v_k^2 = 1/2 (1 − ξ_k / E_k) = 1/2 (1 − ξ_k / √(Δ^2 + ξ_k^2))
u_k^2 = 1/2 (1 + ξ_k / E_k) = 1/2 (1 + ξ_k / √(Δ^2 + ξ_k^2))
v_k^2
1
−ħω_c
−Δ
0
Δ
ħω_c
ξ_k
ξ_k = ε_k − μ = ε_k − E_F
Δ ~ k_B T_c ~ 2ħω_c e^(−1/(V g(E_F)))
ω_phonons = 2 √(K/M) |sin(ka/2)|
T_c ∝ M^(−1/2)



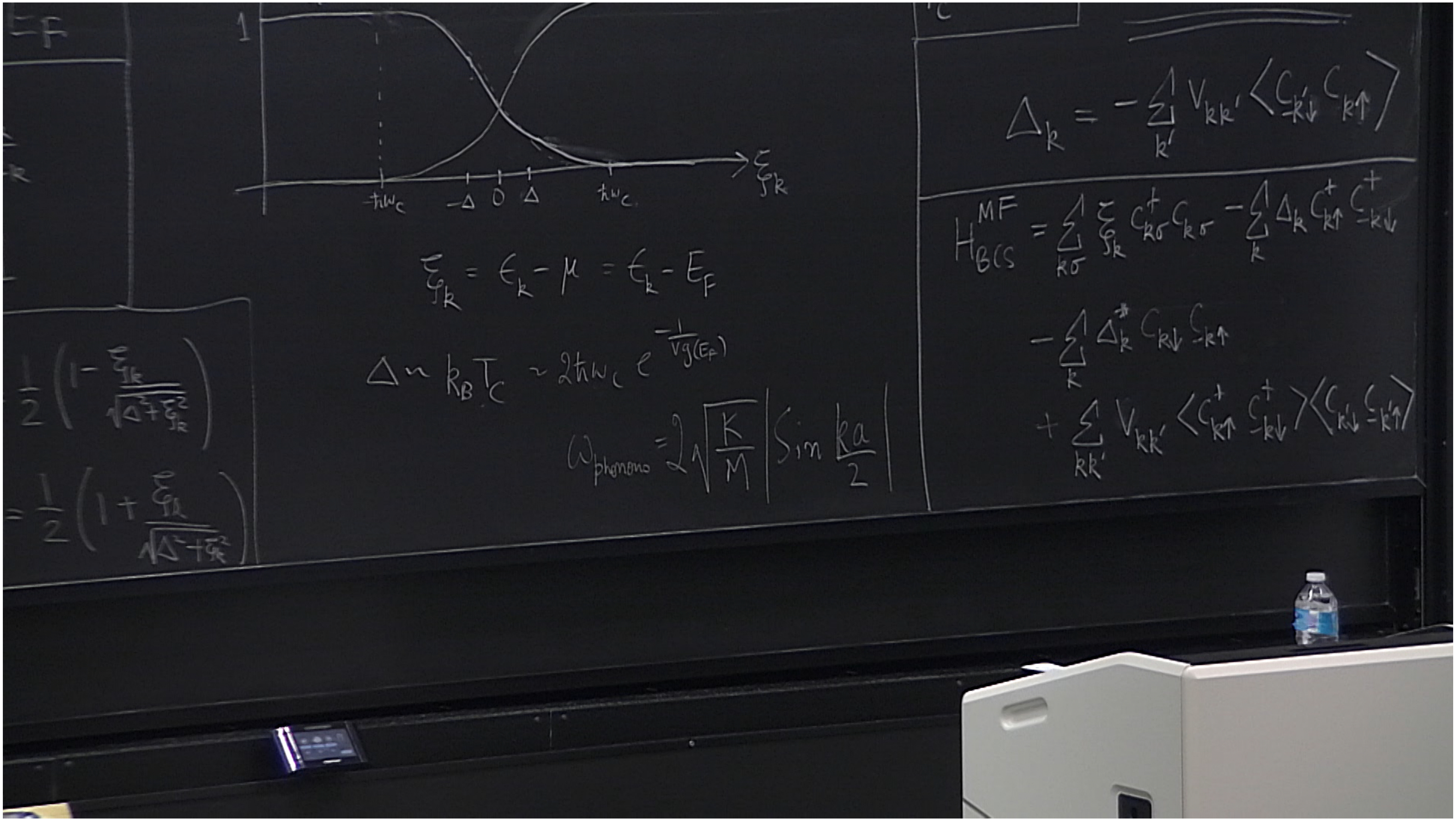
Tc ∝ M^{-1/2}
Mean-Field
ξk = εk − μ = εk −
Δ ~ kB Tc ~ 2ħω
−ħωc
−Δ
0
Δ
ħωc
= 1/2 (1 − ξk / √(Δ²+ξk²))
= 1/2 (1 + ξk / √(Δ²+ξk²))

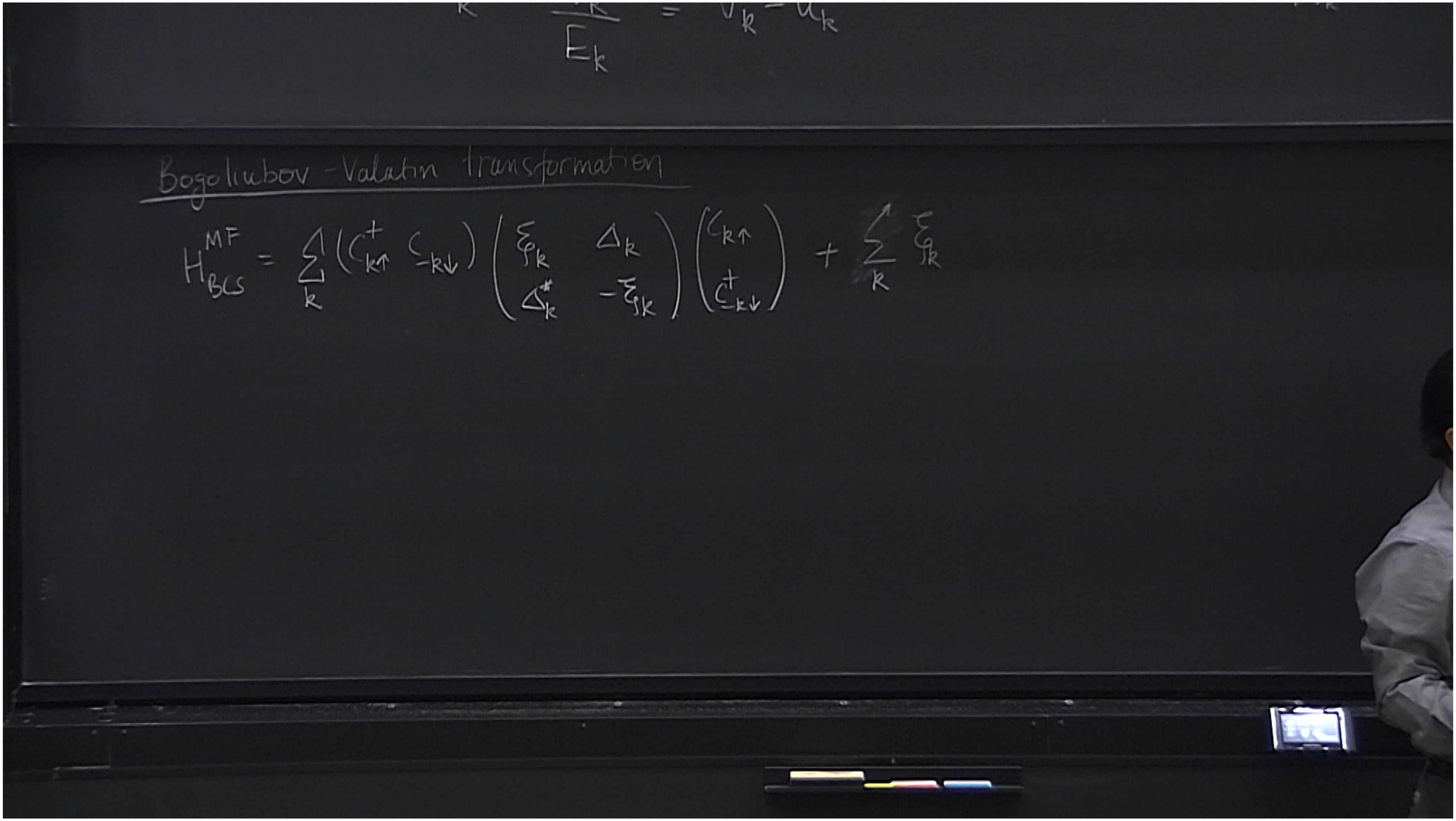


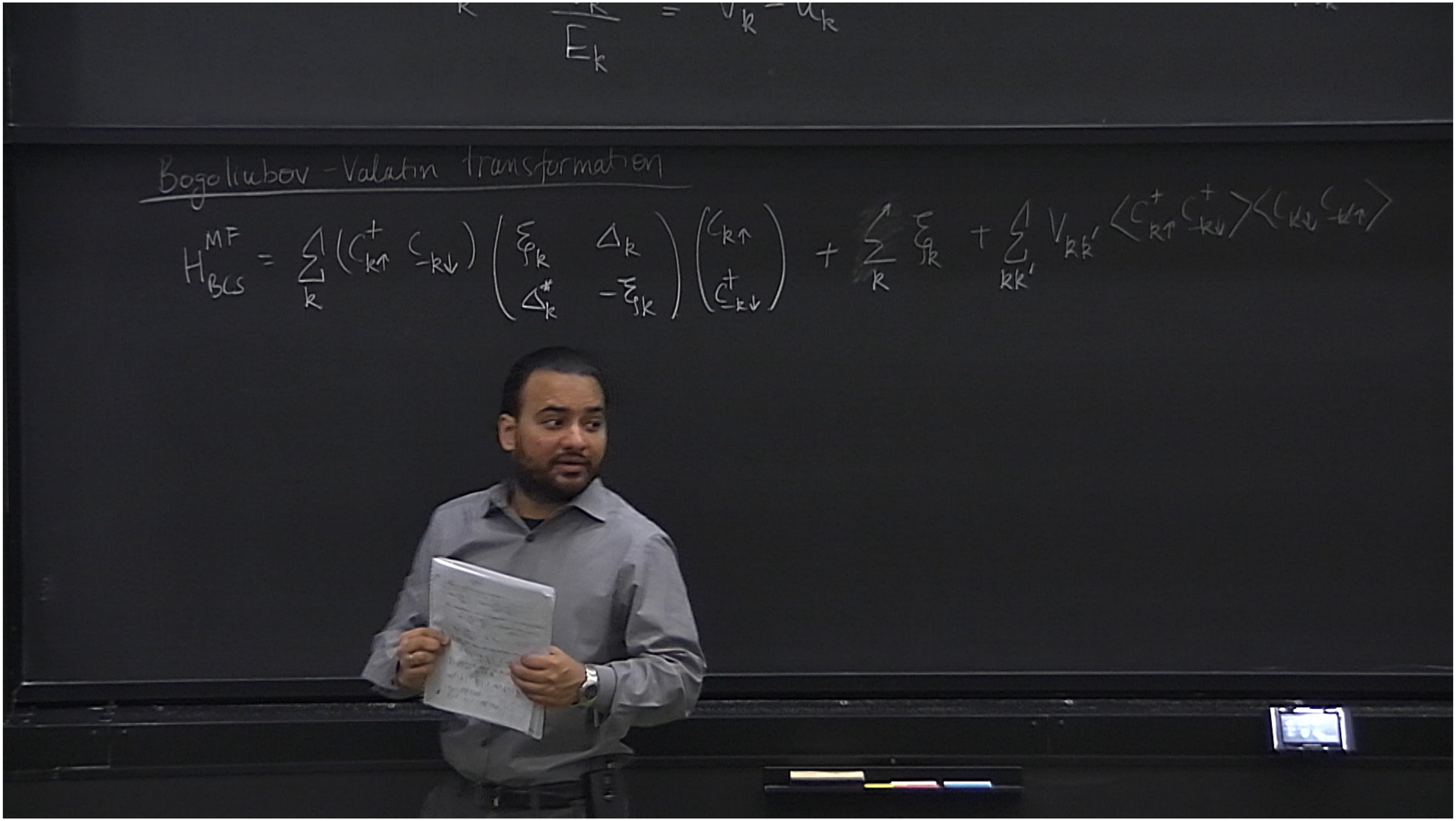
Mean-Field
Tc ∝ M^-1/2
Δk = − Σk' Vkk' ⟨C−k'↓ Ck'↑⟩
ξk = εk − μ = εk − EF
Δ ~ kB Tc ~ 2ħωc e^(−1/Vg(EF))
ωphonons = 2 √(K/M) |Sin ka/2|

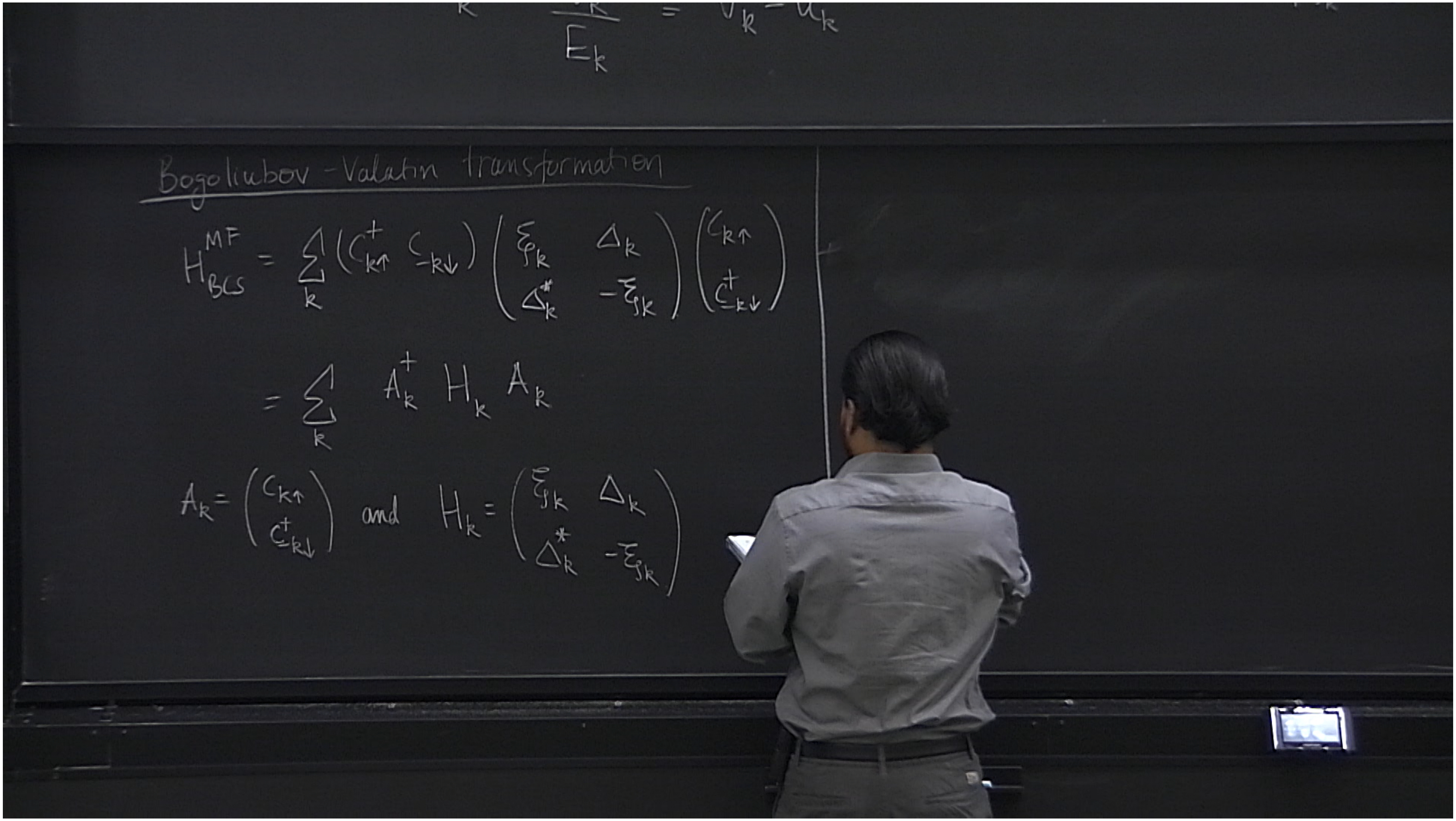


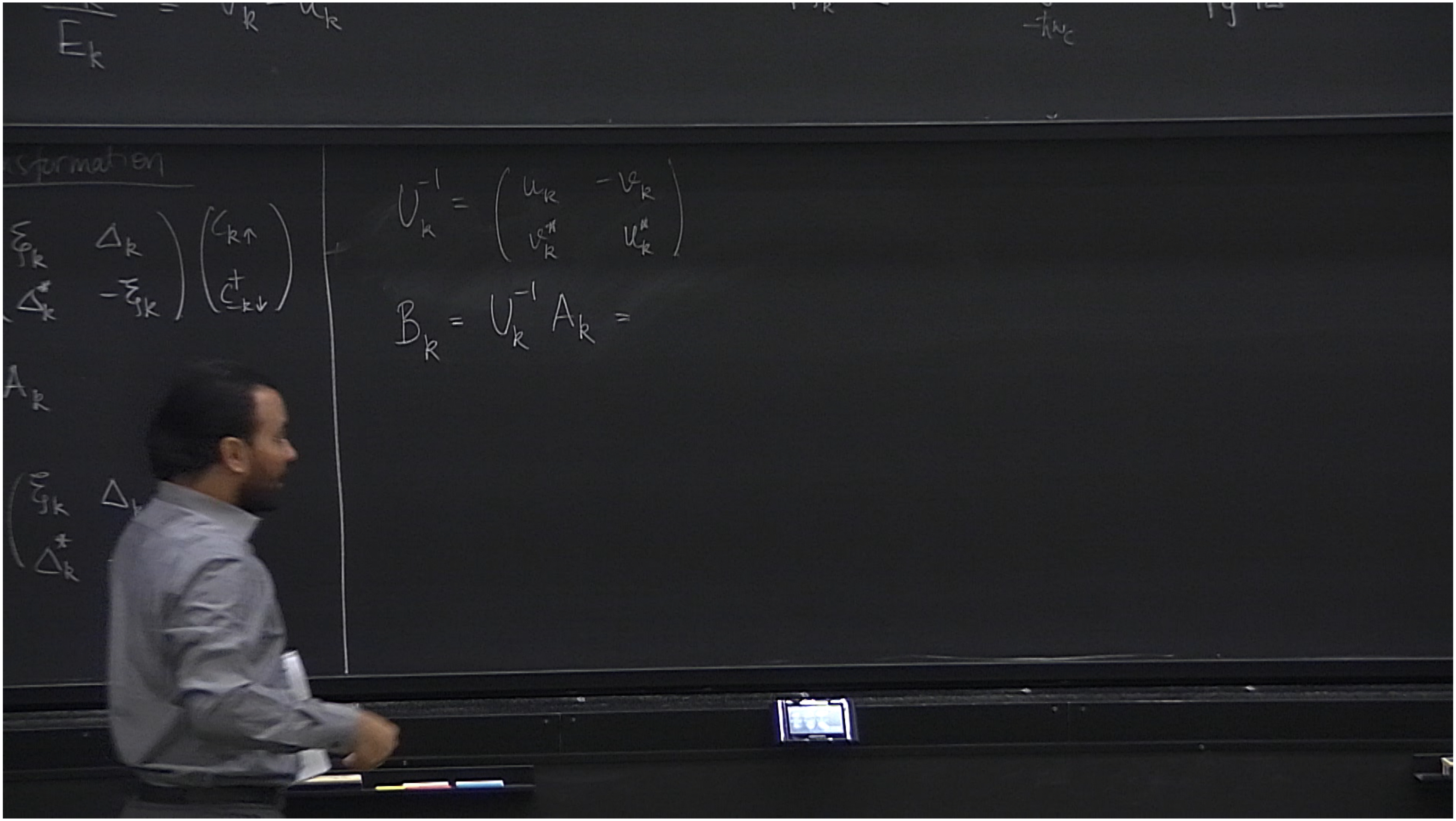
1
−ħω_c −Δ 0 Δ ħω_c ξ_k
ξ_k = ε_k − μ = ε_k − E_F
Δ ~ k_B T_c ~ 2ħω_c e
Δ_k = − Σ_k' V_kk' ⟨c_-k'↓ c_k'↑⟩
H^MF_BCS = Σ_kσ ξ_k c†_kσ c_kσ − Σ_k Δ_k c†_k↑ c†_-k↓
− Σ_k Δ*_k c_-k↓ c_k↑
+ Σ_kk' V_kk' ⟨c†_k↑ c†_-k↓⟩⟨c_-k'↓ c_k'↑⟩

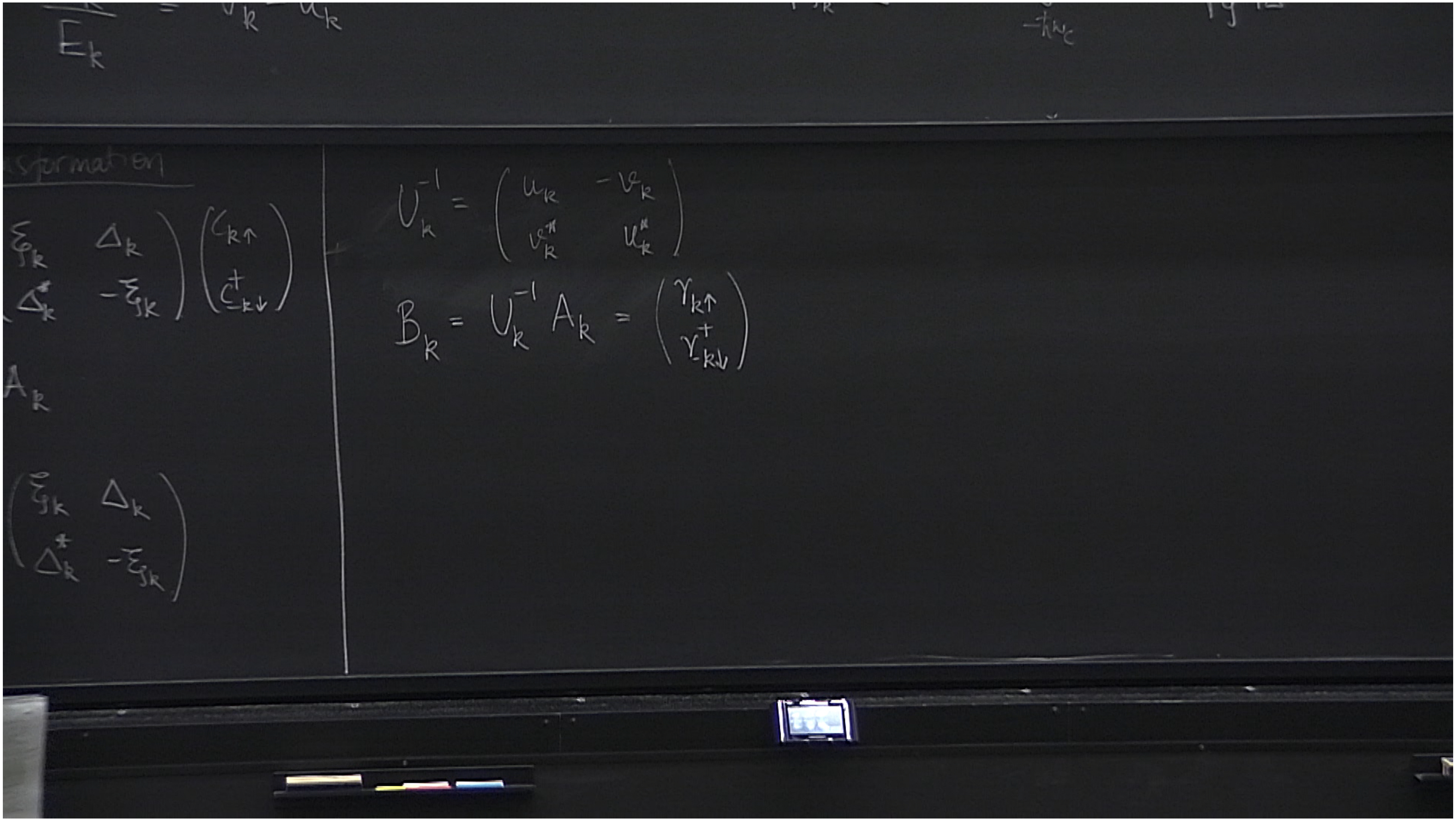

ξ_k = ε_k − μ = ε_k − E_F
Δ ~ k_B T_c ~ 2ħω_c e^{−1/(Vg(E_F))}
ω_phonon = 2√(K/M) |sin(ka/2)|
−ħω_c −Δ 0 Δ ħω_c
ξ_k
Δ_k = − Σ_k' V_kk' ⟨c_−k'↓ c_k'↑⟩
H_BCS^MF = Σ_kσ ξ_k c†_kσ c_kσ − Σ_k Δ_k c†_k↑ c†_−k↓
− Σ_k Δ*_k c_−k↓ c_k↑
+ Σ_kk' V_kk' ⟨c†_k↑ c†_−k↓⟩⟨c_−k'↓ c_k'↑⟩
1/2 (1 − ξ_k/√(Δ² + ξ_k²))
1/2 (1 + ξ_k/√(Δ² + ξ_k²))

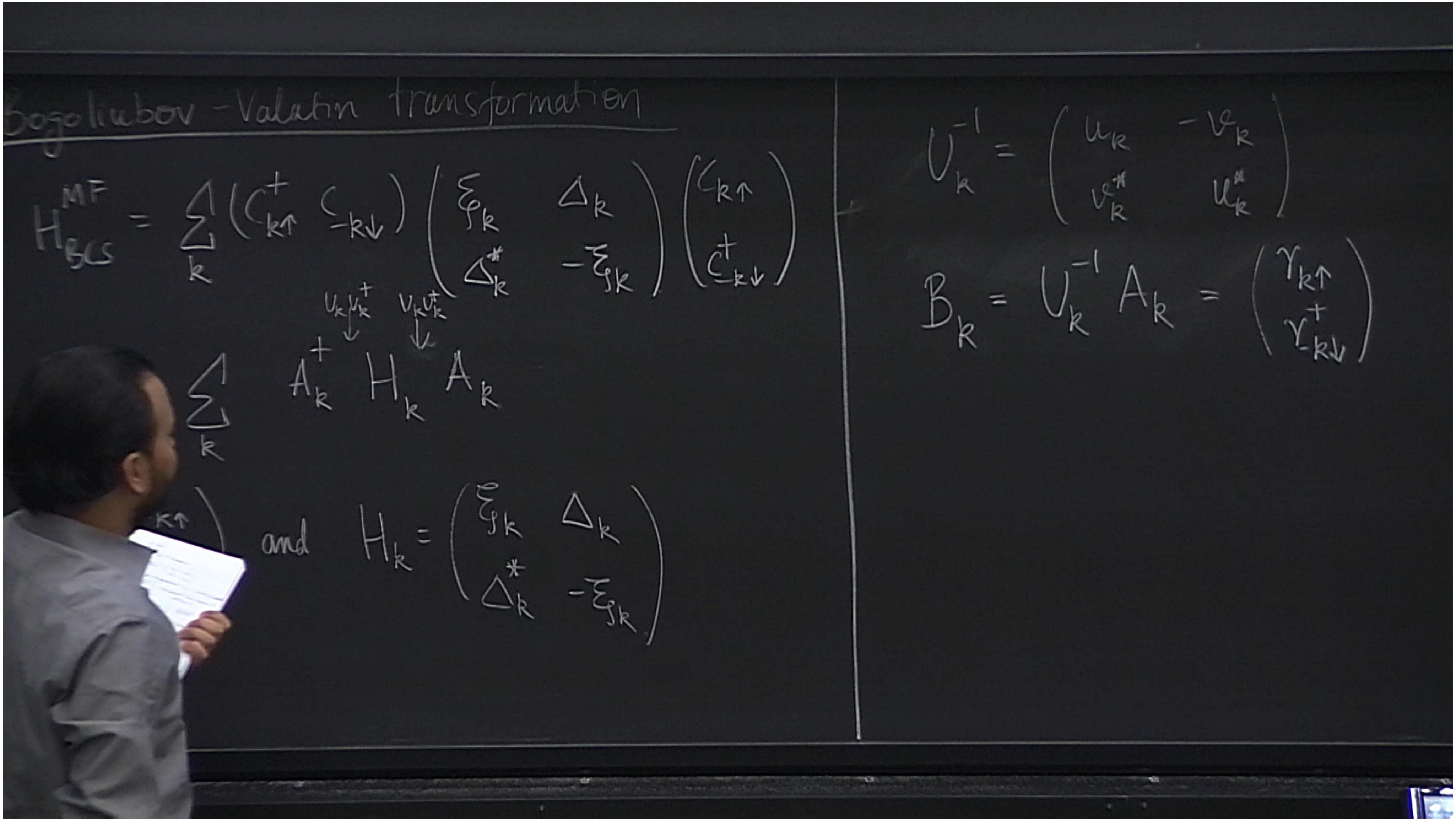

Bogoliubov - Valatin transformation
$H^{MF}_{BCS} = \sum_k (c^\dagger_{k\uparrow} \; c_{-k\downarrow}) \begin{pmatrix} \xi_k & \Delta_k \\ \Delta^*_k & -\xi_k \end{pmatrix} \begin{pmatrix} c_{k\uparrow} \\ c^\dagger_{-k\downarrow} \end{pmatrix} + \sum_k \xi_k$

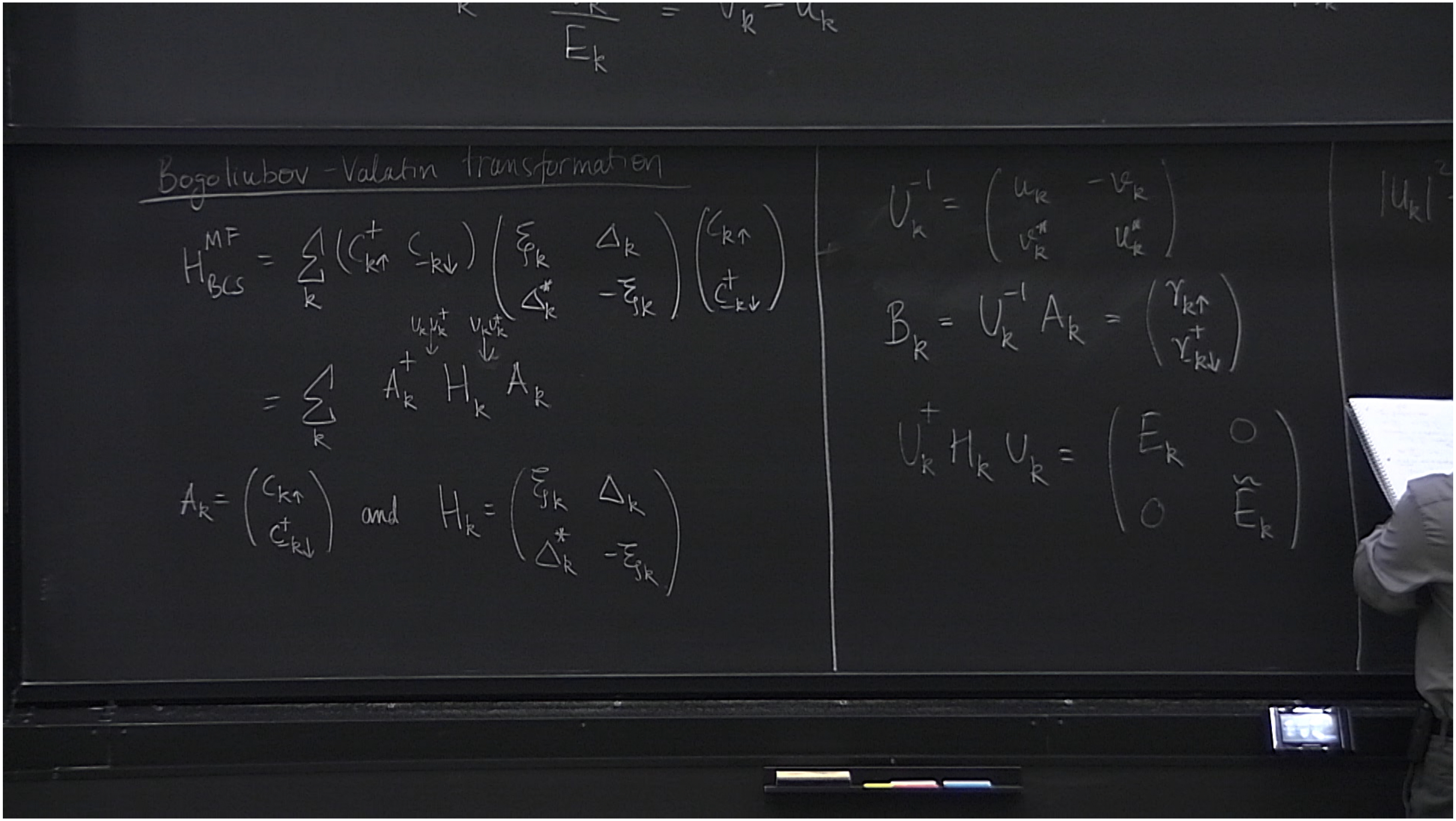

Bogoliubov - Valatin transformation
$H^{MF}_{BCS} = \sum_k (c^\dagger_{k\uparrow} \; c_{-k\downarrow}) \begin{pmatrix} \xi_k & \Delta_k \\ \Delta^*_k & -\xi_k \end{pmatrix} \begin{pmatrix} c_{k\uparrow} \\ c^\dagger_{-k\downarrow} \end{pmatrix} + \sum_k \xi_k + \sum_{kk'} V_{kk'} \langle c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow} \rangle \langle c_{k\downarrow} c_{-k\uparrow} \rangle$


Bogoliubov - Valatin transformation
$H^{MF}_{BCS} = \sum_k \left(c^\dagger_{k\uparrow} \; c_{-k\downarrow}\right) \begin{pmatrix} \xi_k & \Delta_k \\ \Delta^*_k & -\xi_k \end{pmatrix} \begin{pmatrix} c_{k\uparrow} \\ c^\dagger_{-k\downarrow} \end{pmatrix}$
$= \sum_k A^\dagger_k H_k A_k$
$A_k = \begin{pmatrix} c_{k\uparrow} \\ c^\dagger_{-k\downarrow} \end{pmatrix}$ and $H_k = \begin{pmatrix} \xi_k & \Delta_k \\ \Delta^*_k & -\xi_k \end{pmatrix}$

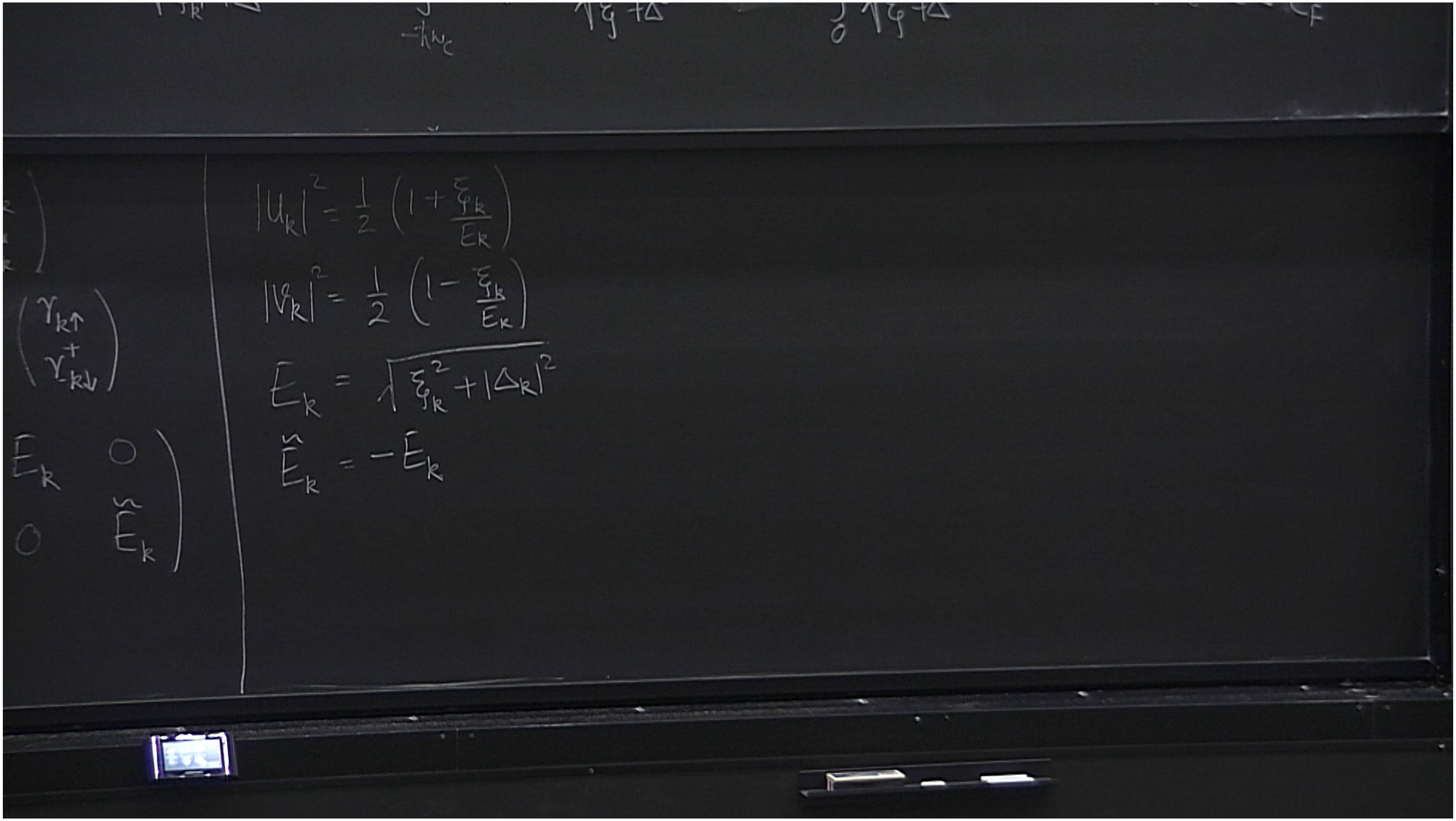

U_k^{-1} = (u_k -v_k ; v_k^* u_k^*)
B_k = U_k^{-1} A_k =

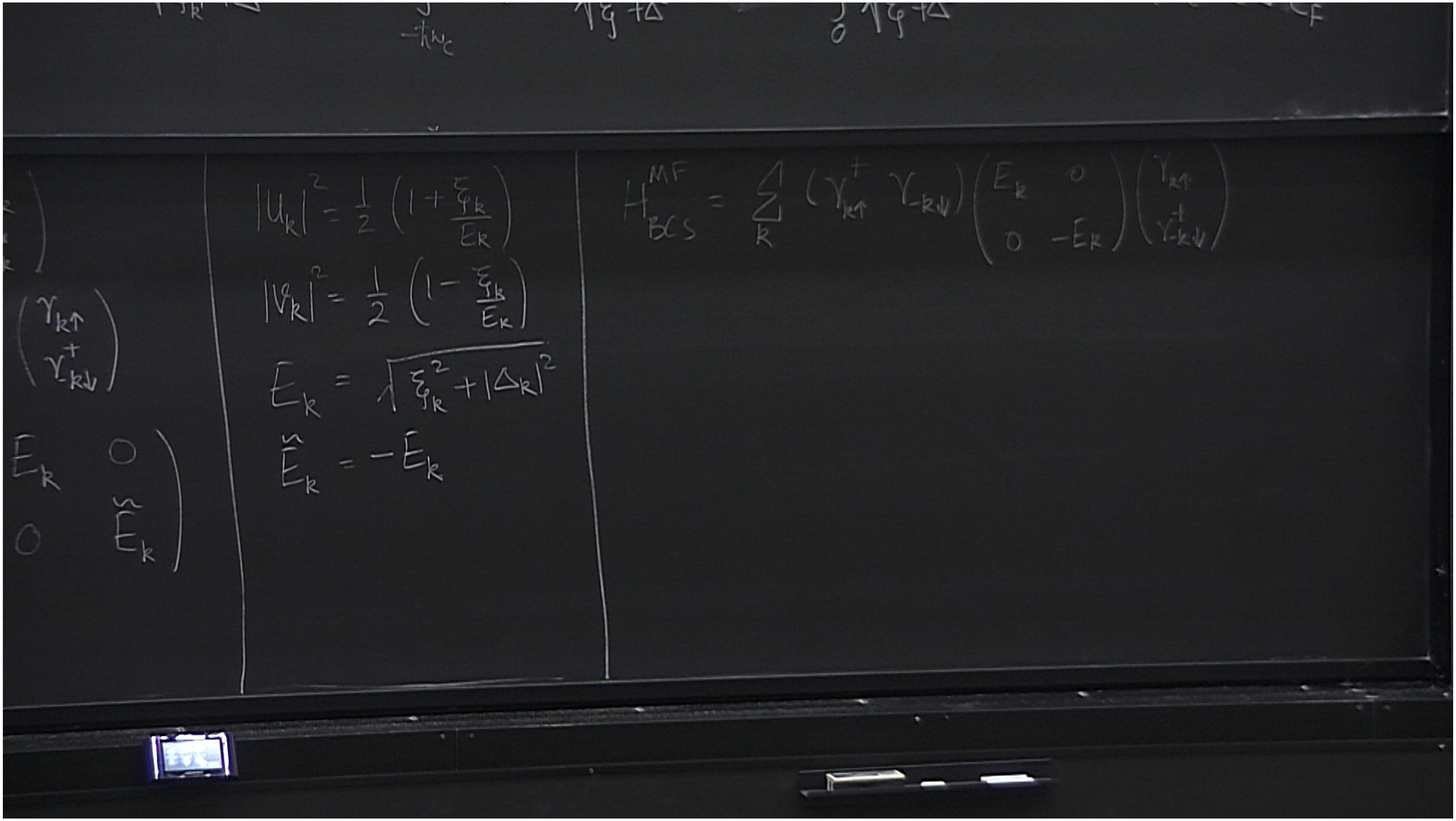

$U_k^{-1} = \begin{pmatrix} u_k & -v_k \\ v_k^* & u_k^* \end{pmatrix}$
$B_k = U_k^{-1} A_k = \begin{pmatrix} \gamma_{k\uparrow} \\ \gamma^\dagger_{-k\downarrow} \end{pmatrix}$

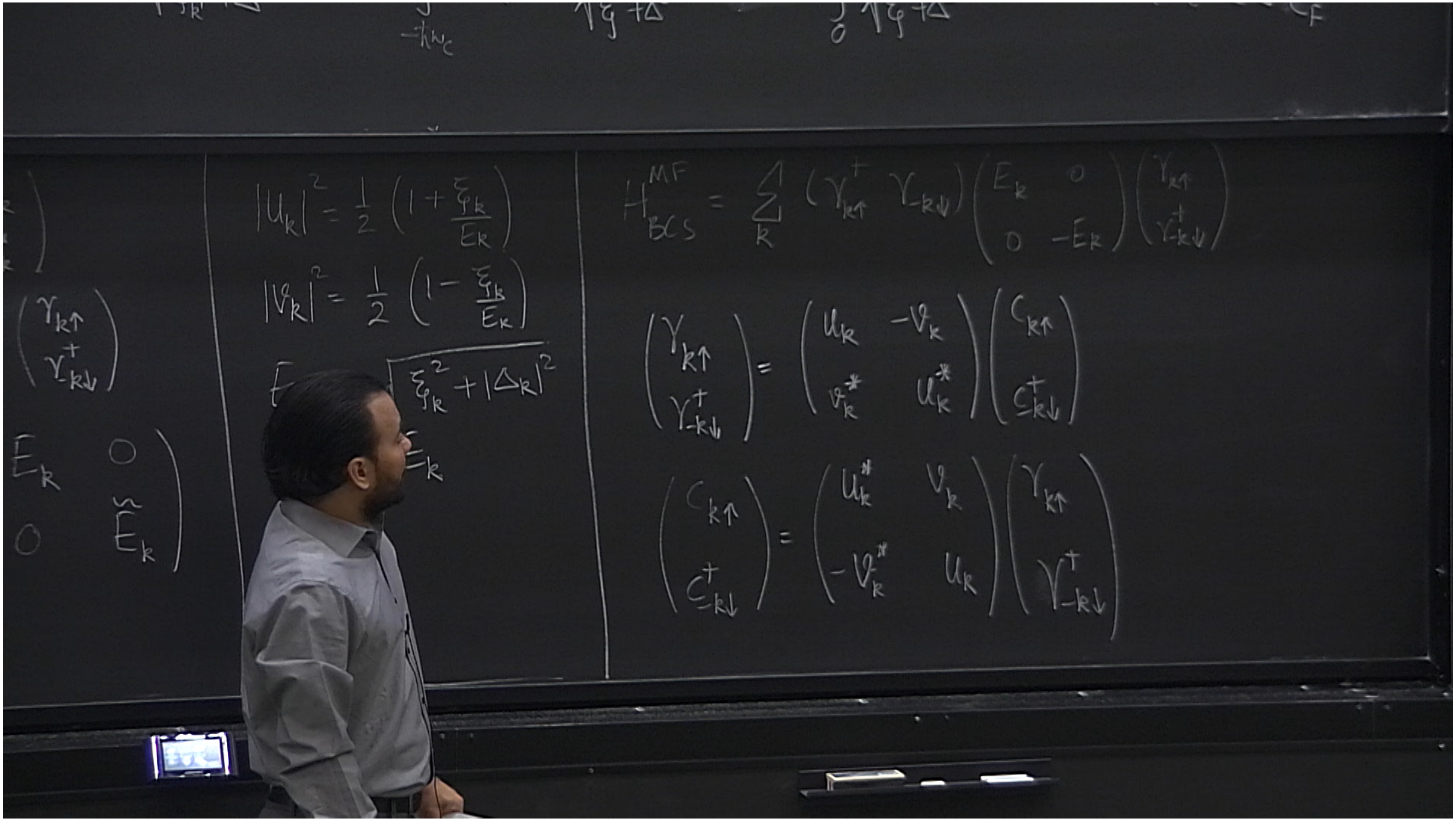

Bogoliubov-Valatin transformation
$H^{MF}_{BCS} = \sum_k (c^\dagger_{k\uparrow}\ c_{-k\downarrow}) \begin{pmatrix}\xi_k & \Delta_k\\ \Delta_k^* & -\xi_k\end{pmatrix}\begin{pmatrix}c_{k\uparrow}\\ c^\dagger_{-k\downarrow}\end{pmatrix}$
$U_k U_k^\dagger$
$V_k V_k^\dagger$
$\sum_k A_k^\dagger H_k A_k$
and $H_k = \begin{pmatrix}\xi_k & \Delta_k\\ \Delta_k^* & -\xi_k\end{pmatrix}$
$U_k^{-1} = \begin{pmatrix}u_k & -v_k\\ v_k^* & u_k^*\end{pmatrix}$
$B_k = U_k^{-1} A_k = \begin{pmatrix}\gamma_{k\uparrow}\\ \gamma^\dagger_{-k\downarrow}\end{pmatrix}$

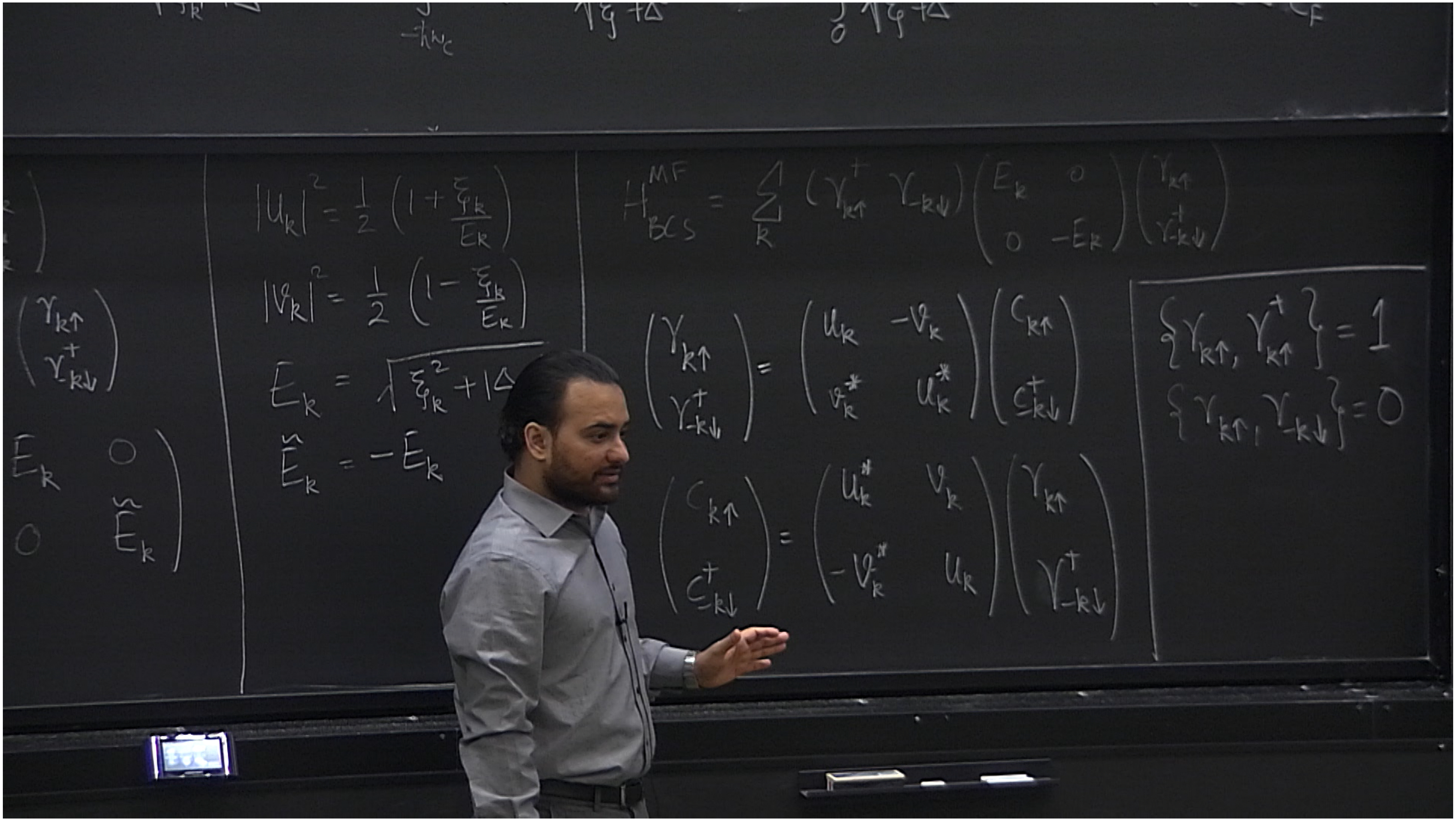


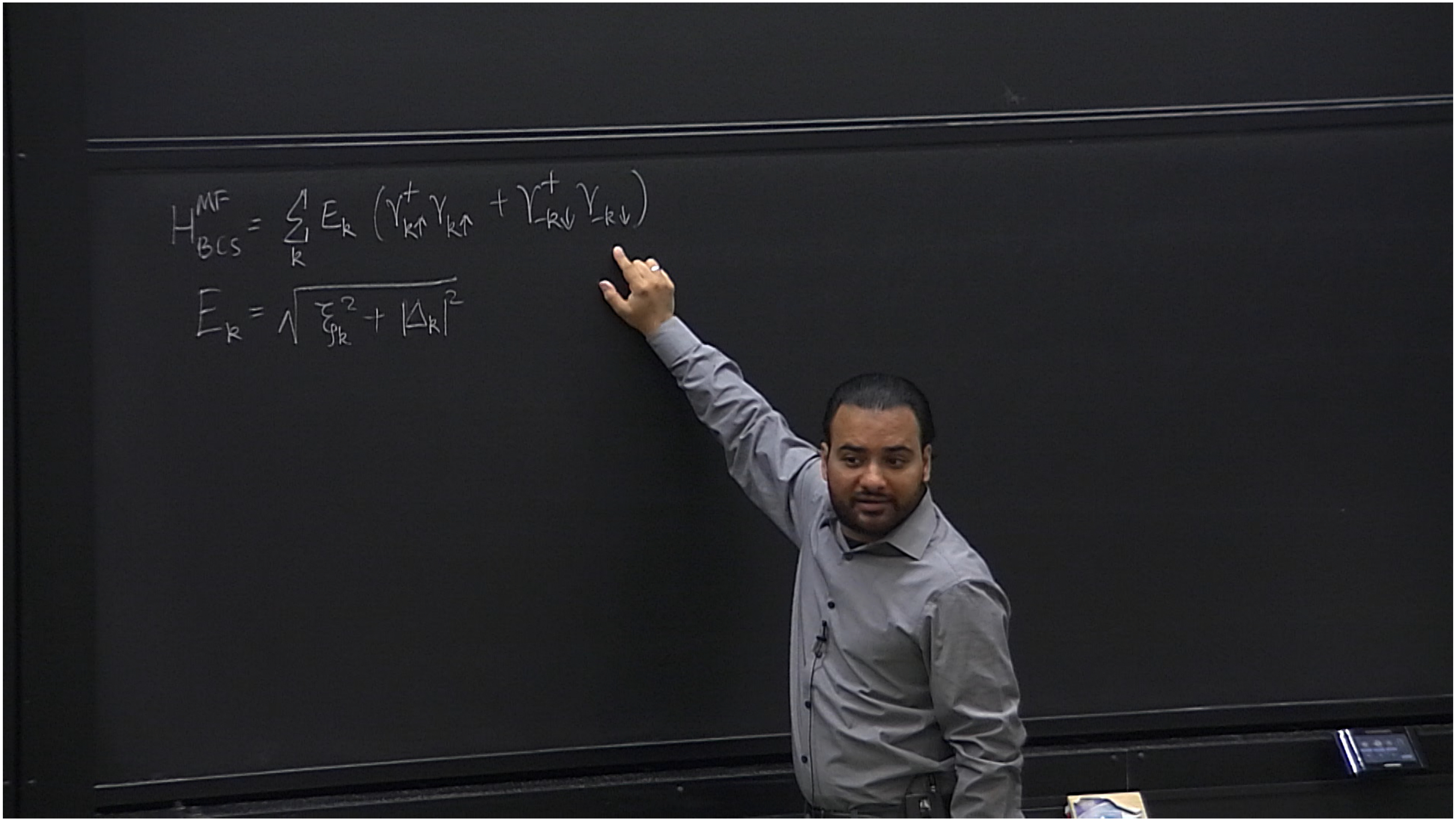
Bogoliubov-Valatin transformation
$H_{BCS}^{MF} = \sum_k (c^\dagger_{k\uparrow}\ c_{-k\downarrow}) \begin{pmatrix} \xi_k & \Delta_k \\ \Delta^*_k & -\xi_k \end{pmatrix} \begin{pmatrix} c_{k\uparrow} \\ c^\dagger_{-k\downarrow} \end{pmatrix}$
$= \sum_k A^\dagger_k H_k A_k$
$A_k = \begin{pmatrix} c_{k\uparrow} \\ c^\dagger_{-k\downarrow} \end{pmatrix}$ and $H_k = \begin{pmatrix} \xi_k & \Delta_k \\ \Delta^*_k & -\xi_k \end{pmatrix}$
$U_k^{-1} = \begin{pmatrix} u_k & -v_k \\ v^*_k & u^*_k \end{pmatrix}$
$B_k = U_k^{-1} A_k = \begin{pmatrix} \gamma_{k\uparrow} \\ \gamma^\dagger_{-k\downarrow} \end{pmatrix}$
$U^\dagger_k H_k U_k = \begin{pmatrix} E_k & 0 \\ 0 & \tilde{E}_k \end{pmatrix}$

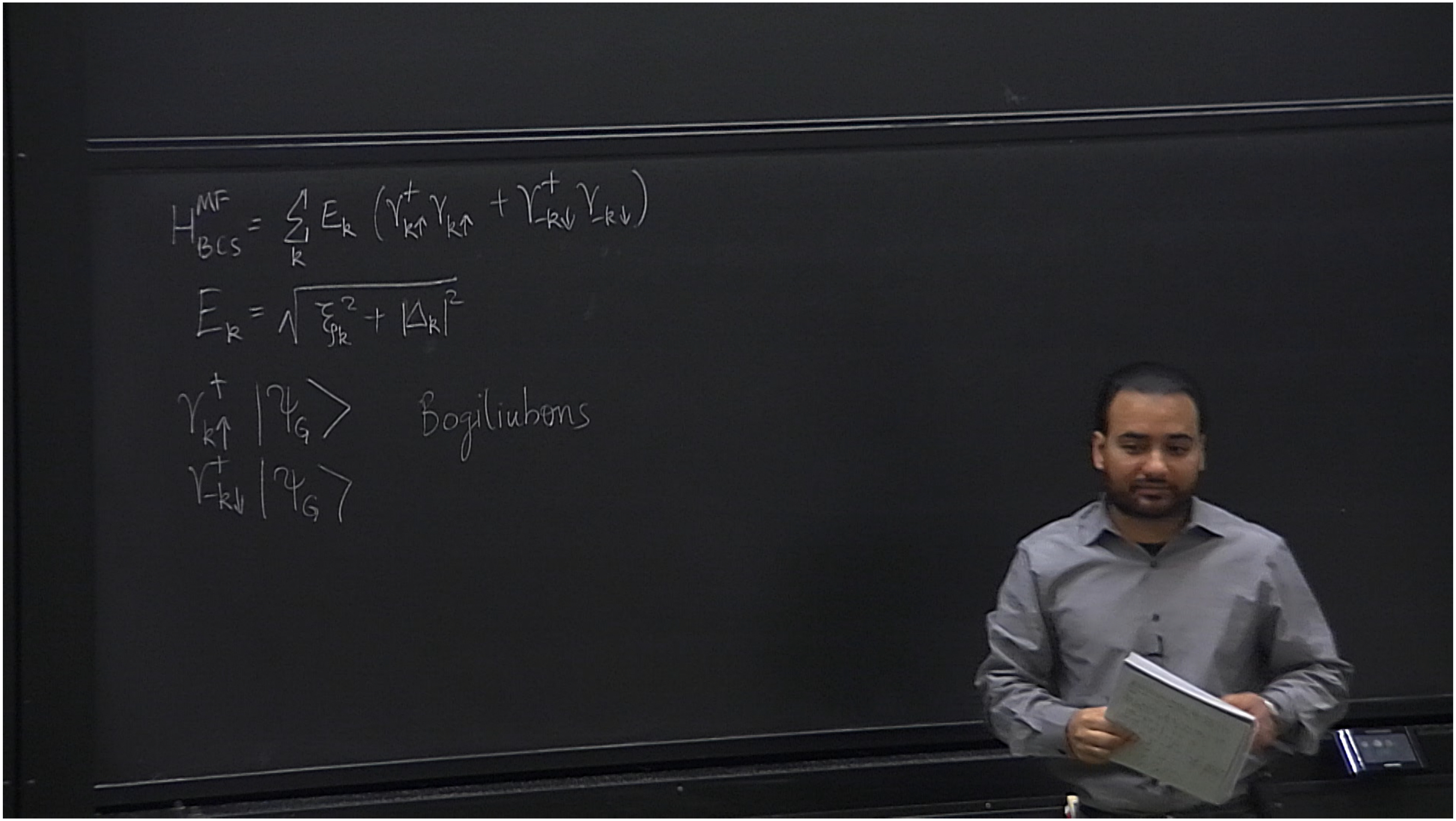

$|u_k|^2 = \frac{1}{2}\left(1 + \frac{\xi_k}{E_k}\right)$
$|v_k|^2 = \frac{1}{2}\left(1 - \frac{\xi_k}{E_k}\right)$
$E_k = \sqrt{\xi_k^2 + |\Delta_k|^2}$
$\tilde{E}_k = -E_k$

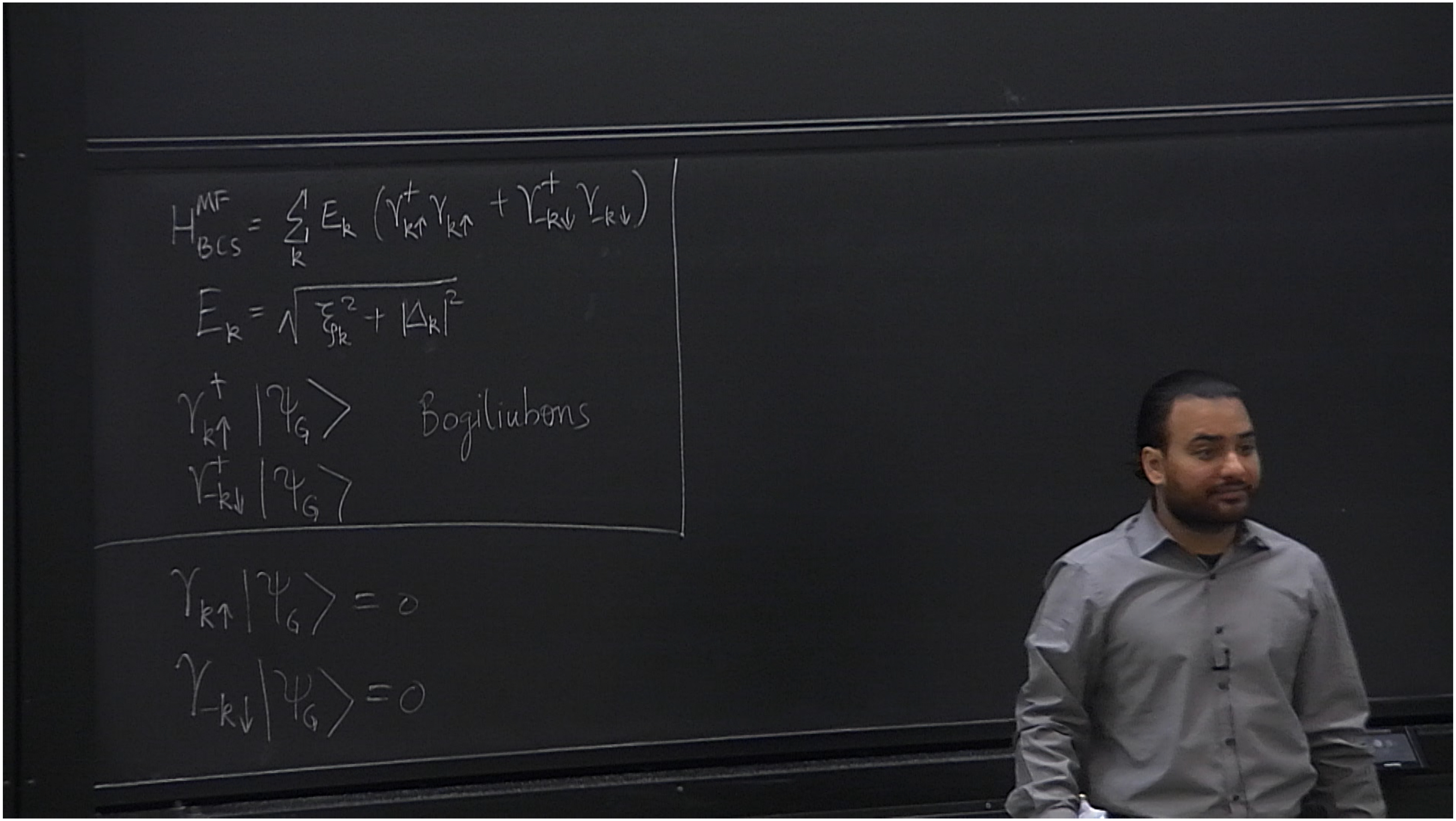

$|u_k|^2 = \frac{1}{2}\left(1 + \frac{\xi_k}{E_k}\right)$
$|v_k|^2 = \frac{1}{2}\left(1 - \frac{\xi_k}{E_k}\right)$
$E_k = \sqrt{\xi_k^2 + |\Delta_k|^2}$
$\tilde{E}_k = -E_k$
$H^{MF}_{BCS} = \sum_k (\gamma^+_{k\uparrow} \ \gamma_{-k\downarrow}) \begin{pmatrix} E_k & 0 \\ 0 & -E_k \end{pmatrix} \begin{pmatrix} \gamma_{k\uparrow} \\ \gamma^+_{-k\downarrow} \end{pmatrix}$

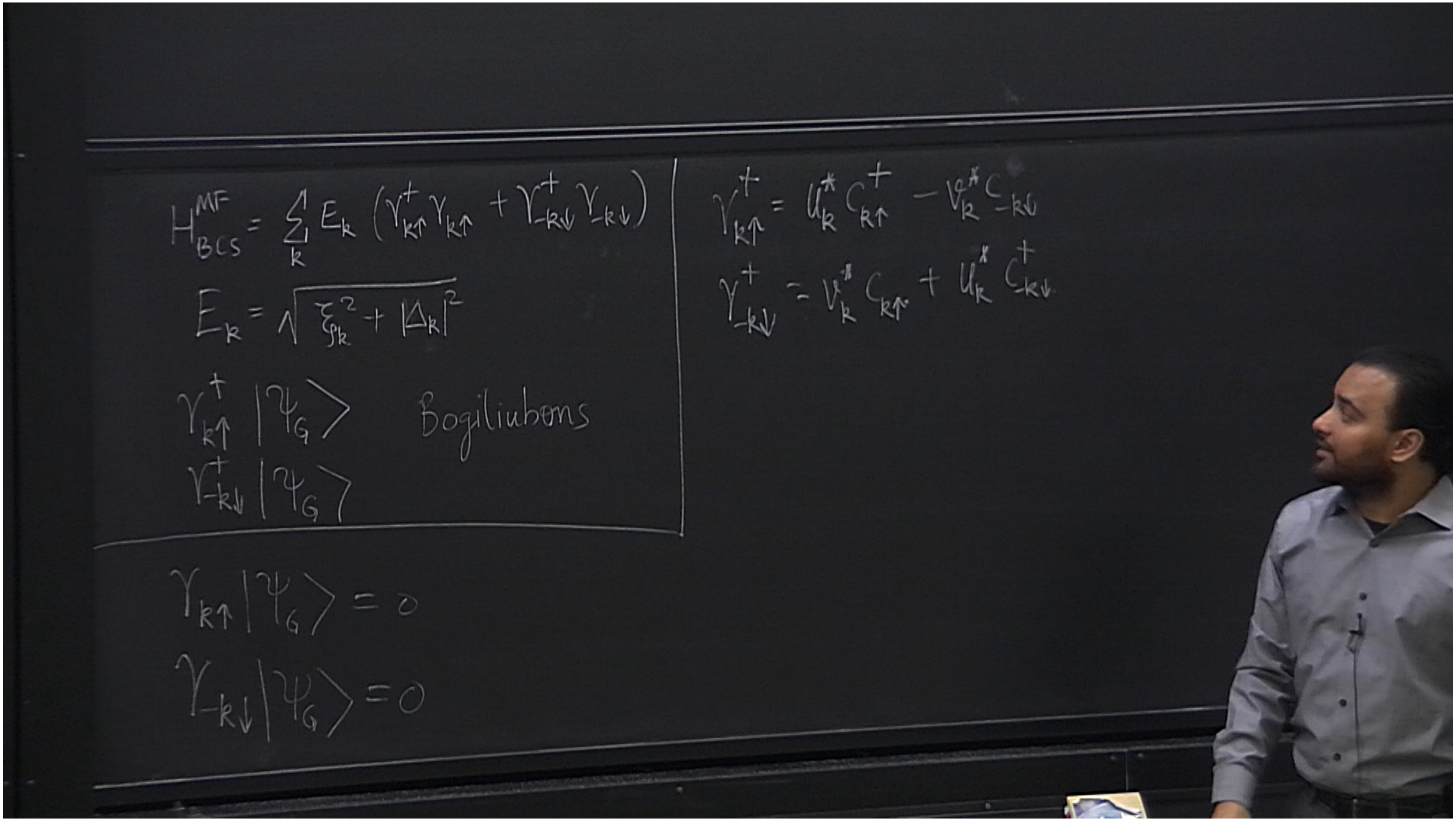

|u_k|^2 = 1/2 (1 + ξ_k/E_k)
|v_k|^2 = 1/2 (1 − ξ_k/E_k)
H^MF_BCS = Σ_k (γ†_k↑ γ_−k↓) (E_k 0; 0 −E_k) (γ_k↑; γ†_−k↓)
(γ_k↑; γ†_−k↓) = (u_k −v_k; v*_k u*_k) (c_k↑; c†_−k↓)
(c_k↑; c†_−k↓) = (u*_k v_k; −v*_k u_k) (γ_k↑; γ†_−k↓)

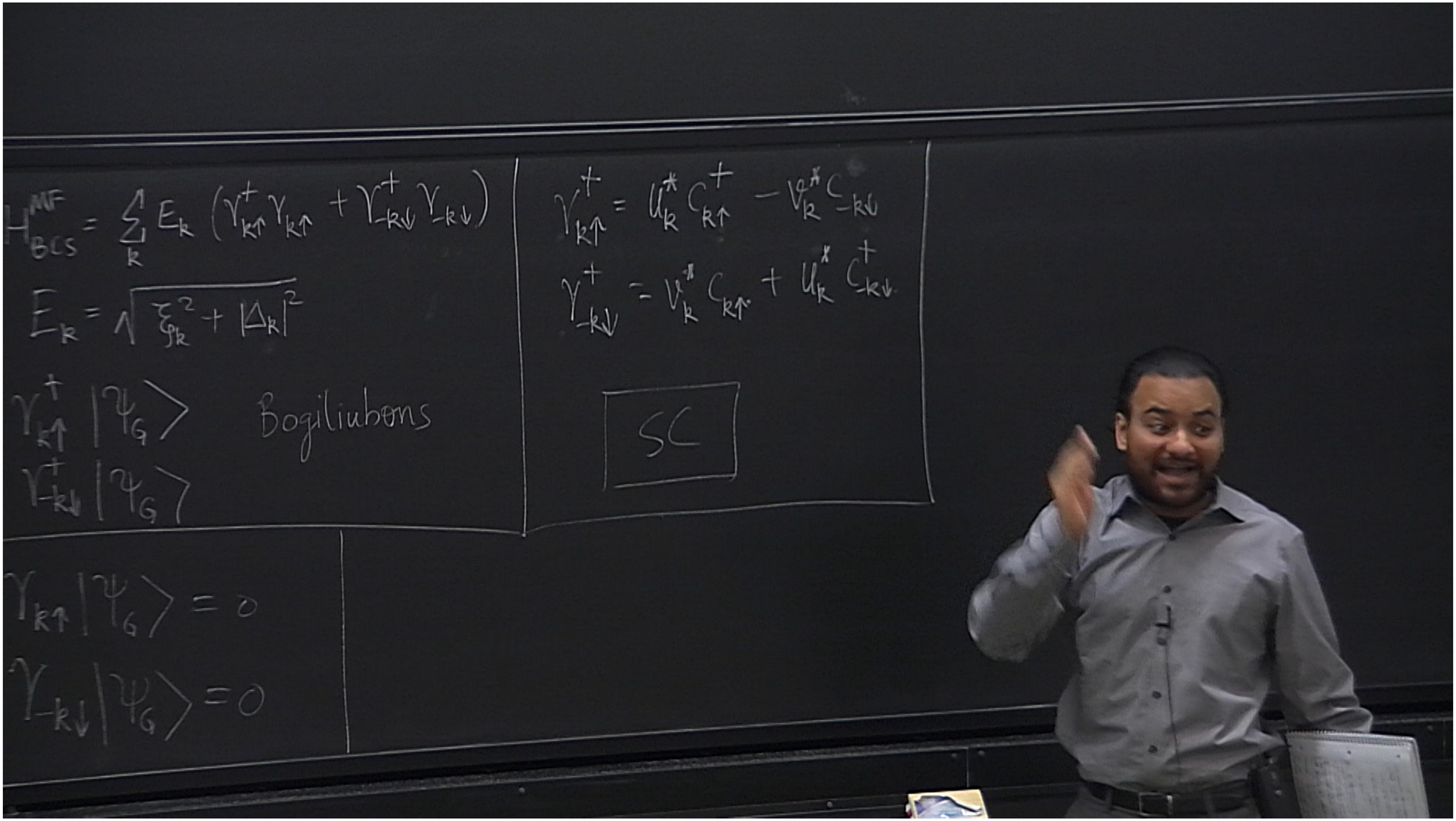

|u_k|^2 = 1/2 (1 + ξ_k/E_k)
|v_k|^2 = 1/2 (1 − ξ_k/E_k)
E_k = √(ξ_k^2 + |Δ
Ẽ_k = −E_k
H^MF_BCS = Σ_k (γ†_k↑ γ_−k↓) (E_k 0; 0 −E_k) (γ_k↑; γ†_−k↓)
(γ_k↑; γ†_−k↓) = (u_k −v_k; v*_k u*_k) (c_k↑; c†_−k↓)
(c_k↑; c†_−k↓) = (u*_k v_k; −v*_k u_k) (γ_k↑; γ†_−k↓)
{γ_k↑, γ†_k↑} = 1
{γ_k↑, γ_−k↓} = 0

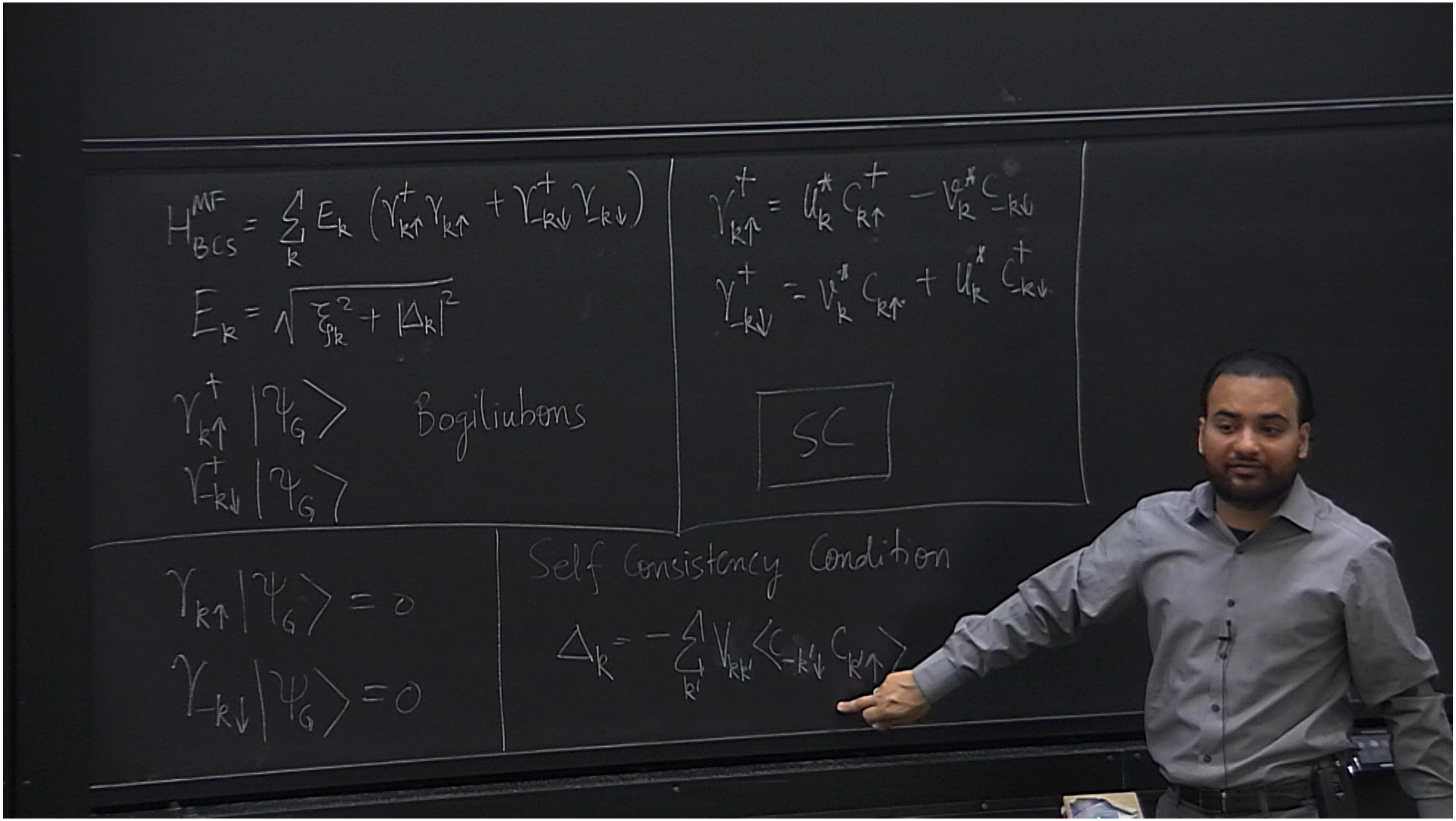

$H^{MF}_{BCS} = \sum_k E_k \left(\gamma^\dagger_{k\uparrow} \gamma_{k\uparrow} + \gamma^\dagger_{-k\downarrow} \gamma_{-k\downarrow} \right)$
$E_k = \sqrt{\xi_k^2 + |\Delta_k|^2}$

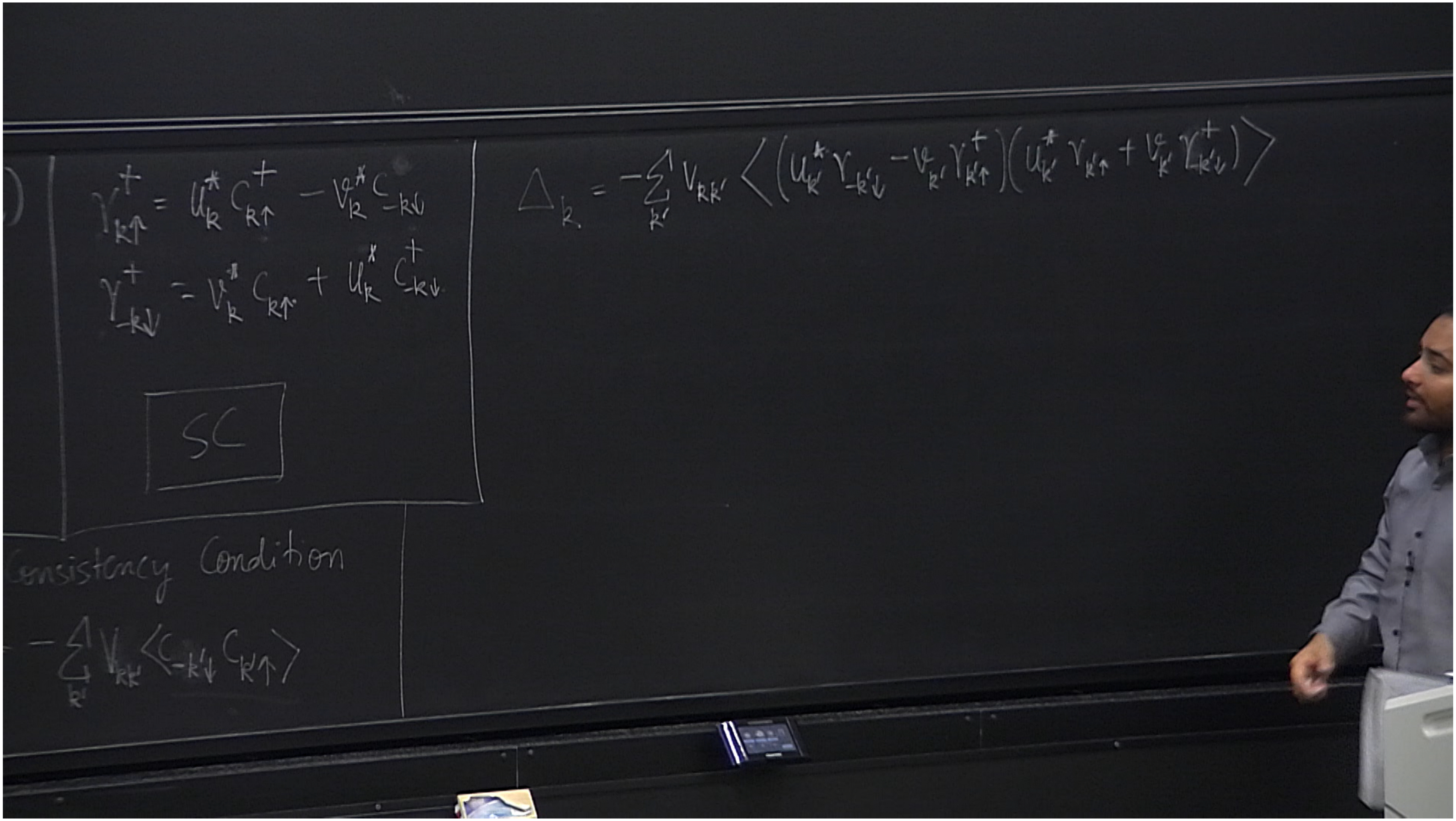

$H^{MF}_{BCS} = \sum_k E_k \left(\gamma^\dagger_{k\uparrow} \gamma_{k\uparrow} + \gamma^\dagger_{-k\downarrow} \gamma_{-k\downarrow} \right)$
$E_k = \sqrt{\xi_k^2 + |\Delta_k|^2}$
$\gamma^\dagger_{k\uparrow} |\psi_G\rangle$
Bogiliubons
$\gamma^\dagger_{-k\downarrow} |\psi_G\rangle$

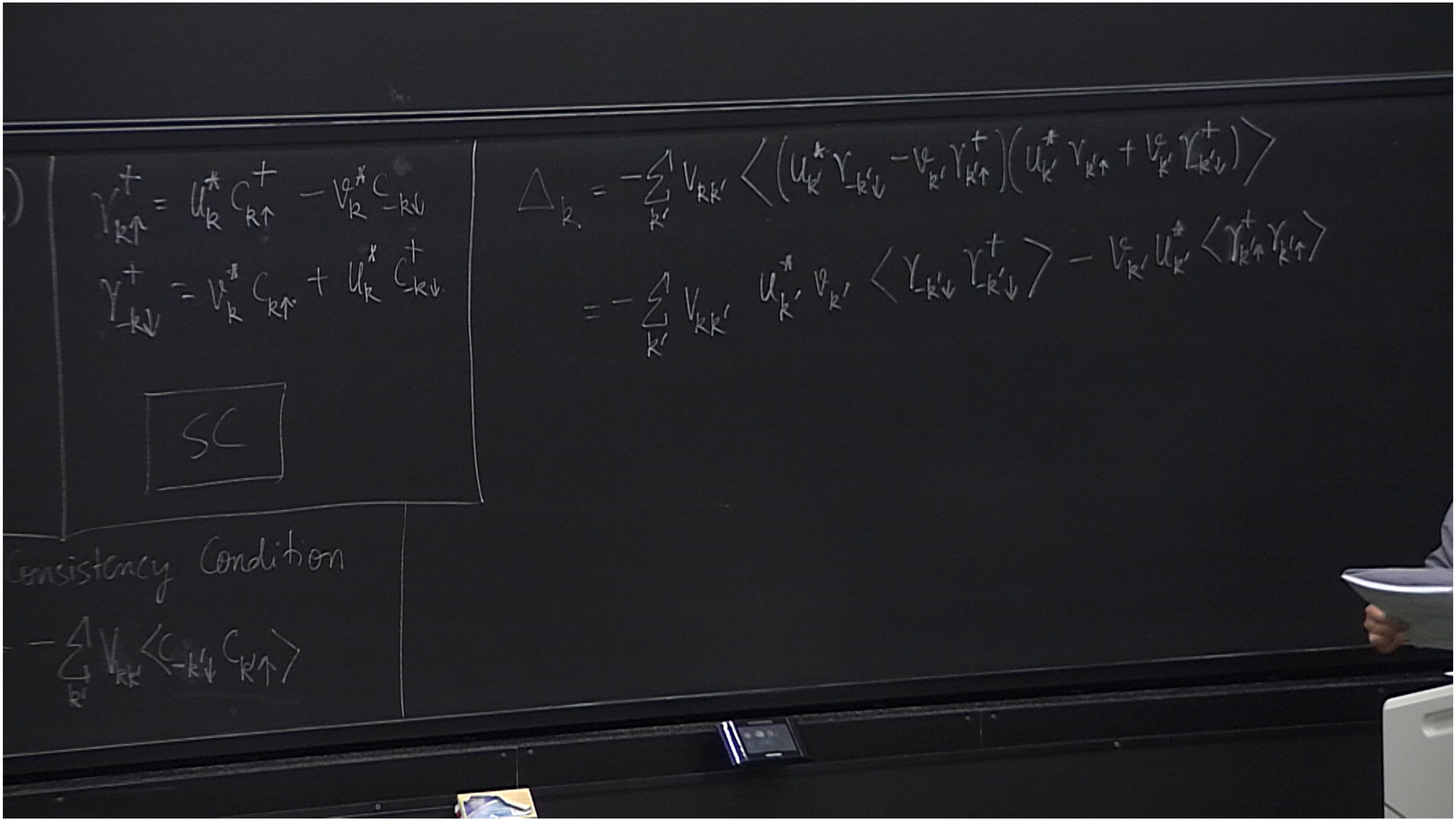

$H^{MF}_{BCS} = \sum_k E_k (\gamma^\dagger_{k\uparrow}\gamma_{k\uparrow} + \gamma^\dagger_{-k\downarrow}\gamma_{-k\downarrow})$
$E_k = \sqrt{\xi_k^2 + |\Delta_k|^2}$
$\gamma^\dagger_{k\uparrow}|\psi_G\rangle$
Bogiliubons
$\gamma^\dagger_{-k\downarrow}|\psi_G\rangle$
$\gamma_{k\uparrow}|\psi_G\rangle = 0$
$\gamma_{-k\downarrow}|\psi_G\rangle = 0$

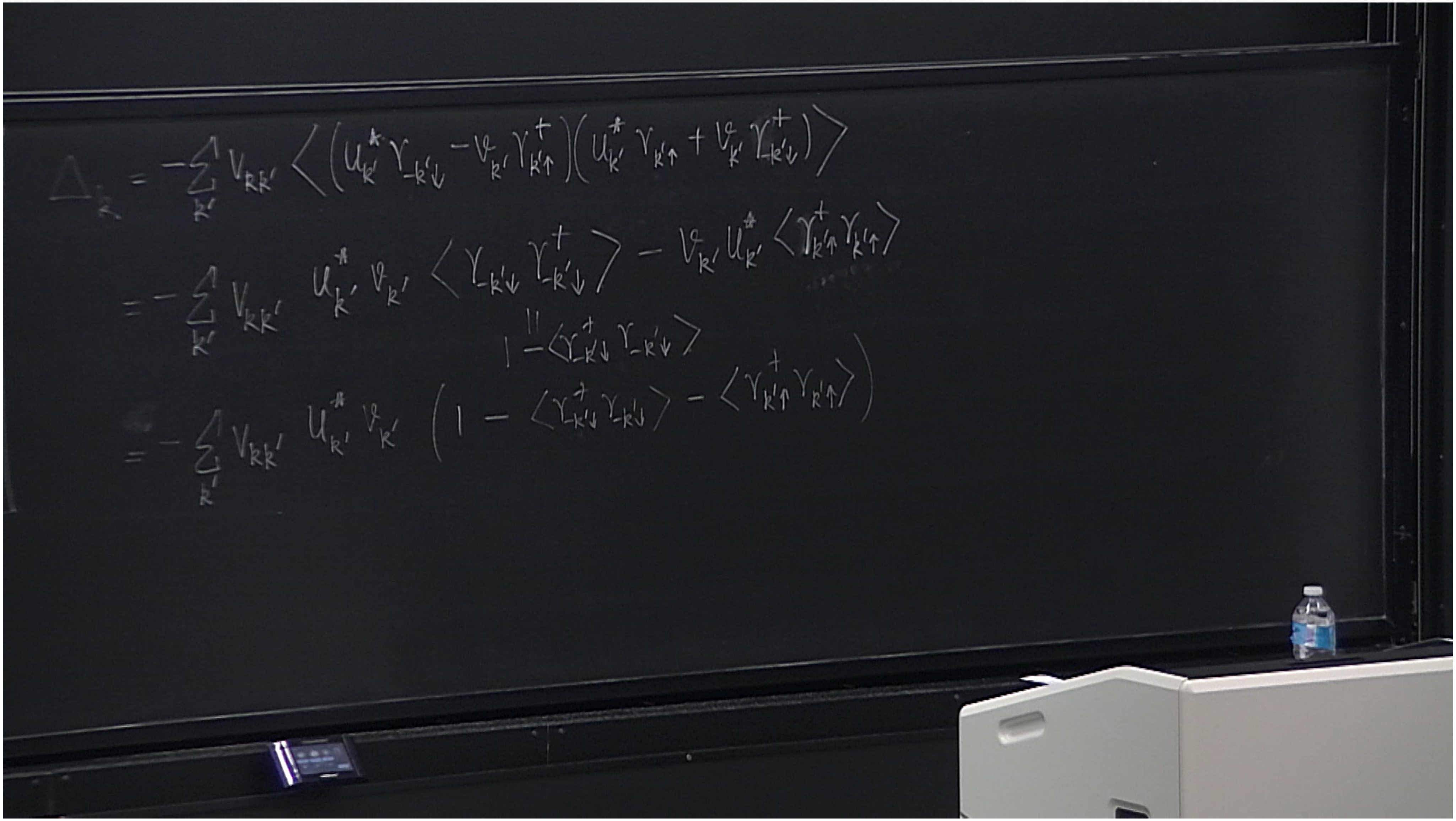

H^MF_BCS = Σ_k E_k (γ†_k↑ γ_k↑ + γ†_-k↓ γ_-k↓)
E_k = √(ξ_k² + |Δ_k|²)
γ†_k↑ |Ψ_G⟩
γ†_-k↓ |Ψ_G⟩
Bogiliubons
γ_k↑ |Ψ_G⟩ = 0
γ_-k↓ |Ψ_G⟩ = 0
γ†_k↑ = u*_k c†_k↑ − v*_k c_-k↓
γ†_-k↓ = v*_k c_k↑ + u*_k c†_-k↓

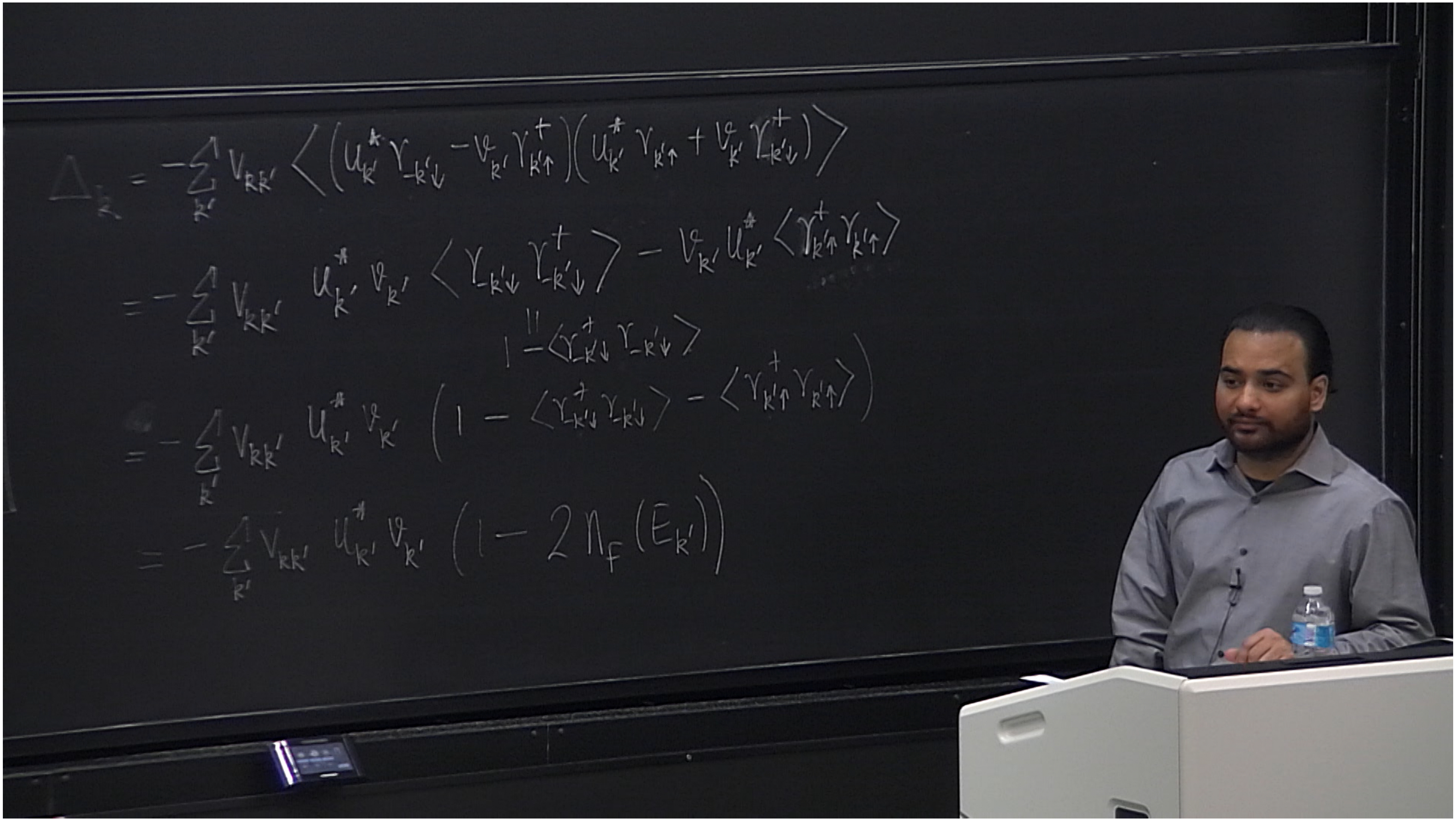

$H^{MF}_{BCS} = \sum_k E_k (\gamma^\dagger_{k\uparrow}\gamma_{k\uparrow} + \gamma^\dagger_{-k\downarrow}\gamma_{-k\downarrow})$
$E_k = \sqrt{\xi_k^2 + |\Delta_k|^2}$
$\gamma^\dagger_{k\uparrow}|\psi_G\rangle$
Bogiliubons
$\gamma^\dagger_{-k\downarrow}|\psi_G\rangle$
$\gamma^\dagger_{k\uparrow} = u^*_k c^\dagger_{k\uparrow} - v^*_k c_{-k\downarrow}$
$\gamma^\dagger_{-k\downarrow} = v^*_k c_{k\uparrow} + u^*_k c^\dagger_{-k\downarrow}$
SC
$\gamma_{k\uparrow}|\psi_G\rangle = 0$
$\gamma_{-k\downarrow}|\psi_G\rangle = 0$

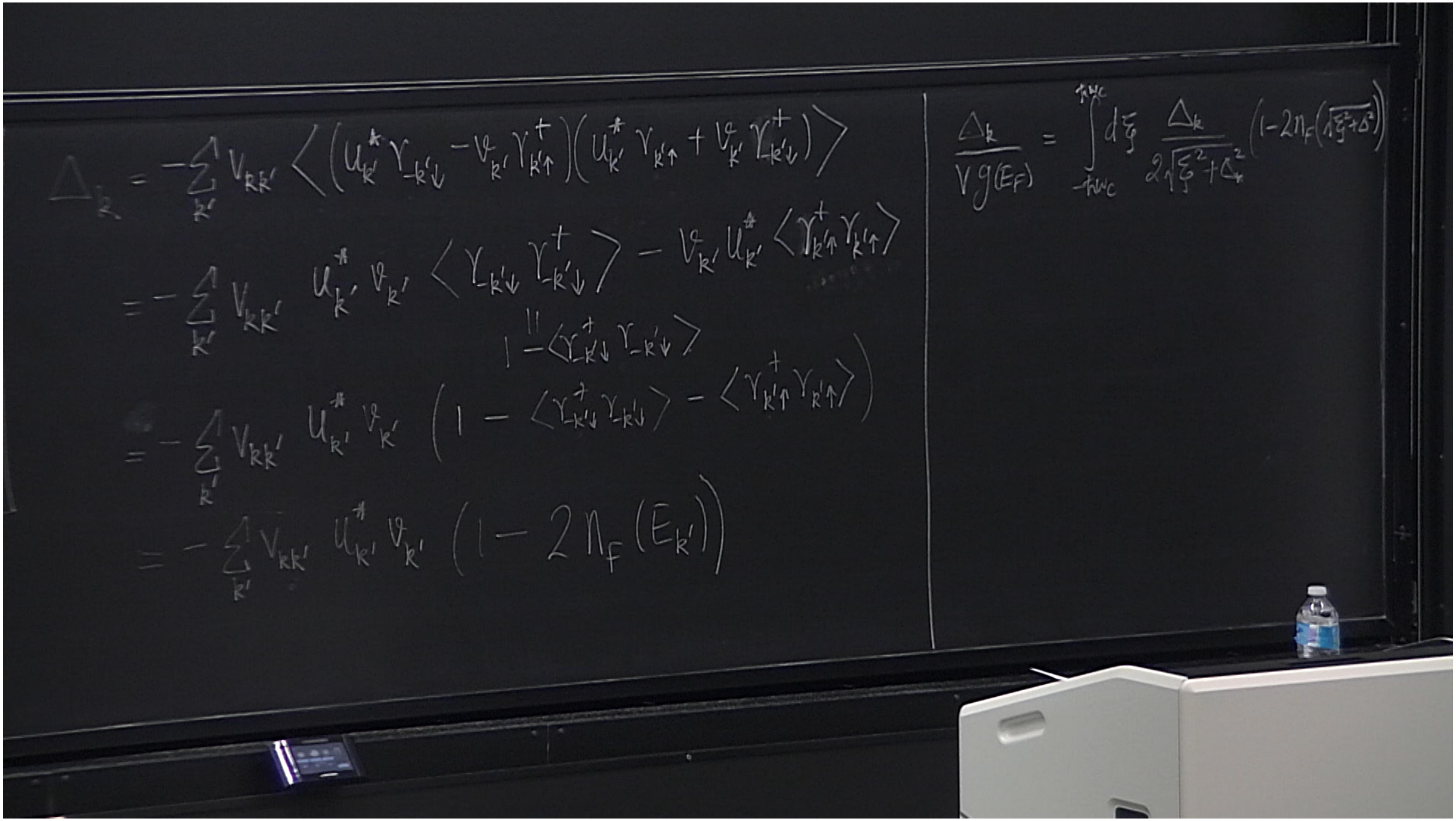

$H^{MF}_{BCS} = \sum_k E_k (\gamma^\dagger_{k\uparrow}\gamma_{k\uparrow} + \gamma^\dagger_{-k\downarrow}\gamma_{-k\downarrow})$
$E_k = \sqrt{\xi_k^2 + |\Delta_k|^2}$
$\gamma^\dagger_{k\uparrow}|\Psi_G\rangle$
Bogiliubons
$\gamma^\dagger_{-k\downarrow}|\Psi_G\rangle$
$\gamma^\dagger_{k\uparrow} = u^*_k c^\dagger_{k\uparrow} - v^*_k c_{-k\downarrow}$
$\gamma^\dagger_{-k\downarrow} = v^*_k c_{k\uparrow} + u^*_k c^\dagger_{-k\downarrow}$
SC
$\gamma_{k\uparrow}|\Psi_G\rangle = 0$
$\gamma_{-k\downarrow}|\Psi_G\rangle = 0$
Self Consistency Condition
$\Delta_k = -\sum_{k'} V_{kk'}\langle c_{-k'\downarrow} c_{k'\uparrow}\rangle$

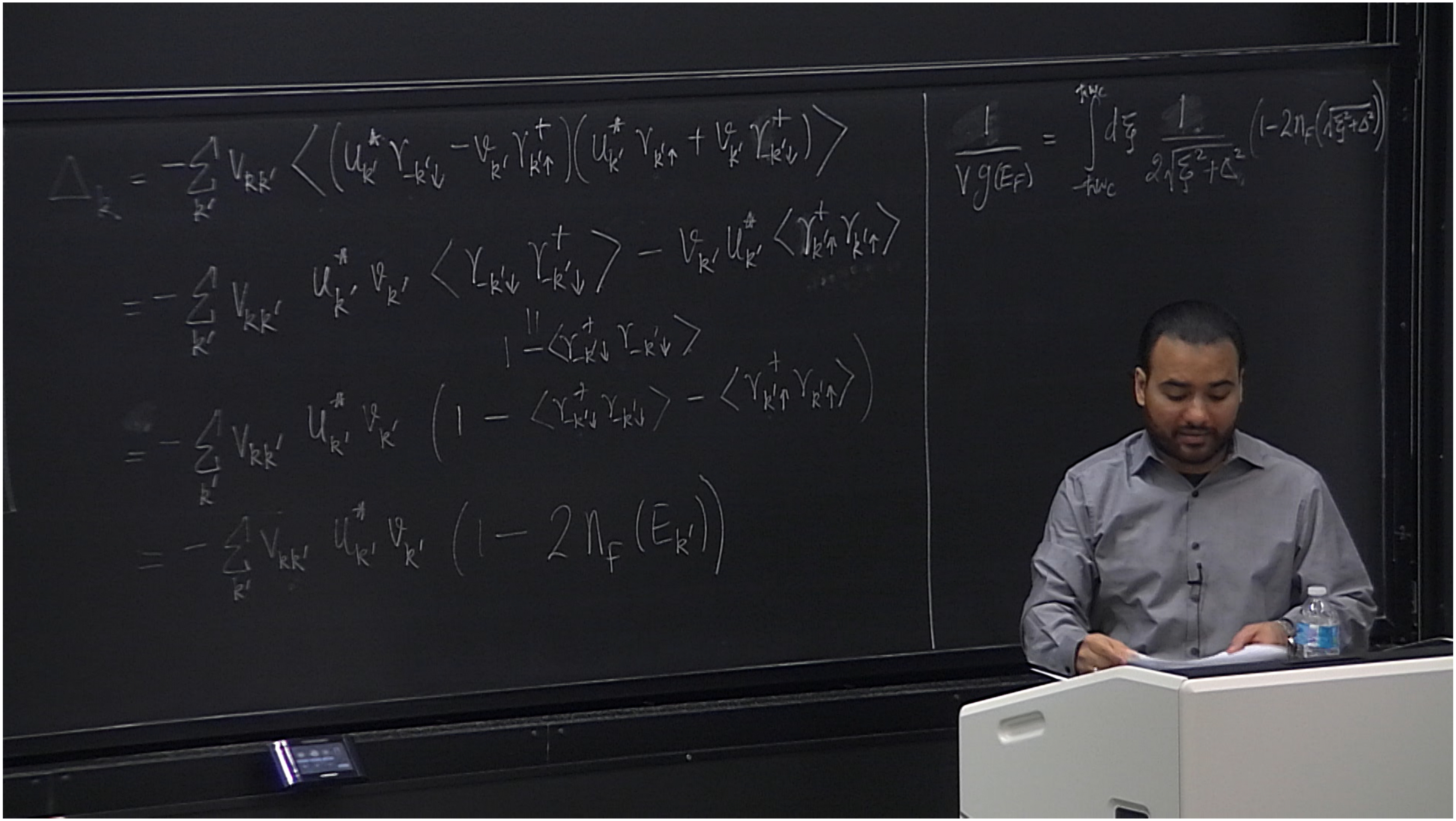

$\gamma^\dagger_{k\uparrow} = u_k^* c^\dagger_{k\uparrow} - v_k^* c_{-k\downarrow}$
$\gamma^\dagger_{-k\downarrow} = v_k^* c_{k\uparrow} + u_k^* c^\dagger_{-k\downarrow}$
SC
Consistency Condition
$-\sum_{k'} V_{kk'} \langle c_{-k'\downarrow} c_{k'\uparrow} \rangle$
$\Delta_k = -\sum_{k'} V_{kk'} \langle (u_{k'}^* \gamma_{-k'\downarrow} - v_{k'} \gamma^\dagger_{k'\uparrow})(u_{k'}^* \gamma_{k'\uparrow} + v_{k'} \gamma^\dagger_{-k'\downarrow}) \rangle$

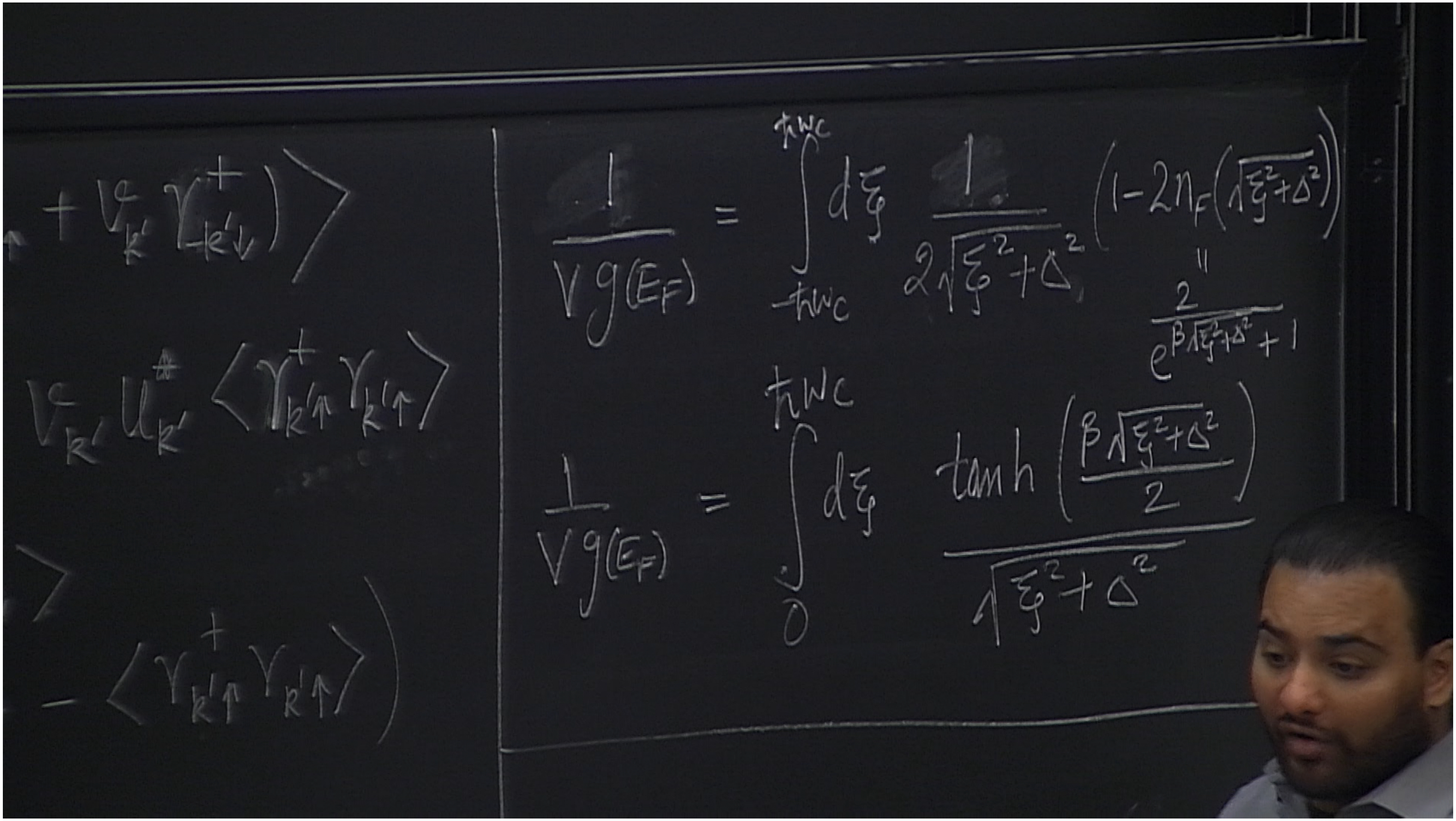

$\gamma_{k\uparrow}^{\dagger} = u_k^* c_{k\uparrow}^{\dagger} - v_k^* c_{-k\downarrow}$
$\gamma_{-k\downarrow}^{\dagger} = v_k^* c_{k\uparrow} + u_k^* c_{-k\downarrow}^{\dagger}$
SC
Consistency Condition
$- \sum_{k'} V_{kk'} \langle c_{-k'\downarrow} c_{k'\uparrow} \rangle$
$\Delta_k = -\sum_{k'} V_{kk'} \langle (u_{k'}^* \gamma_{-k'\downarrow} - v_{k'} \gamma_{k'\uparrow}^{\dagger})(u_{k'}^* \gamma_{k'\uparrow} + v_{k'} \gamma_{-k'\downarrow}^{\dagger}) \rangle$
$= -\sum_{k'} V_{kk'} u_{k'}^* v_{k'} \langle \gamma_{-k'\downarrow} \gamma_{-k'\downarrow}^{\dagger} \rangle - v_{k'} u_{k'}^* \langle \gamma_{k'\uparrow}^{\dagger} \gamma_{k'\uparrow} \rangle$

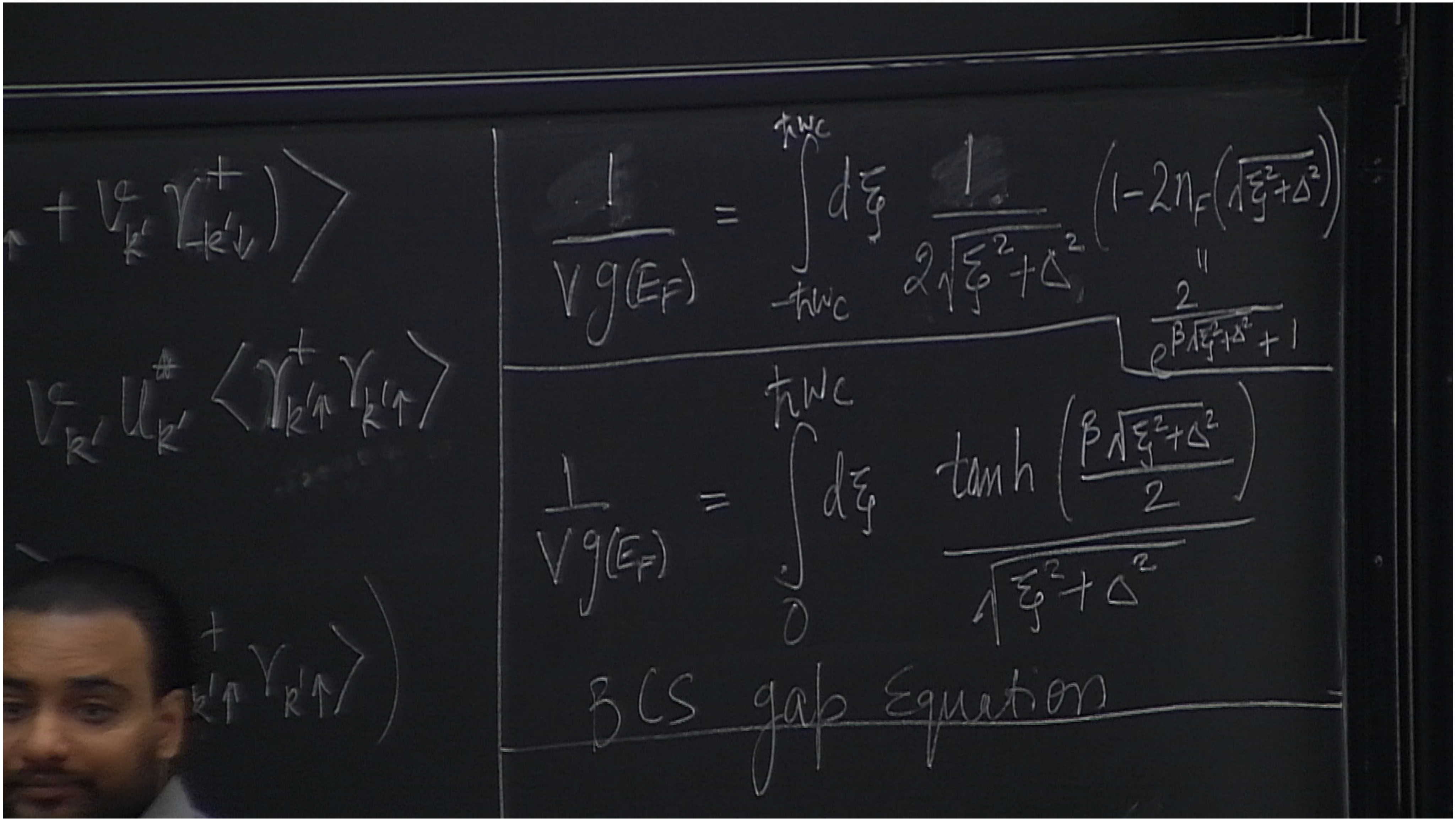

$\Delta_k = -\sum_{k'} V_{kk'} \langle (u^*_{k'} \gamma_{-k'\downarrow} - v_{k'} \gamma^\dagger_{k'\uparrow})(u^*_{k'} \gamma_{k'\uparrow} + v_{k'} \gamma^\dagger_{-k'\downarrow}) \rangle$
$= -\sum_{k'} V_{kk'} \; u^*_{k'} v_{k'} \langle \gamma_{-k'\downarrow} \gamma^\dagger_{-k'\downarrow} \rangle - v_{k'} u^*_{k'} \langle \gamma^\dagger_{k'\uparrow} \gamma_{k'\uparrow} \rangle$
$= 1 - \langle \gamma^\dagger_{-k'\downarrow} \gamma_{-k'\downarrow} \rangle$
$= -\sum_{k'} V_{kk'} \; u^*_{k'} v_{k'} \left(1 - \langle \gamma^\dagger_{-k'\downarrow} \gamma_{-k'\downarrow} \rangle - \langle \gamma^\dagger_{k'\uparrow} \gamma_{k'\uparrow} \rangle \right)$


$\Delta_k = -\sum_{k'} V_{kk'} \langle (u^*_{k'} \gamma_{-k'\downarrow} - v_{k'} \gamma^\dagger_{k'\uparrow})(u^*_{k'} \gamma_{k'\uparrow} + v_{k'} \gamma^\dagger_{-k'\downarrow}) \rangle$
$= -\sum_{k'} V_{kk'} \, u^*_{k'} v_{k'} \langle \gamma_{-k'\downarrow} \gamma^\dagger_{-k'\downarrow} \rangle - v_{k'} u^*_{k'} \langle \gamma^\dagger_{k'\uparrow} \gamma_{k'\uparrow} \rangle$
$1 - \langle \gamma^\dagger_{-k'\downarrow} \gamma_{-k'\downarrow} \rangle$
$= -\sum_{k'} V_{kk'} \, u^*_{k'} v_{k'} \left(1 - \langle \gamma^\dagger_{-k'\downarrow} \gamma_{-k'\downarrow} \rangle - \langle \gamma^\dagger_{k'\uparrow} \gamma_{k'\uparrow} \rangle\right)$
$= -\sum_{k'} V_{kk'} \, u^*_{k'} v_{k'} \left(1 - 2 n_F(E_{k'})\right)$

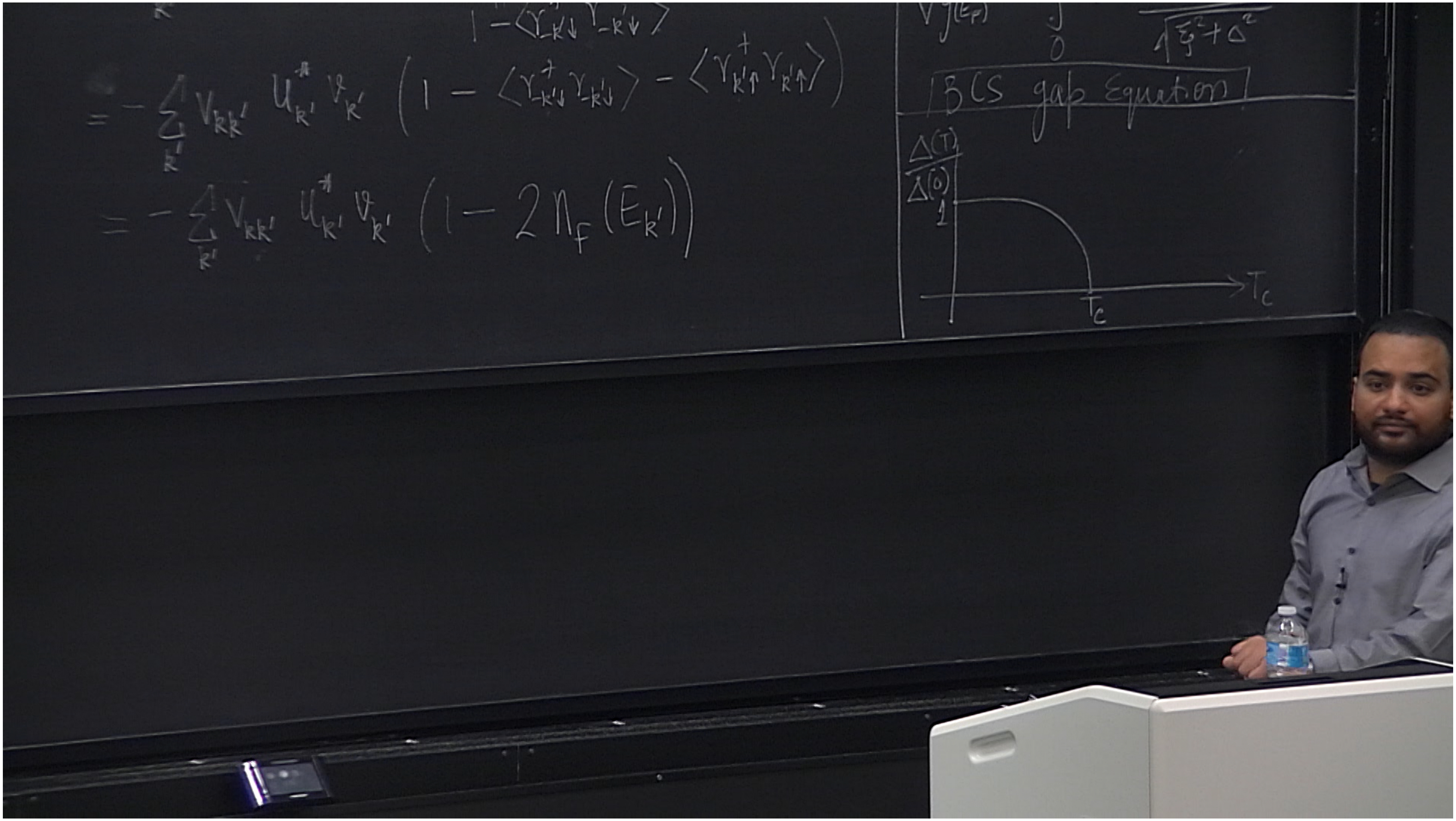

$\Delta_k = -\sum_{k'} V_{kk'} \langle (u^*_{k'} \gamma_{-k'\downarrow} - v_{k'} \gamma^\dagger_{k'\uparrow})(u^*_{k'} \gamma_{k'\uparrow} + v_{k'} \gamma^\dagger_{-k'\downarrow}) \rangle$
$= -\sum_{k'} V_{kk'} \; u^*_{k'} v_{k'} \langle \gamma_{-k'\downarrow} \gamma^\dagger_{-k'\downarrow} \rangle - v_{k'} u^*_{k'} \langle \gamma^\dagger_{k'\uparrow} \gamma_{k'\uparrow} \rangle$
$1 - \langle \gamma^\dagger_{-k'\downarrow} \gamma_{-k'\downarrow} \rangle$
$= -\sum_{k'} V_{kk'} \; u^*_{k'} v_{k'} \left(1 - \langle \gamma^\dagger_{-k'\downarrow} \gamma_{-k'\downarrow} \rangle - \langle \gamma^\dagger_{k'\uparrow} \gamma_{k'\uparrow} \rangle \right)$
$= -\sum_{k'} V_{kk'} \; u^*_{k'} v_{k'} \left(1 - 2 n_F(E_{k'})\right)$
$\frac{\Delta_k}{V g(E_F)} = \int_{-\hbar\omega_c}^{\hbar\omega_c} d\xi \frac{\Delta_k}{2\sqrt{\xi^2 + \Delta_k^2}} \left(1 - 2 n_F(\sqrt{\xi^2 + \Delta^2})\right)$

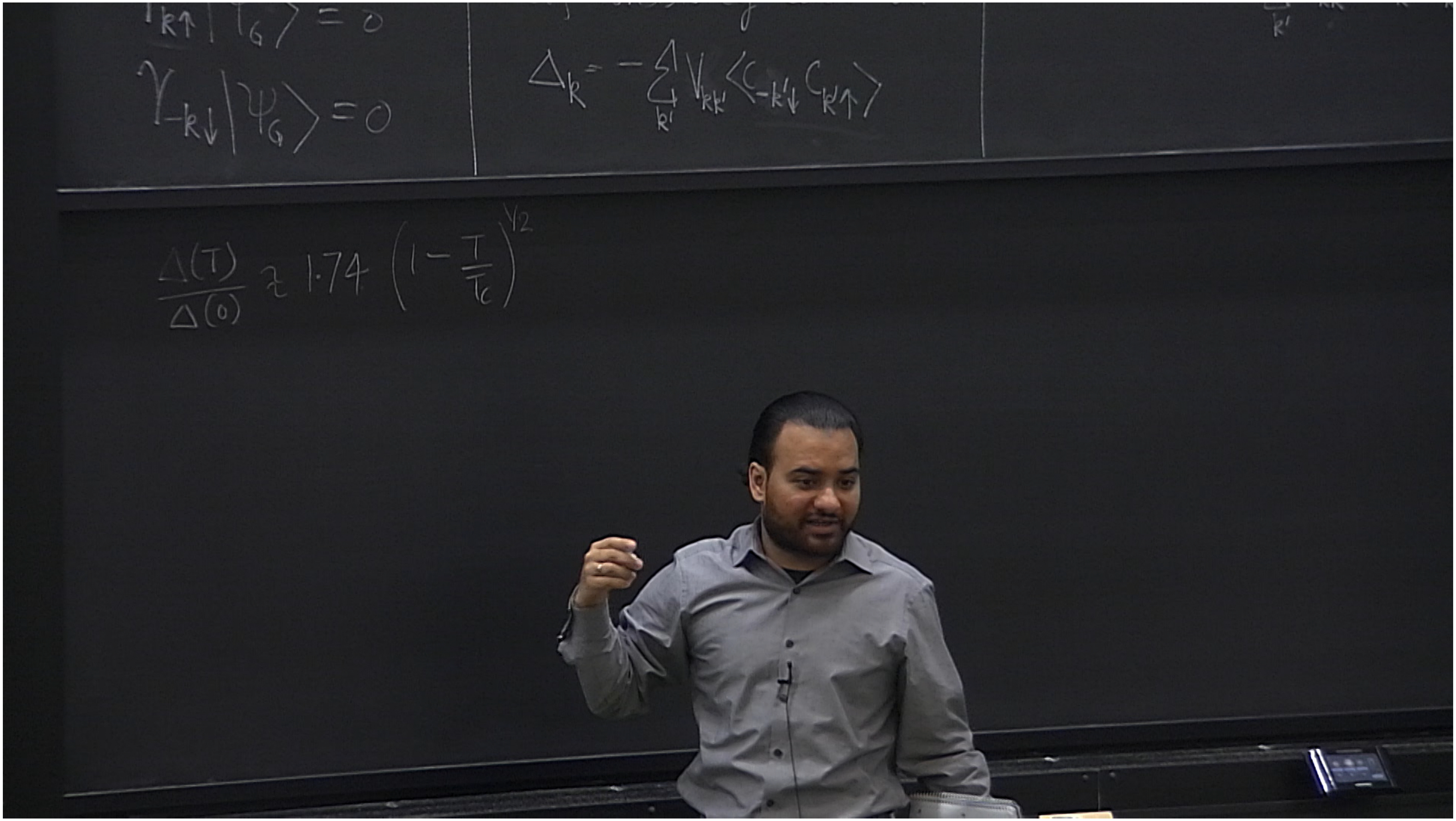

$\Delta_k = -\sum_{k'} V_{kk'} \langle (u^*_{k'} \gamma_{-k'\downarrow} - v_{k'} \gamma^\dagger_{k'\uparrow})(u^*_{k'} \gamma_{k'\uparrow} + v_{k'} \gamma^\dagger_{-k'\downarrow}) \rangle$
$= -\sum_{k'} V_{kk'} \; u^*_{k'} v_{k'} \langle \gamma_{-k'\downarrow} \gamma^\dagger_{-k'\downarrow} \rangle - v_{k'} u^*_{k'} \langle \gamma^\dagger_{k'\uparrow} \gamma_{k'\uparrow} \rangle$
$1 - \langle \gamma^\dagger_{-k'\downarrow} \gamma_{-k'\downarrow} \rangle$
$= -\sum_{k'} V_{kk'} \; u^*_{k'} v_{k'} \left(1 - \langle \gamma^\dagger_{-k'\downarrow} \gamma_{-k'\downarrow} \rangle - \langle \gamma^\dagger_{k'\uparrow} \gamma_{k'\uparrow} \rangle\right)$
$= -\sum_{k'} V_{kk'} \; u^*_{k'} v_{k'} \left(1 - 2 n_F(E_{k'})\right)$
$\frac{1}{V g(E_F)} = \int_{-\hbar\omega_c}^{\hbar\omega_c} d\xi \frac{1}{2\sqrt{\xi^2+\Delta^2}} \left(1 - 2 n_F(\sqrt{\xi^2+\Delta^2})\right)$


$\frac{1}{V g(E_F)} = \int_{-\hbar\omega_c}^{\hbar\omega_c} d\xi \frac{1}{2\sqrt{\xi^2+\Delta^2}} \left(1 - 2n_F\left(\sqrt{\xi^2+\Delta^2}\right)\right)$
$\frac{2}{e^{\beta\sqrt{\xi^2+\Delta^2}}+1}$
$\frac{1}{V g(E_F)} = \int_{0}^{\hbar\omega_c} d\xi \frac{\tanh\left(\frac{\beta\sqrt{\xi^2+\Delta^2}}{2}\right)}{\sqrt{\xi^2+\Delta^2}}$



$\frac{1}{V g(E_F)} = \int_{-\hbar\omega_c}^{\hbar\omega_c} d\xi \frac{1}{2\sqrt{\xi^2+\Delta^2}} \left(1-2n_F\left(\sqrt{\xi^2+\Delta^2}\right)\right)$
$\frac{2}{e^{\beta\sqrt{\xi^2+\Delta^2}}+1}$
$\frac{1}{V g(E_F)} = \int_0^{\hbar\omega_c} d\xi \frac{\tanh\left(\frac{\beta\sqrt{\xi^2+\Delta^2}}{2}\right)}{\sqrt{\xi^2+\Delta^2}}$
BCS gap Equation


= − Σ_k' V_kk' u*_k' v_k' (1 − ⟨γ†_−k'↓ γ_−k'↓⟩ − ⟨γ†_k'↑ γ_k'↑⟩)
= − Σ_k' V_kk' u*_k' v_k' (1 − 2 n_F (E_k'))
BCS gap Equation
Δ(T)/Δ(0)
1
T_c


γ_{-k↓} |ψ_G⟩ = 0
Δ_k = − Σ_{k'} V_{kk'} ⟨c_{-k'↓} c_{k'↑}⟩
Δ(T)/Δ(0) ≈ 1.74 (1 − T/T_c)^{1/2}

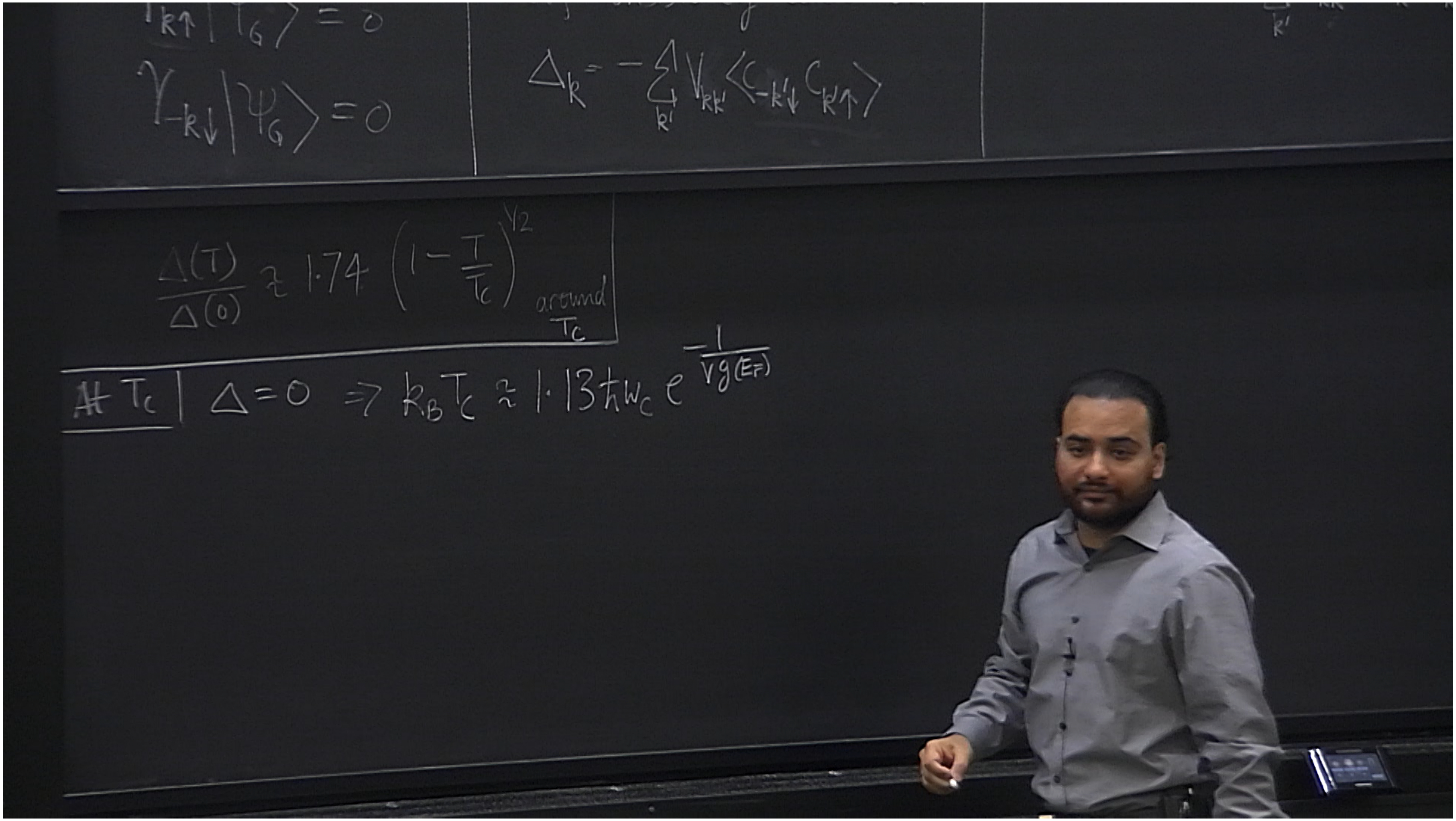
$\gamma_{k\uparrow}|\psi_G\rangle = 0$

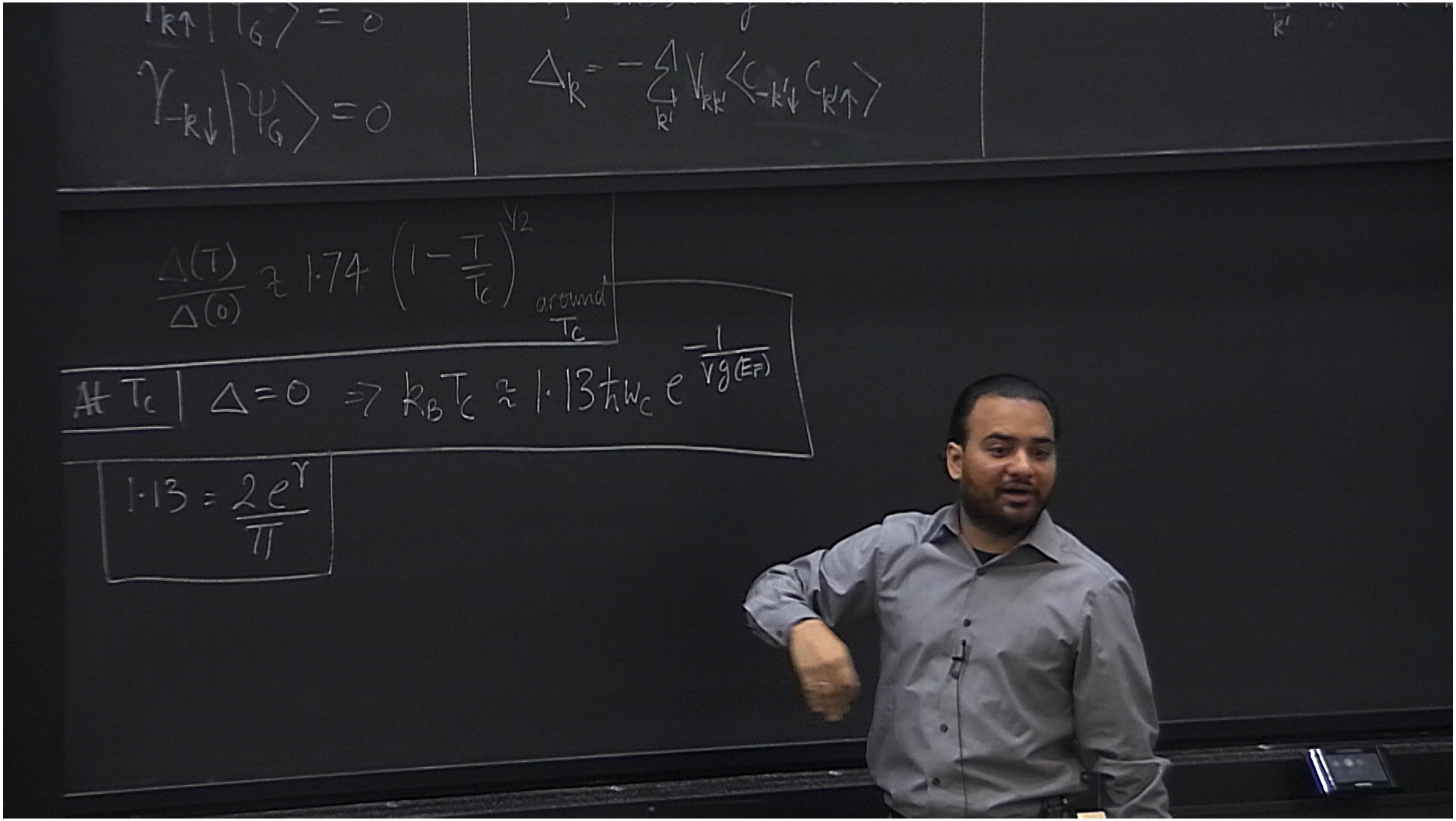
$\gamma_{-k\downarrow}|\psi_G\rangle = 0$

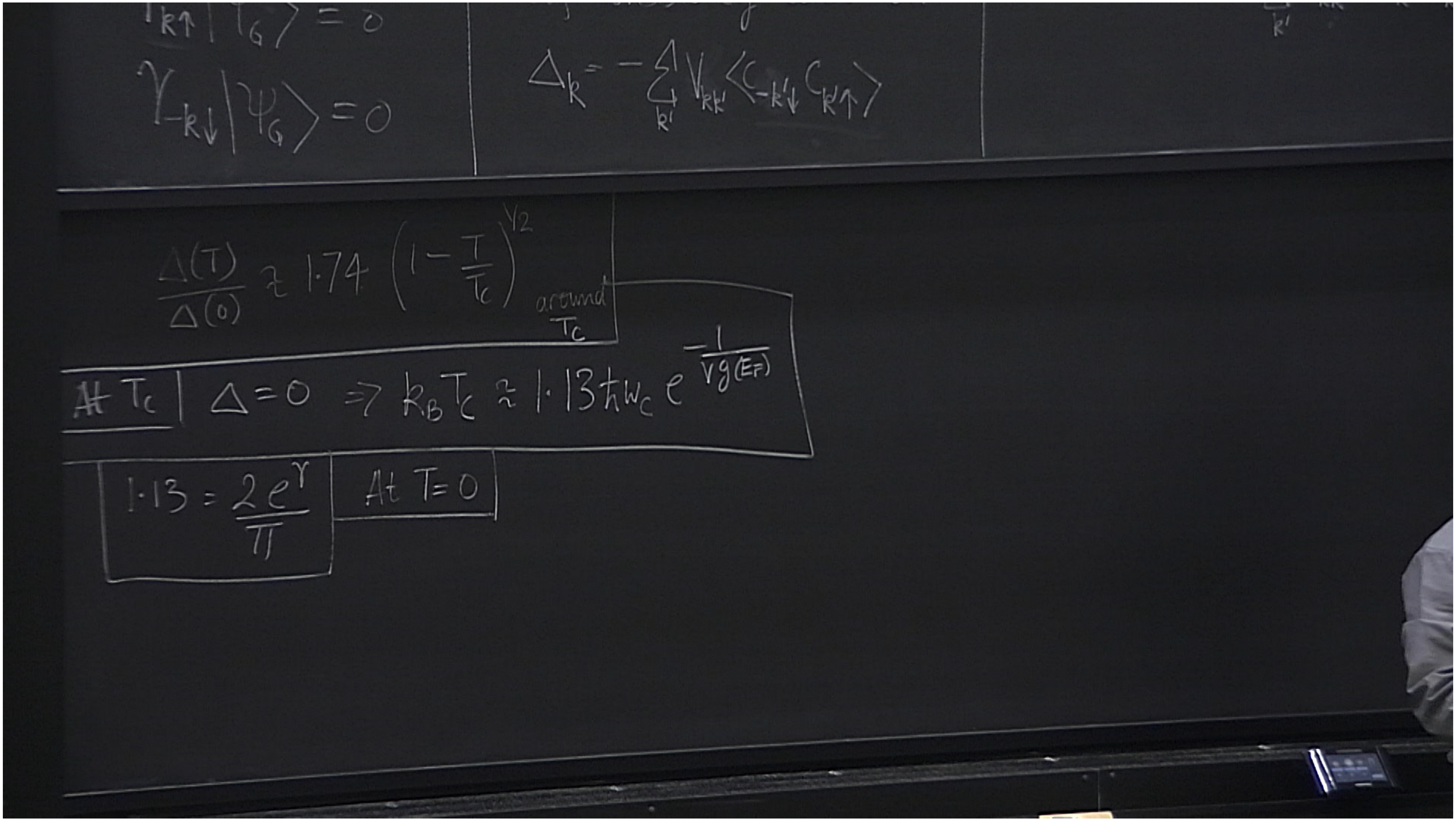
$$\Delta_k = -\sum_{k'} V_{kk'} \langle c_{-k'\downarrow} c_{k'\uparrow} \rangle$$

$$\frac{\Delta(T)}{\Delta(0)} \approx 1.74 \left(1 - \frac{T}{T_c}\right)^{1/2} \quad \text{around } T_c$$

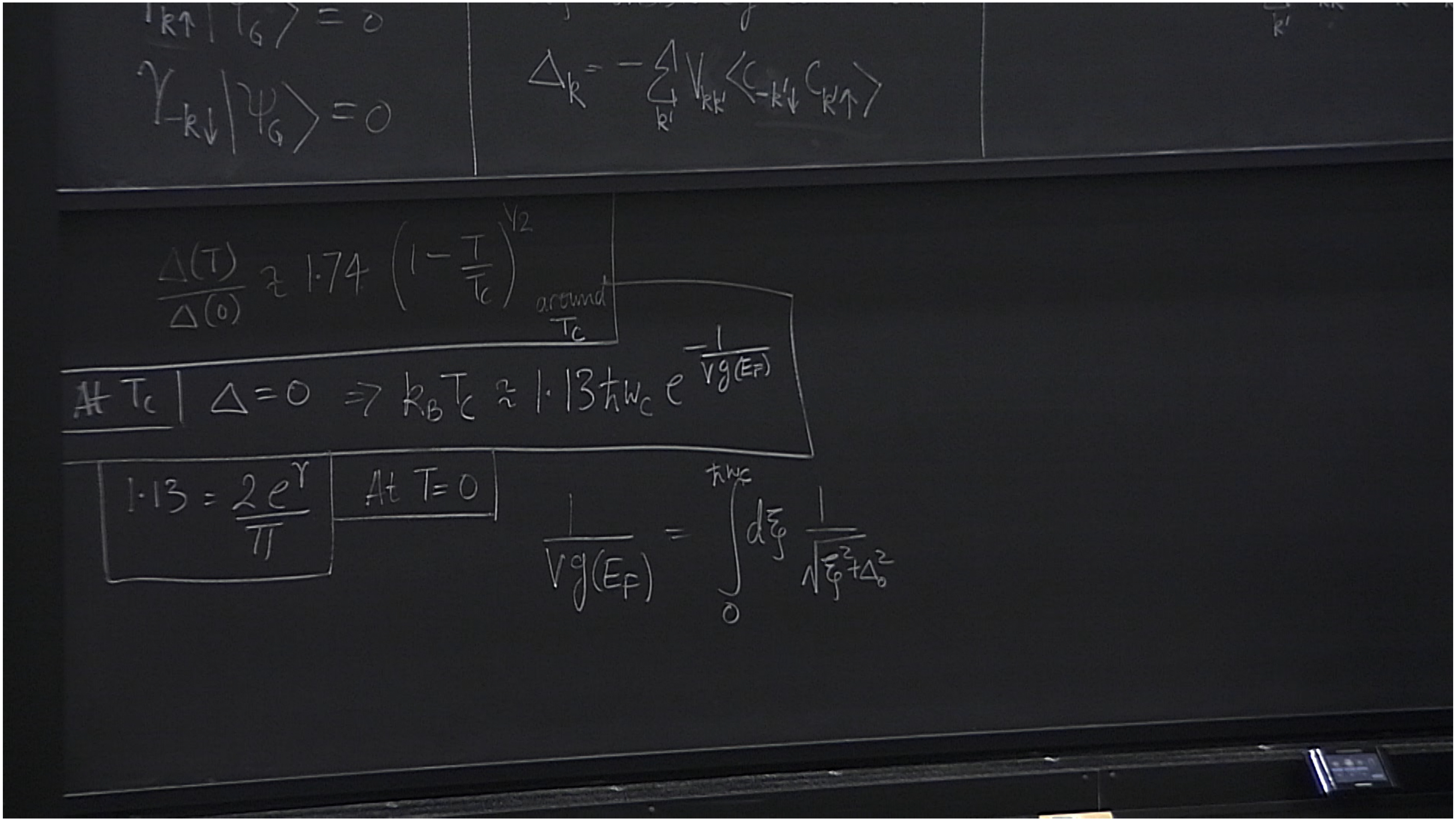
At T_c | $\Delta = 0$

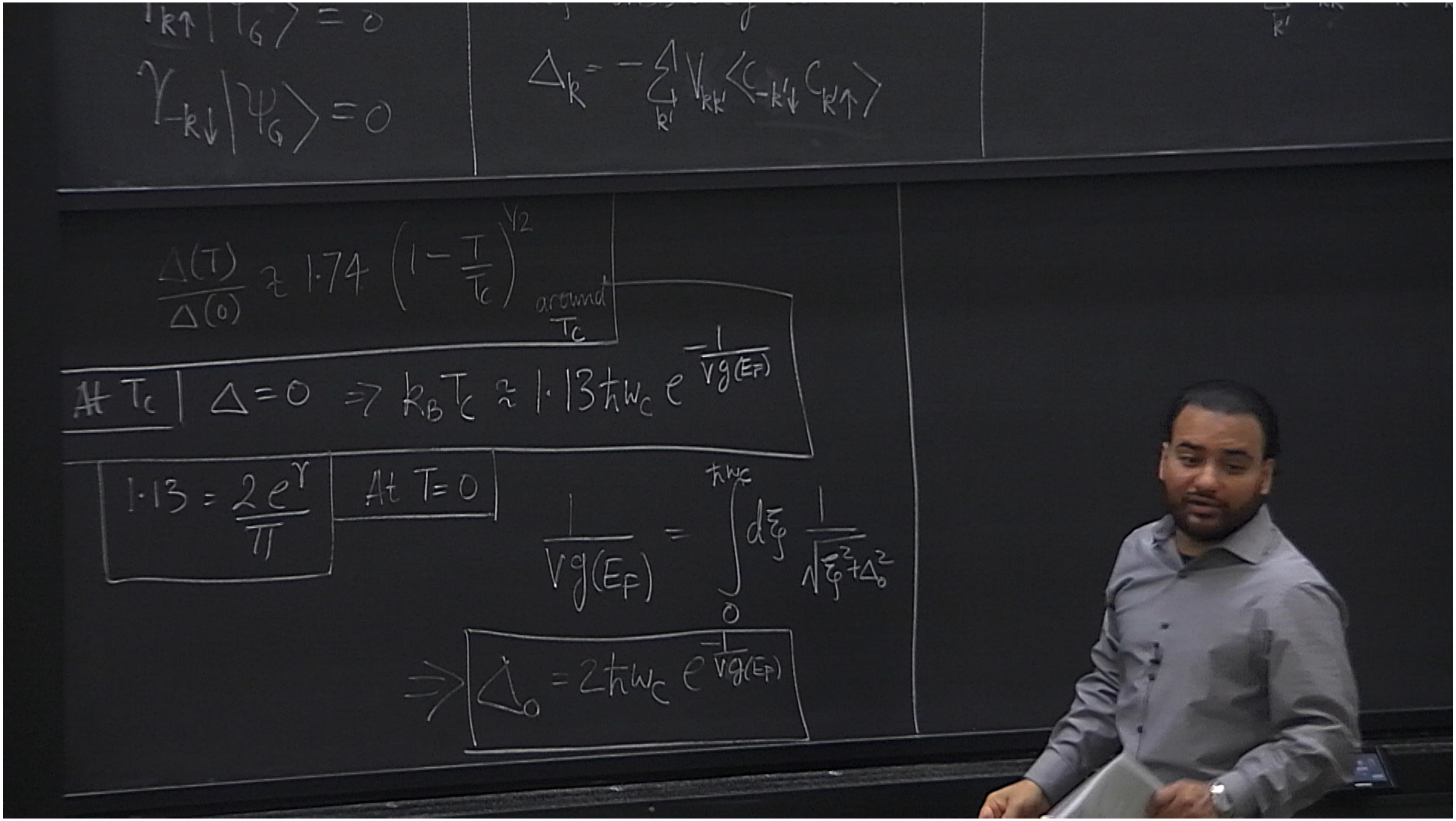

γ_-k↓ |ψ_G⟩ = 0
Δ_k = − Σ_k' V_kk' ⟨c_-k'↓ c_k'↑⟩
Δ(T)/Δ(0) ≈ 1.74 (1 − T/T_c)^1/2
around T_c
At T_c | Δ = 0 ⇒ k_B T_c ≈ 1.13 ħω_c e^(−1/(V g(E_F)))


γ_{-k↓}|ψ_G⟩ = 0
Δ_k = −Σ_{k'} V_{kk'} ⟨C_{-k'↓} C_{k'↑}⟩
Δ(T)/Δ(0) ≃ 1.74 (1 − T/T_c)^{1/2}
around T_c
At T_c | Δ = 0 ⇒ k_B T_c ≃ 1.13 ħω_c e^{−1/(V g(E_F))}
1.13 = 2e^γ/π



$\gamma_{-k\downarrow}|\psi_G\rangle = 0$
$\Delta_k = -\sum_{k'} V_{kk'} \langle c_{-k'\downarrow} c_{k'\uparrow} \rangle$
$\frac{\Delta(T)}{\Delta(0)} \approx 1.74 \left(1 - \frac{T}{T_c}\right)^{1/2}$
around T_c
At T_c
$\Delta = 0 \Rightarrow k_B T_c \approx 1.13 \hbar \omega_c e^{-\frac{1}{V g(E_F)}}$
$1.13 = \frac{2e^{\gamma}}{\pi}$
At $T=0$


$\gamma_{k\uparrow}|\psi_G\rangle = 0$
$\gamma_{-k\downarrow}|\psi_G\rangle = 0$
$\Delta_k = -\sum_{k'} V_{kk'} \langle c_{-k'\downarrow} c_{k'\uparrow} \rangle$
$\frac{\Delta(T)}{\Delta(0)} \approx 1.74 \left(1 - \frac{T}{T_c}\right)^{1/2}$
around T_c
At T_c | $\Delta = 0 \Rightarrow k_B T_c \approx 1.13 \hbar \omega_c e^{-\frac{1}{V g(E_F)}}$
$1.13 = \frac{2e^{\gamma}}{\pi}$
At $T=0$
$\frac{1}{V g(E_F)} = \int_0^{\hbar\omega_c} d\xi \frac{1}{\sqrt{\xi^2 + \Delta_0^2}}$



$\gamma_{-k\downarrow}|\psi_G\rangle = 0$
$\Delta_k = -\sum_{k'} V_{kk'} \langle c_{-k'\downarrow} c_{k'\uparrow} \rangle$
$\frac{\Delta(T)}{\Delta(0)} \approx 1.74 \left(1 - \frac{T}{T_c}\right)^{1/2}$
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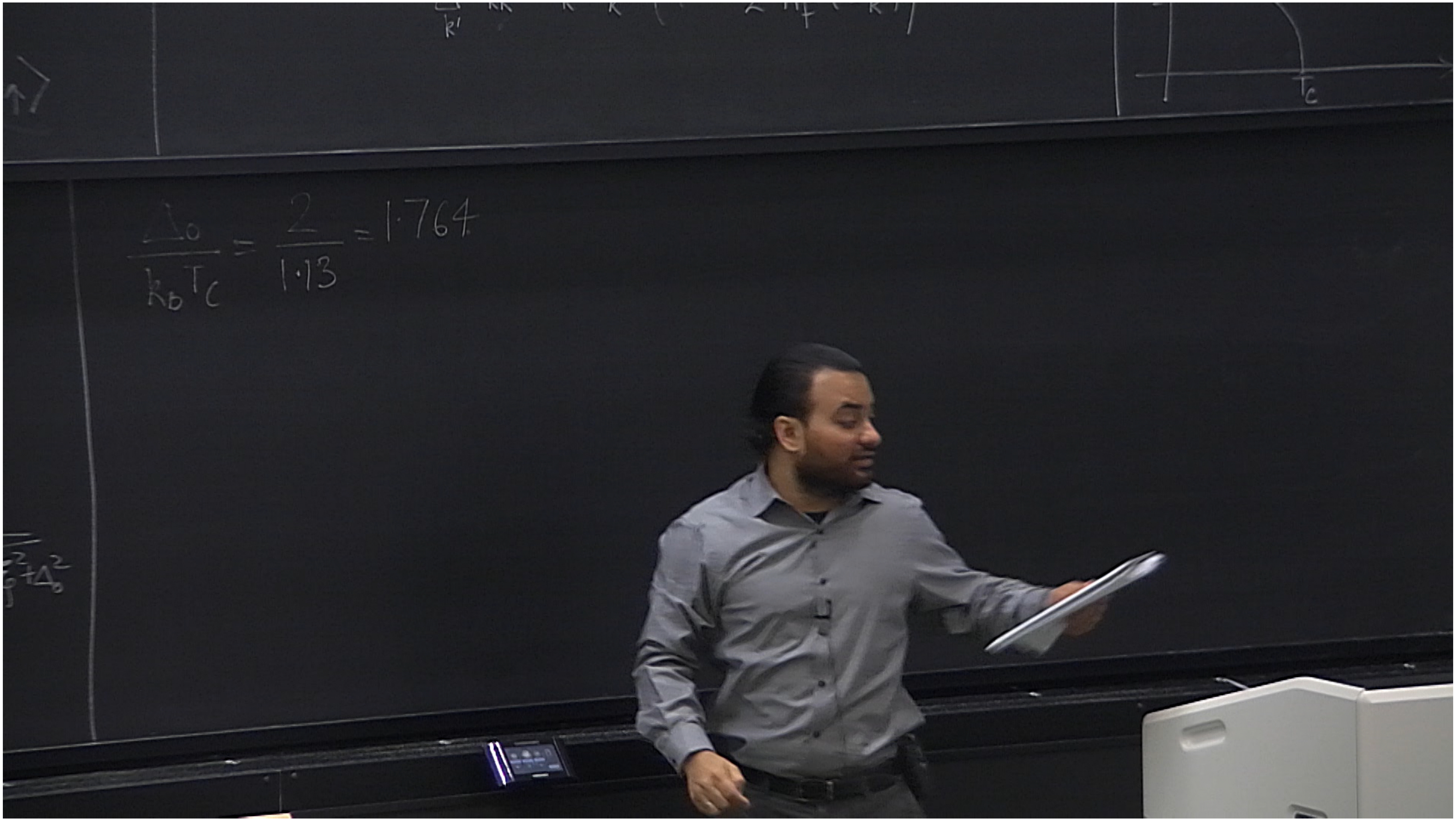
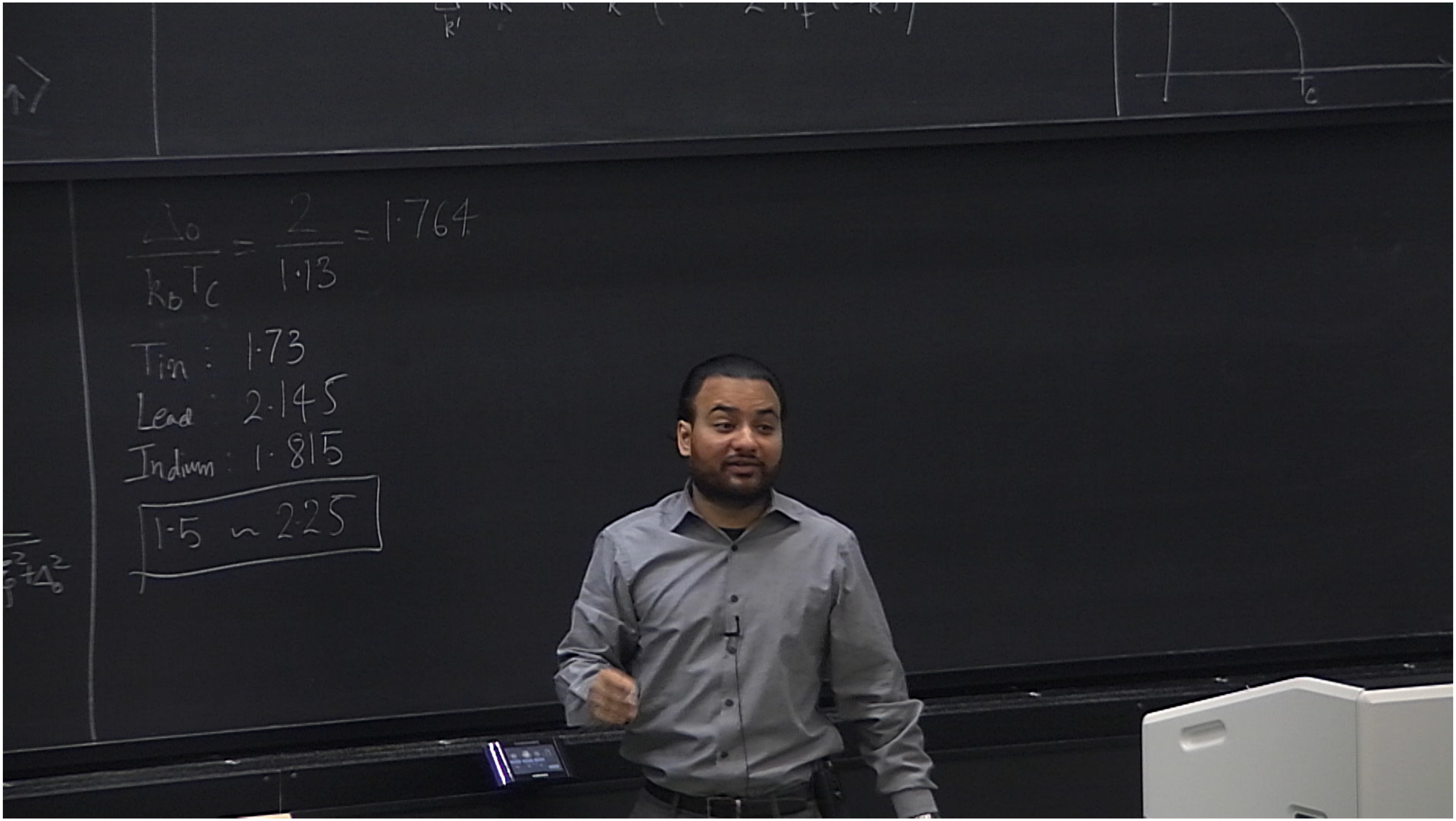
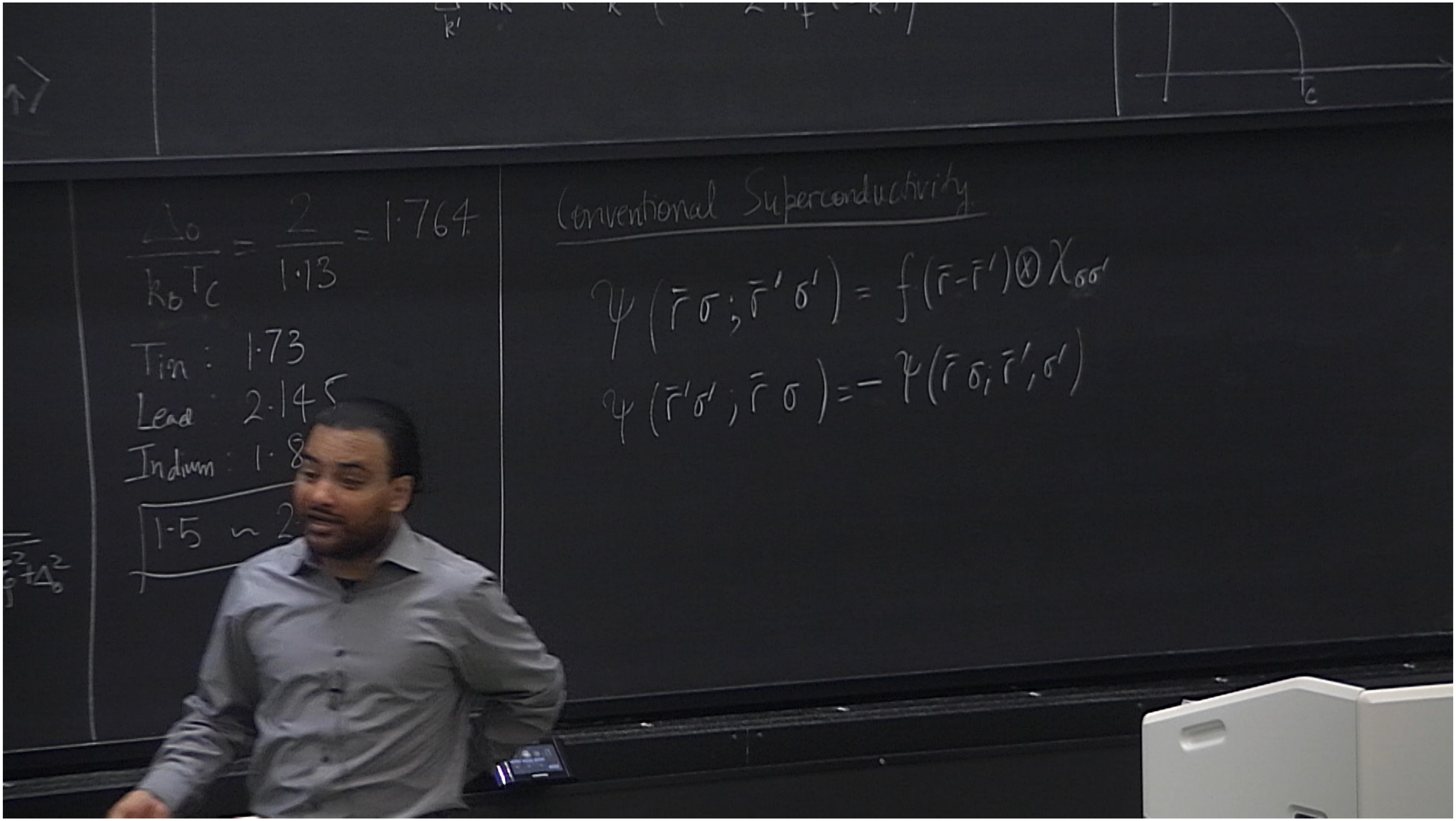
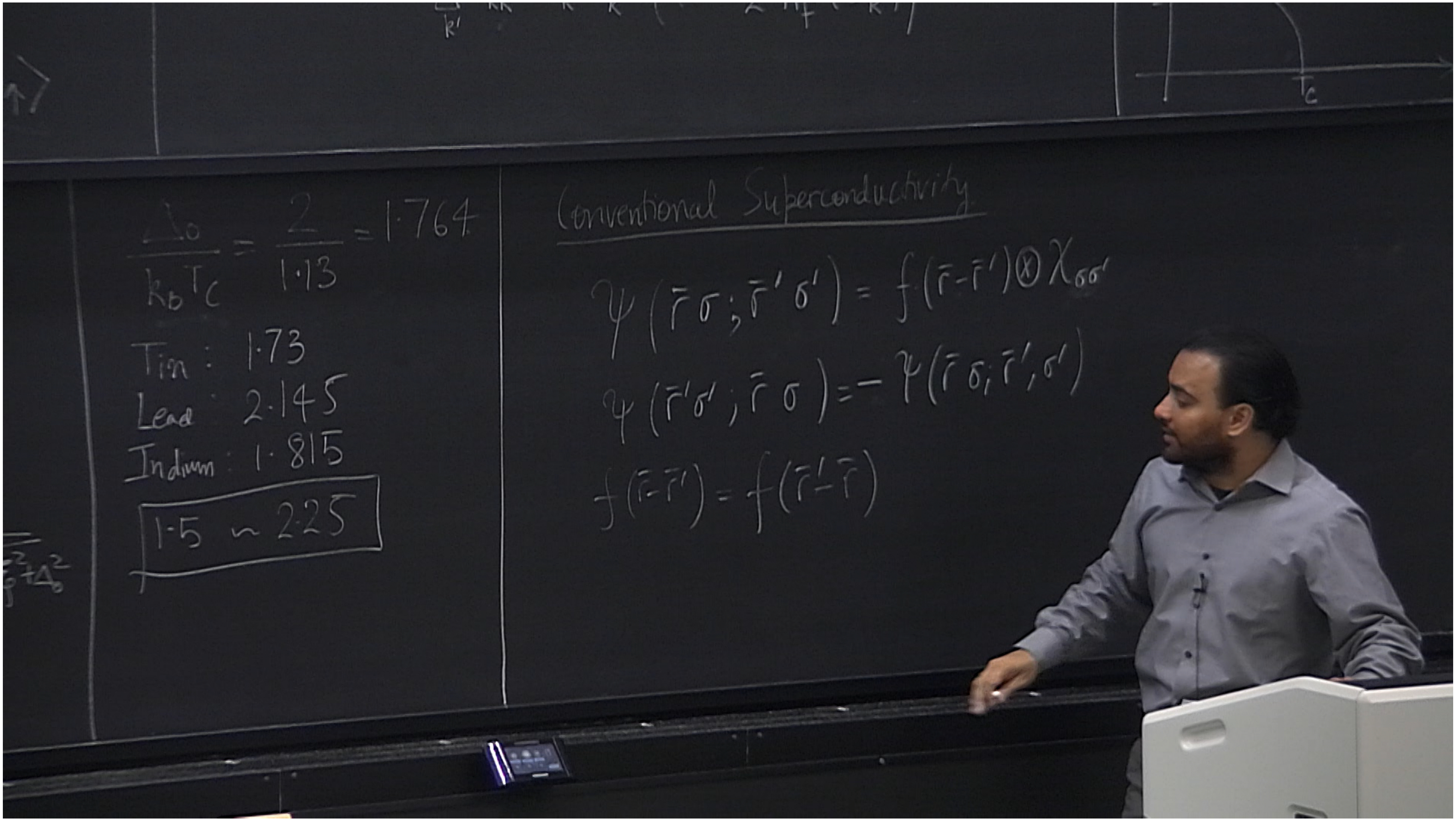
At T_c
$\Delta = 0 \Rightarrow k_B T_c \simeq 1.13 \hbar \omega_c e^{-\frac{1}{V g(E_F)}}$
$1.13 = \frac{2e^\gamma}{\pi}$
At T=0
$\frac{1}{V g(E_F)} = \int_0^{\hbar\omega_c} d\xi \frac{1}{\sqrt{\xi^2 + \Delta_0^2}}$
$\Rightarrow \Delta_0 = 2\hbar\omega_c e^{-\frac{1}{V g(E_F)}}$



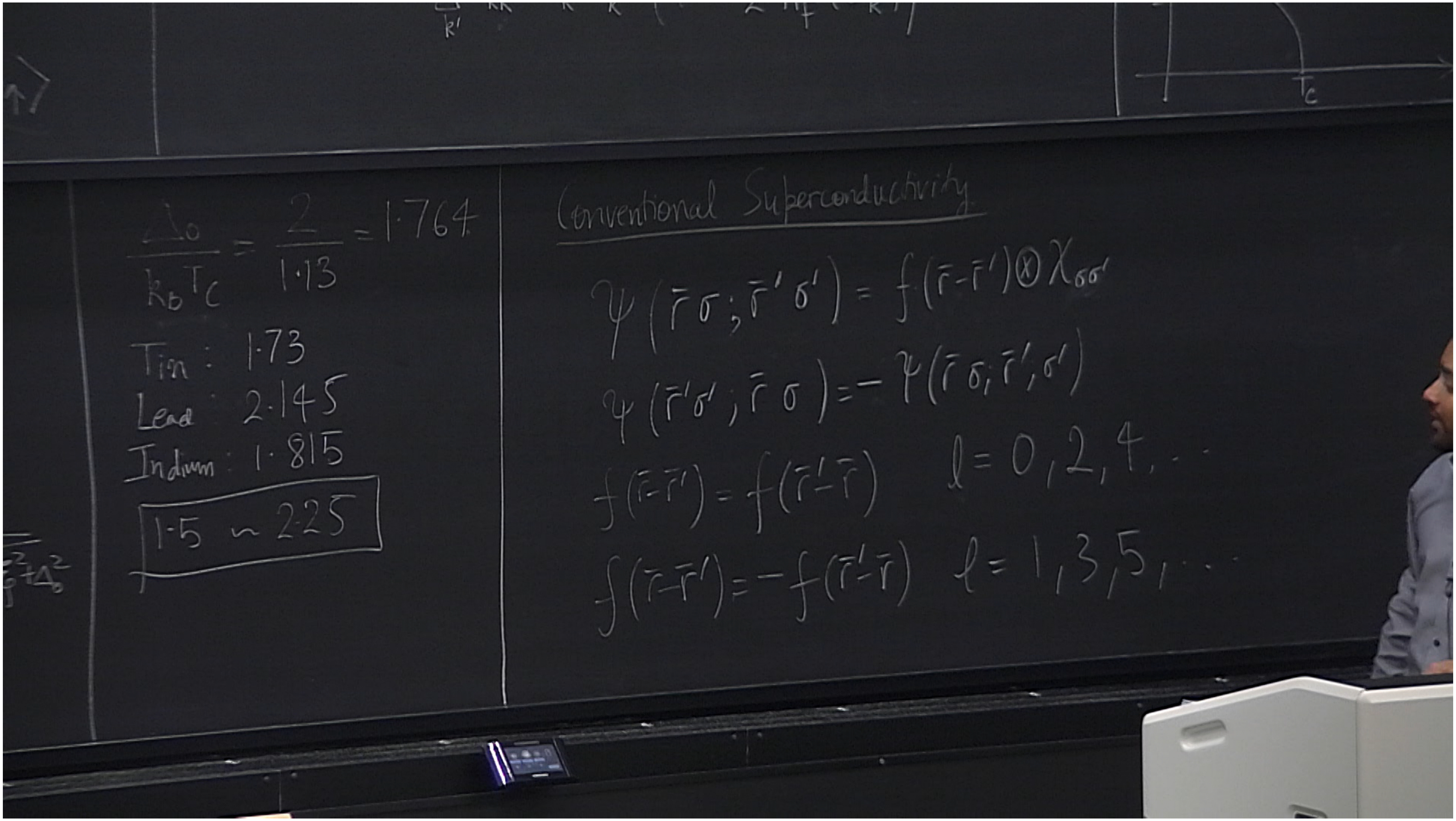
$\frac{\Delta_0}{k_b T_c} = \frac{2}{1.13} = 1.764$
T_c

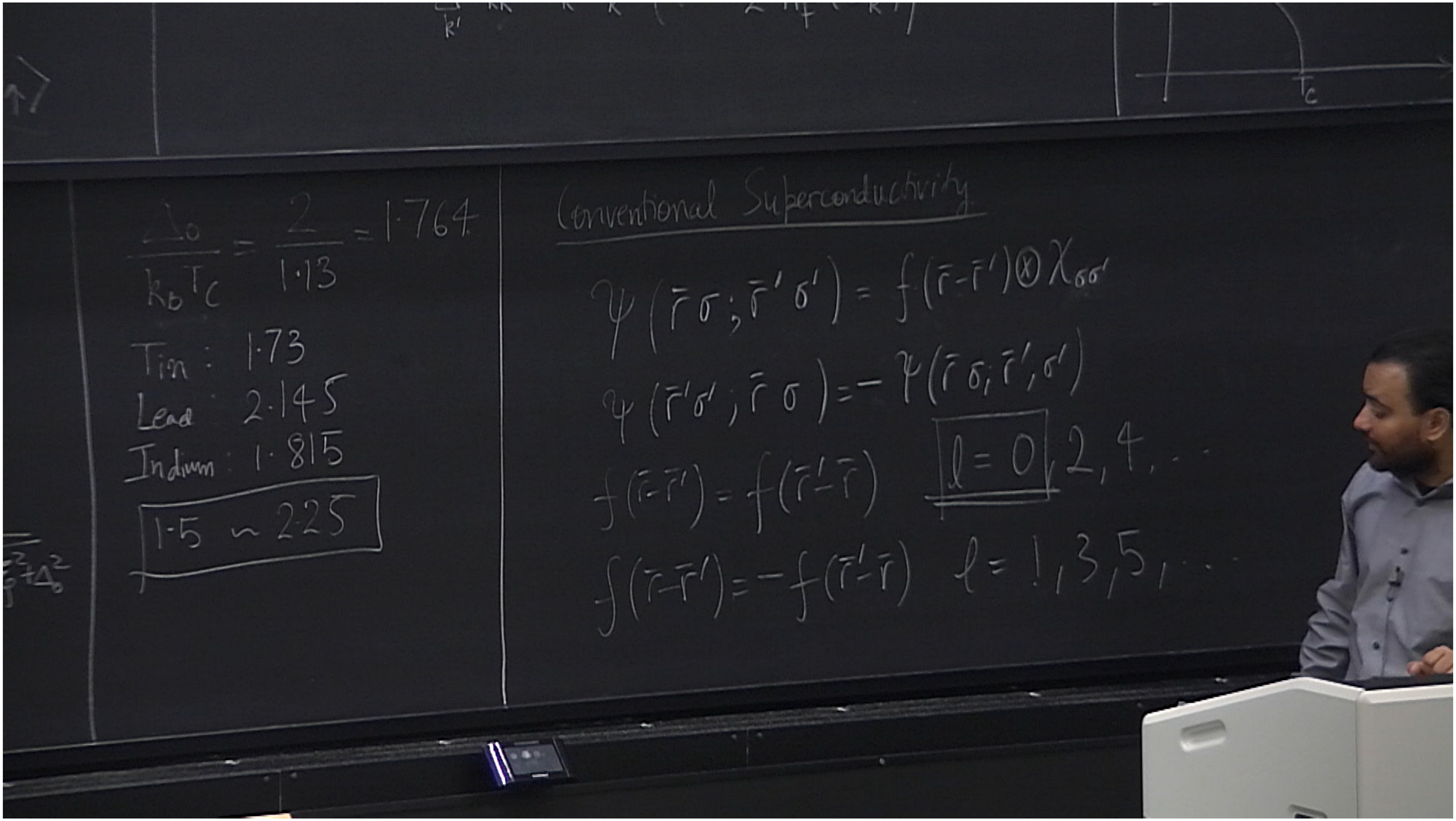

Δ0 / kb Tc = 2 / 1·13 = 1·764
Tin : 1·73
Lead : 2·145
Indium : 1·815
1·5 ~ 2·25


Conventional Superconductivity
$\frac{\Delta_0}{k_B T_c} = \frac{2}{1.13} = 1.764$
Tin : 1.73
Lead 2.145
$\psi(\bar{r}\sigma; \bar{r}'\sigma') = f(\bar{r}-\bar{r}') \otimes \chi_{\sigma\sigma'}$
$\psi(\bar{r}'\sigma'; \bar{r}\sigma) = -\psi(\bar{r}\sigma; \bar{r}', \sigma')$



Δ₀ / k_B T_c = 2 / 1.13 = 1.764
Tin : 1.73
Lead : 2.145
Indium : 1.815
1.5 ~ 2.25
Conventional Superconductivity
ψ(r̄σ; r̄'σ') = f(r̄ − r̄') ⊗ χ_σσ'
ψ(r̄'σ'; r̄σ) = − ψ(r̄σ; r̄'; σ')
f(r̄ − r̄') = f(r̄' − r̄)


Conventional Superconductivity
Δ₀ / k_B T_c = 2 / 1.13 = 1.764
Tin : 1.73
Lead : 2.145
Indium : 1.815
1.5 ~ 2.25
ψ(r̄σ; r̄'σ') = f(r̄−r̄') ⊗ χ_σσ'
ψ(r̄'σ'; r̄σ) = −ψ(r̄σ; r̄'σ')
f(r̄−r̄') = f(r̄'−r̄)   l = 0, 2, 4, . . .
f(r̄−r̄') = −f(r̄'−r̄)   l = 1, 3, 5, . . .
T_c


Δ₀ / k_B T_C = 2 / 1·13 = 1·764
Tin : 1·73
Lead 2·145
Indium : 1·815
1·5 ~ 2·25
Conventional Superconductivity
ψ(r̄σ ; r̄′σ′) = f(r̄ − r̄′) ⊗ χ_σσ′
ψ(r̄′σ′ ; r̄σ) = − ψ(r̄σ ; r̄′, σ′)
f(r̄ − r̄′) = f(r̄′ − r̄)   l = 0, 2, 4, . . .
f(r̄ − r̄′) = − f(r̄′ − r̄)   l = 1, 3, 5, . . .
T_c